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Dynamic Determinacy and the Existence
of Sunspot Equilibria

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Abstract: This paper relates the existence of stationary sunspot equilibria in the vicinity of a conventional stationary state to the phase diagram surrounding the latter. We find that the local condition required for a unique equilibrium path returning to the stationary state following any slight perturbation is also sufficient to exclude stationary sunspot equilibria in the state's immediate vicinity or (if the model includes predetermined variables) to exclude local sunspot outcomes which are not virtually indistinguishable from the stationary state itself. In that sense, an eigenvalue condition can tell us something about the possibilities of local sunspot activity.

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Dynamic Determinacy and the Existence of Sunspot Equilibria

If we can think of an economic model as defining a function from agents' preferences, endowments, and technologies to market prices and quantities, comparative statics exercises will be straightforward. Two complications revealed in recent work, however, are the possibility of large numbers of sunspot solutions — even arbitrarily close to more conventional outcomes — and indeterminacy. In the former instance, an "extrinsic random variable"— a variable with realizations not directly affecting the underlying characteristics of households or production — can influence equilibrium outcomes. Essentially, if all agents think a sunspot variable is important, it may become so — see, for example, Cass and Shell [6] and Azariadis [1]. In the case of indeterminacy, a model (generically) may exhibit a continuum of equilibria (unrelated to extrinsic randomness) consistent with given initial conditions, perfect foresight, and market clearing restrictions. In fact, a stationary solution may have a continuum of equilibrium paths in its local vicinity converging to it — see, for instance, Calvo [7], Laitner [13], Woodford [15], and Kehoe and Levine [12]. In either case, we are left unsure about which equilibrium a given model should direct us to after a parameter change.

The purpose of this paper is to show that a condition needed for avoiding indeterminacy in the vicinity of a stationary solution, say, x^*, also rules out stationary sunspot equilibria arbitrarily close to x^*, or at least it rules out local stationary sunspot solutions which are not virtually indistinguishable from x^*. Thus, an eigenvalue condition characterizing local (equilibrium) dynamics also yields results about sunspots. More precisely, if a model with no historical (or "predetermined") endogenous variables has a stationary solution x^* which is determinate in the sense that every possible given initial condition in some open neighborhood of it constitutes the starting point of a single stable equilibrium time path converging to it, then Theorem 1 below establishes that for any extrinsic random variable there exists a neighborhood of x^* not containing (entirely within it) any stationary sunspot equilibrium based on the random variable. Theorem 2 presents similar, but slightly weaker results for models containing historical as well as non–historical arguments: a stationary sunspot equilibrium near a determinate stationary solution x^* must assign very small probabilities to states other than those extraordinarily close to x^*. Our framework allows an arbitrary (finite) number of state variables, and it encompasses extrinsic random variables with an arbitrary (finite) number of states.

Our theorems complement a number of existing articles. For example, Azariadis [1], Azariadis and Guesnerie [2], and Guesnerie [11] show, in the context of consumption loan frameworks with no historical variables, that a sufficient condition for local stationary sunspot equilibria is the existence of one or more eigenvalues of modulus less than 1 for a model's forward dynamics in the vicinity of a conventional stationary solution — in fact, a condition leading to indeterminacy (see Section 1). Woodford [16] considers a model with infinite lived agents. The local dynamics can be analyzed in terms of two state variables — one historical and the other nonhistorical. He derives a similar result — showing that having two stable eigenvalues, which implies dynamic indeterminacy, is sufficient for the existence of a stationary sunspot in each neighborhood of a given stationary solution. Grandmont [9, 10] obtains results for both existence and non–existence: for a consumption loan model having a single nonhistorical state variable in each period, he proves that an eigenvalue condition implying local instability in the backward dynamics (in other words, the case with indeterminacy — see Section 1) is necessary and sufficient for local sunspot activity. Woodford [15] summarizes a large number of examples in the same vein.[1]

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[1] As revisions on this paper proceeded, the author became aware of Woodford [17], which independently (and with a somewhat different technology) reaches conclusions analogous to ours.

The macroeconomics literature on linear rational expectations models (with intrinsic randomness) provides parallel results. In particular, Blanchard and Kahn [4] show that eigenvalue configurations leading to dynamic indeterminacy imply the existence of solutions containing extrinsic random variables (see also Gourieroux, Laffont, and Monfort [8] and Broze, Gourieroux, and Szafarz [5]).

The organization of the present paper is as follows. Section 1 sets up our framework of analysis. Section 2 introduces sunspots and presents our theorems, and Sections 3–4 prove them.

1. Local Dynamics

Until the introduction of extrinsic uncertainty, the model we consider consists of the following elements: a time–t vector of state variables $x_t \in R^n$, a (time–autonomous) system of excess demand functions $Z(x_t, x_{t+1}) \in R^n$, and a set of initial conditions. The vector x_t may contain prices and/or quantity variables such as capital stock figures. Time is discrete. We restrict our attention to equilibrium time paths.

Our analysis begins at time 0. The sequence of vectors $\{x_0, x_1, ...\}$ is an "equilibrium" if $Z(x_t, x_{t+1}) = 0$ all $t \geq 0$ and if x_0 satisfies initial conditions. This definition implicitly requires perfect foresight. The vector $x^* \in R^n$ defines a "stationary equilibrium" if $Z(x^*, x^*) = 0$. Assume such an x^* exists. Normalize variables so that $x^* = 0$.

To consider non–stationary equilibrium paths in the vicinity of x^*, assume that $Z(.,.)$ is twice continuously differentiable in some open neighborhood of (x^*, x^*). Define

$$A_1 \equiv \partial Z(x^*, x^*)/\partial x_t, \quad A_2 \equiv \partial Z(x^*, x^*)/\partial x_{t+1}.$$

Assume

$$\det(A_2) \neq 0. \tag{A1}$$

Using the implicit function theorem and (A1), $Z(.,.)$ uniquely defines a difference equation

$$x_{t+1} = \varsigma(x_t) \quad \text{all} \quad t \geq 0 \tag{1}$$

with

$$Z(x_t, \varsigma(x_t)) = 0$$

all x_t in some open neighborhood of x^*. Well-known theorems show we can study the behavior of (1) locally using the linear system

$$x_{t+1} = A \cdot x_t \quad \text{with} \quad A \equiv -(A_2)^{-1} \cdot A_1. \tag{2}$$

For simplicity, we assume below that the eigenvalues of A are distinct and that none have modulus 1.

Consider the makeup of the vector $x_t = (x_{1t}, ..., x_{nt})$. Some components may be "historical" variables: variables with time–t levels fixed by events of prior dates (for example, lags may mean the time–t capital stock depends only on earlier investment behavior). The remaining elements of x_t are "non–historical" — their values being determined during period t. Without loss of generality let x_{it}, $i = 1, ..., h$ be historical, $u_t \equiv (x_{1t}, ..., x_{ht})$, and $v_t \equiv (x_{h+1,t}, ..., x_{nt})$. Then given initial conditions will consist only of values for u_0; our analysis must determine v_0 and $x_t = (u_t, v_t)$ all $t > 0$.

If h^* eigenvalues of A have modulus less than 1, consider three cases: (i) $h^* < h$; (ii) $h^* = h$; and (iii) $h^* > h$. Stationary equilibria have, of course, long been a cornerstone of dynamic analysis. For x^* to warrant special attention, however, we presumably want to insist on being able to reach it (via an equilibrium time path) from any u_0 in an open neighborhood of $0 \in R^h$ — ruling out, in general, case (i). In case (iii), for any u_0 in a small enough open set containing $0 \in R^h$ there will be a continuum of values $v_0 \in R^{n-h}$ such that $x_{t+1} = \varsigma(x_t)$ all $t \geq 0$ implies $|x_t - x^*| \to 0$ as $t \to \infty$. Thus, for the possibility of determinacy within the class of convergent equilibrium paths, we are left with only case (ii).[2] Laitner [13], Kehoe and Levine [12], and Begg [3, ch. 3] discuss similar issues.

A precise condition for a unique convergent equilibrium path originating from each x_0 in some neighborhood $\mathcal{V}$ of x^* and lying entirely within some small given open set $\mathcal{U}$ is

Condition 1. *The matrix A has h distinct eigenvalues e_i with $|e_i| < 1$ and $n - h$ distinct eigenvalues e_j with $|e_j| > 1$. The projection of the stable manifold for (2) onto the space containing all historical subvectors u has dimension h.*

2. Sunspot Equilibria

Suppose we have an extrinsic random variable with outcomes $a_1, ..., a_M$. As stated, "extrinsic" means the variable's realizations have no affect on agents' preferences, on production technologies, or on endowments. Let the random variable have transition matrix $\Pi = [\pi_{ij}]$ — the probability of transiting from state a_i at time t to state a_j at time $t + 1$ being π_{ij}. Assume all agents in the model can observe the current realization a_i and know Π. We now define a stationary sunspot equilibrium and present our theorems.

Following Guesnerie [11], if all agents anticipate time-$(t + 1)$ state vector $x_{t+1}(a_j)$ conditional on a_j (in that time period), if all maximize expected utility, and if we have outcome a_i at time t, let time-t excess demand be

$$\tilde{Z}(x_t(a_i), x_{t+1}(a_1), ..., x_{t+1}(a_M); \Pi) \in R^n.$$

Our definition is conventional:

Definition 1. *For any $\infty > M \geq 2$, the vectors $x(a_1), ..., x(a_M) \in R^n$, at least two of which differ from one another, and the $n \times n$ probability transition matrix Π for the extrinsic random variable having realizations $a_1, ..., a_M$ determine a "stationary M-sunspot equilibrium" if for all $i = 1, ..., M$, $\tilde{Z}(x(a_i), x(a_1), ..., x(a_M); \Pi) = 0$.*

Thus, $x(a_1), ..., x(a_M)$ and Π characterize a stationary sunspot equilibrium if given a time-t realization of the extrinsic variable a_i, $x_t = x(a_i)$ clears all current markets provided agents anticipate $x_{t+1} = x(a_j)$ in the event of random realization a_j next period.

Assume $\tilde{Z}(.)$ is twice continuously differentiable. In the event that $x_{t+1}(a_1) = x_{t+1}(a_2) = ... = x_{t+1}(a_M) \equiv \bar{x}$, for any x we should have

$$\tilde{Z}(x, \bar{x}, ..., \bar{x}; \Pi) = Z(x, \bar{x}). \tag{A2}$$

---

[2] Nonconvergent paths may lead to contradictions of market clearing conditions within a finite number of periods — or they may define legitimate equilibria (see, for example, Woodford [15]).

Two derivative properties follow from the underlying model's first-order conditions in the same special case: for any Π,

$$\tilde{Z}_1(x, \bar{x}, ..., \bar{x}; \Pi) = Z_1(x, \bar{x}), \qquad (A3)$$

$$\tilde{Z}_{j+1}(x, \bar{x}, ..., \bar{x}; \Pi) = \pi_{ij} \cdot Z_2(x, \bar{x}) \quad \forall \, j = 1, ..., M. \qquad (A4)$$

We treat these as assumptions — see Guesnerie [11, p. 108].

Let $\| \, . \, \|$ be the Euclidean norm. Fix any $M < \infty$. Define the "radius" of any M–sunspot equilibrium characterized by

$$s \equiv (x(a_1), ..., x(a_M), \Pi) \qquad (3)$$

as

$$r(s) \equiv \sum_{j=1}^{M} \|x(a_j)\|. \qquad (4)$$

Our first result is

Theorem 1. *Suppose $h = 0$ — so that a model's endogenous variables are all non–historical. Suppose assumptions (A1)-(A4) and Condition 1 hold for stationary state $x^* = 0$. Fix any $M < \infty$. Then there exists $\epsilon_M > 0$ such that no stationary M–sunspot equilibrium has radius less than ϵ_M.*

Section 1 shows that Condition 1 leads to the saddlepoint configuration in the neighborhood of x^* desired for a unique convergent equilibrium time path. Theorem 1 shows that in the context of a model with no predetermined variables — such as the popular consumption loan systems mentioned in the introduction — for any M, the same saddlepoint implies the existence of an open set $\mathcal{V}$ containing x^* such that no stationary M–sunspot equilibrium exists having all state vectors, $x(a_i)$, in $\mathcal{V}$. In the latter particular sense Condition 1 insures that x^* is "isolated" from sunspot equilibria.[3]

To allow historical variables as well, we need an additional definition. Let s be as in (3). For $m = 1, 2, ...$, use the notation

$$(\Pi)^m = \Pi \cdot \Pi \cdot ... \cdot \Pi \equiv [\pi_{ij}(m)].$$

Then define

$$p(i, T, s) \equiv \sum_{m=1}^{T} \pi_{ii}(m). \qquad (5)$$

If we have a stationary M–sunspot equilibrium s, and if the extrinsic random variable's current realization is a_i, $p(i, T, s)$ gives the probability of at least one repeat of a_i over the next T periods. If the realization a_i is visited very infrequently, $p(i, T, s)$ will be near 0 even for a large T.

Our second theorem shows that if Condition 1 holds at $x^* = 0$, even if $h > 0$, for any T and δ we can find $\epsilon_M > 0$ such that s as in (3) and $r(s) < \epsilon_M$ implies

$$\max_{i=1,...,M} [x(a_i)/r(s)] \cdot p(i, T, s) < \delta. \qquad (6)$$

In other words, in a small enough neighborhood of x^* states $x(a_i)$ in a stationary sunspot equilibrium s either (i) are nearly transitory (with $p(i, T, s)$ very small) or (ii) have a norm $\| \, x(a_i) \, \|$ which is a tiny faction of the sunspot's radius. A stationary sunspot equilibrium with the latter two properties is virtually indistinguishable from the conventional stationary solution x^* — giving us our analogue to Theorem 1.

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[3] The theorem does not rule out sunspots with some, but not all, state vectors arbitrarily close to x^* — see, especially, Grandmont [10, p. 23].

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Theorem 2. *Suppose $h > 0$. Suppose assumptions (A1)–(A4) and Condition 1 hold for stationary state $x^* = 0$. Fix any $M < \infty$. Fix any integer $T < \infty$ and constant $\delta > 0$. Then there exists an $\epsilon_M > 0$ such that for any stationary M-sunspot equilibrium s — see (3) — with $r(s) < \epsilon_M$, inequality (6) holds.*

The next two sections present proofs.

3. Proof of Theorem 1

The proof of Theorem 1 is straightforward. We begin with a preliminary result, Lemma 1.

Fix any M. Suppose each open neighborhood $\mathcal{V}$ of x^* contains a stationary M-sunspot equilibrium. Then for each $k = 1, 2, \ldots$ we can find a stationary M-sunspot equilbrium, characterized by $s_k \equiv (x_k(a_1), \ldots, x_k(a_M), \Pi_k)$, with

$$r(s_k) \le 1/k.$$

Define

$$X^k(a_i) \equiv x_k(a_i) / \sum_{j=1}^{M} \| x_k(a_j) \| . \tag{7}$$

(Definition 1 shows we are not dividing by 0.) Then since $(X_k(a_1), \ldots, X_k(a_M))$ for each k lies in the compact set

$$S \equiv \{(X_1, \ldots, X_M) \mid \text{for all } i, X_i \in R^n \text{ and } \sum_{j=1}^{M} \| X_j \| = 1\} \subset R^{n \cdot M},$$

and since each row of Π_k lies in the unit simplex in R^M, there is a vector $(X^*(a_1), \ldots, X^*(a_M)) \in S$, a probability transition matrix Π^*, and a subsequence

$$s_{k(\ell)} \equiv \{(X_{k(\ell)}(a_1), \ldots, X_{k(\ell)}(a_M), \Pi_{k(\ell)}\} \quad \text{all} \quad \ell = 1, 2, \ldots,$$

with

$$\lim_{\ell \to \infty} s_{k(\ell)} = (X^*(a_1), \ldots, X^*(a_M), \Pi^*). \tag{8}$$

We now use a Taylor series approximation. Let $\tilde{Z}_i(.)$ be the derivative of $\tilde{Z}(.)$ with respect to its i'th (vector) argument. Using linear approximations and (A2),[4]

$$\tilde{Z}_1(x^*, x^*, \ldots, x^*, \Pi^*) \cdot x_k(a_i) + \sum_{j=1}^{M} \tilde{Z}_{j+1}(x^*, x^*, \ldots, x^*, \Pi^*) \cdot x_k(a_j) \approx$$

$$\tilde{Z}(x_k(a_i), x_k(a_1), \ldots, x_k(a_M); \Pi_k) = 0 \quad \forall \, i = 1, \ldots, M. \tag{9}$$

The error is second order. Thus, dividing by $\sum_{j=1}^{M} \| x_k(a_j) \|$ and taking limits as in (8) (using subsequences if necessary),

$$\tilde{Z}_1(x^*, x^*, \ldots, x^*, \Pi^*) \cdot X^*(a_i) + \sum_{j=1}^{M} \tilde{Z}_{j+1}(x^*, x^*, \ldots, x^*, \Pi^*) \cdot X^*(a_j) = 0 \;\; \forall \, i = 1, \ldots, M. \tag{10}$$

---

[4] Note that (A2) implies the derivatives of $\tilde{Z}(x^*, x^*, \ldots, x^*, \Pi^*)$ with respect to the elements of Π^* are all 0.

Employing (A3)-(A4), (10) becomes

$$A_1 \cdot X^*(a_i) + \sum_{j=1}^{M} \pi_{ij}^* \cdot A_2 \cdot X^*(a_j) = 0 \quad \forall\, i = 1, ..., M, \tag{11}$$

where $\Pi^* \equiv [\pi_{ij}^*]$. Using (A1) and (11),

$$A \cdot X^*(a_i) = \sum_{j=1}^{M} \pi_{ij}^* \cdot X^*(a_j) \quad \forall\, i. \tag{12}$$

We can use (12) to show that $X^*(a_i)$ all $i = 1, ..., M$ must lie in the stable manifold for (2).

Lemma 1. *Let $X^*(a_i)$ all $i = 1, ..., M$ be as above. Let (A1)-(A4) hold. Then each $X^*(a_i)$ lies in the stable manifold for (2).*

Proof. Define $(\Pi^*)^m = [\pi_{ij}^*(m)]$ all $m = 1, ...$ Multiplying (12) through by A,

$$A^2 \cdot X^*(a_i) = \sum_{j=1}^{M} \pi_{ij}^* \cdot A \cdot X^*(a_j) =$$

$$\sum_{j=1}^{M} \pi_{ij}^* \cdot \sum_{\ell=1}^{M} \pi_{j\ell}^* \cdot X^*(a_\ell) = \sum_{\ell=1}^{M} (\sum_{j=1}^{M} \pi_{ij}^* \cdot \pi_{j\ell}^*) \cdot X^*(a_\ell) =$$

$$\sum_{\ell=1}^{M} \pi_{i\ell}^*(2) \cdot X^*(a_\ell) \quad \forall\, i = 1, ..., M.$$

Repeating the process, for any $m = 2, 3, ...,$

$$A^m \cdot X^*(a_i) = \sum_{j=1}^{M} \pi_{ij}^*(m) \cdot X^*(a_j) \quad \forall\, i = 1, ..., M. \tag{13}$$

Now $(\Pi^*)^m$ is itself a probability transition matrix. Thus, for any m,

$$\| \sum_{j=1}^{m} \pi_{ij}^*(m) \cdot X^*(a_j) \| \le \sum_{j=1}^{m} \pi_{ij}^*(m) \cdot \| X^*(a_j) \| \le \sum_{j=1}^{m} \| X^*(a_j) \| = 1. \tag{14}$$

If $X^*(a_i)$ is not in the stable manifold for (2), some component of the vector on the left-hand side of (13) must diverge for large enough m — contradicting (14). ∎

In words, if $X_t = X^*(a_i)$, the right-hand side of (12) gives the expected value (as of time t) for X_{t+1}. Multiplying through by A, we generate the expected value (as of time t) for X_{t+2}. Continuing in this way, if we began off of the stable manifold for (2), the expected values must explode — an impossibility for for $(X^*(a_1), ..., X^*(a_M))$ derived from "stationary" sunspot equilibria. This result generalizes Guesnerie [1986, Thm. 3, part 2].

The proof of Theorem 1 follows immediately:

Proof of Theorem 1. Fix M. Suppose each open neighborhood $\mathcal{V}$ of x^* contains a stationary M-sunspot equilibrium. Construct $X^*(a_i)$ all $i = 1, ..., M$ as above. Since $h = 0$, the stable manifold for (2) is only the origin. Thus, Lemma 1 implies $X^*(a_i) = 0 \in R^n$ all i. But, by construction, $(X^*(a_1), ..., X^*(a_M)) \in S$, contradiction. ∎

4. Proof of Theorem 2

We begin with a second lemma, establishing a general connection between non–historical variables and existence possibilities for stationary sunspot equilibria. Lemmas 1–2 yield Theorem 2.

For a model with only historical variables, sunspot equilibria are impossible: if all the components of state variable x are historical ($x_t = u_t$), then all the elements of x_{t+1} are determined by period t; hence, at time t agents cannot rationally anticipate several distinct outcomes for x_{t+1} related to extrinsic sunspot realizations occurring at $t+1$. Similarly, if the model has a combination of historical and nonhistorical variables, agents at time t can anticipate sunspot variations only in the elements v_{t+1} of x_{t+1}.

Thus, we can see

Lemma 2. *Let* $(x(a_1), ..., x(a_M), \Pi)$ *determine a stationary M–sunspot equilibrium. Let* $\Pi = [\pi_{ij}]$. *Suppose* $x(a_i) \equiv (u(a_i), v(a_i))$ *with* $u(a_i) \in R^h$ *being a subvector of historical variables. Then for any* $i = 1, ..., M$, *there must be* $u_i \in R^h$ *such that for every* $j = 1, ..., M$ *with* $\pi_{ij} > 0$, *the subvector* $u(a_j) = u_i$.

The idea of the proof of Theorem 2 is as follows. Suppose that Condition 1 holds and that we have a stationary M–sunspot equilibrium in every open neighborhood $\mathcal{V}$ of x^*. Construct a convergent subsequence as in Section 3. If $\pi_{ij}^* > 0$ and $\pi_{i\ell}^* > 0$, the argument of Lemma 2 shows that $X^*(a_j)$ and $X^*(a_\ell)$ must have common historical components. Lemma 1 shows they must both be in the stable manifold for (2). Given the last part of Condition 1, we can then see that $X^*(a_j) = X^*(a_\ell)$. Thus, (12) collapses to

$$A \cdot X^*(a_i) = X^*(a_j).$$

Since the same reasoning applies for all i, we must be able to move among $X^*(a_1), ..., X^*(a_M)$ with multiplication by A. With each $X^*(.)$ on the stable manifold for (2), however, such multiplication leads to convergence to 0 for all states which are not transitory (under Π^*). .

Formally,

Proof of Theorem 2. Fix $M < \infty$. Suppose (A1)–(A4) and Condition 1 hold. Fix any T and δ.

If there exists some open neighborhood of $x^* = 0$ not containing any stationary M–sunspot equilibrium, we are done. Otherwise for any $k = 1, 2, ...$ we can find such an equilibrium s_k. As in (8) there must be a convergent subsequence $s_{k(\ell)}$.

Consider any $i = 1, ..., M$. Let $\Pi_k \equiv [\pi_{ij}^k]$ be the transition matrix in s_k. Let $\pi^* \equiv [\pi_{ij}^*]$. If $\pi_{ij}^* > 0$ and $\pi_{i\ell}^* > 0$, by the definition of Π^* we can find L so large that $\ell > L$ implies $\pi_{ij}^{k(\ell)} > 0$ and $\pi_{i\ell}^{k(\ell)} > 0$. Let $x_k(a_m) = (u_k(a_m), v_k(a_m))$. Then Lemma 2 implies there exists u_i such that $u_{k(\ell)}(a_j) = u_i = u_{k(\ell)}(a_\ell)$ all $\ell \geq L$. Let $X^*(a_m) = (U^*(a_m), V^*(a_m))$. Then by construction, $U^*(a_j) = U^*(a_\ell)$.

Lemma 1 shows $X^*(a_m)$ is in the stable manifold for (2) each $m = 1, ..., M$. Hence, the preceding paragraph and the last part of Condition 1 imply $X^*(a_j) = X^*(a_\ell)$ when $\pi_{ij}^* > 0$ and $\pi_{i\ell}^* > 0$. Hence, for any i, if $\pi_{ij}^* > 0$,

$$A \cdot X^*(a_i) = X^*(a_j).$$

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Repeating, if $\pi_{j\ell}^* > 0$,

$$A^2 \cdot X^*(a_i) = X^*(a_\ell).$$

Suppose Theorem 2 is incorrect. Then we could have chosen s_k with

$$H(s_k) \equiv \max_{i=1,\ldots,M} [x_k(a_i)/r(s_k)] \cdot p(i,T,s_k) \geq \delta$$

all k. Define

$$s^* \equiv (X^*(a_1),\ldots,X^*(a_M),\Pi^*).$$

Then

$$\lim_{\ell \to \infty} H(s_{k(\ell)}) = \max_{i=1,\ldots,M} X^*(a_i) \cdot p(i,T,s^*) \equiv H^*.$$

So,

$$H^* \geq \delta. \tag{15}$$

Suppose for some i that $p(i,T,s^*) > 0$. Then $\pi_{ii}^*(m) > 0$ some $m \in \{1,\ldots,T\}$. Thus, going back two paragraphs,

$$A^m \cdot X^*(a_i) = X^*(a_i). \tag{16}$$

Multiply by A^m over and over, and using (16), for any $v = 1,2,\ldots$,

$$A^{m \cdot v} \cdot X^*(a_i) = X^*(a_i). \tag{17}$$

So, using Lemma 1,

$$X^*(a_i) = \lim_{v \to \infty} A^{m \cdot v} \cdot X^*(a_i) = 0. \tag{18}$$

Hence, for any $i = 1,\ldots,M$,

$$p(i,T,s^*) = 0 \quad \text{or} \quad X^*(a_i) = 0.$$

Thus, $H^* = 0$ — contradicting the supposition that Theorem 2 is incorrect. ∎

5. Conclusion

Theorems 1 and 2 establish a relationship between determinacy and the existence of sunspot equilibria. This enhances the value of learning about a given stationary state's (local) determinacy (see, for example, [14]) — although it unfortunately seems to have little bearing on the existence of stationary sunspot equilibria with at least some state outcomes isolated from x^*.

Bibliography

1. C. Azariadis, Self–Fulfilling Prophecies, *Journal of Economic Theory* 25 (1981), 380–396.

2. C. Azariadis and R. Guesnerie, Sunspots and Cycles, *The Review of Economic Studies* 53 (1986), 725–738.

3. D. Begg, "The Rational Expectations Revolution in Macroeconomics", The Johns Hopkins University Press, Baltimore, 1982.

4. O. Blanchard and C. Kahn, The Solution of Linear Difference Models Under Rational Expectations, *Econometrica* 48 (1980), 1305–1311.

5. L. Broze, C. Gourieroux and A. Szafarz, Solutions of Linear Rational Expectations Models, *Econometric Theory* 1 (1985), 315–340.

6. D. Cass and K. Shell, Do Sunspots Matter? *Journal of Political Economy* 91 (1983), 193–227.

7. G. Calvo, On the Indeterminacy of Interest Rates and Wages with Perfect Foresight, *Journal of Economic Theory* 19 (1978), 321–337.

8. C Gourieroux, J. Laffont and A. Monfort, Rational Expectations in Dynamic Linear Models: Analysis of Solutions, *Econometrica* 50 (1982), 409–426.

9. J. Grandmont, Stabilizing Competitive Business Cycles, *Journal of Economic Theory* 40 (1986), 57–76.

10. J. Grandmont, Local Bifurcations and Stationary Sunspots (Part I), mimeo, CNRS and CEPREMAP Paris (1986).

11. R. Guesnerie, Stationary Sunspot Equilibria in an N Commodity World, *Journal of Economic Theory* 40 (1986), 103–127.

12. T. Kehoe and D. Levine, Comparative Statics and Perfect Foresight in Infinite Horizon Economies, *Econometrica* 53 (1985), 433–454.

13. J. Laitner, The Definition of Stability in Models with Perfect Foresight, *Journal of Economic Theory* 28 (1982), 347–353.

14. J. Laitner, Transition Time Paths for Overlapping–Generations Models, *Journal of Economic Dynamics and Control* 7 (1984), 111–129.

15. M. Woodford, Indeterminacy of Equilibrium in the Overlapping Generations Model: A Survey, mimeo, Columbia University, 1984.

16. M. Woodford, Stationary Sunspot Equilibria in a Finance Constrained Economy, *Journal of Economic Theory* 40 (1986), 128–137.

17. M. Woodford, Stationary Sunspot Equilibria, mimeo, University of Chicago (1986).

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