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Abstract

We examine the welfare effects of price limits and trading halts in a linear mean-variance model of stock and futures trading. We suppose that shocks to liquidity and fundamentals occur between the time investors decide to trade and the time their orders are executed and cannot be insured with contingent claims. This gives rise to *implementation risk* that is not transferred optimally among investors. Price limits (or price contingent trading halts) serve to partially insure implementation risk.

We show that when price fluctuations are due solely to news about fundamentals, judiciously chosen price limits are (ex ante) pareto superior to unconstrained trade. When liquidity shocks are large, then some speculators lose, but hedgers still benefit from price limits.

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I Introduction

The large decline in stock and stock index futures markets on October 19 and 20, 1987, and a smaller, but equally startling, decline two years later have fueled cries that financial markets have become too volatile. One solution to problems evoked by periods of extreme volatility, proposed originally by the Brady Commission and later reiterated by the Working Group on Financial Markets, is coordinated "circuit breakers." Circuit breaker mechanisms, as envisaged by the Brady Commission, "cushion the impact of market movements, which would otherwise damage market infrastructure. They protect markets and investors."[1] Stock and futures exchanges have adopted several types of circuit breakers on an experimental basis; among them are price limits and trading halts.[2]

Free market advocates question how regulations that impede price adjustment can improve market performance, especially in asset markets, which are thought to be highly competitive. They argue that price limits and trading halts prevent mutually advantageous trades that would occur voluntarily and that limited or "stale" prices fail to convey information to investors. As a result, they feel circuit breakers can only make investors worse off.

These arguments are not compelling. While circuit breakers do prevent mutually advantageous trades, it does not follow that investors are better off without them. In fact, history suggests the opposite might be true. Futures exchanges have long employed daily price limits to constrain commodity futures price movements. The New York Stock Exchange hires specialists, who limit price movements by absorbing the difference between external supply and demand, to "maintain price continuity." To the extent that these exchanges serve the interests of their investing clientele, it would seem that investors prefer limited trade over unconstrained trade.[3]

Despite this long history of self regulation, current advocates of stronger circuit breakers have only recently confronted the inefficiency arguments of their opponents.[4] The usual

[1]Report of the Presidential Task Force on Market Mechanisms, January 1988, p. 66.

[2]Price limits are upper and lower bounds placed on price movements over some specified period of time. Trading halts prohibit trade for some specified period of time after the price has moved some specified amount.

[3]There are at least two reasons why exchanges are likely to adopt policies that increase the value of investing. One reason is to increase the number of investors and hence the demand for exchange services. Another is to increase the amount exchange members can collect from investors through the non-linear pricing of brokerage services.

[4]One benefit provided by circuit breakers outlined by the Brady Commission is that they limit credit risk and loss of financial confidence by providing a "time-out" during which participants can settle up and ensure the solvency of other players. The idea that price limits, in conjunction with margin requirements, help ensure contract performance has long been a rationale for price limits in the futures markets. Brennan (1986) formalized this notion. Greenwald and Stein (1989) address how "time-outs" reduce transactional risk by allowing traders to become better informed about market liquidity. We contrast our model with that of Greenwald and Stein below.

rejoinder is that without coherent, coordinated circuit breakers, a disorderly market will create its own circuit breakers in the form of clogged order delivery systems, ad hoc trading halts, jammed communication systems, and unresponsive specialists, locals, and market makers. Circuit breakers are seen as faciliting more orderly exchange during periods of peak demand for exchange services.

This paper incorporates price limits and trading halts into a standard model of asset pricing and considers whether these circuit breaker mechanisms increase efficiency. We show that when the risk of price movements exists between the time the investor decides to trade and the time the order is executed, i.e. during order implementation, circuit breakers may provide for more efficient risk sharing among investors than unconstrained trade. The risk that payoff relevant shocks (to liquidity or fundamentals) occur after an investor decides to place an order but before the order is executed is exacerbated in periods of market stress and is the type of risk that circuit breakers are designed to address.

Today's markets do not efficiently transfer this risk for two reasons. First, organized markets for insurance may fail to exist.[5] Second, insurance markets may be subject to the same implementation risks as the underlying assets for which they are providing insurance.[6] Consequently insurance orders may become obsolete by the time they are executed.[7] The role of circuit breakers in our model is to partially insure implementation risks that are not optimally transferred due to the absence of a complete set of markets for contingent claims.

We consider a group of investors wishing to trade futures contracts either to hedge their endowments of an underlying asset or for purely speculative reasons.[8] To keep the model simple, we assume that investors have constant absolute risk aversion and that returns are normally distributed. For the very reason the futures market exists (i.e. to transfer risk among heterogeneous investors), shocks to fundamentals and liquidity that occur during order implementation affect different investors in different ways. This fact gives rise to potential gains from sharing implementation risk that are not exploited when contingent claims are absent.

We first consider situations in which futures price movements are generated solely by news about fundamentals. "Good news" about the fundamental value of the underlying

[5]For example, an investor who decides after Friday's close to take an agricultural futures position cannot insure against drought information released on Sunday.

[6]For example, trading a stock option, which may be used to insure against fluctuations in the value of a cash market position, is subject to the same implementation delays during volatile times as trading the underlying asset.

[7]Hypothetically, investors who incur no delays in placing orders but who face delays in having them executed could eliminate implementation risk by instructing their brokers how to change their orders under every possible contingency. But as Miller (1990) points out, this would be "impossibly costly under present or any likely foreseeable technology"(p. 11). Furthermore, investors do incur delays in placing their orders during volatile times. For example, phone lines jammed during the October 1987 crash, so that investors could not place their desired orders before additional information hit the market.

[8]Our model is simple enough to be reinterpreted as a stock market.

asset increases the endowment value of those hedgers who have positive endowments in the underlying asset and hedge by selling short. Since good news is associated with a high futures price, hedgers who sell futures contracts are better off when the futures price is high than when it is low. On the other hand, good news about fundamentals reduces the endowment value of those hedgers who have negative endowments and hedge by taking long futures positions. Thus hedgers who buy futures contracts are better off when the futures price is low. If contingent claims were feasible, pareto optimal insurance would transfer wealth from sellers to buyers when price is high and from buyers to sellers when price is low. But as noted above, contingent claims may well be infeasible.

Consider, as an alternative to contingent claims, price limits (or price contingent trading halts) that place upper and lower bounds on the price at which investors are allowed to trade. By artificially reducing price when the true (Walrasian) price is high, and raising price when the true price is low, price limits transfer wealth from sellers to buyers when price is high and from buyers to sellers when price is low. These transfers move in the direction of welfare enhancing insurance. While the price limits involve some costs, in the form of rationed buyers at the upper limited price and rationed sellers at the lower limited price, the dead weight loss from rationing is of second order importance when the limits are set wide enough. We show that when all investors are hedgers, price limits can be chosen to yield (ex ante) pareto improving insurance. If some investors are speculators, then price limits are pareto improving provided that investors have mean-variance utility with constant absolute risk aversion.

We then consider the case in which price fluctuations are generated by liquidity shocks as well as news about fundamentals. When liquidity shocks are small relative to investors' endowments, such that each investor trades on the same side of the market (i.e. each investor is either always long or always short) regardless of the value of the shocks, pareto superior price limits exist. Alternatively, when liquidity shocks are large, price limits still benefit hedgers, but they are no longer pareto improving. In this case some speculators will take short positions when price is high and long positions when price is low. Price limits force them to sell at lower prices when the true price is high and to buy at higher prices when the true price is low. This reduces the speculator's wealth when price is both high and low and therefore reduces its ex ante expected utility.

It is important to bear in mind that while price limits may increase ex ante welfare, in our framework they necessarily reduce ex post efficiency. This is because impediments to price adjustment after information is released prevent some ex post gains from trade. Thus, although it may be optimal to commit to price limits in advance, those market participants who are constrained ex post may complain bitterly. In this sense price limits represent "sand in the gears" of the Walrasian mechanism in the ex post asset market, but they may nevertheless improve ex ante efficiency.

In a recent paper, Greenwald and Stein (1989) also examine the usefulness of circuit breakers. They focus on imperfections in the transaction mechanisms for executing orders and examine the risk that investors' face between the time they have submitted an order and its execution. In particular, they suppose that investors are uncertain as to the number of other investors who will want to trade at the same time. They argue that to reduce transactional risk and the associated information externalities that arise when investors cannot anticipate the arrival of other investors, circuit breakers can provide a "time out" to allow traders to become better informed about the response of other traders to large volume shocks. They argue that circuit breakers may improve the Walrasian pricing mechanism in the ex post asset market, after volume shocks have been realized, and thus should not necessarily be interpreted as "sand in the gears."

Our approach differs from Greenwald and Stein's in two main respects. First, we evaluate the welfare effects of price limits by evaluating investors' expected utility at the time they decide to trade but before they know whether a price limit will be hit. Thus we address the question of whether investors would prefer ex ante to trade in a market with price limits or in one in which price adjustments are unconstrained.

Second, the source of the inefficiency in our model is the absence of a complete set of contingent claims to insure against shocks that may occur during order implementation, not information asymmetries among investors. Price limits do represent sand in the gears of the Walrasian mechanism in the ex post asset market, but they transfer risk more efficiently from an ex ante point of view.

The paper is organized as follows. Section II sets up the model and discusses the implementation risks that are the motivation for circuit breakers in our paper. In section III, we characterize asset market equilibrium when trade is unconstrained and identify the role for insurance in the context of implementation risk. Section IV incorporates price limits and derives necessary and sufficient conditions for limits to increase welfare. Section V discusses the implications, some extensions, and some caveats of our analysis. Section VI concludes the paper.

II A Model of Implementation Risk

A critical feature of asset markets that has received little attention in the theoretical literature, but which is the main focus of our paper, is the delay incurred in carrying a trade through to completion. While short delays are relatively innocuous when prices are stable, they have important consequences when prices become volatile. Between the time an investor decides to place an order and the time it is actually executed, new information may

be released concerning the future value of the asset or the volume of trading activity at the time the order is executed. This information affects the utility investors expect to receive by participating in the market and thus introduces risk. Inefficiency arises when the potential for transferring this risk among investors is not fully exploited. Before proceeding to the formal model, we explore the nature of these *implementation risks* and motivate why they may remain uninsured.

We distinguish two types of implementation risk associated with delays. First, *transactional risk* concerns shocks to volume or fundamentals that lead to price adjustments between the time orders are placed and the time they are executed. Second, *initiation risk* concerns shocks to volume or fundamentals that lead to price adjustments between the time an investor decides to place an order and the time the order is placed. There are several reasons why both risks are present, especially in volatile markets.

The most obvious reason is market closure. An investor endowed with some asset may decide to place a new order on Saturday, based on the morning news, but not be able to place the order or have it executed until Monday. In the meantime, information may be released on Sunday that changes the value of the investor's endowment and the value of trading on Monday. Typically, investors could increase their ex ante expected utilities (as of Saturday) by trading insurance claims that would make transfers on Sunday contingent on any new information. Absent a market on Saturday for such claims, gains from trade cannot be fully exploited.

If market closure were the only source of transactional and initiation risks, then both risks could be avoided with 24 hour trading. However, even when markets are open continuously, initiation risk is still a factor for at least two reasons. First, some investors find it too costly to stay close enough to their phones or computer terminals to be able to place orders the instant information is released. Hence costs of monitoring the market can give rise to delays that lead to initiation risk. Second, current technology does not allow orders to be placed instantaneously in volatile markets, regardless of how close investors are to the market. For example, during the October 1987 decline, phone lines jammed as too many investors attempted to place orders at once. Such technological limitations give rise to transactional risk as well as initiation risk. For example, on October 19, 1987 some orders were successfully relayed to the floor of the Chicago Mercantile Exchange but became obsolete before being executed in the futures pit.

Initiation risk may be present even when it appears that markets are complete. Suppose insurance claims were available, in principle allowing investors to establish ex ante pareto optimal positions. Since buying insurance (e.g. an option) is subject to the same initiation delays as buying or selling an asset, conditions may change before the insurance position is established. Therefore, when initiation risk is present in every market, including insurance markets, it is not insurable via contingent claims.

We now formalize these ideas. Consider a three period futures market with risk averse investors, as illustrated in figure 1. At time 0, which we refer to as *ex ante*, investors contemplate allocating their portfolios between a risk free asset and a risky futures contract that will yield a random return. Between time 0 and time 1, information is released that gives rise to initiation risk, which we formalize shortly. At time 1 investors submit limit orders (in the sense of their entire demand schedules) to maximize their expected utility of terminal wealth. These orders are executed at the competitive equilibrium price or limited futures price without delay. Thus we ignore transactional risk in our formal presentation in order to focus on initiation risk. We return to transactional risk in section V. At time 2 the (random) terminal spot market price V of the underlying asset is determined, and the futures contract expires. We refer to times 1 and 2 as the *ex post* asset market.

We now present the information structure of our model and the formalization of initiation risk. At time 0, when investors contemplate taking futures positions, their common expectation of the terminal spot market price of the underlying asset is $\bar{\nu}$, and investor i's expectation of its terminal endowment is $\bar{\nu}\theta_i$. The (fixed) parameter θ_i can be interpreted as the cash market position that investor i wishes to hedge (where a positive value of θ_i indicates a long position and a negative value indicates a short position). For example, a producer of grain has a positive endowment while a firm that uses grain in production has a negative endowment. Alternatively, θ_i may simply reflect how investor i's future profit or wealth depends on the future value of the underlying asset.[9]

Between time 0 and time 1, news about fundamentals is released that induces investors to update their expectations of the terminal spot price to $n+\bar{\nu}$ and their terminal endowments to $(n+\bar{\nu})\theta_i$. This news is uninsured and represents a source of initiation risk. For simplicity we assume that $n \in \{-\epsilon, 0, \epsilon\}$ $(\epsilon > 0)$ and that positive and negative shocks to fundamentals each occur with equal probability β_n.[10]

In addition to uncertainty regarding news releases, investors are also uncertain as to the volume of trading activity at time 1. We model this additional source of initiation risk as a random liquidity shock $s \in \{-\mu, 0, \mu\}$ $(\mu \geq 0)$ between time 0 and time 1 where s is the number of short positions taken by liquidity traders. We assume that positive and negative supply shocks occur with equal probability β_s. For simplicity we assume that s and n are independently distributed.

To close the model, we assume that at time 2 the terminal spot price is determined as $V = n+\nu$, where $E(\nu|s,n) = \bar{\nu}$. Since the futures and spot prices are equal at contract expiration,

[9] Under this interpretation the linear dependence of the value of the endowment on ν is restrictive, but this assumption allows us to generate our main insights within a tractable model.

[10] Assuming a continuous distribution for n results in the same insights but adds unneeded complexity to the model.

the futures contract expires at the price V. Finally, we make assumptions that justify the linear mean-variance certainty equivalence approach. We assume that conditional on n and s, ν is normally distributed with variance σ_ν^2, and investor i has a negative exponential utility function $U_i(W_i) = -e^{-\gamma_i W_i}$ where W_i is investor i's terminal wealth and γ_i is its coefficient of absolute risk aversion.

III No Circuit Breakers

In this section we begin by characterizing the futures market equilibrium in the absence of circuit breakers. We then consider the extent to which risk sharing opportunities are left unexploited in the presence of initiation risk.

To facilitate the evaluation of investors' ex ante expected utility, it proves convenient to examine their certainty equivalent values of participating in the market at time 1. Investor i's certainty equivalent at time 1 is the amount of money the investor would accept at time 1 for his endowment and the ability to trade at time 1. Since this is an increasing function of expected utility at time 1, choosing a futures position to maximize the certainty equivalent is the same as choosing a position to maximize expected utility.

Under the assumption of normal returns and negative exponential utility, investor i's certainty equivalent is given by

$$C_i(x_i, P, n) = E_\nu\left(W_i(x_i, P, n, \nu)\right) - \frac{\gamma_i}{2} Var_\nu\left(W_i(x_i, P, n, \nu)\right)$$

where P is the futures price, $W_i(x_i, P, n, \nu) = P\theta_i + (n+\nu-P)(\theta_i + x_i)$ is i's terminal wealth, and E_ν and Var_ν are the expectation and variance operators with respect to the distribution of ν.[11] Choosing a futures position, x_i, to maximize C_i yields an optimal position for investor i of[12]

$$x_i(P, n) = \frac{n + \bar{\nu} - \sigma_\nu^2 \gamma_i \theta_i - P}{\gamma_i \sigma_\nu^2}.$$

Setting aggregate net futures demand equal to zero, $\sum_i x_i(P,n) - s = 0$, yields the competitive equilibrium futures price

$$P^*(s, n) = n + \bar{\nu} - \sigma_\nu^2 A(s) \tag{1}$$

where

$$A(s) = \frac{\sum_i \theta_i + s}{\sum_i \frac{1}{\gamma_i}}.$$

[11] This model can be interpreted as a simple stock market by assuming that the cash market supply is $s_c = 1 + s$ (where expected supply is normalized to 1) and x_i is the additional amount of stock purchased by investor i.

[12] We assume that investors' endowments in the risk free security are large enough that interior solutions characterize their optimal futures positions.

Substituting P^* into the equations for x_i and C_i yields investor i's competitive equilibrium position

$$x_i^*(s) = \frac{A(s)}{\gamma_i} - \theta_i \qquad (2)$$

and its equilibrium certainty equivalent

$$C_i^*(s,n) = \left(n + \bar{\nu} - \sigma_\nu^2 A(s)\right)\theta_i + \frac{\sigma_\nu^2 A(s)^2}{2\gamma_i}. \qquad (3)$$

At time 0 investor i cares about the expected utility of $C_i^*(s,n)$ over all realizations of the liquidity shock s and the news term n. These shocks affect the certainty equivalent through their effects on the value of the endowment and the expected value of the equilibrium position. The first term in equation (3) is the value of investor i's endowment as of time 1. The second term is the value of investor i's equilibrium position.

The effect of news about fundamentals on the value of investor i's endowment depends on whether the endowment is positive or negative. For example, an increase in the expected cash price (n positive) would reduce the certainty equivalent if the endowment were negative. However, news about fundamentals does not affect the value of investor i's equilibrium position. This is a result of the fact that investor i's net futures demand is a function of $n-P$. Therefore, the value $n - P$ is determined in equilibrium as a function of s. Consequently, the equilibrium price adjusts one-for-one to reflect fully any news about fundamentals, and investor i's position is unaffected by the news.

Fluctuations in liquidity affect investor i's certainty equivalent by altering both the value of the endowment and the value of the equilibrium position. Differentiating C_i^* with respect to s yields $\partial C_i^*/\partial s = A'(s)\sigma_\nu^2[(A(s)/\gamma_i) - \theta_i] = \sigma_\nu^2 x_i^*(s)/\sum_i(1/\gamma_i)$, which is positive if and only if $x_i^*(s)$ is positive. Hence, an increase in supply benefits investors who take long positions, since they can buy at a lower price, but hurts sellers who must sell at a lower price.

We now examine the extent to which risk sharing possibilities concerning s and n are left unexploited. Pareto optimal risk sharing would entail state contingent transfers, $T_j(s,n)$ (which may be positive or negative), from each investor $j \neq i$ to investor i so as to equate marginal rates of substitution across states of nature. That is, transfers would be chosen so that for all $j \neq i$, s, s', n, and n'

$$\frac{U_i'\left(C_i^*(s,n) + \sum_{k\neq i} T_k(s,n)\right)}{U_i'\left(C_i^*(s',n') + \sum_{k\neq i} T_k(s',n')\right)} = \frac{U_j'\left(C_j^*(s,n) - T_j(s,n)\right)}{U_j'\left(C_j^*(s',n') - T_j(s',n')\right)}$$

where $U_i' = \partial U_i/\partial C_i$. Using the negative exponential utility function, substituting for C_i^* and C_j^* from equation (3), and rearranging, this expression becomes

$$\begin{aligned}(\gamma_i\theta_i - \gamma_j\theta_j)[n' - n + \sigma_\nu^2(A(s) - A(s'))] &= (\gamma_i + \gamma_j)[T_j(s,n) - T_j(s',n')] \\ &+ \gamma_i\left(\sum_{k\neq i,j}[T_k(s,n) - T_k(s',n')]\right). \qquad (4)\end{aligned}$$

Equation (4) is the necessary and sufficient condition for pareto optimal sharing of initiation risk.

Observe from (4) that if state contingent transfers are infeasible ($T_k = 0\ \forall k$), then risk sharing is pareto optimal if and only if $\gamma_i\theta_i = \gamma_j\theta_j\ \forall j \neq i$. But using the expression for the value of investors' equilibrium positions in equation (2), this would imply that $\gamma_i x_i^* = \gamma_j x_j^*$, or that every investor would want to take a position on the same side of the futures market. If this were true, the futures market would cease to exist. We therefore have the following proposition.

Proposition 1 *Suppose an active futures market exists. Then if ex post transfers cannot be conditioned on shocks to liquidity and fundamentals that occur during order initiation, there are potential (ex ante) gains from sharing initiation risk that are not exploited.*

Intuitively, if there are gains from transferring risk among heterogeneous investors in the futures market, then there also will be potential gains from transferring initiation risk.

IV Circuit Breakers

When markets for contingent claims fail to exist, investors may seek alternative means to insure initiation risk. This section incorporates circuit breakers into the model to examine how they can partially insure initiation risk. We begin by describing some of the price limits and trading halts employed by stock and futures exchanges before and after the October 1987 market decline. We then model the effects of these circuit breakers and derive necessary and sufficient conditions for welfare to increase when they are employed. For cases in which welfare does not increase, we determine the types of traders who benefit and lose from circuit breakers.

Most commodity futures exchanges have historically employed daily price limits constraining the maximum upward and downward price movements allowed over the course of a given trading day. More recently, following the 1987 market decline, the Chicago Mercantile Exchange (CME) adopted a 3-tier system of price limits constraining upward and downward movements of the S&P 500 futures price over different lengths of time during different times of the day. The first tier requires the prices to be within 5 S&P points of the previous day's close. If the price reaches the 5- point limit before 10 minutes elapses, then trading can continue at or within the 5-point bound. If the price is at the limit 10 minutes after the open, then trading is halted for two minutes, and a new price is established after the halt by open outcry. The second tier establishes a lower limit 12 points below the previous day's close for 30 minutes or until 2:30 PM Chicago time, whichever comes first. The third tier establishes limits 20 points above and 20 points below the previous day's close. The 20-point limit acts as a "traditional" daily limit (in the sense that trading may continue if price is within the 20-point bound) unless trading has been halted on the New York Stock Exchange (NYSE), in which case trading is also halted on the futures exchange for the same duration.

The NYSE circuit breakers take the form of price-contingent trading halts. If the Dow Jones Industrial average declines by 250 points, trading in all stocks halts for 1 hour. If the Dow falls by another 150 points after reopening, trading is halted for another 2 hours.

In our simple static framework, we model both price limits and trading halts as a single tier of upper and lower bounds on the price at which investors are allowed to trade.[13] Although the static model does not allow us to address the rationale for the multi-tier structure of the circuit breaker mechanisms cited above, we believe that our framework nevertheless yields important insights about the welfare effects of circuit breakers.

Formally, suppose price limits increase price in the lowest price state by L and reduce it in the highest price state by αL where $\alpha \geq 0$. The parameter α represents the relative amount limits are reduced at the upper limited price for any increase in the lower limited price. Setting $\alpha = 0$ would imply the absence of an upper bound, as seen, for example, in the second tier of the limits on the S&P 500 contract and both tiers of the NYSE trading halts.

We will examine the effects of locally tightening the price limit policy by increasing L from zero. When $\alpha > 0$, a small increase in L from zero causes limits to bind only in the highest and lowest price states. Recall that the high price state corresponds to a negative liquidity shock and positive fundamentals shock, $(s,n) = (-\mu, \epsilon)$, and the low price state corresponds to the opposite shocks, $(s,n) = (\mu, -\epsilon)$. To economize on notation in what follows, we use the superscripts h and ℓ to represent the high price and low price states respectively; henceforth we suppress s and n from the notation and use these superscripts whenever they cause no confusion. Following this convention, when price limits are imposed, the futures price is given by

$$\begin{array}{ll} P^h(\alpha, L) = P^*(-\mu, \epsilon) - \alpha L & \text{if } (s,n) = (-\mu, \epsilon), \\ P^\ell(L) = P^*(\mu, -\epsilon) + L & \text{if } (s,n) = (\mu, -\epsilon), \text{ and} \\ P^*(s,n) & \text{otherwise.} \end{array}$$

[13]While, price limits typically allow trades to occur at the upper and lower bounds, trading halts shut down trade altogether. In our static framework, however, the mechanisms are equivalent if as many orders as possible are filled before trade is halted. In fact, specialists on the NYSE are instructed to maintain an orderly market by filling orders to the extent possible before they halt trade.

When the upper price limit is reached, the demand for futures contracts exceeds the supply, and when the lower limit is reached, supply exceeds demand. Thus to evaluate the effects of limits on investors we need to address how buyers are rationed at the upper limit and how sellers are rationed at the lower limit. At this point we assume that when the upper limit is reached, total supply, $x_S = \sum_{\{i|x_i^h(P^h)<0\}} x_i^h(P^h)$, is rationed across buyers according to a fixed and deterministic rule whose properties we specify below. Similarly, when the lower limit is reached, total demand, $x_B = \sum_{\{i|x_i^\ell(P^\ell)>0\}} x_i^\ell(P^\ell)$, is rationed across sellers according to a fixed and deterministic rule. In section V we show that our results continue to hold when investors are rationed on a first-come-first-serve basis.

To formalize the deterministic rationing rule, let $y_i^h(\alpha, L) \leq x_i^h$ be the rationed position of buyer i at the upper limited price, and let $y_i^\ell(L) \geq x_i^\ell$ be the rationed position of seller i at the lower limited price. We make the following assumptions on the rationing rule. When L equals zero, no investor is rationed; i.e., for all α, $y_i^h(\alpha, 0) = x_i^h$ and $y_i^\ell(0) = x_i^\ell$. For any given (α, L), the rationed positions of buyers sum to the total amount supplied, and the rationed position of sellers sum to the total amount demanded; i.e., $\sum_{\{i|x_i^h(P^h)>0\}} y_i^h(\alpha, L) = x_S$ and $\sum_{\{i|x_i^\ell(P^\ell)<0\}} y_i^\ell(L) = x_B$. Finally, we assume that y_i^ℓ and y_i^h are continuously differentiable and that for all α, the derivitives $\partial y_i^h(\alpha, 0)/\partial L$ and $\partial y_i^\ell(0)/\partial L$ are finite. This implies that rationing is "fair" in the sense that each investor's position changes at a finite rate as the price limits are tightened.

Below we will calculate the effects on investor i of locally tightening the limits by differentiating ex ante expected utility with respect to L and evaluating the resulting expression at $L = 0$. Since this will require knowing how the certainty equivalents at the upper and lower limits are affected by L, we calculate and interpret these derivatives now for later use.

The limited value of investor i's certainty equivalent at the lower limit is equal to $C_i^\ell(y_i^\ell(L), P^\ell(L))$. Differentiating with respect to L and evaluating at $L = 0$ gives

$$\begin{aligned} \left.\frac{dC_i^\ell\left(y_i^\ell, P^\ell\right)}{dL}\right|_{L=0} &= \left.\left\{\frac{\partial C_i^\ell\left(y_i^\ell, P^\ell\right)}{\partial x_i}\frac{\partial y_i^\ell}{\partial L} + \frac{\partial C_i^\ell\left(y_i^\ell, P^\ell\right)}{\partial P}\frac{\partial P^\ell}{\partial L}\right\}\right|_{L=0} && (5) \\ &= \left.\left\{\frac{\partial C_i^\ell\left(x_i^\ell, P^\ell\right)}{\partial x_i}\frac{\partial y_i^\ell}{\partial L} + \frac{\partial C_i^\ell\left(x_i^\ell, P^\ell\right)}{\partial P}\frac{\partial P^\ell}{\partial L}\right\}\right|_{L=0} && (6) \\ &= \quad 0 \qquad\qquad - \quad x_i^\ell && (7) \\ &= \quad -x_i^\ell. && (8) \end{aligned}$$

Equation (6) uses the fact that investors are not rationed when $L = 0$, i.e., $y_i^\ell(0) = x_i^\ell$. Equation (7) uses the facts that $\partial C_i^\ell/\partial x_i = 0$, by the first order condition for optimal behavior, $\partial y_i^\ell/\partial L$ is finite, by the fair rationing assumption, $\partial C_i^\ell/\partial P = -x_i^\ell$, by the envelope theorem, and $\partial P^\ell/\partial L = 1$.

Equation (8) intuitively means that a local increase of dL in the lower limited price reduces investor i's certainty equivalent by dL times the amount of its net futures position.[14] In the aggregate, this transfers $x_B^\ell dL$ units of wealth from buyers to sellers. Since all the wealth transferred from buyers goes to sellers, there is no "deadweight loss" from rationing when the limits just begin to bind.

Similar reasoning can be used to determine how C_i^h is affected by local changes in L. Since a local increase in L reduces the upper limited price by αdL, we have

$$\frac{dC_i^h}{dL} = \alpha x_i^h. \tag{9}$$

Equation (9) implies that in the aggregate, locally tightening the limits transfers $\alpha x_S^h dL$ from sellers to buyers when the futures price is high. Again, since the transfers are complete, there is no deadweight loss from a small change in L at $L = 0$.

We are now ready to examine the effects of locally tightening the limits on investor i's ex ante expected utility. Under the assumption that s and n are independent, $\beta_s\beta_n$ =Prob{High Price State}=Prob{Low Price State}. Since the limits affect the certainty equivalent only in limited states, a local increase in L from $L = 0$ changes buyer i's ex ante expected utility by

$$\frac{dE_{s,n}U_i(C_i)}{dL} = \beta_s\beta_n \left(U_i'(C_i^h)\frac{dC_i^h}{dL} + U_i'(C_i^\ell)\frac{dC_i^\ell}{dL} \right)\bigg|_{L=0} \tag{10}$$

$$= \beta_s\beta_n \left(U_i'(C_i^h)\alpha x_i^h - U_i'(C_i^\ell)x_i^\ell \right)\big|_{L=0} \tag{11}$$

where equation (11) makes use of the expressions for dC_i^h/dL and dC_i^ℓ/dL in equations (8) and (9).

We can use equation (11) to determine the welfare effects of price limits in the simplest case, which results when investment occurs solely for hedging purposes and price movements are due entirely to news about fundamentals. In this case sellers hedge their positive endowments and buyers hedge their negative endowments. Sellers are better off when the futures price is high, which corresponds to good news about their endowments, and buyers are better off when the price is low, which corresponds to bad news about their endowments. When there are no liquidity shocks, $\mu = 0$, and investor i's equilibrium position, $A(0)/\gamma_i - \theta_i$, is the same in both the high and low price states; i.e. $x_i^h = x_i^\ell$.

Now consider the effects of a tightening the limits by the same amount ($\alpha = 1$) at the upper and lower limited prices. Suppose first that hedger i is a buyer. Then, since $x_i^h = x_i^\ell > 0$ and $C_i^\ell > C_i^h$, equation (11) becomes $dE_{s,n}U_i(C_i)/dL = \beta_s\beta_n x_i^h[U_i'(C_i^h) - U_i'(C_i^\ell)] > 0$, by the concavity of U_i. That is, a symmetric local tightening of the limits increases investor i's ex ante expected utility. Alternatively suppose investor i is a seller. Then $x_i^h = x_i^\ell < 0$,

[14]This result is simply an extension of Roy's Law to a portfolio choice problem in which the marginal certainty equivalent of wealth equals one.

$C_i^\ell < C_i^h$, and $dE_{s,n}U_i(C_i)/dL$ is again greater than zero by the concavity of U_i. Hence, tightening the limits increases the ex ante expected utility of both buyers and sellers. Since this result emerges as a special case of Proposition 2 below, we state it as a corollary here.

Corollary 1 *Suppose that fluctuations in the futures price are due entirely to news about fundamentals. Then a symmetric ($\alpha = 1$) local tightening of the limits increases the ex ante expected utility of those investors who trade solely to hedge their endowments of the underlying asset. If all investors are hedgers, then there exist price limits that are pareto superior to unconstrained trade.*

Despite critics' claims that circuit breakers represent sand in the gears of the (ex post) Walrasian mechanism, Corollary 1 demonstrates that establishing price limits before volatile times occur can increase investors' ex ante expected utility. The intuition is straightforward. When limits are tightened symmetrically, a local increase in L transfers $x_i^h dL$ units of wealth from the sellers to buyer i when price is high, and it transfers $x_i^\ell dL = x_i^h dL$ units of wealth from buyer i to the sellers when price is low. Since the high and low prices are equally likely, and since buyers are better off when the futures price is low, these transfers reduce the variance of buyer i's random wealth across news realizations without changing the mean. In the discrete distribution formulation of the liquidity and news shocks that we consider, this is sufficient for buyer i's ex ante expected utility to increase. A similar intuition holds for sellers of futures contracts.

We have highlighted the pure-hedging and no-liquidity-shock case because it is the most intuitive. However price limits are pareto improving for a wider class of cases. To see this we examine more carefully how investor i's ex ante expected utility depends on L.

Rearranging (11) yields the following necessary and sufficient condition for a local increase in L to increase investor i's expected utility:

$$\frac{dE_{s,n}U_i(C_i)}{dL} > 0$$

$$\text{if and only if} \quad M_i = \frac{U_i'(C_i^h)x_i^h}{U_i'(C_i^\ell)x_i^\ell} = \frac{dP^\ell}{dP^h}\bigg|_{EU_i} \begin{matrix} > \\ < \end{matrix} \frac{1}{\alpha} \quad \text{as } x_i^\ell \begin{matrix} > \\ < \end{matrix} 0. \tag{12}$$

For a buyer in the low price state ($x_i^\ell > 0$), M_i is the amount the buyer is willing to pay, in terms of an increase in the low price, for an equal reduction in the high price. The parameter $1/\alpha$ is the amount the buyer actually pays, in terms of an increase in the low price, for a reduction in the high price. The buyer benefits from a local tightening of the limits if the buyer is willing to accept a greater increase in the low price for a reduction in the high price than is required, as reflected by the upper set of inequalities in condition (12). For a seller in the low price state ($x_i^\ell < 0$), M_i is the amount the seller is willing to accept, in terms of an increase in the low price, for a reduction in the high price. $1/\alpha$ is the increase in the low price that the seller actually receives for a reduction in the high price. The seller benefits from a local tightening of the limits if the seller is willing to accept a smaller increase in the low price for a reduction in the high price than the amount required, as reflected by the lower set of inequalities in condition (12).

A local tightening of the limits is pareto improving if and only if there exists an α such that condition (12) holds for every investor i. Let the subscript i_B refer to a buyer at the low price and i_S a seller at the low price. Putting the inequalities in condition (12) together to isolate $1/\alpha$ gives the following condition for pareto improving price limits: A local increase in L (from 0) is pareto improving if and only if, for every buyer, i_B, and every seller, i_S,

$$M_{i_S} < \frac{1}{\alpha} < M_{i_B}. \tag{13}$$

Substituting for the equilibrium certainty equivalents from equation (3), using the negative exponential utility function, and recognizing that we are free to choose any α satisfying (13), condition (13) becomes

$$M_{i_S} = Ke^{-2\gamma_{i_S}\theta_{i_S}[\sigma_\nu^2\mu+\epsilon]}\left\{\frac{A(-\mu)-\gamma_{i_S}\theta_{i_S}}{A(\mu)-\gamma_{i_S}\theta_{i_S}}\right\} < Ke^{-2\gamma_{i_B}\theta_{i_B}[\sigma_\nu^2\mu+\epsilon]}\left\{\frac{A(-\mu)-\gamma_{i_B}\theta_{i_B}}{A(\mu)-\gamma_{i_B}\theta_{i_B}}\right\} = M_{i_B} \tag{14}$$

for every buyer at the low price, i_B, and every seller at the low price, i_S, where $K = exp\{2\sigma_\nu^2\mu\sum_i\theta_i/\sum_i(1/\gamma_i)\}$. We now give the formal conditions for pareto superior limits in the presence of shocks to both news and liquidity.

Proposition 2 *Suppose that each investor i takes a position that is either always long or always short, regardless of the value of shocks to liquidity and fundamentals. That is, $\forall i_B, i_S$, $\gamma_{i_B}\theta_{i_B} < A(-\mu) < A(\mu) < \gamma_{i_S}\theta_{i_S}$. Then there exist price limits that are pareto superior to unconstrained trade. On the other hand, if some investor i_B sells futures at the upper limited price and buys futures at the lower limited price ($A(-\mu) < \gamma_{i_B}\theta_{i_B} < A(\mu)$), then a local increase in L from zero is not pareto improving.*

Proof: Suppose first that buyers always buy and sellers always sell. That is, $\gamma_{i_B}\theta_{i_B} < A(-\mu) < A(\mu) < \gamma_{i_S}\theta_{i_S}$. Then M_{i_S} and M_{i_B} are both positive in condition (14). It is easy to verify that M_i is strictly decreasing in $\gamma_i\theta_i$. Since $\gamma_{i_B}\theta_{i_B} < \gamma_{i_S}\theta_{i_S}$, it follows that condition (14) is satisfied. Hence pareto superior price limits exist in this case.

Next suppose that some speculator buys when price is low and sells when price is high. For this speculator, M_{i_B} is negative since $A(-\mu) < \gamma_{i_B}\theta_{i_B} < A(\mu)$. Since M_{i_S} is positive (because any investor i_S who sells at the low price will also sell at the high price, i.e., $A(-\mu) < A(\mu) < \gamma_{i_S}\theta_{i_S}$), condition (14) cannot be satisfied. Hence a local tightening of the limits is not pareto improving. **Q.E.D.**

Several interesting corollaries follow from Proposition 2. Corollary 1, which stated that pareto superior price limits exist in the pure-hedging and no-liquidity-shock case, follows from the fact that hedgers each take positions on the same side of the market in order to offset their endowed positions. Since this is true whether liquidity shocks are present or absent, the no-liquidity-shock requirement is superfluous when investors are hedgers. This gives Corollary 2.

Corollary 2 *There exist price limits that increase the ex ante expected utility of every hedger. Hence if all investors are hedgers, then there exist price limits that are pareto superior to unconstrained trade.*

On the other hand, suppose that some investors are speculators, in the sense that their optimal positions do not offset their endowments, and that there are no liquidity shocks ($\mu = 0$). Then, since $x_i^h = x_i^\ell = A(0)/\gamma_i - \theta_i \ \forall i$, each investor always takes a position on the same side of the market. We therefore have the following.

Corollary 3 *Suppose that price movements are due entirely to news about fundamentals. Then there exist price limits that are pareto superior to unconstrained trade.*

Corollary 3 implies that even if some investors trade for purely speculative reasons, judiciously chosen price limits are pareto improving provided there are no liquidity shocks. When fluctuations in price are due solely to news about fundamentals, sellers have lower marginal rates of substitution between price decreases in the high and low price states than buyers. Thus, α can be chosen high enough that a reduction of αdL in the upper price benefits each buyer by more than an increase of dL in the lower price, yet low enough that the reduction in the upper price harms sellers by less than they benefit from the increase in the lower price.

The final case of interest, already described in Proposition 2, occurs when liquidity shocks are large enough to induce a speculator to buy when price is low and sell when price is high. It is easy to see that a local tightening of the limits is not pareto improving in this case. Price limits require such a speculator to sell at a lower price when price is high and buy at a higher price when price is low. This decreases the speculator's time 1 certainty equivalent both when price is high and low and therefore reduces ex ante expected utility.

V Implications and Extensions

Although we have not characterized the optimal price limits that satisfy specific objectives, we have shown that investors can benefit from price limits that constrain both upward and downward price movements. In general, futures exchanges employ limits that allow equal upward and downward movements in price. This is consistent with Proposition 2, which demonstrates that symmetric limits make hedgers better off when price movements are due entirely to news about fundamentals.

In the stock market, we observe only lower limits. In our model this corresponds to $\alpha = 0$. To see when the model is consistent with lower limits alone, recall the necessary and sufficient condition (13) for locally tightening the limits to be pareto improving:

$$M_{i_S} < \frac{1}{\alpha} < M_{i_B}.$$

As α approaches 0, $1/\alpha$ approaches infinity. Thus lower limits alone are pareto improving only when all investors are sellers. In the futures market, all investors cannot be sellers, since the market would cease to exist. In the stock market, however, the presence of a specialist theoretically enables all other investors to sell to the specialist. Thus, if the specialist's welfare is ignored, then lower limits alone could be welfare enhancing.[15]

A common criticism of price limits is that limited or "stale" prices fail to convey all available information to investors. Note that in our model, the limited prices fully reveal the news shock. This is true because we consider limits that bind only in the highest and lowest price states; such limits are associated with particular values of the news. This will be true of any model in which the liquidity and news shocks have discrete distributions and price limits bind in the highest and lowest price states. While optimally chosen price limits may be tighter, and hence the limited prices may fail to convey all information, this criticism does not negate our results establishing when limits are pareto improving.

We have assumed that when prices reach a limit, investors are rationed according a deterministic rule. A more realistic assumption might be that investors' orders are filled in the order that they arrive. As we show in the appendix, none of our results are altered when investors are rationed on a first-come first-serve basis. The intuition is similar to that in the proportional rationing case. A local increase of dL in the lower limited price (at $L = 0$) transfers $x_B^\ell dL$ from buyers to sellers. There is no deadweight loss since the marginal seller, whose order arrives last, is not rationed when $L = 0$. Similarly, a local decrease in the upper limited price transfers $\alpha x_S^h dL$ from sellers to buyers without any deadweight loss. These transfers are pareto improving according to the conditions we have already established. Of course, in practice limits may be chosen tight enough that some buyers (sellers) are rationed completely out of the market at the upper (lower) limited price. Nevertheless, we only need to consider local changes in L at $L = 0$ to demonstrate whether *some* set of price limits is pareto improving.

We ignored transactional risk, which arises when shocks occur between the time orders are submitted and the time they are executed. Allowing for transactional risk exacerbates the efficiency problem in two ways. When it arises from liquidity shocks, it simply worsens the ex ante inefficiency that we have identified. When transactional risk arises from news shocks, it creates inefficiency in the ex post asset market by making the orders submitted prior to the news release sub-optimal by the time they are executed. We ignored transactional risk, and the associated ex post inefficiency, in order to focus on ex ante inefficiency. In our setting, price limits cannot provide insurance for news about fundamentals that is released after investors' orders are submitted but before they are executed.

Assuming that investors' preferences are important in determining price limit policy, our model suggests conditions under which limits are more or less likely to be used. Since judiciously chosen price limits benefit hedgers, we are more likely to see price limits when hedgers dominate the market. Similarly, we are more likely to see price limits used when price movements are driven largely by news about fundamentals. However, when the variance of liquidity shocks is large and induces a large number of speculators to buy when price is low and sell when price is high, then the harmful effects of limits for these investors may preclude their use.

An interesting extension of our model would be to solve for optimal price limits under alternative assumptions about the exchange's objectives. This might generate testable predictions concerning how "optimal" price limits vary with parameters such as the variance of liquidity shocks, the variance of news, the risk aversion of investors, the number of hedgers and speculators, etc.

Another interesting extension would consider how circuit breakers should be coordinated across stock and futures markets. Competing exchanges may choose their price limit policies to attract investors. It would be interesting to consider whether some futures exchanges choose different policies than stock exchanges and other futures exchanges and whether requiring coordinated circuit breakers would be beneficial.

We view our static model as a first attempt at examining the effects of price limits and trading halts. Important dynamic issues were ignored that may be fruitful areas for further research.

Investors with long horizons may be inclined to wait-out price limits until they are no longer in effect. For example, when the lower limited price is reached, investors planning to hedge by buying and holding may choose to wait for tomorrow's price if they expect it to be lower. Our model abstracts from this possibility by assuming a single trading period.

Another limitation of the static model is that price limits and trading halts are treated

[15] Inequality (13) is inappropriate for analyzing the stock market when there are no liquidity shocks and investors' endowments are chosen optimally in the period prior to the release of news. For then no change in the investor's position is desired (i.e. $x_i^h = x_i^\ell = 0$) since investor i's competitive equilibrium position is independent of the news (see section III), and hence M_i is undefined. However it is clear that (non-local) lower limits increase the welfare of sellers by allowing them to sell stock to the specialist at artificially high prices.

identically. While they are equivalent in our static framework, provided that as many orders as possible are executed before trade is halted, it may well be that some orders on both sides of the market arrive after trade is halted. This may induce investors to alter their orders in different ways depending on whether they anticipate a trading halt or a price limit. These issues can only be addressed in a dynamic framework.

VI Conclusion

Recent proponents of circuit breakers as a means to "cushion the impact of market movements..." and "protect markets and investors,"[16] have met frequent criticism from free market advocates who view asset markets as efficient. The efficiency argument rests on the assumption that orders are initiated and executed without delay. During periods of volatile price movements, however, delays, both in initiating and executing orders, appear to be widespread. We argue that such delays give rise to implementation risk that cannot be transferred optimally among investors; hence, even when the market is ex post efficient, it may nevertheless be ex ante inefficient.

We examine the case in which investors have constant absolute risk aversion and returns are normally distributed. We show that price limits and trading halts can serve to partially insure implementation risk and in some cases are pareto superior to unconstrained trade. One such case is when price movements are due entirely to news regarding fundamentals. In this case price limits provide pareto improving transfers of wealth across realizations of the news in such a way as to "smooth" each investor's wealth.

When shocks are due to both fundamentals and liquidity, price limits are pareto improving when each investor is either always a buyer or always a seller regardless of the nature of the shocks. This condition holds when all investors are hedgers or when liquidity shocks are small enough that no speculators choose to buy at low prices and sell at high prices. If some speculators prefer to buy at low prices and sell at high prices, price limits will not benefit this set of investors and thus will be pareto inferior to unconstrained trade.

It is not surprising that "second best" (self) regulation, such as price limits and trading halts, can improve market performance when a complete set of contingent claims is absent. Perhaps other exchange-imposed impediments to mutually advantageous trade are motivated by risk inherent in the trading process.

[16]Report of the Presidential Task Force on Market Mechanisms, January, 1988, p. 66.

APPENDIX

This appendix shows that all of our results continue to hold when investors are rationed according to the arrival of their orders.

Consider first the rationing of buyers at the upper limited price. Suppose that buyers' orders arrive randomly and are filled in the order that they arrive. When L is small, the probability that buyer i is the *marginal* buyer, who is rationed, is equal to $q_B = 1/$(Total Number of buyers). At the lower limited price buyers are not rationed. Since the limits affect investors only in the highest and lowest price states, a local increase in L from $L = 0$ changes buyer i's ex ante expected utility by

$$\frac{dE_{s,n}U_i(C_i)}{dL} = \beta_s\beta_n\left((1-q_B)U_i'(C_i^h)\frac{dC_i^h}{dL} + q_B U_i'(C_i^h)\frac{dC_i^h}{dL} + U_i'(C_i^\ell)\frac{dC_i^\ell}{dL}\right) \quad (15)$$

$$= \beta_s\beta_n\left(U_i'(C_i^h)\alpha x_i^h - U_i'(C_i^\ell)x_i^\ell\right) \quad (16)$$

Equation (15) uses the fact that when $L = 0$, buyer i's certainty equivalent is the same whether he is rationed or not. Equation (16) uses the expressions for dC_i^h/dL and dC_i^ℓ/dL in equations (8) and (9).

Expression (16) is identical to expression (11) in the main body of the paper, and a similar expression holds for sellers. Since all of our results in the paper follow from manipulations of expression (11), they continue to hold when investors are rationed according to the arrival of their orders.

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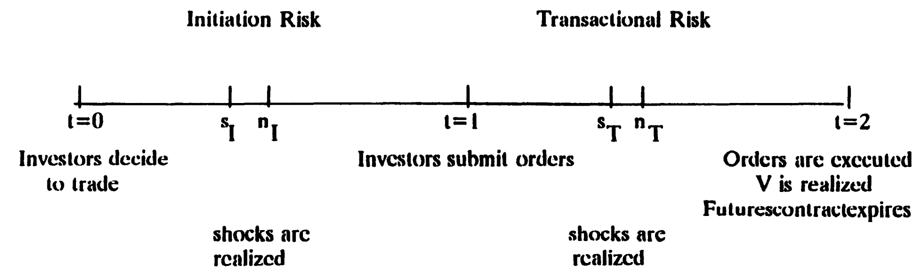
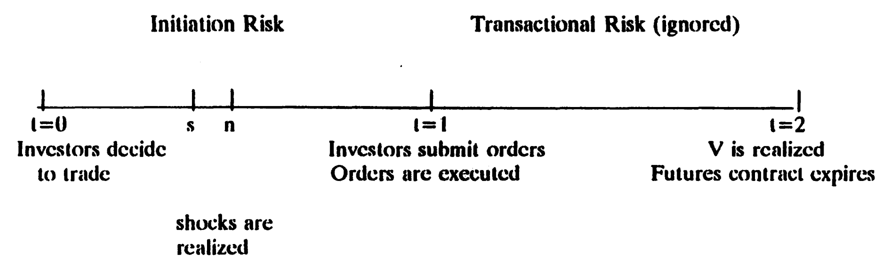
Figure 1



In our model, we ignore transactional risk in order to focus on initiation risk. Thus, the timing in our model is as follows:



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