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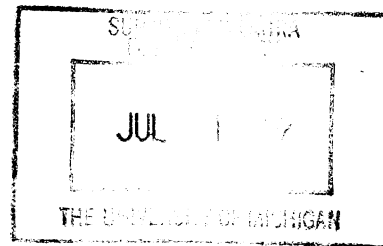
Cartel Quotas Under Majority Rule

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Abstract

We examine the choice of quotas by legal volume-restricting organizations: domestic and international cartels, commodity organizations, U. S. Federal agricultural marketing boards, and prorationing boards. Unlike their illegal counterparts, legal cartels have published regulations and broader enforcement capabilities. Differences in costs and size among cartel members, however, still make quota selection contentious. Conflicts over quotas are typically resolved by voting. Cartel regulations usually require that quotas be chosen in the following manner: a scalar (depending on context capacity, inventory, historical output or historical exports) is assigned to each entity subject to regulation. Cartel members then vote on the common percentage of each scalar which is the maximum the entity may sell. Side-payments to influence votes are prohibited. We examine the predicted effects of this real-world institution on prices and welfare and compare the equilibrium outcomes to what would occur if joint profits were instead maximized. We also show the economic consequences of exogenous political changes such as alterations in the voting weights or in the identity of the voters.

1. Introduction

The recent literature on cartels has focussed on how production quotas can be enforced. Enforcement is particularly tricky when the cartel is illegal since the machinery of government is then unavailable to penalize cheating. It has been shown that the credible threat of price wars in the future in response to cheating, observed directly or rationally inferred, can be sufficient to deter quota violations even if such behavior would garner substantial profits in the short run.¹

We focus here on a different aspect of cartels: their choice of production quotas by voting. We limit ourselves to legal cartels. Enforcement is often a secondary problem for such cartels since the machinery of government can be used to punish cheating. Their primary problem is to resolve often fierce internal conflicts and to choose collectively a compromise set of quotas.²

As their published regulations reflect, legal cartels almost invariably choose quotas in the following manner: a scalar (for example capacity, inventory, historical output or historical exports) is assigned to each entity subject to regulation. Cartel members then vote on the common percentage of each scalar which is the maximum the entity may sell. Sidepayments to influence the votes of cartel members are prohibited.

Many domestic cartels, international cartels, agricultural marketing boards, and prorationing boards governing common properties choose quotas in this manner. A few examples spanning different epochs and continents will illustrate the diverse circumstances in which this institution arises.

Domestic Cartels The Japanese Cotton Textile Association (Dai Nippon Bōseki Rengōkai) regulated every firm in the cotton-spinning industry in Japan during the late 19th and early 20th century (until 1940). According to Shoji [1930], every firm was assigned a benchmark capacity and was allowed to produce up to a common fraction of that capacity. No trading was permitted in unused capacity. Enforcement was effective.³ The common fraction was chosen by majority rule. Every regulated firm voted and each vote received equal weight. Although all Japanese spinners were regulated, rival producers in other countries constituted an unregulated sector.

International Cartels The International Rubber Regulation Agreement signed in 1934 by the United Kingdom, India, the Netherlands, France and Siam, fixed basic quotas for each member country. The International Rubber Regulation Commit-

¹For a review of this literature, see Chapter 6 of Tirole [1988].

²For a similar assessment of legal cartels, see Porter [1991, p. 559].

³After the common fraction of benchmark capacity was decided, officials from the Textile Association visited each firm and sealed spindles to prevent production beyond the legal limit. Other safeguards were taken. Since production could not occur without imported cotton and imported cotton came via shipping lanes controlled by a confederate cartel, the Textile Association entered into agreements with the shipping cartel to regulate and monitor imports of raw cotton. All told, the Japanese Cotton Textile Association was able to closely monitor the employment, wages, imported inputs and output of each of each Japanese spinning firm to insure compliance.

tee determined a uniform percentage of the basic quotas which could be exported.⁴ The “Sugar Cartel” which regulated 90% of world sugar production before the second world war and the ongoing International Coffee Organization which regulates all but a handful of small coffee exporters reflect this same structure. In each of these cases, voters are government officials and countries with larger basic quotas have more votes.⁵

Agricultural Marketing Boards The Agricultural Marketing Act of 1937 permits growers and handlers of fruits, vegetables, and tree nuts grown in the U.S. to form cartels. If an allocation or prorated scheme is adopted, each handler in a designated geographical region must report the inventory he has under contract (the analog to the basic quota’s of the international cartels). An administrative committee then votes by majority rule on the common maximum fraction of each handler’s inventory which may be sold on the “primary” market. The remaining inventory may be sold only in unregulated outlets. Only a small subset of the regulated handlers are members of the administrative committee. Monitoring is strict and cheating, a violation of federal law, is rare. Foreign firms and domestic firms outside the regulated area comprise an unregulated sector.

Prorating Boards Governing Common Properties The Texas Railroad Commission (and counterpart regulatory commissions in Louisiana, Oklahoma, New Mexico, and Kansas) regulate oil extraction within the state. A “yardstick” capacity is assigned to each well (the analog of the “basic quota”) and the commission sets a common percentage of the yardstick—the “market demand factor”—that each well is permitted to produce during the month. Commission members choose this percentage by majority rule. Members of the commission are government officials not market participants. In the past, when import restrictions protected the domestic market, this regulation not only reduced congestion externalities but also affected the domestic price. In the absence of import quotas, such prorating would not affect the price but still mitigates congestion externalities.

These volume-restricting organizations bear little resemblance to the standard model of cartels in the economics literature.⁶ In particular, although all of these real-world cartels select quotas by majority rule, no model in the literature incorporates voting.⁷ Our goal in this research is to determine whether it is always appropriate

⁴For a tabulation of the “basic quota” of each member country and the common percentage which could be exported from 1934-1943, see Table VII and VIII of Knorr [1945].

⁵For a discussion of the distribution of votes in the Sugar Council, see Stocking and Watkins, p. 45. For a discussion of the distribution of votes in the International Coffee Organization, see Bates and Lien [1985] and Schrag [1986].

⁶Despite the mismatch between the received model and the real-world institution, these diverse organizations are typically referred to as cartels. For example, Varian [1990, p.458] refers to agricultural marketing boards as cartels and Blair [1976, Ch. 7] regards prorating boards as cartels. As for the sugar and rubber agreements, in the opinion of the expert retained by the Temporary National Economic Committee of the U.S. Congress in its investigations of cartels “... it would be difficult to find a foreign observer that would not call either the sugar or rubber agreement a cartel.” [TNEC, p. 13066].

⁷The only exception is our own preliminary paper [Cave and Salant, 1987], which is limited to

to abstract from the voting aspect of cartels. It is our conclusion that sensible predictions about prices in markets with legal cartels cannot be made without considering the political aspects of quota selection. As has long been appreciated, members of cartels with larger capacities or higher costs want tighter quotas than members with smaller capacities or lower costs.⁸ This insight is often mentioned in informal discussions of internal conflicts in cartels. It is precisely these two factors of cost and capacity that—in the absence of sidepayments— influence the voting on quotas. As we show, the compromises reached under majority rule need not be profit-maximizing.

In the next section, we describe the economic equilibria that would arise in response to alternative quotas. We assume that each cartel member foresees the profit consequences of alternative quotas and votes to maximize his subsequent profits. We show that—for any given committee of voters and voting weights—one quota is always preferred by a majority of the voters to any other; moreover, we show how exogenous political changes in the identity of the voters or how their votes are weighted will affect the market price. Section 3 compares the price predicted in our model to the price which would arise if the quota maximized profits. Section 4 concludes the paper.

the special case of agricultural marketing boards where marginal costs are identical and hence all constrained firms must completely agree about which of two binding quotas is the more profitable.

⁸See, for example, Chapter 13 of Stigler [1966].

2. The Model

Consider an industry of n firms that simultaneously choose outputs and produce perfect substitutes. We assume initially that all firms are regulated by a cartel. Associated with firm i is a firm-specific scalar (q_i) used to translate the cartel's prior choice (F) into restrictions on that particular firm. If the cartel chooses the quota F , then firm i 's output (denoted y_i) must not exceed Fq_i .

Firm i is assumed to produce at constant marginal cost c_i and to incur no fixed cost. Index the firms in order of ascending marginal cost. If two firms have the same marginal cost, index them in order of increasing capacity. That is, set $i < j$ if (a) $c_i < c_j$ or (b) $c_i = c_j$ and $q_i < q_j$.⁹

Denote aggregate output as $Y (= \sum_i^n y_i)$ and the induced price as $P(Y)$. Firm i wishes to

$$\max_{0 \leq y_i \leq Fq_i} y_i [P(y_i + Y_{-i}) - c_i] \quad (1)$$

where $Y_{-i} = Y - y_i$. At a Cournot equilibrium, each firm's choice maximizes its payoff given the choices of the other firms. The equilibrium profits of each firm will depend on the cartel's prior choice F . We refer to such profit functions as "induced preferences" (denoted $\pi_i(F)$) since they determine the preferences of market participants over alternative quotas.

The reader should note that our formulation is sufficiently general to encompass each of the cases discussed in the introduction—not merely domestic and international¹⁰ cartels but also prorationing boards and agricultural marketing boards.¹¹ As discussed in the introduction, these volume-restricting organizations have in the past based their quotas on a wide variety of firm-specific scalars. For simplicity, we refer to q_i as "capacity" and the volume-restricting organization as a "producer cartel" until the next section when welfare comparisons require us to distinguish the case with congestion externalities.

Assume that the inverse demand curve is strictly decreasing and twice continuously differentiable. In addition, assume that the Novshek condition holds.¹²

⁹If the firms are identical in both respects, arbitrarily assign them successive indices.

¹⁰The model applies to international cartels if each country is assumed to contain a single firm (or, alternatively, a collection of identical firms) and if the voters representing each country are assumed to reflect the interests of its firm. While admittedly imperfect, this seems like a useful first approximation that abstracts from the way the country quota is allocated internally and from the incentives of elected representatives who are not market participants.

¹¹In the case of an agricultural marketing board, handler i would maximize $y_i P(y_i + Y_{-i}) + (q_i - y_i)c$, subject to $0 \leq y_i \leq Fq_i$ where $P(\cdot)$ is the volume-sensitive price in the primary market and c is the exogenous price in the secondary market. To obtain this from (1), simply add to (1) the constant term cq_i . In the case of a prorationing board, extractor i would maximize $y_i(P - A(y_i + Y_{-i}))$ subject to $0 \leq y_i \leq Fq_i$, where P is the exogenously fixed price of the resource and $A(Y)$ is the average cost of extraction on the particular field due to a congestion externality. To obtain this from (1), replace the positive, decreasing function $P(Y)$ in (1) by the positive, decreasing function $c + P - A(Y)$.

¹²The Novshek condition is that $P'(Y) + YP''(Y) \leq 0$ for all $Y \geq 0$. In the appendices, we dispense with this condition and require only that the total revenue function be concave in aggregate

Finally, assume that there would be positive demand if the lowest cost firm produced at cost ($D(c_1) > 0$) and that demand disappears at sufficiently high prices ($\lim_{P \rightarrow \infty} D(P) = 0$). Given these assumptions, there exists a unique Cournot equilibrium in pure strategies induced by a given quota F .¹³ This equilibrium is characterized by an aggregate output (Y) divided into a vector of outputs (y_1, \dots, y_n) satisfying one of the following conditions for $i = 1, \dots, n$:

production.

¹³For a proof, see Appendix I.

$$\text{Inactive} \quad y_i = 0 \quad \text{and} \quad P(Y) - c_i \leq 0 \quad (2a)$$

$$\text{Unconstrained} \quad 0 < y_i < q_i F \quad \text{and} \quad P(Y) + y_i P'(Y) - c_i = 0 \quad (2b)$$

$$\text{Constrained} \quad y_i = q_i F \quad \text{and} \quad P(Y) + q_i F P'(Y) - c_i \geq 0. \quad (2c)$$

Consider the Cournot equilibrium in the output market induced by F . Denote by $Y(F)$ the aggregate equilibrium production induced by F . Denote the quota which would just bind on firm j as F_j . Then F_j is implicitly defined as follows:

$$P(Y(F_j)) + q_j F_j P'(Y(F_j)) - c_j = 0. \quad (3)$$

We refer to F_j as j 's "cutoff."¹⁴

Suppose the following regularity condition holds for each pair of firms i and j such that $i < j$:

$$P'(Y(F_j)) F_j (q_j - q_i) \leq c_j - c_i. \quad (4)$$

Then any quota binding on one firm must also bind on all firms with smaller indices. For suppose the quota F is just binding on firm j . Consider some firm i where $i < j$. Subtracting (4) from (3), we obtain:

$$P(Y(F_j)) + q_i F_j P'(Y(F_j)) - c_i \geq 0. \quad (5)$$

Hence, firm i would also be constrained at this quota. The cutoffs are therefore "nested":

$$F_n \leq \dots \leq F_2 \leq F_1.$$

The regularity condition (4) admits many cases of interest. It holds if two firms have equal marginal costs or if they have unequal marginal costs but the capacity of the firm with the smaller marginal cost is not too much larger than the firm with the larger marginal cost.¹⁵ The regularity condition is necessary and sufficient for the cutoffs to be nested and, as shown below, is sufficient but not necessary for the existence of a Condorcet quota.¹⁶

When cutoffs are nested, the induced preferences display a property we refer to as "partial agreement." For any two firms i and j such that $i < j$ and any pair of quotas \hat{F} and F such that $\hat{F} < F < F_j < F_i$:

$$\text{If } \hat{F} \succ_i F \text{ then } \hat{F} \succ_j F; \quad (6)$$

$$\text{If } F \succ_j \hat{F} \text{ then } F \succ_i \hat{F}. \quad (7)$$

¹⁴As shown in Appendix II, each cutoff is unique.

¹⁵For example, suppose demand is linear and the slope has the magnitude m . If $c_i < c_j$ but $q_i \leq q_j + (c_j - c_i)/m$, then the regularity condition will still hold.

¹⁶A Condorcet quota (F) is one which a majority of the voters prefers to any alternative. To see that the regularity condition is not necessary for existence, suppose every firm but one is identical and the final firm's cost and capacity violate the regularity condition. The common ideal point of the identical firms will still be the Condorcet quota.

That is, if the firm with the larger cutoff strictly prefers the smaller quota then so must the firm with the smaller cutoff; reciprocally, if the firm with the smaller cutoff strictly prefers the larger quota then so must the firm with the larger cutoff. The agreement in preference is said to be “partial” rather than “complete” since no restrictions are placed on the preferences if the firm with the larger cutoff prefers the larger quota or, alternatively, if the firm with the smaller cutoff prefers the smaller quota.¹⁷

To verify (6), note that since $\hat{F} \succ_i F$, we have

$$\hat{F}q_i (P(Y(\hat{F})) - c_i) > Fq_i (P(Y(F)) - c_i). \quad (8)$$

Note also that since $c_i \leq c_j$ and $\hat{F} < F$

$$-\hat{F}(c_j - c_i) \geq -F(c_j - c_i). \quad (9)$$

Dividing (8) by q_i , adding (9), and multiplying the result by q_j we obtain:

$$\hat{F}q_j (P(Y(\hat{F})) - c_j) > Fq_j (P(Y(F)) - c_j), \quad (10)$$

which confirms that $\hat{F} \succ_j F$.

To verify (7), note that since $F \succ_j \hat{F}$, we have

$$Fq_j (P(Y(F)) - c_j) > \hat{F}q_j (P(Y(\hat{F})) - c_j). \quad (11)$$

Dividing (11) by q_j , adding (9), and multiplying the result by q_i we obtain:

$$Fq_i (P(Y(F)) - c_i) > \hat{F}q_i (P(Y(\hat{F})) - c_i), \quad (12)$$

which confirms that $F \succ_i \hat{F}$.

In addition to nesting and partial agreement, the preferences display two other noteworthy characteristics. First, $\pi_i(F)$ is a continuous function. Moreover, if firm i is unconstrained and at least one firm is constrained ($F_i \leq F \leq F_1$), then i 's induced preference ($\pi_i(F)$) is strictly decreasing in F . We refer to these two properties as “continuity” and “unconstrained monotonicity” of the induced preferences.¹⁸ Unconstrained monotonicity is intuitively plausible since a reduction in a quota binding only on one's rivals shifts out the residual demand curve facing the firm.

Some readers may find it convenient to visualize geometrically the impact of quotas on the Cournot equilibrium. We therefore conclude this subsection with a graph which—although unorthodox—is useful in this (and many other) exercises

¹⁷Firms with identical marginal costs must rank the two quotas identically even in these cases. Agreement is then said to be “complete.” If firm i strictly prefers F to \hat{F} and both quotas bind on i then $Fq_i(P(Y(F)) - c) > \hat{F}q_i(P(Y(\hat{F})) - c)$. Multiplying by the positive number q_j/q_i , we obtain $Fq_j(P(Y(F)) - c) > \hat{F}q_j(P(Y(\hat{F})) - c)$. Therefore, if both quotas also bind on firm j , firm j strictly prefers F also. Cave and Salant [1987] is limited to this special case.

¹⁸Proofs are relegated to Appendix II.

with the Cournot model. Since $P'(Y) = 1/D'(P(Y))$, we can re-write the first order conditions in (2a-2c) as follows:

$$y_i = \begin{cases} \min(q_i F, -D'(P)(P - c_i)) & \text{if } P_i > c_i \\ 0 & \text{otherwise} \end{cases} \quad (13)$$

We refer to this function as firm i 's pseudosupply curve. The prefix "pseudo" is added to remind readers that the function would shift if the demand curve changed and is therefore not a "true" supply curve.

Since each firm's first-order condition must hold at every Cournot equilibrium, each firm must be on its pseudosupply curve at a Cournot equilibrium. Since we assume that the Novshek condition holds, the second-order condition holds as well.¹⁹ Furthermore, the Novshek condition implies that every pseudosupply curve will be upward-sloping (where it is unconstrained). Each pseudosupply curve has a vertical intercept at $P = c_i$, is strictly increasing until $y_i = q_i F$ and is vertical thereafter as illustrated in panel (a) of Figure 1.²⁰ Given our assumptions, there must exist a unique intersection of the demand curve and the horizontal sum of the pseudosupply curves as illustrated in panel (b) of Figure 1.

[Figure 1]

This intersection point is the only candidate for a Cournot equilibrium since aggregate production must equal demand at every equilibrium. Since the second-order condition of each firm is satisfied at this candidate solution, it must correspond to one (or more) Cournot equilibria. If it corresponded to more than one Cournot equilibrium, all would have to result in the same market price and aggregate production. But since no pseudosupply function is ever horizontal, only one output vector satisfies the first-order conditions for the given price. Hence, the Cournot equilibrium is unique.

It is now straightforward to prove that a reduction in F must raise the Cournot equilibrium price as long as at least one firm is constrained. For, if the price remained fixed, demand would be unchanged. But at that price the supplies of the firms producing the legal limit would be reduced while the supplies of the other firms would be unchanged. Hence, demand would exceed pseudosupply at the old equilibrium price and only a price increase could eliminate the imbalance.

¹⁹Differentiating $D(P(Y)) = Y$, we conclude that $P'(Y) = 1/D'(P(Y))$ and $P''(Y) = -D''(P(Y))/[D'(P(Y))]^3$. We can use these equations to express the second-order condition ($2P'(Y) + y_i P''(Y) < 0$) and the Novshek condition ($P'(Y) + Y P''(Y) < 0$) in terms of the demand curve. The second-order condition becomes $2D'(P(Y)) + (P - c_i)D''(P(Y)) < 0$ while the Novshek condition becomes $D'(P(Y)) + (P - c_i)D''(P(Y)) < 0$ for all P and $i = 1, \dots, n$.

²⁰The regularity condition is satisfied in the case represented in Figure 1. Geometrically, the regularity condition requires that any horizontal line crossing the vertical segment (the increasing segment) of firm i 's pseudosupply curve must cross the vertical segment (the increasing segment) of the pseudosupply curves of firms with smaller (larger) indices. In addition, we note that as a simplification we have drawn the rising segments of the pseudosupply functions in Figure 1 as linear; this would be the case only if the demand curve were linear.

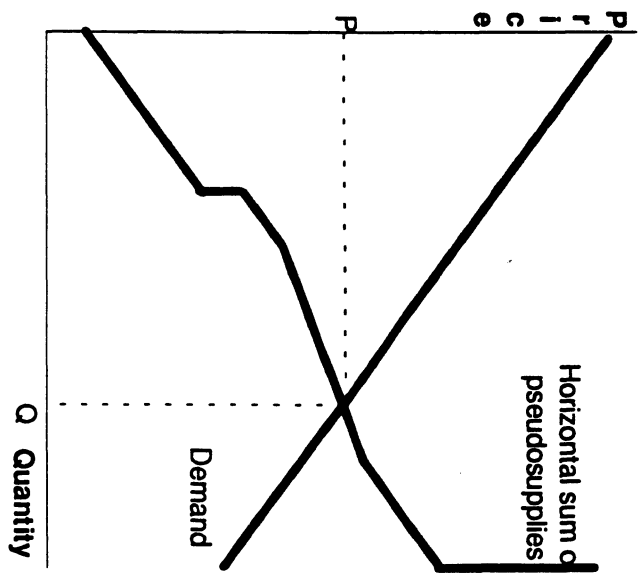
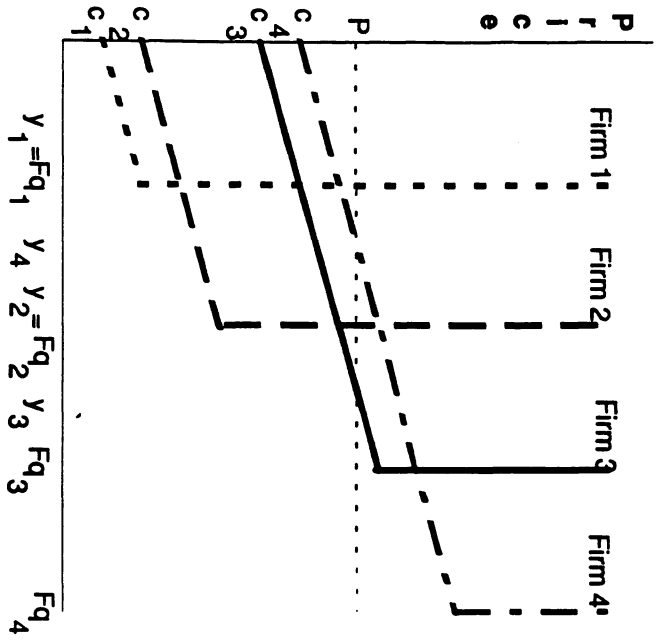


FIGURE 1

Since each individual pseudosupply function is increasing, the reduction in F must induce an increase in the output of each unconstrained firm. Since the contraction in F increases firm i 's average profit $(P - c_i)$ as well as its output (y_i) , its total profit must increase—implying “unconstrained monotonicity.” Since both factors are continuous functions of F , profit must also be continuous in F . Finally, since each firm's pseudosupply function is strictly increasing, a firm constrained at F will also be constrained at any smaller quota; conversely, a firm unconstrained at F will be unconstrained at any larger quota. Readers who prefer more conventional proofs of these claims are directed to the appendices.

2.1 Existence of a Condorcet Quota

Having derived several properties of the induced preferences, we now ask what quota (F) the cartel will select. We refer to a quota which would win a majority of votes when paired against any alternative as a “Condorcet” winner. If a Condorcet quota exists, we assume that it will be chosen under majority rule.²¹ The induced preferences in our model are not single-peaked even in the simplest case where demand is linear and costs are identical. Hence, the standard condition sufficient for the existence of a Condorcet quota [Black, 1958] cannot be applied. Nonetheless, the structure of the cartel problem insures that the induced preferences display nesting of cutoffs, unconstrained monotonicity, partial agreement, and continuity. These characteristics turn out to be sufficient for the existence of a Condorcet quota. We first prove this assertion and then distinguish our result from other sufficiency conditions in the collective choice literature.

Since $\pi_i(F)$ is continuous, there exists for each firm i an “ideal point,” denoted I_i , such that $\pi_i(I_i) \geq \pi_i(F)$ for all F . Moreover, it follows from unconstrained monotonicity that $I_i \leq F_i$.

Assume that n is odd and that voting is by simple majority rule. We now verify the following fundamental proposition:

Median Cutoff Theorem: THE IDEAL POINT OF THE FIRM WITH THE MEDIAN CUTOFF IN THE INDUSTRY WILL BE PREFERRED TO ANY OTHER QUOTA BY A MAJORITY OF THE VOTERS.

Suppose there are n voters, where n is an odd integer. Denote the median index by $m(= \frac{n+1}{2})$. If $F < I_m$, voters $1, 2, \dots, m$ (a majority) would prefer I_m . This follows since $F < I_m \leq F_m \leq \min(F_1, F_2, \dots, F_{m-1})$ and voters partially agree. If instead $F > I_m$, it will be shown that voters $m, m+1, \dots, n$ (a majority) would prefer I_m . Recall that the cutoffs of these firms are no larger than F_m and that $I_m \leq F_m$. Any i such that $F_i \leq I_m$ must prefer I_m to $F > I_m$ (unconstrained monotonicity). As for any i such that $I_m < F_i \leq F_m$, he prefers I_m to any $F \in (I_m, F_i]$ (since

²¹That is, many extensive-form voting games which have payoffs with a Condorcet winner will result in that outcome being chosen under majority rule. For an example of one such game and also experimental evidence that the Condorcet winner will be chosen, see Salant and Goodstein [1990].

preferences partially agree) and F_i to any $F > F_i$ (unconstrained monotonicity). Hence, he prefers I_m to any $F > I_m$ (continuity).

Finally, if I_m is the *unique* ideal point of the voter with the median cutoff, then it will be the unique Condorcet point.

It follows that where our lexicographic indexing turns out to order firms by marginal cost, the choice of the cartel will be the ideal point of the firm with the median marginal cost. Similarly, where it orders firms by capacity, the choice of the cartel will be the ideal point of the firm with the median capacity.

2..2 Our Conditions Compared to Others Sufficient for a Condorcet Winner

There are many preference restrictions sufficient to guarantee existence of a Condorcet point. Some, like “single-peakedness” [Black, 1958] apply to individual preferences. More general conditions restrict the distribution of preferences in the voting population. The most powerful, “value restriction” [Sen and Pattanaik, 1969], is both necessary and sufficient for the existence of a Condorcet point. Obviously, our conditions—like any others sufficient for the existence of a Condorcet point—imply value restriction. Alternatives to value restriction have been sought because that restriction is difficult to verify.

For example, Epple and Romer [1987], Grandmont [1978], Roberts [1977] and others have formulated alternative conditions sufficient for the existence of a Condorcet winner within the context of specific models. Rothstein [1990] defines the concept of “order restriction” and then verifies that the preferences in each of the foregoing specific models are order-restricted.

Moreover, recently Rothstein [1991] has shown that whenever preferences are order-restricted, there must exist a “representative voter.” A representative voter is an individual whose preference between *any* two alternatives coincides with the preference of a majority of the voters. Clearly, the ideal point of the representative voter must be a Condorcet winner.

If our conditions implied order-restriction (or more generally the existence of a representative voter), then the existence of a Condorcet quota would be assured and our “median cutoff theorem” would be superfluous. As we show, however, our conditions—nesting of cutoffs, partial (or even complete) agreement, unconstrained monotonicity, and continuity—do *not* imply the existence of a representative voter. Hence, Rothstein’s results cannot be used to deduce the existence of a Condorcet winner from our conditions.

It suffices to consider an example. Assume there are 3 voters with the continuous preferences represented in Figure 2.

[Figure 2]

Assume the cutoffs are as follows: $F_1 = 1$, $F_2 = .5$, and $F_3 = .25$. Hence, the cutoffs are nested. As Figure 2 reflects, each voter’s preference is strictly decreasing to the

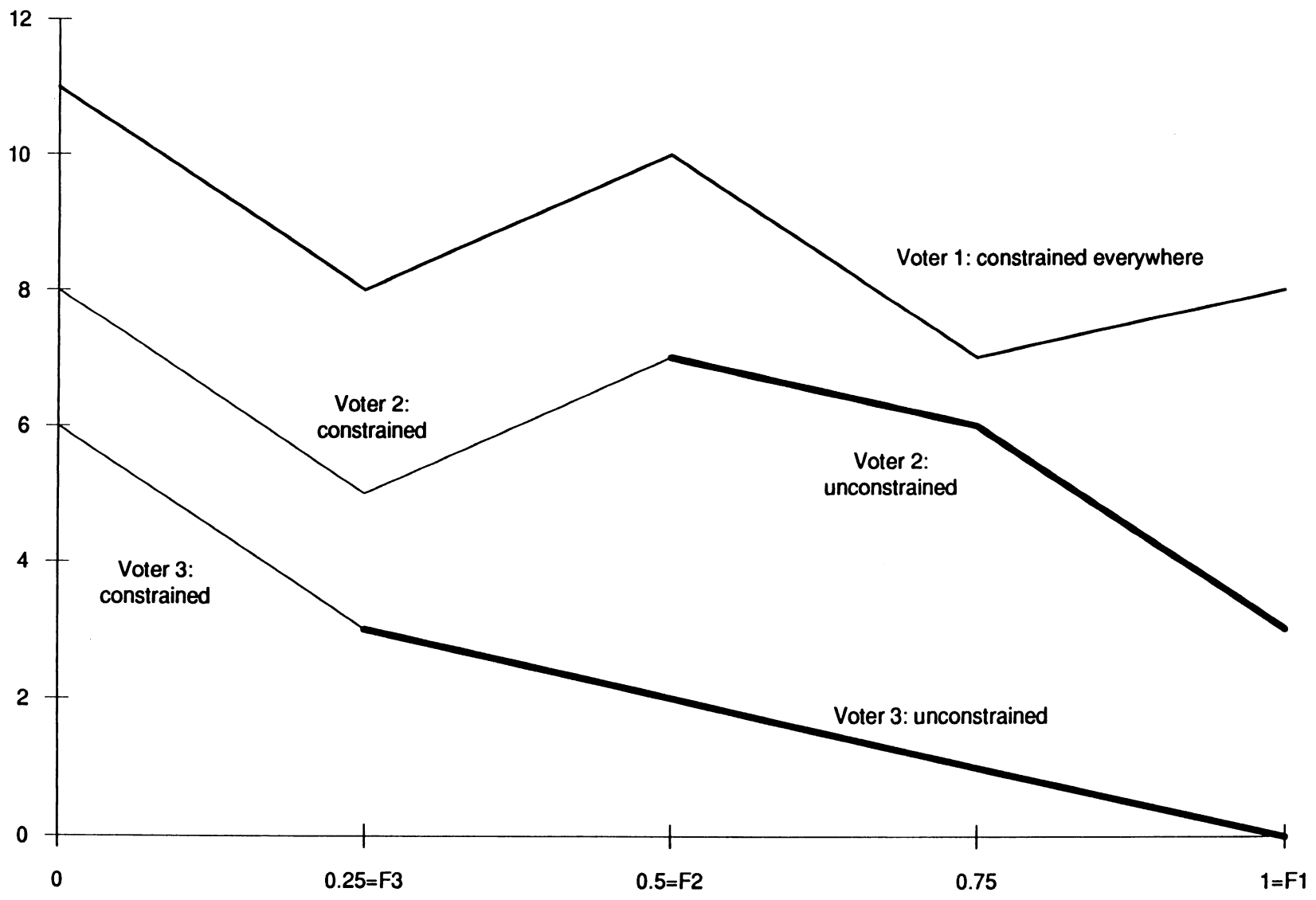


FIGURE 2

right of his cutoff—satisfying unconstrained monotonicity. It remains to verify from Figure 2 that any two voters agree between any two alternatives binding on both of them. Voter 2 and 3 are both constrained to the left of F_3 and completely agree among alternatives in that range. Voter 1 and 3 are both constrained to the left of F_3 and completely agree among alternatives in that range. Voter 1 and 2 are both constrained to the left of F_2 and completely agree among alternatives in that range. Hence, the preferences display not merely partial but “complete agreement.” It follows from our theorem (and from inspection) that the ideal point of Voter 2 ($I_2 = 0$) is the Condorcet winner.

Nonetheless, no voter is representative. Voter 3 is not representative since, unlike the other voters, he prefers .25 to .5. Voter 1 is not representative since, unlike the other voters, he prefers 1.0 to .75. It may at first seem surprising that the voter with the median cutoff, Voter 2, also fails to be “representative.” While he must reflect majority opinion in comparisons between his own ideal point and any alternative, he does not reflect majority opinion in pairwise comparisons which *exclude* his ideal point. For example, Voter 2 is not representative of the majority in his preference of .75 over .25. Hence, while the preferences satisfy our restrictions, there is no representative voter. We conclude, therefore, that our median cutoff theorem differs from the assorted collective choice results in the literature.

2.3 The Economic Effects of Political Changes: Comparative-Statics

As mentioned in the introduction, some cartels permit *all* firms subject to quotas to vote but weight the votes unequally—so-called “weighted majority rule” voting. Typically, firms with larger “capacity” or larger production in the benchmark year receive more votes. Other cartels such as U.S. Federal agricultural marketing boards permit only a subset of regulated firms to vote but the vote of each committee member is given equal weight. The latter scheme is really a special case of weighted majority rule with the votes of some regulated firms receiving zero weight.

Weighted majority rule can be represented within our framework by replicating the preferences of those firms given more than one vote and deleting from consideration the preferences of those firms given zero weight. Since each of the preferences of the “new set” of voters has the characteristics sufficient for the existence of a Condorcet quota, existence is assured under weighted majority rule.

Changing the weights assigned to the different voters or, in the extreme case, zeroing out one subset of voters rather than another changes the identity of the median voter on the cartel and displaces the political-economic equilibrium. To determine the comparative-static effects of changing the index of the median voter, we now show that a firm with a lower index must also have a higher ideal point. That is, if $i < j$, then $I_i \geq I_j$. For, a quota equal to firm j 's ideal point must bind on firm j (unconstrained monotonicity) and must therefore bind on firm i (nesting of cutoffs). If $I_i < I_j$ were possible, then a quota equal to firm i 's ideal point would also bind on both firms. Since i has the loser of the two cutoffs and must strictly prefer its own ideal point, partial agreement would require that j also strictly prefer

I_i to I_j . But since I_j is firm j 's own ideal point, such a conclusion is absurd. Thus, $I_i \geq I_j$.

The significance of this observation is the following. If a re-weighting scheme results in the index of the new median voter being smaller, then the committee will vote for a looser quota and the induced price will decrease. Thus, for example, if the Secretary of Agriculture were to remove from an agricultural marketing board a member larger than the median voter and were to replace him with someone smaller than the median voter then the identity of the median voter would change and—in the new political-economic equilibrium—the market price would (weakly) decline. Similarly, if votes were reapportioned in a commodity agreement so that the new median voter had a lower index, then the committee would select a less restrictive quota and the market price would decline as a consequence.

3. Does the Condorcet Quota Maximize Industry Profits?

In this section, we compare the economic equilibrium which arises under majority rule voting with what would occur if F were set to maximize industry profits. In this comparison, it is important to distinguish the case of identical marginal costs from the case where costs differ.

3.1 Identical Marginal Costs

Suppose each firm in an industry has the same marginal cost. Then, the aggregate production (denoted Y^*) that maximizes industry profits is defined implicitly as the unique solution to the following first-order condition:

$$P(Y^*) + Y^*P'(Y^*) - c = 0. \quad (14)$$

Suppose the cartel were to select the quota $F^* = Y^*/Q_n$. Then, (14) can be rewritten as:

$$P(Q_n F^*) + Q_n F^* P'(Q_n F^*) - c = 0. \quad (15)$$

As long as there is more than one firm subject to regulation, any firm i will have an individual capacity smaller than the aggregate capacity ($q_i < Q_n$) and (15) implies:

$$P(Q_n F^*) + q_i F^* P'(Q_n F^*) - c > 0.$$

It follows that F^* would induce every firm in the industry to produce its legal limit. Hence, F^* would result in the monopoly output (Y^*).

Since every firm is constrained at F^* , the monopoly quota must be (weakly) smaller than the cutoff of the largest firm ($F^* \leq F_n$). Indeed, the monopoly quota must coincide with the ideal point of the largest firm ($F^* = I_n$). To verify this, note that firm n strictly prefers its own ideal point to any other quota and, *a fortiori*, to any other quota binding on all firms. Furthermore, since with identical marginal costs all firms rank quotas binding on everyone in the same way (complete agreement),²² each firm in the industry must prefer I_n to any other quota binding on everyone. Industry profit is, therefore, maximized if and only if the ideal point of the largest firm in the industry is chosen by the cartel.

The monopoly outcome could occur under various circumstances. It would, of course, occur if the *only* voter on the cartel were the largest firm in the industry. But it would also occur if firms were identical, no matter which subset of them voted. For, then—no matter how infinitesimal the capacity of each firm—the Condorcet quota would be the common ideal point of the identical firms and would coincide with the ideal point of the “largest” firm. Homogeneity among firms—not their absolute size—leads to monopoly profits in this model. Heterogeneity leads to conflict within the cartel and reduced profits in the voting equilibrium.

Whenever the cartel votes for a quota different from F^* , it will select a looser quota. Indeed, since the firms would unanimously prefer F^* to any other quota binding on everyone, any quota which the voters prefer must *exceed* the cutoff of the largest firm ($F > F_n \geq I_n = F^*$). If the monopoly quota is not chosen, therefore, the resulting equilibrium price must be lower and some firms (firm n and perhaps others) will produce less than the legal limit. The presence of any firm producing less than its legal limit constitutes proof, within the context of the identical-cost case of the model, that industry profits are not being maximized by the cartel.

The welfare implications of these results depend on the interpretation of the model. Consider the following example. Suppose demand is linear and the industry

²²See footnote 17.

consists of three firms with different capacities but identical marginal costs:

$$\begin{aligned} P &= 10 - Y; \\ c_1 &= c_2 = c_3 = 2; \\ q_1 &= q_2 = 3; \quad \text{but} \quad q_3 = 10. \end{aligned}$$

Since costs are identical, industry profit can be expressed as a function of aggregate production: $Y(10 - Y - 2) = Y(8 - Y)$. It is straightforward to verify that the monopoly output is $Y^* = 4$, the monopoly price is $P^* = 6$, and the maximum industry profit is 16.

Since aggregate capacity is $Q_3 = 16$, the monopoly outcome can be achieved if the cartel were to select a quota of $F^* = Y^*/Q_3 = 1/4$. In that case, the legal limits for the three firms would be, respectively, $Fq_1 = 3/4$, $Fq_2 = 3/4$, and $Fq_3 = 10/4$. It is a Nash equilibrium for each firm to produce the maximum it is allowed. For then the resulting price would be 6 and, as the reader can verify, no firm would have an incentive to reduce its output. The distribution of the monopoly profits in this equilibrium would be as follows: $\pi_1 = \pi_2 = (6 - 2) \cdot 3/4 = 3$ while $\pi_3 = (6 - 2) \cdot 10/4 = 10$.

If each firm casts one vote, then firms 1 and 2 would reject the monopoly quota (and any other alternative) in favor of $F = 2/3$.²³ This quota would induce an equilibrium with firms 1 and 2 constrained but firm 3 unconstrained. Specifically, $y_1 = y_2 = y_3 = 2$. To verify this, note that the resulting price would be 4 and the respective first-order conditions would be:

$$\begin{aligned} 4 &- 1 \cdot 2 - 2 \geq 0 \text{ and } y_1 \text{ is at its upper bound } (y_1 = 2); \\ 4 &- 1 \cdot 2 - 2 \geq 0 \text{ and } y_2 \text{ is at its upper bound } (y_2 = 2); \\ 4 &- 1 \cdot 2 - 2 = 0 \text{ and } y_3 \text{ is unconstrained } (y_3 = 2 < 10 \cdot 2/3). \end{aligned}$$

Hence, neither firm 1 nor firm 2 has an incentive to contract output unilaterally; and firm 3 has no incentive to change its output unilaterally in either direction. Compared to the monopoly quota, industry profits fall 25%. But two of the firms (a majority) are better off: $\pi_1 = \pi_2 = \pi_3 = (4 - 2) \cdot 2 = 4$. Moreover, the movement toward the competitive output increases social surplus. The gain of the two smaller firms *plus* the gain of the consumers from the lower price exceeds the loss to the larger firm.

This example can be re-interpreted as applying to prorationing boards governing common properties. We will examine two cases. In the first, sole ownership achieves the social optimum while a heterogeneous prorationing board would allow too much extraction. In the second, a heterogeneous prorationing board achieves the social optimum while a sole owner would excessively restrain extraction.

Suppose first that the price of oil is fixed exogenously at $P = 8$ and the average cost of extracting from any well on a field ($A(Y)$) depends on the aggregate extraction (Y) from that field due to a pure congestion externality. In particular, assume that $A(Y) = Y$. Suppose three extractors have yardstick capacities of $q_1 = q_2 = 3$ and $q_3 = 10$. Then the socially optimal extraction rate maximizes $Y(8 - Y)$, namely $Y_s = 4$. A sole owner would induce socially optimal extraction. A

²³That is, $F = 2/3$ is the ideal point of the firm with the median capacity (firm 2) and hence is the Condorcet point. To find the best F for firm 2 among the set of quotas which constrain firms 1 and 2 but not 3, verify that $F = 2/3$ solves:

$$\begin{aligned} \max_F & \quad q_2 F(10 - 3F - 3F - y_3 - 2) \\ \text{subject to:} & \quad 10 - 6F - 2y_3 - 2 = 0 \end{aligned}$$

and that at $F = 2/3$ only firm 3 is unconstrained.

board could achieve the social optimum by choosing a quota of $F = 1/4$. However, under majority rule such a quota would be rejected in favor of the Condorcet quota of $F = 2/3$.

On the other hand, suppose that a sole owner could influence the price. Then he would exploit this ability and would extract less than is socially optimal. Since the voting equilibrium results in more extraction, a prorationing board may be socially preferable to sole ownership.

To illustrate, suppose that $P = 8 - 2/3Y$ and $A(Y) = 1/3Y$ while the capacity of each well described above remains unchanged. Then a sole owner would still maximize $Y(P(Y) - A(Y)) = Y(8 - Y)$ and would again set $Y = 4$. The committee could still achieve this output with a quota of $1/4$ but would extract $Y = 6$ by voting for a quota of $F = 2/3$ instead. In this modified example, the social optimum is no longer $Y = 4$ but is exactly the amount chosen by the prorationing board. To verify this, note that $Y = 6$ maximizes $\int_0^Y P(x)dx - YA(Y)$ which is net social surplus.

3.2 Different Marginal Costs

When marginal costs differ among firms, full maximization of industry profit requires that the only firms which produce are those with the lowest marginal costs; the rest shut down. It is not always possible to achieve this outcome with a quota. Nonetheless, we can compare the quota that results in the largest industry profits (the “most profitable” F) to the Condorcet quota. As a separate matter, we can also compare the price that results when industry profits are fully maximized to the price associated with the Condorcet quota.

As in the previous subsection, the Condorcet quota may be looser than the most profitable quota and may in addition result in a lower price than would occur if industry profit were fully maximized.²⁴ But a new phenomenon may also arise when marginal costs differ: the quota chosen by the cartel may be *more restrictive* than the most profitable quota and, as a result, the price may be *higher* than would occur if industry profits were fully maximized. As the following example illustrates, high-cost firms may use their voting power to restrict the output of firms with lower costs.

Suppose again that demand is linear and the industry consists of three firms. Assume now, however, that the firms differ in cost rather than in capacity:

$$\begin{aligned} P &= 10 - Y; \\ c_1 &= 2 \text{ but } c_2 = c_3 = 6; \\ q_1 &= q_2 = q_3 = 5. \end{aligned}$$

A profit-maximizing monopolist would shut down firms 2 and 3 and would produce $y_1 = 4$ with the remaining firm. As a result, the price would be $P = 6$ and monopoly profits would be $\pi_1 = 16$.

The committee could achieve the same result by setting $F = 4/5$ since this would induce an equilibrium with $y_2 = y_3 = 0$ and $y_1 = 4$.²⁵ To verify this, note

²⁴To verify this, slightly perturb the first example in the previous subsection so that the marginal cost of the largest firm differs trivially from that of the other two firms. Continuity assures that the results will change trivially from before. The cartel will still select a quota which is less restrictive than the most profitable one; in addition, the price that results from voting will be lower than the profit-maximizing price.

²⁵To construct an example where *no* quota can fully maximize profits, suppose the marginal cost of the two high cost firms were instead slightly lower (e.g. $c_2 = c_3 = 5$). To mimic the monopoly solution, the high cost firms must be shut down. This would require a quota larger than $F = 4/5$; but any such quota would depress the output of firm 1 below what is required in the monopoly solution.

that the price resulting from this quota would be 6 and the respective first-order conditions for the three firms would be satisfied:

$$\begin{aligned} 6 - 1 \cdot 4/5 \cdot 5 - 2 &\geq 0 \text{ and } y_1 \text{ is at its upper bound;} \\ 6 - 1 \cdot 0 - 6 &\leq 0 \text{ and } y_2 = 0; \\ 6 - 1 \cdot 0 - 6 &\leq 0 \text{ and } y_3 = 0. \end{aligned}$$

Hence, neither firm 2 nor firm 3 has an incentive to increase its production unilaterally and firm 1 has no incentive to reduce its production unilaterally. Note that in this equilibrium firms 2 and 3 earn zero — as they would at any looser quota. Firms 2 and 3 will therefore reject this quota in favor of something tighter (unconstrained monotonicity).

Firms 2 and 3 would reject the most profitable quota (and any other alternative) in favor of $F = 2/15$.²⁶ If $F = 2/15$, all three firms will be constrained at the Cournot equilibrium. Hence, $y_1 = y_2 = y_3 = 2/3$ and $P = 8$. $\pi_2 = \pi_3 = (8 - 6) \cdot 2/3 = 4/3$ while $\pi_1 = (8 - 2) \cdot 2/3 = 4$. Compared to the most profitable quota, industry profits fall by 58%. But two of the firms (a majority) are better off. Firms 2 and 3 collectively gain $8/3$ but in the process firm 1's profit declines from 16 to 4.

Since in this example the cartel could fully maximize industry profit by using the most profitable quota, the voting equilibrium results *both* in a quota which is more restrictive than the most profitable quota and a price which is higher than would arise if profits were fully maximized.

²⁶It is straightforward to verify that $F = 2/15$ is the ideal point of the firm with the median marginal cost and hence is the Condorcet point. The best F for firm 2 must bind on firm 2 and hence will also constrain firms 1 and 3. Hence, $I_2 = \arg \max_F 5F(10 - 15F - 6) = 2/15$.

4. Conclusion

As mentioned in the introduction, in many applications only a subset of the firms in the industry are subject to the cartel's restrictions. Therefore, we conclude by showing how to adapt the analysis if some firms are unregulated.

Assume some of the n firms in the industry are unregulated. Assume the cartel selects F by majority rule and then, having observed F , the regulated and unregulated firms choose outputs simultaneously. Suppose F has been chosen. Compare the Cournot equilibrium when all firms are regulated to the equilibrium when some of them are unregulated. When a firm ceases to be regulated, its pseudosupply curve in Figure 1 (a) loses its vertical segment and instead is strictly increasing throughout. The previous arguments extend to establish the existence of a unique Cournot equilibrium. In addition, they imply that the profits in that equilibrium (the so-called "induced preferences") once again display unconstrained monotonicity and continuity in F . Since at any price P the pseudosupply of every unregulated firm is at least as large as when it was regulated, however, the equilibrium price associated with a given F must be weakly less than when all firms are regulated. Hence, since the demand curve slopes down, the aggregate production associated in the Cournot equilibrium with a given F must be weakly larger than when everyone is regulated. That is, $\tilde{Y}(F) \geq Y(F)$, where $\tilde{Y}(F)$ denotes the equilibrium aggregate production induced by F when the specified set of firms are unregulated. Since the regularity condition necessary for the nesting of cutoffs involves aggregate production, (4) must be modified so that $Y(F)$ is replaced by $\tilde{Y}(F)$. If the regularity condition holds with this modification, then all of our results continue to hold.

The cutoffs will be nested and the induced preferences will display partial agreement. As a consequence, a Condorcet quota will again exist; moreover the quota will respond as before to changes in the voting weights. If firms have identical marginal costs, then when some firms are unregulated the induced production may exceed the monopoly output ($\tilde{Y}(0) > Y^*$) even with $F = 0$. In that case, of course, the price induced by voting will be strictly smaller than the monopoly price. When there exists a quota which would induce Y^* that quota will again be binding on every regulated firm and will coincide with the ideal point of the regulated firm with the largest index. If that quota is not selected the market price will be, as before, lower than the monopoly price. If firms have different marginal costs, then as before the price may be either higher or lower than that associated with the most profitable F . In short, as long as the modified regularity condition holds, none of our results changes when some of the firms are unregulated.

Appendix I: Existence and Uniqueness of a Pure-Strategy Nash Equilibrium

I. Existence.

Let $Q_N = \sum_{i=1}^N q_i$, the sum of the exogenous “capacities”.

Let $\beta_i(Y) = \min \left\{ \max \left(0, \frac{P(Y) - c_i}{-P'(Y)} \right), Fq_i \right\}$ for $Y \in [0, FQ_N]$. Define $\beta(Y) = \sum_{i=1}^N \beta_i(Y)$. Hence $\beta(Y)$ is the “aggregate best reply”. Since $P(Y)$ and $P'(Y)$ are continuous and $P'(Y) < b < 0$, $\beta(Y)$ is continuous. Moreover as long as $P(0) > c_1$, $\beta(0) > 0$. Finally, $\beta(FQ_N) \leq FQ_N$. It follows that there exists at least one fixed point $Y^* \in [0, FQ_N]$ such that $\beta(Y^*) = Y^*$.

Assume that total revenue is strictly concave:

$$2P'(Y) + YP''(Y) < 0 \quad \text{for all } Y \in [0, FQ_N].$$

Then, $2P'(Y^*) + \beta_i(Y^*)P''(Y^*) < 0$ for every $Y^* \in [0, FQ_N]$ and each firm’s second-order condition will be satisfied at each fixed point. Hence, every fixed point of the mapping $\beta(\cdot)$ is a pure-strategy Nash equilibrium. ■

II. Uniqueness.

We now verify that the left and right-hand derivatives of $\beta(\cdot)$ evaluated at any fixed point Y^* are strictly less than 1. This implies that there exists a *unique* fixed point.

If i is unconstrained,

$$\beta_i(Y) = \frac{P(Y) - c_i}{-P'(Y)}.$$

Hence $\beta'_i(Y) = -\frac{P'(Y) + \beta_i(Y)P''(Y)}{P'(Y)}$. Assume that as $Y \rightarrow Y^*$ from the right that u_R firms are unconstrained. Summing over these unconstrained firms we obtain:

$$\beta'(Y^*)^+ = - \left\{ u_R + \frac{P''(Y^*)}{P'(Y^*)} [\beta(Y^*) - FQ_c] \right\}$$

where Q_c is the aggregate capacity of the constrained firms — firms for which $\beta_i(Y^*) = Fq_i$ and $P(Y^*) + \beta_i(Y^*)P'(Y^*) - c_i > 0$. Since $P'(Y^*) < 0$ and $\beta(Y) \geq FQ_c$, $\beta'(Y^*)^+ \leq 0 < 1$ provided $P''(Y) \leq 0$. It remains to show that $\beta'(Y^*)^+ < 1$ if $P''(Y) > 0$.

At any fixed point, $2P'(Y^*) + \beta_i(Y^*)P''(Y^*) < 0$. Hence, summing over the unconstrained firms:

$$2u_R P'(Y^*) + [\beta(Y^*) - FQ_c] P''(Y^*) < 0.$$

Recall that $2P'(Y^*) + Y^*P''(Y^*) < 0$. Thus $2P'(Y^*)[u_R + 1] + 2 \left\{ \beta(Y^*) - \frac{FQ_c}{2} \right\} P''(Y^*) < 0$. Dividing by $-2P'(Y^*) > 0$, we obtain:

$$-[u_R + 1] - \left\{ \beta(Y^*) - \frac{FQ_c}{2} \right\} \frac{P''(Y^*)}{P'(Y^*)} < 0$$

or

$$-\left\{u_R + \frac{P''(Y^*)}{P'(Y^*)}[\beta(Y^*) - \frac{FQ_c}{2}]\right\} < 1 .$$

If $P''(Y^*) > 0$ and the foregoing inequality holds, then adding the negative quantity $\frac{FQ_c}{2} \frac{P''(Y^*)}{P'(Y^*)}$ to the left-hand side will not reverse the inequality:

$$-\left\{u_R + \frac{P''(Y^*)}{P'(Y^*)}[\beta(Y^*) - FQ_c]\right\} < 1 . \quad (\text{A1})$$

Hence $\beta'(Y^*)^+ < 1$. Now assume that as $Y \rightarrow Y^*$ from the left that u_L firms are unconstrained. Summing over these unconstrained firms, we obtain:

$$\beta'(Y^*)^- = -\left\{u_L + \frac{P''(Y^*)}{P'(Y^*)}[\beta(Y^*) - FQ_c]\right\} .$$

Repeating the same steps as for the right-hand derivative, we obtain:

$$-\left\{u_L + \frac{P''(Y^*)}{P'(Y^*)}[\beta(Y^*) - FQ_c]\right\} < 1 . \quad (\text{A2})$$

Hence $\beta'(Y^*)^- < 1$.

It follows that there exists a unique Nash equilibrium in pure strategies induced by any F . ■

Appendix II: Unconstrained Monotonicity and Convexity of the Set of Quotas Binding on Firm i

Let i be an unconstrained firm with profit

$$\pi_i = [P(Y(F)) - c_i] \cdot y_i(F) .$$

A change in F will affect i 's profits:

$$\frac{d\pi_i}{dF} = \frac{dY}{dF} \left\{ [P(Y(F)) - c_i] \frac{dy_i}{dY} + y_i(F) P'(Y(F)) \right\} .$$

Since i is unconstrained, $P(Y(F)) - c_i + y_i P'(Y(F)) = 0$. Substituting this in the expression for $\frac{d\pi_i}{dF}$, we obtain:

$$\begin{aligned} \frac{d\pi_i}{dF} &= \frac{dY}{dF} \left\{ [-y_i P'(Y(F))] \frac{dy_i}{dY} + y_i P'(Y(F)) \right\} \\ &= \frac{dY}{dF} \left\{ 1 - \frac{dy_i}{dY} \right\} y_i P'(Y(F)) . \end{aligned}$$

The last factor is strictly negative. Moreover, the factor in braces is strictly positive since

$$1 - \frac{dy_i}{dY} = 1 + \frac{P'(Y) + y_i P''(Y)}{P'(Y)} = \frac{2P'(Y) + y_i P''(Y)}{P'(Y)}.$$

Since the demand curve is downward-sloping and the second-order condition of firm i holds, the last fraction is the ratio of two strictly negative numbers and is therefore strictly positive.

$$\text{Hence } \text{sgn} \frac{d\pi_i}{dF} = -\text{sgn} \frac{dY}{dF}.$$

To show that $\frac{d\pi_i}{dF} < 0$ as long as some firm is constrained (clearly $\frac{d\pi_i}{dF} = 0$ if no firm is constrained) we verify that $\frac{dY}{dF} > 0$.

Let U be the set of unconstrained firms and u be the number of elements in this set. For each unconstrained firm $i \in U$

$$P(Y) + y_i P'(Y) - c_i = 0.$$

Summing over the set of unconstrained firms and using the fact that $\sum_{i \in U} y_i = Y - FQ_c$ we obtain:

$$uP(y) + [Y - FQ_c]P'(Y) - \sum_{i \in U} c_i = 0.$$

Total differentiation gives:

$$\frac{dy}{dF} = \frac{Q_c P'(Y)}{(u+1)P'(Y) + (Y - FQ_c)P''(Y)}. \quad (\text{A3})$$

This expression is zero if no firm is constrained. Suppose $Q_c > 0$. The numerator is negative since demand is downward-sloping. To sign the denominator, note that equations (A1) and (A2) can be written:

$$u + [Y - FQ_c] \frac{P''(Y)}{P'(Y)} > -1 \quad (\text{A4})$$

where Y is the Nash equilibrium total quantity induced by F and u is the number of firms unconstrained at F . This immediately implies that the denominator of (A3) is negative, and hence $dY/dF > 0$.

We can use these results to verify that a firm constrained (unconstrained) at F will remain constrained (unconstrained) at a tighter (looser) quota. This will be the case if $y_i(F)$, the optimal output for a firm i at any F where it is unconstrained does not increase faster than the maximum allowable output, $q_i F$, as F increases. This can be written:

$$\frac{dy_i}{dF} = \frac{dy_i}{dY} \frac{dY}{dF} = - \frac{[y_i P''(Y(F)) + P'(Y(F))] Q_c}{(u+1)P'(Y(F)) + [Y - FQ_c]P''(Y(F))} \leq q_i. \quad (\text{A5})$$

The denominator is negative by the argument used to establish that $dY/dF > 0$. The numerator is negative if $P''(Y(F)) \leq 0$. Suppose instead that $P''(Y(F)) > 0$. Substituting Fq_i for the smaller value y_i in the numerator and rearranging terms gives the following sufficient condition for (A5):

$$0 \geq [(u+1)P'(Y(F)) + YP''(Y(F))] + Q_c P''(Y(F)). \quad (\text{A6})$$

Concavity of total revenue combined with downward-sloping demand imply that, for every $u \geq 1$,

$$(u + 1)P'(Y) + YP''(Y), \quad (\text{A7})$$

so the first and second terms on the RHS of expression A6 are negative, and inequality A5 is verified.

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