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In their analysis of the consequences of research joint ventures, Claude d'Aspremont and Alexis Jacquemin (1988) deduce from the hypothesis of joint profit maximization the behavior of two firms that are permitted to coordinate their research investments while anticipating that they will subsequently compete as Cournot rivals in the product market. After modifying slightly their assumptions about the effects of R&D on production costs, Morton Kamien, Eitan Muller, and Israel Zang (forthcoming) also consider this same case of "R&D Cartelization". Both papers assume that firms in the industry have a common "spillover" parameter ($\beta \in [0,1]$). Investing \$$\beta$ in one's own R&D lowers production costs by the same amount as \$1 of investment by a rival.

Both papers presume that—at the joint profit maximum—each firm makes identical decisions regarding investment and subsequent output. This presumption may seem plausible since the cost functions for production and research for firms are identical and their outputs are perfect substitutes. Nonetheless, the presumption is erroneous.[1]

Instead of investing identical amounts and then slugging it out as equally matched rivals, the firms can often earn strictly higher joint profits by investing at the first stage to induce monopoly (or at least asymmetric duopoly) at the second stage.

I D'Aspremont and Jacquemin's Model

D'Aspremont and Jacquemin consider a market with two firms facing an inverse demand of $P = a - b(q_1 + q_2)$, where P is the market price and q_i is the quantity produced by firm i, $i = 1,2$. They assume that any investment by firm i which lowers its own constant marginal cost of production by \$1 spills over to the other firm and lowers the rival's constant marginal cost by $\beta \in [0,1]$. If x_i denotes the amount of investment that firm i undertakes and x_j the amount of investment by

[1]Meister (1992) reports numerical simulations where two identical firms simultaneously choose one of three levels of R&D and then compete as Cournot duopolists. Some of these simulations suggest that joint profits might be higher if the two firms did not invest identically. Despite these simulations, however, Meister commits the same error as the rest of the literature both in extending d'Aspremont and Jacquemin to the n firm case and in investigating research joint ventures involving only a subset of the firms.

its rival, then the production cost function for firm i is given by $(A - x_i - \beta x_j)q_i$, for $i, j = 1, 2$, and $i \neq j$. The cost of investment is also symmetric for both firms and is given by $\gamma x_i^2/2$. The exogenous parameters must satisfy the following: $0 < A < a$, $0 < \beta < 1$, and $\gamma > 0$.

Assume that both firms cooperate in their choice of investment levels to maximize joint profits, knowing that they will subsequently compete noncooperatively in the output market. If both outputs in the second stage are strictly positive and form a Nash equilibrium given the investments at the first stage, then joint profits may be written as the following function of these investments:

$$\hat{\Pi} = \sum_{i=1}^{2} \left\{ \frac{1}{9b} [(a - A) + (2 - \beta)x_i + (2\beta - 1)x_j]^2 - \gamma \frac{x_i^2}{2} \right\}, \quad j \neq i. \tag{1}$$

D'Aspremont and Jacquemin define their proposed solution on p.1134 as

$$\hat{x}_i = \frac{(\beta + 1)(a - A)}{4.5b\gamma - (\beta + 1)^2}$$

and

$$\hat{q}_i = \hat{Q}/2 = \frac{(a - A)}{3b} \left[\frac{4.5b\gamma}{4.5b\gamma - (1 + \beta)^2} \right].$$

The following parameter restriction, reported by them in their footnote 7, insures that these decision variables are strictly positive: $b\gamma > (1 + \beta)^2/4.5$. Provided $\beta < .5$, however, the exogenous parameters may simultaneously satisfy the following condition: $b\gamma < 2(1 - \beta)^2$. If so, the symmetric solution that d'Aspremont and Jacquemin identify cannot be optimal.[2] Indeed, *any* other allocation of the same total investment between the two firms that still induces duopoly in the second stage will generate higher profits.

The stationary point cannot be optimal in this region because joint profits are strictly convex in firm 1's investment assuming that firm 2's investment is altered so as to maintain a fixed sum (and

[2]In an earlier comment on the d'Aspremont and Jacquemin model, Henriques (1990) considered the game where firms act noncooperatively in both output and R & D. She proposed restricting attention to the region of parameter space where the noncooperative solution is stable in the sense that the reduced form first stage reaction functions in R & D space cross "correctly". See equation (6) on p.639 of her article. Our analysis concerns a different game since cooperation occurs at the first stage. Moreover, her parameter restriction is not sufficient to eliminate the problem we have identified.

assuming that both firms produce in the second stage). To verify this, we substitute $x_2 = k - x_1$ into (1), where k is any positive constant. Joint profits can then be expressed as:

$$\hat{\Pi}(x_1) = F + \frac{x_1(x_1 - k)(2(1 - \beta)^2 - b\gamma)}{b},$$

where F depends only on the exogenous parameters. Differentiating the profit function twice with respect to firm 1's investment, we obtain:

$$\frac{d^2\hat{\Pi}}{dx_1^2} = \frac{2\,(2(1 - \beta)^2 - b\gamma)}{b}.$$

Hence, joint profits ($\hat{\Pi}(x_1)$) are strictly convex in the region of parameter space we identified above. Figure 1 depicts this region as well as the one identified by d'Aspremont and Jacquemin as necessary for the symmetric solution to be positive in output and investment. Since any local change in the neighborhood of their proposed solution does not increase joint profits and since joint profits are strictly convex where total investment is constant, nonlocal changes in the allocation of the same total investment must strictly increase joint profits.

To identify what pair of investment strategies *is* optimal in the shaded region of Figure 1, the joint profit maximization problem must be analyzed. The following preliminary observations reduce the number of cases that must be considered. It is never optimal for both firms to produce zero output. Denote the firm producing the weakly larger output at the optimum as firm 2. Hence, firm 2 will produce strictly positive output. Since firm 2's output is set to maximize its profits at the second stage for any pair of investments and for any output of firm 1, no local increase *or decrease* in firm 2's output will affect its profits. This is reflected in the expression for q_2 below. Since firm 2 produces the weakly larger output, it turns out to make the weakly larger investment. Indeed, since it can be shown that it is never optimal for both firms to make zero investment, firm 2 always makes a strictly positive investment in R&D. We incorporate these observations in our formulation of the joint profit maximization problem:

$$\max_{x_1, q_1, x_2, q_2} (a - bq_1 - bq_2)(q_1 + q_2) - (A - x_1 - \beta x_2)q_1 - (A - x_2 - \beta x_1)q_2 - .5\gamma x_1^2 - .5\gamma x_2^2$$

where $q_2 = (a - bq_1 - [A - x_2 - \beta x_1])/2b \geq q_1$, and

$$x_1 \geq 0,\ x_2 > 0,\ q_2 > 0, \tag{2}$$

$$q_1 \geq 0, \tag{3}$$

$$a - 2bq_1 - bq_2 - (A - x_1 - \beta x_2) \leq 0, \tag{4}$$

$$q_1[a - 2bq_1 - bq_2 - (A - x_1 - \beta x_2)] = 0. \tag{5}$$

Condition (4) states that firm 1's marginal revenue must be less than or equal to its marginal cost at an optimum. We will henceforth refer to the left hand side of (4) as firm 1's marginal profits ($M\Pi_1$). Equation (5) is the complementary slackness condition: If firm 1 produces a strictly positive amount at an optimum, then its marginal revenue must equal its marginal cost; if its marginal cost strictly exceeds its marginal revenue at an optimum, then it must produce nothing.

Firm 1 may or may not produce at the second stage and may or may not invest at the first stage. Hence, in addition to the case d'Aspremont and Jacquemin considered, there are three other possibilities for firm 1: firm 1 (a) produces but does not invest ($q_1 > 0, x_1 = 0$), (b) neither produces nor invests ($q_1 = 0, x_1 = 0$), or (c) invests but does not produce ($q_1 = 0, x_1 > 0$).

In principle, there are two possible subcases associated with each case where firm 1 produces nothing. That is, cases (b) and (c) can arise in two ways: a local increase in firm 1's production above zero can strictly decrease firm 1's profit on the one hand ($M\Pi_1 < 0$) or can leave it unchanged on the other ($M\Pi_1 = 0$). Hence, in addition to the solution candidate proposed by d'Aspremont and Jacquemin, there are, in principle, five other candidates where an optimum might occur.

However, one of these possibilities need not be considered in the context of their example. Although it remains a logical possibility that firm 1 might invest nothing when subsequent production

is anticipated to be strictly unprofitable ($M\Pi_1 < 0$), it is straightforward to show that such behavior is jointly suboptimal. For suppose firm 1 did no investment in these circumstances. Then a marginal increase in its R&D would create spillover benefits to firm 2 without bringing firm 1 into competition; and given d'Aspremont and Jacquemin's assumption about the cost of investing in R&D, the marginal cost of doing the first bit of R&D is zero. Since a marginal increase in R&D by firm 1 would have strictly positive marginal benefits and zero marginal cost, it is suboptimal for firm 1 to do no investment in these circumstances (for any strictly positive spillover parameter). Hence, in the region we have identified there remain four *candidates* for the optimal solution. In three of the four cases, the firms invest at the first stage in such a way that monopoly occurs at the second stage.

Each of these four candidate solutions can in fact be optimal as we illustrate below. We assume that $a = 1.5A$. and $\beta = .1$. In each of four exogenous specifications of $b\gamma$, distinguished by dots in Figure 1, we identify the optimal solution and report the associated joint profit in Table 1. In addition, we report the profit that would arise at the symmetric solution proposed by d'Aspremont and Jacquemin.[3]

$b\gamma$	$\Pi/(A^2\gamma)$	x_1/A	x_2/A	$q_1/(A\gamma)$	$q_2/(A\gamma)$	$M\Pi_1$	$\hat{\Pi}/(A^2\gamma)$	$\hat{x}_i/A$	$\hat{q}_i/(A\gamma)$
1.5	.046	0	.220	.072	.204	$= 0$	.045	.099	.135
1.0	.121	0	.625	0	.562	$= 0$	.076	.167	.228
.9	.157	.019	.671	0	.652	$= 0$	.088	.194	.264
.8	.212	.085	.847	0	.847	< 0	.105	.230	.314

Table 1

[3]Given the specification of the production cost function, it is possible to invest so much that marginal costs become negative. However, it is straightforward to verify that for each case in Table 1, marginal costs are strictly positive at both the optimal solution and at the symmetric solution proposed by d'Aspremont and Jacquemin.

In three of the four cases, firm 1's marginal profit is zero at the optimum and hence joint profit in the first stage is given by (1). Notice that (1) is equal to the sum of duopoly profits in the second stage minus investment costs in the R&D stage. The cheapest way to accomplish a given total investment is to set $x_1 = x_2$. However, with linear demand, the sum of duopoly profits in the second stage is always maximized by handicapping one firm's marginal cost as much as possible. This is achieved by doing no R&D at one of the firms. On balance, whether or not an asymmetric solution obtains depends on the relative tradeoff between these opposing effects.

When γ is high enough, minimizing investment costs is relatively more important, and the d'Aspremont and Jacquemin solution holds. The symmetric solution also holds when the sum of duopoly profits in the second stage diminishes in importance, as will be the case when demand is low enough (large b).

For example, when $a = 3A/2$ and $\beta = .1$, the symmetric solution holds for $b\gamma > 1.62$. See Figure 1. However, at $b\gamma = 2(1-\beta)^2 = 1.62$, identified by the upper most dot in Figure 1, we have shown that (1) is linear in the sum of the investment levels. This means that once the level of industry investment has been decided upon, its allocation between the firms is inconsequential. The two opposing effects are exactly offsetting. For $b\gamma < 1.62$, maximizing joint profits in the second stage assumes prominence.

In our example with $b\gamma = 1.5$, the optimal choices of investment are given by $x_1 = 0$ and $x_2 = .22$. The resulting wedge between the two firms' marginal production costs, however, is not enough to induce firm 1 to exit. It may seem counterintuitive that joint profits are maximized even though marginal investment by firm 1 would have a zero marginal cost. However, the marginal benefit of such an investment to joint profits in the second stage is strictly negative at the equilibrium.

In the example with $b\gamma = 1.0$, the gap created between the two firms' marginal production costs is sufficient to induce firm 1 to exit. Once again joint profit maximization calls for $x_1 = 0$, even

though marginal investment by firm 1 would have a zero marginal cost. The intuition is the same as before. Holding firm 2's investment constant at $x_2 = .625$, any positive investment by firm 1 would induce entry in the second stage.

Entry in the second stage can only be forestalled as long as the gap in the two firms' marginal production costs is sufficiently great. Any incremental investment by firm 1 would have to be offset by a proportionately larger investment by firm 2 to maintain the monopoly. In the previous case, joint profits were maximized by setting $x_1 = 0$. For low enough $b\gamma$, however, the amount of *R&D* that firm 2 would undertake if it were constrained to do all of the investing would be more than sufficient to induce firm 1's exit in the output market. Formally, marginal profits of firm 1 would be strictly negative ($M\Pi_1 < 0$). But if this were so, joint profits could be improved by shifting some of the cost of investment to firm 1. This is illustrated for the case $b\gamma = .9$. Here, the solution yields $x_1 = .019$ and $x_2 = .671$. Investment levels are chosen so that firm 1 is just indifferent between entering the market or staying out ($M\Pi_1 = 0$).

Finally, for sufficiently low $b\gamma$, equilibria are reached in which the allocation of investment between the two firms is governed solely by the cost efficiency of reducing firm 2's marginal production cost. The example we give is $b\gamma = .8$. Here, the optimal x_1 is exactly ten percent of x_2. This is because a \$1 investment by firm 1 is assumed to lower firm 2's marginal production cost by 10 cents ($\beta = .1$). In equilibrium, firm 1 strictly prefers to stay out of the output market.

II Kamien, Muller, and Zang's Model

Kamien, Muller, and Zang reformulate d'Aspremont and Jacquemin's treatment of "R&D Cartelization" in such a way that monopoly never results at the second stage. For purposes of comparison, we specialize their model to the case of two firms producing perfect substitutes. Assume a linear demand of $P = a - q_1 - q_2$ and unit cost of production given by $A - f(X_i)$, where $f(X_i)$ is the R&D production function and $X_i = x_i + \beta x_j$ is the effective level of firm i's R&D investment. The first stage R&D expenditure levels for the two firms are given by x_1 and x_2 respectively. In addition, Kamien, Muller, and Zang place the following restrictions on the R&D production function:

Assumption I The R&D production function $f(X)$ is twice differentiable and concave, $f(0) = 0$ and $f(X) \leq A$, $f'(X) > 0$ for all $X \geq 0$.

Assumption II The R&D production function $f(X)$ satisfies

$$\lim_{x \to \infty} f(X) < a - A, \ \ and \ \ f'(0) > \frac{9}{2(a-A)}.$$

Assumption III For all $X \geq 0$, $(a - A + f(X))f''(X) + f'(X)^2 < 0$.

It is straightforward to derive the second-stage Cournot quantities as a function of the first-stage investments:

$$q_i = \frac{a - A + 2f(X_i) - f(X_j)}{3} \quad \text{for} \quad i,j = 1,2, \quad \text{and} \quad i \neq j. \tag{6}$$

Assumptions I and II insure that both outputs are strictly positive for any nonnegative investments.

In their "R&D Cartelization" game, Kamien, Muller, and Zang allow firms to coordinate their R&D decisions so as to maximize joint profits. The firms' problem is given by

$$\max_{x_1, x_2} \ (a - q_1 - q_2)(q_1 + q_2) - (A - f(X_1))\,q_1 - (A - f(X_2))\,q_2 - x_1 - x_2, \tag{7}$$

where outputs are given in (6).

A stationary point for this function occurs where $x_1 = x_2 = x^*$ and is implicitly defined as follows:

$$\frac{4(1+\beta)(a - A + f((1+\beta)x^*))\,f'((1+\beta)x^*)}{9} - 2 = 0. \tag{8}$$

Kamien et al. presume that the function given in (7) achieves a global optimum at the unique symmetric solution to (8).[4] If so, then the function must be weakly concave there.

[4] Uniqueness is implied by Assumption III.

Writing outputs as a function of the two investment levels, fixing their sum at what Kamien et al. regard as optimal but allowing reallocations of investment between the two firms, we discover joint profits need not be weakly concave at the presumed optimum. Hence, the alleged optimal solution can be dominated.

Differentiating (7) twice, we obtain:

$$\frac{4(\beta-1)^2\left((a - A + f((1+\beta)x^*))\,f''((1+\beta)x^*) + 9f'((1+\beta)x^*)^2\right)}{9}, \tag{9}$$

Assumption III is insufficient to insure that (9) is weakly negative. For example, let $f(X_i) = AX_i/(X_i + \theta)$, $i = 1,2$. For $a > 2A$ and $\theta < (a - A)A/4.5$, it can be verified that assumptions I, II, and III hold. For concreteness, let $A = 10$, $a = 30$, $\theta = 40$ and $\beta = .1$. Then the solution proposed by Kamien, Muller, and Zang yields $x_1 = x_2 = 5.04$, giving joint profit of 89.96. But at that point, the second derivative defined in (9) equals $.056 > 0$. Hence, the proposed solution can be strictly dominated.[5] For example, reallocating the same total investment as $x_1 = 0$, $x_2 = 10.08$, gives joint profit of 90.69.

Indeed, it can be verified that for $a = 3A$ and $\theta = 2A^2/5$, (9) is strictly positive *for all* spillover parameters $\beta \in (0,1)$ and all values of the production cost parameter $A > 0$. The symmetric solution they identify is never optimal for these parameters.

III Conclusion

Contrary to the implicit assumptions of the literature, identical participants in research joint ventures may often increase their *collective* profits by investing asymmetrically in cost-reducing R&D.

[5] Joint profits equal industry gross sales revenue minus research costs minus production costs. At the symmetric stationary points which the respective papers identify as optimal, industry sales revenue minus research costs is a strictly concave function of the investments of the cooperating firms. This follows since the cost of research is a convex function of the investments (strictly convex in d'Aspremont and Jacquemin's formulation and linear in Kamien et al.'s formulation) while industry revenue is a quadratic function of industry output which, in turn, is a concave function of the investments (strictly concave in Kamien et al.'s formulation and linear in d'Aspremont and Jacquemin's formulation). In both formulations, joint profits fail to be strictly concave because the remaining term in the sum — minus joint production costs — is sufficiently convex in the investments to outweigh the other terms in the sum.

By doing so, they avoid slugging it out in the output market as equally matched rivals. If sidepayments are costless, such asymmetric investments should occur rather than the symmetric investments characterized in the literature. If, on the other hand, sidepayments are not costless, then investment decisions cannot be separated from decisions about how the proceeds will be distributed and the assumption of joint profit maximization is inappropriate. Predicting investment decisions then requires taking explicit account of the costliness of making sidepayments.

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Figure 1: Region of Optimal Asymmetric Strategies in d' Aspremont and Jacquemin's Model