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Abstract. The study of sequential decision processes has been important for theories of statistical decisionmaking, dynamic programming, optimal stopping, search, and investment decisions. These literatures have emphasized the benefits of incrementalism and the resulting value of information. However, the costs arising from asymmetries in access to newly-arriving information have been uniformly ignored. This paper presents a preliminary inquiry into the costs of information in sequential economic decisionmaking problems. An investment project with several stages is modeled, in which the project manager and the investors have differential access to new information about project value. An optimal contract is derived specifying financing terms and project management rules. Two general conclusions are developed: first, asymmetric information in sequential decision problems can be quite costly; second, modeling investment decisions as sequential problems with asymmetric information can lead to optimal contracts that are simple, robust and realistic. Applications to a broad range of economic questions are suggested.

Keywords. sequential decisions, principal-agent, dynamic contracts, investment

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1. Introduction

This paper presents a preliminary inquiry into a rather broad question: what are the *costs* of information in a sequential decision-making problem? A rapidly growing economic literature emphasizes the benefits of sequential decision-making, but ignores the costs. Making decisions incrementally allows parties to use newly-arriving information, but interested parties are likely to have differential access to new information. Thus, spreading decisions over time creates opportunities to exploit informational asymmetries. Dynamic asymmetric information generates costs.

The study of sequential decision processes was largely initiated in the statistical decision literature, by Wald [1947]. This field flourished during the 1950s and 1960s, including in particular Bellman's work on dynamic programming [1957] and the extensive work on optimal stopping rules (*e.g.*, Chow *et al.* [1971]). Interest in sequential decision problems has been growing in the economics literature, particularly for search theory and investment modeling applications.[1]

The various literatures on sequential decision problems have uniformly ignored informational asymmetries between different interested parties.[2] Since the essence of sequential

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[1] Sequential decision search theory papers typically model search for a good price, be it for a consumer good or a labor contract (*e.g.*, Rothschild [1974]; Morgan and Manning [1985]; Morgan [1985]). Sequential-decision investment applications emphasize high-risk projects, such as research and development or large projects with long lead times (*e.g.*, Weitzman [1979]; Roberts and Weitzman [1981]; Weitzman, Newey and Rabin [1981]; Bernanke [1983]; Reinganum [1983]; MacDonald and Siegel [1983]; Majd and Pindyck [1985]).

[2] One recent exception is Tirole [1986], who studies procurement contracts.

decisionmaking problems is the value of information, the possibility of asymmetric information costs is fundamental to a complete analysis.

In economic contexts, it seems as natural to presume that there *will* be informational asymmetries as that there will not. For example, a typical investment project will involve the interests of managers and investors. Wage contracting in a labor search model depends on the differential information available to workers and firms. In general, any firm activity which uses dynamically changing information will be affected by asymmetries between the parties to the firm's "nexus of contracts": employees, managers, shareholders, creditors, regulators, etc. Thus, the results below are applicable to such varied problems as the role of the bankruptcy mechanism and the costs of external financing (such as debt or limited partnerships); the design of optimal patent policy; and the decision to close a factory.

Our purpose is to investigate optimal decision rules in a sequential decision problem when information is obtained asymmetrically. The effect of asymmetries on the value of information is of special interest.

We model a prototypical investment problem, which captures the central characteristics of much of the economics literature cited above. Following Roberts and Weitzman [1981], the problem will be called a sequential development project (*SDP*). The project takes T periods to develop before it produces any revenues, and requires investment of I_t during period t of development. The value of the project is uncertain, but as each development stage is completed, new information about project value is obtained. At each stage, the decision-maker can choose to continue or abandon the project.

This framework is relatively general, yet simple enough to accommodate an intuitive understanding of the optimal decision problem in a dynamic stochastic setting with asymmetric information. Some generalizations and applications are discussed at the end.

Previous studies of *SDP*shave assumed that a project's investors and manager are the same economic actor. In fact, often the financing and management of an *SDP*are separated. When more than one actor has an interest in project outcomes, conflicts of interest can arise. Such conflicts are common in reality.[3] Analyses of *SDP*swhich ignore informational

[3] So common that limited partnership prospectuses are required by law to have a section detailing possible conflicts. Much of the modern finance literature distinguishes between securities as contracts which specify different asymmetries of information and control.

asymmetries exaggerate the beneficial value of information. Existing and proposed policies which subsidize or otherwise encourage decision schemes involving interest conflicts may be costly and ineffective.

Two general conclusions are developed below. First, the agency problem in sequential decision problems may be quite severe, leading to high efficiency costs. This result indicates the need to consider asymmetric information when evaluating *SDPs*. Second, the characteristics of an *SDP*, when put into a dynamic principal-agent model, generate theoretically optimal contracts which are simple, robust and realistic. An emerging view in the principal-agent literature holds that theoretically optimal contracts are often unrealistically complex and nonrobust.[4] The optimal decision contract derived below bears a striking resemblance to contracts actually observed for financing at least some *SDPs*. This result follows from restricting the dynamic problem to one of incentives provision, without insurance characteristics. The implications are wide-ranging. For instance, the analysis suggests reasons for the existence of a bankruptcy mechanism, rather than more complex schemes for the transfer of organizational control and resources.

The paper proceeds as follows. Section 2 presents a description of the *SDP*. Descriptions of the full-information solution, and of a static (one-period) asymmetric information solution are presented as benchmarks for comparison with the dynamic second-best contract. The *SDP* contracting problem is formally treated in Section 3. The results are summarized in Section 4, which then discusses implications and presents a numerical example. Section 5 considers generalizations and other applications of the model; Section 6 concludes.

2. A Model of Sequential Decision Making

This section presents a formal model of the sequential development problem with asymmetric information, and discusses the solution of the problem in two special cases: a two-decision model with full information, and a one-decision model with asymmetric information. These two cases will be referred to as the first-best and static contracts. The

[4] See, *e.g.*, Holmström and Milgrom [1985]; Hart and Holmström [1095].

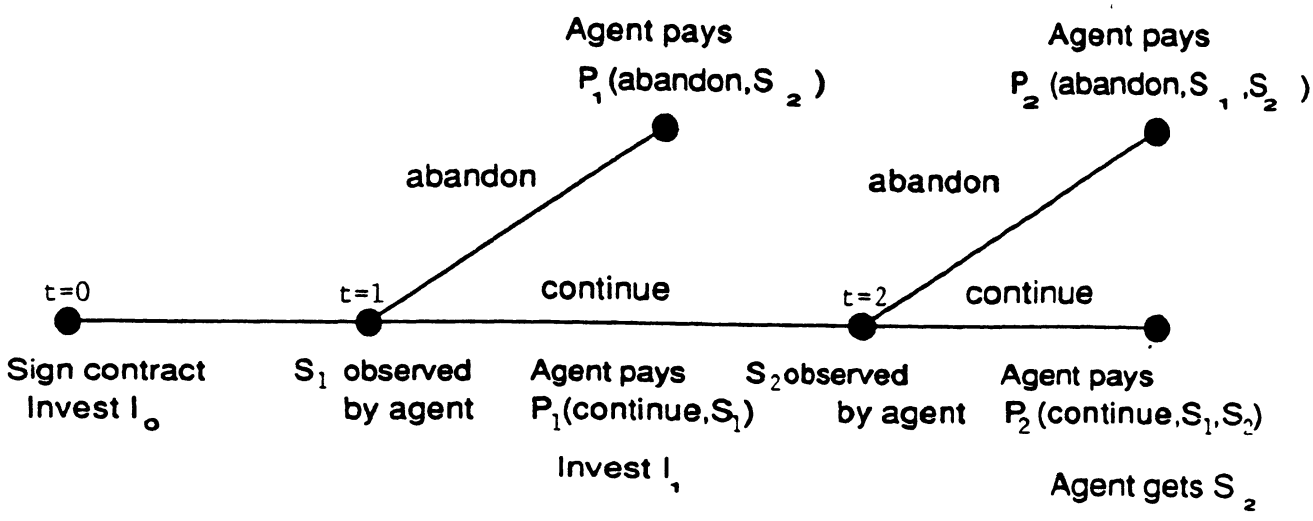

Figure 1. Timing of the Sequential Development Project

next section derives the solution to the multiple-decision problem with asymmetric information.

The problem will be specifically stated for a two-decision, three-period problem. Section 3 first solves the contract for the two-decision case. The model is then generalized to T periods.

Timing

At time t_0, the investor and the firm meet to sign a contract, which specifies the terms according to which the firm will undertake a risky development project requiring a total investment of I dollars (see Figure 1).[5] No revenues are received until the project is completed.[6] The investor puts up I_0 dollars ($I_0 \leq I$) to fund the development effort during the first period, $t \in [0,1]$. A contract's observable terms are legally enforceable at zero cost for the duration of the game.

[5] We will refer to the parties as a firm and an investor to lend concreteness to the discussion. However, the model is generally applicable to any type of principal and agent relationship.

[6] This assumption is made to emphasize the nature of development projects, consistent with the existing literature, but is not necessary for the contract results below. See Section 5.

At time t_1, the firm privately receives some information S_1. (The information structure is detailed below.) The firm then decides whether to continue or stop project development ($D_1 = 1, 0$ respectively). If stopped, the project is abandoned forever.[7] The investor knows whether the firm abandons development. The firm can announce $\hat{S}_1$, a public message concerning its private information. Contracted payments, $P_1(\hat{S}_1, D_1)$, can be made between the parties at t_1. The courts can enforce contract terms which are functions of common knowledge (the message and the continuation decision), but the courts have no access to private information (such as the truthfulness of the message).

We can think of the newly-arriving information as being observable by both parties, but not verifiable by the courts. The important thing is that contracts can only be contingent on *messages*, or public claims about the information, not the information itself. Messages are important because the problem is dynamic: future payments can be contingent on past payments, which can deter the firm from making frivolous or empty announcements.

If the firm continues development, the investor now (t_1) puts up the remaining investment, I_1. At time t_2, the project development is completed. The firm receives another private signal, S_2, and decides whether to complete or abandon ($D_2 = 1, 0$). Another message can be sent to the investor; both message and continuation are again common knowledge. Time t_2 is the end of the sequential investment game, so the parties make the final contracted payments (which can depend on all past and present common knowledge). If the firm completes the project it receives the present (t_2) value of the project.

Information

There is a one-dimensional signal generated by a stochastic process, $S_t \in [0, \bar{S}_t]$.[8] The cumulative distribution of S_t conditional on all prior realizations (denoted by S_{-t}) is given by $F(S_t \mid S_{-t})$. Generalized density functions[9] are assumed to exist, $dF(S_t \mid S_{-t}) =$

[7] In a similar problem, but without asymmetric information, Majd and Pindyck [1985] evaluate a project for which development can be both stopped and restarted. We leave this generalization of our results to further research.

[8] All of the results go through for any general lower bound, $\underline{S}_t$, on the support of S_t. Working with zero merely simplifies the description of the first-best project management.

[9] See DeGroot [1970].

$f(S_t \mid S_{-t})dS_t$. Higher values of S_t represent "good news" about the distribution of S_{t+1}, in the sense of first-order stochastic dominance:

Assumption (*FSD*). *$\int U(S_t)dF(S_t \mid S_{t-1}, S_{t-2}, \ldots, S_0) > \int U(S_t)dF(S_t \mid S'_{t-1}, S_{t-2}, \ldots, S_0) \forall S_t, \forall S_{t-1} > S'_{t-1}, \forall S_{-(t-1)}$, for nondecreasing functions $U(Q)$.*

We also make an assumption on the hazard rate for the conditional distribution:

Assumption (*DHR*). *The hazard rate of S_t is decreasing; i.e,*

$$\frac{\partial}{\partial S_t}\left(H(S_t, S_{-t})\right) \equiv \frac{\partial}{\partial S_t}\left(\frac{1 - F(S_t \mid S_{-t})}{f(S_t \mid S_{-t})}\right) < 0$$

This assumption is common in the incentives literature, and is true for many distributions (*e.g.*, normal and lognormal). The condition needed below is actually weaker than (*DHR*); the role of the assumption will be highlighted when it is used.

The last signal, S_2 in the two-decision problem, is identically equal to the value of the project if completed (*e.g.*, net revenues). S_0 is known before contracts are signed at t_0. S_0 and $F(\cdot \mid \cdot)$ are common knowledge.

The information structure can be thought of in the following way. At t_0, both parties have equal access to what is known about the project, say through published patent materials, or in the case of natural resource exploration, from public records of geological surveys and production from adjacent fields.[10] However, after development is underway, the firm obtains new information from its development work, such as updated estimates of unit production cost. The investor can only learn of the development results through messages ($\hat{S}_t$) from the firm.

Given the timing and information structure above, the payment notation in Figure 1 can be made clear. Stated in general for a many-period *SDP*, let $\hat{\mathbf{S}}_\tau = (\hat{S}_\tau, \hat{S}_{\tau-1}, \ldots, \hat{S}_0)$, and likewise for $\mathbf{D}_\tau$, where D_τ represents the continue/abandon decision. The contract can specify that at time τ the firm will pay the investor an amount $P_\tau(\hat{\mathbf{S}}_\tau, \mathbf{D}\tau)$ which is a function of the firm's messages, $\hat{\mathbf{S}}_\tau \in [0, \bar{S}_\tau]$ and the continuation decisions, $D_\tau \in \{0, 1\}$.

[10] The possibility of *ex ante* asymmetries of information is discussed in Section 5, below.

Preferences and Wealth

The firm is assumed to be risk neutral, to have limited liability, and to be able to lend any wealth it accrues at its discount rate. The firm has zero wealth at time zero, and its borrowing is observable (hence controllable) by the investor. Limited liability with risk neutrality for the firm is often assumed as a tractable way of capturing managerial risk aversion.[11] The investor has substantial wealth and no legal liability limit. The investor is presumed to be risk neutral. Relaxation of the preferences is discussed in Section 5.

First-Best Solution

Suppose the investor has free access to the same information as the firm, and a free right to undertake the project (i.e, the firm does not own the idea for the project). Then, the investor could operate the project herself. (Alternatively, suppose the firm has sufficient wealth to pay the development costs; no investor is needed.) At time t_2, all investment costs are sunk; since S_2 is bounded below by zero, the first-best decision is to complete any project which reaches the t_2 stage. Solving backwards for the t_1 decision, the choice is to abandon for a zero return, or make the incremental investment I_1 to continue development. The optimal choice is to continue if and only if $\delta\,\mathrm{E}[S_2 \mid S_1] \geq I_1$, where δ is the discount factor. That is, the project should be continued at t_1 if the observed signal exceeds a critical value, $S_1 \geq S_1^{FB}$, where S_1^{FB} satisfies the previous condition with equality. The first-best decision rule is extremely simple for the two-decision *SDP*; for certain specifications, full-information rules can also be derived analytically in many-period problems.[12] To summarize, for a project which is commenced at time t_0, we have:[13]

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[11] Limited liability is assumed to hold for each period; i.e, courts will not enforce contract terms which make the agent's wealth go negative in some states of nature. Sappington [1983] has pointed out that risk neutrality with limited liability is related to the more general model, in that limited liability will be a necessary condition in equilibrium for optimal contracts if an agent is risk-neutral for outcomes above the liability limit, L, but infinitely risk averse for outcomes below L.

[12] See, *e.g.*, Roberts and Weitzman [1981]; Weitzman, Newey, and Rabin [1981]. See also the analytical example in Section 4.

[13] We are suppressing the time t_0 decision, which is to initiate the project iff $S_0 \geq S_0^{FB}$, where S_0^{FB} is defined analogously to S_1^{FB}, and guarantees that the expected future discounted cash flows (conditional on future optimal stopping) just cover the incremental investment I_0.

Full-Information Decision Rule for *SDP. At* t_1, *invest* I_1 *and continue iff* $S_1 \geq S_1^{FB}$, *where* S_1^{FB} *is a critical value, defined above. At* t_2, *continue in all states.*

Static Contract

Suppose that no new information is received until the end of the entire development period (or that the investor acts as if the firm learns nothing until the end). After I dollars are sunk, the firm observes the project value, S_2; the investor observes only announcements and whether or not the firm decides to abandon the developed project. Payments only take place at the end of the project, after the continuation decision is observed.

In the next section we shall appeal to the Revelation Principle to justify restricting contracts to those which induce the firm to truthfully report its private knowledge about project value. Making that restriction here, it is clear that the payment from the firm to the investor cannot depend in any way on the value of the project. Suppose payments were contingent on the firm's announcement: then the firm would lie about S_2 so as to minimize its obligations to the investor. Since the game is over, and the investor never directly observes S_2, there is no way to detect or punish the firm for such a lie. Therefore, the firm will only tell the truth as a matter of indifference; i.e, payments cannot depend on the announced project value.

The optimal static contract specifies a fixed payment, X, which the firm makes if it completes the project (and receives S_2). Otherwise, the project is abandoned, and no transfers occur (since the firm has limited liability, the investor cannot demand compensation for her sunk investment costs). The firm's decision is to complete if S_2 exceeds X, or abandon otherwise. The firm's value at the final decision is $\max[S - X, 0]$. An optimal contract solves a constrained Pareto optimizing problem for X^*.

The static contract distorts the final decision (in the first-best, the project is always completed once all costs are sunk), and ignores any opportunity to make decisions incrementally. That is, the static contract suffers from information asymmetries, but does not capture the benefits of dynamic decision-making. The full-information dynamic contract above benefits from incrementalism, but ignores the possibility of asymmetric information. The remainder of this paper is concerned with problems which involve both sequential decision-making and asymmetric information. The next section undertakes the theoretical analysis. The discussion resumes (with a summary of the theoretical results) in Section 4.

3. The Optimal Contract

In this section the optimal financing contract for a sequential investment project is formally characterized. The optimal contracts literature typically solves static contract problems in two stages: first characterizing the set of actions by the agent which a feasible contract can implement, and then solving the principal's problem of optimizing over the set of implementable actions.[14] This approach will be generalized to the dynamic problem and followed here.

We shall first analyze the problem when there are three periods: a contracting period, and two development periods with one continuation decision and one completion decision. Then the results are extended to a problem with any finite number of decisions.[15]

The surprising result shown in the next section is the simplicity of the optimal financing contract for this complicated dynamic problem. A feasible contract can both induce the firm to truthfully report its private information and condition payments on all announcements except the last one. However, an optimal contract will not use the firm's private information. Rather, the optimal dynamic contract names a sequence of constant termination fees, P_τ^0 (the investor pays the firm P_τ^0 if the firm abandons the project at time τ), and a completion fee P_T^1 (paid by the firm to the investor if it completes the project at time T). Whether the firm reports its information or not is a matter of indifference to the investor.

Implementability

The solution of the game is considerably simplified by invoking a basic result from the literature:

[14] See, e.g., Guesnerie and Laffont [1984]. A related two-stage approach is taken in the "hidden action" literature; see, e.g., Grossman and Hart [1983].

[15] This approach is followed because there are few results on dynamic principal-agent contracts in the literature; therefore, a clear presentation of the results requires a careful formal development. The formal proofs are simpler and more intuitive in the two-decision problem than in the T-decision model.

The Revelation Principle. *Without loss of generality, the class of feasible contracts, $\Psi(\hat{\mathbf{S}},\mathbf{D})$ can be restricted to contracts in which the messages sent by a utility maximizing firm are truthful reports of the signals observed (S_1, S_2).*[16]

The revelation principle is exploiting the fact that everything is known by the investor/principal except (S_1, S_2). A contract Ψ is a game form mapping strategies to outcomes. Since the firm/agent's objective is common knowledge, the agent's strategy mapping $\hat{\mathbf{S}}(\mathbf{S})$ is common knowledge for any game form. The truthtelling game merely relabels the strategies from $\hat{S}$ to S.

Define:

$$D_t = \begin{cases} 1, & \text{if the project is continued at time } t \\ 0, & \text{otherwise.} \end{cases}$$

We want to implement $D_1(S_1)$, $D_2(S_1, S_2)$.[17] Without loss of generality, we restrict the analysis to deterministic incentive schemes.[18] Denote the agent's utility from time t_2 on by

$$\omega_2 = -P_2\left(D_2(\hat{S}_1, \hat{S}_2), \hat{S}_1, \hat{S}_2\right) + S_2 D_2\left(\hat{S}_1, \hat{S}_2\right)$$

where carets (ˆ) indicate announcements, and variables without carets indicate true values. At time t_2, the past is history, and past utility cannot directly affect time t_2 decisions (i.e, we shall solve the finite game by backwards recursion). To implement a time t_2 decision which involves time t_2 truthtelling (regardless of the time t_1 message, $\hat{S}_1$) requires:

$$S_2 D_2(\hat{S}_1, S_2) - P_2\left(D_2(\hat{S}_1, S_2), \hat{S}_1, S_2\right) \geq S_2 D_2(\hat{S}_1, S_2') - P_2\left(D_2(\hat{S}_1, S_2'), \hat{S}_1, S_2'\right) \quad (1)$$

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[16] *Informal Proof*: The firm observes $(S_1, S_2) \equiv \mathbf{S}$ and reports $(\hat{S}_1(S_1), \hat{S}_2(S_2)) = \hat{\mathbf{S}}$ to maximize utility. Suppose a contract $\Psi(\hat{\mathbf{S}})$ is feasible. Offer a contract $\Psi^*(\hat{\mathbf{S}}^{-1}(\hat{\mathbf{S}}))$; i.e, if the particular message $\hat{\mathbf{S}}(\mathbf{S}') = \hat{\mathbf{S}}'$, let the payoffs $\Psi^*(\mathbf{S}')$ be identical to $\Psi(\hat{\mathbf{S}}')$. Then, if when the true state is (S_1', S_2') the optimal choice for the firm under Ψ is to get the outcomes associated with $(\hat{S}_1', \hat{S}_2')$, it follows that given choice over the same array of outcomes, the same outcome will be optimal under Ψ^*, and that outcome is available by reporting $\hat{\mathbf{S}}_1 = \mathbf{S}_1'$, the true state. See Dasgupta, Hammond and Maskin [1979] for a more formal statement of the theorem and proof.

[17] We will not be stuck with noncontingent payments as in the static contract described in Section 2. It will become clear that because the problem is dynamic, it will be possible to construct an intertemporal scheme which alternately punishes and rewards the firm in a way which makes truthtelling optimal.

[18] That is, we do not contemplate schemes in which the agent announces $\hat{S}_t$ and the principal gives a *probability* of continuing the project. Usually, if both parties are risk-neutral with respect to income, random schemes are not optimal (see, *e.g.*, Moore 1985). It can be shown in the present case that if $D_t \in [0, 1]$, an optimal contract will always specify either $D_t = 1$ or $D_t = 0$. Jean Tirole pointed out the possibility of random schemes to me.

$$S_2' D_2(\hat{S}_1, S_2') - P_2\left(D_2(\hat{S}_1, S_2'), \hat{S}_1, S_2'\right) \geq S_2' D_2(\hat{S}_1, S_2) - P_2\left(D_2(\hat{S}_1, S_2), \hat{S}_1, S_2\right) \quad (2)$$

for almost all S_2, S_2' and almost all $\hat{S}_1$ such that $D_1(\hat{S}_1) = 1$. Adding (1) and (2) yields

$$(S_2 - S_2')\left[D_2(\hat{S}_1, S_2) - D_2(\hat{S}_1, S_2')\right] \geq 0$$

so $D_2(\hat{S}_1 \hat{S}_2)$ must be nondecreasing in $\hat{S}_2$. This implies that an optimal contract can name a critical value of S_2 (which may be a function of $\hat{S}_1$), $S_2^*(\hat{S}_1)$, such that if S_2 is greater, the project is completed, and is abandoned otherwise:

Proposition 1. *A truthtelling second-period action, $D_2(\hat{S}_1, S_2)$, is implementable only if $\exists\, S_2^*(\hat{S}_1)$ such that $D_2(\hat{S}_1, \hat{S}_2) = 1 \Leftrightarrow S_2 \geq S_2^*$.*

Therefore, payments at time t_2 (P_2) must be independent of $\hat{S}_2$; otherwise, at t_2, given $D_2(\hat{S}_1, \hat{S}_2) = D_2(\hat{S}_1, S_2)$ (i.e, within the region of $\hat{S}_2 \gtrless S_2^*$, for $S_2 \gtrless S_2^*$ respectively), the agent will lie about S_2 so as to minimize its payments to the principal.

Now fold the game back to time t_1. The agent's expected utility is

$$\omega_1 = -P_1\left(D_1(\hat{S}_1), \hat{S}_1\right) + \delta D_1(\hat{S}_1)\,\mathrm{E}\left[\omega_2(\hat{S}_1, S_2) \mid S_1\right]$$

where δ is the one-period discount factor, and the expectation of ω_2 incorporates the implementation of $D_2(\hat{S}_1, S_2)$ and $P_2(D_2, \hat{S}_1)$. To simplify notation, define

$$W(S_1, \hat{S}_1) \equiv \mathrm{E}\left[\omega_2(\hat{S}_1, S_2) \mid S_1\right] = \int_0^{\bar{S}_2}\left[S_2 D_2(\hat{S}_1, S_2) - P_2\left(D_2(\hat{S}_1, S_2), \hat{S}_1\right)\right] f(S_2 \mid S_1)\, dS_2 \quad (3)$$

Applying Proposition 1, it is clear that the integrand in (3) is nondecreasing in S_2; therefore, by first-order stochastic dominance (*FSD*), $W(S_1, \hat{S}_1)$ is strictly increasing in S_1.

To implement $D_1(S_1)$ we require

$$-P_1(D_2(S_1), S_1) + \delta D_1(S_1) W(S_1, S_1) \geq -P_1(D_1(S_1'), S_1') + \delta D_1(S_1') W(S_1, S_1') \quad (4)$$

$$-P_1(D_1(S_1'), S_1') + \delta D_1(S_1') W(S_1', S_1') \geq -P_1(D_1(S_1), S_1) + \delta D_1(S_1) W(S_1', S_1) \quad (5)$$

for almost all S_1, S_1'. Adding (4) and (5) yields

$$D_1(S_1)\Big[W(S_1,S_1) - W(S_1',S_1)\Big] - D_1(S_1')\Big[W(S_1,S_1') - W(S_1',S_1')\Big] \geq 0 \quad \forall S_1,\ S_1' \qquad (6)$$

Suppose $S_1 > S_1'$; since $W(\cdot,\cdot)$ is increasing in its first argument, and $D_1 \in \{0,1\}$, implementability of $D_1(S_1)$ will require that $D_1(\hat{S}_1)$ be nondecreasing. Thus, analogously to Proposition 1, we obtain a critical value result for S_1:

Proposition 2. *A truthtelling first-period action, $D_1(S_1)$, is implementable only if $\exists\ S_1^*$ such that $D_1(\hat{S}_1) = 1 \Leftrightarrow S_1 \geq S_1^*$.*

To complete the characterization of implementability, it is necessary to consider the choice of $\hat{S}_1$ to implement $S_2^*(S_1)$ in a truthtelling equilibrium. Consider all S_1, S_1' such that $D_1(S_1) = D_1(S_1') = 1$. Then from (6) we require

$$\Big[W(S_1,S_1) - W(S_1',S_1)\Big] - \Big[W(S_1,S_1') - W(S_1',S_1')\Big] \geq 0 \qquad (7)$$

Condition (7) states that to ensure truthtelling, the gain in expected time t_2 utility from telling the truth must be at least as great when the truth is good news as when the truth is bad news (i.e, $S_1 > S_1'$). To investigate (7) first consider the definition of $S_2^*(\hat{S}_1)$ in Proposition 1: the project is continued at time t_2 (i.e, $D_2 = 1$) if $S_2 \geq S_2^*$. Let $P_2^1(\hat{S}_1)$ denote the payment $P_2\left(D_2(\hat{S}_1, S_2), \hat{S}_1\right)$ when $D_2 = 1$; similarly define $P_2^0(\hat{S}_1)$. That is, P_2^1 is the completion payment, and P_2^0 the payment if the project is abandoned at t_2. Then, an optimizing agent will continue at time t_2 (i.e, choose $D_2 = 1$) if and only if $S_2 - P_2^1 \geq P_2^0$, or $S_2 \geq P_2^1 - P_2^0$. Together with Proposition 1, this implies that $S_2^*(\hat{S}_1) = P_2^1 - P_2^0$. Writing out time t_2 utility gives

$$\begin{aligned} \mathrm{E}[\omega_2 \mid S_1] &= -P_2^0(\hat{S}_1)F\left(S_2^*(\hat{S}_1) \mid S_1\right) + \int_{S_2^*(\hat{S}_1)}^{S_2} \Big[S_2 - P_2^1(\hat{S}_1)\Big] f(S_2 \mid S_1)dS_2 \\ &= -P_2^0(\hat{S}_1) + \int_{S_2^*(\hat{S}_1)}^{S_2} \Big[S_2 - S_2^*(\hat{S}_1)\Big] f(S_2 \mid S_1)dS_2 \end{aligned} \qquad (8)$$

Then (7) can be rewritten as:

$$\int_{S_2^*(S_1)}^{\bar{S}_2} \left[S_2 - S_2^*(S_1)\right]\left[f(S_2 \mid S_1) - f(S_2 \mid S_1')\right] dS_2$$
$$- \int_{S_2^*(S_1')}^{\bar{S}_2} \left[S_2 - S_2^*(S_1')\right]\left[f(S_2 \mid S_1) - f(S_2 \mid S_1')\right] dS_2 \geq 0 \quad (9)$$

Now, integrate (9) by parts to get

$$\left\{\left[F(S_2 \mid S_1) - F(S_2 \mid S_1')\right]\left[S_2 - S_2^*(S_1)\right]\right\}\Big|_{S_2^*(S_1)}^{\bar{S}_2}$$
$$- \int_{S_2^*(S_1)}^{\bar{S}_2} \left[F(S_2 \mid S_1) - F(S_2 \mid S_1')\right] dS_2 - \left\{\left[F(S_2 \mid S_1) - F(S_2 \mid S_1')\right]\left[S_2 - S_2^*(S_1')\right]\right\}\Big|_{S_2^*(S_1')}^{\bar{S}_2}$$
$$- \int_{S_2^*(S_1')}^{\bar{S}_2} \left[F(S_2 \mid S_1) - F(S_2 \mid S_1')\right] dS_2 \geq 0 \quad (10)$$

But $[F(S_2 \mid S_1) - F(S_2 \mid S_1')]$ evaluated at the upper limit of S_2's support, $\bar{S}_2$, equals zero, as does $\left[S_2 - S_2^*(\tilde{S}_1)\right]$ evaluated at $S_2 = S_2^*(\tilde{S}_1)$ for any $\tilde{S}_1$. Thus the first and third terms of (10) are identically zero. If we then suppose $S_1 > S_1'$, $[F(S_2 \mid S_1) - F(S_2 \mid S_1')] < 0$ by (FSD), so implementability of $S_2^*(S_1)$ requires $S_2^*(\hat{S}_1)$ nonincreasing in $\hat{S}_1$:

Proposition 3. *Given the condition in Proposition 1, $D_2(S_1, S_2)$ is implementable (equivalently, $S_2^*(S_1)$ is implementable) only if $S_2^*(\hat{S}_1)$ is nonincreasing.*

The intuition for Proposition 3 is important for understanding the optimal sequential investment contract: since news is monotonically better as S_1 increases, the agent knows the project is more likely to go through to completion the higher is S_1. The agent thus has a greater interest in lower net completion payments S_2^* if S_1 is higher; truthful reporting of S_1 will only occur if the contract reflects the agent's interest in decreasing S_2^*.

Propositions 1–3 establish the necessary conditions for a sequential investment financing contract to be incentive-compatible. In general, it is feasible to induce truthful revelation of the time t_1 private information, S_1, and to condition contract terms on the value of S_1. However, the next result will demonstrate that it is not optimal to do so; i.e, that $\dot{S}_2^*(S_1) = 0$ in an optimal contract (where "dots" over variables indicate time differentiation).

The Principal's Optimization

Consider the principal's optimization problem:

$$\max_{S_1^*, S_2^*(S_1)} -I_0 + \mathop{\mathrm{E}}_{S_1, S_2} \{\delta P_1(D_1[S_1], S_1) - \delta I_1 D_1(S_1) + \delta^2 D_1(S_1) P_2(D_2[S_1, S_2], S_1)\} \quad (11)$$

$$\text{s.t.} \quad D_1 = \begin{cases} 1, & \text{if } S_1 \geq S_1^* \\ 0, & \text{otherwise} \end{cases}$$

$$D_2 = \begin{cases} 1, & \text{if } S_2 \geq S_2^* \\ 0, & \text{otherwise} \end{cases}$$

$$\dot{S}_2^* \leq 0 \quad \text{almost everywhere (a.e.)}$$

(The agent's individual rationality constraint is never binding because the agent always chooses D_t to obtain the expected maximum of something and zero; therefore the agent's expected utility must always be nonnegative.) Following the discussion leading to equation (8), $\mathrm{E}[P_2(D_2, S_1) \mid S_1]$ can be written as $P_2^0(S_1) + S_2^*(S_1)[1 - F(S_2^* \mid S_1)]$. Thus, the optimizing choice of the function $S_2^*(S_1)$ must maximize

$$\mathop{\mathrm{E}}_{S_1} \left\{ \delta^2 D_1(S_1) S_2^*(S_1) \Big[1 - F(S_2^*(S_1) \mid S_1\Big] \right\} \qquad \text{s.t. } \dot{S}_2^* \leq 0 \quad \text{a.e.}$$

Defining the control variable $\mu(S_1) = \dot{S}_2^*(S_1)$, and the constraint $\mu(S_1) \leq 0$, we can write the Hamiltonian for this subsidiary problem

$$H = \delta^2 D_1(S_1) S_2^*(S_1) \left\{ 1 - F\Big[S_2^*(S_1) \mid S_1\Big] \right\} f(S_1) - \lambda(S_1)\mu(S_1)$$

The necessary and sufficient conditions for an optimal $\mu(S_1)$ are:

$$-\frac{\partial H}{\partial S_2^*} = \dot{\lambda}(S_1) = -\delta^2 D_1(S_1) \left\{ \Big[1 - F(S_2^* \mid S_1)\Big] - S_2^* F(S_2^* \mid S_1) \right\} f(S_1)$$

$$\frac{\partial H}{\partial \mu} = -\lambda(S_1) \begin{cases} \geq 0, & \text{if } \mu \geq 0 \\ = 0, & \text{if } \mu < 0 \text{ (strictly)} \end{cases}$$

$$\lambda(\bar{S}_1) = \lambda(0) = 0 \qquad \text{(transversality)}$$

$$\mu \leq 0$$

where sufficiency follows from the concavity (i.e, linearity) of H^* in μ.

Now we shall demonstrate that the optimal critical value of $S_2^*(S_1)$ is independent of S_1:

Proposition 4. *$\dot{S}_2^*(S_1) = 0$ for all $S_1 \geq S_1^*$ (wlog for all S_1).*

Proof. Suppose $\dot{S}_2^* < 0$ strictly for some interval $S_1 \in [a, b]$. Then, by the complementary slackness condition, $\lambda(S_1) = 0$ for $S_1 \in [a, b]$, which implies $\dot{\lambda} = 0$ for $S_1 \in [a, b]$. If $\dot{\lambda} = 0$ then by the costate equation

$$S_2^*(S_1) = \frac{1 - F\left[S_2^*(S_1) \mid S_1\right]}{f\left[S_2^*(S_1) \mid S_1\right]} \qquad \forall\, S_1 \in [a, b]$$

Implicitly differentiating:

$$\dot{S}_2^* = -\frac{-\frac{\partial}{\partial S_1}\left[H(S_2^*, S_1)\right]}{1 - \frac{\partial}{\partial S_1^*}\left[H(S_2^*, S_1)\right]} \qquad \text{where } H(S_2, S_1) \equiv \frac{1 - F\left[S_2 \mid S_1\right]}{f\left[S_2 \mid S_1\right]} \tag{12}$$

Since the hazard rate $H(S_2, S_1)$ is assumed to be decreasing in S_2,

$$\operatorname{sgn}(\dot{S}_2^*) = \operatorname{sgn}\left\{\frac{\partial}{\partial S_1}\left[H(S_2^*, S_1)\right]\right\}$$

The (FSD) condition implies

$$\frac{f(\tilde{S}_2 \mid S_1)}{f(S_2 \mid S_1)} > \frac{f(\tilde{S}_2 \mid S_1')}{f(S_2 \mid S_1')} \qquad \forall\, \tilde{S}_2 > S_2,\ S_1 > S_1'$$

(see Milgrom [1981]), so

$$\frac{\int_{\tilde{S}_2 > S_2} f(\tilde{S}_2 \mid S_1) dF(\tilde{S}_2 \mid S_1)}{f(S_2 \mid S_1)} > \frac{\int_{\tilde{S}_2 > S_2} f(\tilde{S}_2 \mid S_1') dF(\tilde{S}_2 \mid S_1')}{f(S_2 \mid S_1')}$$

which implies

$$\frac{1 - F(S_2 \mid S_1)}{f(S_2 \mid S_1)} > \frac{1 - F(S_2 \mid S_1')}{f(S_2 \mid S_1')} \qquad \forall\, S_2,\ S_1 > S_1'$$

so $H(S_2, S_1)$ is increasing in S_1, and $\dot{\lambda} = 0$ implies $\dot{S}_2^* > 0$. This result contradicts the incentive-compatibility constraint however, so we conclude that $\dot{\lambda} \neq 0$ for any interval $S_1 \in [a, b]$, which means $\lambda(S_1) \neq 0$ on any interval, so $\mu = \dot{S}_2^* = 0$ for almost all $S_1 \geq S_1^*$

by complementary slackness. For $S_1 < S_1^*$, $D_1(S_1) = 0$, so we can take $S_2^*(S_1) = \dot{S}_2^* = 0$ for $S_1 < S_1^*$ wlog.[19] ∎

The intuition behind this result is straightforward. The higher is the truthful report of S_1, the better is the posterior distribution of project value, S_2. Therefore, the investor/principal wishes to extract more rents when S_1 is higher, requiring a higher S_2^* (the net completion payment). This is similar to the monopoly pricing problem with unknown reservation prices, or to the Ramsey pricing problem. To see the latter analogy, note that in a first-best situation, the cutoff value of S_2 would be zero (all investment costs are sunk). Therefore, setting a net completion payment $S_2^* > 0$ creates a distortion. However, the higher is S_1, the higher is S_2 likely to be, so the principal wishes to minimize distortions by extracting higher rents in those states of the world where the distortion is less likely to be binding.

It is important to emphasize this result: *it is feasible* for a contract to exploit the firm's private first-period information, and that information is valuable. The conflict between what the investor wants to do with the information and what she can do with it yields a severe inefficiency in *SDP*s with conflicts of interest.[20]

Implementation

If we do not assume that the agent has limited liability and begins with zero wealth, it would be simple to show that the first-best allocation described in Section 2 could be implemented. For example, the principal/investor could sell the project to the firm for its time t_0 expected value, and let the firm make all of the investments and decisions.[21]

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[19] Note that the (*DHR*) assumption was stronger than we need for the result; the proposition follows if $1 - (\partial/\partial S_2^*)\left[H(S_2^*, S_1)\right] > 0$, which is true if the hazard rate rises less rapidly than at rate unity. But this condition to obtain a positive denominator in (12) is identical to the second-order condition for an optimal $S_2^*(S_1)$ for the problem in which $\dot{S}_2^*$ is unconstrained. Therefore, (*DHR*) is not a restrictive condition.

[20] It is not a general result in dynamic problems that contract payments cannot depend on messages about new information. Allen [1985] presents a dynamic model with no actions—just information—in which it is optimal to condition second-period payments on first-period messages. See the discussion below, in Section 4.

[21] Alternatively, with a different distribution of bargaining power at t_0, a contract could specify that

With limited liability the first-best outcome is not implementable. However, the propositions above imply that the structure of the optimal contract is simple indeed. A second-best contract which implements $D_1 = 1 \Leftrightarrow S_1 \geq S_1^*$ and $D_2 = 1 \Leftrightarrow S_2 \geq S_2^*$ (given $D_1 = 1$), is equivalent to a contract which specifies just two constant payments: a termination fee paid by the investor to the firm if the project is abandoned at t_1, and a completion payment from the firm to the investor if the project is completed at t_2.

Proposition 5. *The optimal contract can be implemented by naming two constant payments, $P_1^0 < 0$ and $P_2^1 > 0$, where P_1^0 is paid if $D_1 = 0$, and P_2^1 is paid if $D_2 = 1$.*

Proof. By Proposition 1, implementation of D_2 requires that the firm have incentives to complete the project iff $S_2 \geq S_2^*$. At time t_2, the firm in fact chooses to complete iff $S_2 \geq P_2^1 - P_2^0$. Set $P_2^1 = S_2^*$ and $P_2^0 = 0$. If the firm reaches time t_2 with nonnegative wealth, these payments cannot violate limited liability in any state of the world.

By Proposition 2, an optimal implementation of D_1 requires that the firm continue the project iff $S_1 \geq S_1^*$. At time t_1, the firm continues if $\mathrm{E}[D_2(S_2 - S_2^*) \mid S_1] \geq P_1^1 - P_1^0$. By (FSD), the left-hand side of the inequality is monotonically increasing in S_1. Therefore, set $-P_1^0 = \mathrm{E}[D_2(S_2 - S_2^*) \mid S_1^*]$ and $P_1^1 = 0$. $D_2(S_2 - S_2^*)$ is nonnegative everywhere, so $P_1^0 \leq 0$. Therefore the firm has nonnegative wealth in all states of the world at t_1. ∎

Finally, certain straightforward welfare comparisons can be made between the optimal second-best dynamic contract, the first-best, and a static contract as described earlier. Most of these results are simple and intuitive; formal proofs are omitted for brevity. Recall that a static contract is one that ignores intermediate decisions and information, writing terms only for the completion of the project.

Proposition 6. *(Comparison of a dynamic contract to a static contract.)*

(a) The value of a dynamic contract weakly dominates a static contract.

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the principal would make the investments. The contract would then set $S_2^* = 0$ by letting time t_2 payments be zero. Recall that at time t_1, the first-best decision is to continue if $S_1 \geq S_1^{FB}$, where S_1^{FB} is defined implicitly by $\mathrm{E}\left[S_2 \mid S_1^{FB}\right] = I_1$. Thus, the firm would pay $\mathrm{E}\left[S_2 \mid S_1^{FB}\right]$ to the investor at t_1 if the project were continued. The investor gets her expected reservation utility (zero) and the firm collects all of the project rents. In either solution to the first-best, the agent/firm is the residual claimant on the project.

(b) A long-term (2-period) static contract is equivalent to repeated short-term contracting (i.e, recontracting at the beginning of each period).

(c) If $I_1 > 0$, there will be underinvestment under static contracts relative to dynamic contracts; i.e, some projects for which dynamic contracting is feasible will not be commenced with static contracting.

Proposition 6(a) is true because a dynamic contract can duplicate a static contract by setting $S_1^* = 0$ and S_2^* appropriately; but $S_1^* \neq 0$ in general. The repeated short-term contracts in part (b) work as follows: at time t_0, the parties write a contract specifying time t_1 payments. After the contract terms are fulfilled at t_1, a new contract is written specifying time t_2 payments. Since the agent has no wealth, he can't pay the principal anything at t_1. Since the agent has limited liability, he will always sign any new contract offered at t_1, so the principal need not pay the agent anything at t_1. Then, at t_1, since there have been no payments or observable actions, the principal has no new information; she will offer the agent the same t_2 terms that she would have offered at t_0. The result is identical to writing a contract at t_0 for t_2 which ignores the t_1 continuation decision (the firm goes ahead in all states at t_1 under either static contract scheme).

Proposition 6(c) follows from showing that the dominance in part (a) is in fact strict for projects with $I_1 > 0$; projects with an expected value of zero to the principal under dynamic contracting must have a negative value under static contracting, which violates the investor's rationality constraint.

Similar arguments establish analogous comparisons between the asymmetric and full-information dynamic contracts. We prove one further characteristic to highlight the inferiority of second-best contracts: for projects that are commenced at t_0 under either first or second-best contracting, abandonment is inefficiently high under the second-best at *both* t_1 and t_2.

Proposition 7. *(Comparison of a constrained dynamic contract to the first-best.)*

(a) The value of a first-best dynamic contract strictly dominates a constrained dynamic contract if $I_1 > 0$; therefore some projects feasible in the first-best are infeasible in the second-best (underinvestment).

(b) Both continuation cutoffs are higher under the constrained dynamic contract ($S_1^* > S_1^{FB}$, $S_2^* > S_2^{FB}$) so abandonment is inefficiently high at both decision points.

Proof. (Proposition 7(b)) See Appendix 1.∎

Thus, with a second-best contract, some projects are abandoned at t_1 which would be continued if financing were first-best; likewise at t_2.

Extending the Results to T Periods

The results on an optimal contract for financing sequential investment projects were developed above for a three-period problem (t_0, t_1, t_2); they can be extended to a general T-period problem.[22] We need only add an assumption that the signal process is Markov:

Markov Signal Assumption (MS). $F(S_t \mid S_{t-1}, S_{t-2}, \ldots S_0) = F(S_t \mid S_{t-1})$.

The reason the result goes through is quite simple. Incentive-compatible payments can now be contingent on everything the firm has learned over several past decision periods (not just S_1), which provides more flexibility in designing reward-punishment schemes to induce truthtelling. However, because signals are Markovian, the period $t = \tau - 1$ signal is a sufficient statistic for the expected value of the current $t = \tau$ signal. In static principal-agent problems it is well-known that incentive schemes based on sufficient statistics weakly dominate those based on the underlying information.[23] We generalize the sufficient statistic result to a dynamic problem below, with the result that we can restrict consideration to payment schemes which are based on only current and one-period-prior announcements. Then, since the project life is finite, the game can be solved by backwards recursion. At each stage of the recursion, because of the sufficient statistic result, the structure of the problem is identical to the two-decision problem considered above. After defining somewhat more general notation and establishing the sufficient statistic result, the optimal

[22] Our notational convention specifies that problems have one more period (period 0) than abandonment decision. However, we will casually refer to the general problem as having T periods or T decisions.

[23] This is not what drives the result in Proposition 4. There *is* information useful at time t_2 in signal S_1; the conflict between how the principal would use it and when the agent will truthfully reveal it makes actual use of the information nonoptimal.

contract results follow precisely as before. Proofs are somewhat tedious, however, and are relegated to Appendix 1.

To facilitate symmetry of notation, assume that the final project value, if the project is completed at time T, is S_{T+1} rather that S_T.[24] Then, denoting the agent's utility at time $t = \tau$ for the remainder of the game by ω_τ, we can write the final period utility as

$$\omega_T = -P_T\left(D_T[\hat{S}_T, \hat{S}_{-T}], \hat{S}_T, \hat{S}_{-T}\right) + \delta D_T(\hat{S}_T, \hat{S}_{-T})\, \mathrm{E}\left[\omega_{T+1}(\hat{S}_T, \hat{S}_{-T}, S_{T+1}) \mid S_T\right]$$

where $\omega_{T+1} \equiv S_{T+1}$, and $\hat{S}_{-T} \equiv \{\hat{S}_{T-1}, \hat{S}_{T-2}, \ldots, \hat{S}_1\}$. Applying Bellman's Optimality Principle for dynamic programming, we can write period $t = \tau$ utility as

$$\omega_\tau = -P_\tau\left(D_\tau[\hat{S}_\tau, \hat{S}_{-\tau}], \hat{S}_\tau, \hat{S}_{-\tau}\right) + \delta D_\tau(\hat{S}_\tau, \hat{S}_{-\tau})\, \mathrm{E}\left[\omega_{\tau+1}(\hat{S}_\tau, \hat{S}_{-\tau}, S_{\tau+1}) \mid S_\tau\right] \qquad (13)$$

Proceeding recursively, it is possible to show that implementable abandonment/continuation decisions require critical values, $S_t^*(\hat{S}_{-t})$, as before:

Proposition 8. *$D_t(S_t, S_{-t})$ is implementable only if $\exists\, S_t^*(\hat{S}_{-t})$ such that $D_t(\hat{S}_t, \hat{S}_{-t}) = 1 \Leftrightarrow S_t \geq S_t^*(\hat{S}_{-t})$.*

The next step is to obtain conditions for the implementability of $S_t^*(\hat{S}_{-t})$. To do this, it is helpful to establish the sufficiency of $\{S_t, S_{t-1}\}$ as a basis for payments P_t. Generalizing Milgrom [1981], define a sufficient statistic for the dynamic contracting problem as follows:

Definition. *The function $\Gamma(S_t, S_{-t})$ is called sufficient for $\{S_t, S_{-t}\}$ with respect to the agent's decision D_t, if there exist functions $\eta_t(\cdot) \geq 0$, $h_t(\cdot) \geq 0$ such that*

$$f\left(S_t, S_{t-1}, \ldots S_1 \mid D_t, D_{-t}\right) = \eta_t\left(\Gamma_t[S_t, S_{-t}] \mid D_t\right) \times h_t(S_{-t} \mid D_{-t})$$

for all S and D in the support of $f(\cdot)$.

Then, because S_t is Markovian, the following is true:

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[24] We could without loss of generality assume that $\mathrm{E}[S_{T+1} \mid S_T] = S_T$ and the problems would be identical.

Proposition 9. $\Gamma_t(S_t, S_{-t}) \equiv \{S_t, S_{t-1}\}$ *is a sufficient statistic for* $\{S_t, S_{-t}\}$ *with respect to the agent's decision* D_t.

The notion of a sufficient statistic is that for any prior distribution on the agent's action D_t, the posterior distribution of D_t given $\{S_t, S_{-t}\}$ depends on the observed (truthfully reported) values of $\{S_t, S_{-t}\}$ only through $\Gamma_t(S_t, S_{-t})$. Therefore, the principal gains no additional control over the agent's action D_t by basing payments P_t on information other than Γ_t, or the current and prior value of the signal. This allows us to show that without loss of generality we can restrict the contract to schemes in which payments P_t are contingent only on S_t and S_{t-1}.

Proposition 10. *For the T-period SDP contract,* $\exists$ *an appropriately chosen scheme* $P_t(\Gamma_t)$ *which weakly dominates any scheme* $P_t(S_t, S_{-t})$*, for* $t_1, \ldots, T$.

The intuition behind Proposition 10 is that all of the statistical information about current and future signals at time τ is contained in $T_\tau = \{S_\tau, S_{\tau-1}\}$; conditioning time τ payments on anything other than Γ_τ adds extraneous noise. Using the (weak) risk aversion of the agent, we can maintain the agent's expected utility while (weakly) increasing the principal's expected utility by reducing the extraneous noise in the incentives scheme.

Now, to implement $S_t^*\left(S_{t-1}, \hat{S}_{-(t-1)}\right)$, note that the continuation decision is given by $D_t = 1$ iff $\delta D_t E[\omega_{t+1} \mid S_1] \geq P_t$. Therefore, since S_t^* is equivalent to implementable D_t, we can consider only $S_t^*(\hat{S}_{t-1})$. Proceeding as before, the necessary condition for implementability of $S_t^*(S_{t-1})$ is

Proposition 11. $S_t^*(S_{t-1})$ *is implementable only if* S_t^* *is nonincreasing in* S_{t-1}.

The conditions for implementability of a given action vector D have now been extended from the two-decision to the T-decision *SDP*. It is a straightforward matter to show, as before, that the optimal scheme under limited liability can be implemented by a contract which names a vector of constants:

Proposition 12. *The optimal contract for the T-period SDP is equivalent to one which names a vector of constants* $\left(-P_1^0, \ldots, -P_{T-1}^0; P_T^1\right)$. *The first* $T-1$ *values are the termination fees* P_τ^0 *for each period* $\tau = 1, \ldots, T-1$ *respectively which the investor pays the* *agent if the project is abandoned at time τ. The last value is the completion payment P_T^1 from the firm to the investor if the project is completed at time T.*

By generalizing to a T-decision *SDP*, a sequence of termination fees has been introduced, rather than just one. As before, these fees should not be interpreted as payoffs to the firm, but as incentives to ensure that the firm doesn't go ahead and spend the investor's money if conditions have turned sour. In effect, the value of making a sequence of incremental investment decisions rather than just a once-and-for-all investment comes from the value of the option to abandon the project early. In the present case, when the investor's and project manager's interests diverge, the investor offers the project manager a share of the abandonment option value in an attempt to align their interests.

The abandonment option intuition is borne out neatly in two results we can prove which characterize the termination fee sequence $\{P_t^0\}$. First, we know that because of limited liability, the abandonment payments must be nonpositive, $P_t^0 \leq 0$. In fact, if we assume that $S_t = 0$ is an absorbing state of the Markov process (i.e, $\Pr[S_\tau = 0 \mid S_t = 0] = 1 \quad \forall\, \tau \geq t$), then, when a period's investment is nonzero, the termination fee is strictly negative:

Proposition 13. *If $I_t > 0$ and $S_t = 0$ is an absorbing state of the Markov sequence $\{S_t\}$, then $P_t^0 < 0 \quad \forall\, t < T$.*

The simple intuition for Proposition 13 is that as long as $I_\tau > 0$, there is some abandonment option value at time τ, and the investor is better off sharing that value with the firm than not (if $P_\tau^0 = 0$, the investor would go ahead at time τ in all states of the world).

Even more importantly, we can establish that the termination fee sequence is nonincreasing. Formally,

Proposition 14. $-P_t^0 \geq -P_{t+1}^0 \qquad \forall\, t = 1, \ldots, T.$

This result also accords with the abandonment option value notion: the greater the remaining number of stages in the project, the greater the remaining investment. Therefore, the opportunity cost of continuing a bad project at time τ is greater than at time $\tau + 1$; the investor accordingly wants to give the firm a larger incentive to abandon at time τ.

4. Discussion and Example

In this section we summarize the results characterizing the optimal principal-agent contracts for sequential investment projects. Then, following a brief discussion of the results, we shall present a numerical example for a three-period problem. The example suggests the relative importance of agency costs in evaluating *SDPs*, and is useful for discussing the comparative statics of the problem.

Summary

We have examined a prototypical *SDP*: a development project which will take T periods to develop before it produces any revenues, and which requires development expenditures of I_t each period before completion. The theoretical analysis of the previous section yields the following description of the optimal contract between investor and firm for an *SDP*:

Optimal *SDP* Contract. *The investor pays I_0, the development investment for the period between t_0 and t_1. At t_1, after obtaining information from the first development stage, the firm decides whether or not to continue the project. If the firm abandons, the investor pays a fixed termination fee to the firm, P_1^0. If the firm continues, the investor pays I_1, the development cost for the next stage. The sequence is repeated at each t, with the firm facing the choice to abandon and take P_t^0 or to continue. If the project passes the final development stage, at time T, the firm can abandon and receive nothing; or continue, receiving the (certain) project value and paying the investor a flat completion fee, P_T^1.*

Therefore, given the investment sequence $\{I_t\}$, the optimal contract is completely specified by the payment sequence $\{-P_t^0; P_T^1\}$ (for $t = 1, \ldots, T-1$). From the firm's point of view, after the contract is signed the *SDP* proceeds precisely as it does in models which ignore agency problems; the opportunity cost of each development stage, however, is the foregone termination fee. It is straightforward to analyze the behavior of the firm, and to perform comparative statics on that behavior, after the contract is signed.[25]

In our simple model, the conflicts of interest change not the qualitative description of project management, but rather the efficiency of the investment decisions in such projects.

[25] See, *e.g.*, Roberts and Weitsman [1981] for such an exercise.

It was shown in the previous section that the firm will abandon a project in more states of the world at any development stage under a dynamic contract than it would if financing were first-best. Since the identical project is less likely to be completed under a dynamic contract than in the first-best, it is clear that some projects will not even be started with external financing which should be initiated according to a social efficiency criterion. It was also shown that for a project which *is* initiated, expected abandonment at each stage is inefficiently high. Thus, a world with external financing will have inefficiently low levels of investment in risky development projects.

The emphasis on the value of sequential decision-making in the existing literature is exaggerated if asymmetric access to information is present, because much of the information made available by sequential decision-making is not used efficiently in optimal project management. In principle, the contract can induce the firm's manager to truthfully reveal his private information, and to condition payments on that information. A fundamental conflict prevents this, however. The better is the first-period signal, the more likely is the firm to ultimately complete the project. Thus, to induce the manager to truthfully reveal S_1, he must be offered a lower payment schedule for t_2; since he knows he's more likely to complete, he has a greater interest in lower payments. On the other hand, the investor/principal wants to extract maximal rents from the project, by minimizing distortions in the final decision. In the first-best, a project which reaches the final decision is always completed. Like a Ramsey-price setter, the principal wants to extract higher completion payments when S_2 is likely to be high, because that is less likely to distort the completion decision. S_2 is likely to be higher when S_1 is higher, so the principal wants higher payments when S_1 is high. These two interests directly conflict; the result is that payments (and actions) are conditioned on the wrong cutoff signal.

Implications for Contract Complexity

It has been considered a general result in the literature that optimal contracts use all of the information available. In contrast, we have developed a general model in which contracting parties can do better by essentially ignoring some information; the outcome is a simple and intuitive contract.[26]

The crucial assumption is that of the agent's risk neutrality.[27] With risk-neutral agents, the problem between principal and agent is one of providing only incentives, rather than both incentives and insurance, or risk-sharing. By contrast, Allen [1985] presents a model of pure risk-sharing (there are no actions whatsoever, and hence, no incentives problems), which yields dynamic contracts which are complex and difficult to implement (i.e, second-period payments are a continuous function of truthful announcements of the first-period signal).

When the contracting problem is one of dynamic incentives provision, the complexity of the contract is determined not by the complexity of the stochastic state space (a continuously-distributed signal process in this case) and the associated message game, but by the specification of the verifiable *action* space. In the present model, the action space is quite simple — a stop-go decision is made at each juncture — and the optimal contract is correspondingly simple (a single decision payment at each node).

The implications of these results for more general dynamic agency problems depend on what we think is important about individual contracting relationships — insurance or incentives — and what we think the complexity of the verifiable action space is. If we think principal-agent contracts are a setting for behavioral incentives, and that insurance is obtained elsewhere (*e.g.*, in futures markets), then the first condition for simple contracts is met. If we believe that full contingency specification in contracts is expensive, and that the unwillingness of courts to second-guess "reasonable business judgment" reduces the verifiability of more than a few, clearly-defined actions, then the second condition might be met, and our theoretical model might appropriately be interpreted as a general statement about the relative simplicity of actual contracts.

Numerical Example

We have constructed a numerical example of a three-period *SDP* managed under an optimal contract. The example illustrates several of the points discussed above. First, the contract

26 Guesnerie and Laffont [1984] obtain a similar simplification in their example of a labor-managed firm, but their result follows from an unusual objective function.

27 As mentioned earlier, the principal's risk aversion is inessential to the results.

specification is simple. As a result, the value of the contract to each party has a intuitive interpretation in terms of familiar notions from financial options theory. Second, the efficiency costs of external financing are illustrated, and prove to be quite dramatic. It turns out that in the example below, investment will not take place for projects which, in a first-best setting, yield a 60% expected excess rate of return. Third, comparative statics exercises are straightforward, and indicate the sensitivity of the efficiency costs to different parameters of the problem.

Suppose that the signal S_t is generated by a Wiener diffusion process, $dS_t = \alpha S dt + \sigma^2 S dz$; α and σ constant imply S_t is distributed lognormally.[28] Denoting the termination fee by X_1, the completion fee by X_2, and the rate of time preference by r, it can be shown that the *ex ante* expected value of the contract to the firm is given by[29]

$$W_2(\cdot) = S_0 e^{-\mu\tau_2}\Phi_2(h_2, k_2; \rho) - X_2 e^{-r\tau_2}\Phi_2(h_1, k_1; \rho) + X_1 e^{-r\tau_1}\left[1 - \Phi(h_1)\right] \tag{14}$$

where τ_i is the length of time from contract signing until the completion of stage i ($\tau_1 = 1$, $\tau_2 = 2$ in the theoretical presentation earlier); $\Phi_2(\cdot, \cdot; \rho)$ is the bivariate standard normal c.d.f. with correlation coefficient ρ; and $\Phi(\cdot)$ is the univariate normal c.d.f., with

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[28] See, e.g., Merton [1973], or Chow [1981].

[29] The derivation was presented in the earlier version of this paper, but involves the evaluation of double integrals, and is now omitted for brevity. Details available from the author on request.

$$
\begin{aligned}
h_1 =& \frac{\ln(S_0/\bar{S}) + \left[r - \mu - \frac{\sigma^2}{2}\right]\tau_1}{\sigma\sqrt{\tau_1}} \\
h_2 =& h_1 + \sigma\sqrt{\tau_1} \\
k_1 =& \frac{\ln(S_0/X_2) + \left[r - \mu - \frac{\sigma^2}{2}\right]\tau_2}{\sigma\sqrt{\tau_2}} \\
k_2 =& k_1 + \sigma\sqrt{\tau_2} \\
\rho =& \sqrt{\tau_1/\tau_2} \\
\bar{S} & \text{ solves } S\Phi(d_2) - X_2 e^{-r(\tau_2-\tau_1)}\Phi(d_1) - X_1 = 0 \\
d_1 =& \frac{\ln(S_0/X_2) + \left[r - \mu - \frac{\sigma^2}{2}\right](\tau_2 - \tau_1)}{\sigma\sqrt{\tau_2 - \tau_1}} \\
d_2 =& d_1 + \sigma\sqrt{\tau_2 - \tau_1} \\
\mu =& r - \alpha
\end{aligned}
$$

Alternatively, we can write equation (14) as

$$
W_2(\cdot) = C_2(S_0, \tau_1, \tau_2, X_1, X_2) + X_1 e^{-r\tau_1} \tag{15}
$$

where $C_2(\cdot)$ is a function known in finance theory which gives the value of a compound call option. That is, $C_2(\cdot)$ represents the value of an option held at t_0 to buy another option at t_1 for price X_1. The second option is the right to pay X_2 at t_2 to obtain an asset worth S_2.[30]

Equation (15) is a convenient way to view the firm's problem. At τ_0, the firm is guaranteed $X_1 \exp[-r\tau_1]$ (it can costlessly quit at τ_1 and receive the termination payment). At a price of X_1 (giving up the termination fee) it can exercise its option to "stay in the game" at τ_1. If the firm continues the project, then, at a price of X_2 (the payment to the investor), it can obtain the value of the project at τ_2 (S_2).

[30] See Geske [1979] on the theory of compound options. The function $C_2(\cdot)$ is "mispriced" because it is not valued "as if" the expected return on S_t were r instead of α. The mispricing occurs here because S_t cannot be freely traded—the current value of S_t is private information—so arbitrage cannot force the contingent claim price to reflect a risk-free equilibrium rate of return. This discussion is not essential for what follows in the paper. For more on the theory of contingent claim pricing and "mispriced" claims, see Merton [1973], Cox and Ross [1976], and Constantinides [1978].

The value of the project to the investor is also easy to interpret. Let β be the fraction of the total investment I which is spent during the second stage ($\beta I = I_1$). Then the value to the investor is given by:

$$V_2(\cdot) = e^{-r\tau_2} X_2 \Phi_2(h_1, k_1; \rho) - e^{-r\tau_1} X_1 \left[1 - \Phi(h_1)\right] - I\{1 - \beta\left[1 - e^{-r\tau_1}\Phi(h_1)\right]\} \quad (16)$$

$V(\cdot)$ is the present value of the completion payment, X_2, discounted by the probability that the project won't be completed; less the present value of the termination fee discounted by the probability that the project won't be abandoned at τ_1; less the present value of the investment costs discounted for the probability that the project is abandoned before βI is spent.

The total value of the project in the first-best is given by

$$V^* = S_0 e^{-\mu\tau_2}\Phi(m_2) - I\left\{1 - \beta\left[1 - e^{-r\tau_1}\Phi(m_1)\right]\right\}$$

where:

$$m_1 = \frac{\ln(S_0/\bar{S}) + \left[r - \mu - \frac{\sigma^2}{2}\right]\tau_1}{\sigma\sqrt{\tau_1}}$$
$$m_2 = m_1 + \sigma\sqrt{\tau_1}$$

The solution to the optimal contract problem can be obtained numerically given values for the exogenous parameters $(S_0, I, \beta, r, \tau_1, \tau_2, \alpha, \sigma)$. For the examples below and in Appendix 2, we let $S_0 = 100$, and varied the other parameters.[31] Suppose the total undiscounted investment cost is $I = 60$, and that 50% must be paid at both $t = 0$ and τ_1. Let $\tau_1 = 1$, $\tau_2 = 2$. Let the real risk-free rate of return be $r = 0.02$, and the expected rate of gross capital gain on the project be $\alpha = 0.03$.

For these values, an optimal contract will specify that the investor pay the firm a termination fee of $X_1 = 3.98$ if the firm abandons the project at t_1. This fee is 13% of the

[31] For these examples, we made the arbitrary assumption that the capital market conditions and ownership rights to develop the project are such that the firm maximizes its project value, subject to the investor receiving a reservation expected utility of $V^0 = 0$ (i.e, there is an infinitely elastic supply of risk-neutral capital).

second-period investment cost, which the investor would save if the firm terminated the project. The firm agrees to pay the investor $X_2 = 65.7$ at τ_2 if the project is completed.

We define the efficiency cost of asymmetric information as the loss in total surplus, or $V^* - (W_2 + V_2)$. In this example, the value of the project $(W_2 + V_2)$ is 39.3. Under full information, the project is worth $V^* = 42.6$, so the efficiency cost of the agency problem is 3.3, which is nearly 8% of the first best project value. An efficiency cost of 8% might be significant, and as is shown in Appendix 2, the efficiency loss increases rapidly under only slightly less favorable project conditions. Projects with first-best excess returns of 60% may not even be feasible in the second-best.

We have also evaluated the described project with asymmetric and full information under the assumption that there is no sequential decisionmaking opportunity; i.e, the investment costs are still spread over time, but once undertaken, a project continues until the final decision at time τ_2. (Under asymmetric information, this yields the static contract discussed in Section 2.) With no sequential decision making, the second-best project value falls to 36.4; however, the first-best value is still 42.6.[32]

Recent papers have argued that the opportunity to spread operating decisions over the life of a project can substantially increase the value of the project.[33] In the *SDP*, the increase in project value is due to the opportunity to stop a project if the environment turns sour before all of the investment funds are committed. However, the increase in full-information project value due to the benefits of sequential decision making is essentially nil in this example. The project is very attractive in a full-information world: for a present value investment of about 59, the firm can undertake a project with an expected present value of about 101 (recall that $\alpha > r$). The *SDP* will almost surely be completed even with an early abandonment option.

On the other hand, while sequential decision-making is rather unimportant in a full-information world for the example project, the introduction of asymmetric information is quite significant. The efficiency cost of asymmetric information is 8% of first-best value

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[32] To three significant digits. In fact, the no-sequential-decision first-best is slightly less than V^*.

[33] See Majd and Pindyck [1985], Roberts and Weitsman [1981].

in the sequential decision case, and 14.5% in the no-sequence case. Previous papers which ignore asymmetric information report the benefits of sequential decision making without considering the costs.

The comparative statics of asymmetric information costs in this example are discussed in Appendix 2.

5. Extensions

Some generalizations of the model have been considered in MacKie-Mason [1986]. For example, all of the results go through if the firm has some wealth to invest in the project, as long as that wealth is less than the total investment cost. Whatever wealth is available will be invested.[34] Also, the results hold if revenues are received before completion of the decision sequence. The only difference is that the contract also specifies a sequence of continuation payments, in order to transfer part of the revenues to the investor. In another direction, adding *ex ante* asymmetric information leads to a standard "lemons" problem: some "bad" firms will offer fraudulent projects, which leads to an equilibrium with even great investment inefficiency.

Other Applications

The *SDP* is similar to other long-term dynamic economic problems which may involve divergent interests. Long-term debt and patent policy were mentioned at the beginning of the paper. The former is essentially the *SDP* with revenues flowing in during the investment period as discussed above. It might be interesting to generalize even further for the long-term debt problem. Suppose the lender could observe the private firm's information directly, but at substantial cost (say, by writing into the contract a provision allowing the bank to place representatives inside the firm). Suppose further that if the lender forecloses on the debt, the project/firm is not worthless; i.e, it has salvage value (though less than if the present management had been allowed to continue).[35]

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[34] This has been called "maximum equity participation" in the one-period bankruptcy model of Gale and Hellwig [1985].

[35] What we are describing is essentially the same as the model in Gale and Hellwig [1985] but extended

In the situation described, the lender may not want to set a stream of fixed payments which the firm must pay or face automatic liquidation. Instead, the lender may be willing to let the firm default, but let the firm continue in operation with the lender observing full information. Then, if conditions improve, the firm can buy its way back into an arm's-length debt contract, but only on terms less favorable for the remaining periods than the original contract. The foregoing is a rough description of long-term loans with restrictive covenants and bank intervention of the sort observed in reality, and intuition suggests that such a contract might emerge as optimal from the model in this present paper. Alternatively, the model could be interpreted as a theory of bankruptcy which allows for both liquidation and reorganization.

In the case of patent policy, there is a striking resemblance between the contracts embodied in European patent policies and the model developed in this paper. Consider the European patent with renewal fees. The firm is developing a project with a sequential option to abandon. Each year it must pay the renewal fee to stay in the game. The government receives a series of payments, unless the firm lets the patent lapse. In some countries, there are no renewal fees, but a firm must "work" a patent to retain exclusive rights. "Working" may require a sequence of investments. The government wants to optimize some objective function by choosing renewal fees or minimum "working" expenditures subject to self-interested behavior by the firm.

The model is directly applicable to analysis of a firm's decision on when to close a factory, or to exit an industry altogether.[36] The firm's manager faces a sequence of decisions of when to optimally close the factory or the firm, based on newly-arriving information.[37] Interests are likely to diverge between manager and owners, especially when managers have substantial human capital invested in the firm, the rents on which are not residually

beyond a single-period, static contract problem to consider the dynamics of information and decision-making. Aghion and Bolton [1986] have presented a model of long-term debt which is one example of the general problem developed in this paper.

[36] Schary [1986] studies the role of abandonment option value without informational asymmetries in declining industries with application to the cotton spinning industry.

[37] Recall that the model generalizes to the case where revenues are received in every period; see MacKie-Mason [1986].

claimed by the shareholders. Severance payments (or, *e.g.*, relocation assistance) are a natural prediction in this case.

The results also address the capital structure puzzle (i.e, what determines the choice of financing instruments?). The opportunity to exploit asymmetric learning makes external capital more costly to firms than internal capital. In general, internal finance should be preferred to external. In many instances, however, external capital is subsidized, such as through tax code provisions for limited partnerships, interest payment deductibility, and loan guarantees. When subsidies are substantial, there may be a preference for external finance, if asymmetric information costs are not too severe. However, when information that emerges during the life of the project is especially valuable, and one party has better access to that information than the other, internal financing may predominate, despite subsidies to external capital. This notion is consistent with the observation that most high-tech start-ups are financed with venture (equity) capital, while limited partnerships are often used for more predictable, less volatile development projects, or natural resource exploration projects sponsored by established firms.[38]

6. Conclusion

Some answers to the questions motivating this paper have now been suggested. Asymmetric learning in a sequential decision problem may substantially reduce the benefits of incremental decisionmaking. We have derived an optimal sequential decision contract which demonstrates the nature of the resulting inefficiencies. However, the rather stark conflict of interest which can be so costly also leads to simple and realistic contract terms. The emphasis on the value of sequential decision-making in the existing literature is exaggerated if asymmetric access to information is present, because much of the insider information made available by sequential decision-making is not used efficiently in optimal project management.

In a more general model, in which other elements of project design can be specified in the contract, information asymmetries are likely to also affect project management. For

[38] Another, related argument is that the reputation which established firms can demonstrate helps overcome agency costs; start-up ventures have more limited access to reputation.

instance, in an R&D or oil exploration project, the number of tests to run in a given period may be chosen differently with information asymmetries. The differences between fixed-sample-size and sequential strategies is examined in Morgan and Manning [1985].

Perhaps the two most important problems absent from the analysis are the two which have been most studied in the literature: hidden effort and *ex ante* hidden information.[39] Some advances into the area of dynamic contracting have been made for these problems, especially the former.[40]

This paper has presented a model of optimal stopping when there are conflicting interests in the outcome of the optimal stopping problem. Agency costs have been ignored in the economic literature on sequential decision-making. Many economic problems, including bankruptcy and patent or R&D policy, contain elements of an optimal stopping problem.[41] The results and approach of this paper can be profitably applied to such problems.

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[39] Often referred to as "moral hazard" and "adverse selection." Arrow's [1985] "hidden" terminology is more expressive, general, and accurate.

[40] See, e.g., Laffont and Tirole [1985], Holmström and Milgrom [1985], Baron and Besanko [1984], Tirole [1985], Myerson [1985], and Allen [1985].

[41] See, for yet another example, Bollier [1985], who considers the problem of optimal default by third-world debtor nations which can also control the rate of national investment, and thus of return on national debt.

Appendix 1

Proof of Proposition 7(B). Under the first-best, the cutoff is $S_2^{FB} = 0$; $S_2^* > 0$ strictly because the investor gets a return on I only through $S_2^* = P_2^1$.

At t_1, the agent continues the project if

$$\delta \int_{S_2^*}^{\bar{S}_2} S_2 f(S_2 \mid S_1) dS_2 - \delta S_2^* \Big[1 - F(S_2^* \mid S_1)\Big] \geq -P_1^0 \tag{A1.1}$$

Therefore, equation (A1.1) is satisfied with equality at $S_1 = S_1^*$. However, the principal's problem is to choose S_1^* to maximize

$$P_1^0 + \delta \int_{S_1^*}^{\bar{S}_1} \Big\{ -P_1^0 - I_1 + \delta S_2^* \Big[1 - F(S_2^* \mid S_1)\Big] \Big\} f(S_1) dS_1$$

with the necessary condition that

$$-P_1^0 + \delta S_2^* \Big[1 - F(S_2^* \mid S_1^*)\Big] = I_1 \tag{A1.2}$$

The first-best decision at t_1 is to continue if $\delta\, \mathrm{E}[S_2 \mid S_1] > I_1$, so the cutoff S_1^{FB} is the solution to

$$\delta \int_0^{\bar{S}_2} S_2 f(S_2 \mid S_1^{FB}) dS_2 - I_1 = 0$$

or, substituting from (A1.2),

$$\delta \int_0^{\bar{S}_2} S_2 f(S_2 \mid S_1^{FB}) dS_2 - \delta S_2^* \Big[1 - F(S_2^* \mid S_1)\Big] + P_1^0 = 0 \tag{A1.3}$$

Comparison of (A1.2) evaluated at $S_1 = S_1^*$ with (A1.3), applying *FSD*, establishes that $S_1^* > S_1^{FB}$ ∎

Proof of Proposition 8. Define $W_t = W_t\Big[S_t, \hat{S}_t, \hat{S}_{-t}\Big] \equiv \mathrm{E}\Big[\omega_{t+1}(\hat{S}_t, \hat{S}_{-t}, S_{t+1}) \mid S_t\Big]$. (Recall that this convention was used in the two decision problem.) Starting with the last period, $W_T = \int S_{T+1} f(S_{T+1} \mid S_T) dS_{T+1}$, so $W_T\Big[S_T, \hat{S}_T, \hat{S}_{-T}\Big]$ is increasing in S_T by *FSD*. Implementability of $D_T(S_T, \hat{S}_{-T})$ requires

$$-P_T\left[D_T(S_T,\hat{S}_{-T}),S_t,\hat{S}_{-T}\right]+\delta D_T(S_T,\hat{S}_{-T})W_T(S_T,S_T,\hat{S}_{-T})$$
$$\geq -P_T\left[D_T(S'_T,\hat{S}_{-T}),S'_T,\hat{S}_{-T}\right]+\delta D_T(S'_T,\hat{S}_{-T})W_T(S_T,S'_T,\hat{S}_{-T})$$

$$-P_T\left[D_T(S'_T,\hat{S}_{-T}),S'_t,\hat{S}_{-T}\right]+\delta D_T(S'_T,\hat{S}_{-T})W_T(S'_T,S'_T,\hat{S}_{-T})$$
$$\geq -P_T\left[D_T(S_T,\hat{S}_{-T}),S_T,\hat{S}_{-T}\right]+\delta D_T(S_T,\hat{S}_{-T})W_T(S'_T,S_T,\hat{S}_{-T})$$

for almost all S_T, S'_T and almost all $\hat{S}_{-T}$ such that $D_{T-j}(\hat{S}_{-T})=1$, for $j=1,\ldots,T-1$. Adding yields

$$D_T(S_T,\hat{S}_{-T})\left[W_T(S_T,S_T,\hat{S}_{-T})-W_T(S'_T,S_T,\hat{S}_{-T})\right]$$
$$-D_T(S'_T,\hat{S}_{-T})\left[W_T(S_T,S'_T,\hat{S}_{-T})-W_T(S'_T,S'_T,\hat{S}_{-T})\right]\geq 0 \tag{A1.4}$$

Since W_T is increasing in S_T, implementability requires $D_T(\hat{S}_T,\hat{S}_{-T})$ nondecreasing in $\hat{S}_T$.

Now consider W_{T-1}:

$$W_{T-1}\left(S_{T-1},\hat{S}_{T-1},\hat{S}_{-(T-1)}\right)$$
$$=\int_0^{S_T}\Big\{-P_T\left(D_T(S_T,\hat{S}_{-T}),S_T,\hat{S}_{-T}\right)$$
$$+\delta D_T(S_T,\hat{S}_{-T})W_T(S_T,S_T,\hat{S}_{-T})\Big\}f(S_T\mid S_{T+1})dS_T$$

where I presume $\hat{S}_T=S_T$ has been implemented from above. Since D_T and W_T are nondecreasing in S_T, the integrand of W_{T-1} is nondecreasing, and W_{T-1} is increasing in S_{T-1} by *FSD*. Writing the implementability conditions for $D_{T-1}(S_{T-1},\hat{S}_{-(T-1)})$ as for D_T above, and adding, yields

$$D_{T-1}(S_{T-1},\hat{S}_{-(T-1)})\left[W_{T-1}(S_{T-1},S_{T-1},\hat{S}_{-(T-1)})-W_{T-1}(S'_{T-1},S_{T-1},\hat{S}_{-(T-1)})\right]$$
$$-D_{T-1}(S'_{T-1},\hat{S}_{-(T-1)})\left[W_{T-1}(S_{T-1},S'_{T-1},\hat{S}_{-(T-1)})-W_{T-1}(S'_{T-1},S'_{T-1},\hat{S}_{-(T-1)})\right]\geq 0 \tag{A1.5}$$

which requires D_{T-1} nondecreasing in S_{T-1}. This last recursive step is perfectly general for the T-decision problem; solving recursively by first demonstrating that W_{t+1} is increasing in S_{t+1}, then checking the implementability conditions for D_t completes the proof.∎

Proof of Proposition 9. The definition of a sufficient statistic with respect to an action is that the joint density of S conditional on the action vector D be separable as follows:

$$f(S_t,S_{t-1}\ldots,S_1;D_t,D_{-t})=\eta_t\Big(\Gamma_t(S_t,S_{-t});D_t\Big)\times h(S_{-t};D_{-t})$$

where η_t and h are nonnegative (see Holmström [1982]). The statistical notion is that for any prior distribution on D, the Bayesian posterior depends on S only through the statistic Γ. By Proposition 5, D_t is equivalent to a critical value S_t^*, such that $D_t=1\leftrightarrow S_t\geq S_t^*$. Therefore, $f(S_t,S_{-t};D_t,D_{-t})=f(S_t\mid S_{t-1};D_t)\times f(S_{t-1},S_{t-2},\ldots,S_1;D_{-t})$ by the Markovian assumption, and $\Gamma_t=\{S_t,S_{t-1}\}$ is sufficient for D_t.∎

Proof of Proposition 10. Define $\tilde{P}_t(\Gamma_t)$ by (where S denotes $\{S_t,S_{-t}\}$):

$$\begin{aligned}\omega_t\left(\tilde{P}_t(T_t)\right)&=\int_{\Gamma_t(S)=\Gamma_t}\omega_t\Big(P_t(S_t,S_{-t})\Big)\Big[f(S_t,S_{-t};D_t,D_{-t})\;/\;\eta_t(S_t,S_{t-1};D_t)\Big]dS\\&=\int_{T_t(S)=T_t}\omega_t\Big(P_t(S_t,S_{-t})\Big)h(S_{-t};D_{-t})dS\end{aligned} \tag{A1.6}$$

Now, $\mathrm{E}_S\,\omega_t\left(\tilde{P}_t(\Gamma_t)\right)=\mathrm{E}_S\,\omega_t\Big(P_t(S_t,S_{-t})\Big)$, so $\omega_t\left(\mathrm{E}_S\,\tilde{P}_t(\Gamma_t)\right)=\mathrm{E}_S\,\omega_t\Big(P_t(S_t,S_{-t})\Big)\geq\omega_t\Big(\mathrm{E}_S\,P_t(S_t,S_{-t})\Big)$ by Jensen's Inequality (in fact, in the present case, the weak inequality is an equality because ω is linear in P). Therefore, from this fact and (A1.6),

$$\int\tilde{P}_t(\Gamma_t)h(S_t;D_{-t})dS\geq\int P_t(S_t,S_{-t})h(S_t;D_{-t})dS \tag{A1.7}$$

By construction, the agent chooses the same action at each time t, and receives the same expected utility. However, the principal is weakly better off by (A1.7).∎

Proof of Proposition 11. Proceeding as before, we can implement $S_t^*(S_{t-1})$ if and only if (from equation (A1.4))

$$\left[W_t(S_t, S_t, \hat{S}_{-t}) - W_t(S_t', S_t, \hat{S}_{-t})\right] - \left[W_t(S_t, S_t', \hat{S}_{-t}) - W_t(S_t', S_t', \hat{S}_{-t})\right] \geq 0 \qquad (A1.8)$$

for almost all S_t,S_t' such that $D_t(S_t, S_{t-1}) = D_t(S_t', S_{t-1}) = 1$, and almost all $\hat{S}_{t-1}$ such that $D_{t-1} = 1$, given $D_{t-j} = 1$ for all $j = 1, \dots, t-1$. As before, we can write W_t as

$$\begin{aligned} W_t(S_t, \hat{S}_t, \hat{S}_{t-1}) =& \mathrm{E}\left[\omega_{t+1}(S_{t+1}, \hat{S}_t, \hat{S}_{t-1}) \mid S_t\right] \\ =& - P_{t+1}^0(S_{t+1}, \hat{S}_t) \\ &+ \int_{S_{t+1}^*(\hat{S}_t)}^{\bar{S}_{t+1}} \left[W_{t+1}(S_{t+1}, S_{t+1}, \hat{S}_t) - S_{t+1}^*(\hat{S}_t)\right] f(S_{t+1} \mid S_t) dS_{t+1} \end{aligned}$$

where, by backwards recursion, $\hat{S}_{t+1} = S_{t+1}$ is presumed. It now becomes evident that, because of Proposition 9, $W_t(S_t, \hat{S}_t, \hat{S}_{t-1})$ reduces to $W_t(S_t, \hat{S}_t)$. Equation (A1.8) can be rewritten as

$$\begin{aligned} &\int_{S_{t+1}^*(S_t)}^{\bar{S}_{t+1}} \left[W_{t+1}(S_{t+1}, S_{t+1}) - S_{t+1}^*(S_t)\right] \left[f(S_{t+1} \mid S_t) - f(S_{t+1} \mid S_t')\right] dS_{t+1} \\ &\quad - \int_{S_{t+1}^*(S_t')}^{\bar{S}_{t+1}} \left[W_{t+1}(S_{t+1}, S_{t+1}) - S_{t+1}^*(S_t')\right] \left[f(S_{t+1} \mid S_t) - f(S_{t+1} \mid S_t')\right] dS_{t+1} \geq 0 \end{aligned}$$

QBecause $W_t(\cdot, \cdot)$ is monotonic in its first argument, we know that it is differentiable almost everywhere. Therefore, integration by parts, with cancellation of terms which evaluate to zero, yields

$$\begin{aligned} &- \int_{S_{t+1}^*(S_t)}^{\bar{S}_{t+1}} \left[F(S_{t+1} \mid S_t) - F(S_{t+1} \mid S_t')\right] \frac{\partial W_{t+1}(S_{t+1}, S_{t+1})}{\partial S_{t+1}} dS_{t+1} \\ &\qquad + \int_{S_{t+1}^*(S_t')}^{\bar{S}_{t+1}} \left[F(S_{t+1} \mid S_t) - F(S_{t+1} \mid S_t')\right] \frac{\partial W_{t+1}(S_{t+1}, S_{t+1})}{\partial S_{t+1}} dS_{t+1} \end{aligned}$$

For $S_t > S_t'$, $[F(S_{t+1} \mid S_t) - F(S_{t+1} \mid S_t')]$ is negative by *FSD*. The partials in each integrand are identical (by virtue of the independence of W_{t+1} from $\hat{S}_t$), so implementability of $S_{t+1}^*(\hat{S}_t) = S_{t+1}^*(S_t)$ requires S_{t+1}^* nonincreasing in $\hat{S}_t$.∎

Proof of Proposition 12. Available from author upon request.

Proof of Proposition 13. (Sketch) Using Proposition 12, and the necessary conditions of the principal's optimization, we can solve for the optimal termination fee, P_t^0:

$$P_t^0 = \mathrm{E}\left[\sum_{\tau=t+1}^{T-1} \delta^{\tau-t}\left[P_\tau^0(1-D_\tau^*) - I_\tau D_\tau^*\right] + \delta^{T-t} P_T^1 D_T^* \mid S_t = S_t^*\right] - I_t$$

where $D_t^* \equiv D_t D_{t-1}$. By the agent's optimal stopping rule, $P_t^0 = 0 \leftrightarrow S_t^* = 0$. Since $S_t = 0$ is absorbing, $\Pr[D_\tau^* = 1 \mid S_t^* = 0] = 1 \quad \forall\, t < t < T$, and $\Pr[D_T^* = 0 \mid S_t = 0] = 1$. So

$$P_t^0 = \sum_{\tau=t+1}^{T-1} \delta^{\tau-t} P_\tau^0 - I_t < 0 \quad \text{for } I_t < 0.$$

∎

Proof of Proposition 14. (Sketch) By the agent's optimal stopping decision, $-P_t^0 = \delta\,\mathrm{E}\left[\omega_{t+1}(S_{t+1}, S_t^*) \mid S_t^*\right]$. We can prove (see Shiryayev [1978]) that the solution ω to the agent's optimal stopping functional equation

$$\omega_{t+1} = \max\left\{-P_{t+1}^0, \delta\,\mathrm{E}\left[\omega_{t+1}(S_{t+2}, S_{t+1}) \mid S_{t+1}\right]\right\}$$

exists, is unique, and is the least δ-excessive majorant of the $-P_{t+1}^0$. In particular, since $P_{t+1}^0 \leq 0$ almost everywhere (by limited liability), we know that $\omega_{t+1} \geq -P_{t+1}^0 \geq 0$ a.e. Therefore, $-P_t^0 \geq -\delta P_{t+1}^0$.∎

Appendix 2

Comparative Statics for Numerical Example in Section 4

For the following examples, the parameters reported in the text are used as the "base case"; one parameter at a time is varied.

The results of varying the total undiscounted investment cost (I) are shown in Figure 2.[42] For an investment only slightly larger than 60, the efficiency cost increases rapidly. At about $I = 63.5$, contracting becomes infeasible altogether, and the project is never undertaken; yet the full information present value of such a project is about 39. Thus, it turns out that Proposition 7, which states that some socially desirable projects will not be undertaken if financing is external, is not a close question. The cost of the agency problem is so high that, in this example, contracting is infeasible for a project which has a full information excess rate of return of 66%.

Agency costs increase with the investment level because the value to the investor of abandoning the project increases. Therefore, the investor wants to increase the termination fee; i.e, as the abandonment option value increases, offer more to the firm to encourage the firm's interest in abandonment. However, at the same time, a larger total investment requires a larger promised completion payment from the firm to the investor (X_2), to cover both the total investment and the expected value of the termination fee (which is an investment in incentive alignment). Since the likelihood of project completion falls as the completion payment increases, it eventually becomes impossible to increase the expected value of the completion payment. Hence, a contract becomes infeasible.

The pressure on the completion payment to cover both total investment and the termination fee investment is evident in Figure 3. As the investment cost increases, the termination fee initially follows, reflecting the increasing abandonment option value. However, at about $I = 61$, the termination fee begins to decline again, to ease the pressure on the completion payment X_2 to cover the investment cost, I.

[42] The figures below show the effects of project parameters on efficiency cost as a percentage of the first-best project value, to remove scale effects.

The effect of the length of the project development period is shown in Figure 4. For this figure, we varied the final date, τ_2, while holding the relative length of the first period constant ($\tau_2/\tau_1 = 2$). As τ_2 increases, the efficiency cost increases, because the abandonment option becomes more valuable.

The role of project riskiness is illustrated in Figure 5, which plots the efficiency cost against the volatility parameter, σ.[43] It is not surprising that agency costs increase rather rapidly with riskiness. For higher σ, the likelihood of completion falls, increasing the abandonment option value to the investor. In the example shown, contracting becomes infeasible at rather mild values for σ (about 0.23); the full information value of such a second-best infeasible project is 42.6, which provides a 72% excess economic rate of return on investment.

Other comparative statics are presented in Figures 6–9. As the date of the second period investment, τ_1, is delayed (Figure 6), agency costs fall because the abandonment option falls as its life shortens. The effects of the risk-free interest rate, r, and the expected growth rate in S_t, α, have opposite signs (Figures 7, 8). Increasing the discount rate, r, reduces the present value of the completion payment, X_2, making it more difficult to cover the project investment and termination fee, so the parties are less able to align their incentives through the termination fee. Increasing a has precisely the opposite effect, as well as making it relatively more likely that S_2 will be high enough for the project to be completed.

Somewhat surprisingly, the efficiency costs fall as the fraction invested in the second period (β) increases (Figure 9). What is happening here is that the net present value of total investment is declining as more is delayed until τ_1; thus, although the abandonment option value is increasing, the investor is able to invest substantially more in the termination fee because there is more slack in the completion payment, X_2. This is indicated in Figure 10, which shows the termination fee increasing as a percentage of the present value of the τ_1 investment.

[43] An increase in σ, is equivalent to an increase in riskiness in the Rothschild-Stiglitz [1970] sense. See Merton [1973].

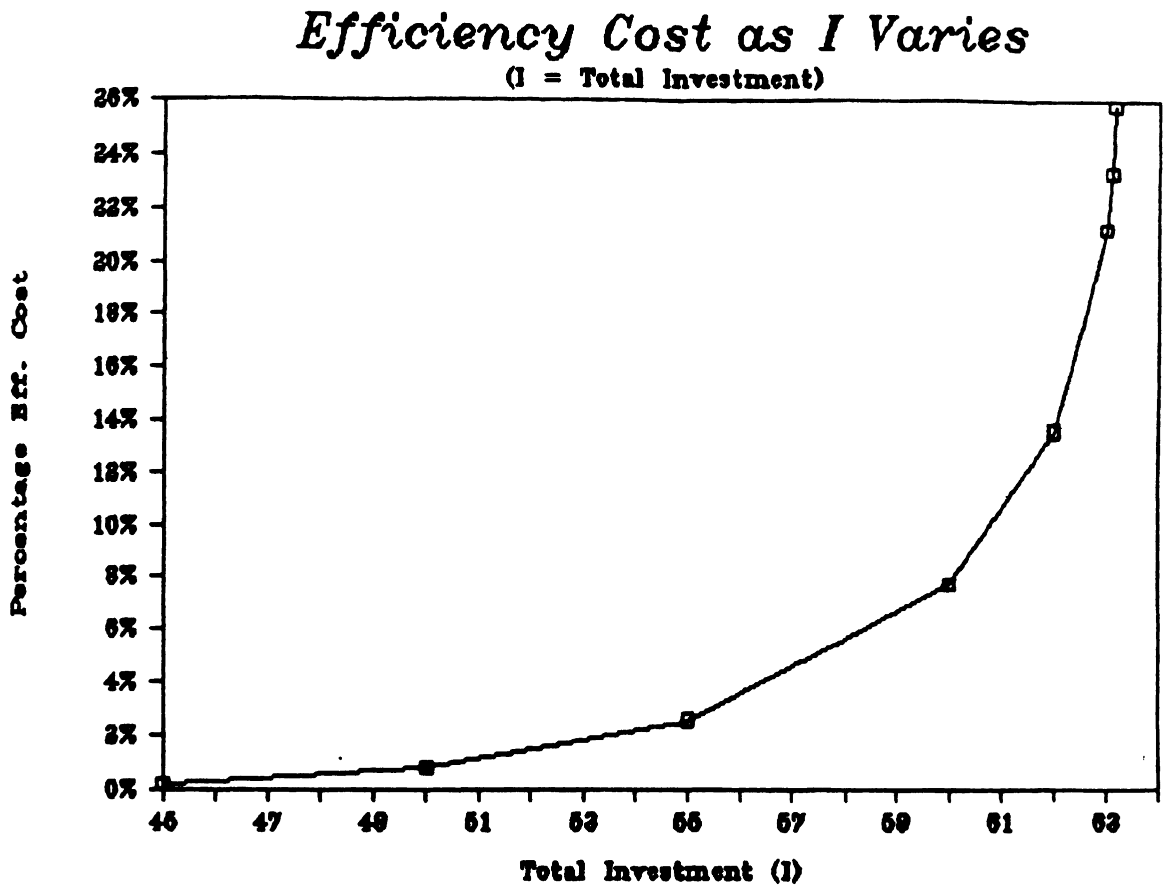

Figure 2

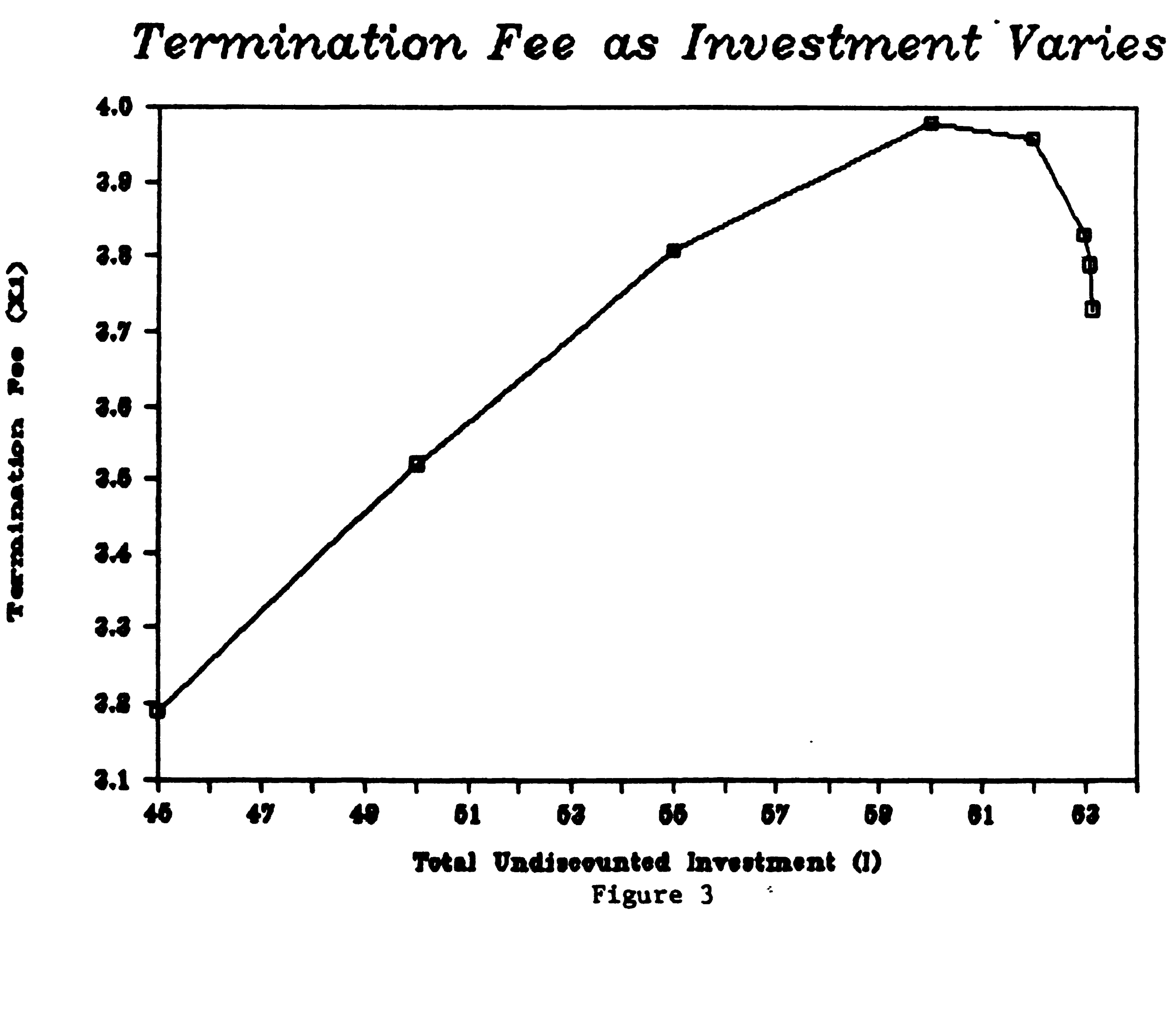

Figure 3

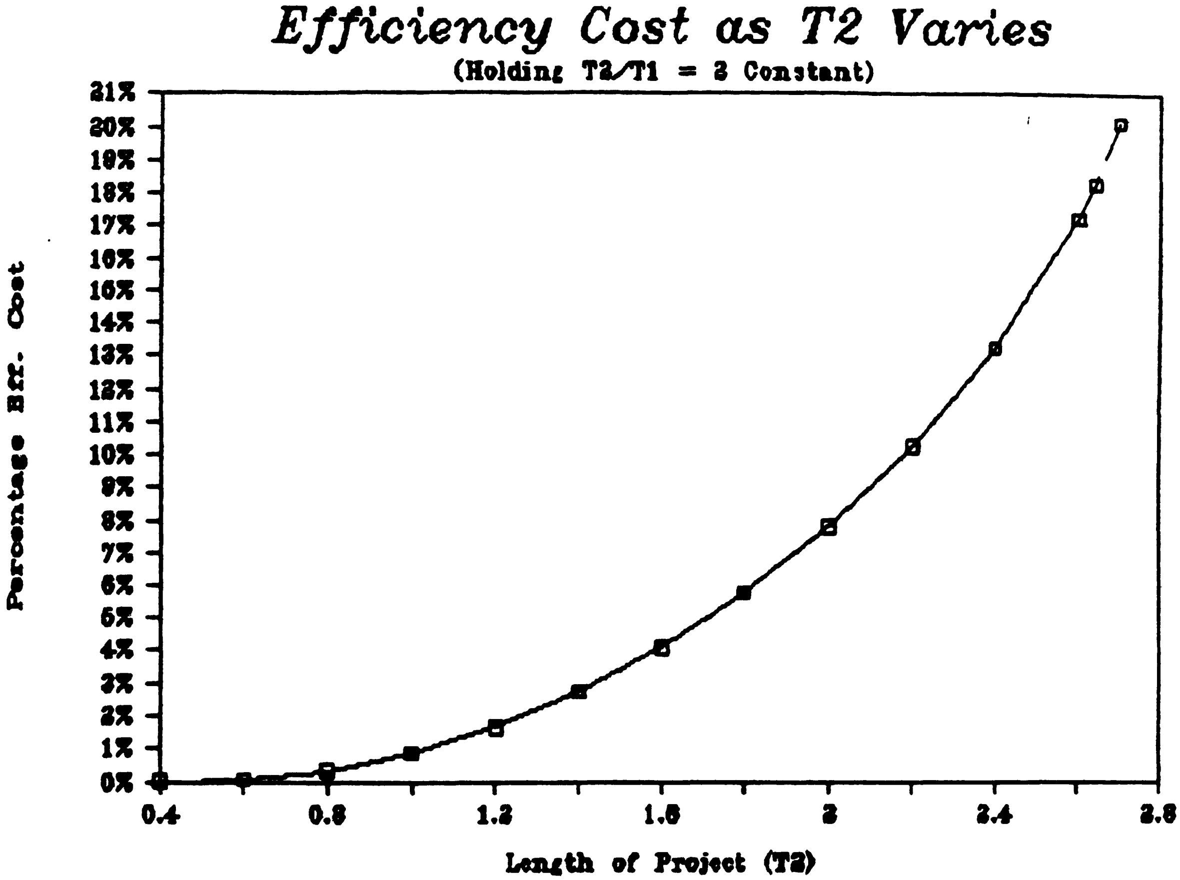

Figure 4

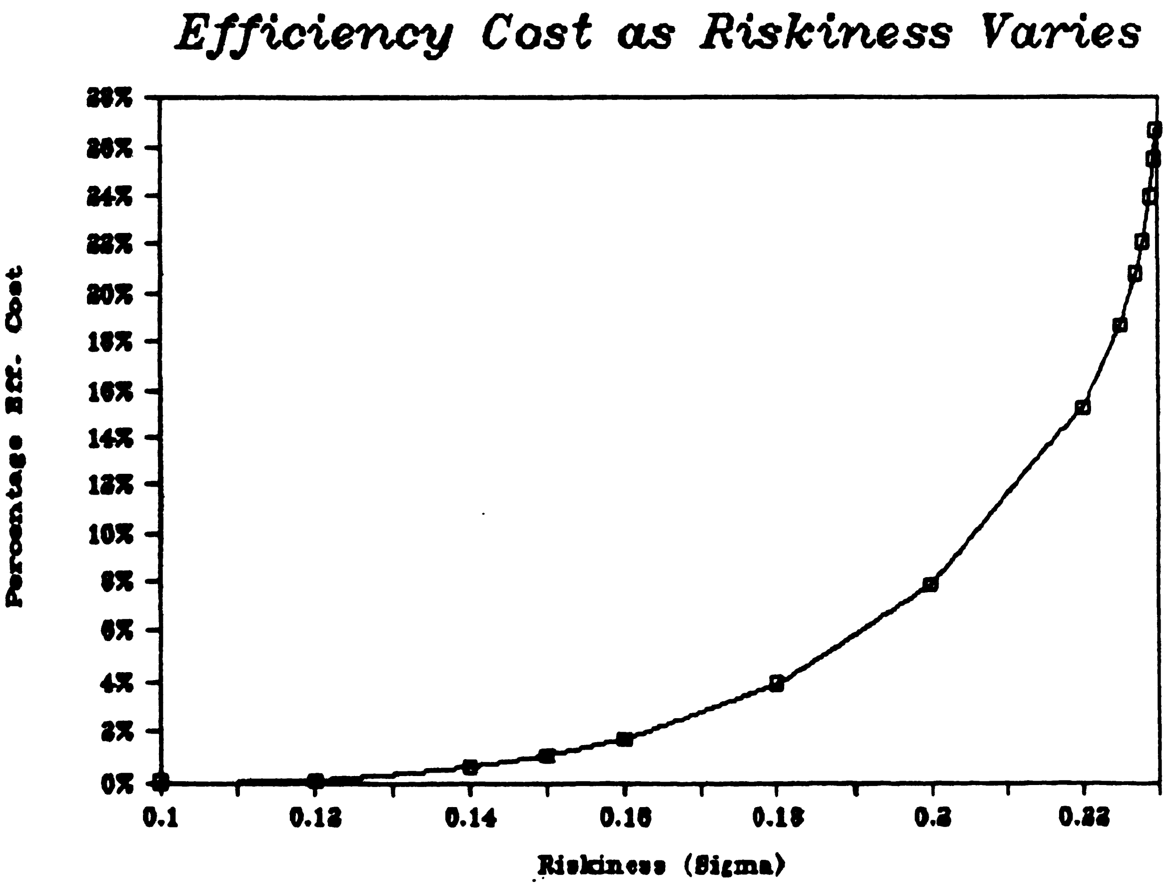

Figure 5

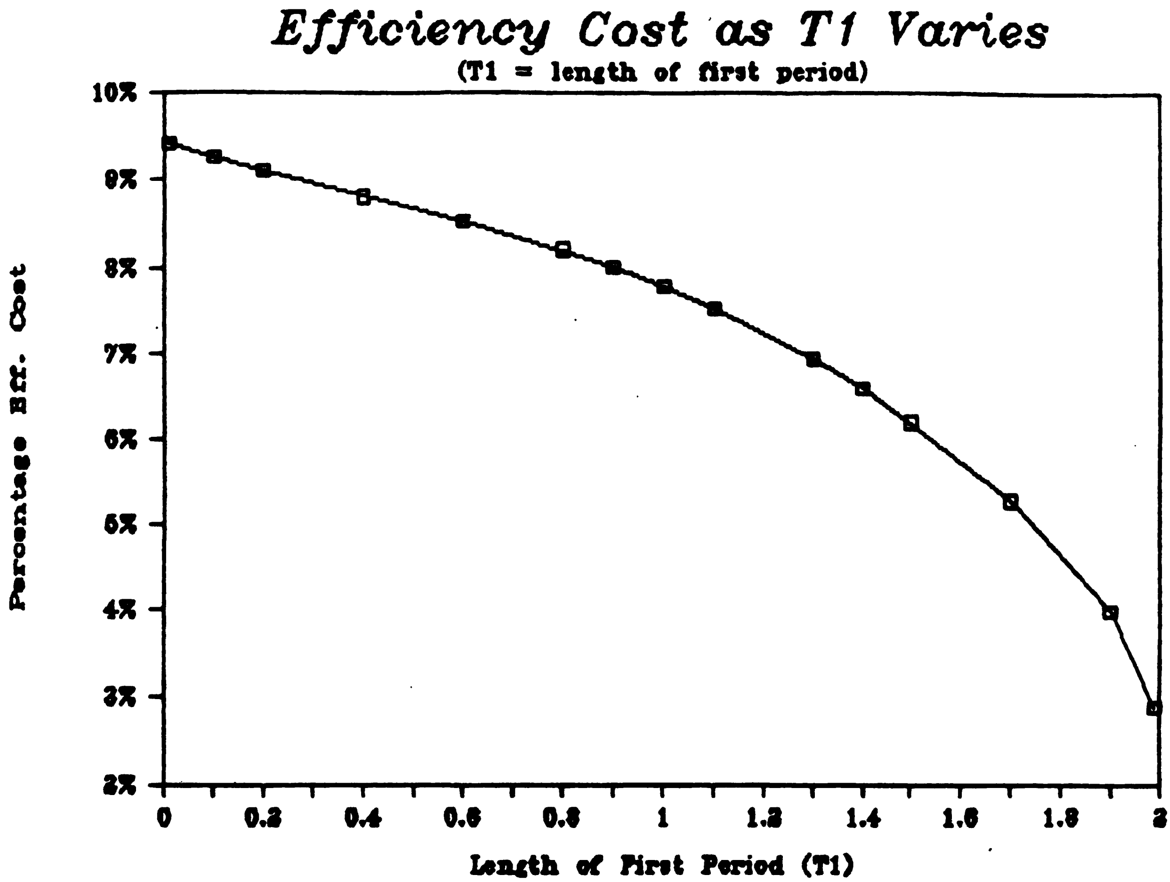

Figure 6

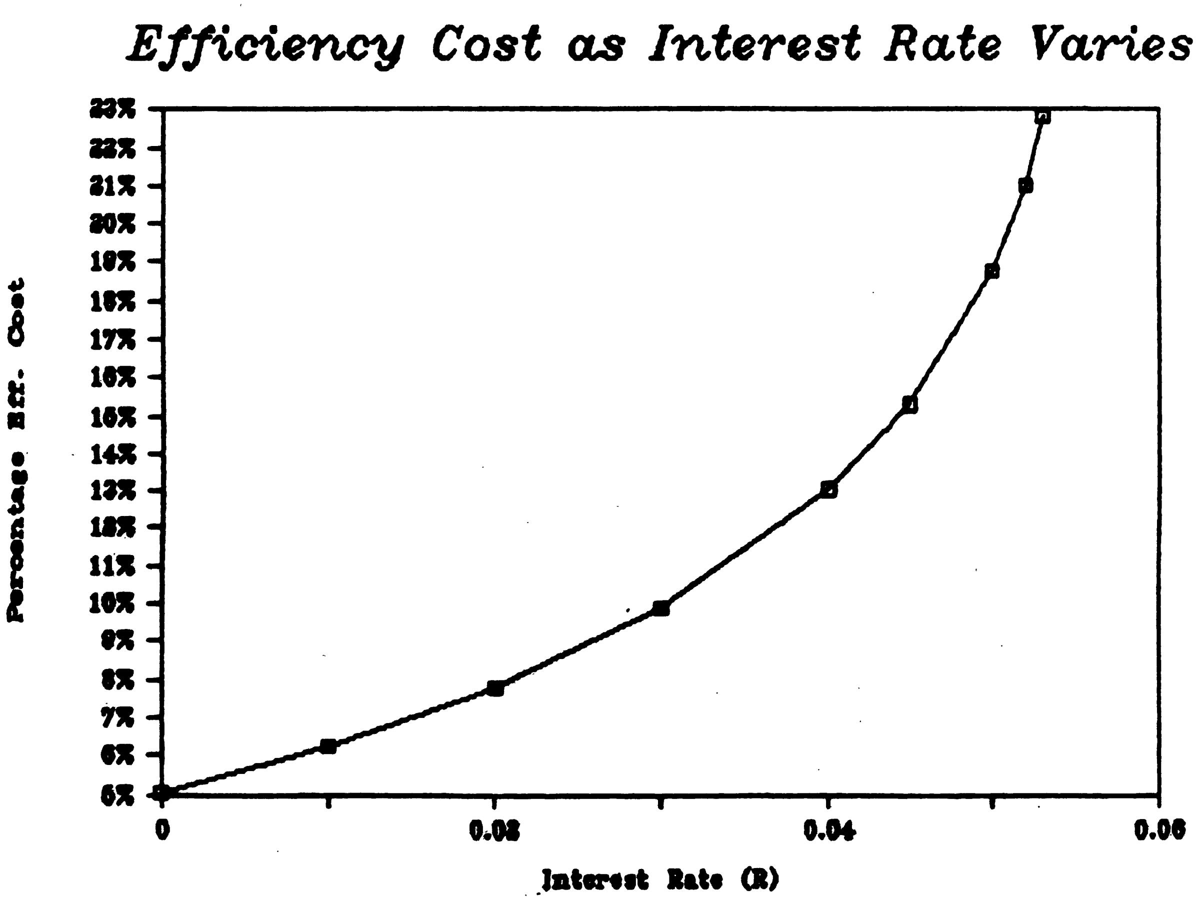

Figure 7

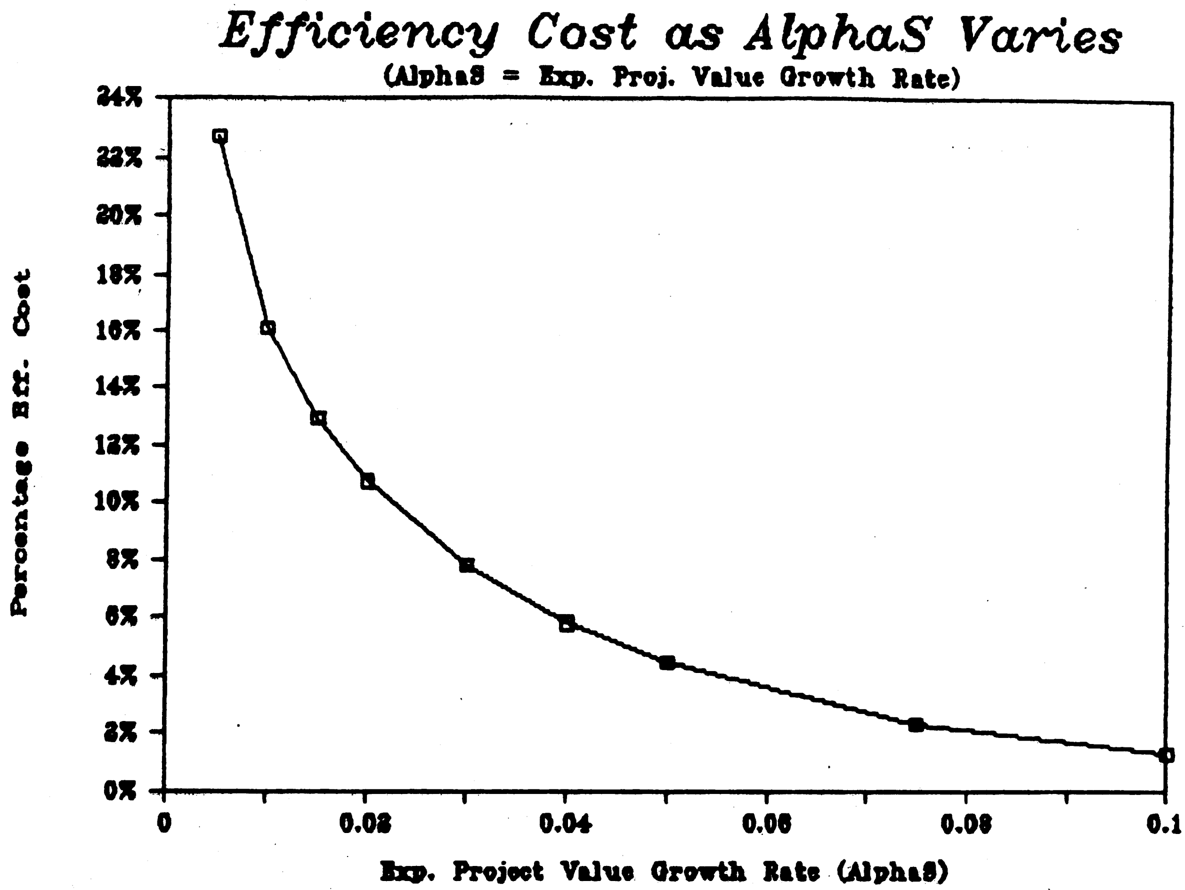

Figure 8

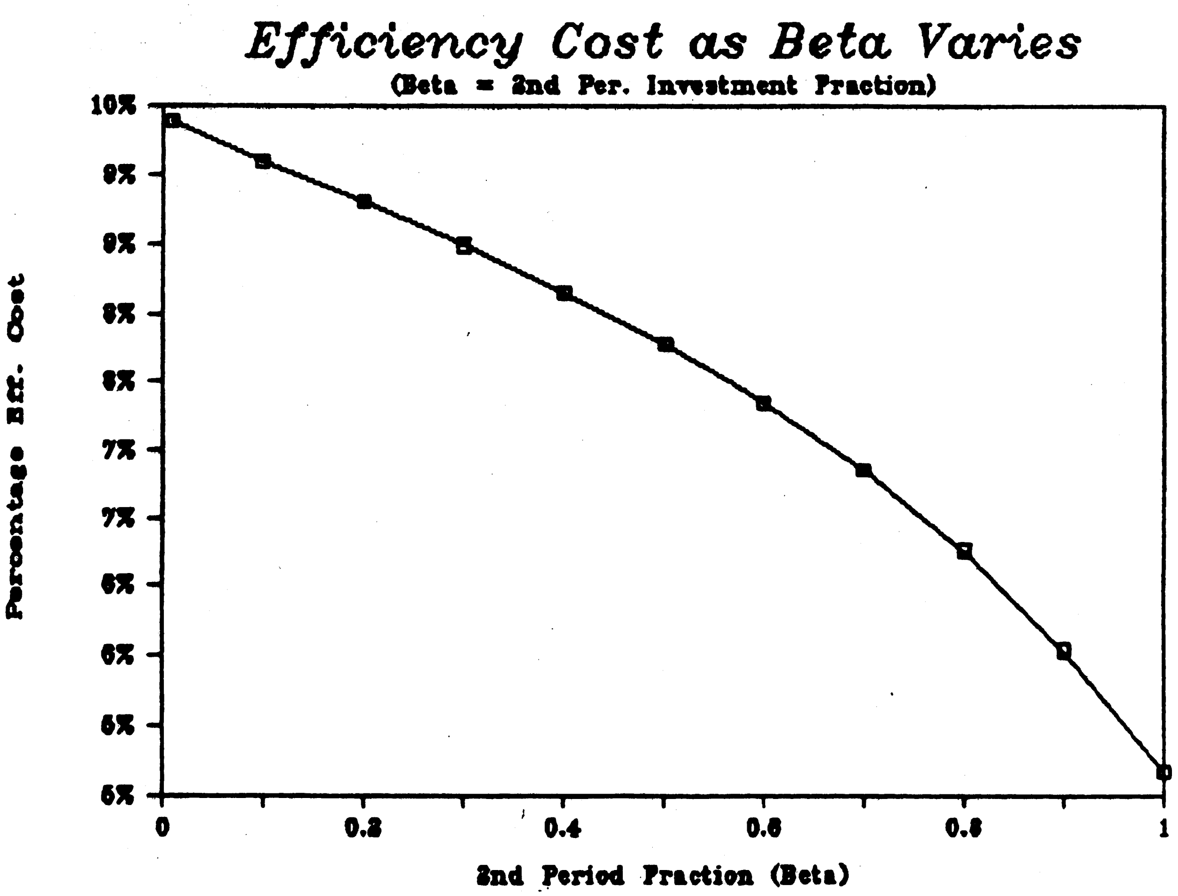

Figure 9

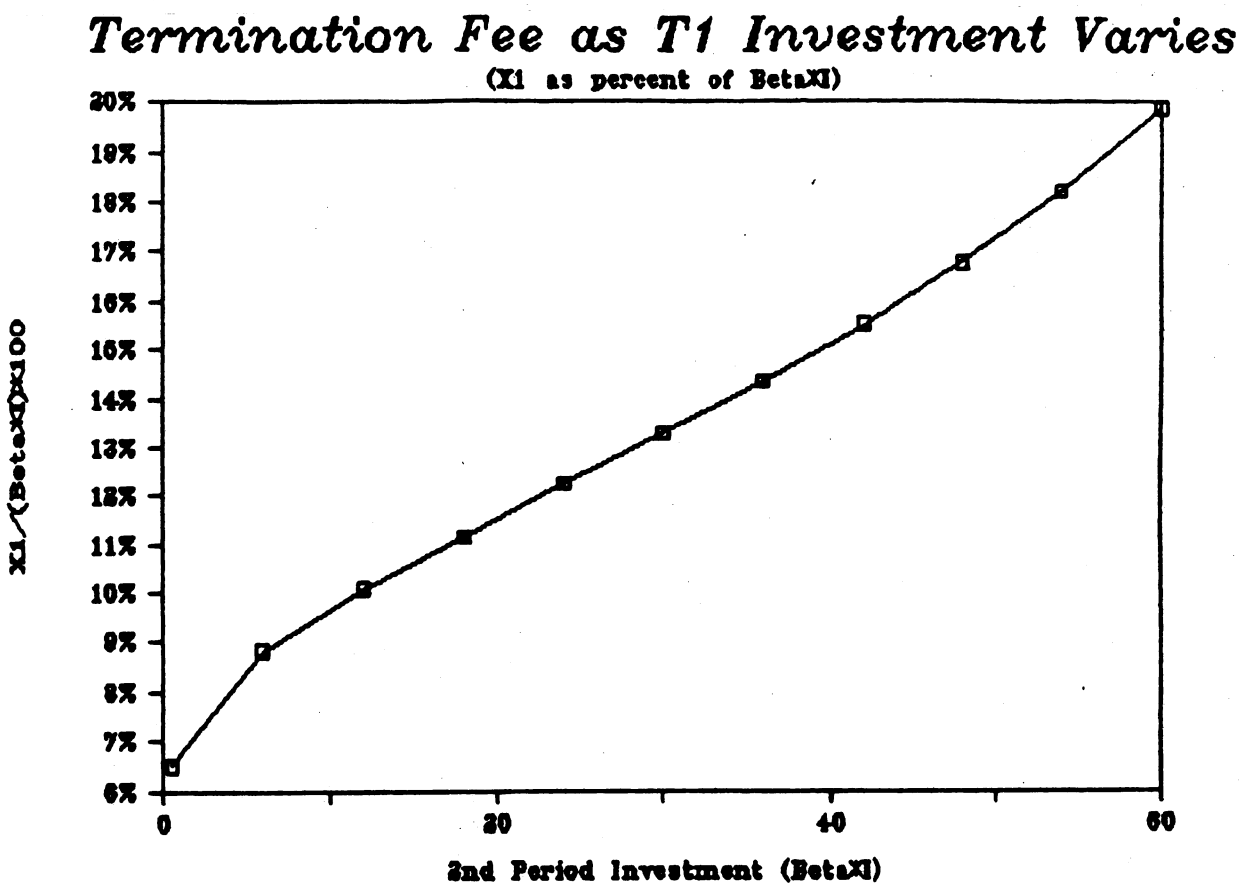
Termination Fee as T1 Investment Varies
(X1 as percent of BetaXI)
20%
19%
18%
17%
16%
15%
14%
13%
12%
11%
10%
9%
8%
7%
6%
0
20
40
60
2nd Period Investment (BetaXI)

Figure 10

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