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by

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Section 1. Introduction

The major criticisms of the work on disequilibrium macroeconomics[1] are that (1) one is unable to explain why firms set non–market clearing prices and given that they have done so, (2) do the prices adjust through time to equilibrate the market and if so, how. In this paper, I provide a simple model which illustrates that the following intuition may provide a partial answer to both criticisms. The basic idea is that firms may learn about the market in which they compete by observing their own sales. If their own sales provide additional information and if that information is valuable, then the firm may use non–market clearing prices to acquire this information. This possibility may arise because the firm may be unable to infer whether demand was just sufficient to buy all that were for sale at the price he was charging or whether the firm could have raised its price and still sold every unit it had produced. If the demand states are correlated, then this information has value as the firm can make more informed choices in the future. The model provided shows that this intuition is supportable as an potential, partial explanation for non–market clearing pricing and provides a (potentially) over simple explanation of the adjustment process to equilibrating prices in the future.

An additional reason for studying this type of model is that it allows me to explore one of the less competely studied issues in games with incomplete information, how learning through repeated play affects the outcomes in different stages of the game.[2] Within the set of games intended to describe competition between firms, one would like to better understand how learning takes place. This is especially important because a firm's rivals are able, by their chosen strategies, to affect their opponent's inference problem. In this paper, I intend to focus on a relatively simple game that allows me to study these issues.

The idea is to study a game which, depending upon an easily adjusted assumption, is either a game of complete or incomplete information. To understand the learning effects, the game is chosen so that when the game is a game of complete information, its equilibrium has simple, well known properties. In fact, the game is designed so that in this case, the perfectly competitive equilibrium price and quantity are traded in each period. The assumption is then altered so that the game is a game of incomplete information but has the property that information obtained in

[1] See, for example, Howitt [1979]. For an excellent description of disequilibrium macroeconomics, see Benassy, [1982].

[2] Some papers on this subject include Riordan [1985], Kumar [1985], Mailath [1984] and Bagnoli [1987].

the previous period has no value. The differences in the equilibrium outcomes are studied and it is shown that the differences in the firms' information results in an equilibrium price dispersion in each period. Basically, the idea is that, in the separating equilibrium, the firms' who believe that the demand state is good will choose a higher price than the firms that believe that the demand state is bad. They do so because the probability of the good state conditional on their signal is large enough so that they are willing to accept a "gamble" in which they sell none if the demand state is bad but they sell at a high price if the demand state is good. The firms that choose the low price do so because the probability of the good demand state (in their opinion) is low enough given their signal that they are unwilling to take the gamble.[3] However, in equilibrium, the market clears in each period. That is, the quantity demanded equals the quantity supplied at the price at which trades occur. It is not true that all units produced are sold with probability one, though. A more complete explanation will be provided after a brief description of the model which follows below.

Finally, in the version of the game with incomplete information and the property that information acquired in previous periods has value, I will show that two types of equilibria are possible. The first is similar in nature to the equilibrium described above. That is, in each period there is an equilibrium price dispersion and the quantity demanded equals the quantity supplied at the price at which trade occurs. The second has the property that the first period prices chosen by some firms are *designed* not to clear the market. Basically, these firms opt to charge a price which is not myopically profit maximal in order to acquire the valuable information. By charging a price which ensures that there is excess supply in the bad demand state, these firms are able to infer from their own sales what the demand state is. Opting to charge the price that would clear the market under the bad demand state would not enable them to infer the demand state given the strategy choices of their rivals. Charging such a price means that they would sell every unit produced regardless of the demand state and thus be unable to infer it. Thus, it appears that this equilibrium arises because the firms that choose to charge the low price pay to learn the true demand state by intentionally choosing a price that earns smaller first period profits. This occurs because they do not sell every unit that they have produced. Having learned the demand state, the second period game is a game of complete information and, as a result, the competitive equilibrium

[3] There are also pooling equilibria in which firms receiving the same signal choose different prices. Two cases arise depending upon the exact parameter values assumed. In the first case, those receiving the signal that makes the bad demand state more probable all choose to charge a low price and some of the other firms choose to charge a high price. In the second case, those receiving the signal that makes the good demand state more probable all choose to charge a high price and some of the other firms choose to charge a low price.

arises. Thus, this model provides an equilibrium explanation for non–market clearing pricing and a (probably over simple) mechanism for future price adjustment to a market clearing price.

As I mentioned above, the objective is to study a relatively simple model in which the issues of learning and the effects of one's opponents' strategy choices are captured. In addition, the idea that firms might not choose equilibrium prices which clear the market is one worth exploring. To do so requires that the model not impose market clearing as part of the equilibrium. Thus the standard models in which the price is determined by setting the quantity demanded to the sum of the quantities chosen or setting the quantity sold equal to the quantity demanded at the chosen prices are inappropriate. Instead, I employ a model in which the firms choose both prices and quantities. In particular, in each of two periods, the firms are required to first choose how many units to produce and then choose the price that they will charge. Thus, except for the repeated nature of the model, the timing is similar to the timing employed by Kreps and Scheinkman [1983], Brock and Scheinkman [1985] and Davidson and Deneckre [1986]. Two important differences are that first, they assume that the quantity choices are common knowledge at the price choosing stage while I do not. Second, they do not admit incomplete information. This aspect of the game is similar in spirit to the work triggered by Novshek and Sonnenschein [1980].[4] Thus, one might wish to think of this structure as an industry with firms that must pre–produce their output and then sell it at the price of their choosing in each period.

It is assumed that none of the firms know the true demand state which is either good or bad and that each receives a private signal about the true demand state. Thus, firms know that no one knows the true demand state and that each has received a private signal about it. I assume that the signal is received after the current period's production decision is made but prior to the current period's pricing decision. Each firm then observes the number of units that it sells at its chosen price and the prices charged by its rivals. It does not observe the quantity that its rivals sell. Thus, each firm uses its own sales to learn about the true state of demand. Obviously, its own sales depend upon the prices its rivals charge and its rivals' sales depend upon the price it charges. It is in this way that the ability of a firm to learn is affected by the decisions of its rivals. Thus, the information obtained that was alluded to above is how many units the firm sold.

Earlier, I alluded to the case in which there was complete information and the case in which

[4] See for example, Clarke [1983], Vives [1984], Samet [1984] and Gal–Or [1985].

the information had no value. I can now state more clearly what each of these cases are. The first case arises when the firms know the true demand state and this fact is common knowledge. In this case, they play a game of complete information and I show that the equilibrium outcome corresponds to the standard perfectly competitive outcome. The second case arise when the signals do not reveal the true demand state (and this is common knowledge) and the demand states are uncorrelated across periods. Then any inference a firm might make about the first period demand state based on the quantity it sold is of no value. If it provided additional information about the second period demand state, then it would have value. However, in the case in which the demand states are uncorrelated, it does not have value. In this situation, the differences in information that arise from the different signals that the firms receive results in an equilibrium price dispersion.

The second section of the paper lays out the model in detail and solves the two cases just described. The third section of the paper solves the model when the information has value and the fourth section is the conclusion.

Section 2. The Model and Preliminaries.

I assume that the firms produce a homogeneous, perishable product with a common cost function, $c(q)$. Let $c(q)$ be continuously differentiable with derivative c' a bijective function with $c' \colon \Re_+ \rightarrow \Re_+$. In addition, assume that $c(0) = 0$, $c'(q) \geq 0$and $c'(q) > 0$. Throughout, the consumers are assumed to be perfectly informed, utility maximizers and their behavior will be completely characterized by a demand function $p = f(Q) + \delta$, where p is the price, Q is the number of units purchased, $f'(Q) < 0$, $f(Q) \rightarrow 0$ as $Q \rightarrow \infty$ and $\delta \in \{\delta_g, \delta_b\}$ is a shift parameter with $\delta_g > \delta_b$. Further, I assume that the firms are indexed by $i \in Z \equiv [0, I]$ with measure m such that[5] $\int_0^I \Xi_Z(i)di = m(Z) = I$ where $\Xi_Z(i)$ is the indicator function.

The firms make decisions in each of two periods.[6] In the first period, each firm first chooses how many units to produce, then observes a private signal on δ, and then chooses the price that it will charge in this period. After the firms have all chosen a price, the market opens.[7] In the second

[5] Throughout, for an arbitrary measurable set $A \subset Z$, $\int_A \Xi_A(i)di = m(A)$.

[6] They will be constrained to choose pure strategies.

[7] If the firms do not all choose the same price an assumption concerning the allocation of the low priced units is needed. Following Kreps and Scheinkman, I assume that the residual demand curve facing a high price seller is simply the market demand curve less the number of units produced by the lower priced sellers. As Davidson and Deneckere [1983] have shown, this is not necessarily an innocuous assumption.

period, the firms again choose how many units to produce, potentially observe another signal, and then choose a price. As always, an important assumption is what the firms know when a decision is to be made. I assume that the firms choose their price without observing the quantity choices of their rivals. Further, after units have been exchanged in the first period, each firm is able to observe only the number of units it sold. No other market information is observable. Consequently, unless the firm's actions allow it to infer the state of demand, it remains imperfectly informed.[8] Clearly, this is a restrictive specification but it is made so that the underlying uncertainty may be modelled in a simple way.[9]

I assume that there are two possible signals, s_1 and s_2, where

$$\phi_1 \equiv \text{Prob}[\delta = \delta_g \mid s_1] \in (\frac{1}{2}, 1),$$

$$\phi_2 \equiv \text{Prob}[\delta = \delta_g \mid s_2] \in (0, \frac{1}{2}).$$

The firm that observes s_1 believes that it is more probable that $\delta = \delta_g$ and vice versa. I assume that a firm observes s_1 with probability θ and that the firms' signals are independent of one another. Further, I adopt the convention that a firm's first period type will refer to the signal that it has observed. Thus, there are two types of firms, t_1 and t_2, each firm's type is drawn independently and θ is the probability that a firm is a t_1. Finally, it is assumed that all of the above is common knowledge.

Before solving this game, two simplified versions will be considered which serve as useful benchmarks. The first version will eliminate all of the imperfections in the firms' information and it will be shown that the outcome is simply the competitive equilibrium. The second version assumes that the state of demand in period 2 is independent of the state of demand in period 1. Thus, instead of assuming that δ is known and unchanging through the two periods, I assume that it is drawn independently, period by period. Contrasting this version with the results in Section 3, highlights the effects of the possibility of learning.

So, consider the first version of the model. Here, it is assumed that δ is known prior to the firms' choosing their first period quantities, thereby making the signals redundant. The appropriate

8 Firms with different signals will, in general, choose different prices. Thus, some of them will be able to use the number of units they sell to infer the value of δ.

9 As will become clear in the next section, for any specification in which not every firm can deduce the state of demand from observable first period market data, similar results would obtain.

equilibrium concept is the subgame perfect Nash equilibrium. To find it, consider the second period pricing decision for the i^{th} firm. For any set of quantity choices in the second period, total supply is $\hat{Q} = \int_0^I \hat{q}(i)di$ with $p(\hat{Q}) = f(\hat{Q}) + \delta$. Given $\hat{Q}$ the Nash equilibrium in the final choice stage has every firm choosing to charge $p(i) = p(\hat{Q})$. To see this, note that a firm's profits when the other firms charge $p(\hat{Q})$ are

$$\pi(p, p(\hat{Q}), i) = \begin{cases} p\hat{q}(i) - c(\hat{q}(i)) & p \leq p(\hat{Q}) \\ -c(\hat{q}(i)) & p > p(\hat{Q}), \end{cases}$$

because the firm sells no units if it charges a price larger that $p(\hat{Q})$, but sells all that it has produced if it charges not more than $p(\hat{Q})$. Obviously, i's best reply is to charge $p^*(i) = p(\hat{Q})$. Since this is true for every firm, all charging $p(\hat{Q})$ is the Nash equilibrium given $\hat{q}(i)$.[10]

Knowing the equilibrium set of prices charged for each $\hat{Q}$ permits one to find the second period equilibrium quantity choices for the firms. Firm j, $j \in [0, I]$, chooses how many units to produce to maximize

$$q(j)\left[f(\int_0^I q(i)di) + \delta\right] - c(q(j)).$$

For any given set of quantity choices by firm j's rivals, the assumptions on $c(q)$ ensure that the second order conditions for this maximization problem are satisfied. Thus, the first order condition, given below, defines j's best reply.

$$f(\int_0^I q(i)di) + \delta - c'(q^*(j)) = p(Q) - c'(q^*(j)) = 0$$

In other words, each firm, j, chooses q^* so as to equate the second-period price to its marginal costs. This yields a second period equilibrium of $\langle q^*(j), p(Q^*)\rangle$ which is obviously the same equilibrium that would have been obtained had the firms been price takers, *i.e.*, the competitive equilibrium..

Since the firms are perfectly informed, the firms' second period profits are independent of their first period actions. This, together with the uniqueness of the period equilibrium, implies that the unique subgame perfect Nash equilibrium has the firms choosing the same output in each period and the same price in each period. Thus, without imperfect information, the outcome is the same as the competitive equilibrium which means that the sequencing of choices of quantity and then price, and the ability to set prices are not forcing the solution away from the competitive outcome. This result is summarized as Proposition 1.

10 It is straightforward to show that for each $\hat{Q}$, this is the unique equilibrium in this subgame under the assumption that the firms choose pure strategies.

Proposition 1: *When the firms are perfectly informed, in each period they set quantity such that price equals marginal cost, and the common price charged equates demand with supply.*

The second version of the model differs in that instead of assuming that the state of demand is known and the same in the two periods, I assume that the state of demand in period two is independent of the state of demand in period 1 and that the firms receive a signal in each period. Recall that, the state of demand is unknown to the firms when they choose both the quantity that they will produce as well as the price that they will charge. Thus, there are two types of firms in each period, those that have observed s_1 after their production decision but before their pricing decision, and those that have observed s_2. Further, since the state of demand in period 2 is independent of the state of demand in period 1, a firm's second period type is independent of that firm's first period type. Here, an appropriate equilibrium concept is the extended subgame perfect equilibrium described in Kreps and Wilson [1982]. In other words, the strategies that constitute an equilibrium are equilibrium strategies in each subgame, where each subgame is a game of incomplete information. Notice that because a player's type in the second period is independent of his type in the first period, the players' beliefs trivially satisfy Bayes' rule.

Recall that θ is the probability that s_1 is the signal a firm observes. Thus, in period two, the measure of the set of type 1 firms is θI and the measure of the set of type 2 firms is $(1-\theta)I$. Let $T_2 \equiv \{i : t(i) = t_2\}$; *i.e.*, T_2 is the set of firms that receive the signal s_2.

Consider the firms' second period pricing decision given any set of outputs $q(j)$, $j \in [0, I]$. As will be true throughout the remainder of this paper, the equilibrium will depend on the values of the exogenous parameters. In particular, the equilibrium strategy choices of the firms in the second period depend on $q(j)$ and on the values of ϕ_1 and ϕ_2. Define $p_b(\eta_1, \eta_2) = f(\eta_1 Q_1 + \eta_2 Q_2) + \delta_b$ and[11] $p_g(\eta_1, \eta_2) = f(Q_1 + Q_2) + \delta_g$, where $Q_2 = \int_{j\in T_2} q(j)\,dj$, $Q_1 = \int_0^I q(j)\,dj - Q_2$ and $\eta_k \in [0,1]$ is a fraction of the t_k's, for $k = 1, 2$. In other words, η_k is the fraction of the t_k's charging p_b, and Q_k is the aggregate output sold at the low and high price respectively.

[11] Obviously, p_g is independent of the values of η_1 and η_2. It is written as a function of them only to remind the reader of the type of equilibrium under consideration.

Lemma 1: *If $\phi_1 \geq p_b(0,1)/p_g(0,1) \geq \phi_2$*[12]*, then the equilibrium pricing strategy for a firm is*

$$\sigma^*(t) = \begin{cases} p_b(0,1) & t = t_2 \\ p_g(0,1) & t = t_1, \end{cases}$$

and $\eta_1^* = 0, \eta_2^* = 1$.

Note that assuming that $\phi_2 \leq p_b(0,1)/p_g(0,1) \leq \phi_1$ implies that $p_b(0,1) < p_g(0,1)$ and that the profits of a t_1 firm when its rivals adopt $\sigma^*(t)$ are

$$\pi(p, \sigma^*, t_1) = \begin{cases} pq - c(q) & p \leq p_b(0,1) \\ \phi_1 pq - c(q) & p_b(0,1) < p \leq p_g(0,1) \\ -c(q) & p > p_g(0,1). \end{cases}$$

This is because the consumers are assumed to buy first from the firms charging the lowest price, and σ^* has the t_2 firms charging a lower price than the t_1 firms. Thus, if $\delta = \delta_b$ then, by the definition of $p_b(0,1)$, the t_2 firms sell all units that are demanded. Consequently, if a t_1 firm chooses $p \leq p_b(0,1)$, it is sure to sell all of the units that it has produced. If it chooses $p_b(0,1) < p \leq p_g(0,1)$, it sells all of its output in the event that $\delta = \delta_g$, and sells none otherwise. Finally, if this firm chooses $p > p_g(0,1)$, it sells none regardless of the state of demand. The profits of a t_2 firm are analogous except that ϕ_2 replaces ϕ_1.

Proof: For $\sigma^*(t)$ to be an equilibrium, it must be each firm's best reply when its rivals adopt it. From the profit function above, it is clear that a firm chooses to charge either $p_b(0,1)$ or $p_g(0,1)$. Since $p_b(0,1)q - c(q) - [\phi_1 p_g(0,1)q - c(q)] = [p_b(0,1) - \phi_1 p_g(0,1)]q$, and $p_b(0,1)/p_g(0,1) \leq \phi_1$, then a t_1's profit maximizing price is $p_g(0,1)$. Similarly the difference in profits of a t_2 firm is $p_b(0,1)q - c(q) - [\phi_2 p_g(0,1)q - c(q)]$. Since $p_b(0,1)/p_g(0,1) \geq \phi_2$, its profit maximizing choice is $p_b(0,1)$. ∎

In the equilibrium described in Lemma 1, all of the t_2 firms sell all of their output at a low price and the t_1 firms charge a high price and "gamble" that the state of demand is δ_g. If it is, they "win" and sell all of their output at a high price. If they "lose", they sell none. Their beliefs about the probability that $\delta = \delta_g$ are such that they prefer to take the gamble while the t_2's beliefs are such that they do not wish to take the gamble.

Obviously, Lemma 1 does not characterize all of the possibilities. The remaining possibilities

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12 Note that the dependence upon Q is captured by the value of $p_b(0,1)/p_g(0,1)$.

are $\phi_1 > \phi_2 > p_b(0,1)/p_g(0,1)$ and $p_b(0,1)/p_g(0,1) > \phi_1 > \phi_2$. These are taken up in Lemmas 2 and 3 respectively. In the remaining two possibilities, given the strategy defined above, either both types of firms prefer the gamble or both do not. The method for constructing an equilibrium in these cases involves altering the gamble so that one group is indifferent between taking it and not.

Define a subset of T_2, A^* implicitly by

$$f(Q(A^*)) + \delta_b = \phi_2[f(Q) + \delta_g]$$

where $Q(A^*) \equiv \int_{j \in A^* \subseteq T_2} q(j)dj$.[13] Also, let η_2 be the proportion of the t_2 firms in A^*, *i.e.*, $\eta_2 = m(A^*)/m(T_2)$. Thus, those $t_2 \in A^*$ are to be the firms charging the low price, $p_b(0,\eta_2)$. Further, since $\phi_2 < \frac{1}{2}$, $p_b(0,\eta_2) \equiv f(Q(A^*)) + \delta_b < [f(Q) + \delta_g] \equiv p_g(0,\eta_2)$.

Lemma 2: *If $\phi_1 > \phi_2 > p_b(0,1)/p_g(0,1)$, then the equilibrium pricing strategy for a firm is*

$$\sigma^*(i,t) = \begin{cases} p_b(0,\eta_2) & t = t_2 \text{ and } i \in A^* \\ p_g(0,\eta_2) & t = t_1 \text{ or } t = t_2 \text{ and } i \notin A^*, \end{cases}$$

with $\eta_1 = 0$ and $\eta_2 = \frac{m(A^*)}{m(T_2)}$.

Proof: From the definition of A^*, a t_2 type firm is indifferent between charging $p_b(0,\eta_2)$ and $p_g(0,\eta_2)$ when all of the firms adopt $\sigma^*(i,t)$. Further, since $\phi_1 > \phi_2$, the t_1 firms prefer to charge $p_g(0,\eta_2)$. ∎

Now, define $T_1 = \{i : t(i) = t_1\}$ and define a subset of T_1, B^* implicitly by

$$f(Q_2 + Q(B^*)) + \delta_b = \phi_1[f(Q) + \delta_g]$$

where $Q(B^*) \equiv \int_{j \in B^* \subseteq T_1} q(j)dj$.[14] Let η_1 be the proportion of the t_1 firms in B^*, *i.e.*, $\eta_1 = m(B^*)/m(T_1)$. Further, since $\phi_1 < 1$, $p_b(\eta_1,1) \equiv f(Q_2 + Q(B^*)) + \delta_b < [f(Q) + \delta_g] \equiv p_g(\eta_1,1)$.

Lemma 3: *If $p_b(0,1)/p_g(0,1) > \phi_1 > \phi_2$, then the equilibrium pricing strategy for a firm is*

$$\sigma^*(i,t) = \begin{cases} p_b(\eta_1,1) & t = t_2 \text{ or } t = t_1 \text{ and } i \in B^* \\ p_g(\eta_1,1) & t = t_1 \text{ and } i \notin B^*, \end{cases}$$

with $\eta_2 = 1$ and $\eta_1 = \frac{m(B^*)}{m(T_1)}$.

13 That such a subset exists follows directly from the assumptions on f.

14 If at $B^* = T_1$, it is still true that $f(Q_1(B^*) + Q_2) + \delta_b > \phi_1[f(Q_1 + Q_2) + \delta_g]$, then all firms charging p_b is the equilibrium. However, if $f(0)$ is finite and $[f(0) + \delta_b]/[f(0) + \delta_g] < \phi_1$, then B^* is a strict subset of T_1 satisfying the definition in the text. Hereafter, I will simply take it that B^* is a strict subset of T_1.

Proof: From the definition of B^*, a t_1 is indifferent between charging $p_b(\eta_1, 1)$ and $p_g(\eta_1, 1)$ when all of the firms adopt $\sigma^*(i,t)$. Further, since $\phi_1 > \phi_2$, the t_2 firms prefer to charge $p_g(0, \eta_2)$. ▮

Each of the equilibria described in the Lemmas is presented as an equilibrium in pure strategies. In other words, each type chooses a pure strategy rather than a mixed strategy. The equilibria in Lemmas 2 and 3 are asymmetric in the sense that some firms of one type choose a different strategy than the other firms of the same type. Often, such an equilibrium is thought to be unsatisfactory. Such criticism is unwarranted in this case because equivalent symmetric equilibria in mixed strategies may be constructed from the asymmetric pure strategy equilibria presented. For example, consider the equilibrium described in Lemma 2. A symmetric mixed strategy equilibrium which is equivalent has the t_2 firms choosing $p_b(0, \eta_2)$ with probability η_2 and $p_g(0, \eta_2)$ with probability $1 - \eta_2$. The t_1 firms choose $p_g(0, \eta_2)$ with probability one. An analogous construct may be derived for the equilibrium presented in Lemma 3.

It remains to determine the firms' output choices and to show that both an equilibrium exists and that it may be any of the types described in the previous lemmas depending on the parameter values of exogenous variables. Since a firm has not received its signal (does not know its type) when it chooses q, the firm chooses it to maximize its expected profits

$$[\theta\phi_1 P_g + (1-\theta)P_b]q - c(q),$$

for any prices P_g, P_b. As before, the first order condition characterizes the profit maximizing quantity $q(P_b, P_g, \phi_1)$. To show the existence of an equilibrium, it will be convenient to define

$$Q_1(\alpha_1, \alpha_2, \phi_1, P_b, P_g) = [(1-\alpha_1)\theta + (1-\alpha_2)(1-\theta)]Iq(P_b, P_g, \phi_1)$$

$$Q_2(\alpha_1, \alpha_2, \phi_1, P_b, P_g) = [\alpha_1\theta + \alpha_2(1-\theta)]Iq(P_b, P_g, \phi_1),$$

for α_1, α_2 both contained in $[0,1]$ and at least one not equal to zero. Obviously, we have the following interpretation: Q_1 is the number of units for sale at P_g and Q_2 is the number of units for sale at P_b given ϕ_1 and (α_1, α_2) which should be thought of as the proportion of the type i firms charging P_b.

This gives

$$P_b(\alpha_1, \alpha_2, \phi_1, P_g) = f(Q_2) + \delta_b$$

$$P_g(\phi_1, P_b) = f(Q_1 + Q_2) + \delta_g.$$

Note that P_g is independent of (α_1, α_2) because $Q_1 + Q_2 = Iq(P_b, P_g, \phi_1)$. The properties of these functions are inherited from the functions f and c, the latter because the properties of q are inherited from c. In particular,

$$q(0,P_g,\phi_1)>0,\quad q(P_b,0,\phi_1)>0\quad q(P_b,P_g,\phi_1)\to\infty \text{ as } P_b\to\infty$$

$$\frac{\partial q(P_b,P_g,\phi_1)}{\partial P_b}>0,\quad \frac{\partial q(P_b,P_g,\phi_1)}{\partial P_g}>0\quad q(P_b,P_g,\phi_1)\to \text{ as } P_g\to\infty.$$

Thus, because $f'(Q)<0$, we have

$$\frac{\partial P_b(\alpha_1,\alpha_2,\phi_1,P_g)}{\partial P_g}<0,\quad \frac{\partial P_g(\phi_1,P_b)}{\partial P_g}<0,$$

as well as $P_b(\alpha_1,\alpha_2,\phi_1,0)>0, P_g(\phi_1,0)>0$ and $P_g(\phi_1,P_b)\to 0$ as $P_b\to\infty, P_b(\alpha_1,\alpha_2,\phi_1,P_g)\to 0$ as $P_g\to\infty$. Obviously, these guarantee a solution P_b^*, P_g^* such that

$$P_b^*(\phi_1)=f(Q_2(\alpha_1,\alpha_2,\phi_1,P_b^*,P_g^*))+\delta_b$$

$$P_g^*(\phi_1)=f(Q_1(\alpha_1,\alpha_2,\phi_1,P_b^*,P_g^*)+Q_2(\alpha_1,\alpha_2,\phi_1,P_b^*,P_g^*))+\delta_g,$$

which are depicted in figure 1. To complete the verification that an equilibrium exists, one simply employs Lemmas 1, 2, and 3 to determine that for all exogenous parameter values satisfying my assumptions, there are (α_1,α_2) which are consistent with one of these lemmas. If $\phi_1\geq\frac{P_b^*(0,1,\phi_1)}{P_g^*(\phi_1)}\geq\phi_2$, then we have an equilibrium of the form described in Lemma 1. If $\phi_1>\phi_2>\frac{P_b^*(0,1,\phi_1)}{P_g^*(\phi_1)}$, then we have an equilibrium of the type described in Lemma 2.[15] Similarly, if $\frac{P_b^*(0,1,\phi_1)}{P_g^*(\phi_1)}>\phi_1>\phi_2$, then as argued in an earlier footnote, either there exists an α_1^* such that $0<\alpha_1^*<1$ and $\frac{P_b^*(\alpha_1^*,1,\phi_1)}{P_g^*(\phi_1)}=\phi_1$[16] or all firms charging $P_b^*(1,1,\phi_1)$ is the equilibrium.

As in the previous version of this model, the firms' decisions in the first period do not affect their second period profits. In this version, this follows from the assumption that the state of demand in period two is independent of the state of demand in period one. Thus, each firm's second period profits are independent of the firms' first period decisions. As a result, the extended subgame perfect Nash equilibrium has the firms adopting the strategy choices described above in each of the two periods and this result is summarized as Proposition 2.

Proposition 2: *When the states of demand are uncorrelated, one of the three types of equilibria*

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15 Recall that $P_b^*(0,\alpha_2,\phi_1)/P_g^*(\phi_1)\to 1$ as $\alpha_2\to 0$. Since $\phi_2<1/2$, there exists an α_2^* such that $0<\alpha_2^*<1$ and $\phi_2=P_b^*(0,\alpha_2^*,\phi_1)/P_g^*(\phi_1)$. Setting α_2 equal to η_2 completes the demonstration.

16 This occurs if $[f(0)+\delta_b]/[f(0)+\delta_g]<\phi_1$.

described in Lemmas 1, 2, and 3 arises in each of the two periods.

Some features of the equilibria are straightforward. In all of them, a price dispersion arises as a result of the differential information on the firms' side rather than from the lack of perfect information on the consumers' side. For given values of ϕ_1 and ϕ_2, as θ increases, it becomes more likely that $P_b^*(0,1)/P_g^*(0,1) > \phi_1 > \phi_2$ and conversely, as θ becomes smaller, it becomes more likely that $\phi_1 > \phi_2 > P_b^*(0,1)/P_g^*(0,1)$. The reason is that θ is the fraction of firms that see s_1, and these firms are more likely to be the firms charging $P_g^*(0,1)$. In addition, for a given value of θ, the larger is ϕ_1 and the smaller is ϕ_2, the more likely it is that $\phi_1 > P_b^*(0,1)/P_g^*(0,1) > \phi_2$.

Finally, it is important to realize for the results in the next section, that only some of the firms are able to infer the state of demand. In all of the equilibria, the low price sellers sell all of the units that they have produced regardless of the state of demand while the high price sellers sell only if $\delta = \delta_g$. Thus, since a firm only observes its own sales, those choosing to charge the low price are unable to infer the value of δ. Their unsold stock is zero regardless of the value of δ while a high price seller's stock is zero only if $\delta = \delta_g$. This means that in the equilibrium described by Lemma 2 some of the t_2 firms learn the state of demand while others do not. Likewise, in Lemma 3, some of the t_1 firms learn the state of demand while others do not.

3. Perfectly Correlated Demand States.

In this section, I introduce the possibility of valuable learning by assuming that it is known that the state of demand in period one will also be the state of demand in period two,[17] and that the only signal the firms receive is received in the first period. Under these assumptions, the firms' types in the second period depend on more than just the signal observed in the first period. Recall that in some of the equilibria described in the previous section, firms that had observed the same signal chose different actions. As a result, only some of them charged the high price, *i.e.*, only some of them learned the state of demand. Since knowledge of the state of demand may now be valuable, the firm's type in the second period will also depend on whether or not it has learned the state of demand in period one. To keep the distinction clear, the firms' types in period one will still be described by t_1 and t_2 but in period two, their types will be described by τ_1, τ_{21} and

[17] The model can be readily extended to the case in which the demand states are imperfectly correlated.

τ_{22}. I will identify the τ_1 type as a firm that has deduced the state of demand from observable first period data, the τ_{21} type as a firm that has not deduced the state of demand and saw s_1 in period 1, and the τ_{22} type as a firm that has not deduced the state of demand and saw s_2 in period 1.

Again, an appropriate equilibrium concept is the extended subgame perfect Nash equilibrium. To employ it, I consider the second period first. Unlike the previous versions of the game, the firm's decisions in the first period may affect its second period profits because its decisions partially determine whether or not the firm is a τ_1 type or not. One way to proceed is to solve the second period problem for each possible specification of types and then solve the first–period problem.

Proceeding in that fashion, assume that the set of firms that have deduced the state of demand, $\Lambda_1 \equiv \{i : \tau(i) = \tau_1\}$, is given. The remaining firms are those that have not deduced the state of demand and they make up the set $\Lambda_2 = \Lambda_{21} \cup \Lambda_{22}$. Let $\lambda_k = m(\Lambda_k)$ for $k = 1, 2, 3$. Thus, $\sum_k \lambda_k = I$. From the structure of the game, these firms know that any firm belonging to Λ_1 knows the state of demand.

Consider first the second period pricing decisions. As before, take the firms' quantity decisions as given. Let Q_{2k} represent that aggregate output of the τ_{2k} firms for $k = 1, 2$, and let Q_1 represent the aggregate output of the τ_1 firms. Since the τ_1 firms are those that know the state of demand, Q_1 is potentially dependent upon δ, so conjecture that it actually takes on one of two values, $Q_1(\delta_g)$ or $Q_1(\delta_b)$. Since the firms are unable to observe the quantity choices of their rivals, the τ_{2i}'s cannot infer the state of demand from the τ_1's quantity decisions.

Define[18]

$$\begin{aligned} \hat{p}_b(\alpha_1, \alpha_2) &= f(Q_1(\delta_b) + \alpha_1 Q_{21} + \alpha_2 Q_{22}) + \delta_b \\ \hat{p}_g(\alpha_1, \alpha_2) &= f(Q_1(\delta_g) + Q_{21} + Q_{22}) + \delta_g. \end{aligned}$$

As before, for $\alpha_1 = \alpha_2 = 1$, these prices have been defined so that $\hat{p}_b(1,1)$ clears the market in the event that the state of demand is δ_b and the τ_{2i}'s all charge $\hat{p}_b(1,1)$. Clearly, $\hat{p}_g(1,1)$ clears the market if $\delta = \delta_g$ and the τ_2's all charge $\hat{p}_b(1,1)$.

[18] As in the previous section, $\hat{p}_g$ is actually independent of both α_1 and α_2. It is written as a function of them only to remind the reader of the type of equilibrium under discussion.

Lemma 4: *If $\hat{p}_b(1,1)/\hat{p}_g(1,1) \geq \phi_1$, then the equilibrium pricing strategy is*

$$\sigma^*(\tau_{2i}) = \hat{p}_b(1,1) \quad i = 1,2$$

$$\sigma^*(\tau_1, \delta) = \begin{cases} \hat{p}_b(1,1) & \delta = \delta_b \\ \hat{p}_g(1,1) & \delta = \delta_g, \end{cases}$$

with $\alpha_1 = \alpha_2 = 1$.

Before proving the lemma, consider the firms' profits if they all choose to adopt the given strategies. The profits of a type τ_{21} are

$$\pi(p, \tau_{21}) = \begin{cases} pq - c(q) & p \leq \hat{p}_b(1,1) \\ \phi_1 pq - c(q) & \hat{p}_b(1,1) < p \leq \hat{p}_g(1,1) \\ -c(q) & p > \hat{p}_g(1,1). \end{cases}$$

The profits of a type τ_{22} are similar except that the probability of selling units when $\hat{p}_b(1,1) < p \leq \hat{p}_g(1,1)$ is ϕ_2 rather than ϕ_1. The profits of a type τ_1 firm are

$$\pi(p, \tau_1, \delta_b) = \begin{cases} pq - c(q) & p \leq \hat{p}_b(1,1) \\ -c(q) & p > \hat{p}_b(1,1) \end{cases}$$

$$\pi(p, \tau_1, \delta_g) = \begin{cases} pq - c(q) & p \leq \hat{p}_g(1,1) \\ -c(q) & p > \hat{p}_g(1,1). \end{cases}$$

Proof: Since $\hat{p}_b(1,1)/\hat{p}_g(1,1) \geq \phi_1$, both $\hat{p}_b(1,1) \geq \phi_1 \hat{p}_g(1,1)$ and $\hat{p}_b(1,1) > \phi_2 \hat{p}_g(1,1)$. Thus if a τ_{21}'s rivals all adopt the given strategy, a τ_{21}'s profits are maximized if it chooses $\hat{p}_b(1,1)$. Finally, note that a τ_1 chooses to charge $\hat{p}_b(1,1)$ in the event that $\delta = \delta_b$ and $\hat{p}_g(1,1)$ in the event that $\delta = \delta_g$ because each is the largest price that it can charge and sell its output in each demand state. ∎

This lemma corresponds to Lemma 1 in that the firms' beliefs are such that the uninformed, those firms that do not know the state of demand (the τ_{2i}'s), prefer to ensure that they sell the units that they have produced rather than gamble that the demand state is δ_g. Since the τ_{2i}'s are not a homogeneous group, their beliefs differ about the probability that $\delta = \delta_g$, implying that for other parameter values, other types of equilibria may exist. The remaining possibilities are examined in the following lemmas. As before, the different possibilities arise for different values of

$\phi_1, \phi_2, Q_{21}, Q_{22}$ and Q_1. Summarizing them is facilitated by defining

$$r(\alpha_1, \alpha_2) = \frac{\hat{p}_b(\alpha_1, \alpha_2)}{\hat{p}_g(\alpha_1, \alpha_2)}.$$

In other words, for different values of α_1, and α_2, $r(\alpha_1, \alpha_2)$ gives the ratio of market clearing prices in the two demand states, when α_1 is the fraction of the τ_{21} types charging the low price, and α_2 is the fraction of the τ_{22} types charging the low price. Further, let the set of τ_{2k} charging the low price be L_k for $k = 1, 2$. Define $\alpha_1 = m(L_1)/m(\Lambda_1)$ implicitly by $f(Q_1(\delta_b) + \alpha_1 Q_{21} + Q_{22}) + \delta_b = \phi_1[f(Q_1(\delta_g) + Q_{21} + Q_{22}) + \delta_g]$ if there is an α_1 such that $r(\alpha_1, 1) = \phi_1$.[19]

Lemma 5: *If $\phi_1 \geq r(0,1) \geq \phi_2$, or if $r(0,1) > \phi_1 > \phi_2$ and*[20] *$\exists$ an $\alpha_1 > 0$ such that $\phi_1 = r(\alpha_1, 1) > \phi_2$, then the equilibrium pricing strategy is*

$$\sigma^*(\tau_{22}) = \hat{p}_b(\alpha_1, 1)$$

$$\sigma^*(\tau_{21}, i) = \begin{cases} \hat{p}_b(\alpha_1, 1) & i \in L_1 \\ \hat{p}_g(\alpha_1, 1) & i \notin L_1 \end{cases}$$

$$\sigma^*(\tau_1, \delta) = \begin{cases} \hat{p}_b(\alpha_1, 1) & \delta = \delta_b \\ \hat{p}_g(\alpha_1, 1) & \delta = \delta_g, \end{cases}$$

where $\alpha_1 = 0$ if $\phi_1 \geq r(0,1)$ or, if $r(0,1) > \phi_1$ then L_1 is a subset of Λ_1 such that $\phi_1 = r(\alpha_1, 1)$ and $\alpha_1 = \frac{m(L_1)}{m(\Lambda_1)}$.

Proof: As before, it is obvious from the profit functions that the firms will charge either $\hat{p}_b(\alpha_1, 1)$ or $\hat{p}_g(\alpha_1, 1)$. Since $\phi_2 \leq r(0,1)$, the τ_{22}'s (weakly) prefer charging $\hat{p}_b(\alpha_1, 1)$ to $\hat{p}_g(\alpha_1, 1)$. Further, if $\phi_1 \geq r(0,1) \geq \phi_2$, the τ_{21}'s (weakly) prefer to charge $\hat{p}_g(0,1)$ to $\hat{p}_b(0,1)$. On the other hand, if there is an $\alpha_1 > 0$ such that $\phi_1 = r(\alpha_1, 1) > \phi_2$ then, for that value of α_1, the τ_{21}'s are indifferent between charging either price. As a result, a fraction α_1 of them are willing to charge $\hat{p}_b(\alpha_1, 1)$ while the rest are willing to charge $\hat{p}_g(\alpha_1, 1)$. Finally, the τ_1's know the state of demand, and given the definitions of $\hat{p}_b(\alpha_1, 1)$ and $\hat{p}_g(\alpha_1, 1)$, find that their profit maximizing strategy is as described. The reason for this is that in each state of demand, the price just clears the market given the strategy choices of the others. Thus, these firms can sell every unit that they have produced by charging any price less

19 Obviously, L_1 is not uniquely defined by its measure. This is unimportant because any division of the set of τ_{21}'s, Λ_1, into L_1 and $\Lambda_1 - L_1$ is an equilibrium so long as $m(L_1) = \alpha_1$. The indeterminacy is simply in the identity of which of the identical τ_{21} firms charge one price versus another when they are indifferent between charging either price.

20 See footnote 13.

than or equal to the relevant market clearing price, and can sell none if they charge a higher price. Clearly, they maximize revenues, and thus profits if they choose to charge the market clearing price. ▮

The idea of this lemma may be illuminated by comparing the case where $\phi_1 \geq r(0,1) \geq \phi_2$ with the case where $r(0,1) > \phi_1 > \phi_2$. In the first case, all of the τ_{21}'s prefer to gamble and charge the high price because ϕ_1 is large enough. In the second case, if there is an $\alpha_1 > 0$ the equilibrium involves a proportion of the τ_{21}'s, α_1 charging the low price, the remainder charging the high price and $\hat{p}_b(\alpha_1, 1) = \phi_1 \hat{p}_g(\alpha_1, 1)$. Thus, each of the τ_{21}'s is indifferent between charging these two prices.[21]

Let L_2 be the set of τ_{22}'s charging the low price. Then as shown in the previous section, if $\phi_1 > \phi_2 > r(0,1)$ then there exists a set $L_2 \subset \Lambda_2$ with $m(L_2)/m(\Lambda_2) = \alpha_2$ such that $\phi_2 = r(0, \alpha_2)$.

Lemma 6: *If $\phi_1 > \phi_2 > r(0,\alpha_2)$, then the equilibrium pricing strategy is*

$$\sigma^*(\tau_{22}, i) = \begin{cases} \hat{p}_b(0,\alpha_2) & i \in L_2 \\ \hat{p}_g(0,\alpha_2) & i \notin L_2 \end{cases}$$

$$\sigma^*(\tau_{21}) = \hat{p}_g(0, \alpha_2)$$

$$\sigma^*(\tau_1, \delta) = \begin{cases} \hat{p}_b(0,\alpha_2) & \delta = \delta_b \\ \hat{p}_g(0,\alpha_2) & \delta = \delta_g \end{cases}$$

where $L_2 \subset \Lambda_2$ is such that $\alpha_2 = m(L_2)/m(\Lambda_2)$ and $\phi_2 = r(0, \alpha_2)$.

The proof of this lemma is omitted but the line of argument follows replicates the reasoning used to prove Lemma 5 with the roles of the τ_{21} and τ_{22} types reversed.

In all of these lemmas, the motivation for the choices made by the τ_{2i}'s is very much the same as the motivation for the firms' choices in the previous version of the model captured in Lemmas 1, 2, and 3. The real difference here is the choices made by the τ_1 firms. Their actions can be understood by recognizing that (1) they know δ and know the measure of the set of firms that

21 Obviously this is very similar in spirit to Lemma 3 in the previous section.

are τ_{21}'s and τ_{22}'s, and (2) they can infer the pricing choice of the other firms from the equilibrium. This enables the τ_1's to ensure that any units that are not sold are those produced by the other types. In other words, given δ and the pricing choices of their rivals, the τ_1's ensure that there is no excess supply at the appropriate market clearing price. Consequently, any unsold units must have been produced by the τ_{2i}'s who are attempting to sell them at to high a price and the τ_{2i}'s know this.

Having characterized the equilibrium second period pricing decisions, I must now determine the second period quantity choices for the firms. Since there is no signal in the second period, the firms' types remain as described earlier in this section.

The firms choose their second period quantity so as to maximize expected profits given the equilibrium pricing strategies just described. As before, the first order condition for profit maximization depends on the firm's type. That is, the quantity chosen depends upon the price the firm will charge. Thus, if the firm is a τ_{22}, then its quantity satisfies $p_\ell - c'(q) = 0$ and will be denoted as $q(p_\ell, p_h, \tau_{22})$.[22] Similarly, if the firm is a τ_{21} then the quantity chosen satisfies either $p_\ell - c'(q) = 0$ or $\phi_1 p_h - c'(q)$ depending upon which of the cases described by the lemmas arises. Again, this quantity will be denoted by $q(p_\ell, p_h, \tau_{21})$. Finally, if the firm is a τ_1 type, then it chooses $q(p_\ell, p_h, \tau_1, \delta)$ to satisfy $p_\ell - c'(q) = 0$ if $\delta = \delta_b$ and $p_h - c'(q) = 0$ if $\delta = \delta_g$. Note that a τ_1 firm chooses a quantity that depends upon the state of demand. Since $p_h > p_\ell$, $q(p_\ell, p_h, \tau_1, \delta_g) > q(p_\ell, p_h, \tau_1, \delta_g)$. Thus, the original conjecture that the quantity sold by the τ_1's depended on δ is appropriate.

To facilitate the characterization of the equilibrium quantity choices, the following definitions are used. Define

$$\begin{aligned}
Q_\ell(\alpha_1, \alpha_2, \phi_1, \phi_2, p_\ell, p_h) &= \lambda_1 q(p_\ell, p_h, \tau_1, \delta_b) + \alpha_1 \lambda_{21} q(p_\ell, p_h, \tau_{21}) + \alpha_2 \lambda_{22} q(p_\ell, p_h, \tau_{22}) \\
Q_h(\alpha_1, \alpha_2, \phi_1, \phi_2, p_\ell, p_h) &= \lambda_1 q(p_\ell, p_h, \tau_1, \delta_g) + (1-\alpha_1)\lambda_{21} q(p_\ell, p_h, \tau_{21}) + (1-\alpha_2)\lambda_{22} q(p_\ell, p_h, \tau_{22}) \\
p_\ell &= f(Q_\ell) + \delta_b \\
p_h &= f(Q_h + Q_\ell) + \delta_g .
\end{aligned}$$

In other words, Q_ℓ and Q_h are the quantities sold at the low and high prices, p_ℓ and p_h, respectively. As before, Lemmas 4, 5, and 6 constrain the values that α_1 and α_2 may take on. In particular,

[22] It is unnecessary to separate the quantity decisions of the τ_{22}'s that choose different prices in some equilibrium because in the equilibrium, the firms must be indifferent between charging the low and the high price. That is, by the previous lemmas, if the τ_{22}'s charge different prices then $r(0, \alpha_2) = \phi_2$ which is the same as $p_\ell = \phi_2 p_h$.

they may both be zero (Lemma 4), or if one is a fraction, the other must be either one or zero (Lemmas 5 and 6).

As before, these may be solved for $P_\ell(P_h, \alpha_1, \alpha_2, \phi_1, \phi_2)$ and $P_h(P_\ell, \phi_1, \phi_2)$ with the following properties:

$$\frac{\partial P_\ell(P_h, \alpha_1, \alpha_2, \phi_1, \phi_2)}{\partial P_h} < 0 \quad \frac{\partial P_h(P_\ell, \phi_1, \phi_2)}{\partial P_\ell} < 0.$$

In addition: $P_\ell(0, \alpha_1, \alpha_2, \phi_1, \phi_2) > 0$, $P_h(0, \phi_1, \phi_2) > 0$, $P_h(P_\ell, \phi_1, \phi_2) \to 0$ as $P_\ell \to \infty$. and $P_\ell(P_h, \alpha_1, \alpha_2, \phi_1, \phi_2) \to 0$ as $P_h \to \infty$. Again, these guarantee a solution (P_ℓ^*, P_h^*) such that

$$P_\ell^*(\alpha_1, \alpha_2, \phi_1, \phi_2) = f(Q_\ell(P_\ell^*, P_h^*)) + \delta_b$$

$$P_h^*(\phi_1, \phi_2) = f(Q_h(P_\ell^*, P_h^*) + Q_\ell(P_\ell^*, P_h^*) + \delta_g.$$

As in the previous section, to complete the verification that an equilibrium exists, one simply uses lemmas 4, 5, and 6 to determine that for all exogenous parameter values which are consistent with my assumptions, there are (α_1, α_2) which are consistent with one of these lemmas. If $\phi_1 \geq r(0,1) \geq \phi_2$ then we have an equilibrium by lemma 4. If $\phi_1 > \phi_2 > r(0,1)$ then, by exactly the same argument used in the previous section, there is an α_2 such that $\phi_2 = r(0, \alpha_2)$ which is the α_2 required to satisfy Lemma 6. Also, by the argument used in the previous section, if $r(0,1) > \phi_1 > \phi_2$ then either all firms charge P_h^* or there is an α_1 such that $r(\alpha_1, 1) = \phi_1$ thus satisfying Lemma 5.

Before solving for the firms' first period choices, it is worth noting that the τ_1 firms earn profits that are at least as large as those earned by the τ_{2i}'s. To see this, first consider the firms that charge the low price in any of the equilibria described above. The uninformed who charge the low price have chosen to produce (and sell) $q(P_\ell^*, P_h^*, \tau_{21}) = q(P_\ell^*, P_h^*, \tau_{22})$, which obviously satisfies $P_\ell^* - c'(q) = 0$. Clearly, if the τ_1 type knows that $\delta = \delta_b$, the quantity chosen is exactly the same. Thus, if $\delta = \delta_b$ the uninformed firms charging the low price earn the same profits as the informed firms do. However, if $\delta = \delta_g$ this is no longer true because the τ_1 firms choose to produce a larger quantity and sell it at a higher price than the low price firms. Similarly, the uninformed firms that charge the high price produce the quantity that satisfies $\phi_i P_h^* - c'(q) = 0$ for $i = 1$ or 2. For both types, the quantity chosen is smaller than that chosen by the τ_1's when $\delta = \delta_g$. As a result, in this event the informed firms earn larger profits than the uninformed. In the event that the true state of demand is δ_b, the informed firms' profits are larger because the uninformed firms charging the high price sell no units. Thus, in every possible outcome, the informed firms' profits are at least as great as the uninformed firms' profits.

This observation explains why the firms' first period choices affect their second period profits. For example, consider the per–period pricing equilibrium of the previous section in which some members of a type choose to charge the low price while the remainder choose the high price. Those choosing the high price had expected profits just equal to the profits of the firms charging the low price. However, now that the states of demand are perfectly correlated, the firms' first period profits are still equal but their expected second period profits differ (in expectation). The firms that charge the high price learn the state of demand and are able to employ this information to earn larger profits in the second period than the firms that saw the same signal but charged the low price. Since the arguments above imply that in every second period equilibrium, the expected difference between the profits of an informed firm and the profits of an uninformed firm is positive, there will be two potential types of first period equilibria. Each corresponds to a different division of the firms into second period types.

In the first equilibrium that I will present, every firm is a τ_1 in the second period, *i.e.*, every firm learns the state of demand in the first period by observing their own unsold stocks. Every first period pricing equilibrium will roughly correspond to one of Lemmas 1, 2, or 3.

Consider first the pricing equilibrium that corresponds to the pricing equilibrium in Lemma 1. Recall that for a given set of quantity choices $q(j)$ $j \in [0, I]$, $p_b(\eta_1, \eta_2) = f(\eta_1 Q_1 + \eta_2 Q_2) + \delta_b$ and $p_g(\eta_1, \eta_2) = f(Q_1 + Q_2) + \delta_g$, where $Q_2 = \int_{j \in T_2} q(j)\,dj$, $Q_1 = \int_0^I q(j)\,dj - Q_2$ and $\eta_k \in [0, 1]$ is a fraction of the t_k's, for $k = 1, 2$. Also, assume that there is an exogenous sharing rule in the event that the firms' pricing decisions do not clear the market.[23] In particular, if the firms' pricing decisions are such that there is excess supply at a quoted price, assume that each firm charging the same price is unable to sell exactly the same fraction of the units that they have for sale. Thus, if the price is ε larger than the market clearing price, then $\xi(\varepsilon, n)$ is the percentage of a firm's supply that it does not sell if it is one of the set of firms of measure n, quoting the price that is ε larger than the market clearing price.

[23] Such an assumption is unfortunate. However, with no differences between the firms and a consumer sector modelled simply as a demand function, there is no means of determining (endogenously) how many units a given firm is selling. One could imagine that there is a measure of consumers each of whom chooses to purchase from any given low price firm with equal probability. This would generate exactly the sharing rule that I impose.

Lemma 7: *If $\phi_1 \geq (p_b(0,1)+\varepsilon)/p_g(0,1) \geq \phi_2$ for some ε small, then the equilibrium pricing strategy for a firm is*

$$\sigma^*(t) = \begin{cases} p_b(0,1)+\varepsilon & t = t_2 \\ p_g(0,1) & t = t_1, \end{cases}$$

and $\eta_1 = 0, \eta_2 = 1$.

Note that the profits of a t_1 firm when its rivals adopt $\sigma^*(t)$ are

$$\pi(p,\sigma^*,t_1) = \begin{cases} pq - c(q) & p < p_b(0,1)+\varepsilon \\ pq[1-(1-\phi_1)\xi(\varepsilon, m(T_2))] - c(q) & p = p_b(0,1)+\varepsilon \\ \phi_1 pq - c(q) & p_b(0,1)+\varepsilon < p \leq p_g(0,1) \\ -c(q) & p > p_g(0,1). \end{cases}$$

Since the consumers are assumed to buy from the firms charging the lowest price first, σ^* has the t_2 firms charging a lower price than the t_1 firms. The profits of a t_2 firm are analogous except the ϕ_2 replaces ϕ_1.

Proof: If all of the firms adopt the pricing strategy described, all learn the state of demand. Consequently, conditional on this, their best reply is the price that makes their first period profits largest. Since $\phi_1 \geq (p_b(0,1)+\varepsilon)/p_g(0,1) \geq \phi_2$, for some ε small, the t_1's prefer to charge $p_g(0,1)$ while the t_2's (weakly) prefer $p_b(0,1)+\varepsilon$ to $p_g(0,1)$. Further, if a t_2 chose to charge a price less than $p_b(0,1)+\varepsilon$, it would sell every unit but would not know the state of demand. Thus, for this firm to prefer to adopt the specified strategy, ε must be small enough so that the loss in profits of not selling every unit is more than offset by the gain in profits from being able to learn the state of demand. Since ε determines the amount of excess supply and thus $\xi(\varepsilon, m(T_2))$, and since f is continuous, then the cost of charging $p_b(0,1)+\varepsilon$ can be made arbitrarily small. Since the benefits of learning the state of demand are strictly positive (as explained above), there is an ε such that each t_2 prefers to charge $p_b(0,1)+\varepsilon$ and learn the state of demand rather than a price arbitrarily smaller than $p_b(0,1)+\varepsilon$ given that its rivals adopt the strategy defined above. Thus, given the first period quantity choices of the firms, there is an ε such that the strategies above constitute a first period pricing equilibrium. ∎

Before exploring the possibilities that arise from other parameter values, a few words about the above equilibrium are necessary. First, notice that ε is not uniquely determined in the proof.

That is, there are many ε's that form an equilibrium. Second, every one of them is a strong equilibrium. That is, they satisfy all of the available criteria for robustness of an equilibrium. They are both (trembling–hand) perfect and proper equilibria. Third, they are Pareto comparable. That is, no firm is worse off and some firms are better off in an equilibrium with a lower ε than a higher ε. Consequently, if one is willing to add Pareto efficiency into the definition of equilibrium then this non–market clearing effect is minimized. It is unusual to do this and the fact that the equilibria are strong Nash equilibria provides a sound basis for believing that the non–market clearing effect is minimal.

As before, the other possibilities result in other types of first period pricing equilibria and these possibilities are described in the next two lemmas. For this section, define a subset of T_2, A^* implicitly by

$$[f(Q(A^*)) + \delta_b + \varepsilon][1 - (1 - \phi_2)\xi(\varepsilon, m(A^*))] = \phi_2[f(Q) + \delta_g]$$

where $Q(A^*) \equiv \int_{j \in A^* \subseteq T_2} q(j)dj$ and η_2 be the proportion of the t_2 firms in A^*, *i.e.*, $\eta_2 = \frac{m(A^*)}{m(T_2)}$. Again, A^* is defined so that a member of A^* is indifferent between charging the low price (and bearing the possibility of not selling all of the units it has produced) and charging the high price (selling units only if the state of demand is δ_g). Note that the firm is indifferent because it learns the state of demand regardless of which price it chooses and that the firm's expected, first period profits of choosing either price are the same. From the discussion in the previous section, such an A^*, a strict subset of T_2 exists which makes $(p_b(0, \eta_2) + \varepsilon)/p_g(0, \eta_2) = \phi_2$. Also for this section, define a subset of T_1, B^* implicitly by

$$[f(Q_2 + Q(B^*)) + \delta_b + \varepsilon][1 - (1 - \phi_1)\xi(\varepsilon, m(B^*))] = \phi_1[f(Q) + \delta_g]$$

where $Q(B^*) \equiv \int_{j \in B^* \subseteq T_1} q(j)dj$ and η_1 be the proportion of the t_1 firms in B^*, *i.e.*, $\eta_1 = m(B^*)/(T_1)$. Again, the discussion in the previous section implies that either such a B^* exists[24] or that all firms charge the high price is the first period equilibrium pricing strategy when $(p_b(0,1) + \varepsilon)/p_g(0,1) > \phi_1 > \phi_2$.

Lemma 8: *If $\phi_1 > \phi_2 > (p_b(0,1) + \varepsilon)/p_g(0,1)$ for some ε small, then the equilibrium pricing strategy for a firm is*

$$\sigma^*(i,t) = \begin{cases} p_b(0,\eta_2) + \varepsilon & t = t_2 \text{ and } i \in A^* \\ p_g(0,\eta_2) & t = t_1 \text{ or } t = t_2 \text{ and } i \notin A^*, \end{cases}$$

[24] See footnote 13

with $\eta_1 = 0$ *and* $\eta_2 = \frac{m(A^*)}{m(T_2)}$.

Proof: From the definition of A^*, a t_2 earns the same first period expected profits from charging either $p_b(0, \eta_2) + \varepsilon$ or $p_g(0, \eta_2)$ when all of the firms adopt $\sigma^*(i, t)$. Since it learns the state of demand in either case, the t_2's are indifferent between charging these prices. Further, since $\phi_1 > \phi_2$, the t_1 firms prefer to charge $p_g(0, \eta_2)$. ∎

Lemma 9: *If* $(p_b(0,1) + \varepsilon)/p_g(0,1) > \phi_1 > \phi_2$ *for some* ε *small, then the equilibrium pricing strategy for a firm is*

$$\sigma^*(i,t) = \begin{cases} p_b(\eta_1, 1) + \varepsilon & t = t_2 \text{ or } t = t_1 \text{ and } i \in B^* \\ p_g(\eta_1, 1) & t = t_1 \text{ and } i \notin B^*, \end{cases}$$

with $\eta_2 = 1$ *and* $\eta_1 = \frac{m(B^*)}{m(T_1)}$.

Proof: From the definition of B^*, a t_1 earns the same first period expected profits from charging either $p_b(\eta_1, 1) + \varepsilon$ or $p_g(\eta_1, 1)$ when all of the firms adopt $\sigma^*(i, t)$. Since it learns the state of demand in either case, the t_1's are indifferent between charging these prices. Further, since $\phi_1 > \phi_2$, the t_2 firms prefer to charge $p_b(\eta_1, 1) + \varepsilon$. ∎

The analysis of the firms' first period quantity choices is analogous to the analysis of the previous section. A firm's profit maximizing output choice, $q(p_b + \varepsilon, p_g)$, satisfies $\theta\phi_1 p_g + (1-\theta)(p_b - \varepsilon)(1 - \xi(\varepsilon, n)(1 - \phi_2)) = c'(q)$ where θ is still the probability that a firm is a t_1. Using exactly the same arguments employed earlier, for each pair (η_1, η_2), there is an equilibrium pair of prices $P_b^* + \varepsilon, P_g^*$ which induce an equilibrium first period quantity choice $q(P_g^*, P_b^* + \varepsilon)$.

As before, $P_b^*(0,1) + \varepsilon$ and $P_g^*(0,1)$ are uniquely defined. Thus, since ϕ_1 and ϕ_2 are exogenous, they may be chosen so as to satisfy any one of the conditions for Lemmas 7, 8, or 9. Thus, any of the three possibilities may result:

(i) all of the t_2's charging a low price and all of the t_1's charging a high price,

(ii) some (potentially none) of the t_2's charging a low price and the remaining t_2's and all of the t_1's charging the high price,

or (iii) all of the t_2's and some of the t_1's charging the low price and the remaining t_1's charging the high price, may result.

In all of the first period equilibria above, every firm learns the state of demand. Therefore, the second period equilibrium has every firm adopting the strategy choice of a τ_1 firm. In other words, the second period equilibrium quantity and price choices are exactly those described in the previous section when the complete information case was considered. There (Proposition 1), I showed that the equilibrium is the same as the standard competitive equilibrium. Thus, this equilibrium in the two period game has one of the first period pricing equilibria described by Lemmas 7, 8, and 9 arising in the first period; and it has the competitive outcome in the second period. Let $p_\ell(\phi_1, \phi_2)$ and $p_h(\phi_1, \phi_2)$ represent the equilibrium prices charged in the first period, let $q(p_\ell, p_h)$ be the first period quantity choice of the firms, and let $\eta_1(\phi_1, \phi_2)$ and $\eta_2(\phi_1, \phi_2)$ be the equilibrium fractions of the t_1's and t_2's charging p_ℓ in the first period. For example, if ϕ_1, ϕ_2 are such that the pricing equilibrium of Lemma 7 arises, $\eta_1(\phi_1, \phi_2) = 0, \eta_2(\phi_1, \phi_2) = 1, p_\ell(\phi_1, \phi_2) = P_b^*(0,1) + \varepsilon$ and $p_h(\phi_1, \phi_2) = P_g^*(0,1)$. With this notation, the equilibrium is summarized in Theorem 1.

Theorem 1: *An equilibrium has all of the firms producing $q(p_\ell, p_h)$ in the first period, has $\eta_1(\phi_1, \phi_2)$ of the t_1's and $\eta_2(\phi_1, \phi_2)$ of the t_2's charging $p_\ell(\phi_1, \phi_2)$ and the remaining firms charging $p_h(\phi_1, \phi_2)$ in period 1, and has the firms producing q^* and charging $p^*(Q^*)$ in the second period.*[25]

Before turning to a brief description of another type of equilibrium that may arise, consider the implications of theorem 1. It says that the firms that charge the low price rationally choose to charge the low price and avoid the gamble that the demand state is good. However, in avoiding the gamble, they choose not to remain uninformed. They do this by choosing a price that is high enough so that if the demand state is bad (the more likely event they believe) the quantity demanded at that price is smaller than the quantity produced. Thus, if they sell every unit produced, they know that the demand state is good and if not, then the demand state is bad. It is not in any firm's short run best interest to charge this price because it would earn larger profits by reducing its price. However, doing so ensures that the firm will not learn the state of demand and therefore lose profits in the second period. In the equilibrium, these future losses exceed the present gains which induces the firm to charge a non–market clearing price. In other words, they purchase information about the demand state by absorbing, with positive probability, some unsold stock. In addition, the equilibrium has the property that learning does occur and that the firms choose the market clearing price in the second period. That is, there is a very simple dynamic structure that has the

[25] Recall that q^* solves $p^*(Q^*) - c'(q^*) = 0$ when $Q^* = \int_0^I q^* di \equiv q^* I$.

firms achieving the market clearing price after having chosen a non–market clearing price. It is true that the amount of excess supply is not large but this follows from the very simplistic uncertainty (two demand states) that I posited. Also, the dynamic that leads to market clearing pricing in the second period is overly simple. Again, this follows from the simple uncertainty posited (perfectly correlated demand states) and the simple two period structure studied.

Another possible type of equilibrium in this model has the firms which charge the low price in the first period, remaining uninformed about the state of demand. This form of equilibrium is most easily described when the t_2's choose to charge the low price in period one and the t_1's choose to charge the high price. Let $\Delta(\lambda_1)$ be the difference in expected profits of knowing versus not knowing the state of demand in period 2 when λ_1 is the measure of the set of firms that know it. One possible first period pricing equilibrium arises if $\phi_1 p_g(0,1)q > p_b(0,1)q - \Delta(\lambda_1) > \phi_2 p_g(0,1)q$.

Lemma 10: *If* $\phi_1 p_g(0,1)q > p_b(0,1)q - \Delta(\lambda_1) > \phi_2 p_g(0,1)q$, *then the equilibrium pricing strategy for a firm is*

$$\sigma^*(t) = \begin{cases} p_b(0,1) & t = t_2 \\ p_g(0,1) & t = t_1, \end{cases}$$

and $\eta_1 = 0, \eta_2 = 1$.

Proof: Clearly, this pricing strategy leaves the t_2's uninformed about the state of demand while the t_1's learn it. It is also clear that the firms will only choose one of $p_b(0,1)$ or $p_g(0,1)$. Now, consider a t_1 firm. Since it has learned the state of demand, switching to $p_b(0,1)$ results in a loss in profits as $\phi_1 p_g(0,1)q > p_b(0,1)q - \Delta(\lambda_1)$. Similarly, if a t_2 switched to $p_g(0,1)$ it would also lose profits as $p_b(0,1)q - \Delta(\lambda_1) > \phi_2 p_g(0,1)q$ ▌

It is also straightforward to characterize the first period equilibrium pricing strategies if $\phi_1 p_g(0,1)q > \phi_2 p_g(0,1)q > p_b(0,1)q - \Delta(\lambda_1)$. This is because, as in the previous case, the only uninformed firms in the second period are t_2's as they are the only firms charging the low price. Now, let η_2 be the fraction of the t_2's charging $p_b(0,\eta_2)$ which leaves them indifferent between (a) charging this low price, guaranteeing that they sell all of the units that they produced in the first period but not learning the state of demand and (b) charging the high price $p_g(0,\eta_2)$, selling only if the state of demand is δ_g but reaping the benefits of learning the state of demand. Further, let the set of t_2's that charge the low price be A^*.

Lemma 11: *If $\phi_1 p_g(0,1)q > \phi_2 p_g(0,1)q > p_b(0,1)q - \Delta(\lambda_1)$, then the equilibrium pricing strategy for a firm is*

$$\sigma^*(t,i) = \begin{cases} p_b(0,\eta_2) & t = t_2 \text{ and } i \in A^* \\ p_g(0,\eta_2) & t = t_1, \text{ or } i \notin A^*. \end{cases}$$

with $\eta_2 = \frac{m(A^*)}{m(T_2)}$.

Analogous first period pricing equilibria exist when neither of the inequalities above are satisfied, *i.e.*, when $p_b(0,1)q - \Delta(\lambda_1) > \phi_1 p_g(0,1)q > \phi_2 p_g(0,1)q$. The equilibrium will have some of the t_1's and all of the t_2's charging the low price. The tedious statement of these equilibria is omitted as the number of possibilities is quite large. This is because the fraction of firms which are indifferent between charging the low price and the high price depends on which of the second period pricing equilibria are achieved. Since δ_b and δ_g are exogenous, they may be chosen so that any of the second period equilibria follow the first period equilibrium pricing strategy.

Basically, all of the equilibria of this type are very similar to the equilibria when the demand states are uncorrelated. The firms charge one of two prices. In the "separating" equilibrium, the firms charging the low price are the firms which received the bad signal and the firms charging the high price are the firms which received the good signal. Each chooses to charge the stated price because, conditional on their signal, their expected profits are larger. In the "pooling" equilibria, the type of firms that are indifferent between charging the low and high price take into account the lost profits in the second stage from remaining uninformed. Therefore, relative to the corresponding equilibria when the demand states were uncorrelated, the low price is, in general, higher. This is necessary to compensate the firms charging the low price for the lost profits in the second stage which are due to their subsequent informational disadvantage.

Unfortunately, the equilibria of Theorem 1 and these equilibria cannot be ranked by the Pareto criteria. In the equilibrium in which every firm learns the state of demand in the first period, the firms earn larger profits than they do in the other type of equilibrium. However, in the second period, the profits of an informed firm decline as the number of informed firms rises. This is because when some firms are uninformed, there are cases in which they fail to sell any units in the bad demand state. Consequently, in this event, the informed firms sell their units at a higher price and their profits decline as the number of informed rises. Therefore, for a firm that would be informed in both types of equilibria, its profits in the equilibrium with uninformed firms are larger than its profits in the equilibrium with only informed firms.

4. Conclusions

My objective was to consider dynamic oligopoly pricing when the firms were imperfectly informed about their rivals. I built a model which had the firms choosing a quantity to produce and then a price to charge in each period. They observed a signal about the state of demand after they had produced the units but prior to choosing the price they wished to sell them for. I began by showing that without imperfect information, the firms' decisions replicated the traditional competitive equilibrium, period by period.

Secondly, I explored a version of the model in which the states of demand in each period were uncorrelated and the firms received a signal about the demand state in each period. I showed that the equilibrium was the same, period by period, and that the firms charged different prices based on their different signals. In each period, the market cleared in the sense that every unit for sale at the price quoted for the last unit sold, was purchased. However, if the demand state was bad, the low price sellers sold their units and the high priced firms sold none. An interesting feature of this version was that some of the firms learned the state of demand after the units were sold by observing their own unsold stocks. In other words, only the high price firms were able to infer the state of demand from personal market data.

This feature was exploited in Section 3. There, I considered a version of the model in which the state of demand was unknown to the firms in the first period but remained the same for the second period. In other words, if knowing the state of demand was valuable, those that learned it in the first period could exploit their information advantage in the second period of this model. I showed that knowing the state of demand was, in fact, valuable. The result was that two types of equilibria were possible.

The first type of equilibrium had all of the firms, regardless of their type, pricing so that they learned the state of demand. To do this, the firms chose to charge a low price in the first period, chose it high enough so that they too would learn the state of demand. This was accomplished by choosing a price that *did not* clear the market in the event that the demand state was bad. In other words, the firms' profit maximizing decisions resulted in excess supply of the product in the bad demand state at the price at which trade occurs. In the second period, all of the firms had learned the state of demand which meant that the second period decisions replicated the standard competitive equilibrium.

In other words, profit maximizing behavior by the firms led to a positive probability of excess supply and provided a simple adjustment to an "equilibrium" state, *i.e.*, one in which the market cleared with probability one. Unfortunately, the simple specification of the possible demand states meant that the firms that chose the low price did not choose to induce very much excess supply. A very small amount was sufficient to permit them to infer the state of demand. Having provided an explanation for why profit maximizing firms would choose a price that would result in excess supply (with positive probability) an explanation for a movement back to market clearing pricing is also available. Because the firms chose the non–market clearing prices to learn the state of demand. and because I had the very simple assumption that the demand states were perfectly correlated. the firms choose market clearing prices in the very next period. Had it been the case that the demand states were only imperfectly correlated, then one would have had non–market clearing prices in all but the last period. However, the difference between the market clearing prices and the non–market clearing prices would shrink each period. Thus, in a more complete model with less stark assumptions, one would expect to obtain a more sensible explanation for the convergence to market clearing pricing. Again, let me emphasize that the explanation for non–market clearing pricing and the convergence to market clearing pricing flow from the firms attempts to learn about their environment in a situation in which what they may learn is affected by the decisions of their rivals. An alternate specification of the game with a more complicated demand structure is the focus of my current research.

Another type of equilibrium was possible, in which all of those firms charging the low price in the first period, failed to learn the state of demand. In every possible outcome, one of two things can happen: Either the firms' beliefs about the relative probabilities of the good and bad state were sufficient to offset the second period informational advantage of charging the high price; or the prices were different enough to compensate the firms charging the low price for not learning the state of demand.

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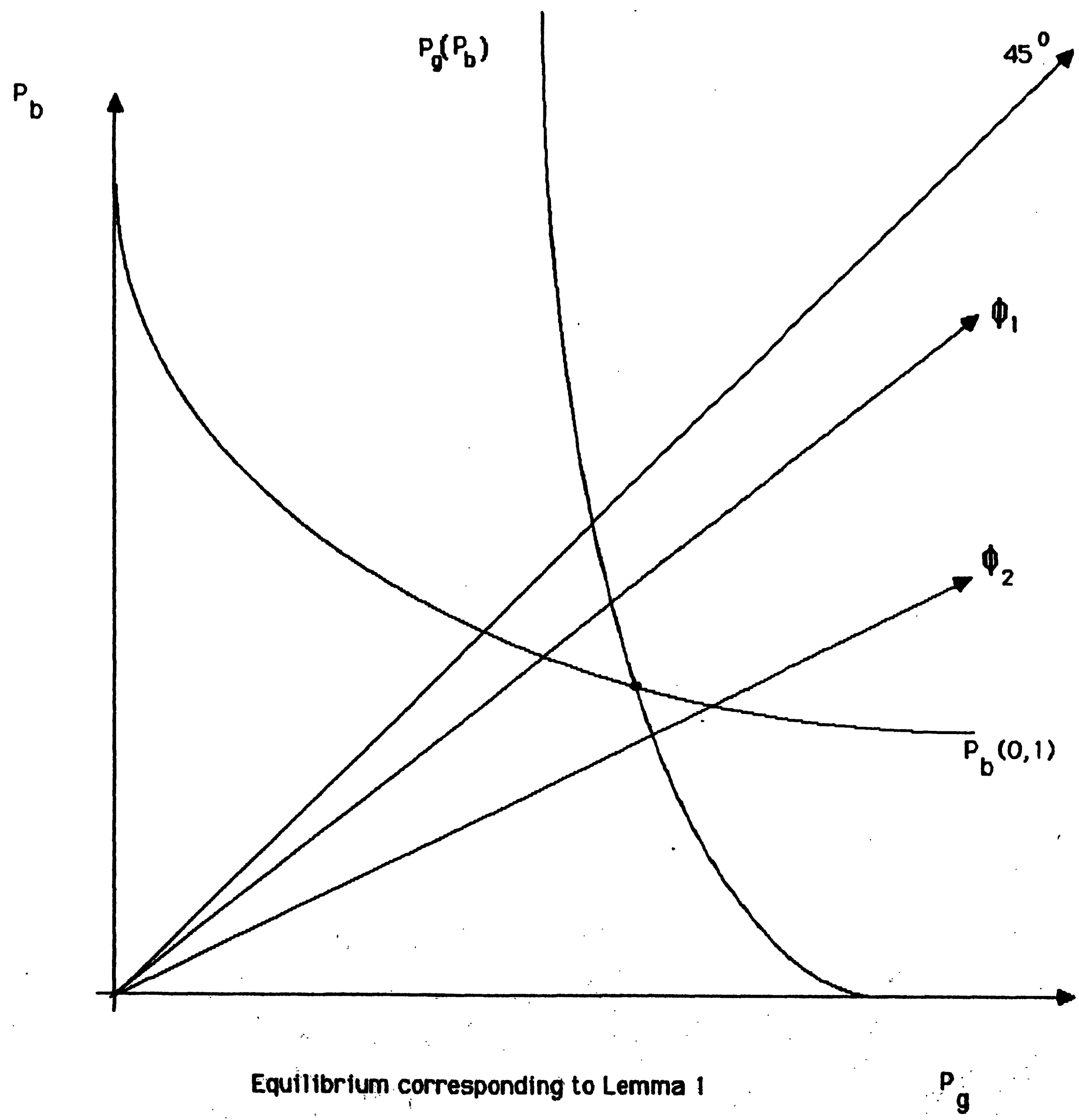
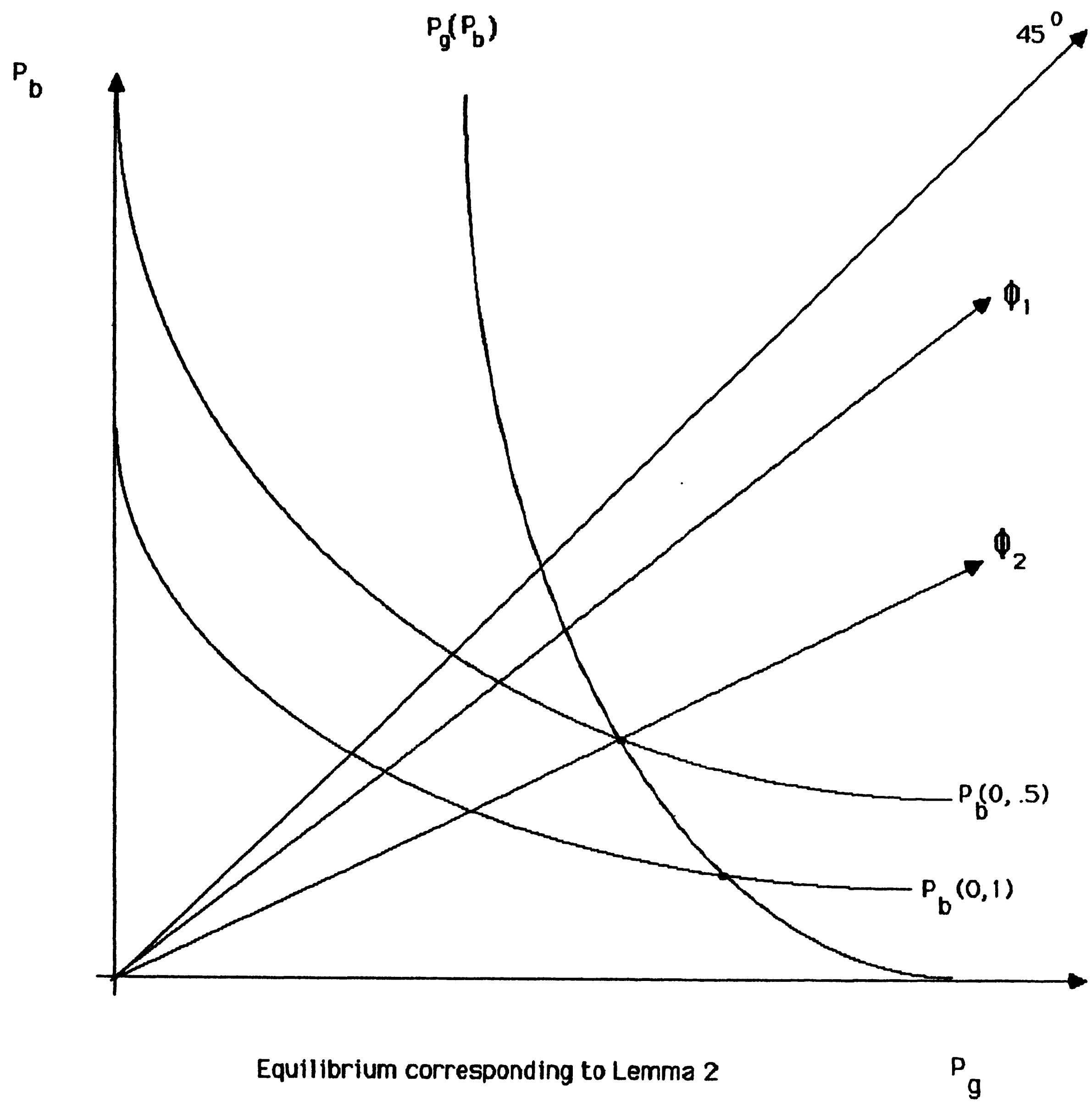
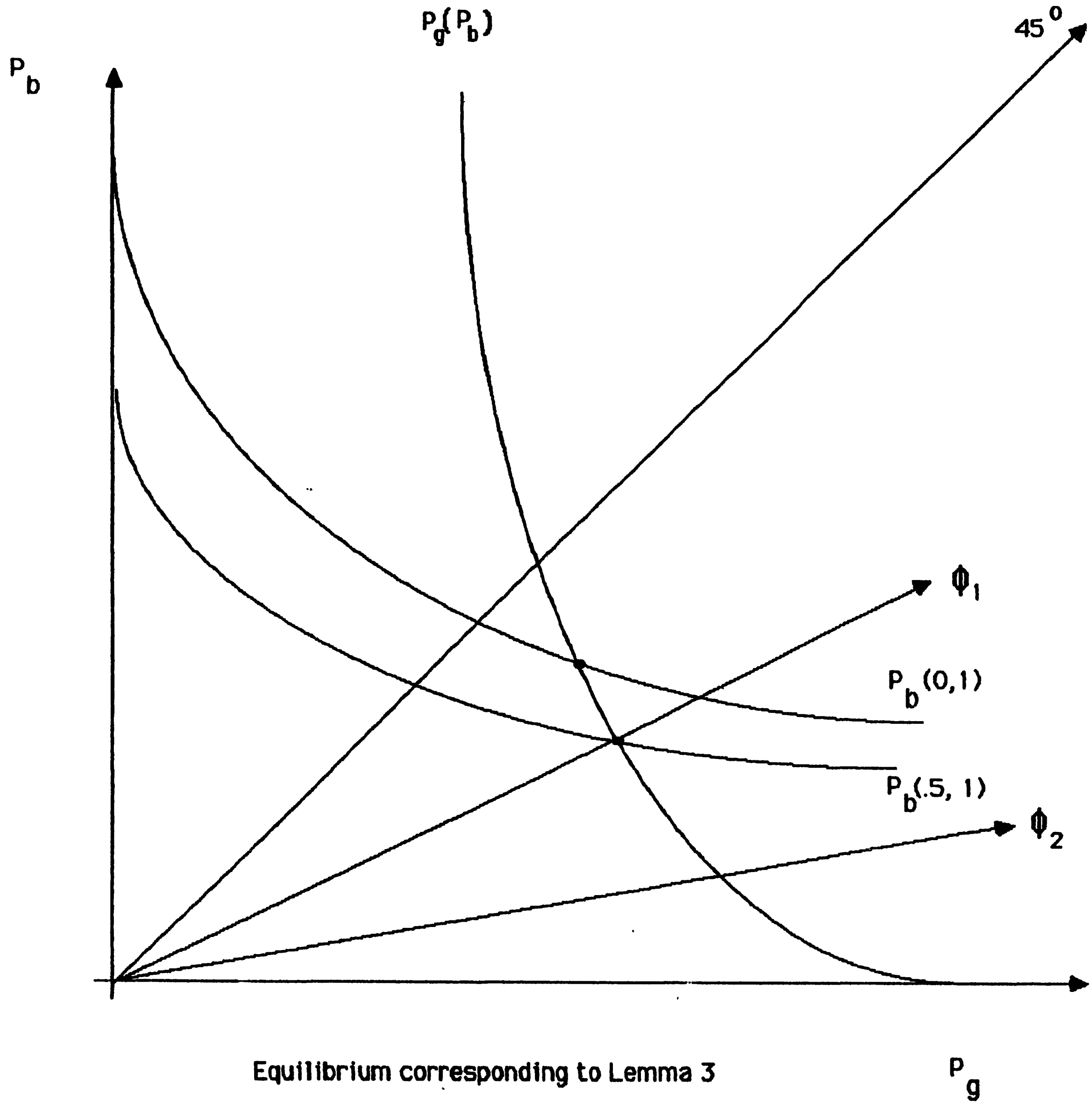


Equilibrium corresponding to Lemma 1



Equilibrium corresponding to Lemma 2



Equilibrium corresponding to Lemma 3

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