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Monitoring Agents with Other Agents

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Abstract. I investigate the multiple agency problem when agents can monitor the performance of other agents. A particularly interesting incentive scheme of this sort has been used by the Grameen Bank of Bangladesh and I use this example to investigate some general questions involving group incentive schemes. For example, I show that a principal prefers a monitor who can reduce the costs of desirable actions rather than increase the cost of undesirable actions. I also consider when it is beneficial to the principal for agents to mutually insure each other. Finally, I examine a sequential incentive plan in which agents form a group and first serve as monitors and later are monitored by other agents. [Prepared for the *Seventh International Seminar on the New Institutional Economics,* University of Saarbrücken, May 31–June 2, 1989. To appear in *Journal of Institutional and Theoretical Economics.*]

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Monitoring Agents with Other Agents

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The principal-agent literature typically assumes that principal is unable to observe the characteristics or the actions of the agents whom they monitor. The inability of the principal to observe characteristics or actions leads to complications in the design of incentive schemes. For surveys of this literature, see Hart and Holmström [1987] and Rees [1985*a*-*b*].

However, in reality, it is often not the case that agents' characteristics or effort levels are really *unobservable;* rather, they simply may be very costly to observe. One may choose to model high-costs actions as being unfeasible actions, but in doing so, one may miss some interesting phenomena. In particular, simply because information is costly to the principal doesn't mean that it is costly to everyone. It may happen that the agents themselves are in good positions to monitor or advise each other.

In reality is is common to find incentive mechanisms that involve agents monitoring each other. For example, the authorities often post rewards for citizens who turn in criminals or report violations of crimes. Similarly, principals may create task forces or working committees so that agents can jointly engage in some activity. We will discuss a particularly interesting example of such a group incentive device below.

The literature on group incentive mechanisms has tended to focus on issues involving situations where the actions of one agent provide information to the principal about the actions of other agents. Holmström [1979] showed that a signal will be valuable in constructing an incentive scheme if and only if it provides information about the hidden action of an agent. Therefore, when the output of one agent is correlated with the actions of the other agents, the optimal incentive scheme will typically involve making the payments to one agent depend on the outputs of the other agents.

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Several authors have investigated the form that such optimal incentive contracts will take, among them Bohn [1987], Crocker [1985], Sappington and Demski [1983], Demski and Sappington [1984], Demski, Sappington, and Spiller [1987], Holmstrom [1982], Ma, Moore, and Turnbull [1988], Malcomson [1986], Mookherjee [1984], Rasmusen [1987], and Rasmusen and Zenger [1989].

In general, interactions among agents may take more complex forms that simply signaling productivity of the other agents. For example, agents may take actions that directly affect each others' costs or utility, they may provide advice or information to each other, and they may insure each other. In addition, the principal and some of the agents may be able to influence the composition of the group of other agents with which they are involved.

1. The Grameen Bank

A particularly interesting example of such group monitoring schemes arises in the provision of credit in developing countries. A village moneylender in Bangladesh may charge 150% interest a year. World interest rates are on the order of 10–20%, so there are many potentially profitable projects that are not undertaken due to the excessive local interest charges.

Apparently the village moneylender has a local monopoly; but why do private lenders not compete away the local monopoly of the village moneylender? One part of the answer is that the projects involved are often of a very small scale and the transactions costs of choosing recipients of loans and monitoring their performance can easily outweigh the profits involved. The village moneylender knows the reputations of his clients and can monitor their activities first-hand. These may be very costly activities for potential competitors.

A Bengali economist named Muhammad Yunas founded the Grameen Bank in order to overcome some of these problems. The Grameen Bank uses an innovative incentive scheme to "internalize" the selection and monitoring costs.

> "Although loans are made to individual entrepreneurs, each individual is in a group of four or five others who are in line for similar credits. Together they act as co-guarantors. If one individual is unable to make timely payments, credit for the entire group is jeopardized which results in heavy peer-group pressure on the

delinquent. At first only two members of the group are allowed to apply for a loan. Depending on their repayments, the next two borrowers can apply, and then the fifth." [Farnsworth [1988]]

According to Hossain [1989],

> "The group functions as an institution to ensure mutual accountability. The individual is kept in line by a considerable amount of pressure from the other members of the group. The existence of the group thus acts as collateral for the bank loans. The credibility of the group as a whole and its future benefits in terms of new loans are in jeopardy if one member breaks the discipline and defaults on loan payments... Other members of the group also extend financial support to a member in times of genuine difficulty when the member cannot pay the installment." [Hossain [1989], p. 26]

The Grameen Bank has been quite successful. It makes about 475,000 loans per month with an average size of $70. They have a loan recovery rate of 98 percent, while conventional lenders in Bangladesh have a recovery rate of about 30 to 40 percent. (Figures are taken from Lurie [1988].)

The Grameen program involves several of the aspects of group incentive mechanisms that we alluded to above:

1) mutual monitoring — agents influence the other agents' costs of engaging in desirable and undesirable aspects.
2) mutual insurance — members are expected to assist each other in time of need.
3) formation of the group — the group is formed by the individual members.
4) mutual assistance — agents are expected to provide information and assistance to other agents.
5) sequential decisions — the incentive system takes an explicitly sequential nature; members of the group are awarded loans depending on the outcome of the other members projects.

We will examine some simple models of these points below. I investigate these issues in a fairly general setting and do not attempt to model in detail the functioning of the

Grameen Bank. In particular, I should emphasize that the Grameen bank is not a profit-maximizing organization; it is a semi-private bank whose purpose is to encourage rural economic development. However, in what follows we shall analyze the behavior a profit-maximizing provider of loans. I do this because I believe that it is important to examine the purely economic incentives inherent in plans such as that used by the Grameen bank.

In other countries and other periods, different economic institutions have been created to address the problem of local credit provision. Bonus and Schmidt [1990]

2. Choice of monitors

In some group agency problems there is a hierarchical structure: some people do the monitoring and some people do the producing. The principal typically chooses these assignments, presumably in the way that is most appropriate for his objectives. It seems reasonable to ask what characteristics are desirable in a monitor. For example, would the principal want a monitor that can punish deviations, or one that can award desirable behavior?

In order to address this question, let us consider the standard principal-agent problem as described, say, in Hart and Holmström [1987]. The principal is risk-neutral and the agent is risk averse; the principal wishes to determine the least expensive pattern of transfer payments that induces a given action by the agent. For simplicity, we suppose that there are only two actions open to the agent, the "best" action, b, and the "alternative" action, a. The costs of the actions to the agent are denoted by c_a and c_b, and (π_{ia}) and (π_{ib}) are the probability distributions over output induced by the two actions.

Suppose that the principal wishes to induce action b. Then principal's problem is

$$
\begin{aligned}
V = \max_{s_i} \; & \sum_{i=1}^{n} \pi_{ib}(x_i - s_i) \\
\text{such that} \; & \sum_{i=1}^{n} \pi_{ib} u(s_i) - c_b \geq 0 \\
& \sum_{i=1}^{n} \pi_{ib} u(s_i) - c_b \geq \sum_{i=1}^{n} \pi_{ia} u(s_i) - c_a.
\end{aligned}
$$

The first constraint is the *participation constraint.* It says that the principal must provide the agent with a level of utility that it as least as large as his next best alternative.

For simplicity we have normalized the utility of the next best alternative to be zero. The second constraint is the *incentive compatibility* constraint. It says that the utility of the action the principal is trying to induce must be at least as large as the utility of the alternative action.

Most of the attention in the principal-agent problem has focused on the shape of the optimal incentive scheme (s_i); see Hart and Holmström [1987] for a survey of the results in this area and Grossman and Hart [1983] for a detailed treatment. Here we consider somewhat different issues.

In particular, let us ask what sort of changes in the costs of the actions would the principal prefer. If a principal could hire a monitor who would lower the cost of the principal's preferred action by a dollar or one that could raise the cost of the alternative action by a dollar, which would be prefer? More colloquially: *which is better, the carrot or the stick?*

This question can be answered easily using the envelope theorem. The Lagrangean for the principal-agent problem is

$$\mathcal{L} = \sum_{i=1}^{n} \pi_{ib}(x_i - s_i) - \lambda \left[c_b - \sum_{i=1}^{n} \pi_{ib} u(s_i) \right] - \mu \left[c_b - c_a - \sum_{i=1}^{n} u(s_i)(\pi_{ib} - \pi_{ia}) \right]$$

Differentiating this Lagrangean with respect to c_b and c_a, we have

$$\frac{\partial V}{\partial c_a} = \mu \tag{1}$$

$$\frac{\partial V}{\partial c_b} = -(\lambda + \mu) \tag{2}$$

It follows from the Envelope Theorem that the principal is better off if the cost of the action he wishes to induce is lowered by a dollar than if the cost of the alternative action is increased by a dollar. In the terms introduce above, *the carrot is better than the stick.* Making the best action more attractive aids the principal in two ways: it makes the participation constraint less binding as well as making the incentive constraint less binding.

Agents may also influence the probability distribution facing other agents. The marginal effect on the principal's utility of a perturbation $(d\pi_{ia})$ and $(d\pi_{ib})$ is given by

$$dV = \sum_{i=1}^{n} (x_i - s_i) d\pi_{ib} + (\lambda + \mu) \sum_{i=1}^{n} u(s_i) d\pi_{ib} \tag{3}$$

$$dV = -\mu \sum_{i=1}^{n} u(s_i) d\pi_{ia}. \tag{4}$$

Equation (4) shows that any change in (π_{ai}) that makes the expected utility of the alternative action less attractive—such as a mean-preserving spread—is better for the principal. In order to interpret equation (3), consider a change in the probability distribution of the preferred action that leave the net payments to the principal unchanged. Then the first term in (3) vanishes, and we see that for such changes in the probability distribution the principal and the agent have no conflict of interest: the principal will be better off if and only if the agent is made better off.

3. Mutual insurance

Another thing that the agents might do is to provide insurance for each other. One of the explicit "Sixteen Decisions" in the Grameen bank program states: "We shall always be ready to help each other. If anyone is in difficulty, we shall all help." (Hossain [1989], p. 28.) It is natural to ask how mutual provision of insurance affects the utility of the principal. As it turns out, mutual insurance can make the principal better off or worse off, depending on the form that it takes.

Arnott and Stiglitz [1987] have investigated the related question of the desirability of non-market insurance in a moral hazard context. Arnott and Stiglitz show that in a standard model of a competitive insurance industry with moral hazard, the provision of non-market insurance by the agents may make them all worse off. Here we shall show that in a principal-agent problem involving moral hazard, provision of certain sorts of insurance will unambiguously make the principal worse off, while other sorts of insurance will make the principal better off.

Essentially, if the agents offer insure each other across the same states of nature that the principal observes, then the principal is definitely made worse off. However, if the agents can insure across states that are not available to the principal, then the principal will unambiguously be made better off if the agents provide such mutual insurance.

The principal and the agents observe the same states

Let us first consider the case where the agents can only insure across the same states that the principal can insure, i.e., the states $i = 1, \ldots, n$. The provision of this sort of insurance makes the principal worse off in two distinct ways. First, suppose that the principal designs an incentive scheme for the agents which does not take into account the fact that the agents may decide to insure each other. Then whenever the two agents have different marginal utilities of income in a particular state they will be able to find a mutually beneficial insurance plan.

However, the principal had it in his power to provide any insurance plan the agents can provide. By revealed preference any insurance plan provided by the agents must make the principal worse off.

Now suppose that the principal recognizes the fact that the agents may want to insure each other. In order to remove this incentive, the principal must design an incentive payment plan that satisfies the constraint that the marginal utility of income in each state is the same for each agent. However, this adds additional constraints to the original problem and hence the principal can be made no better off, and will typically be made worse off.

The agents observe a finer set of states than the principal

Next we ask what will happen if the agents are capable of insuring each other in ways that are not available to the principal. In this case, the principal will be unambiguously better off when the agents provide such insurance. The proof is an application of the observation due to Holmström [1979] that the principal never benefits from providing a contract involving randomization.[1]

Here is an easy way to see why this theorem is true. Suppose to the contrary that the optimal contract takes the form $(\tilde{s}_i)$, where $\tilde{s}_i$ is some (nondegenerate) random payment plan with expected value $\bar{s}_i$. Let r_i be the risk premium defined by the condition

$$u(\bar{s}_i - r_i) = Eu(\tilde{s}_i).$$

[1] Grossman and Hart [1983] provide an extension of Holmström's theorem.

Note that if the utility function is strictly concave the risk premium is strictly positive.

Now consider the incentive plan $(\bar{s}_i - r_i)$. Since this provides the same expected utility to the agent as the plan $(\tilde{s}_i)$ this plan is feasible. But it yields a higher expected net payoff to the principal than the plan $(\tilde{s}_i)$.

Let us now apply this logic to the problem at hand. Suppose that the optimal incentive scheme for the principal involves some agent receiving a payment of s_i in state i. However, suppose that the agent also receives a random shock of $\tilde{\epsilon}_i$ in state i. If this agent insures himself against the shock $\tilde{\epsilon}_i$, conditional on the state i, we suppose that he can achieve the income $s_i + \tilde{\delta}_i$ where

$$Eu(s_i + \tilde{\delta}_i) > Eu(s_i + \tilde{\epsilon}_i).$$

Again, define the risk premium r_i by

$$Eu(s_i + \tilde{\delta}_i - r_i) = Eu(s_i + \tilde{\epsilon}_i),$$

and consider the incentive payment plan $(s_i - r_i)$. By construction this yields the same expected utility to the agent and so satisfies the original set of incentive constraints. On the other hand, the principal receives the (positive) expected value of the risk premiums. It follows that the principal would always desire that the agents engage in this sort of insurance. Essentially, the agents insure themselves to make themselves better off (conditional on each output level), but the principal manages to get all the surplus from their doing so.

Note that the relationship is monotonic—the more risk that can be eliminated, in terms of the increase in expected utility, the better off the principal becomes. This follows since the risk premium r_i is increasing in the level of expected utility.

4. Formation of the group

We now turn to a very simple model of another aspect of the Grameen bank lending behavior, namely the stage of group formation. The bank explicitly requires the borrowers to form a group of five people who then simultaneously apply for a loan. The bank then either approves or rejects the loan for the entire group. However, the projects of the borrowers are all independent.

Presumably the motivation for this arrangement is that the agents have a better information than the bank has about who is a good credit risk. The fact that their loans are conditional on the impression made by the other members of the group induces them to select good credit risks, thereby making the evaluation activity by the bank easier.

Consider the following simple model of this kind of scheme. There are two types of agents, good and bad. The good agents have an investment scheme that will produce $V > 1$ from an initial investment of 1. The bad type produce 0 from an investment of 1.

The loan officer can determine whether an agent is good or bad by spending some fixed cost K, which we interpret as the cost of an interview or a credit report. Clearly if $K > V - 1$ the bank will not find it worthwhile to interview the agents. If the fraction of bad types is high enough, it will not find it in its interest to grant loans without an interview. Hence, no loan contracts will be possible with or without an interview.

Now suppose that the bank offers the following plan: the members form groups of size n and come into the bank. The lending officer chooses one of the group members and interviews him at cost K. If this member is determined to be good, then all members of the group receive the loan. If this person is judged to be bad, then none of the members of the group receive a loan.

Assume for the moment that the members of the group find it in their interest to associate only with good agents. In this case, the investment projects will be profitable with probability 1, so that this scheme will be profitable if $n(V - 1) \geq K$, which will certainly be satisfied for large enough n.

Let us now investigate the incentives of the group members to form homogeneous groups. If there is no cost to locating good agents, and there are no side payments allowed between agents, then the agents will certainly find it in their interest to only group themselves with good agents. Thus the problem is of interest only if there are costs to locating good agents, or if side payments or bribes between agents are possible.

It seems clear that our assumption of zero cost of locating good agents isn't crucial; as long as the agents can recognize other good agents at a lower cost than the loan officer a similar scheme should be possible. The interesting variation is then the possibility of side payments.

Consider a group of size n that is contemplating adding a member who is a bad risk. Assuming that unanimous consent is necessary, the bad credit risk can offer a bribe of up to $1/n$ to each person. What are the incentives facing a typical member of the group?

If the bad person is chosen for the interview, each other member will get a payoff of zero; this event happens with probability $1/(n+1)$. If one of the good members of the group is chosen, each member gets a payoff of $V-1+1/n$; this is the payoff from the loan, plus the bribe from the bad agent. If the agents reject the bad agent entirely, each agent is assured $V-1$.

The benefits of rejecting the bad agent exceed the benefits of accepting him if

$$\frac{0}{n+1}+\frac{n}{n+1}\left[V-1+\frac{1}{n}\right]\leq V-1.$$

Some algebra shows that this condition holds only if $V\geq 2$. Hence this procedure will induce complete sorting only if the project has a sufficiently high value. Put another way, the bank has to ensure that each group member receives a value of 2 from the project in order for the members to have the correct incentives to maintain homogeneity of the group.

However, note that the method just described lacks one essential feature of the Grameen Bank method: in the Grameen model, the loans are granted sequentially, not simultaneously. I receive a loan only if the loan is granted to the group *and* the other members of the group who proceed me succeed.

Let us add this feature to the model. If the loan is approved then there is a probability of $1/2$ that the bad agent will precede me in the ordering. If this happens, I get only the $1/n$ bribe. If it doesn't happen I get $V-1+1/n$. The condition to reject the bad member now becomes

$$\frac{0}{n+1}+\frac{n}{n+1}\left[\frac{1}{2}\frac{1}{n}+\frac{1}{2}\left(V-1+\frac{1}{n}\right)\right]\leq V-1.$$

Some calculations show that this reduces to

$$\frac{2}{n+2}\leq V-1.$$

Note two things. First, the left-hand side of this equation is always less than 1. Hence the sequential scheme will accept projects that the other scheme would reject for every group size n. Secondly, as n goes to infinity, a project with value arbitrarily close to 1 can be supported if a large enough group size is chosen. Roughly speaking, this scheme is asymptotically efficient.

5. Sequential incentive problems

We turn now to analyzing the sequential nature of the Grameen Bank incentive scheme. In order to do this, it will be convenient to examine the standard one-agent hidden-type incentive problem. The treatment here follows the exposition of Varian [1989] which in turn is based on Tirole [1989] and Maskin and Riley [1984].

The one-period problem

We now suppose that the sorting induced by the loan granting procedure is not perfect and that there is some heterogeneity remaining in the group of agents. For simplicity, we suppose that there are two types of agents, one with high productivity and one with low productivity. The high productivity agent can produce an output x at a cost of $c_1(x)$ while the low-productivity produces the same output at a cost of $c_2(x)$. We assume that $c_2(x) > c_1(x)$ and $c_2'(x) > c_1'(x)$ for all x so that the high-productivity agent has both lower total and lower marginal costs than the other type of agent. The high-productivity agents are a fraction π of the population, and the low-productivity agents therefore comprise $1-\pi$ of the population.

Since there are only two types of agents, the principal will want to present two incentive schemes, each one targeted toward each of the two agent. The two schemes will have the form (s_1, x_1) and (s_2, x_2) where s_i is the payment for x_i units of output. The scheme (s_1, x_1) is meant for the high-productivity agent and the scheme (s_2, x_2) is meant for the low-productivity agent. In order for each agent to select the scheme meant for him, the principal must design the system of payments so that they satisfy the *self-selection conditions:*

$$s_1 - c_1(x_1) \geq s_2 - c_1(x_2) \tag{5)a}$$

$$s_2 - c_2(x_2) \geq s_1 - c_2(x_1). \tag{5)b}$$

In addition, the principal must ensure that each agent receives his reservation level of utility, which we take to be zero:

$$s_1 - c_1(x_1) \geq 0 \tag{6)a$$

$$s_2 - c_2(x_2) \geq 0. \tag{6)b$$

The expected profit to the principal is

$$\pi(x_1 - s_1) + (1-\pi)(x_2 - s_2). \tag{7}$$

The principal's problem is to maximize (7) subject to (5) and (6).

From inspecting equations (5-6) it is clear that only one of (5*a*–6*a*) will be binding, and only one of (5*b*–6*b*) will be binding. In fact, our assumptions about the cost function imply that that (5*a*) and (6*b*) are the binding equations. The proof is a standard one and is presented in the Appendix.

Substituting the binding constraints into the objective function, we can now write the principal's maximization problem as

$$\max_{x_1, x_2} \pi[x_1 - c_1(x_1) - c_2(x_2) + c_1(x_2)] + (1-\pi)[x_2 - c_2(x_2)].$$

The first-order conditions to this problem can be rearranged to yield the following two conditions:

$$\begin{aligned} c_1'(x_1^*) &= 1 \\ c_2'(x_2^*) &= 1 - \frac{\pi}{1-\pi}[c_2'(x_2^*) - c_1'(x_2^*)]. \end{aligned} \tag{8}$$

The second of these equations can also be written as

$$1 - c_2'(x_2^*) = \pi(1 - c_1'(x_2^*)). \tag{9}$$

From the facts about which constraints are binding, and the first-order conditions, we can immediately state the essential features of the optimal incentive scheme.

1) The high-productivity agent produces an efficient amount of output; i.e., where price equals marginal cost.

2) The high-productivity agent receives more than his reservation level of utility.

3) The low-productivity agent produces an inefficient amount of output since the price exceeds marginal cost.

4) The low-productivity agent receives exactly his reservation level of utility for this production.

The reason for these results can be seen by examination of Figure 1. Here I have illustrated the marginal cost curves—the supply curves if you will—of each of the two agents. The fully efficient plan involves each agent producing at x_1^* and x_2^{**} respectively, while the principal takes all the surplus. This means that the incentive payment to the high-productivity agent is $s_1^* = A + B$ and the incentive payment to the low-productivity agent is $s_2 = A + D$.

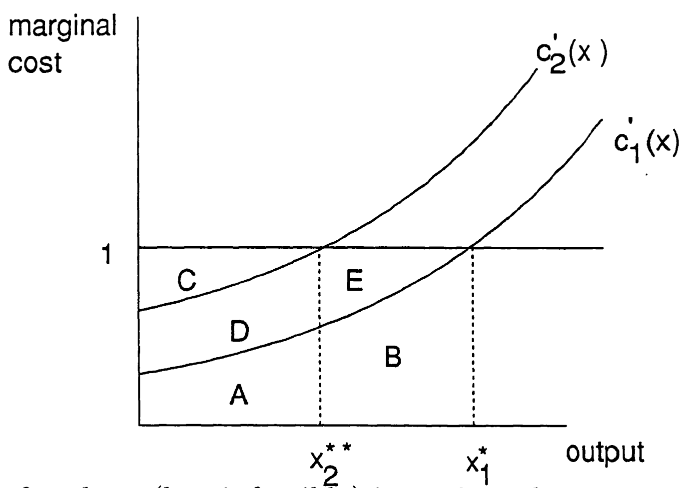

Figure 1. The first-best (but infeasible) incentive scheme.

However, this scheme does not satisfy the self-selection constraint. The high-productivity agent should realize that by choosing (s_2^*, x_2^*) he would receive a surplus of D, while sticking with (s_1^*, x_1^*) yields a surplus of zero.

Suppose instead the the principal kept the same levels of output but now paid the high-productivity agent $A + B + D$, yielding him the same positive surplus if he produces either x_1^* or x_2^*. This plan satisfies the self-selection constraints and is therefore feasible, but it is not optimal. By reducing the target level of output for the low-productivity agent, the principal loses the dark triangular area in Figure 2a from the low productivity agent. But he *gains* the area of the shaded rectangle from the high-productivity agent.

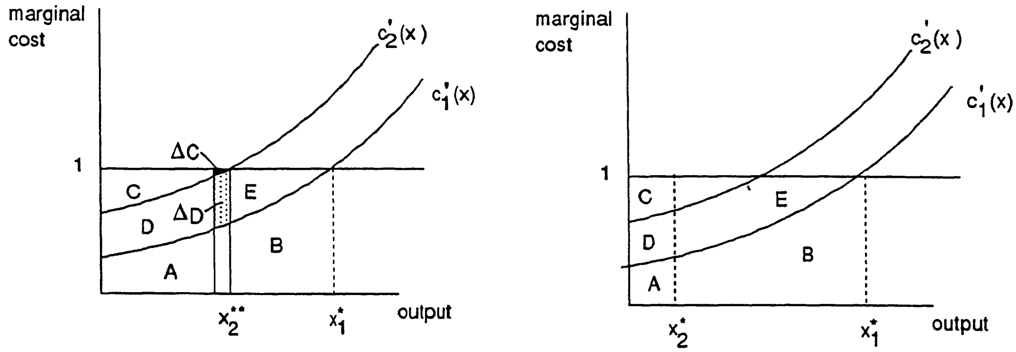

Figure 2. Panel a : A small decrease from x_2^{**} increases profits. Panel b : The optimal incentive scheme.

Hence reducing the target level of output for the low-productivity agent must increase the principal's profits.

The optimal production level for the low-productivity agent occurs where these two effects just balance out—where $1 - c_2'(x_2^*) = \pi(1 - c_1'(x_2^*))$, as in Figure 2$b$. At this point the lost output from reducing the target output of the low-productivity agent just balances the gain from the reduction in the payment to the high-productivity agent. The high-productivity agent produces x_1^* and receives $A + B + D$ while the low-productivity agent produces x_2^* and receives $A + D$.

The two-period problem

We turn now to a two-period problem. We suppose that the high-productivity agent can convey the knowledge that gives him the high-productivity to the other agent, assuming, of course, that he has the incentive to do so. For simplicity we will make the extreme assumption that the high-productivity agent can *perfectly* convey this information to the low-productivity agent so that the cost-function of the low-productivity agent becomes $c_1(x)$ after suitable instruction. We also assume initially that there is no cost to the high-productivity agent of conveying this information.

The incentive plan we will consider is loosely modeled on the Grameen Bank plan. Suppose that several agents apply to the principal for a position. The principal will randomly partition the agents into several groups of two agents each. He will then choose one agent from each group and offer him the incentive schemes (s_{01}, x_{01}) and (s_{02}, x_{02}). The other agent has the opportunity to advise and assist the chosen agent at this point.

Based on the performance of the first agent, the second agent is offered an incentive scheme. If the first agent produces a high level of output, the second agent is presented with the schemes (s_{11}, x_{11}) and (s_{12}, x_{12}). If the first agent produces a low level of output, then the second agent is presented with the schemes (s_{21}, x_{21}) and (s_{22}, x_{22}). The problem the principal faces is to design the incentive payments so as to maximize his profits.

The first observation to make is that we can always offer the two-part plan described above twice in a row. This will certainly be feasible, but in general we will be able to improve on this plan by taking account of the externality between the two agents.

In order to see how to design the optimal plan please refer to Table 1.

Productivity of		Probability
first agent	second agent	of observing
high	high	π^2
high	low	$\pi(1-\pi)$
low (high)	high	$(1-\pi)\pi$
low	low	$(1-\pi)^2$

Table 1. Probability of different production patterns.

If both agents have high productivity, then we will get high output in the first period, regardless of the behavior of the monitoring agent. Similarly, if the first agent is highly productive we will get a high output even if the monitoring agent has low productivity. It is only in the third row of the table that things get interesting: if the first agent has low productivity and the second agent advises him, then we will observe high productivity in the first period. Hence, if we are to have any improvement over simply offering the same two-part package twice, we must induce the high-productivity agent to convey the correct information to the low-productivity agent.

Suppose that we do succeed in inducing this communication. Then by inspection, if we observe a low output in period 1, it must be that the remaining agent in period 2 is a low-productivity agent. Hence we will not have to worry about self-selection in this case: only one plan needs to be offered second period if the output in the first period is low.

Given this observation, we can write down the entire set of constraints facing the principal when he designs the incentive scheme:

$$s_{01} - c_1(x_{01}) \geq s_{02} - c_1(x_{02}) \qquad {}^*(10)a$$

$$s_{01} - c_1(x_{01}) \geq 0 \qquad (10)b$$

$$s_{02} - c_2(x_{02}) \geq s_{01} - c_2(x_{01}) \qquad (11)a$$

$$s_{02} - c_2(x_{02}) \geq 0 \qquad {}^*(11)b$$

$$s_{11} - c_1(x_{11}) \geq s_{12} - c_1(x_{12}) \qquad (12)a$$

$$s_{11} - c_1(x_{11}) \geq 0 \qquad (12)b$$

$$s_{12} - c_2(x_{12}) \geq s_{11} - c_2(x_{11}) \qquad (13)a$$

$$s_{12} - c_2(x_{12}) \geq 0 \qquad {}^*(13)b$$

$$s_{22} - c_2(x_{22}) \geq 0 \qquad {}^*(14)$$

$$s_{11} - c_1(x_{11}) \geq s_{22} - c_1(x_{22}) \qquad (15)$$

Equations ($10ab$) and ($11ab$) are the self-selection constraints and participation constraints in period 1. Equations ($12ab$) and ($13ab$) are the self-selection constraints in period 2 if a high output was observed in period 1. Equation (14) is the participation constraint in period 2 if a low output was observed in period 1. As already observed, there is no need for a self-selection constraint in this case. Finally, equation (15) is the incentive compatibility constraint for inducing the high-productivity agent to reveal his information to the low-productivity agent. In order to understand this constraint, observe that if the high-productivity agent reveals his knowledge to the low-productivity agent in the first period he will guarantee that he will receive $s_{11} - c_1(x_{11})$ in the second period. On the other hand, if he does not reveal his information, he can mimic the behavior of a low-productivity agent and face the incentive scheme (s_{22}, x_{22}) second period. Equation (15) requires that the high-productivity agent receives more utility from revealing his knowledge than from not revealing it.

The objective of the principal is to maximize profits subject to the constraints in (10-15). This profit function turns out to be given by

$$\pi(2-\pi)[x_{01}-s_{01}]+(1-\pi)^2[x_{02}-s_{02}+x_{22}-s_{22}]+\pi[x_{11}-s_{11}]+\pi(1-\pi)[x_{12}-s_{12}].$$

The only tricky thing about this expression are the probabilities with which the various incentive schemes will be accepted. The first term is simply the probability that either the first agent is a high-productivity agent or that the second agent is a high-productivity agent. The second term is the probability that the first agent is a low-productivity agent and the second agent is a low-productivity agent.

The third term is the probability that the second agent is a high-productivity agent, *and* that the high output was observed in the first period. From examination of Table 1, this probability is

$$\pi^2+(1-\pi)\pi=\pi.$$

The final term is the probability that the second agent is a low-productivity agent *and* high output was observed in the first period. Examining the table, we see that this is simply $\pi(1-\pi)$.

We now consider which of the 10 constraints will be binding. Recall from our examination of the single period problem that the pattern of binding constraints there only depended on the properties of the cost functions. It follows that the binding constraints in this problem will follow the same pattern so that (10*a*), (11*b*), (13*b*), and (14) will be binding. This is the reason for the asterisks in (10–15). The same sort of reasoning implies that (12*b*) cannot be binding, so we are left with the comparison between (12*a*) and (15).

Normally one would expect that one of these constraints will be binding and one won't. However, I show in the appendix that this is not the case: in the optimal solution, *both* constraints will be binding.

It follows that

$$s_{12}-c_1(x_{12})=s_{22}-c_1(x_{22}).$$

Applying our observations about (13*b*) and (14) we can write this as

$$c_2(x_{12})-c_1(x_{12})=c_2(x_{22})-c_1(x_{22}).$$

Since $c_2(x) - c_1(x)$ is a monotonic function of x, it follows that $x_{12} = x_{22}$. Inserting this constraint into the maximization system given by (10–15), we can rewrite the profit function as

$$\begin{aligned}\pi(2-\pi)[x_{01} - c_1(x_{01}) - c_2(x_{02}) + c_1(x_{02})] \\ + (1-\pi)^2[x_{02} - c_2(x_{02}) + x_{12} - c_2(x_{12})] \\ + \pi[x_{11} - c_1(x_{11}) - c_2(x_{12}) + c_1(x_{12})] \\ + \pi(1-\pi)[x_{12} - c_2(x_{12})].\end{aligned}$$

This problem has first-order conditions which can be arranged to yield

$$c_1'(x_{01}^*) = 1 \tag{16)a$$

$$c_2'(x_{02}^*) = 1 - \frac{\pi(2-\pi)}{(1-\pi)^2}[c_2'(x_{02}^*) - c_1'(x_{02}^*)] \tag{16)b$$

$$c_1'(x_{11}^*) = 1 \tag{16)c$$

$$c_2'(x_{12}^*) = \frac{\pi}{1-\pi}[c_1'(x_{12}^*) - c_2'(x_{12}^*)] \tag{16)d$$

These equations should be compared with (8). The only equation that is different from the corresponding equation in (8) is (16*b*). This is due to the fact that the fraction of high-productivity agents available first period is larger in this problem than in the one-period problem. Hence $x_{02}^* > x_2^*$, which simply means that the output assigned to the low-productivity agent in the first period is higher in the two-period problem than in the one-period problem, since there are fewer of them.

The second observation is that $x_{22}^* = x_{12}^* = x_2^*$, which says that the output assigned to the low-productivity agent in the second period, if low output is observed the first period, is the same as the output assigned to the low-output agent if *high* output observed first-period.

To see this reason for this equality, put yourself in the position of the high-productivity agent. If you reveal your technology to the low-productivity agent, then you will be presented with output targets of x_{11}^* and x_{12}^* next period. The optimal incentive payments make you just indifferent between these two choices. If you fail to reveal your technology, you face x_{22}^*. Therefore, you must also be indifferent to facing x_{22}^*.

It follows that the optimal incentive scheme is *not* time-consistent. Once the principal observes that there is a low output in the first period he *knows* that the remaining agent is a low-productivity agent. Nevertheless he must refrain from fully exploiting the agent, since an attempt to do so would create the wrong incentives for the high-productivity agent to reveal his information. If the principal were to fully exploit the low-productivity agent, he would want him to produce a larger amount of output. But this would then induce the high-productivity agent to refrain from revealing his information to the low productivity agent in order to mimic the behavior of a low-productivity agent.

Note that the two period incentive scheme has two effects on the principal's profits. First, it increases the number of agents with high-productivity, due to the information transmission. Second, and more subtly, the signal of first-period production helps to sort the remaining agents more effectively. Despite this sorting, the principal is unable to fully exploit the low-productivity agents in some circumstances, even though he knows their type for certain.

6. Markets for cost reduction

In the previous section we have analyzed the optimal incentive scheme for the principal which induces the high-productivity agent to impart his knowledge to the low-productivity agent. However, we can ask whether this information transmission is in the private interest of the agents.

Consider a world in which the principal only offers standard two-part, single period incentive schemes. In this framework it is always in the interest of the low-productivity agents to purchase knowledge from the high-productivity agents. This is true because the low-productivity agents get no surplus while the high-productivity agents receive surplus in the optimal scheme. Hence they would be willing to pay a positive price in order to become high-productivity agents.

However, if all low-productivity agents purchase the cost reduction from the high-productivity agents then they all are worse off. Why? If the principal knows that all agents are high productivity agents, then he can use a *one-part* pricing scheme that extracts all of their surplus—which means that it is not worthwhile to extract all of their information.

It follows from this argument that when private transmission of is possible, it will be in the interest of the principle to offer only a single-part plan, leaving it to the agents to set up a system to disseminate information among themselves. However, note that we have implicitly made very strong assumptions about the costs of imparting information. If the costs of organization and information transmission are high, then it would certainly make sense to consider the kinds of schemes described earlier in which the principal takes responsibility for organizing this information transmission.

Appendix 1

In this appendix I consider the equations defining the constraints in the one-period problem and show which of these will be binding. This is a standard result and is included only for the sake of completeness.

For convenience, here are the equations

$$s_1 - c_1(x_1) \geq s_2 - c_1(x_2) \tag{17a}$$

$$s_2 - c_2(x_2) \geq s_1 - c_2(x_1). \tag{17b}$$

$$s_1 - c_1(x_1) \geq 0 \tag{18a}$$

$$s_2 - c_2(x_2) \geq 0. \tag{18b}$$

Clearly one of (17a–18a) and one of (17b–18b) will be binding. Let us first consider (17a–18a). By the monotonicity of the cost functions and (18b):

$$s_2 - c_1(x_2) > s_2 - c_2(x_2) \geq 0.$$

Combining this inequality with (17a) we have $s_1 - c_1(x_1) > 0$. Hence (18a) is not binding which means that (17a) is binding.

Now consider equations (17b) and (18b). Is it possible that (17b) is binding? Assume so and use the fact that (17a) is binding to write

$$s_2 = s_1 + c_2(x_2) - c_2(x_1) = s_2 + c_1(x_1) - c_1(x_2) + c_2(x_2) - c_2(x_1).$$

Rearranging, we have

$$c_1(x_2) - c_1(x_1) = c_2(x_2) - c_1(x_1),$$

which contradicts the assumed properties of the cost function.

Appendix 2

In this appendix I show that constraints (12*a*) and (15) will both be binding in the optimal solution.

Part 1. *The incentive compatibility constraint, equation (15), will always be binding.*

Proof. Suppose that (15) is not binding and that (12*a*) is binding. Then we simply have two two-part pricing problems. Since x_{22} only appears in one constraint, the optimal choice x_{22}^* will be efficient, and therefore $x_{22}^* > x_{12}^*$. We now have

$$\begin{aligned} s_{11} - c_1(x_{11}^*) &= c_2(x_{12}^*) - c_1(x_{12}^*) \quad \text{From equation (12}a\text{)} \\ &< c_2(x_{22}^*) - c_1(x_{22}^*) \quad \text{from assumption about marginal cost} \\ &= s_{22} - c_1(x_{22}^*) \quad \text{since (14) is binding.} \end{aligned}$$

This violates the assumption that (15) is satisfied. ∎

The logic behind this result is instructive. If the principal knows that the agent has low productivity then there is no selection problem and he will want to have him produce an efficient amount of output. If the principal doesn't know the productivity of the agent, then the plan intended for the low productivity agent will be inefficient and the payoff to the high productivity agent is just adequate to make him indifferent between choosing either plan. But this means that the high productivity agent will always be better off by failing to instruct the low-productivity agent in the first period and choosing the plan intended for the low-productivity agent in the second period.

Part 2. *The information revelation constraint, (12a), is binding.*

Proof. In order to show that (12*a*) is binding, we suppose that this not the case so that only (15) is binding.

In this case the maximization problem of the principal takes the form

$$\begin{aligned} \max\ & \pi(2-\pi)[x_{01} - c_1(x_{01}) - c_2(x_{02}) + c_1(x_{02})] \\ & + (1-\pi)^2\left[(x_{02} - c_2(x_{02})) + (x_{22} - c_2(x_{22}))\right] \\ & + \pi[x_{11} - c_1(x_{11}) - c_2(x_{22}) + c_1(x_{22})] \\ & + \pi(1-\pi)[x_{12} - c_2(x_{12})]. \end{aligned}$$

Differentiating with respect to the choice variables, we have the first-order conditions

$$1 - c_1'(x_{01}) = 0 \qquad \text{(19)a}$$

$$(2\pi - \pi^2)[c_1'(x_{02}) - c_2'(x_{02})] + (1-\pi)^2[1 - c_2'(x_{02})] = 0 \qquad \text{19b}$$

$$(1-\pi)^2[1 - c_2'(x_{22})] + \pi[c_1'(x_{22}) - c_2'(x_{22})] = 0 \qquad \text{19c}$$

$$1 - c_1'(x_{11}) = 0 \qquad \text{19d}$$

$$1 - c_2'(x_{12}) = 0. \qquad \text{19e}$$

Conditions (19*c*) and (19*e*) imply that $x_{22} < x_{12}$. We now have

$s_{11} - c_1(x_{11}) = c_2(x_{22}) - c_1(x_{22})$ assumption that (15) is binding

$< c_2(x_{12}) - c_1(x_{12})$ from assumption about marginal cost and fact that $x_{22} < x_{12}$

$= s_{12} - c_1(x_{12})$ from (13*b*)

This violates (12*a*) and completes the argument. ∎

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