RESEARCH SEMINAR IN INTERNATIONAL ECONOMICS

Department of Economics
The University of Michigan
Ann Arbor, Michigan 48109-1220

SEMINAR DISCUSSION PAPER NO. 241

On the Likelihood of Factor Price Equalization with Nontraded Goods

by

Alan V. Deardorff

and

Paul N. Courant

The University of Michigan

August 23, 1989

Our thanks to Bill Ethier, Jim Levinsohn, Jim Melvin, Alasdair Smith, Bob Staiger, Bob Stern, Marie Thursby, and Ian Wooton for their helpful comments. Deardorff received partial support in writing this paper from the Ford Foundation.

ABSTRACT

On the Likelihood of Factor Price Equalization with Nontraded Goods

by

Alan V. Deardorff

and

Paul N. Courant

The University of Michigan

It is argued that nontraded goods reduce the likelihood of Factor Price Equalization (FPE). Specifically, the addition of nontraded goods to a small-open-economy Heckscher-Ohlin model with any number of goods reduces the size of the cone of diversification by the fraction of income spent on nontraded goods. This in turn may be regarded as reducing by a comparable amount the likelihood that a country's factor endowments will lie within that cone, and thus the likelihood of FPE.

Address correspondence to:

Alan V. Deardorff
Institute of Public Policy Studies
The University of Michigan
440 Lorch Hall
Ann Arbor, MI 48109-1220

August 23, 1989

On the Likelihood of Factor Price Equalization with Nontraded Goods

by

Alan V. Deardorff

and

Paul N. Courant

The University of Michigan

In the factor-proportions model of international trade, free trade equalizes factor prices in a pair of countries so long as factor endowments of those countries are sufficiently similar to permit nonspecialization. Formally the endowments must lie in the same "cone of diversification." If the model also includes nontraded goods that are produced using the same factors of production as the traded goods, then factor price equalization (FPE) implies that the prices of these nontraded goods are equalized as well. It is tempting to conclude, therefore, that the addition of nontraded goods to the factor proportions model does not interfere with the ability of international trade to eliminate differences between countries. We will argue, however, that the addition of nontraded goods to an otherwise conventional factor proportions model reduces the likelihood of FPE. Furthermore, using a particular method of measuring this likelihood, the size of the reduction is equal to the fraction of income spent on nontraded goods. This occurs because the presence of nontraded goods reduces the size of the cone of diversification by this fraction, and thus reduces the chance that two countries with arbitrary factor endowments will lie in the same cone.

Komiya (1967) was apparently the first to note that the tendency for FPE extends to equalization of the prices of nontraded goods, although he acknowledged that the result was implicit in Samuelson (1953). In a model of two traded goods and one nontraded good, the result requires that all three goods be produced, and Ethier (1972) pointed out that this can be a difficult condition to satisfy.[1] More recently, Helpman and Krugman (1985, pp. 19–22) derived precisely our result of a reduction in the likelihood of FPE due to nontraded goods in their analysis of a two-country model and its relationship to the integrated world economy. They did not however quantify the reduction in likelihood, as we do here; nor did they extend the result to more than two goods and factors. In addition, their comparison was slightly different from the one done here. They held the number of goods fixed and noted the effect of making one of them nontraded. We hold the number of traded goods fixed, and note the effect of adding additional, nontraded goods.[2]

There is no unique way of defining likelihood in this context. Our approach will be to take the technologies of producing goods and the tastes for consuming them as given, and then allow factor endowments to be, in some sense, randomly selected. Alternatively one could fix endowments and randomly select technologies, or even tastes, and we do not know what these approaches would imply about the likelihood of FPE.

We will also control for another consideration that might alter our result by assuming that countries share identical and homothetic preferences. Without this assumption countries might systematically prefer the nontraded goods produced with their own abundant factors, and this, it has been suggested, might lead to an increase in the likelihood of FPE.[3]

---

[1]Ethier (1971) came very close to the result of this paper in a geometric analysis of the two-traded good case, though then in Ethier (1972) he seems to have been diverted from it by a case where FPE appeared to be impossible.

[2]Since FPE requires that at least as many goods be traded as there are factors, it is obvious that reducing the number of traded goods sufficiently will undermine FPE. Helpman and Krugman showed, however, that the likelihood of FPE falls when traded goods become nontraded, even when the number of tradeds remains as large as the number of factors. We show here that the likelihood of FPE falls with the introduction of nontraded goods, even without any reduction in the number of traded goods.

[3]This was pointed out to us by Bill Ethier and will be explained further in a footnote below. Note however that by assuming identical preferences we do not contradict the common observation that countries do in fact consume disproportionately the nontraded goods that rely heavily on their abundant factors. This behavior could well arise with identical preferences due to the *absence* of FPE and the consequent effects on prices of nontraded goods. One could also imagine a model in which differences in demands with

Consider, then, a country that uses n factors, $v_1,...,v_n$, to produce n traded goods.[4] We make the usual assumptions needed for FPE, including perfect competition, the absence of factor intensity reversals, and that goods are produced under constant returns to scale.[5] Given the prices of the traded goods, there will therefore exist a unique vector[6] of factor prices $w = (w_1,...,w_n)'$ consistent with zero profit in all n industries. Let $v^1=(v_1^1,...,v_n^1)'$, ..., $v^n=(v_1^n,...,v_n^n)'$ be least-cost vectors of inputs for each industry, subject to these factor prices and corresponding to one dollar's worth of each of the outputs. Thus

$$w'v^i = 1 \quad , \quad i = 1,...,n. \tag{1}$$

In the absence of nontraded goods, these vectors of inputs define the diversification cone, which is the set of factor endowment vectors consistent with full employment of all factors, nonnegative outputs of all goods, and the use in all industries of only the techniques v^i. Thus the cone can be constructed as the set of all factor demand vectors, v, that arise from nonnegative value-of-production levels, x_i, in each industry. Restricting attention to those endowments worth one dollar, a cross-section of these endowments (for the case of tradables only) is therefore given by

$$E^T = \{v|w'v = 1, v = \sum_{i=1}^{n} x_i v^i, x_i \geq 0\} \ . \tag{2}$$

---

identical prices could result, not from differences in tastes, but rather from different past experiences of relative prices, as in Stigler and Becker's (1977) account of stability of tastes in changing environments.

[4]If there are fewer goods than factors, then FPE is not an issue. If there are more traded goods than factors, then our argument applies to the diversification cone corresponding to any n of them. See Vanek and Bertrand (1971) for the culmination of an old debate on the likelihood of FPE when goods outnumber factors.

[5]See Samuelson (1949), Ethier (1984), Deardorff (1986).

[6]Our vectors are column vectors except when transposed by a prime.

From (1) and (2) together we note that the x_i must sum to unity in order to get $w'v = 1$, and thus that E^T is simply the set of convex combinations of the vectors v^i:

$$E^T = \{v | v = \sum_{i=1}^{n} \lambda_i v^i, \lambda_i \geq 0, \sum_{i=1}^{n} \lambda_i = 1\} \ . \tag{3}$$

The set E^T is a subset of the unit-value simplex, i.e., the set of all vectors whose values at wages w sum to one. We will identify the likelihood of FPE with the probability that a country's endowment vector lies in the diversification cone, and thus that the normalization of its endowment vector to the unit-value simplex lies in the set E^T. Taking all endowments on the simplex as equally likely — a strong assumption that we relax below — we will therefore use the size of the diversification cone as our measure of the likelihood of FPE.

The meaning of size here depends upon the number of factors. The two factor case is illustrated in Figure 1. The unit-value simplex here is the line AF. With v^1 and v^2 as the cost-minimizing techniques in the two industries, the set E^T is the line segment BE. In this case the size of the set is given by the length of this segment, and the likelihood of FPE is the ratio BE/AF.

With three factors the simplex becomes a triangle, as does the intersection of the diversification cone with the simplex, and the likelihood of FPE is the ratio of the areas of these two triangles. In still higher dimensions, both sets become tetrahedrons, and then the higher dimensional analogues of tetrahedrons, and the likelihood becomes the ratio of their volumes.

Now consider adding nontraded goods to the model. The factor price vector w implies prices of both traded and nontraded goods, and therefore a quantity of nontraded goods demanded for each dollar of the country's income.[7] Let the factors used to produce

[7]It is not necessary for this construction that preferences be homothetic so that all increments to income be spent the same. Since national income will remain fixed in the analysis of a single country, this bundle of nontraded goods can be the country's total

this bundle of nontraded goods at factor prices w be given by a vector v^N. Then for each dollar's worth of the country's endowment the vector v^N must be employed in producing nontradeds. The range of factor endowments consistent with diversification among traded goods is therefore altered.

Specifically, let E^N be the set of factor endowments on the unit-value simplex consistent with FPE in the presence of nontraded goods. These endowments must have the property that what is left after production of nontraded goods, $v-v^N$, can be allocated across traded-good industries to achieve full employment. That is,

$$E^N = \{v|w'v = 1, v = v^N + \sum_{i=1}^{n} x_i v^i, x_i \geq 0\} \ . \tag{4}$$

This set is shown in Figure 1 as the segment CD, which has been constructed by drawing two lines from the tip of the v^N vector parallel to v^1 and v^2. The figure shows clearly, for the case drawn, how the presence of nontraded goods has shrunk the diversification cone from HOK to IOJ and reduced the likelihood of FPE from BE/AF to CD/AF.

To see the same result for the general case, let

$$\theta = 1 - w'v^N \ . \tag{5}$$

We now show that

$$\begin{aligned} E^N &= \{v|v = v^N + \theta v^T, v^T \epsilon E^T\} \\ &= v^N + \theta E^T \ , \end{aligned} \tag{6}$$

which is the main result of the paper. To see it, consider any $v^0 \epsilon E^N$. From (4)

$$w'v^0 = 1 \ , \tag{7}$$

and there exists some vector $x^0 \geq 0$ such that

---

demand for nontradeds, simply divided by the value of national income. However, we do find it useful to assume homotheticity below in order to make this demand for nontraded goods independent of the distribution of factor endowments.

$$v^0 = v^N + \sum_{i=1}^{n} x_i^0 v^i \ . \tag{8}$$

Let $y^0 = x^0/\theta$. Then rearranging (8) and dividing by θ,

$$\frac{v^0 - v^N}{\theta} = \sum_{i=1}^{n} y_i^0 v^i \ . \tag{9}$$

Multiplying both sides by w, this becomes

$$\frac{w'v^0 - w'v^N}{\theta} = \sum_{i=1}^{n} y_i^0 w'v^i \ , \tag{10}$$

in which the left-hand side, by (7) and (5), is one, while the right-hand side, by (1), is the sum of the y_i^0's. Thus

$$\sum_{i=1}^{n} y_i^0 = 1 \ . \tag{11}$$

It therefore follows from (3) and (9) that

$$v^{T^0} \equiv \frac{v^0 - v^N}{\theta} \epsilon \, E^T \tag{12}$$

or that

$$v^0 = v^N + \theta v^{T^0} \text{ where } v^{T^0} \epsilon E^T. \tag{13}$$

This proves (6).

Equation (6) says that the set E^N of normalized endowments consistent with FPE is identical to the set E^T, except that it is scaled down by multiplying by the fraction θ and displaced by the vector v^N. Therefore the size of E^N is θ times the size of E^T, and the likelihood of FPE is also multiplied by the fraction θ when nontraded goods are introduced. From (5) it then follows that the likelihood of FPE is reduced by the presence of nontraded goods, and that the size of this reduction is the fraction of income that would be spent on nontradeds at the prices implied by FPE.

A Caveat

Before concluding we should note the importance of the assumption that all factor endowments on the unit-value simplex are equally likely. This has made possible the strong result of being able to quantify the reduction in likelihood of FPE, and in some exceptional circumstances it has been needed to get that likelihood to reduce at all.

Suppose, then, instead of all factor endowments being equally likely, that there were a probability density function (pdf) for factor endowments defined over the unit-value simplex.[8] And suppose in addition that the vector v^N lay outside of the cone of E^T. As illustrated in Figure 2, the interval of factor endowments consistent with FPE would still be reduced in length, so that our earlier analysis would still apply if all factor endowments were equally likely. However, this interval would also be shifted to include endowments that were not a part of E^T. If the probability density of factor endowments happened to be heavily concentrated in the segment CD, outside of E^T but inside E^N, then the probability of FPE could rise, not fall, with the introduction of nontraded goods.[9]

This possibility arises, however, only if v^N lies outside the cone of E^T. If instead the factor intensity of nontraded goods is intermediate between those of traded goods, as in Figure 1, then the probability of FPE may not fall by the exact amount $(1-\theta)$, but it will

[8]The unit-value simplex depends on world prices which in turn depend on factor endowments in the world economy. Thus this pdf is not independent of endowments elsewhere.

[9]A variant of this possibility was suggested by Bill Ethier if preferences for nontraded goods are not identical across countries. If two countries systematically prefer those nontraded goods that rely heavily on their own abundant factors, then countries whose endowments lay on opposite sides of the cone OBE in Figure 2 might also use factors in nontraded-goods production on opposite sides of the cone as well. Thus they could each shift the region of nonspecialization but in opposite directions, approaching their respective factor endowments, and perhaps making FPE more likely.

surely fall, not rise, as long as there is any probability density at all for those portions of E^T that are not in E^N. For it is easily seen that[10]

$$\frac{v^N}{1-\theta} \epsilon E^T \Rightarrow E^N \subset E^T . \tag{16}$$

---

[10]Suppose $v^0 \epsilon E^N$. Then from (6), $v^0 = v^N + \theta v^{T^0}$ for some $v^{T^0} \epsilon E^T$. Thus

$$v^0 = (1-\theta)\frac{v^N}{1-\theta} + \theta v^{T^0} ,$$

and, since E^T is convex from (3), it follows that $v^0 \epsilon E^T$.

References

Deardorff, Alan V. (1986) "FIRless FIRwoes: How Preferences Can Interfere with the Theorems of International Trade," *Journal of International Economics*, 20 (February), pp. 131–142.

Ethier, Wilfred J. (1971) "The Two Sector Neoclassical Production Model: A Geometric Note and an Extension to Nontraded Goods," *Australian Economic Papers* 13 (December), pp. 188–195.

Ethier, Wilfred J. (1972) "Nontraded Goods and the Heckscher-Ohlin Model," *International Economic Review* 13 (February), pp. 132–147.

Ethier, Wilfred J. (1984) "Higher Dimensional Issues in Trade Theory," in Ronald W. Jones and Peter B. Kenen (eds.), *Handbook of International Economics*, Vol. I (North-Holland, Amsterdam), pp. 131–184.

Helpman, Elhanan and Paul R. Krugman (1985) *Market Structure and Foreign Trade: Increasing Returns, Imperfect Competition, and the International Economy* (MIT Press, Cambridge, Mass.).

Komiya, Ryutaro (1967) "Non-Traded Goods and the Pure Theory of International Trade," *International Economic Review* 8 (June), pp. 132–152.

Samuelson, Paul A. (1949) "International Factor-Price Equalization Once Again," *Economic Journal* 59 (June) pp. 181–197.

Samuelson, Paul A. (1953) "Prices of Factors and Goods in General Equilibrium," *Review of Economic Studies* 21 (October), pp. 1–20.

Stigler, George J. and Gary S. Becker (1977) "De Gustibus Non Est Disputandum," *American Economic Review* 67 (March), pp. 76–90.

Vanek, Jaroslav and Trent Bertrand (1971) "Trade and Factor Prices in a Multi-Commodity World," in J. N. Bhagwati, R. W. Jones, R. A. Mundell, and J. Vanek, eds., *Trade, Balance of Payments and Growth* (North-Holland, Amsterdam).

Deardorff & Courant
#241

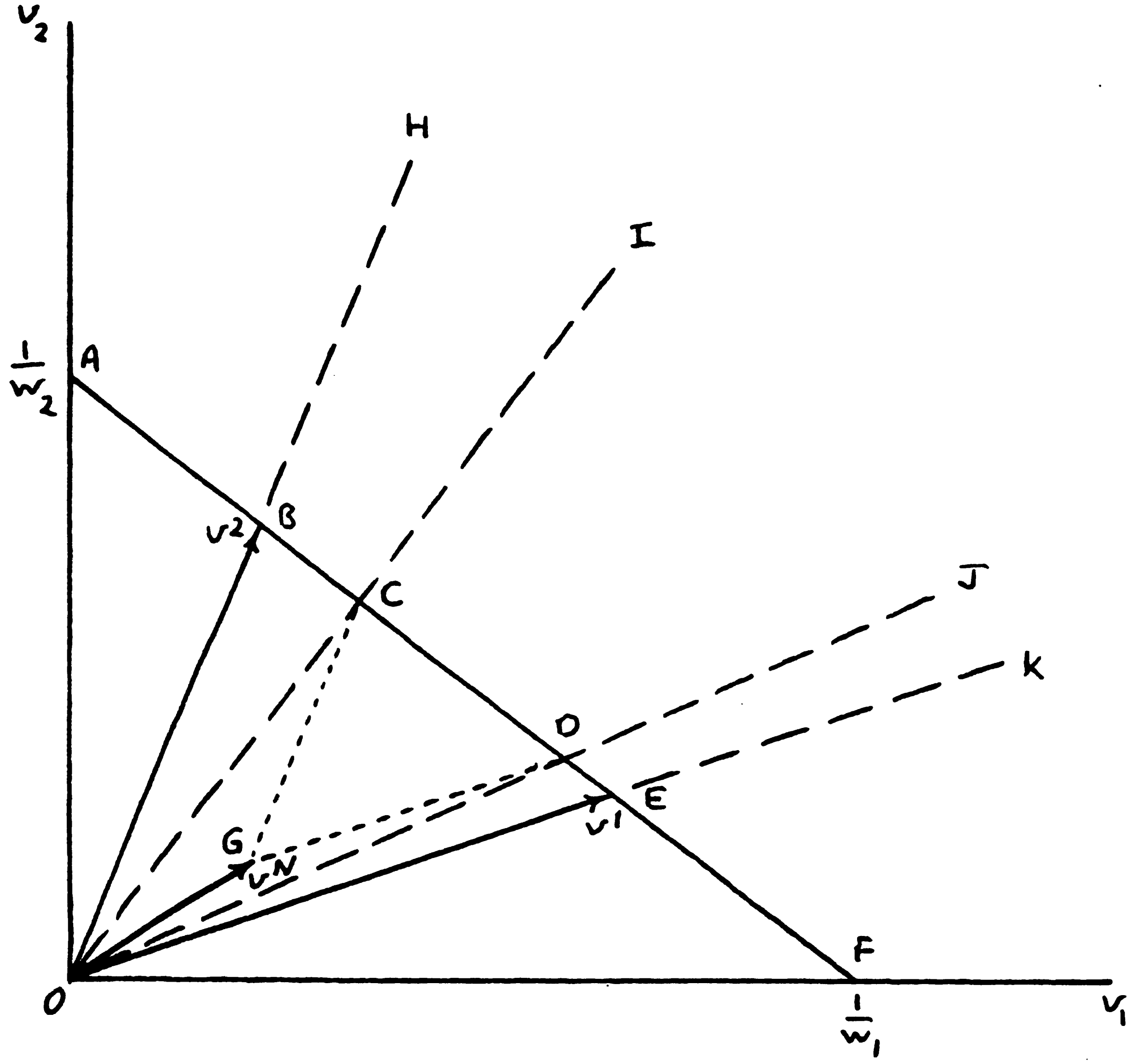

Figure 1

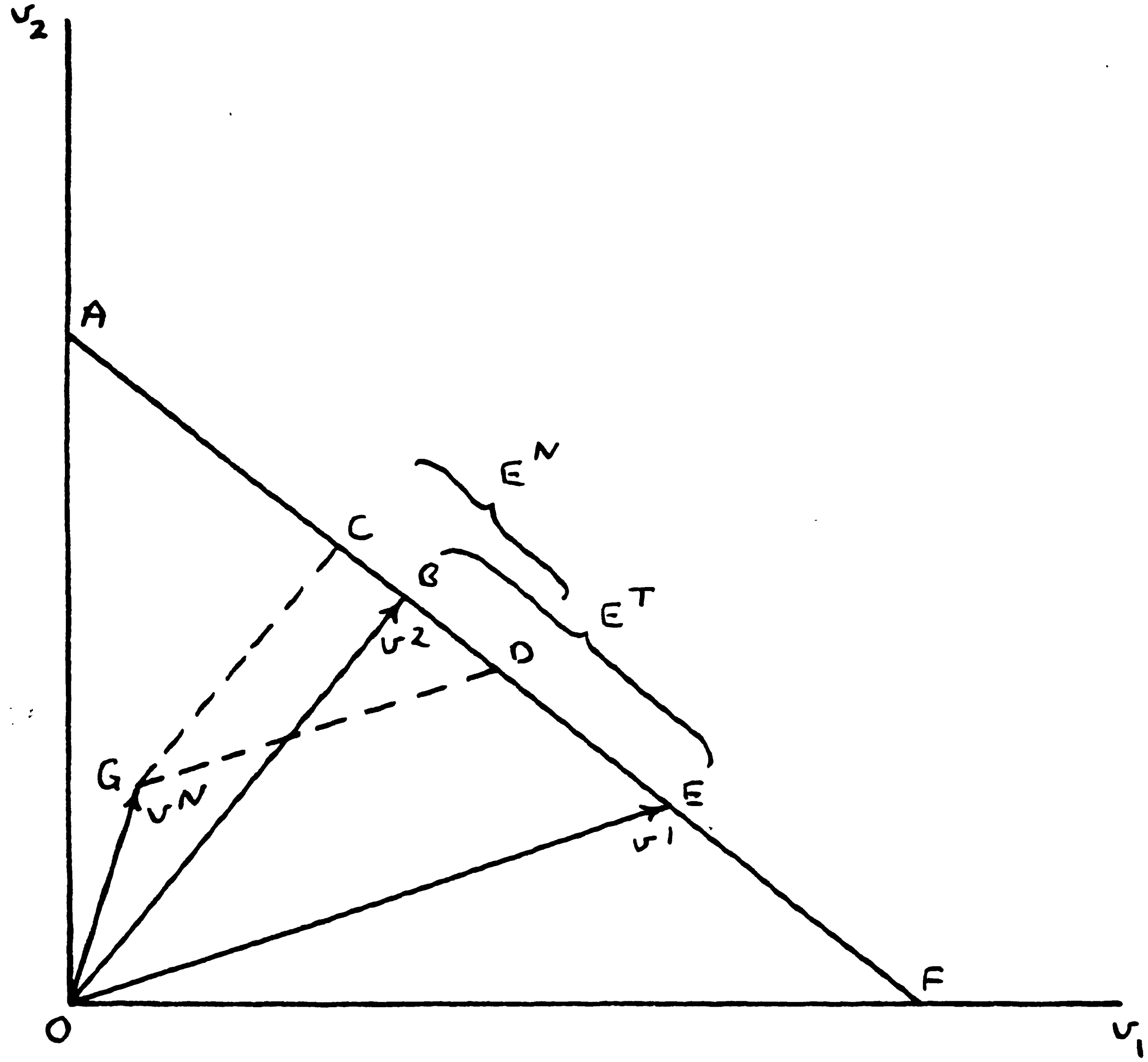

Figure 2