Strategic Pricing in Service Industries

by

Yao Cui

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Doctoral Committee:

Professor Izak Duenyas, Co-Chair
Associate Professor Ozge Sahin, Co-Chair, Johns Hopkins University
Assistant Professor Stefanus Jasin
Assistant Professor Yesim Orhun
Assistant Professor Cong Shi

For my parents

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ABSTRACT

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Co-Chairs: Izak Duenyas, Ozge Sahin

This dissertation studies several strategic-level pricing decisions of firms that are motivated by recent changes of pricing policies in several service industries. It consists of three essays, each analyzing a different problem of selecting the optimal pricing policy for firms in a certain service industry. All three essays contribute to the arising areas of strategic-level revenue management and consumer-driven operations management.

The first essay studies whether preventing resale of tickets benefits the ticket providers for sporting and entertainment events. Different from what common wisdom may suggest, I find that this event organizers can benefit from reductions in consumers' (and speculators') transaction costs of resale in many cases. Further, I propose ticket options (where consumers would initially purchase an option to buy a ticket and then exercise at a later date) as a novel ticket pricing policy, and show that ticket options naturally reduce ticket resale and increase event organizers' revenues.

The second essay studies a conditional upgrade strategy that has recently become common in the travel industry. A consumer can accept a conditional upgrade offer after making a reservation and pay the fee to upgrade at check-in if the higher-quality

product type is still available. I identify multiple benefits of conditional upgrades including demand expansion, price correction, and risk management. Moreover, I find that using conditional upgrades can generate higher revenues than having the ability to optimize product prices and use dynamic pricing.

The third essay studies the firm's strategic decision of whether to bundle the ancillary service (e.g., baggage delivery) into the main service (e.g., air travel) or to unbundle and charge separate prices. I find that a firm that can price-discriminate when selling the main service should unbundle the ancillary service because consumers' likelihoods of purchasing the ancillary service are low, or a large proportion of consumers are myopic instead of forward-looking, or the firm is dependent on intermediaries to make sales. I also find that the firm's ability to price-discriminate when selling the main service reverses some classic bundling results for a uniform-pricing firm.

CHAPTER I

Introduction

Traditionally, research in the area of revenue management has focused on using models with exogenous demand to optimize firms' pricing decisions on the tactical level. The key question to answer has been *what price to charge*. This dissertation applies the idea of revenue management to firms' strategic-level pricing decisions. The key question to answer is *what pricing policy to use*. In the three essays included in this dissertation, I study three of firm's optimal pricing policy selection problems in the contexts of different service industries. The research questions are motivated by recent changes of pricing policies in these industries, and the goal of this dissertation is to provide firms with insights about innovating their pricing policies in the right directions. When choosing the right pricing policy, it is important for firms to take the consumer response into consideration. I use economic models and consumer-driven operations management models to capture the strategic interactions between firms and consumers.

In the first essay, I am interested in whether preventing resale of tickets benefits the capacity providers for sporting and entertainment events. Common wisdom suggests that ticket resale is harmful to event organizers' revenues and event organizers have tried to prevent resale of tickets. For instance, Ticketmaster has recently proposed paperless (non-transferrable) ticketing which would severely limit the opportunity to resell tickets. I consider a model that allows resale from both consumers and speculators with different transaction costs for each party. Surprisingly, I find that this wisdom is incorrect when event organizers use fixed pricing policies, in fact event organizers benefit from reductions in consumers' (and speculators') transaction costs of resale. Even when multiperiod pricing policies are used, I find that an event organizer may still benefit from ticket resale if his capacity is small. While paperless ticketing is suggested as a way to reduce ticket resale and prevent speculators from buying tickets, my results suggest that it may reduce the capacity providers' revenues in many situations. Instead, I propose ticket options as a novel ticket pricing mechanism. I show that ticket options (where consumers would initially buy an option to buy a ticket and then exercise at a later date) naturally reduce ticket resale significantly and result in significant increases in event organizers' revenues. Furthermore, since a consumer only risks the option price (and not the whole ticket price) if she cannot attend the event, options may face less consumer resistance than paperless tickets.

In the second essay, I study a conditional upgrade strategy that has recently become very common in the travel industry. After a consumer makes a reservation for a product (e.g., a hotel room), she is asked whether she would like to upgrade her product to a more expensive one at a discounted price. The upgrade, however, is not fulfilled immediately. The firm fulfills upgrades at check-in if there are still more expensive products available, and the upgrade fee is only charged to the consumer if she gets upgraded. Consumers decide which product type to book and whether to accept an upgrade offer or not based on the anticipated upgrade probability. I model the consumers' decisions using a Poisson-arrival game framework with incomplete information and prove the existence of Bayesian Nash equilibrium. To further study the firm's optimal upgrade pricing strategy and develop managerial insights, I also analyze a fluid model which is the asymptotic version of the stochastic model. My numerical studies validate that my theoretical results derived from the fluid model carry through to the stochastic model.

My analysis identifies multiple benefits of conditional upgrades. First, the firm is able to capture more demand by offering conditional upgrades, i.e., the consumers who value original product types lower than the original prices but value higher-quality products higher than the discounted price with upgrades. Second, conditional upgrades enable the firm to improve its market segmentation by inducing more consumers to purchase higher-quality products. Third, conditional upgrades give the firm more flexibility in better matching fixed capacities to stochastic demands. For a firm that is a price taker in the market, offering conditional upgrades is effective in compensating for the firm's lack of ability in setting its prices optimally, and can sometimes generate even higher revenues than being able to optimize product prices. For a firm that has the ability to optimize product prices, conditional upgrades can generate higher revenues than dynamic pricing.

In the third essay, I consider a setting where the firm sells a main service (e.g., air travel) as well as an ancillary service (e.g., baggage delivery) to two types of consumers with different valuations (e.g., business travelers and leisure travelers). I study the firm's strategic decision of whether to bundle the ancillary service into the main service and charge a single price, or to unbundle and charge separate prices. I consider both a firm that price-discriminates when selling the main service and a firm that charges a uniform main service price. For a price-discriminating firm, I find that it may be more profitable to unbundle the ancillary service for the following three cases: 1) consumers' likelihoods of purchasing the ancillary service are low, 2) a large proportion of consumers are myopic (i.e., they do not consider future purchase of the ancillary service when purchasing the main service in advance) instead of forward-looking (i.e., considering future utilities), 3) the firm is dependent on intermediaries to make sales.

Moreover, I study how the firm's ability to price-discriminate when selling the main service affects the optimal strategy for the ancillary service. For a uniform-pricing firm, it is optimal to unbundle the ancillary service if the consumers that value the main service higher have a high enough likelihood of purchasing the ancillary service. However, for a price-discriminating firm, it is optimal to unbundle the ancillary service if the consumers that value the main service higher have a low enough likelihood of purchasing the ancillary service. The ability to price-discriminate when selling the main service makes bundling (unbundling) more likely to be the optimal ancillary service strategy when consumers' valuations for the main service and the ancillary service are positively (negatively) correlated. Finally, I characterize how firms' use of main service price discrimination and consumers' valuation structure (i.e., the correlation between consumers' valuations for the main service and the ancillary service) jointly determine the ancillary service pricing strategies in an industry.

CHAPTER II

Should Event Organizers Prevent Resale of Tickets?

2.1 Introduction

Consumer resale behavior plays an important role in ticket sales of concerts and sporting events. For live music and sporting events, ticket sales in the primary markets generate \$20 billion per year in the US. On the other hand, resale markets generate \$3 billion each year in the US, and this number is expected to grow over the next several years (*Mulpuru et al.*, 2008). For popular concerts, the resale market revenue can be as much as 37% of the primary market revenue, and 46% of the resale activity is generated by consumers (*Leslie and Sorensen*, 2011). Consumer resale is prevalent in event ticket sales for the following reasons. First, event capacity providers make tickets available early in advance to satisfy the needs of those highly dedicated fans who want to secure the rights to attend the events they are interested in (*Courty*, 2003a and *Moe et al.*, 2011). Second, event tickets are usually transferrable. Third, most tickets are non-refundable and consumers purchasing event tickets usually have high valuation uncertainties. A sporting fan may not know whether her favorite team will get into the final game or not when she buys the ticket for it. A consumer may also find the event conflicting with some other appointment of higher priority after she buys the ticket. In addition to consumer resale, there may be speculators who purchase tickets solely for the purpose of reselling later hopefully at a higher price.[1]

A consumer who cannot attend the event can resell the ticket directly to another consumer or through a broker, among which StubHub, eBay, RazorGator are major players. Brokers obtain profits by charging transaction fees that can be as high as 25% of the ticket resale value to the seller and the buyer. The development of online transactions on the Internet has provided more opportunities for such brokers to thrive. No matter how consumers resell their tickets, traditionally the perception is that resale (secondary) markets are bad for the event organizers and ticket distributors and need to be prevented. As the largest ticket sales and distribution company in the US, Ticketmaster attempted to prevent resale of tickets by influencing ticketing legislation. The battle between firsthand ticket sellers and brokers has produced two nonprofit groups (*Sisario*, 2011). The Fans First Coalition, financed by Live Nation Entertainment which is the parent company of Ticketmaster, supports paperless ticketing. On the other hand, The Fan Freedom Project, financed by StubHub, supports the use of paper tickets. Paperless ticketing works like an airline e-ticket, with no traditional ticket printed when a customer places an order. Instead, a fan shows his credit card at the box office to enter the event, guaranteeing that the person who originally placed the order is the same one attending the event. Paperless ticketing is an instrument to make the tickets non-transferrable while paper tickets are transferrable. However, in 2010, Ticketmaster failed to prevent a change to New York's scalping law which required that consumers have the option for transferrable tickets. So far, there is no federal regulation regarding event ticket resale in the US. Some states restrict resale, but anti-scalping laws are rarely enforced. In 2010, non-transferrable tickets made up only 0.01 percent of all the tickets Ticketmaster processed (*Rovell*, 2011). Moreover, it is not clear when and under what conditions resale markets are harmful

[1] Speculators can be thought as consumers with zero valuations for attending the event.

to event capacity providers, as many college athletics departments have recently partnered with brokers to create fan-to-fan ticket exchange marketplaces and encouraged their fans to use these platforms to resell their tickets.

There are two major goals of event capacity providers in order to maximize revenue in this challenging environment: first, tie prices to demand; second, capture the revenues from the resale markets. Indeed, Nathan Hubbard, the CEO of Ticketmaster, said that 2010 taught them they have real challenges as an industry and one of them is pricing (*Smith*, 2011). While the level of analytics and technology in event revenue management is far behind travel and retail revenue management, in recent years, event capacity providers started to use multiperiod pricing (i.e., changing the ticket price over time) which has been used by airlines for 30 years. For example, Ticketmaster has partnered with MarketShare to bring multiperiod pricing to events.[2] The event capacity providers are hoping that rather than fixed pricing (i.e., keeping the same ticket price over time) which was used as the major pricing strategy, a more flexible pricing strategy can help them capture more of the revenue potential, especially the revenue generated by the resale markets. Recent dynamics of the event ticketing industry and the resale markets motivate my research questions: (i) How does ticket resale affect the event capacity providers' prices and revenues (i.e., is resale harmful to event capacity providers?), and ii) Which pricing strategy is more effective in capturing the resale market revenues?

	Period 1: tickets	Period 1: ticket options
Price fixed over time	Fixed pricing (Section 2.4)	N/A
Price changes over time	Multiperiod pricing (Section 2.5)	Ticket options (Section 2.6)

Table 2.1: Pricing strategies studied in this essay

To answer these questions, I study whether an event capacity provider is indeed harmed by or in fact can benefit from resale of tickets from consumers as well as

[2]According to LiveAnalytics (March 3, 2012 MIT Sports Analytics Conference Presentation), 57% of NBA, 50% of MLB, 37% of NHL teams use multiperiod pricing.

speculators under different pricing strategies. As described by Table 2.1, I consider whether the capacity provider keeps the price fixed over the selling period (fixed pricing) or can change the price (multiperiod pricing) and whether the capacity provider actually sells tickets or ticket options in period 1 (clearly, if the capacity provider sells ticket options initially but tickets later, the prices over time cannot be fixed). Fixed pricing is the pricing mechanism used by most college athletics departments and concert organizers, and multiperiod pricing has started to be used by professional sports teams. I find that the capacity provider's optimal revenue from fixed pricing increases when ticket resale is easier for either consumers or speculators, and paperless (non-transferrable) ticketing actually hurts the capacity provider's revenue. Under multiperiod pricing, when the provider's capacity is small, similar to fixed pricing, he benefits from consumer resale. On the other hand, if an event capacity provider uses multiperiod pricing and his capacity is large, then he indeed may benefit from making tickets non-transferrable. Finally, motivated by recent industry practice, I study ticket options that are offered by OptionIt. When an event capacity provider sells options, the consumer pays a fee to get an option to buy a ticket later, and she pays an execution fee when she finally buys the ticket. An advantage to the consumer is that if the consumer cannot attend the event, she only loses the option fee instead of the whole ticket price. I show that options generate higher revenues for event capacity providers by significantly reducing ticket resale and capturing the resale market revenues. However, the capacity provider improves revenues further if tickets are non-transferrable under option pricing. My numerical results indicate that while switching to selling options from multiperiod pricing results in a large revenue increase, making tickets non-transferrable in addition does not result in a revenue increase that is as significant. Therefore, the revenue gains from switching to selling options can be very significant for event capacity providers. Thus, this essay offers a different route to increasing revenues and shrinking the resale market (than non-transferrable tickets) that is likely to generate less adverse consumer reaction.

2.2 Literature Review

This essay is related to the general revenue management literature (see *Talluri and van Ryzin*, 2005 for a review). In particular, the advance pricing literature, *Gale and Holmes* (1993), *DeGraba* (1995), *Dana* (1998), *Shugan and Xie* (2000), is relevant to event ticket sales. However, these papers assume tickets are non-transferrable and there is no secondary market. There is not much literature in operations management that deals with issues regarding event ticket pricing in particular. To my knowledge, this essay is one of the very few works that study event ticket pricing (*Su*, 2010, *Balseiro et al.*, 2011, *Tereyagoglu et al.*, 2012) and the first one that studies the consumer resale behavior in the context of event ticket pricing (perishable product pricing).

Streams of economics and marketing literature investigate several aspects of the ticket industry. Table 2.2 summarizes the papers, including this essay, that study ticket resale and are closely related to event revenue management. *Courty* (2003b) studies monopolistic ticket selling to consumers who learn new information about their demands over time. He assumes no capacity constraint and shows that rationing and inter-temporal sales are never optimal. He also shows that the monopolist cannot do strictly better by allowing resale. I assume the provider has limited capacity and the resellers incur resale transaction costs, and study how the capacity level and the resale transaction cost influence the provider's optimal pricing decisions. Moreover, I study a general ticket options model and show that options help event capacity providers capture more resale market revenues. *Leslie and Sorensen* (2011) study a similar problem empirically and find that while consumer resale improves allocative efficiency, some of the welfare gain from reallocation is offset by increases in efforts and transaction costs in the resale market. *Moller and Watanabe* (2010) briefly study consumer resale with price commitment and with period 1 arrivals only. They show that the relative profitability of clearance sales with respect to advance purchase discounts increases with resale.

Geng et al. (2007) study a two-period model where the capacity provider changes the price in period 2 (multiperiod pricing) and assume consumers are only allowed to resell before the capacity provider's price change (they call this pricing scheme "partial resale"). In contrast, in this essay, I assume that initially tickets can only be sold by the capacity provider, but after a later date, tickets are also available from the secondary market till the event takes place (currently it is possible to buy a ticket from StubHub only a few hours before the start of an event). Furthermore, *Geng et al.* (2007) assume no resale transaction cost. In contrast, I am interested in whether increases in the resale transaction cost benefit or hurt the capacity provider. These differences in modeling lead to different conclusions. For example, *Geng et al.* (2007) predict that resale before the price change is beneficial to the capacity provider only if he sells advance tickets at a premium. If advance tickets are discounted, they find that resale should not be allowed. I find that premium advance selling is not an equilibrium if resale occurs till event takes place, and the only equilibrium is discounted advance selling. Finally, I also study ticket resale in the context of fixed pricing and option pricing, in addition to multiperiod pricing. Therefore, the focus and the insights of this essay are different.

Paper	Source of resale	Resale transaction cost	Capacity constraint	Pricing strategies	Findings
Courty (2003b)	- Consumers	No	No	- Multiperiod pricing	- The provider cannot do strictly better by allowing resale.
Moller and Watanabe (2010)	- Consumers	No	Yes	- Multiperiod pricing	- Resale makes clearance sales (i.e., high-to-low pricing) more profitable. - Resale increases the relative profitability of clearance sales with respect to advance purchase discounts (i.e., low-to-high pricing).
Geng et al. (2007)	- Consumers	No	Yes	- Multiperiod pricing	- Partial resale (i.e., consumers can resell tickets till the capacity provider announces the period 2 price) is beneficial to the capacity provider only if the seller sells advance tickets at a premium.
Karp and Perloff (2005)	- Speculators	Yes	Yes	- Multiperiod pricing	- If the speculators are able to perfectly price discriminate and extract all consumer surplus, then speculators may increase capacity provider profits when their transaction costs are low.
Su (2010)	- Speculators	No	Yes	- Fixed pricing - Dynamic pricing	- Speculators increase capacity provider profits under fixed pricing. - Speculators do not increase capacity provider profits under dynamic pricing.
This essay	- Consumers - Speculators	Yes (consumers and speculators can incur different resale transaction costs)	Yes	- Fixed pricing - Multiperiod pricing - Option pricing	- Option pricing is the most effective pricing strategy to reduce consumer and speculator resale and to capture resale market revenues. - Consumer and speculator resale always increases provider profits under fixed pricing and always decreases under option pricing. - Speculator resale decreases provider profits under multiperiod pricing; consumer resale increases provider profits under multiperiod pricing unless the capacity is large.

Table 2.2: Comparison of papers studying ticket resale

There is also a stream of literature on ticket scalping and speculative behavior. Different from consumer resale, speculators purchase tickets solely for the purpose of reselling later hopefully at a higher price. *Courty* (2003a) provides a survey of this literature. *Karp and Perloff* (2005) assume scalpers are able to perfectly price discriminate and extract maximal consumer surplus. Therefore, they find that speculators do not reduce and may increase monopoly profits when their transaction costs are low under multiperiod pricing. I find that if the speculators cannot perfectly price discriminate and consumer resale is possible, speculator resale is never beneficial to the capacity provider under multiperiod pricing. Different from *Karp and Perloff* (2005), *Su* (2010) captures the possibility that scalpers may incur a loss (e.g., if demand turns out to be weak). He finds that the presence of speculators increases the firm's expected profits from fixed pricing but does not change the profits if dynamic pricing is used. This essay is complementary to *Su* (2010), as I study resale from both consumers and speculators. I show that while his finding regarding speculator resale remains true for consumer resale as well if fixed pricing is used, consumer resale can sometimes be a benefit to the capacity provider when multiperiod pricing is used. Under multiperiod pricing, consumer resale can create competition in the secondary market and drive down the capacity provider's price, but it also increases consumers' willingness to pay in the advance selling period. Thus, consumer resale can sometimes be beneficial to the capacity provider. On the other hand, speculator resale is never beneficial and may even decrease the revenues of the provider under multiperiod pricing. Therefore, interestingly, I find that effects of consumer resale and speculator resale on provider revenues are not identical. Moreover, unlike previous papers, I allow consumers to have inter-temporal valuation uncertainties, and allow both consumers and speculators to incur transaction costs for ticket resale. Finally, for the first time in the literature, I show that ticket options result in higher revenues for event capacity providers than fixed and multiperiod pricing due to significant reduction of the resale markets.

Finally, there are a few papers that study options for services. *Xie and Gerstner* (2007) show that a capacity-constrained service provider can profit from offering partial refunds for service cancellations. Selling ticket options is similar to allowing service cancellations, with the advance price equal to the sum of the option price and the execution fee, and the refund equal to the execution fee. However, in *Xie and Gerstner* (2007), the refund is set upon receiving a cancellation notification. With ticket options, the service provider commits to the refund upfront as he pre-announces both the option price and the execution fee. It is easy to show that commitment results in higher profits. More importantly, my focus is the benefit of tickets options in capturing the resale market revenues. *Xie and Shugan* (2001) show that with infinite capacity, advance selling with refund is more profitable than both advance selling without refund and spot selling. *Gallego and Sahin* (2010) study real options with limited capacity. They show that the capacity provider earns significantly higher revenues by selling real options on capacity than low-to-high pricing. Similarly, *Balseiro et al.* (2011) show that offering team-based options for sporting events benefits the provider and the consumers. *Sainam et al.* (2010) find that consumer options can protect consumers from the downside risk related to uncertain outcomes and enhance seller profits by enabling superior market segmentation and increasing consumer willingness to pay. They empirically demonstrate that consumer willingness to pay increases and profits from option pricing can exceed those from advance selling and spot selling. However, none of these papers considers consumer or speculator resale in secondary markets. My main focus is how resale markets and transaction costs affect the capacity providers' revenues and optimal pricing strategies. I am interested in whether the capacity provider has an incentive to prevent ticket resale under different pricing strategies where pricing with ticket options is one of these strategies. I show that the capacity provider can significantly reduce resale hence capture more resale market revenues with options, while under fixed and multiperiod pricing, he has only limited control over resale markets.

2.3 Model

Consider an event capacity provider that sells his capacity C over two periods. As in *Courty* (2003a), I "assume that the audience is composed only of two types of consumers: 'diehard fans,' who plan their social calendars well in advance, and 'busy professionals,' who make decisions at the last minute. This consumer characterization does not suggest that busy professionals enjoy the event less than diehard fans, only that these two market segments plan their social calendars differently. Indeed, a consumer could qualify as a diehard fan for one event and as a busy professional for another." λ_1 consumers arrive in period 1 to purchase advance tickets. λ_2 consumers arrive in period 2, who make their purchasing decisions at the last minute because they may want to wait until some uncertainties in their schedules or regarding the event are settled (e.g., a diehard soccer fan can buy a ticket for the World Cup final without knowing who will be in the final; others will only buy if their country is in the final). $\lambda_1 + \lambda_2$ measures the magnitude of the consumer base for the event. In my analysis, I focus on the case of $C < \lambda_1 + \lambda_2$ which is the more realistic and interesting scenario. When $C \geq \lambda_1 + \lambda_2$, the prices decrease to the lowest possible level $v_{\min}$ (i.e., consumer valuation lower bound) no matter which strategy is used because the market is over-supplied. In that case, every pricing strategy results in the same outcome.

Consumers have an ex ante i.i.d. valuation V which has a continuous support and is bounded below by $v_{\min} > 0$. Let $F(\cdot)$ and $f(\cdot)$ denote the cumulative distribution function and probability density function of V, respectively. Without loss of generality, I assume that consumers arriving in different periods have the same valuation distribution, my analysis can be easily generalized to the case where period 2 consumers' valuations follow a different distribution.[3] Consumers learn their valuations at the beginning of period 2. If a period 1 consumer purchases an advance ticket, she can either use the ticket to attend the event or resell it in period 2, depending on her realized valuation. A period 1 consumer may also decide to postpone her purchasing decision to period 2 when she gains more information about her valuation. In this case, she can buy from either the capacity provider or the resale market. I assume efficient rationing, i.e., given the same price, consumers who value the ticket the most are served first and resellers who value the ticket the least make sales first. This assumption is common in economics literature and is also made in papers studying event ticketing such as *Su* (2010).

Consumers incur a transaction cost when they resell tickets. This transaction cost can represent the commission paid to the broker and can also represent the search or inconvenience cost when looking for the buyer. In reality, brokers charge commissions which are typically percentages of the ticket resale prices. For example, StubHub charges a 15% commission to the seller and a 10% commission to the buyer. To make sales, the resellers have to reduce the resale price so that buyers find the price competitive to the capacity provider's price after paying the buyers' commission. Without loss of generality, I use a single transaction cost $\tau > 0$ which is a percentage of the resale price and define the resale price as the one in the case where only the resellers pay the commission.[4]

Besides regular consumers who have a genuine interest in potentially attending the event, I also allow an infinite pool of speculators who do not value attending

[3] All my results in Sections 2.4 - 2.6 hold and all my managerial insights remain valid if the valuations of the two classes of consumers are different but both follow uniform or shifted exponential distributions.

[4] To see the equivalence of using a single transaction cost and using separate transaction costs, let r denote the resale price, and let τ_s and τ_b denote the transaction costs (as percentages) that the broker charges to the seller and the buyer, respectively. With separate transaction costs, the actual price resellers can charge is $r/(1+\tau_b)$, hence the net gain from resale is $r(1-\tau_s)/(1+\tau_b)$. Thus, using separate transaction costs is equivalent to using a single transaction cost of $\tau = 1-(1-\tau_s)/(1+\tau_b)$. For StubHub, this single transaction cost is equal to $\tau = 1-(1-15\%)/(1+10\%) = 22.73\%$.

the event but may purchase tickets in period 1 and resell tickets in period 2. Since speculators only enter the market if the net payoff from resale is greater than the capacity provider's period 1 price, the number of speculators entering the market in equilibrium is endogenously determined.[5] I use τ' to denote the speculators' resale transaction cost and assume $\tau' \leq \tau$ to capture the fact that speculators usually have less costly channels to resell tickets (e.g., speculators may not have to sell their tickets through well-established brokers such as StubHub but create their own cheaper channels to sell tickets directly to consumers).

The capacity provider's goal is to maximize his revenue from selling his capacity over two periods.[6] To reflect the event ticketing industry practice, I assume that the capacity provider makes tickets available in advance to satisfy the needs of those highly dedicated fans who want to secure the rights to attend events they are interested in (*Courty*, 2003a, *Moe et al.*, 2011). (Under fixed pricing, in my model, the capacity provider may increase his revenues even further by not allowing advance sales, whereas under multiperiod or option pricing, advance sales can be endogenously optimal. Note that it may not be realistic to sell event tickets only on the spot before the event. For example, many college football fans travel from out of state to see their team play. Last minute airfares and hotel prices are a lot more expensive typically. Thus, if the capacity provider does not make tickets available in advance, these fans may not attend the event.) I also assume the provider does not strategically hold back capacity in either period. This is consistent with the practice of most college sports teams, professional sports teams and artists as they intentionally offer all seats available to maximize the entertainment value of the event which is highly correlated with the size of the audience: the bigger the audience, the more enjoyable the expe-

[5] Without loss of generality, in the model I do not include arbitrageurs who buy tickets in period 2 and resell tickets immediately. Similarly, I do not allow period 2 consumers to buy and resell tickets in period 2. It is easy to show that such behavior cannot occur in equilibrium.

[6] I ignore the variable cost because from a production standpoint, events have high fixed costs and low variable costs (*Connolly and Krueger*, 2006).

rience (*Becker*, 1991). In section 2.7.1, I study the case where the provider may hold back part of his capacity in period 1 to sell in period 2 as a model extension.

I first study the pricing strategies that have been commonly used in practice by event organizers. In Section 2.4, I study fixed pricing where the capacity provider sells tickets at price p_f throughout two periods. In Section 2.5, I study multiperiod pricing where the capacity provider sells tickets at price p_1 in period 1 and at price p_2 in period 2. The sequence of events is as follows. First, at the beginning of period 1, the capacity provider announces his advance ticket price. After that, period 1 consumers decide whether to purchase tickets immediately or wait, and speculators decide whether to enter the market or not. Then, in period 2, after consumers realize their valuations, the period 1 consumers who have purchased tickets decide whether to resell or use them, and those choosing to resell the tickets as well as speculators determine the resale price. If the capacity provider uses multiperiod pricing, he determines his period 2 price at the same time. Figure 2.1 describes the period 1 consumers' inter-temporal decision process and the payoff from each decision under fixed and multiperiod pricing. A speculator's decision process is a special case of Figure 2.1 where $V = 0$ with probability one and the resale transaction cost is τ' instead of τ. Throughout this chapter, I add subscripts to the notations to specify which pricing strategy I am considering: "f" for fixed pricing, "m" for multiperiod pricing, "o" for ticket options.

As described above, my main interest is in the effect of ticket resale on the capacity provider's revenues where the capacity provider's goal is to extract as much revenue as possible while selling out the tickets to maximize the entertainment value of the event. On the one hand, allowing resale (or a decrease in resale transaction costs) can increase the value of tickets for consumers since consumers know that they have an option to resell tickets if for some reason they cannot attend the event. On the other hand, resale markets (as well as speculators buying tickets when resale is allowed)

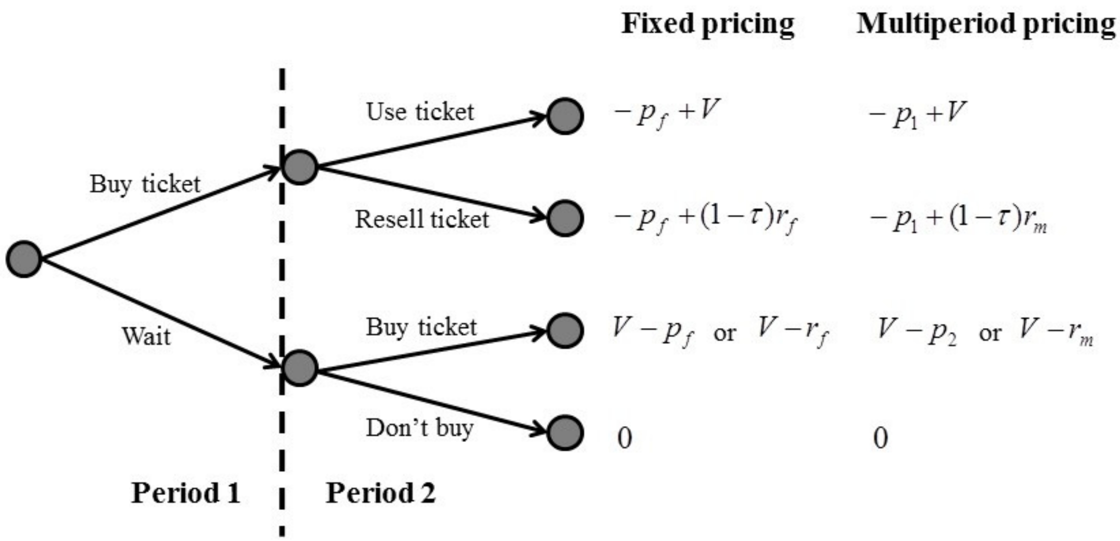


Figure 2.1: Consumer choice model under fixed and multiperiod pricing

may increase competition with the capacity provider and may result in a decrease of ticket revenues. This is the fundamental high-level tradeoff that I am interested in and that I am going to analyze under fixed, multiperiod and option pricing in the following sections.

2.4 Fixed Pricing

In this section, I study the fixed pricing strategy that has been commonly used by event capacity providers such as college sports teams and concert organizers in practice. My result here is that event capacity providers are always hurt by an increase in the transaction costs that either consumers or speculators incur in reselling the tickets, that is, an event capacity provider using fixed pricing prefers consumers and speculators to be able to use resale markets with no transaction cost at all. To analyze this case, I use backward induction to find the subgame perfect equilibrium of the game between the capacity provider, consumers and speculators. More specifically, I first characterize the equilibrium resale price in period 2, then characterize the purchasing decisions of consumers and speculators in period 1, and finally determine the capacity provider's optimal fixed price.

Theorem II.1. *(i) The equilibrium resale price* r_f^* *is given by* $[(\lambda_1 - C)^+ + \lambda_2)]\bar{F}(r_f^*) = (C - \lambda_1)^+ + \min(\lambda_1, C)F((1-\tau)r_f^*)$.

(ii) Define p_s *as the solution to* $(\lambda_1 + \lambda_2)\bar{F}(p_s) = C$ *and define* p_f^n *as the solution to* $p_f^n + \int_{p_s}^{\infty}(v - p_f^n)\,\mathrm{d}F(v) = E[\max(V, (1-\tau)r_f^*)]$. *The capacity provider's optimal fixed price is* $p_f^* = p_f^n$ *if* $p_f^n \geq (1-\tau')p_s$ *and* $p_f^* = \min\big(E[\max(V,(1-\tau)r_f^*)] - E(V - p_s)^+, (1-\tau')p_s\big)$ *otherwise. Moreover,* $p_f^* < r_f^*$.

(iii) For a given τ*, speculators enter the market in equilibrium if and only if* $\tau' < \bar{\tau}_f'(\tau) = 1 - p_f^n/r_f^*$.

Theorem II.1 characterizes the equilibrium outcome under fixed pricing. Given that a period 1 consumer obtains a ticket from the capacity provider, in period 2, if her valuation is smaller than the net payoff from resale, $(1-\tau)r_f^*$, she resells the ticket; otherwise, she uses the ticket herself. Note that the equilibrium resale price is higher than the capacity provider's optimal fixed price. This price inflation in resale markets close to event dates is often observed in reality,[7] and is one of the reasons event capacity providers are sometimes interested in eliminating resale markets. However, as I will show in Theorem II.2 below, this would actually hurt event capacity providers.

Theorem II.1(iii) states that speculators enter the market in equilibrium if their resale transaction cost is small enough. In this case, speculators keep entering the market in period 1 until the provider's capacity is depleted, and they resell the tickets in period 2 instead of the capacity provider at the resale price which is higher than the capacity provider's fixed price. This result provides one explanation for why we see speculators in reality – their transaction cost to resell tickets is smaller than the

[7] In 2012, the average ticket resale price is \$139.71 for the Southeastern Conference and \$132.65 for the Big Ten Conference (*Rovell*, 2012) which is almost double the original ticket price.

transaction cost incurred by regular consumers. A reduced transaction cost to resell tickets gives speculators an advantage and makes speculators more likely to enter the market.

On the other hand, if speculators' resale transaction cost is large enough so that their willingness to pay for advance tickets is lower than regular consumers, by charging a price higher than speculators' willingness to pay, the capacity provider may shut speculators out of the market. Of course, in many events where the capacity provider uses fixed pricing, we see speculators and they are not shut out of the market. A second reason for speculators' existence may be underpricing by event capacity providers. Note that in my model, if $\tau' \geq \bar{\tau}'_f(\tau)$, the capacity provider can shut speculators out of the market if he uses "optimal" pricing. However, it is not clear that event capacity providers always set prices optimally in reality. For example, in the 2012 college football season, the Ohio State University charged \$75 or \$85 per seat for every game (\$85 was charged for better seats) even though some games are known to be much more popular than others, such as the game against the University of Michigan. Even though the Michigan – Ohio State game is one of the most popular games in college football, Ohio State did not charge more for this game. Consequently, ticket prices on the resale markets were at a minimum double the original ticket price, which would offer a great opportunity for speculators to make profits. Thus, underpricing may be another reason for speculators' existence in the market. There is some evidence in the literature that until recently, teams were afraid of offending loyal fans by changing prices according to demand. For example, as *Courty* (2003a) pointed out, "a constant price (same price for all events in a season) may be necessary to attract loyal team fans". Similarly, *Krueger* (2001) cited the NFL vice president for public relations who stated that the league tries to set "a fair, reasonable price" because it wants to maintain an "ongoing relationship with fans and business associates". The NFL vice president for public relations stated that although the NFL could increase its "present-day profit" by raising ticket prices, it prefers to take "a long-term strategic view".[8] The underpricing potentially motivated by these considerations, however, can lead to speculators buying tickets under fixed pricing as I showed above. Interestingly, under fixed pricing at least, speculator and consumer resale do not hurt the capacity provider's optimal revenues, as I show below.

Theorem II.2. *Under fixed pricing, the capacity provider's optimal price and optimal revenue are decreasing in τ and τ'. Thus, the capacity provider achieves the highest revenue when $\tau = \tau' = 0$, and selling non-transferrable tickets harms the capacity provider.*

My primary interest is in whether the capacity provider benefits from a larger or smaller resale transaction cost and whether the capacity provider should prevent resale of tickets. I answer this question by analyzing the most favorable resale transaction costs incurred by consumers and speculators from the capacity provider's point of view. Theorem II.2 states that the capacity provider's optimal fixed price and optimal revenue from fixed pricing are decreasing[9] in both τ and τ'. The decreasing result regarding τ holds for any τ' and is independent of the existence of speculators in the market, and vice versa. Thus, the existence of speculators never hurts the capacity provider under fixed pricing. If τ' is small enough (i.e., $\tau' < 1 - p_f^n/p_s$), the capacity provider's optimal fixed price and revenue are higher with the existence of speculators. This is because when speculators' transaction cost is small enough, they will enter the market even when period 1 consumers do not buy tickets immediately. In this case, if period 1 consumers wait, then in period 2, they will have to buy tickets from speculators at a higher price than the capacity provider. Seeing this threat, period 1 consumers will accept a higher price for advance tickets from the capacity provider, hence the capacity provider can earn more revenue.

[8] Modeling how long-term demand may change because loyal fans may be offended by more demand-driven pricing is beyond the scope of this essay. It is an interesting future research direction.

[9] In this chapter, I use increasing/decreasing in the weak sense.

Moreover, Theorem II.2 implies that the capacity provider actually loses money when it is more costly for consumers to resell tickets. This is exactly the opposite of the belief of many event capacity providers in practice. As τ becomes larger, period 1 consumers value advance tickets less because their payoff in period 2 if purchasing advance tickets, $E[\max(V, (1-\tau)r_f^*)]$, decreases. On the other hand, their payoff from waiting and purchasing in period 2 may increase. To induce them to buy tickets, the capacity provider has to decrease his price.[10] The capacity provider can charge a higher price and earn more revenue when resale is less costly.[11] When $\tau = \tau' = 1$, the net payoff from resale is zero for both consumers and speculators, so this corresponds to the case of selling non-transferrable tickets.[12] I have clearly shown that an event capacity provider using fixed pricing would be hurt by non-transferrable tickets and always benefits from ticket resale, even if some of the tickets will be bought by speculators. Thus, the increase in how much consumers value tickets (and thus the capacity provider being able to charge consumers more because of this increased valuation) dominates the effect of increased competition with the capacity provider from the resale market. I now analyze how these results are affected if the capacity provider charges different prices over time.

[10]Note that the amount that the capacity provider decreases his price is not always equal to the amount that period 1 consumers' payoff from buying tickets decreases. Period 1 consumers' payoff from waiting is $\int_{p_s}^{\infty}(v - p_f^n)\,\mathrm{d}F(v)$ if $p_f^n \geq (1-\tau')p_s$ and $E(V-p_s)^+$ otherwise. $E(V-p_s)^+$ is independent of τ while $\int_{p_s}^{\infty}(v-p_f^n)\,\mathrm{d}F(v)$ is increasing in τ, hence as the capacity provider decreases his price, period 1 consumers' payoff from waiting may increase. Thus, as τ becomes larger, if period 1 consumers' payoff from waiting increases, the capacity provider may have to decrease his price more than the amount that period 1 consumers' payoff from buying tickets decreases.

[11]The University of Michigan signed an agreement with StubHub in July 2011 that makes the company the official fan-to-fan ticket exchange marketplace for Wolverine Athletics. In the following season, Michigan raised ticket prices for the first time in seven seasons (*Shea*, 2012). In fact, StubHub is now the secondary ticketing partner of 20 colleges. In addition to Michigan, StubHub has partnered with the Big Ten Conference, North Carolina, Florida State and Virginia Tech.

[12]Strictly speaking, for every C, selling non-transferrable tickets is equivalent to $\tau \geq \hat{\tau}(C) = \inf\{0 \leq \tau \leq 1 : [(\lambda_1 - C)^+ + \lambda_2]\bar{F}(v_{\min}/(1-\tau)) \leq (C-\lambda_1)^+\}$ and $\tau' \geq 1 - p_f^n/r_f^*$. For the sake of readability, I refer to $\tau = \tau' = 1$ as selling non-transferrable tickets in the main text.

2.5 Multiperiod Pricing

In this section, I study the multiperiod pricing strategies where capacity providers change their ticket prices over time. Multiperiod pricing has started to become the dominant strategy used by capacity providers such as professional sports teams. To study the effects of price changes and demonstrate whether the capacity provider should try to prevent resale or not, I analyze a two-period model. I assume the capacity provider announces his advance ticket price p_1 at the beginning of period 1 and can adjust his price to p_2 in period 2, after consumers learn their valuations, to sell the remaining capacity. In this section, I assume the capacity provider cannot commit to the period 2 price upfront. (I have also analyzed the case where the capacity provider can commit to the period 2 price, and omit this case for space considerations. The insights regarding whether the capacity provider should prevent resale or not do not change if he can commit to the period 2 price under the multiperiod pricing setting.) Clearly, being able to charge different prices over time gives the capacity provider more flexibility, so the fact that multiperiod pricing results in higher revenues than fixed pricing is not too surprising. However, I am more interested in whether the capacity provider benefits from a larger or smaller resale transaction cost for consumers and speculators under multiperiod pricing. Recall that under fixed pricing, I showed that the capacity provider always benefits from a smaller transaction cost. As I will show in this section, this is no longer true under multiperiod pricing.

Theorem II.3. *(i) The capacity provider's optimal period 2 price p_2^* and the equilibrium resale price r_m^* are $p_2^* = r_m^* = r_f^*$.*

(ii) The capacity provider's optimal period 1 price is $p_1^* = E[\max(V, (1-\tau)p_2^*)] - E(V - p_s)^+$. Moreover, $p_1^* < p_2^*$.

(iii) For a given τ, speculators enter the market in equilibrium if and only if $\tau' < \bar{\tau}'_m(\tau) = 1 - p_1^*/p_2^*$.

Similar to fixed pricing, I use backward induction to find the subgame perfect equilibrium of the game between the capacity provider, consumers and speculators. The difference is that in period 2, to determine p_2^*, the capacity provider plays a simultaneous game with the consumers and speculators who have purchased tickets in period 1. Theorem II.3 characterizes the equilibrium outcome under multiperiod pricing. In period 2, the capacity provider's price is equal to the resale price, so the equilibrium outcome in period 2 is equivalent to the situation where the capacity provider participates in the resale market and determines its market clearing price together with the resellers. This is because the capacity provider and the resale market are competitors in period 2. When their prices are different, the party with the lower price will raise the price to gain more margin, and if the party with the higher price is not making sales, it will decrease the price to gain market share. As $p_1^* < p_2^*$, the capacity provider implements a "low-to-high" pricing. He offers a discount for advance tickets but captures a higher margin close to the event date.

Note that Theorem II.3(iii) characterizes the condition on τ' for a given τ such that speculators enter the market in equilibrium. For a given τ', I can also characterize the condition on τ such that speculators enter the market in equilibrium. Define $\bar{\tau}_m(\tau')$ as the τ solving $\tau' = \bar{\tau}'_m(\tau)$, i.e., $\bar{\tau}_m(\tau')$ is the inverse function of $\bar{\tau}'_m(\tau)$. Speculators enter the market in equilibrium if $\tau > \bar{\tau}_m(\tau')$ and do not enter the market in equilibrium if $\tau \leq \bar{\tau}_m(\tau')$. This is because when consumers' transaction cost becomes larger, fewer consumers would like to resell tickets, hence speculators have less competition in the resale market and can make more profits. Recall that under fixed pricing, I showed that the existence of speculators does not hurt the capacity provider and may in fact benefit the capacity provider. Under multiperiod pricing, the result is exactly the opposite – the capacity provider's revenue decreases when speculators enter the market in equilibrium. Speculators hurt the capacity provider's revenue under multiperiod pricing because they force the capacity provider to sell more tickets in period 1 at a lower price than period 2. Without speculators, if the provider has sufficient capacity to satisfy the period 1 consumers and has leftovers, he will then sell the remaining tickets in period 2 at a higher price and earn more revenue. Under fixed pricing, however, the capacity provider does not have the flexibility to change the price and capture a higher margin close to the event date in the first place. Therefore, with the additional price flexibility under multiperiod pricing, an event capacity provider no longer needs speculators as an instrument to boost revenue, he is better off in the absence of speculators. Given these interesting dynamics under multiperiod pricing, the result on whether the capacity provider would like consumer resale to be less or more costly is more complex than under fixed pricing and I characterize it below.

Theorem II.4. *(i) Under multiperiod pricing, the capacity provider's optimal period 1 price is decreasing in τ, while the optimal period 2 price is increasing in τ.*

(ii) For $\tau > \bar{\tau}_m(\tau')$, the optimal revenue from multiperiod pricing is decreasing in τ.

(iii) Assume $f(\cdot)$ is decreasing. For $\tau \leq \bar{\tau}_m(\tau')$, there exists a threshold $\bar{C} > \lambda_1$ such that if $C \leq \lambda_1$, the optimal revenue from multiperiod pricing is decreasing in τ; if $C \geq \bar{C}$, it is increasing in τ; otherwise, it may be decreasing or first decreasing then increasing in τ. The capacity provider achieves the highest revenue either when $\tau = \tau' = 0$ or $\tau = \tau' = 1$. If $C \leq \lambda_1$, $\tau = \tau' = 0$ results in the highest revenue; if $C \geq \bar{C}$, $\tau = \tau' = 1$ results in the highest revenue (i.e., the capacity provider benefits from selling non-transferrable tickets).

I have shown that under multiperiod pricing, the capacity provider prefers speculators' resale transaction cost to be large enough to prevent them from entering the market. Now I analyze what resale transaction cost incurred by consumers is most favorable to the capacity provider. For a given τ', Parts (ii) and (iii) of Theorem II.4 characterize how the capacity provider's optimal revenue from multiperiod pricing changes with respect to τ when speculators exist and do not exist in equilibrium, respectively. When speculators exist in equilibrium (i.e., $\tau > \bar{\tau}_m(\tau')$), decreases in the consumer resale transaction cost increase the capacity provider's revenue. This is because in this case, the provider will sell out his capacity in period 1 (with speculators' help) and he can increase his period 1 price and earn more revenue if consumers incur a smaller resale transaction cost. On the other hand, when speculators do not enter the market in equilibrium (i.e., $\tau \leq \bar{\tau}_m(\tau')$), how the capacity provider's optimal revenue from multiperiod pricing changes with respect to τ depends on the capacity C. If the provider's capacity is small, he sells out his capacity early and most sales occur in period 1. Since a smaller τ results in a higher period 1 price, the capacity provider achieves a higher revenue when the consumers' resale transaction cost is smaller. Thus, if an event capacity provider has a small capacity or the event is popular (a sufficient condition is $C \leq \lambda_1$), I have the same result from fixed pricing that the capacity provider will be better off when consumer resale is less costly. On the contrary, if the provider's capacity is large enough so that the majority of his revenue comes from ticket sales in period 2 (a sufficient condition is when $C \geq \bar{C}$), the effect of a larger τ on the period 2 price will dominate. As I show in Part (i) of Theorem II.4, a larger τ results in a higher period 2 price. Thus, the capacity provider prefers larger resale transaction costs in this case, as he has sufficient remaining tickets to sell in period 2 at a higher margin and the competition from consumers that resell tickets can harm his revenue. This is different from what I found under fixed pricing.

To summarize, my result indicates that the capacity provider may sometimes benefit from non-transferrable tickets when using multiperiod pricing, unlike the fixed pricing case when he will always be hurt by non-transferrable tickets. Whether the capacity provider benefits or not depends on the actual values of demand and capacity. For example, if demand significantly exceeds capacity, then non-transferrable tickets are again a bad idea for capacity providers. However, the problem is that most capacity providers have more than one event in the same venue during a season with each event having a different demand level. For example, an NBA team (where multiperiod pricing is commonly used) typically plays 82 games in a regular season. The Detroit Pistons (who have been performing pretty badly in the last few years), for example, cannot sell out capacity for most games except the games where they play against very popular teams such as Miami Heat. It would be very difficult for a team like Pistons to allow ticket resale for the Miami Heat game but sell non-transferrable tickets for another game.

Interestingly, the primary reason that a team would want to make resale more difficult (or sell non-transferrable tickets) is to increase revenues. In fact, in the next section, I show that for that purpose, there is a much better pricing mechanism than multiperiod pricing. I will show that ticket options always dominate multiperiod pricing in revenue generation for event capacity providers. Furthermore, ticket options naturally reduce ticket resale. Thus, there is in fact a way for capacity providers to reduce the resale market and capture its revenue without resorting to paperless ticketing.

2.6 Ticket Options

So far, I have analyzed fixed and multiperiod pricing which are the pricing strategies that have been commonly used by event capacity providers in practice. I have found that consumer resale is actually beneficial to an event capacity provider in most cases unless he has a large capacity to sell and is using multiperiod pricing. Speculators may benefit the capacity provider under fixed pricing, but they may hurt the capacity provider under multiperiod pricing. Thus, under multiperiod pricing, if the provider has a large capacity (or the event is not popular), he achieves the highest revenue without any ticket resale, where paperless ticketing proposed by Ticketmaster is one way to make tickets non-transferrable and eliminate the resale markets. However, to achieve this benefit in practice, an event capacity provider would have to enforce paperless ticketing for only unpopular events and allow ticket resale for other events in the same season. In this section, I study a novel pricing strategy with ticket options that has emerged recently in practice (e.g., OptionIt sells online ticket options for events). As I will show, this novel pricing strategy is generally more profitable than the current strategies used in practice. It also has the benefit of giving consumers more flexibility, that is, consumers initially only buy an option to attend the event at a much lower price than the regular ticket price and can exercise the option when they know their valuations for the event.

Consumers expose themselves to low valuation risks by purchasing advance tickets as the event may conflict with their schedules that are not known in advance. Also, many sports employ elimination type tournaments, and an advance ticket may become worthless to a consumer if the athlete/team she supports does not qualify for the event (e.g., US Open men's final). On the other hand, if consumers do not purchase tickets in advance, they risk paying high prices in the resale markets or seats being sold out. Options can be very attractive to consumers because options can help them hedge against the valuation uncertainties. For example, a search for tickets for the ice hockey game of Florida Panthers vs. Montreal Canadiens on March 10, 2013 resulted in tickets at \$76.75 – \$87 on Ticketmaster for seats on the lower level of the stadium. On the other hand, OptionIt allows consumers to buy an option (i.e., to reserve a seat) for the seats in the same region for \$8 and pay an additional \$100 if later deciding to actually buy the ticket. By purchasing an option, if a consumer later finds herself unable to attend the event, she loses at most \$8 (she may even be able to resell the ticket and incur a smaller loss if the resale price is high enough), while she may lose up to \$87 if purchasing a regular ticket. With options, consumers can purchase the right but not the obligation to buy tickets closer to the event date. A consumer can pay a relatively small amount (option price) to secure the right of purchase and make her final purchasing decision after the uncertainties are resolved.

She needs to pay an additional amount (strike price) if she exercises the option to obtain a real ticket later. A consumer may exercise the option because her valuation is high enough (e.g., her favorite tennis player qualifies for the final) so she will use the ticket to attend the event, or the resale price is high enough so she will resell the ticket. Otherwise, the consumer will find an event ticket unattractive and let the option expire.

I study a pricing scheme where the capacity provider sells (x, p) options in period 1 and regular tickets at price p_o in period 2. x is the option price. i.e., the price to purchase a ticket option; p is the strike price, i.e., the extra amount to pay if one decides to exercise the option to obtain a real ticket. Both x and p are announced at the beginning of period 1. To reflect the fact that consumers would want to decide whether to exercise the options or not as their uncertainties are resolved, I assume options can be exercised in period 2 after consumers learn their valuations. The capacity provider can sell the expired options again as tickets in period 2. I assume the capacity provider announces his period 2 ticket price p_o after consumers learn their valuations, that is, my ticket options model also has the multiperiod pricing feature. At the same time, the consumers and speculators who choose to resell tickets after exercising the options determine the resale price r_o. The capacity provider's goal is to optimally set the option price, the strike price (both are announced in period 1) and the period 2 price (announced in period 2) so that his revenue is maximized. I do not allow the capacity provider to sell more options than his capacity although one might increase revenues by doing so in the short term. The reason is that there have been consumer backlashes to firms (e.g., Yoonew and FirstDibz) that have sold more options than their available capacities and had to deny consumers' requests to exercise the options. Compared to airline tickets where overselling is standard, event tickets are much less substitutable because an event usually occurs only once.

2.6.1 Consumer Choice Model

Consumers make their purchasing decisions in period 1 based on their expectations on the realizations of valuations and the prices in period 2. Period 1 consumers' inter-temporal decision process and the corresponding payoffs are illustrated in Figure 2.2. A speculator's decision process is a special case of Figure 2.2 where $V = 0$ with probability one and the resale transaction cost is τ' instead of τ. In period 2, the option price x becomes sunk cost; the period 1 consumers who have purchased options decide whether to exercise the options or not and whether to resell or use the tickets. A consumer exercises the option if her valuation is greater than the strike price or the payoff from reselling the ticket is greater than the strike price, i.e., $\max(V, (1-\tau)r_o) > p$; she lets the option expire otherwise. On the other hand, as speculators never use the tickets to attend the event, they exercise the options and resell the tickets if $(1-\tau')r_o > p$ and let the options expire otherwise.

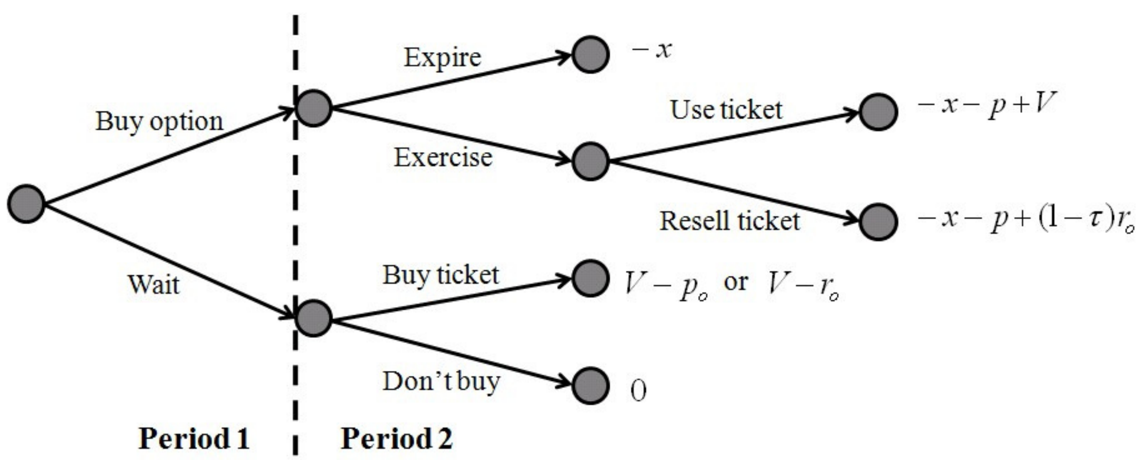


Figure 2.2: Consumer choice model under option pricing

2.6.2 Optimal Option Pricing

I again use backward induction to solve the game between the capacity provider, consumers and speculators. In this section, I assume $F(\cdot)$ has an increasing failure rate. I will show that selling ticket options in period 1 instead of regular tickets can indeed improve the capacity provider's revenue in the multiperiod pricing framework, and I discuss where the benefit of ticket options comes from. Theorem II.5 characterizes the optimal pricing strategy with options as well as how the capacity provider's optimal prices and revenue change as consumers' and speculators' transaction costs are changed. Similar to multiperiod pricing, the capacity provider's period 2 price is equal to the resale price in equilibrium due to competition. Speculators enter the market in equilibrium if their resale transaction cost is small enough. If speculators buy options in period 1, then in period 2, they exercise the options and resell the tickets because they would not enter the market in the first place if they later let the option expire and incur a net loss. In Section 2.5, I showed that with the flexibility to change the price in period 2, the capacity provider prefers the absence of speculators. This is still true if the capacity provider sells ticket options, as without speculators, the capacity provider can sell more tickets in period 2 at a higher margin (i.e., $x^* + p^* < p_o^*$) and increase the revenue.

Theorem II.5. *(i) The capacity provider's optimal strike price p^* is decreasing in τ.*[13] *The optimal options price is $x^* = E(V - p^*)^+ - E(V - p_s)^+$ which is increasing in τ. In equilibrium, period 1 consumers do not choose to resell tickets in period 2.*

(ii) The capacity provider's optimal period 2 price p_o^* and the equilibrium resale price r_o^* are $p_o^* = r_o^* = \inf\{r \geq v_{\min} : [(\lambda_1 - C)^+ + \lambda_2]\bar{F}(r) \leq (C - \lambda_1)^+ + \min(\lambda_1, C)F(p^*)\}$. p_o^* is increasing in τ. Moreover, $x^* + p^* < p_o^*$.

(iii) For a given τ, there exists a threshold $\bar{\tau}_o'(\tau)$ such that speculators enter the

[13] The characterization of p^* is complicated, therefore I omit it in the theorem statement. It can be found in the proof of Theorem II.5 in Appendix A.

market in equilibrium if and only if $\tau' < \bar{\tau}'_o(\tau)$.

(iv) The capacity provider's optimal revenue from option pricing is increasing in τ. *The capacity provider achieves the highest revenue when* $\tau = \tau' = 1$ *(i.e., the capacity provider benefits from selling non-transferrable tickets).*

Different from speculators that exercise the options in equilibrium, the equilibrium number of period 1 consumers that choose to exercise the options after learning their valuations is influenced by the strike price p. If $p < (1-\tau)r_o$, since the payoff from reselling a ticket exceeds the strike price, all consumers exercise the options. In this case, the capacity provider's optimal period 2 price is equal to the optimal period 2 price under multiperiod pricing and selling (x, p) options is equivalent to multiperiod pricing with the period 1 price equal to $x + p$. Thus, the pricing strategy with options I am analyzing cannot result in a lower optimal revenue than multiperiod pricing. On the other hand, if $p \geq (1-\tau)r_o$, since the payoff from reselling a ticket does not exceed the strike price, a consumer will exercise an option only because her valuation is higher than the strike price so that she will use the ticket herself. In this case, consumer resale is eliminated. I find that it is indeed optimal for the capacity provider to set the strike price high enough to eliminate consumer resale (i.e., the optimal strike satisfies $p^* \geq (1-\tau)r_o^*$), as he can achieve a higher revenue without the resale competition from consumers. Moreover, the capacity provider should set the optimal option price x^* at the minimum possible level that induces period 1 consumers to purchase options. Therefore, by appropriately choosing the prices, the capacity provider can prevent resale from consumers with the use of ticket options.

Given the fact that the capacity provider's optimal strike price is high enough to dominate the payoff from resale so that consumers are prevented from reselling tickets in equilibrium, I can explain why the optimal prices and revenue change with respect to the consumers' resale transaction cost τ in the way stated in Theorem II.5. Observing the high strike price, when a period 1 consumer purchases the option, she knows that her chance of reselling the ticket after exercising the option in period 2 is very low. Thus, her payoff in period 2 is merely her realized valuation. As τ becomes larger, the payoff from ticket resale becomes lower, so the capacity provider is able to prevent consumer resale with a lower strike price (i.e., p^* is decreasing in τ). Having a lower strike price to pay later, a period 1 consumer is willing to pay more to purchase an option. Thus, when τ is larger, the capacity provider can charge a higher option price (i.e., x^* is increasing in τ). Since consumers do not resell tickets in equilibrium, a period 1 consumer exercises the option in period 2 if her realized valuation is greater than the strike price. When τ is larger, the capacity provider's optimal strike price is lower, hence more consumers will exercise the options and fewer consumers will let the options expire. As the capacity provider can sell the expired options again as regular tickets in period 2, when τ is larger, he has fewer tickets left to sell and the total supply in period 2 becomes smaller. As a result, the equilibrium resale price as well as the capacity provider's period 2 price is driven up (i.e., p_o^* is increasing in τ).

Finally, Theorem II.5(iv) states that unlike fixed or multiperiod pricing, the capacity provider always benefits when consumers have larger resale transaction costs if he sells ticket options. Recall that under multiperiod pricing, if the provider's capacity is small enough, his revenue increases when consumer resale becomes less costly because he can charge a higher period 1 price. This is not true with ticket options because for all levels of capacity, while the provider's optimal strike price is high enough to eliminate consumer resale, it also guarantees that there are enough consumers letting the options expire in period 2 so that the capacity provider can sell a significant amount of tickets in period 2 at a higher price. Thus, the effect that the optimal period 2 price is increasing in τ dominates and the optimal revenue from option pricing is increasing in τ. Therefore, I have shown that with ticket options, the capacity provider loses revenue when resale is less costly for either consumers or speculators. The capacity provider achieves the highest revenue when $\tau = \tau' = 1$

in which case ticket resale from both speculators and consumers are precluded, that is, if an event capacity provider sells ticket options, he always benefits from making tickets non-transferrable.

Theorem II.6. $\bar{\tau}'_o(\tau) \leq \bar{\tau}'_m(\tau) < \bar{\tau}'_f(\tau)$.

Theorem II.6 points out another interesting feature of option pricing. I have stated before that whereas speculators can benefit the capacity provider under fixed pricing, they can hurt the capacity provider's revenues under multiperiod pricing and option pricing. For speculators to profitably buy and resell tickets, their transaction cost has to be lower than a certain threshold $\bar{\tau}'_i(\tau),\ i = f, m, o$. Theorem II.6 shows that this threshold is lowest under option pricing. Thus, an event capacity provider is most likely to be able to shut speculators out of the market under option pricing.

Finally, I discuss why option pricing is beneficial to event capacity providers. First, option pricing is more effective in reducing resale of tickets, hence the capacity provider can capture more revenue from the resale markets. I have shown that with ticket options, the capacity provider can eliminate consumer resale regardless of the consumers' resale transaction cost. Moreover, Theorem II.6 indicates that speculators are less likely to exist under option pricing, as speculators enter the market in equilibrium for a smaller range of τ' compared to other pricing strategies. Second, as the capacity provider can sell the expired options as tickets in period 2 and can use the strike price to control the number of expired options, this additional price decision gives the capacity provider more flexibility that he can "virtually" allocate capacity to the two periods and earn more revenue from selling more tickets in period 2 at a higher price.

Note that my comparison between multiperiod pricing and option pricing has been for the same τ and τ', that is, if an event capacity provider is currently using multiperiod pricing, he can increase revenues by switching to selling ticket options while consumers and speculators incur the same resale transaction costs. Theorem

II.5(iv) indicates that the capacity provider could increase his revenues even more by switching to ticket options and making tickets non-transferrable. Note that under fixed and multiperiod pricing, consumers lose a lot if they buy non-transferrable tickets and then cannot attend the event, as they lose the whole value of the ticket in this case. Thus, generally, even discussions to initiate non-transferrable tickets have led to significant consumer backlashes (e.g., in a June 18, 2012 op-ed, the Consumer League of New Jersey President Bob Russo stated that "Ticketmaster paperless tickets are anti-consumer and is new ploy by company to take more of fans' hard-earned money"). Negative consumer reaction usually focuses on the fact that consumers would lose the whole value of the ticket if they could not attend the event for some reason. However, with ticket options, a consumer will only lose the option price (which is much less than the regular ticket price) if she buys an option and then decides she does not want the ticket. Even more interestingly, by only switching to ticket options from multiperiod pricing while still allowing resale of tickets, the capacity provider may capture most of the total benefit that he could obtain from option pricing with non-transferrable tickets. For example, if $\lambda_1 = 150$, $\lambda_2 = 100$, $C = 120$, $V \sim U[10, 100]$, $\tau = 0.25$, $\tau' = 0.1$, by switching from multiperiod pricing to option pricing, the capacity provider improves his revenue from 6339 to 7166 (increased by 13%); by further making tickets non-transferrable, the capacity provider's revenue is increased to 7253 (increased by only 1.2% additionally). Thus, compared to making tickets non-transferrable which may result in significant consumer backlashes, the novel pricing strategy of option pricing may be a good choice for event capacity providers to consider.

2.7 Extensions

2.7.1 Strategic Capacity Rationing

In this section, I consider the case where the capacity provider can strategically hold back some of his capacity in period 1 to sell later in period 2. This, however, isn't common for events in practice. As I have stated before, consumers can get very upset if the capacity provider sells tickets later when he claimed tickets were sold out earlier. Although Ticketmaster explicitly claims on its website that it does not divert inventory designated by clients for primary sales into the resale market, the possibility that Ticketmaster does this still has worried consumers and there have been consumer complaints.[14] Nevertheless, it is of interest to understand if any of my main findings regarding whether the capacity provider should prevent resale or not in the previous sections would change in this case.

I define the decision variable $0 < b \leq C$ as the provider's designated capacity to be sold in period 1. For any b, the previous equilibrium analysis for each pricing strategy still holds. Thus, to analyze the optimal pricing problem with strategic capacity rationing, I can write all optimal prices as functions of b and optimize on the b-dimension. For fixed pricing, I can easily show that the optimal revenue is increasing in b. As Theorem II.1 indicates, the equilibrium resale price $r_f^*(b)$ is given by $[(\lambda_1 - b)^+ + \lambda_2)]\bar{F}(r_f^*) = C - \min(\lambda_1, b) + \min(\lambda_1, b)F((1-\tau)r_f^*)$. Since $r_f^*(b)$ is increasing in b, p_f^* is also increasing in b. Thus, under fixed pricing, even if the capacity provider can hold back some capacity, it is optimal to sell as many tickets in period 1 as possible (i.e., not to ration any capacity). Thus, strategic capacity rationing does not improve the capacity provider's revenue and I have the same results in Section 2.4. The capacity provider still benefits when resale of tickets are easier for consumers as well as speculators, and selling non-transferrable tickets hurts his revenue.

[14] See http://www.consumeraffairs.com/entertainment/ticketmaster.htm.

Theorem II.7. *If the consumer valuations are uniformly distributed over* $[v_{\min}, v_{\max}]$*, the optimal revenue from multiperiod pricing with strategic capacity rationing is increasing in* τ*.*

In Section 2.5, I showed that under multiperiod pricing, the capacity provider may still prefer consumers to have a zero resale transaction cost if his capacity is small. Interestingly, Theorem II.7 states that this is no longer true when the provider can strategically ration capacity in period 1. With the additional flexibility from capacity rationing, I find that the optimal revenue from multiperiod pricing is always increasing in τ. Therefore, if an event capacity provider can ration capacity in period 1, he will never benefit from a resale market in period 2. In this case, the capacity provider achieves a higher revenue if the resale market is precluded (e.g., by the enforcement of non-transferrable tickets).

Finally, if the capacity provider sells ticket options, all my numerical results indicate that the optimal revenue is still increasing in τ with strategic capacity rationing. So the capacity provider still benefits when consumers have larger resale transaction costs, and he achieves the highest revenue by making tickets non-transferrable. Moreover, for any b, option pricing reduces to multiperiod pricing if the strike price is low enough (i.e., $p < (1-\tau)r_o(b)$), and the capacity provider can improve his revenue by choosing a high enough strike price that dominates the payoff from ticket resale so that consumer resale is prevented. Therefore, my previous insight that ticket options can help event capacity providers prevent consumers resale of tickets and increase revenues carries through to a capacity rationing provider. As I noted at the beginning, holding back capacity to sell later may cause significant consumer dissatisfaction and may be very hard to implement in practice. Thus, it is interesting to compare its benefit to other strategies (such as ticket options) that I have discussed. Consider the example given at the end of Section 2.6 ($\lambda_1 = 150$, $\lambda_2 = 100$, $C = 120$, $V \sim U[10, 100]$, $\tau = 0.25$, $\tau' = 0.1$). By strategically rationing capacity, the provider can improve his multiperiod pricing revenue from 6339 to 6762, whereas his revenue is increased to 7166 by switching to option pricing. Moreover, after switching to ticket options, the capacity provider does not further increase the revenue by rationing capacity, because it is indeed optimal for the capacity provider to sell as many options as possible in period 1 in this example. Therefore, compared to increasing revenues through rationing capacity, switching to option pricing may be a better way to increase revenues and avoid risking upsetting the fan base.

2.7.2 Heterogeneous Consumers

Similar to other papers in the literature (e.g., *Geng et al.*, 2007, *Courty*, 2003b), my model considered a situation where all period 1 customers have ex ante symmetric valuations. In this section, I consider the case of two types of consumers in period 1 to explore whether the insights from my model are affected. In this case, I assume that among the λ_1 consumers who arrive in period 1, λ_{1H} consumers (the super fans) have higher ex ante valuations (V_H) than the rest λ_{1L} consumers (V_L), where V_H is stochastically larger than V_L. The λ_2 consumers who arrive in period 2 have ex ante valuations V_L. For each consumer type, all the equilibrium analysis in my model still holds. However, characterizing the optimal pricing policy becomes much more complicated, because in period 1 the capacity provider may want only one type or both types of consumers to buy tickets, resulting in a much more complex revenue function. Nevertheless, my numerical results indicate that the main insights regarding when resale markets are beneficial or harmful to the capacity provider do not seem to be affected. For example, suppose the capacity provider is using multiperiod pricing and the problem parameters are as follows: $\lambda_{1H} = 90$, $\lambda_{1L} = 60$, $\lambda_2 = 100$, $V_H \sim U[50, 100]$, $V_L \sim U[10, 80]$. If $C \leq 104$, the capacity provider achieves the highest revenue when $\tau = \tau' = 0$, that is, if the capacity is small enough, the capacity provider's most favorable scenario is when tickets can be resold with zero transaction cost. On the other hand, if $C > 104$, the capacity provider achieves the highest revenue when $\tau = \tau' = 1$, that is, if the capacity is large enough, the capacity provider benefits from making tickets non-transferrable. These observations are consistent with my results given by Theorem II.4. Moreover, intuitively, the capacity provider would like to induce more consumer types to purchase tickets in period 1 when he has a larger capacity. In the above example, when $\tau = \tau' = 1$ which is the best scenario for the capacity provider for $C > 104$, the optimal multiperiod pricing policy induces only the high-valuation consumers to purchase tickets in period 1 if $C \leq 193$; if $C > 193$, the optimal multiperiod pricing policy induces both types of consumers to purchase tickets in period 1. Thus, as the numerical results clearly indicate, my main insights with respect to whether event capacity providers should prevent resale of tickets or not do not change significantly with more complex assumptions about the number of period 1 consumer types.

2.8 Conclusion

In this essay, I study three pricing strategies, fixed pricing, multiperiod pricing, and option pricing, for an event capacity provider that faces resale of tickets. One major contribution of this essay is that I find how the behavior of optimal prices and revenues depend on the resale transaction costs incurred by the consumers and speculators, which indicates whether the capacity provider should prevent resale of tickets or not. I have found that contrary to what common wisdom suggests, event capacity providers do not always benefit from restricting resale.

By appropriately choosing the prices associated with ticket options (i.e., option price and strike price), an event capacity provider can eliminate consumer resale of tickets and significantly reduce the magnitude of the resale market. I conjecture that compared to enforcing paperless ticketing under multiperiod pricing, event capacity providers would have a much easier time convincing consumers to switch to buying options. Ticket options also benefit consumers, because if a consumer buys an option and cannot attend the event, she is risking only the option price instead of the whole ticket price. Furthermore, under multiperiod pricing, whether paperless ticketing is beneficial or not depends on the event's demand, which would imply that to obtain the highest benefit, the capacity provider would have to make some events' tickets paperless and allow ticket resale for other events. This is clearly impractical. While going to paperless ticketing with options would increase the capacity provider's revenues even more, my numerical results indicate that this additional revenue gain is small compared to switching to option pricing from multiperiod or fixed pricing.

Thus, this essay suggests that efforts to move to paperless ticketing are likely to hurt not only consumers but also event capacity providers in many cases. A reason given by Ticketmaster to introduce paperless ticketing is to prevent speculators from entering the market. However, this essay argues that speculators may actually be beneficial to event providers when they use fixed pricing. While speculators are indeed never beneficial to capacity providers under multiperiod pricing, the capacity provider may still lose revenues overall by introducing paperless ticketing. Moreover, I provide the insight that option pricing not only results in the highest revenues for event capacity providers but also has the highest likelihood of shutting down speculators, while giving consumers much greater choice than paperless ticketing. Thus, my research indicates that event organizers should not support paperless ticketing but instead consider novel pricing strategies such as ticket options.

CHAPTER III

Pricing of Conditional Upgrades in the Presence of Strategic Consumers

3.1 Introduction

Like many other industries, a big challenge faced by the travel industry is the mismatch between demand and supply across different types of products. In the travel industry (e.g., hotels, airlines, car rental companies, cruise lines), consumers usually make reservations in advance and the products are perishable in the sense that they do not generate value for the firm after the end of the booking period. The capacity for each type of product is fixed, but due to the stochastic demand across different product types over time, firms frequently find capacity of some product types under-utilized while capacity of other product types in shortage at the end of the booking period. Ideally, firms should be able to eliminate the demand-supply mismatch by having enough flexibility in pricing their products. However, in reality, different industries face different constraints on setting prices.

In the hotel industry, a lot of firms lack the ability to adjust prices dynamically. Due to consumer resistance, dynamic pricing (i.e., adjusting prices for the same product over time) is not as common as in the airline industry. Some hotels do not use dynamic pricing at all but only use variable pricing (i.e., setting different nightly rates for the same room based on expected demand) as their primary pricing strategy. Others use dynamic pricing for their "best available rates" but have had a hard time convincing corporate travel buyers. For example, hotel chains would like to change prices dynamically and give large travel accounts a negotiated discount off the dynamic best available price. However, according to the survey by Business Travel News conducted on 221 travel buyers, more than two-thirds said that they did not use dynamic pricing in their hotel program(*Baker*, 2010). Instead, most travel buyers negotiate a fixed corporate rate which does not change dynamically. 16% of travel buyers used dynamic pricing only with select hotel chains, 9% used dynamic pricing only in low-volume markets, and only 6% reported that their use of dynamic pricing is standard.

Even with variable pricing, hotels still face constraints on setting room rates. In competitive industries such as travel, firms usually have several direct competitors, hence have less flexibility to adjust product prices as they like. Since consumers can compare prices for similar products very easily on the Internet where online travel agencies such as Orbitz and Expedia have provided such services, most firms providing similar products set similar prices for at least some of their products. For example, the following three hotels all reside in Ann Arbor, Michigan: Hilton Garden Inn, Residence Inn by Marriott, Sheraton. These are all upscale mid-priced hotels, and are located within 1 mile from each other. Thus, they are direct competitors in the local market. As a result, all three hotels use exactly the same (variable rather than dynamic) pricing strategy for standard rooms (with either one king-size bed or two queen-size beds). For example, the price in September and October 2013 is $169 for weekdays and $139 for Friday/Saturday nights.

While hotels have struggled with widespread acceptance of dynamic pricing, especially with corporate clients, and some are price takers in the market, many hotels have recently adopted a new type of conditional upgrade policy. This new strategy works in the following way. After a consumer makes a reservation, she is offered an upgrade option which she decides whether to accept or not. If she accepts the upgrade offer, then she will be notified whether she gets upgraded or not during check-in. By accepting the upgrade offer, the consumer agrees that she will pay the associated upgrade fee if her upgrade is fulfilled by the hotel later. The hotel fulfills upgrades if there are high-quality products still available by the check-in date. Many of the hotels use Nor1, a leading technological company, to offer the upgrades and decide the price of the upgrades.[1] These new upgrades are different from the upgrades historically offered by hotels where elite travelers may be upgraded for free at check-in as part of their consumer loyalty program benefits. First, these are paid upgrades instead of free upgrades. Second, they are conditional upgrades because a consumer does not know whether she will be upgraded and pay the upgrade fee when she accepts an upgrade offer; the upgrades are fulfilled conditional on the availability of higher-quality products by the check-in date. Third, they are offered to not only elite members but also regular consumers. Fourth, instead of being offered at check-in, the upgrades we consider are offered in advance, usually right after the original booking.

However, offering conditional upgrades may result in some consumers, who would purchase high-quality products when the firm does not offer conditional upgrades, to deliberately book less expensive products as they hope to get upgraded and pay less than the original price of high-quality products they actually prefer. Thus, conditional upgrades have the potential to cannibalize the high-quality product sales. When using the conditional upgrade strategy, it is important for the firm to carefully account for such consumer behaviors in setting upgrade prices optimally. In this essay, we study how firms can properly manage the trade-off between the conditional upgrade strategy's potential benefits and potential threats such as cannibalization. More specifically, the research questions we investigate are: 1) what is the optimal

[1]Besides hotels, Nor1 is also expanding its business to airlines, cruise lines, car rentals.

conditional upgrade pricing strategy for the firm when consumers may deliberately choose lower-quality products with upgrades? 2) When are conditional upgrades profitable/non-profitable for the firm? 3) How profitable is the conditional upgrade strategy compared to being able to set product prices optimally, in particular, can it replace product price optimization and dynamic pricing?

To answer these questions, we study a model where consumers select which product type to book and whether or not to accept an upgrade offer based on the anticipation of future upgrade probability. Our model analyzes the upgrade policy as currently implemented by Nor1 and hotels, where upgrade prices are static over time. Our analysis indicates that conditional upgrades significantly improve revenues of the firm by "demand expansion", "price correction", and "risk management". The conditional upgrades are "real options" that consumers purchase from the firm to be exercised with an upgrade fee if the high-quality products are still available by the end of the booking period. We find that this option expands the firm's demand by capturing the consumers who are not willing to pay the full price of higher-quality products but still value higher-quality products significantly more than regular products. If the firm does not have pricing flexibility due to competition or other industry constraints, conditional upgrades can be an instrument to correct the firm's original price for higher-quality products and reoptimize the firm's demand segmentation to improve demand-supply matching. Our numerical studies show that by properly using conditional upgrades, the firm can capture at least the revenue potential from being able to optimize the higher-quality product price. Interestingly, we also identify situations where conditional upgrades can generate even higher revenues than the case where the firm can set both product prices optimally but do not offer upgrades. This implies that conditional upgrades can compensate for the firm's lack of ability in setting the optimal product prices by managing prices and capacities in a more flexible way. Moreover, offering conditional upgrades generate higher revenues than offering last-minute upgrades in most cases. Thus, we believe that the novel form of "conditional" upgrades is a worthy initiative for travel industries. Finally, if the firm does have the ability to set product prices optimally, then our numerical results indicate that the revenue improvements with conditional upgrades are generally larger than the revenue improvements with dynamic pricing. By offering conditional upgrades, the firm allocates the consumers who accept the upgrade offers to different types of products at the end of the booking period. One of our interesting findings is that this ex-post allocation flexibility that the firm gains with conditional upgrades is generally more valuable than the pricing flexibility one has in dynamic pricing. Interestingly, these observations hold true even for the case where the firm sets only a static upgrade price, indicating that the potential of conditional upgrades to "correct" for mispricing of product prices may be even higher when dynamic upgrade prices can be used.

3.2 Literature Review

Although upgrades are widely used in service industries such as travel, there is limited academic literature that focuses on upgrades in service industries. Most of the literature studies upgrades in the context of airlines where upgrades are offered to preferred travelers as a perk or if the flight's economy cabin is overbooked (see for example *Karaesmen and Van Ryzin*, 2004). *Gallego and Stefanescu* (2009) is one of a handful of papers that study upgrades in detail. They first study free upgrades by generalizing the traditional network revenue management model (where product prices are fixed and demands for different product types are independent) to explicitly account for upgrades. They also study paid upgrades, and find that if a primary capacity provider has complete freedom to select prices, upgrades cannot improve profits. The result found by *Gallego and Stefanescu* (2009) is based on a fluid model. By considering demand randomness, we find that the firm can strictly improve revenues with conditional upgrades compared to having complete freedom to select product prices. *Biyalogorsky et al.* (2005) study conditional upgrades where the upgrade fee is charged at the time of upgrade request (i.e., a consumer pays the upgrade fee even if she does not get upgraded at the end) and find that upgrades increase the provider's profits when the probability of selling higher-quality units at full price is sufficiently high. The upgrade strategy studied in *Biyalogorsky et al.* (2005) is similar to an industry practice where only passengers who hold more expensive "upgradable class" tickets can be upgraded if there is available capacity at the fulfillment time. In this essay, we analyze a more recent upgrade strategy pioneered by Nor1 for the travel industry (i.e., selling conditional upgrades where the fee is paid if the upgrade is honored). Furthermore, unlike *Gallego and Stefanescu* (2009) and *Biyalogorsky et al.* (2005), we model the strategic consumer behavior and analyze conditional upgrades with a Bayesian game. The strategic consumer behavior significantly changes the insights.

There is also a stream of literature studying multi-product inventory management with provider-driven demand substitution. *Hsu and Bassok* (1999), *Bassok et al.* (1999) study full downward substitution where a consumer can be served by another product with superior quality. *Netessine et al.* (2002), *Shumsky and Zhang* (2009) study single-level upgrades where consumers may be upgraded by at most one product level. Although primarily focusing on inventory management or capacity management, these papers also consider upgrades. The main difference from this essay is that in these papers, the upgrade decision is entirely made by the provider and no additional fee is charged to the consumer, while in this essay, consumers get to decide whether they would like to be upgraded to a better product if it is still available by the end of the booking period. Moreover, in the above papers, consumers are not strategic when making their product purchasing decisions and do not take the future upgrade possibility into consideration, while we model this strategic behavior of consumers.

A growing literature in operations management studies the interaction between consumers' strategic behavior and firm's decisions (see *Netessine and Tang*, 2009 for a detailed review). For example, a problem that has been extensively studied is the consumers' deliberate waiting to purchase later in anticipation of a price decrease when the firm can change prices over time (*Su*, 2007, *Elmaghraby et al.*, 2008, *Gallego et al.*, 2008, *Yin et al.*, 2009, *Levin et al.*, 2010, *Mersereau and Zhang*, 2012). *Aviv and Pazgal* (2008), *Osadchiy and Vulcano* (2010), *Correa et al.* (2013) model the strategic consumers' purchasing decisions as a game with incomplete information and assume Poisson arrival of consumers to capture the randomness in the number of players in the game. We adopt the same assumption to model the random arrival of consumers over time to book different types of products. While the papers mentioned above consider a single product type and focus on the consumers' decision of "buy-now-or-wait", we model a firm selling multiple substitutable product types and study the consumers' decisions on which type of product to book and whether to accept an upgrade offer or not.

Jerath et al. (2010) study the effect of strategic consumer behavior if competing firms offer last-minute sales through opaque channels versus through their direct channels. *Fay and Xie* (2008) study probabilistic selling where the firm creates a probabilistic product by creating uncertainty about the type of product that a consumer will eventually receive. In opaque and probabilistic selling, the different product types are horizontally differentiated (i.e., differentiated based on a single characteristic other than quality), while with conditional upgrades, the different product types are vertically differentiated (i.e., they can be ordered according to quality). With the conditional upgrade strategy, the provider sells an option to the consumer so that the consumer can obtain a higher-quality product if the capacity is available at the fulfillment time. Due to the quality difference between the product types, consumers pay an exercise fee when the upgrade option is fulfilled, which is different from opaque and probabilistic selling. This essay is also methodologically different than the above papers in that we model the consumers' booking decisions as a Bayesian game with Poisson arrivals. In this essay, a consumer forms an expectation about the upgrade probability based on the arrival time and the product availability information, and decides which product type to book and whether or not to accept an upgrade offer. We prove the existence of equilibrium and provide a condition for equilibrium uniqueness. We study both the fully stochastic problem and its asymptotic approximation to propose a heuristic.

3.3 Model

We consider a firm that sells two types of perishable products, regular and high-quality (e.g., standard rooms and suites in a hotel). The firm has K_H high-quality products and K_R regular products. The products are consumed at time T but consumers can book the products any time before T. We refer to $[0, T]$ as the booking period. The products are perishable in the sense that they have no value to the firm after time T. The high-quality products are sold at price p_H and the regular products are sold at price p_R ($p_H > p_R$). After a consumer books a regular product, the firm may offer an upgrade opportunity so that the consumer can pay an additional fee p to upgrade the product to a high-quality one if there are leftovers by the end of the booking period. Although the firm does not guarantee the fulfillment of such an upgrade, a consumer only needs to pay the upgrade fee when she actually obtains an upgrade, and she is obliged to pay in this case. The firm offers upgrades to γ proportion of consumers.[2] Another interpretation is that $(1-\gamma)$ proportion

[2] In reality, travel firms sell through multiple channels and may offer conditional upgrades in selected channels only. For example, Hilton offers conditional upgrades to consumers who book their rooms in hilton.com while it does not offer conditional upgrades if consumers book through online travel agencies. Consumer can infer whether they will be offered upgrades or not from the channels they book through.

of consumers are inattentive (do not consider the upgrade offer) when making their purchasing decisions even if the firm offers them conditional upgrades.[3] We assume, consistent with industry practice, that if the firm does not have enough remaining high-quality products to satisfy all consumers that have accepted the upgrade offers, these consumers are rationed randomly, that is, the probability that a consumer gets upgraded does not depend on her booking time. The firm's goal is to optimally choose the upgrade price given product prices so that its revenue from selling two types of products as well as collecting upgrade fees is maximized. As stated before, settings where firms are price takers on product prices but can set upgrade price are common in practice. In Section 3.7 where we evaluate the revenue performance of conditional upgrades, we will also consider a firm that is not a price taker at all and demonstrate that conditional upgrades have great value for such a firm.

Consumers arrive to the market following a Poisson process with rate λ. Each consumer is characterized by a pair of valuations (v_R, v_H), where v_R denotes her valuation for regular products and v_H denotes her valuation for high-quality products. A consumer privately observes her own valuations before arriving to the market. The valuations of consumers are jointly distributed in the two-dimensional support Ω which is a finite subset of $\mathbb{R}^2_+$. The joint probability density function is denoted by $f(v_R, v_H)$.[4] By allowing a joint distribution of consumers' valuations for different product types, we are able to capture not only the consumers' heterogeneity in the willingness to pay but also their heterogeneity in the valuation differential between different product types which is important in making decisions regarding upgrades.

[3] As studied in the recent economics literature, consumers may pay attention to part of the price, menu of products or offerings. When a firm offers a multi-dimensional product, consumers may take only a subset of these dimensions into consideration. This is exemplified by *Spiegler* (2006), where a consumer samples one price dimension from each firm selling a product with a complicated pricing scheme (e.g., health insurance plans); *Gabaix and Laibson* (2006), where some consumers do not observe the price of an add-on before choosing a firm; *Armstrong and Chen* (2009), who extend the notion of "captive" consumers to those who always consider one dimension of a product but not another (e.g., price but not quality).

[4] Our equilibrium analysis for the stochastic model can be generalized to time-dependent arrival rates and time-dependent consumer valuation distributions.

Thus, the way we model consumer valuations is more general than the traditional approach used by the market segmentation literature (e.g., *Mussa and Rosen*, 1978, *Moorthy and Png*, 1992) where consumers' valuations for different product types are proportional. The Poisson arrival rate, product capacities, valuation distribution, percentage of consumers that are offered upgrades are common information for the firm and the consumers.

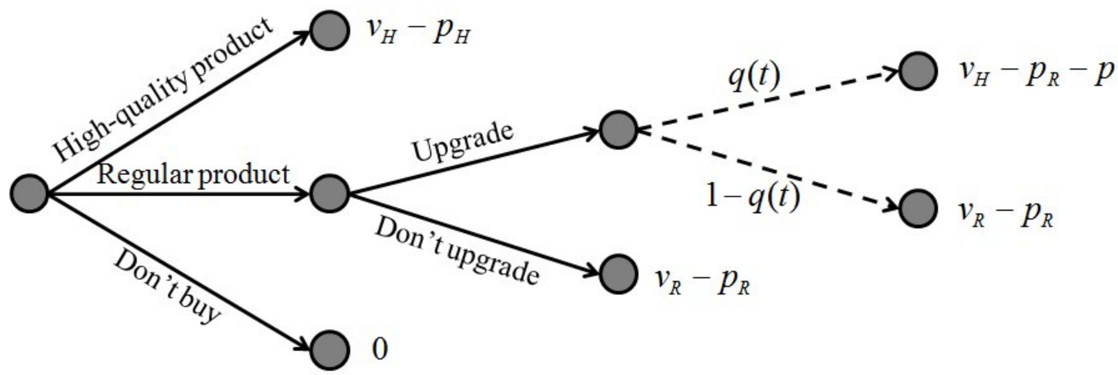


Figure 3.1: Consumer decision process

Consumers are strategic in the sense that a consumer trying to book a product at time t and seeing products are still available can anticipate the probability $q(t)$ of actually obtaining an upgrade if she accepts the upgrade offer. Consumers' rational expectations on the upgrade probability $q(t)$ depend on the arrival time because we allow consumers to infer the upgrade probability from the fact that products have not been fully booked by time t. Figure 3.1 depicts the consumer decision process and the payoffs from each possible decision. We use "H" to denote booking a high-quality product, use "U" to denote booking a regular product and accepting an upgrade offer, use "R" to denote booking a regular product without upgrade, and use "N" to denote not booking any product. The consumers that are not offered upgrades choose from "H", "R", and "N". Note that if $p \geq p_H - p_R$, nobody accepts the upgrade offer because the total price to pay in order to get a high-quality product through upgrade is at least as large as the original price for high-quality products. This is equivalent to the case without upgrades. Thus, we refer to $p \geq p_H - p_R$ as the case where the firm does not offer upgrades.

The firm needs to decide when to stop selling each product type, taking into account the instant booking levels for each product type including upgrades defined as a unique type. Define $N_H(t)$, $N_U(t)$, $N_R(t)$ as the demand stream booking each product type, respectively. Note that $N_U(t)$ is the arrival process of consumers booking a regular product and accepting the upgrade offer, and $N_R(t)$ is the arrival process of consumers booking a regular product and not accepting the upgrade offer, hence $N_U(t)$ and $N_R(t)$ are mutually exclusive. Due to the decomposition property of Poisson processes, $N_H(t)$, $N_U(t)$, and $N_R(t)$ are independent Poisson processes. We assume the firm cannot "bump" consumers upon check-in (i.e., the firm has to accommodate check-in requests of all reservation holders). The firm stops selling high-quality products when $N_H(t) \geq K_H$ and regular products when $N_R(t) \geq K_R$, that is, the firm tries to sell as many products as possible. Moreover, the firm stops selling both product types at the same time when $N_H(t) + N_U(t) + N_R(t) \geq K_H + K_R$. Note that this stopping rule allows the firm to accept more bookings for regular products during the booking period than the capacity (because some of the consumers booking regular products with upgrades may later get upgraded and free up some capacity for regular products) while ensuring no bumping of consumers.

A consumer does not observe the firm's instant capacities (also, how many consumers have arrived and the booking decisions they have made) when she makes her booking decision. However, consumers can observe whether a product type is fully booked or still available when making booking decisions. As the firm stops selling some product type, consumers are restricted to fewer choices. When the high-quality products are unavailable, consumers can only book regular products without upgrades. When the regular products are unavailable, consumers can only book high-quality products. When both types of products are unavailable, consumers cannot book any product. We can see that when at least one product type is unavailable, the consumer decision becomes a simple take-it-or-leave-it decision, so consumers do not anticipate the upgrade probability anymore. Let τ denote the first time when some product type is unavailable ($\tau = T$ if the firm never stops selling any type of product during the booking period), then τ is the (random) stopping time of the consumer booking game that strategic consumers play regarding upgrades.

3.4 Consumer Booking Equilibrium

Before deriving the firm's optimal conditional upgrade policy, we first need to analyze how strategic consumers make their booking decisions. In this section, we derive and characterize the symmetric pure-strategy equilibrium of the consumer booking game for a given upgrade price p. Upon arrival, a consumer observes her valuations for two product types (v_R, v_H) and arrival time t as well as the availability of product types, and books the product type that maximizes her expected utility. For a consumer that is offered an upgrade, the key to her booking decision is the expected upgrade probability $q(\cdot)$ she anticipates which is a function of her booking time t. Let $a_t(v_R, v_H|q(t))$ denote the consumer's utility-maximizing decision if she arrives at time t, has valuations (v_R, v_H), and anticipates the upgrade probability to be $q(t)$.[5] Similarly, let $a'_t(v_R, v_H)$ denote the utility-maximizing decision of a consumer that is not offered an upgrade.

Now we derive $a_t(v_R, v_H|q(t))$ and $a'_t(v_R, v_H)$. Figure 3.1 shows the consumers' utilities from booking different product types. The consumer's utility from booking a high-quality product is $v_H - p_H$, the utility from booking a regular product without upgrade is $v_R - p_R$, the expected utility from booking a regular product with upgrade is $q(t)(v_H - p_R - p) + [1 - q(t)](v_R - p_R)$, the utility from not booking any product is zero. Thus, the consumer chooses to book a high-quality product if $v_H - p_H \geq$

[5] We use $q(\cdot)$ to denote the whole function, and $q(t)$ to denote its value at t.

$\max\{q(t)(v_H - p_R - p) + [1 - q(t)](v_R - p_R), v_R - p_R, 0\}$; she chooses to book a regular product with upgrade if $q(t)(v_H - p_R - p) + [1 - q(t)](v_R - p_R) \geq \max\{v_H - p_H, v_R - p_R, 0\}$; she chooses to book a regular product without upgrade if $v_R - p_R \geq \max\{v_H - p_H, q(t)(v_H - p_R - p) + [1 - q(t)](v_R - p_R), 0\}$; otherwise, she does not book any product. We can simplify the above decision rule to the following:

- If $p \geq p_H - p_R$,

$$a_t(v_R, v_H|q(t)) = \begin{cases} H & \text{if } v_H - v_R \geq p_H - p_R \text{ and } v_H \geq p_H, \\ R & \text{if } v_H - v_R < p_H - p_R \text{ and } v_R \geq p_R, \\ N & \text{otherwise.} \end{cases}$$

- If $0 \leq p < p_H - p_R$,

$$a_t(v_R, v_H|q(t)) = \begin{cases} H & \text{if } v_H - v_R \geq \frac{p_H - p_R - q(t)p}{1 - q(t)} \text{ and } v_H \geq p_H, \\ U & \text{if } p \leq v_H - v_R < \frac{p_H - p_R - q(t)p}{1 - q(t)} \text{ and} \\ & q(t)v_H + [1 - q(t)]v_R \geq p_R + q(t)p, \\ R & \text{if } v_H - v_R < p \text{ and } v_R \geq p_R, \\ N & \text{otherwise.} \end{cases}$$

The utility-maximizing decision of consumers that are not offered upgrades, $a'_t(v_R, v_H)$, is same as $a_t(v_R, v_H|q(t))$ with $p \geq p_H - p_R$. It is easy to see that $a'_t(v_R, v_H)$ is also the equilibrium strategy for consumers that are not offered upgrades. We next focus on consumers that are offered upgrades and find their equilibrium strategy.

It is easy to see that if $0 \leq p < p_H - p_R$, $a_t(v_R, v_H|q(t))$ divides Ω into four subsets. Given $q(\cdot)$, $a_t(v_R, v_H|q(t))$ is uniquely determined for each (v_R, v_H) and each t, and $a_t(v_R, v_H|q(t))$ can be easily computed by plugging $q(t)$ into the equation of $a_t(v_R, v_H|q(t))$. Thus, we use $q(\cdot)$ to define the consumer's strategy in the booking game. The reason for using $q(\cdot)$ as the strategy instead of $a_t(v_R, v_H|q(\cdot))$ is that the corresponding strategy space has fewer dimensions and the computational burden of equilibrium is smaller. The strategy space is then defined as $\mathcal{Q} = \{q(\cdot) : [0, T] \to [0, 1]$, such that $q(\cdot)$ is differentiable$\}$. $\mathcal{Q}$ contains all differentiable functions of $t \in [0, T]$ taking values between 0 and 1.

To find the symmetric equilibrium $q^*(\cdot)$, we first fix one consumer (we call this consumer the acting consumer) and calculate the expected upgrade probability for the acting consumer if she books a regular product and accepts an upgrade offer when all other consumers are making their decisions based on $q(\cdot)$. Denote this resulting upgrade probability for the acting consumer as $b(q(\cdot))$, $b(q(\cdot))$ is also a function of t. Then, $q^*(\cdot)$ is the solution to $b(q^*(\cdot)) = q^*(\cdot)$. We can write $b(q(\cdot))$ as $b(q(\cdot)) = g(q(\cdot))/h(q(\cdot))$, where $g(q(\cdot))$ is the unconditional expected probability that a consumer arriving at time t accepts an upgrade offer and gets upgraded at the end of the booking period, and $h(q(\cdot))$ is the probability that both product types are still available by time t. So, $b(q(\cdot))$ is the expected upgrade probability conditioning on the fact that products are still available at time t.

Now we derive $g(q(\cdot))$ and $h(q(\cdot))$. With a slight abuse of notation, we use $N_H(t|q(\cdot))$, $N_U(t|q(\cdot))$, $N_R(t|q(\cdot))$ to denote the arrival processes of other consumers (as seen by the acting consumer) booking each product type given that the strategy they are using is $q(\cdot)$. Let $\tau(q(\cdot))$ denote the stopping time of the consumer booking game (i.e., the time when the firm stops selling at least one product type) if the acting consumer chooses to book a regular product and accept an upgrade offer and all other consumers use $q(\cdot)$. Then, we have

$$g(q(\cdot)) = \mathop{\mathbb{E}}_{N_H(t|q(\cdot)), N_U(t|q(\cdot)), N_R(t|q(\cdot))} \left\{ \min \left\{ \frac{[K_H - N_H(\tau(q(\cdot))|q(\cdot))]^+}{N_U(\tau(q(\cdot))|q(\cdot)) + 1}, 1 \right\} \cdot \mathbb{1}\left\{ t \le \tau(q(\cdot)) \right\} \right\}$$

where the "+1" term represents the acting consumer, and

$$h(q(\cdot)) = \mathbb{P}(N_H(t|q(\cdot)) < K_H, N_R(t|q(\cdot)) < K_R, N_H(t|q(\cdot)) + N_U(t|q(\cdot)) + N_R(t|q(\cdot)) < K_H + K_R).$$

Note that $g(q(\cdot))$ and $h(q(\cdot))$ both depend on t. To completely characterize $g(q(\cdot))$ and $h(q(\cdot))$, it remains to characterize $N_H(t|q(\cdot))$, $N_U(t|q(\cdot))$, $N_R(t|q(\cdot))$ as well as $\tau(q(\cdot))$.

Lemma III.1. *(Myerson 1998: Environmental equivalence property of games with Poisson arrivals[6]) From the perspective of any one player, the arrival process of other players is also a Poisson process with the same rate as the total arrival rate.*

Lemma III.1 implies that $N_H(t|q(\cdot))$, $N_U(t|q(\cdot))$, $N_R(t|q(\cdot))$ are indeed Poisson processes. Moreover, they have the same distributions as the overall arrival processes. Given $q(\cdot)$, the probabilities of any other consumer that is offered an upgrade booking each type of product are as follows:

$$\begin{aligned}
\xi_H^\gamma(t|q(\cdot)) &= \iint_\Omega \mathbb{1}\{a_t(v_R, v_H|q(\cdot)) = H\} f(v_R, v_H)\, \mathrm{d}v_R\, \mathrm{d}v_H, \\
\xi_U^\gamma(t|q(\cdot)) &= \iint_\Omega \mathbb{1}\{a_t(v_R, v_H|q(\cdot)) = U\} f(v_R, v_H)\, \mathrm{d}v_R\, \mathrm{d}v_H, \\
\xi_R^\gamma(t|q(\cdot)) &= \iint_\Omega \mathbb{1}\{a_t(v_R, v_H|q(\cdot)) = R\} f(v_R, v_H)\, \mathrm{d}v_R\, \mathrm{d}v_H.
\end{aligned}$$

The probabilities of any other consumer that is not offered an upgrade booking each type of product are as follows:

$$\begin{aligned}
\xi'_H(t) &= \iint_\Omega \mathbb{1}\{a'_t(v_R, v_H) = H\} f(v_R, v_H)\, \mathrm{d}v_R\, \mathrm{d}v_H, \\
\xi'_R(t) &= \iint_\Omega \mathbb{1}\{a'_t(v_R, v_H) = R\} f(v_R, v_H)\, \mathrm{d}v_R\, \mathrm{d}v_H.
\end{aligned}$$

Thus, the arrival rates of $N_H(t|q(\cdot))$, $N_U(t|q(\cdot))$, $N_R(t|q(\cdot))$ are $\lambda_H(t|q(\cdot)) = \lambda\gamma\xi_H^\gamma(t|q(\cdot))+$

[6] *Myerson* (1998) first proved the environmental equivalence property of games with Poisson arrivals. *Myerson* (1998) provides a proof for the case of discrete player type set, but it is easily generalized to the case of continuous player type set (in our problem, the player type set is continuous because we assume a continuous valuation support). We refer the readers interested in Poisson games to *Myerson* (1998), *Myerson* (2000) and *Milchtaich* (2004).

$\lambda(1-\gamma)\xi'_H(t)$, $\lambda_U(t|q(\cdot)) = \lambda\gamma\xi^{\gamma}_U(t|q(\cdot))$, $\lambda_R(t|q(\cdot)) = \lambda\gamma\xi^{\gamma}_R(t|q(\cdot)) + \lambda(1-\gamma)\xi'_R(t)$, respectively.

Next, we derive the stopping time $\tau(q(\cdot))$. Define the following auxiliary stopping times:

- $\tau_H(q(\cdot)) = \inf\{t \geq 0 : N_H(t|q(\cdot)) \geq K_H\}$.
- $\tau_R(q(\cdot)) = \inf\{t \geq 0 : N_R(t|q(\cdot)) \geq K_R\}$.
- $\tau_T(q(\cdot)) = \inf\{t \geq 0 : N_H(t|q(\cdot)) + N_U(t|q(\cdot)) + 1 + N_R(t|q(\cdot)) \geq K_H + K_R\}$.

$\tau_H(q(\cdot))$ is the time when high-quality products are fully booked, $\tau_R(q(\cdot))$ is the time when regular products are fully booked, $\tau_T(q(\cdot))$ is the time when the total demand reaches the firm's total capacity so both product types are fully booked simultaneously. Then, the stopping time of the consumer booking game is $\tau(q(\cdot)) = \min\{\hat{\tau}(q(\cdot)), T\}$, where

$$\begin{aligned}
\hat{\tau}(q(\cdot)) &= \min\{\tau_H(q(\cdot)), \tau_R(q(\cdot)), \tau_T(q(\cdot))\} \\
&= \begin{cases} \tau_H(q(\cdot)) & \text{if } \tau_H(q(\cdot)) \leq \tau_T(q(\cdot)), \\ \tau_R(q(\cdot)) & \text{if } \tau_R(q(\cdot)) \leq \tau_T(q(\cdot)), \\ \tau_T(q(\cdot)) & \text{if } \tau_H(q(\cdot)) > \tau_T(q(\cdot)) \text{ and } \tau_R(q(\cdot)) > \tau_T(q(\cdot)). \end{cases}
\end{aligned}$$

$\hat{\tau}(\cdot)$ can be interpreted as the stopping time when $T \to \infty$. Note that the second equality in the above equation follows from the fact that $\tau_H(q(\cdot)) \leq \tau_T(q(\cdot))$ implies $\tau_H(q(\cdot)) < \tau_R(q(\cdot))$ and that $\tau_R(q(\cdot)) \leq \tau_T(q(\cdot))$ implies $\tau_R(q(\cdot)) < \tau_H(q(\cdot))$.

Theorem III.2. *There exists a symmetric pure-strategy equilibrium $q^*(\cdot)$ of the consumer booking game. $q^*(\cdot)$ is increasing in the arrival time of the consumer. Moreover, with $\mathcal{Q}$ equipped with the uniform norm $\|q(\cdot)\|_\infty = \sup_{0\leq t\leq T} |q(t)|$, there exists a constant $\bar{\alpha}$ such that for any $q_1(\cdot), q_2(\cdot) \in \mathcal{Q}$, we have $\|b(q_1(\cdot)) - b(q_2(\cdot))\|_\infty \leq \bar{\alpha}\|q_1(\cdot) - q_2(\cdot)\|_\infty$. Thus, if $\bar{\alpha} < 1$, $b(q(\cdot))$ is a contraction mapping and the equilibrium is unique.*

Theorem III.2 states that the consumer booking game indeed has a symmetric pure-strategy equilibrium $q^*(\cdot)$ which is the solution to $b(q^*(\cdot)) = q^*(\cdot)$. $q^*(\cdot)$ is an increasing function because a consumer that arrives later and still finds both product types are available will have better knowledge that demand has realized to be weak, and hence form a higher probability of getting upgraded. Theorem III.2 also gives a sufficient condition for $q^*(\cdot)$ to be unique.[7] However, due to the complicated structure of our Poisson game, it is not possible to derive the closed-form equilibrium or further analyze the firm's optimal upgrade pricing policy analytically (the firm's revenue function is given in Section 3.9). To study conditional upgrades in greater depth and develop more managerial and policy insights, we are going to analyze a fluid model which is the asymptotic version of our stochastic model (i.e., scale up the capacities and demand rates by n and let $n \to \infty$). One may consider our fluid model as a deterministic approximation of the stochastic model where the consumer booking game is essentially one with perfect information. However, as verified by our numerical examples in Sections 3.5.3 and 3.6, our fluid model is very accurate in approximating the stochastic model and the results and insights derived from the fluid model also hold in the stochastic model. In Section 3.7, we study the stochastic model numerically and derive additional insights from the stochastic model.

3.5 Fluid Model

In this section, we derive and analyze the fluid model. In Section 3.5.1, we derive the asymptotic consumer booking equilibrium by scaling up the problem size by n and letting $n \to \infty$. In the problem instance scaled by n, the consumer arrival rate

[7] The formula of $\bar{\alpha}$ is complicated and is given in the proof of Theorem III.2 in Appendix B. Our numerical studies indicate that $\bar{\alpha} < 1$ is satisfied when the product prices are far apart enough from each other and the capacity-demand ratio is moderately large. Note that $\bar{\alpha} < 1$ is a sufficient but not necessary condition for the equilibrium to be unique. In our extensive numerical studies with joint-uniform valuation distributions, we do not observe multiple equilibria to arise. In fact, as the capacities and demand rates increase proportionally to infinity, the equilibrium is provably unique (Theorem III.4).

is $n\lambda(t)$ and the firm's capacities are nK_H and nK_R. For other variables, we add a subscript of n to specify the problem size. Based on the model in Section 3.5.1, in Section 3.5.2, we study the firm's optimal upgrade pricing strategy. In Section 3.5.3, we evaluate the performance of the fluid model.

3.5.1 Consumer Booking Equilibrium

The following theorem characterizes the equilibrium upgrade probability in the asymptotic scenario of the consumer booking game. As $n \to \infty$, $q^*(\cdot)$ converges to a constant q_f, where the subscript of f denotes the fluid model (we also use s to denote the stochastic model).

Theorem III.3. *(i) As $n \to \infty$, for any $q(\cdot) \in \mathcal{Q}$, the auxiliary stopping times converge to*

$$\begin{aligned}
\tau_H^{\infty}(q(\cdot)) &= \inf\left\{t \geq 0 : \int_0^t \lambda_H(s|q(\cdot))\,\mathrm{d}s \geq K_H\right\}, \\
\tau_R^{\infty}(q(\cdot)) &= \inf\left\{t \geq 0 : \int_0^t \lambda_R(s|q(\cdot))\,\mathrm{d}s \geq K_R\right\}, \\
\tau_T^{\infty}(q(\cdot)) &= \inf\left\{t \geq 0 : \int_0^t \left[\lambda_H(s|q(\cdot)) + \lambda_U(s|q(\cdot)) + \lambda_R(s|q(\cdot))\right]\,\mathrm{d}s \geq K_R + K_H\right\},
\end{aligned}$$

a.s., respectively. The stopping time of the consumer booking game converges to $\tau^{\infty}(q(\cdot)) = \min\{\hat{\tau}^{\infty}(q(\cdot)), T\}$ *a.s., where*

$$\hat{\tau}^{\infty}(q(\cdot)) = \begin{cases} \tau_H^{\infty}(q(\cdot)) & \text{if } \tau_H^{\infty}(q(\cdot)) \leq \tau_T^{\infty}(q(\cdot)), \\ \tau_R^{\infty}(q(\cdot)) & \text{if } \tau_R^{\infty}(q(\cdot)) \leq \tau_T^{\infty}(q(\cdot)), \\ \tau_T^{\infty}(q(\cdot)) & \text{if } \tau_H^{\infty}(q(\cdot)) > \tau_T^{\infty}(q(\cdot)) \text{ and } \tau_R^{\infty}(q(\cdot)) > \tau_T^{\infty}(q(\cdot)). \end{cases}$$

(ii) As $n \to \infty$, the equilibrium upgrade probability $q^{n*}(\cdot)$ converges pointwise to

q_f *which is the (time-independent) solution of the following equation:*

$$q_f = \min\left\{ \frac{\left[K_H - \int_0^{\tau^\infty(q_f)} \lambda_H(t|q_f)\,\mathrm{d}t\right]^+}{\int_0^{\tau^\infty(q_f)} \lambda_U(t|q_f)\,\mathrm{d}t}, 1 \right\}. \tag{3.1}$$

Our primary goal with the fluid model is to derive closed-form solutions which will provide us sharp insights about how consumers make upgrading decisions and how the firm's optimal upgrade price depends on problem parameters. To be able to obtain closed-form solutions, we will assume that the consumers' valuations for two types of products are jointly uniformly distributed in the two-dimensional support $\Omega = \{(v_R, v_H) : 0 \le v_R \le v_H \le u\}$, that is, for consumers that value high-quality products at v_H, their valuations for regular products are uniformly distributed over $[0, v_H]$. $u > p_H$ is the upper bound of consumer valuations. Thus, the valuation support Ω is now an upper triangular subset of $\mathbb{R}_+^2$, and the joint probability density is $f(v_R, v_H) = 2/u^2$. Our analysis can be easily generalized if we move Ω within $\mathbb{R}_+^2$ to allow for different upper and lower bounds of consumer valuations. Moreover, we have numerically tested our results when consumers' valuations follow a bivariate normal distribution, and we find that all results in the paper carry through to the case with bivariate normal distribution.

Now we calculate q_f by solving (3.1). We first need to derive the demand segmentation in the fluid model for a given q (i.e., $\lambda_H(q)$, $\lambda_U(q)$, $\lambda_R(q)$). Figure 3.2 plots all five possible demand segmentations of consumers that are offered upgrades. Throughout this chapter, we use the superscript "a" through "e" consistent with Figure 3.2 to specify which case we are referring to. Case a also gives the demand segmentation of consumers that are not offered upgrades. In each ease, the proportions of consumers booking each product type, $\xi_H(q)$, $\xi_U(q)$, $\xi_R(q)$, can be calculated as the ratio between the area of each region where the consumer decision is to book the corresponding product type and the area of the entire valuation support Ω. The

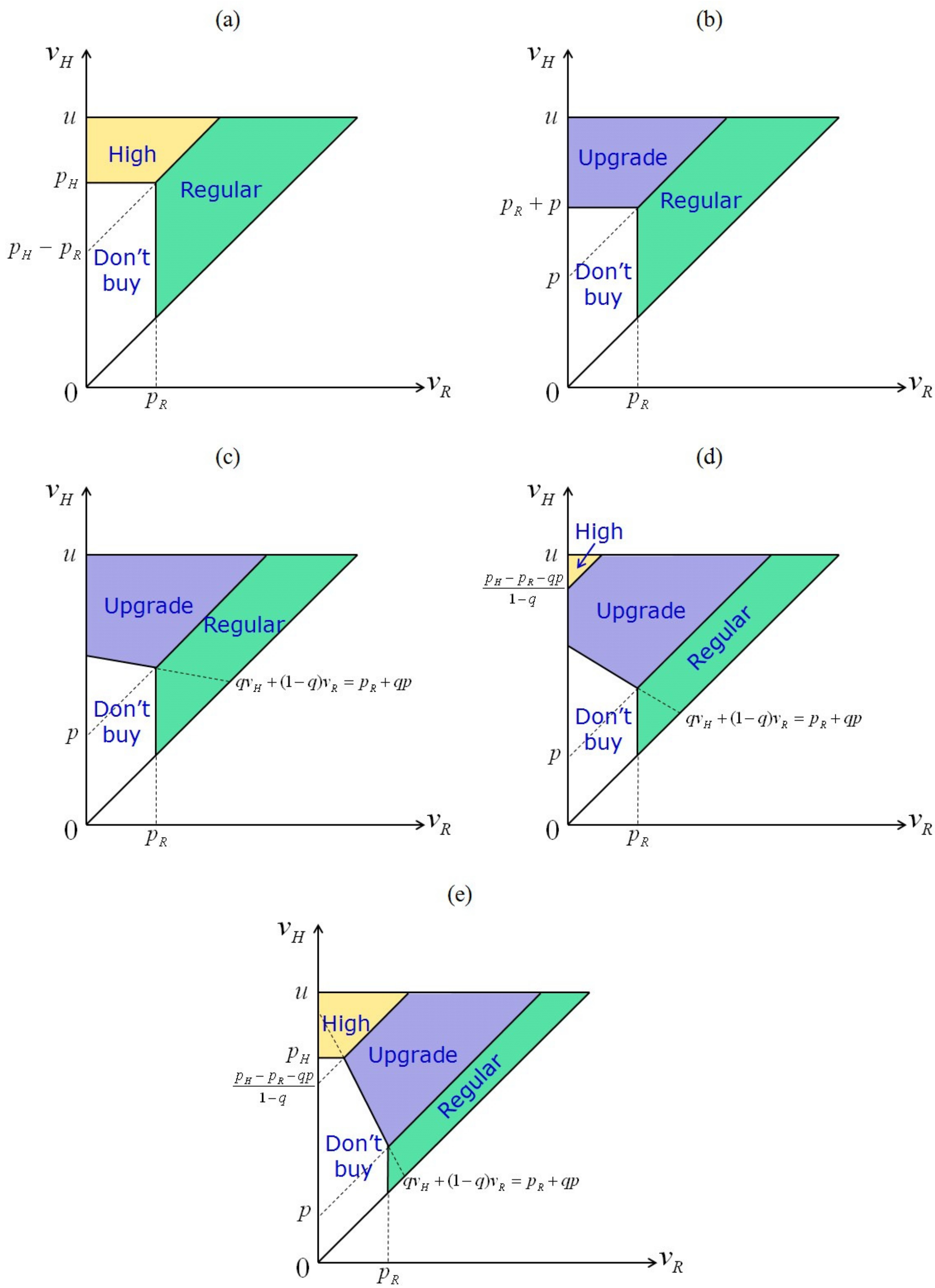


Figure 3.2: Demand segmentation given the upgrade price p and the upgrade probability q: (a) no upgrades offered, or $p \geq p_H - p_R$; (b) $p < p_H - p_R$ and $q = 1$; (c) $p < p_H - p_R$ and $q < 1$ and $(p_H - p_R - qp)/(1-q) \geq u$; (d) $p < p_H - p_R$ and $q < 1$ and $p_H \leq (p_H - p_R - qp)/(1-q) < u$; (e) $p < p_H - p_R$ and $q < 1$ and $(p_H - p_R - qp)/(1-q) < p_H$.

results are shown below. The overall demand rates are $\lambda_H(q) = \lambda\gamma\xi_H^i(q) + \lambda(1-\gamma)\xi_H^a$, $\lambda_U(q) = \lambda\gamma\xi_U^i(q)$, $\lambda_R(q) = \lambda\gamma\xi_R^i(q) + \lambda(1-\gamma)\xi_R^a$ in Case i.

Case a If $p \geq p_H - p_R$ (i.e., the firm does not offer upgrades), the consumer segmentation (of consumers that are offered upgrades) is

$$\xi_H^a = \frac{1}{u^2}(u - p_H + 2p_R)(u - p_H), \qquad \xi_U^a = 0, \qquad \xi_R^a = \frac{1}{u^2}(p_H - p_R)(2u - p_H - p_R).$$

Case b If $p < p_H - p_R$ (i.e., the firm offers upgrades) and $q = 1$, because upgrades are guaranteed to be fulfilled, nobody books a high-quality product directly. The consumer segmentation in this case is

$$\xi_H^b = 0, \qquad \xi_U^b = \frac{1}{u^2}(p_R + u - p)(u - p_R - p), \qquad \xi_R^b = \frac{1}{u^2}\left[-p^2 + 2(u - p_R)p\right].$$

Case c If $p < p_H - p_R$ and $q < 1$ and $(p_H - p_R - qp)/(1-q) \geq u$, since $q < 1$, by booking a regular product and accepting an upgrade offer instead of booking a high-quality product directly, a consumer risks not being upgraded and ending up consuming a regular product. Recall that a consumer books a high-quality product directly if $v_H - v_R \geq (p_H - p_R - qp)/(1-q)$ and $v_H \geq p_H$, where $(p_H - p_R - qp)/(1-q)$ is the minimum valuation differential required to induce one to book a high-quality product directly. If $(p_H - p_R - qp)/(1-q) \geq u$, all consumers that are interested in high-quality products will choose to get them through upgrades. The consumer segmentation in this case is

$$\xi_H^c = 0, \qquad \xi_U^c(q) = \frac{1}{u^2}\left[-\frac{p_R^2}{q} + (u-p)^2\right], \qquad \xi_R^c = \frac{1}{u^2}\left[-p^2 + 2(u - p_R)p\right].$$

Case d If $p < p_H - p_R$ and $q < 1$ and $p_H \leq (p_H - p_R - qp)/(1-q) < u$, since $(p_H - p_R - qp)/(1-q) < u$, the consumers with high enough valuations for high-quality products combined with low enough valuations for regular products will book high-quality products directly. Thus, in this case, high-quality products are sold in both channels (i.e., directly and through upgrades). Further, depending on whether $(p_H - p_R - qp)/(1-q) \geq p_H$ or not, $\xi_H(q)$ and $\xi_U(q)$ take different functional forms. If $(p_H - p_R - qp)/(1-q) \geq p_H$, the consumer segmentation is

$$\begin{aligned}
\xi_H^d(q) &= \frac{1}{u^2}\left(u - \frac{p_H - p_R - qp}{1-q}\right)^2, \\
\xi_U^d(q) &= \frac{1}{u^2}\left[-\left(\frac{p_H - p_R - qp}{1-q}\right)^2 + 2u\left(\frac{p_H - p_R - qp}{1-q}\right) - \frac{p_R^2}{q} + p^2 - 2up\right], \\
\xi_R^d &= \frac{1}{u^2}\left[-p^2 + 2(u - p_R)p\right].
\end{aligned}$$

Otherwise we are in Case e.

Case e If $p < p_H - p_R$ and $q < 1$ and $(p_H - p_R - qp)/(1-q) < p_H$, the consumer segmentation is

$$\begin{aligned}
\xi_H^e(q) &= \frac{1}{u^2}\left[u + p_H - \frac{2(p_H - p_R - qp)}{1-q}\right](u - p_H), \\
\xi_U^e(q) &= \frac{1}{u^2} \cdot \frac{p_H - p_R - p}{1-q} \cdot (2u - p_H - p_R - p), \\
\xi_R^e &= \frac{1}{u^2}\left[-p^2 + 2(u - p_R)p\right].
\end{aligned}$$

We assume $K_H \geq \lambda_H^a T$ and $K_R \geq \lambda_R^a T$, that is, the firm's expected demand when upgrades are not offered does not exceed its capacity for either product type at the prices p_H and p_R. This assumption is reasonable since the utilization rates in travel industries are generally not high (according to Statista[8], the average occupancy rate of the U.S. hotel lodging industry from 2000 to 2013 is only 60%). We would like to note that when the firm offers upgrades, it is still possible under this assumption that

[8] http://www.statista.com/statistics/200161/us-annual-accomodation-and-lodging-occupancy-rate.

the firm's total capacity is fully booked before the end of the booking period, because offering upgrades can generate more demand than the case without upgrades. Thus, our analysis allows for any utilization level with upgrades. Moreover, our numerical analysis indicates that all findings in this essay continue to hold even if the above assumption is not satisfied.

Theorem III.4. *Define*

$$\begin{aligned}
\bar{p} &= u - \sqrt{\frac{1}{\gamma}\left[\frac{K_H}{\lambda T}u^2 - (u-p_H+2p_R)(u-p_H)\right] + (u-p_H+p_R)^2}, \\
\underline{p} &= p_H - p_R - \frac{\frac{K_H}{\lambda T}u^2 - (u-p_H+2p_R)(u-p_H)}{\gamma(u-p_H+p_R)}, \\
\underline{p}' &= -\frac{1}{p_R}\left[\frac{K_H}{\lambda T}u^2 - u^2 + 2(p_H-p_R)u - p_H^2 + p_Hp_R + p_R^2\right] \\
&\quad + \frac{1}{\gamma p_R}\sqrt{\gamma\left[\frac{K_H}{\lambda T}u^2 - (u-p_H+2p_R)(u-p_H)\right]} \\
&\quad \cdot\sqrt{\left[1-\gamma-\frac{(1-\gamma)K_H+K_R}{\lambda T}\right]u^2 + 2\gamma(p_H-p_R)u - \gamma p_H^2 - 2(1-\gamma)p_Hp_R + \gamma p_R^2}.
\end{aligned}$$

(i) If $K_H \geq (\lambda T/u^2)[(u-p_H+2p_R)(u-p_H)+\gamma(p_H-p_R)(2u-p_H+p_R)]$, $q_f = 1$ for all $0 \leq p < p_H - p_R$.

(ii) If $K_H < (\lambda T/u^2)[(u-p_H+2p_R)(u-p_H)+\gamma(p_H-p_R)(2u-p_H+p_R)]$, the equilibrium upgrade probability is uniquely given by the following:

- *If $\underline{p}^+ \geq \underline{p}'^+$ (where $x^+ = \max\{x, 0\}$),*

$$q_f = \begin{cases} 1 & \text{for } \bar{p} \leq p < p_H - p_R, \\ \frac{\frac{K_H}{\lambda T}u^2 - (u-p_H+2p_R)(u-p_H) + \gamma(u-p_H+p_R)^2}{\gamma(u-p)^2} & \text{for } \underline{p}^+ \leq p < \bar{p}, \\ \frac{\frac{K_H}{\lambda T}u^2 - (u-p_H+2p_R)(u-p_H)}{\gamma(p_H-p_R-p)^2 + \frac{K_H}{\lambda T}u^2 - (u-p_H+2p_R)(u-p_H)} & \text{for } 0 \leq p < \underline{p}^+; \end{cases}$$

- *If* $\underline{p}^+ < \underline{p}'^+$,

$$q_f = \begin{cases} 1 & \text{for } \bar{p} \le p < p_H - p_R, \\ \frac{\frac{K_H}{\lambda T}u^2 - (u-p_H+2p_R)(u-p_H)+\gamma(u-p_H+p_R)^2}{\gamma(u-p)^2} & \text{for } \underline{p}'^+ \le p < \bar{p}, \\ \frac{2\gamma\frac{K_H}{K_H+K_R}p_R^2}{-\beta-\sqrt{\beta^2-4\gamma^2\frac{K_H}{K_H+K_R}p_R^2(u-p)^2}} & \text{for } 0 \le p < \underline{p}'^+, \end{cases}$$

where $\beta = (u - p_H + 2p_R)(u - p_H) - \gamma(u - p_H + p_R)^2 + \frac{K_H}{K_H+K_R}[2\gamma p_R p - u^2 + (1-\gamma)(2p_H p_R - p_R^2)]$.

(iii) q_f *is increasing in* p.

Theorem III.4 gives the equilibrium upgrade probability q_f for any upgrade price p set by the firm. If the firm's capacity for high-quality products is very large (i.e., $K_H \ge (\lambda T/u^2)[(u - p_H + 2p_R)(u - p_H) + \gamma(p_H - p_R)(2u - p_H + p_R)]$), consumers accepting upgrade offers are guaranteed to get upgraded. In equilibrium, being aware of the very high chance to get upgraded, all consumers who are interested in high-quality products and offered upgrades choose to book regular products and accept upgrade offers. If the firm's capacity for high-quality products is not very large (i.e., $K_H < (\lambda T/u^2)[(u-p_H+2p_R)(u-p_H)+\gamma(p_H-p_R)(2u-p_H+p_R)]$), the equilibrium upgrade probability q_f increases with the upgrade price p. This is because fewer consumers accept upgrade offers when the upgrade price is higher. As the upgrade price p decreases from $p_H - p_R$ to 0, as shown by the proof of Theorem III.4, the market segmentation takes the form in Cases b, c, d, e in sequence.[9] Case d or e occurs only if the upgrade price is low (i.e., $0 \le p < \underline{p}^+$), meaning the equilibrium upgrade probability is small enough. Thus, the consumers with high enough valuations for high-quality products and low enough valuations for low-quality products will book

[9] $\bar{p}$ is the threshold between Case b and Case c, $\underline{p}$ is the threshold between Case c and Case d when $\tau^\infty(q_f) \ge T$, $\underline{p}'$ is the threshold between Case c and Case d when $\tau^\infty(q_f) < T$. If $\underline{p}^+ \ge \underline{p}'^+$, when Case c switches to Case d, we have $\tau^\infty(q_f) \ge T$; and vice versa. q_f takes the same form in Cases d and e.

high-quality products directly even if they are offered upgrades. In Case b or c, the upgrade probability is large enough so that consumers would like to obtain high-quality products through upgrades if they are given the offers.

3.5.2 Optimal Upgrade Pricing

In this section, based on the equilibrium consumer booking decision characterized in Section 3.5.1, we study the firm's optimal conditional upgrade pricing strategy. The firm's goal is to maximize its revenue from selling both types of products and charging upgrade fees. Recall that $p \geq p_H - p_R$ corresponds to the case without upgrades. In this case, the firm's revenue is $\Pi_{N,f} = p_R \lambda_R^a \min\{K_R/\lambda_R^a, T\} + p_H \lambda_H^a \min\{K_H/\lambda_H^a, T\}$, where the subscript of N denotes no upgrades. If the firm offers upgrades with $p < p_H - p_R$, its revenue is

$$\begin{aligned}
\Pi_f(p) \quad = \quad & p_R[\lambda_U(q_f) + \lambda_R]\tau^\infty(q_f) + p\lambda_U(q_f)\tau^\infty(q_f)q_f + p_H\lambda_H(q_f)\tau^\infty(q_f) \\
& + \mathbb{1}\{\tau^\infty(q_f) = \tau_R^\infty(q_f)\} p_H \min\left\{\lambda_H^a[T - \tau^\infty(q_f)], K_H - [\lambda_H(q_f) + \lambda_U(q_f)]\tau^\infty(q_f)\right\} \\
& + \mathbb{1}\{\tau^\infty(q_f) = \tau_H^\infty(q_f)\} p_R \min\left\{\lambda_R^a[T - \tau^\infty(q_f)], K_R - [\lambda_U(q_f) + \lambda_R]\tau^\infty(q_f)\right\}.
\end{aligned}$$

The first line of $\Pi_f(p)$ is the revenue collected before the consumer booking game stops. The first term is the revenue from selling regular products (including the revenue from consumers accepting upgrade offers), the second term is the revenue from collecting upgrade fees, the third term is the revenue from selling high-quality products. The second line of $\Pi_f(p)$ is the revenue from selling high-quality products after regular products are fully booked, where $\lambda_H^a[T - \tau^\infty(q_f)]$ is the demand and $K_H - [\lambda_H(q_f) + \lambda_U(q_f)]\tau^\infty(q_f)$ is the remaining capacity for high-quality products. The third line of $\Pi_f(p)$ is the revenue from selling regular products after high-quality products are fully booked . Since $\Pi_f(p) = \Pi_{N,f}$ at $p = p_H - p_R$, we limit ourselves to $0 \leq p \leq p_H - p_R$ in studying $\Pi_f(p)$ in the remainder of this chapter. When the optimal upgrade price is achieved at $p_f^* = p_H - p_R$, we know that it is optimal for the firm not to offer upgrades.

Theorem III.5. *The optimal upgrade price is* $p_f^* = \min\left\{\max\left\{(p_{foc}^b)^+, \bar{p}\right\}, p_H - p_R\right\}$, *where*

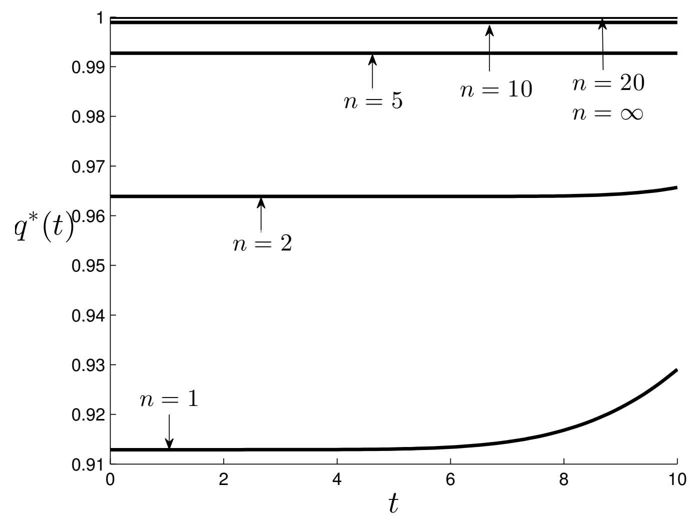
$$p_{foc}^b = \frac{2u - \sqrt{u^2 + 9p_R^2}}{3}.$$

Moreover, the optimal pricing induces $q_f = 1$.

Theorem III.5 characterizes the optimal upgrade price. The optimal upgrade price results in an equilibrium consumer segmentation in Case b (p_{foc}^b is the optimal price in Case b, Case b occurs for $\bar{p} \leq p \leq p_H - p_R$) where the upgrade probability is equal to one. Recall that Theorem III.4 states q_f is increasing in p (or always equal to one if the high-quality product capacity is very large). Thus, Theorem III.5 states that the firm should choose an upgrade price that is high enough. If an upgrade price results in some consumers being rationed for upgrades, that means too many consumers are willing to pay for the upgrades and the current upgrade price is too low. The firm should increase the upgrade price to extract more surplus from consumers while still being able to sell out high-quality products after fulfilling upgrades. Thus, under the optimal upgrade pricing policy, strategic consumers who are offered upgrades purchase high-quality products through upgrades instead of booking directly. Note that because of the deterministic feature, our fluid model captures an ideal situation where the firm and consumers have perfect knowledge about the total demand for each product type. In the stochastic model, because of the demand randomness, the equilibrium upgrade probability may not be exactly equal to one under the optimal upgrade price, so consumers with very high valuations for high-quality products and very low valuations for regular products may choose to book high-quality products directly even if upgrades are offered at the optimal price. However, consistent with the insight we developed from the fluid model, in the stochastic model, the firm should generally charge a high enough upgrade price that results in a high upgrade probability for consumers (Tables 3.1 and 3.2 in the next subsection provide a set of examples).

3.5.3 Performance Evaluation of Fluid Model

We now evaluate how well the fluid model approximates the stochastic model for relatively small values of n (we know that as $n \to \infty$, the fluid model converges to the stochastic model). In Figure 3.3, we provide an illustrative example for the comparison between the consumer purchasing equilibria in the stochastic model for different values of n and the consumer booking equilibrium in the fluid model. For example, in Figure 3.3, we see that the upgrade probability in the fluid model is 1. We also see that when $n = 5$, in the stochastic model, the upgrade probability is 0.9927. We note that in this example, $n = 5$ corresponds to a relatively small hotel with 60 rooms ($n(K_H + K_R) = 60$). Furthermore, in the example in Figure 3.3, we see that when $n = 5$, the percentage of consumers that would make a different decision in the stochastic model (with respect to which type of product to book) than in the fluid model is only 0.73%. In Tables 3.1 and 3.2, we examine the gap between the consumer booking equilibria in the stochastic model and in the fluid model with more examples. Table 3.1 provides examples with different product prices, Table 3.2 provides examples with different product capacities. We can see that the equilibrium upgrade probability in the stochastic model is closer to one when the product price differential is larger, or when the high-quality product capacity is large, both indicating a smaller probability that the firm runs out of high-quality products. Overall, we observe that the equilibrium upgrade probability is increasing in the product price differential, and increasing in the high-quality product capacity. When the equilibrium upgrade probability in the stochastic model is closer to one, the equilibrium consumer segmentation in the stochastic model is also closer to the equilibrium demand segmentation in the fluid model.



n	$\mathbb{E}_t[q^*(t)]$	Demand segmentation			ΔDemand
		High	Upgrade	Regular	
1	0.9152	9.00%	25.20%	30.19%	2.03%
2	0.9640	9.00%	25.51%	30.20%	1.72%
5	0.9927	9.00%	26.49%	29.75%	0.73%
10	0.9989	9.00%	27.02%	29.47%	0.20%
20	1	9.00%	27.21%	29.37%	0.02%
∞	1	9.00%	27.22%	29.36%	–

Figure 3.3: A numerical example on the asymptotic convergence of consumer booking equilibrium under the optimal upgrade price. ($\lambda = 1$, $T = 10$, $K_H = 5$, $K_R = 7$, $p_H = 160$, $p_R = 70$, $\gamma = 0.5$, v_R and v_H are jointly uniformly distributed over $\Omega = \{(v_R, v_H) : 0 \leq v_R \leq v_H \leq 200\}$; "ΔDemand" is defined as the expected percentage of consumers that would make a different booking decision in the stochastic model than predicted by the fluid model)

Table 3.3 provides an illustrative example for the asymptotic convergence of the firm's optimal upgrade price and revenue. The derivation of the stochastic revenue function, $\Pi_s(p)$, is given in Section 3.9. By comparing the stochastic revenues using the optimal upgrade price derived from the fluid model and using the optimal upgrade price for the stochastic model, we can evaluate the performance of the fluid model. From Table 3.3, we clearly see that by using the optimal upgrade price derived from the fluid model, the firm's revenue deviates by an almost negligible amount from the

	$p_H = 130$		$p_H = 140$		$p_H = 150$		$p_H = 160$	
	$\mathbb{E}_t[q^*(t)]$	ΔDemand	$\mathbb{E}_t[q^*(t)]$	ΔDemand	$\mathbb{E}_t[q^*(t)]$	ΔDemand	$\mathbb{E}_t[q^*(t)]$	ΔDemand
$p_R = 60$	0.9879	1.34%	0.9953	0.63%	0.9985	0.24%	0.9996	0.08%
$p_R = 70$	0.9770	2.25%	0.9898	1.21%	0.9962	0.55%	0.9989	0.20%
$p_R = 80$	0.9615	3.42%	0.9809	2.05%	0.9919	1.06%	0.9972	0.45%
$p_R = 90$	0.8680	23.35%	0.9685	3.11%	0.9849	1.79%	0.9939	0.88%

Table 3.1: Numerical examples on the gap between the consumer booking equilibria (under the optimal upgrade price) in the stochastic model and in the fluid model with different product prices: the time-average equilibrium upgrade probability ($\mathbb{E}_t[q^*(t)]$) and the expected percentage of consumers that would make a different booking decision in the stochastic model than predicted by the fluid model (ΔDemand). ($\lambda = 1$, $T = 100$, $K_H = 50$, $K_R = 70$, $\gamma = 0.5$, v_R and v_H are jointly uniformly distributed over $\Omega = \{(v_R, v_H) : 0 \leq v_R \leq v_H \leq 200\}$)

	$K_H = 40$		$K_H = 50$		$K_H = 60$		$K_H = 70$	
	$\mathbb{E}_t[q^*(t)]$	ΔDemand	$\mathbb{E}_t[q^*(t)]$	ΔDemand	$\mathbb{E}_t[q^*(t)]$	ΔDemand	$\mathbb{E}_t[q^*(t)]$	ΔDemand
$K_R = 60$	0.9487	3.05%	0.9919	1.06%	0.9997	0.05%	1.0000	0.00%
$K_R = 70$	0.9487	3.06%	0.9919	1.06%	0.9997	0.05%	1.0000	0.00%
$K_R = 80$	0.9487	3.06%	0.9919	1.06%	0.9997	0.05%	1.0000	0.00%
$K_R = 90$	0.9487	3.06%	0.9919	1.06%	0.9997	0.05%	1.0000	0.00%

Table 3.2: Numerical examples on the gap between the consumer booking equilibria (under the optimal upgrade price) in the stochastic model and in the fluid model with different product capacities: the time-average equilibrium upgrade probability ($\mathbb{E}_t[q^*(t)]$) and the expected percentage of consumers that would make a different booking decision in the stochastic model than predicted by the fluid model (ΔDemand). ($\lambda = 1$, $T = 100$, $p_H = 150$, $p_R = 80$, $\gamma = 0.5$, v_R and v_H are jointly uniformly distributed over $\Omega = \{(v_R, v_H) : 0 \leq v_R \leq v_H \leq 200\}$)

real optimal revenue in the stochastic model even for very small problem sizes (less than or equal to 0.1% even for $n = 1$). The optimal upgrade price itself may have some error especially when the problem size is small, but our numerical studies indicate that the revenue function in the stochastic model is quite flat in the region around the optimal upgrade price, hence the deviation of the optimal revenue is significantly smaller than the deviation of the optimal upgrade price. In Tables 3.4 and 3.5, we examine the deviation of optimal upgrade price and optimal revenue in the stochastic model caused by the fluid solution with more examples. Table 3.4 provides examples with different product prices, Table 3.5 provides examples with different product capacities. We can see that similar to the observation from analyzing the consumer booking equilibrium, the optimal upgrade price and revenue deviations caused by the fluid solution are smaller when the product price differential is larger, or when the high-quality product capacity is larger, both indicating a smaller probability that the firm runs out of high-quality products. Overall we observe that the pricing heuristic derived from the fluid model performs very well in terms of giving the firm close-to-optimal revenues in the stochastic model. Thus, by studying the fluid model, we can develop managerial insights that will carry through to the stochastic model and provide an excellent heuristic for the stochastic problem.

n	Fluid solution		Stochastic solution		$\Delta p^* = \frac{\lvert p_f^* - p_s^* \rvert}{p_s^*}$	$\Delta\Pi^* = \frac{\Pi_s(p_s^*) - \Pi_s(p_f^*)}{\Pi_s(p_s^*)}$
	p_f^*	$\Pi_s(p_f^*)$	p_s^*	$\Pi_s(p_s^*)$		
1	36.7	620.7	40.3	621.1	8.96%	0.07%
2	36.7	1265.4	40.4	1266.4	9.13%	0.08%
5	36.7	3192.2	38.4	3192.8	4.42%	0.02%
10	36.7	6396.9	37.1	6397.0	1.28%	0.00%
20	36.7	12798.0	36.7	12798.0	0.11%	0.00%

Table 3.3: A numerical examples on the asymptotic convergence of optimal upgrade price and revenue. ($\lambda = 1$, $T = 10$, $K_H = 5$, $K_R = 7$, $p_H = 160$, $p_R = 70$, $\gamma = 0.5$, v_R and v_H are jointly uniformly distributed over $\Omega = \{(v_R, v_H) : 0 \le v_R \le v_H \le 200\}$)

3.6 Analysis of Optimal Upgrade Pricing

Now that we have obtained the optimal upgrade pricing strategy, we explore it further and develop managerial and policy insights for firms. We are first interested in when the conditional upgrade policy increases firms' revenues and when it can actually decrease revenues. We identify some benefits of conditional upgrades and show that by optimally deciding when to offer upgrades and at which price to offer upgrades, the firm benefits from offering conditional upgrades to more strategic consumers. Then,

	$p_H = 130$		$p_H = 140$		$p_H = 150$		$p_H = 160$	
	Δp^*	$\Delta\Pi^*$	Δp^*	$\Delta\Pi^*$	Δp^*	$\Delta\Pi^*$	Δp^*	$\Delta\Pi^*$
$p_R = 60$	7.07%	0.07%	3.45%	0.01%	1.36%	0.00%	0.43%	0.00%
$p_R = 70$	12.53%	0.18%	7.20%	0.05%	3.40%	0.01%	1.28%	0.00%
$p_R = 80$	20.25%	0.35%	13.31%	0.14%	7.42%	0.04%	3.34%	0.01%
$p_R = 90$	31.36%	0.62%	22.89%	0.28%	14.77%	0.10%	7.90%	0.02%

Table 3.4: Numerical examples on the gap between the firm's optimal upgrade prices as well as revenues in the stochastic model and in the fluid model with different product prices: the price error ($\Delta p^* = \frac{|p_f^* - p_s^*|}{p_s^*}$) and the revenue error ($\Delta\Pi^* = \frac{\Pi_s(p_s^*) - \Pi_s(p_f^*)}{\Pi_s(p_s^*)}$). ($\lambda = 1$, $T = 100$, $K_H = 50$, $K_R = 70$, $\gamma = 0.5$, v_R and v_H are jointly uniformly distributed over $\Omega = \{(v_R, v_H) : 0 \le v_R \le v_H \le 200\}$)

	$K_H = 40$		$K_H = 50$		$K_H = 60$		$K_H = 70$	
	Δp^*	$\Delta\Pi^*$	Δp^*	$\Delta\Pi^*$	Δp^*	$\Delta\Pi^*$	Δp^*	$\Delta\Pi^*$
$K_R = 60$	16.26%	0.28%	7.42%	0.04%	0.41%	0.00%	0.00%	0.00%
$K_R = 70$	16.27%	0.28%	7.42%	0.04%	0.41%	0.00%	0.00%	0.00%
$K_R = 80$	16.27%	0.28%	7.42%	0.04%	0.41%	0.00%	0.00%	0.00%
$K_R = 90$	16.27%	0.28%	7.42%	0.04%	0.41%	0.00%	0.00%	0.00%

Table 3.5: Numerical examples on the gap between the firm's optimal upgrade prices as well as revenues in the stochastic model and in the fluid model with different product capacities: the price error ($\Delta p^* = \frac{|p_f^* - p_s^*|}{p_s^*}$) and the revenue error ($\Delta\Pi^* = \frac{\Pi_s(p_s^*) - \Pi_s(p_f^*)}{\Pi_s(p_s^*)}$). ($\lambda = 1$, $T = 100$, $p_H = 150$, $p_R = 80$, $\gamma = 0.5$, v_R and v_H are jointly uniformly distributed over $\Omega = \{(v_R, v_H) : 0 \le v_R \le v_H \le 200\}$)

we characterize when it is optimal to offer conditional upgrades for free. Finally, we demonstrate the importance of accounting for strategic consumer behavior with conditional upgrades by evaluating the cost of ignoring strategic consumer behavior.

3.6.1 When to Offer Upgrades?

The following result states when offering conditional upgrades at the optimal price increases or decreases the firm's revenue. For the conditional upgrade policy to be strictly beneficial (i.e., $p_f^* < p_H - p_R$), the product price differential should be large enough. When the product price differential is small, it is optimal not to offer upgrades (or alternatively set the upgrade price at $p_f^* = p_H - p_R$).

Theorem III.6. *Offering conditional upgrades increases the revenue if*

$$p_H > \frac{2u + 3p_R - \sqrt{u^2 + 9p_R^2}}{3}$$

and decreases the revenue otherwise.

The fundamental trade-off regarding whether the firm should offer upgrades is as follows. If the firm offers upgrades, some consumers, who book high-quality products when the firm does not offer upgrades, will now book regular products and accept upgrade offers instead. The firm's revenue from direct sales of high-quality products decreases, that is, the upgrade channel cannibalizes the direct sales of high-quality products. This is the cannibalization effect of conditional upgrades. On the other hand, some consumers who book regular products when the firm does not offer upgrades will now accept upgrade offers, also some consumers who do not book any product when the firm does not offer upgrades will now purchase regular products and accept upgrade offers (these consumers' valuations for regular (high-quality) products are lower than p_R (p_H), but their valuations for high-quality products are higher than or equal to $p_R + p$). These two types of consumers bring additional revenues to the firm. This is the demand improvement effect of conditional upgrades. One important factor that determines which of these two effects is stronger is the product price differential. If the price differential is small and the firm offers upgrades, the cannibalization effect is significant, as a lot of consumers will book high-quality products if the firm does not offer upgrades, and these consumers will shift to upgrades under the optimal upgrade price (Theorem III.5). Moreover, since the high-quality product price is already close to the regular product price, there will not be many consumers who accept the upgrade offers, hence the demand improvement effect is not significant. Therefore, the firm's revenue is hurt if upgrades are offered in this case.

Thus, the firm benefits from offering conditional upgrades if the product price differential is large enough. This finding has important implications for the companies in travel industries regarding whether and when they should use the conditional upgrade strategy. Travel managers tend to believe that upgrades should only be offered between similar product types, as they feel that they may be giving consumers too much benefit by offering them the opportunity to get a product that is much better than the originally booked type. However, this common wisdom does not take into account the consumers' strategic behavior that they may deliberately book a lower-quality product than desired in anticipation of getting upgraded later. Our analysis suggests that as a response to such strategic consumer behavior, the firm should be able to extract more revenues by offering upgrades between product types that are priced not so closely, but also charging sufficiently large amounts for the upgrades. We provide the following example for the stochastic model where as the product price differential becomes smaller, offering upgrades switches from increasing the firm's revenue to decreasing the firm's revenue: $\lambda = 1$, $T = 100$, $K_H = 70$, $K_R = 50$, $p_R = 80$, $\gamma = 0.5$, v_R and v_H are jointly uniformly distributed over $\Omega = \{(v_R, v_H) : 0 \leq v_R \leq v_H \leq 200\}$. For this example, Theorem III.6 would predict that offering upgrades benefits the firm when $p_H \geq 110$ and hurts the firm when $p_H \leq 109$. From the numerical analysis for the stochastic model, we find that offering upgrades benefits the firm when $p_H \geq 111$ and hurts the firm when $p_H \leq 110$, which is very close to the result indicated by the fluid heuristic.

From our analysis above, we have seen two benefits of conditional upgrades. First, the optimal conditional upgrade strategy can lead to demand expansion. Second, offering upgrades can shift some consumers from regular products to high-quality products. We use the following example (in the stochastic model) to illustrate these two benefits of conditional upgrades: $\lambda = 1$, $T = 100$, $K_H = 70$, $K_R = 50$, $p_H = 150$, $p_R = 80$, v_R and v_H are jointly uniformly distributed over $\Omega = \{(v_R, v_H) : 0 \leq v_R \leq$

$v_H \leq 200\}$. For this example, if the firm does not offer upgrades, 26.25% of consumers book high-quality products and 29.75% of consumers book regular products. If the firm offers upgrades to half of the consumers (i.e., $\gamma = 0.5$), 13.13% of consumers book high-quality products directly, 27.41% of consumers book regular products and accept the upgrade offers, and 23.09% of consumers book regular products without upgrades. Compared to the case without upgrades where the total demand is 56%, the firm increases the total demand to 63.63% by offering upgrades to half of the consumers. Thus, the firm captures more demand overall (i.e., demand expansion effect). Moreover, offering upgrades decreases the demand for regular products from 29.75% to 23.09% and increases the demand for high-quality products from 26.25% to 40.54% (including the consumers who accept the upgrade offers). Thus, the firm shifts some consumers from regular products to high-quality products (i.e., demand segmentation reoptimization effect). We will identify more benefits of conditional upgrades in later sections.

Theorem III.7. *The optimal upgrade price and the optimal revenue are increasing in γ.*

How does the firm's revenue change with the proportion of strategic consumers it offers conditional upgrades to? Theorem III.7 states that the firm's revenue becomes higher when it offers conditional upgrades to more strategic consumers. Note that Theorem III.7 incorporates the possibility that it is optimal not to offer conditional upgrades, as the optimal upgrade price and revenue would be constant in γ in this case. For a firm that sells conditional upgrades at the optimal upgrade price, the presence of strategic consumers is actually not a bad thing. Although strategic consumers create the cannibalization effect of conditional upgrades, they also allow the firm to benefit from demand expansion and demand segmentation reoptimization. By appropriately choosing the upgrade price, the firm can compensate the revenue loss due to cannibalization by the revenue gains due to the benefits of conditional upgrades and earn a higher revenue overall. Figure 3.4 plots the firm's optimal revenue in the stochastic model as a function of the proportion of strategic consumers it offers conditional upgrades to, which is an increasing function. Therefore, given that the upgrade price is properly chosen, the firm benefits from offering conditional upgrades to as many consumers as possible even if consumers are strategic.

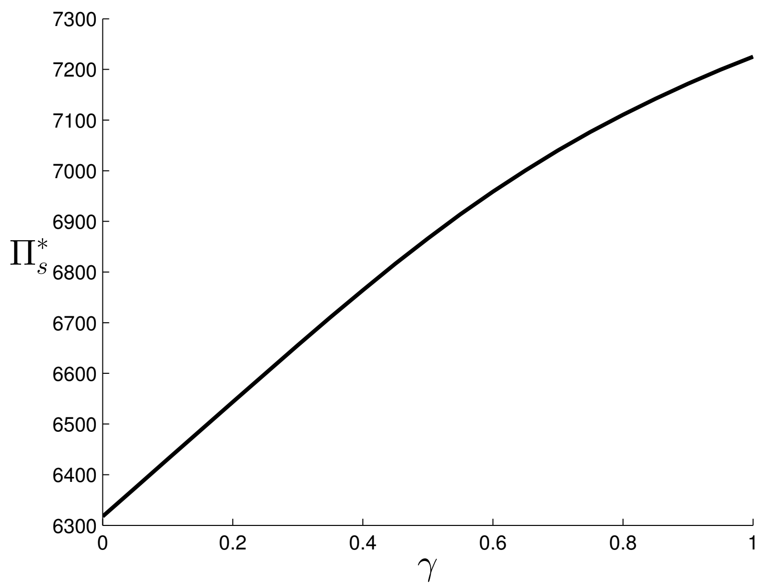


Figure 3.4: Firm's optimal revenue in the stochastic model as a function of the percentage of consumers offered upgrades. ($\lambda = 1$, $T = 100$, $K_H = 50$, $K_R = 70$, $p_H = 150$, $p_R = 80$, v_R and v_H are jointly uniformly distributed over $\Omega = \{(v_R, v_H) : 0 \leq v_R \leq v_H \leq 200\}$)

3.6.2 Free Upgrades

Next, we consider the extreme case where it is optimal for the firm to offer conditional upgrades for free. As we mentioned in the beginning, the recent trend is that firms in the travel industry are offering fewer free upgrades and introducing paid upgrades. The following theorem states that the optimal upgrade price is zero when the regular products are very expensive (i.e., $p_R \geq u/\sqrt{3}$) and the firm has such an overabundant high-quality product capacity (i.e., $K_H \geq (\lambda T/u^2)[(u - p_H + 2p_R)(u - p_H) + \gamma(p_H - p_R)(2u - p_H + p_R)]$) that it could satisfy all demand for both product types in expectation using only the high-quality product capacity when the upgrade price is zero. Clearly, this is a very restrictive condition and is not very likely to be satisfied in reality. Thus, our analysis shows that the conditional upgrades should generally be fulfilled with fees, which is consistent with the industry trend.

Theorem III.8. *$p_f^* = 0$ if and only if $p_R \geq u/\sqrt{3}$ and $K_H \geq (\lambda T/u^2)[(u - p_H + 2p_R)(u - p_H) + \gamma(p_H - p_R)(2u - p_H + p_R)]$.*

The trade-off that the firm is managing when giving free upgrades is as follows. When upgrades are free, the firm will get a number of consumers, who would not have booked any product at a higher upgrade price, to book regular products and accept upgrade offers. In the mean time, the firm will earn less revenue from consumers that would have accepted upgrade offers anyway at a higher upgrade price. As the regular product price p_R becomes higher, we can clearly see from Figure 3.2b that the number of the first type of consumers discussed above becomes larger, and the firm also earns more additional revenue from each of these consumers (at $p = 0$, the firm earns p_R from each consumer). However, the number of the second type of consumers discussed above becomes smaller. Therefore, if the regular product price is high enough (i.e., $p_R \geq u/\sqrt{3}$), the revenue improvement due to the first type of consumers will dominate the revenue loss due to the second type of consumers. Moreover, as Theorem III.5 states, the optimal upgrade price results in the upgrade probability equal to one. Thus, for $p = 0$ to be optimal, we need the high-quality product capacity to be larger than or equal to the expected demand for high-quality products and upgrades, which results in $K_H \geq (\lambda T/u^2)[(u - p_H + 2p_R)(u - p_H) + \gamma(p_H - p_R)(2u - p_H + p_R)]$. We provide the following example for the stochastic model where the optimal policy is to offer free upgrades when the regular product price p_R is high enough: $\lambda = 1$, $T = 100$, $K_H = 70$, $K_R = 50$, $p_H = 150$, $\gamma = 0.5$, v_R and v_H are jointly uniformly distributed over $\Omega = \{(v_R, v_H) : 0 \leq v_R \leq v_H \leq 200\}$. For this example, Theorem III.8 would predict that $p_f^* = 0$ when $p_R \geq 116$. We find the exact same result for the stochastic model ($p_s^* = 0$ if and only if $p_R \geq 116$).

3.6.3 Cost of Ignoring Strategic Consumer Behavior

Finally, we investigate how important it is for the firm to take strategic consumer behavior into consideration when offering conditional upgrades. We measure the importance of accounting for strategic consumer behavior by the revenue loss (in the stochastic model) if the firm mistakenly assumes consumers are myopic while they are strategic. Myopic consumers do not consider future utilities from possibly getting upgrades and make their booking decisions in a two-step way. A myopic consumer first chooses between booking a high-quality product and booking a regular product (ignoring the upgrade opportunity). In the first step, she books a high-quality product if $v_H - p_H \geq \max\{v_R - p_R, 0\}$, books a regular product if $v_R - p_R \geq \max\{v_H - p_H, 0\}$, and does not book any product otherwise. If a myopic consumer books a regular product, then upon receiving an upgrade offer, she accepts the offer if her utility from getting upgraded dominates her utility from consuming the regular product. In the second step, she accepts the upgrade offer if $v_H - p_R - p \geq v_R - p_R$, or equivalently, $v_H - v_R \geq p$. Table 3.6 gives the revenue loss results if the firm mistakenly assumes strategic consumers are myopic. As the results show, the cost of ignoring strategic consumer behavior is non-negligible and can be very significant in most cases (revenue loss exceeding 10%). Across all 16 examples given in Table 3.6, the average revenue loss is 6.79%. According to recent data from Sageworks, a financial information company, the net profit margin of U.S. hotel industry is 5% in 2013 and the five-year average margin is -1% (*Biery*, 2014). Given the low net profit margin in the hotel industry, the cost of ignoring strategic consumer behavior is significant.

	$p_H = 90$	$p_H = 100$	$p_H = 110$	$p_H = 120$
$p_R = 30$	10.80%	8.45%	6.39%	4.52%
$p_R = 40$	10.27%	8.16%	6.17%	4.40%
$p_R = 50$	9.44%	7.57%	5.55%	3.83%
$p_R = 60$	8.09%	6.96%	4.85%	3.19%

Table 3.6: Percentage revenue loss in the stochastic model if the firm prices conditional upgrades assuming consumers are myopic while consumers are strategic. ($\lambda = 1$, $T = 100$, $K_H = 70$, $K_R = 50$, $\gamma = 1$, v_R and v_H are jointly uniformly distributed over $\Omega = \{(v_R, v_H) : 0 \leq v_R \leq v_H \leq 200\}$)

3.7 Revenue Performance of Conditional Upgrades

In this section, we evaluate the conditional upgrade strategy's revenue performance. We will first consider a firm that is a price taker on product prices but can set upgrade price, as we have assumed so far. An interesting question is how much of the revenue potential does the conditional upgrade strategy capture compared to setting product prices optimally? In Section 3.7.1, we compare the conditional upgrade strategy to product price optimization. Our interesting finding is that conditional upgrades as a lever can compensate for the firm's lack of ability to optimize product prices and even generate higher revenues than product price optimization. In Section 3.7.2, we compare conditional upgrades to an alternative way of offering upgrades, in which case the firm offers last-minute upgrades at the end of the booking period and can decide the upgrade price based on demand realizations during the booking period. We find that the value of offering conditional upgrades in advance and collecting consumers' upgrading decisions in advance is greater than the value of pricing flexibility for upgrades in most cases. Moreover, we will also consider a firm that is not a price taker in the market. As dynamic pricing would be another strategy that is naturally considered by such a firm, in Section 3.7.3, we compare the revenue performance of conditional upgrades to the revenue performance of dynamic pricing. Surprisingly, offering conditional upgrades outperforms dynamic pricing.

3.7.1 Conditional Upgrades vs Product Price Optimization

For the fluid model, Corollary III.9 states that when profitable, offering conditional upgrades to all consumers (which is the optimal strategy to offer upgrades, as shown in Theorem III.7) enables the firm to capture all of the revenue potential from optimally setting the price for high-quality products. Recall that as Theorem III.5 indicates, when it is optimal to offer upgrades (i.e., when $p_f^* < p_H - p_R$), consumers choose to obtain high-quality products through upgrades, and the equilibrium outcome is equivalent to the firm selling regular products at price p_R and high-quality products at price $p_R + p_f^*$. Thus, the high-quality product price is replaced by $p_R + p_f^*$ which results in a higher revenue (note that p_f^* does not depend on p_H). In this case, $p_R + p_f^*$ is also the optimal high-quality product price for a firm that is a price taker on regular products. When it is optimal not to offer upgrades (i.e., when $p_f^* = p_H - p_R$), however, the firm may increase the revenue by increasing p_H. Thus, the upgrade price can "correct" the price for high-quality products when it is sub-optimally high. This is consistent with our finding in Section 3.6.1 that offering conditional upgrades can alter consumer segmentation and shift more consumers to high-quality products. By offering upgrades, the firm can offer an effectively lower price for the high-quality products that is somewhat disguised.

Corollary III.9. *Consider two scenarios: 1) the firm takes both product prices as given and offers conditional upgrades, 2) the firm only takes the regular product price as given and does not offer conditional upgrades. With $\gamma = 1$, when it is optimal to offer upgrades in the first scenario, these two scenarios result in the same revenue.*

Next, we explore what happens with stochastic demand. In Table 3.7, we compare the firm's revenue when it is a price taker, $\Pi_{N,s}$, to 1) the revenue when the firm offers upgrades at the optimal price (taking the product prices as given), Π_s^*, and 2) the revenue when the firm is a price taker on only regular products and can set the high-quality product price optimally, $\Pi_{N,s}(p^*_{H,s})$. Interestingly, in all our numerical examples for the stochastic model, we see that optimal upgrade pricing results in strictly higher revenues than optimal high-quality product pricing. When demand is stochastic and no upgrades are allowed, if the realized demand exceeds product capacity for either type, the firm cannot capture this excess demand. However, with upgrades, during the booking period, the firm does not allocate the consumers who accept the upgrade offers to specific product types; after demand is fully realized, the firm then gets to allocate more of these consumers to the product type that has weaker demand. Thus, the firm is able to better match its capacity to demand and improve capacity utilization. For example, suppose the firm is a price taker selling regular products at price 90 and high-quality product at price 130. Suppose now that the firm achieves flexibility to set price optimally for high-quality products. Optimizing p_H results in only a 0.13% improvement in revenue. However, if the firm keeps p_H at 130, p_R at 90, and offers conditional upgrades, it increases revenue by 1.30%. In all of the examples in Table 3.7, the firm is able to obtain higher revenues by offering paid upgrades than by being able to optimize the high-quality product price. Thus, Table 3.7 clearly shows that the conditional upgrade strategy is a very valuable form of flexibility for the firm, and in fact may be at least as valuable as the flexibility to set price for one product type optimally.

In Table 3.8, we go one step further and compare the firm's revenue when it is a price taker, $\Pi_{N,s}$, to 1) the revenue when the firm offers upgrades at the optimal price (taking the product prices as given), Π^*_s, and 2) the revenue when the firm is not a price taker and can set *both* product prices optimally, $\Pi^*_{N,s}$. Interestingly, the flexibility of conditional upgrades in better allocating capacity to stochastic demand may even allow the firm to earn more revenue than optimizing both product prices when the regular product price that the firm is forced to offer is not too far away from optimal. For example, if the firm is forced to offer high-quality products at price 130

	$p_H = 130$		$p_H = 140$		$p_H = 150$		$p_H = 160$	
	$\Delta\Pi_s^*$	$\Delta\Pi_{N,s}(p_{H,s}^*)$	$\Delta\Pi_s^*$	$\Delta\Pi_{N,s}(p_{H,s}^*)$	$\Delta\Pi_s^*$	$\Delta\Pi_{N,s}(p_{H,s}^*)$	$\Delta\Pi_s^*$	$\Delta\Pi_{N,s}(p_{H,s}^*)$
$p_R = 60$	4.57%	4.12%	10.44%	9.96%	18.65%	18.13%	29.75%	29.19%
$p_R = 70$	2.93%	2.36%	8.36%	7.76%	16.28%	15.64%	27.27%	26.57%
$p_R = 80$	1.67%	0.98%	6.58%	5.85%	14.36%	13.59%	25.43%	24.58%
$p_R = 90$	1.30%	0.13%	4.96%	4.09%	12.67%	11.75%	23.97%	22.95%

Table 3.7: Percentage revenue improvements in the stochastic model from $\Pi_{N,s}$ (i.e., the revenue from not offering upgrades and using the given product prices) by 1) optimal upgrade pricing ($\Delta\Pi_s^* = \frac{\Pi_s^* - \Pi_{N,s}}{\Pi_{N,s}}$), and 2) optimal pricing of high-quality products ($\Delta\Pi_{N,s}(p_{H,s}^*) = \frac{\Pi_{N,s}(p_{H,s}^*) - \Pi_{N,s}}{\Pi_{N,s}}$). ($\lambda = 1$, $T = 100$, $K_H = 50$, $K_R = 70$, $\gamma = 1$, v_R and v_H are jointly uniformly distributed over $\Omega = \{(v_R, v_H) : 0 \leq v_R \leq v_H \leq 200\}$)

and regular products at price 100, optimizing p_H and p_R (the optimal product prices are $p_{H,s}^* = 129.1$ and $p_{R,s}^* = 92.7$) results in only a 0.36% improvement in revenue. However, if the firm keeps p_H at 130, p_R at 100, and offers conditional upgrades, it increases revenue by 2.92%. In Table 3.8, $\Pi_s^* > \Pi_{N,s}^*$ for at least $90 \leq p_R \leq 100$. Thus, the conditional upgrade strategy is very effective in capturing the revenue potential from being able to optimize product prices. Additionally, the benefit of conditional upgrades in matching fixed capacities to stochastic demands is more significant when the capacity-demand mismatch without upgrades is more severe. We can see this from the examples given in Table 3.8. The optimal product prices in this case are $p_{H,s}^* = 129.1$ and $p_{R,s}^* = 92.7$. As we move p_H and p_R away from optimal so that the capacity-demand mismatch becomes more severe, the revenue improvement of conditional upgrades increases.

3.7.2 Conditional Upgrades vs Last-Minute Upgrades

Now we consider another type of upgrades that the firm offers to consumers at the last minute and compare it to conditional upgrades that are offered in advance. In this case, the firm offers upgrades at the end of the booking period (e.g., at check-in), and chooses the upgrade price after demand realizations during the booking

	$p_H = 130$		$p_H = 140$		$p_H = 150$		$p_H = 160$	
	$\Delta\Pi^*_s$	$\Delta\Pi^*_{N,s}$	$\Delta\Pi^*_s$	$\Delta\Pi^*_{N,s}$	$\Delta\Pi^*_s$	$\Delta\Pi^*_{N,s}$	$\Delta\Pi^*_s$	$\Delta\Pi^*_{N,s}$
$p_R = 70$	2.93%	5.29%	8.36%	10.84%	16.28%	18.95%	27.27%	30.19%
$p_R = 80$	1.67%	1.81%	6.58%	6.72%	14.36%	14.52%	25.43%	25.61%
$p_R = 90$	1.30%	0.16%	4.96%	4.13%	12.67%	11.78%	23.97%	22.99%
$p_R = 100$	2.92%	0.36%	3.49%	2.73%	11.07%	10.25%	22.71%	21.80%

Table 3.8: Percentage revenue improvements in the stochastic model from $\Pi_{N,s}$ (i.e., the revenue from not offering upgrades and using the given product prices) by 1) optimal upgrade pricing ($\Delta\Pi^*_s = \frac{\Pi^*_s - \Pi_{N,s}}{\Pi_{N,s}}$), and 2) optimal pricing of both product types ($\Delta\Pi^*_{N,s} = \frac{\Pi^*_{N,s} - \Pi_{N,s}}{\Pi_{N,s}}$). ($\lambda = 1$, $T = 100$, $K_H = 50$, $K_R = 70$, $\gamma = 1$, v_R and v_H are jointly uniformly distributed over $\Omega = \{(v_R, v_H) : 0 \leq v_R \leq v_H \leq 200\}$)

period. During the booking period, strategic consumers choose between booking a high-quality product and booking a regular product based on the anticipated upgrade probabilities and prices. We use a similar (stochastic) model to analyze last-minute upgrades; the model and analysis are described in Section 3.10.

In Table 3.9, we compare the firm's revenue when it is a price taker, $\Pi_{N,s}$, to 1) the optimal revenue when the firm offers conditional upgrades, Π^*_s, and 2) the optimal revenue when the firm offers last-minute upgrades, $\Pi^*_{LM,s}$. As Table 3.9 shows, conditional upgrades result in higher revenues than last-minute upgrades in all cases. Across all examples given in Table 3.9, on average, conditional upgrades improve the revenue by 13.08%, whereas last-minute upgrades improve the revenue by only 2.36% (offering last-minute upgrades may even decrease the firm's revenue in some cases). Although last-minute upgrades give the firm more pricing flexibility (i.e., the firm gets to dynamically determine the upgrade price based on demand realizations during the booking period), conditional upgrades give the firm other advantages that appear to be more valuable. First, the firm has better flexibility in managing capacities with conditional upgrades. By offering upgrades in advance and letting consumers reveal their upgrading decisions in advance, the firm is able to better control the stopping time of selling each product type and improve its capacity utilizations. With last-minute upgrades, the firm loses the ability to observe consumers' upgrading decisions in advance, and hence cannot improve capacity utilizations as effectively. Second, with conditional upgrades, by committing to the upgrade price up front, the firm can induce more consumers, who would not purchase any product without upgrades being offered, to purchase from the firm. With last-minute upgrades, however, the demand expansion effect is weakened. Across all examples given in Table 3.9, on average, conditional upgrades generate 13.61% more demand than last-minute upgrades. Additionally, with conditional upgrades, the firm can overbook regular products without having to "bump" consumers during check-in, because by observing consumers' upgrading decisions in advance, the firm can overbook regular products as long as it knows that enough consumers can be moved to high-quality products. However, if upgrades are offered at check-in and the firm overbooks regular products, it has the risk of having to bump some consumers. In this case, the firm chooses the upgrade price at the end of the booking period based on its belief about the probability of consumers (who have booked regular products) accepting the upgrade offer. It may occur that not enough consumers are actually willing to pay for the upgrades at the price chosen by the firm, so the firm will incur penalty costs from bumping consumers. Note that in the examples given in Table 3.9, the penalty cost per consumer, c, is equal to zero. So, we are comparing the conditional upgrade revenue to an upper bound of the last-minute upgrade revenue.

3.7.3 Conditional Upgrades vs Dynamic Pricing

As we have seen, the flexibility of conditional upgrades in better allocating capacity to demand allows the product price-taking firm to achieve higher revenues than being able to optimize product prices and offering last-minute upgrades in many cases. Now, suppose the firm is not a price taker at all and can set both product prices optimally. In Table 3.10, we compare the firm's revenue from optimal product

	$p_H = 130$		$p_H = 140$		$p_H = 150$		$p_H = 160$	
	$\Delta\Pi^*_s$	$\Delta\Pi^*_{LM,s}$	$\Delta\Pi^*_s$	$\Delta\Pi^*_{LM,s}$	$\Delta\Pi^*_s$	$\Delta\Pi^*_{LM,s}$	$\Delta\Pi^*_s$	$\Delta\Pi^*_{LM,s}$
$p_R = 60$	4.57%	0.05%	10.44%	4.03%	18.65%	5.03%	29.75%	14.86%
$p_R = 70$	2.93%	−0.03%	8.36%	0.05%	16.28%	0.68%	27.27%	5.98%
$p_R = 80$	1.67%	0.23%	6.58%	−0.06%	14.36%	0.09%	25.43%	5.69%
$p_R = 90$	1.30%	0.83%	4.96%	0.21%	12.67%	−0.01%	23.97%	0.19%

Table 3.9: Percentage revenue improvements in the stochastic model from $\Pi_{N,s}$ (i.e., the revenue from not offering upgrades and using the given product prices) by 1) offering conditional upgrades ($\Delta\Pi^*_s = \frac{\Pi^*_s - \Pi_{N,s}}{\Pi_{N,s}}$), and 2) offering last-minute upgrades ($\Delta\Pi^*_{LM,s} = \frac{\Pi^*_{LM,s} - \Pi_{N,s}}{\Pi_{N,s}}$). ($\lambda = 1$, $T = 100$, $K_H = 50$, $K_R = 70$, $\gamma = 1$, $c = 0$, v_R and v_H are jointly uniformly distributed over $\Omega = \{(v_R, v_H) : 0 \leq v_R \leq v_H \leq 200\}$)

pricing, $\Pi^*_{N,s}$, to 1) the optimal revenue from the conditional upgrade strategy (using the optimal static product prices), Π^*_s, and 2) the optimal revenue from dynamic pricing, $\Pi^*_{D,s}$. We use the classic multiproduct dynamic pricing model in *Gallego and van Ryzin* (1997) to compute the expected revenue from optimal dynamic pricing.[10] Interestingly, we find that conditional upgrades generate more revenues than dynamic pricing in all examples in Table 3.10. The firm gains different types of flexibility from conditional upgrades and dynamic pricing. By using dynamic pricing, the firm can adjust the allocation of consumers to different product types by changing product prices during the booking period. However, the firm does not have the flexibility to change product assignments after purchase. With conditional upgrades, the firm's product assignments of consumers who have accepted upgrade offers to different product types are made after demand is fully realized. As Table 3.10 shows, the ex-post allocation flexibility created by conditional upgrades has more revenue potential than the pricing flexibility created by dynamic pricing. Therefore, for a firm that is not a price taker and has the ability to set optimal static product prices, the conditional upgrade strategy can serve as a substitute to dynamic pricing and in fact generate

[10] Note that in *Gallego and van Ryzin* (1997), consumers do not postpone their purchases due to the anticipated future price drops. Thus, the dynamic pricing revenue we are comparing the upgrade revenue to is an upper bound on dynamic pricing revenues (*Levin et al.*, 2010).

more revenues.

	$K_H = 20$		$K_H = 30$		$K_H = 40$		$K_H = 50$	
	$\Delta\Pi_s^*$	$\Delta\Pi_{D,s}^*$	$\Delta\Pi_s^*$	$\Delta\Pi_{D,s}^*$	$\Delta\Pi_s^*$	$\Delta\Pi_{D,s}^*$	$\Delta\Pi_s^*$	$\Delta\Pi_{D,s}^*$
$K_R = 50$	6.42%	5.19%	4.02%	2.43%	3.07%	2.21%	1.95%	1.75%
$K_R = 60$	6.42%	5.20%	4.02%	2.43%	3.07%	2.21%	1.95%	1.75%
$K_R = 70$	6.42%	5.20%	4.02%	2.43%	3.07%	2.21%	1.95%	1.75%
$K_R = 80$	6.42%	5.20%	4.02%	2.43%	3.07%	2.21%	1.95%	1.75%

Table 3.10: Percentage revenue improvements in the stochastic model from $\Pi_{N,s}^*$ (i.e., the revenue from optimal product pricing without upgrades) by 1) optimal upgrade pricing given the optimal product prices ($\Delta\Pi_s^* = \frac{\Pi_s^* - \Pi_{N,s}^*}{\Pi_{N,s}^*}$), and 2) optimal dynamic pricing ($\Delta\Pi_{D,s}^* = \frac{\Pi_{D,s}^* - \Pi_{N,s}^*}{\Pi_{N,s}^*}$). ($\lambda = 1$, $T = 100$, $\gamma = 1$, v_R and v_H are jointly uniformly distributed over $\Omega = \{(v_R, v_H) : 0 \leq v_R \leq v_H \leq 200\}$)

Even if the firm is a monopoly in the local market and can freely determine its product prices, implementing variable pricing (i.e., charging different prices for the same product consumed at different times) or dynamic pricing (i.e., changing the price over time for the same product consumed at the same time) may still create consumer dissatisfaction. Recall that hotels are having a hard time to convince consumers, especially corporate travel buyers, to accept dynamic pricing. While variable pricing has become more acceptable over time in travel related industries, most firms still have constraints on how much they can freely adjust prices based on demand. For example, if demand is very low on a given day, optimal pricing for that particular day may result in the hotel setting severely discounted prices for its rooms. But many hotels are reluctant to do that as they believe offering rooms below certain price levels may undercut their image and damage their brand. Compared to changing the product prices, changing the upgrade price may be a more benign strategy. The hotel would not suffer from reputational effects as consumers would usually consider upgrades as a benefit offered to them. Thus, overall we conclude that the conditional upgrade strategy is a good alternative to unconstrained variable/dynamic pricing.

3.8 Conclusion

In this essay, we study the conditional upgrade policy that has become popular especially in the travel industry. We model the consumers' strategic behavior of anticipating the upgrade probability when making booking decisions and derive the firm's optimal upgrade price incorporating the strategic consumer behavior. We find that offering conditional upgrades improves the firm's revenue if and only if the product price differential is not too small. Thus, in practice, firms should carefully decide when to offer conditional upgrades and when to sell high-quality products only directly. We also find that the firm earns more revenue by offering conditional upgrades to more strategic consumers.

We derive managerial insights about why the conditional upgrade strategy is effective in generating more revenues. First, conditional upgrades expand the firm's demand as some consumers, who wouldn't buy any of the products without the upgrade option, start purchasing when conditional upgrades are introduced. Second, the optimal upgrade pricing strategy can work as a product price correction mechanism and reoptimize the firm's demand segmentation, i.e., more consumers become willing to purchase high-quality products (including purchasing through upgrades). This is especially helpful when the firm's high-quality product demand is weak. By properly offering conditional upgrades at the optimal upgrade price, the firm can capture at least the revenue potential from optimizing the high-quality product price. Third, the conditional upgrade strategy is one novel way of risk management. The extra flexibility created by the upgrade channel allows the firm to better allocate its capacities across product types to stochastic demands and improve utilization. We have seen that the conditional upgrade strategy not only can compensate for the firm's lack of ability in setting its product prices optimally, but it can also result in even higher revenues than optimized product prices. If the firm already has the ability of setting static product prices optimally, we have observed that offering conditional upgrades can generate more revenue than using dynamic pricing. We have also seen that conditional upgrades generally outperform last-minute upgrades. Finally, we have shown that a simple fluid model can be very effective in estimating optimal upgrade prices for the underlying stochastic model even in situations with small capacities and demand rates.

3.9 Appendix III.1: Revenue Function in the Stochastic Model

In this section, we derive the stochastic revenue function. To differentiate the demand processes from the ones used in Section 3.4 where we derive the consumer booking equilibrium (which are the number of other consumers as seen by the acting consumer), we use $N_i^*(t)$, $i = H, U, R$, instead of the previous $N_i(t|q^*(\cdot))$ to denote the demand processes for the firm when the consumer booking equilibrium is $q^*(\cdot)$. Since now we are analyzing from the firm's perspective, the environmental equivalence property does not apply, hence the "+1" term in the stopping times does not exist. Again, to represent this difference, we use τ_H^*, τ_R^*, τ_T^*, $\hat{\tau}^*$, and τ^* to denote the stopping times. Moreover, denote $N_H'(t)$ as the demand process for high-quality products after regular products are fully booked ($N_H'(t)$ is a Poisson process with rate λ_H^a). Similarly, denote $N_R'(t)$ as the demand process for regular products after high-quality products are fully booked ($N_R'(t)$ is a Poisson process with rate λ_R^a). The expected revenue in the stochastic model, $\Pi_s(p)$, is as follows:

$$
\begin{aligned}
\Pi_s(p) \quad = \quad & \mathop{\mathbb{E}}_{N_H^*(t), N_U^*(t), N_R^*(t)} \\
& \Big\{ p_R \left[N_R^*(\tau^*) + N_U^*(\tau^*) \right] + p \min\left\{ N_U^*(\tau^*), K_H - N_H^*(\tau^*) \right\} + p_H N_H^*(\tau^*) \\
& + \mathbb{1}\{\tau^* = \tau_R^*\} p_H \mathop{\mathbb{E}}_{N_H'(T-\tau^*)} \Big[\min\left\{ N_H'(T-\tau^*), K_H - N_H^*(\tau^*) - N_U^*(\tau^*) \right\} \Big] \\
& + \mathbb{1}\{\tau^* = \tau_H^*\} p_R \mathop{\mathbb{E}}_{N_R'(T-\tau^*)} \Big[\min\left\{ N_R'(T-\tau^*), K_R - N_U^*(\tau^*) - N_R^*(\tau^*) \right\} \Big] \Big\}.
\end{aligned}
$$

Now we further expand the above revenue function. $\Pi_s(p)$ can be written as $\Pi_s(p) = \Pi_{s1}(p) + \Pi_{s2}(p) + \Pi_{s3}(p) + \Pi_{s4}(p)$, where

$$\begin{aligned}
\Pi_{s1}(p) &= \mathbb{P}(\tau_H^* \le \tau_T^*, \tau_H^* \le T)\Pi_s(p|\tau_H^* \le \tau_T^*, \tau_H^* \le T),\\
\Pi_{s2}(p) &= \mathbb{P}(\tau_R^* \le \tau_T^*, \tau_R^* \le T)\Pi_s(p|\tau_R^* \le \tau_T^*, \tau_R^* \le T),\\
\Pi_{s3}(p) &= \mathbb{P}(\tau_H^* > \tau_T^*, \tau_R^* > \tau_T^*, \tau_T^* \le T)\Pi_s(p|\tau_H^* > \tau_T^*, \tau_R^* > \tau_T^*, \tau_T^* \le T),\\
\Pi_{s4}(p) &= \mathbb{P}(\tau_H^* > T, \tau_R^* > T, \tau_T^* > T)\Pi_s(p|\tau_H^* > T, \tau_R^* > T, \tau_T^* > T).
\end{aligned}$$

Each part of $\Pi_s(p)$ is derived as follows:

$$\begin{aligned}
\Pi_{s1}(p) &= \int_0^T f_{\tau_H^*}(t) \sum_{i_R=0}^{K_R-1} \sum_{i_U=0}^{K_R-i_R} \mathbb{P}(N_R^*(t) = i_R)\, \mathbb{P}(N_U^*(t) = i_U)\\
&\quad \cdot \left\{ p_R(i_R + i_U) + p_H K_H + p_R \mathop{\mathbb{E}}_{N_R'(T-t)} \left[\min\{N_R'(T-t), K_R - i_R - i_U\} \right] \right\} \mathrm{d}t,
\end{aligned}$$

where $f_{\tau_H^*}(t) = \mathbb{P}(N_H^*(t) = K_H - 1)\lambda \xi_H^*(t)$.

$$\begin{aligned}
\Pi_{s2}(p) &= \int_0^T f_{\tau_R^*}(t) \sum_{i_H=0}^{K_H-1} \sum_{i_U=0}^{K_H-i_H} \mathbb{P}(N_H^*(t) = i_H)\, \mathbb{P}(N_U^*(t) = i_U)\\
&\quad \cdot \Bigg\{ p_R(K_R + i_U) + p i_U + p_H i_H\\
&\quad + p_H \mathop{\mathbb{E}}_{N_H'(T-t)} \left[\min\{N_H'(T-t), K_H - i_H - i_U\} \right] \Bigg\} \mathrm{d}t,
\end{aligned}$$

where $f_{\tau_R^*}(t) = \mathbb{P}(N_R^*(t) = K_R - 1)\lambda \xi_R^*(t)$.

$$\begin{aligned}
\Pi_{s3}(p) &= \int_0^T f_{\tau_T^*}(t) \sum_{i_H=0}^{K_H-1} \sum_{i_R=0}^{K_R-1} \binom{K_H + K_R}{i_H} \binom{K_H + K_R - i_H}{i_R}\\
&\quad \cdot \left\{ \mathop{\mathbb{E}}_{0 \le s \le t} [\xi_H^*(s)] \right\}^{i_H} \left\{ \mathop{\mathbb{E}}_{0 \le s \le t} [\xi_R^*(s)] \right\}^{i_R} \left\{ \mathop{\mathbb{E}}_{0 \le s \le t} [\xi_U^*(s)] \right\}^{K_H + K_R - i_H - i_R}\\
&\quad \cdot \Big[p_R(K_H + K_R - i_H) + p(K_H - i_H) + p_H i_H \Big] \mathrm{d}t,
\end{aligned}$$

where $f_{\tau_T^*}(t) = \mathbb{P}(N_H^*(t) + N_U^*(t) + N_R^*(t) = K_R + K_H - 1)\lambda\,[\xi_H^*(t) + \xi_U^*(t) + \xi_R^*(t)]$.

$$\Pi_{s4}(p) \;=\; \sum_{i_H=0}^{K_H-1}\sum_{i_R=0}^{K_R-1}\sum_{i_U=0}^{K_H+K_R-i_H-i_R-1} \mathbb{P}(N_H^*(T)=i_H)\,\mathbb{P}(N_R^*(T)=i_R)\,\mathbb{P}(N_U^*(T)=i_U) \cdot\Big[p_R(i_R+i_U) + p\min\{i_U, K_H - i_H\} + p_H i_H\Big].$$

3.10 Appendix III.2: Last-Minute Upgrades

In this section, we introduce the (stochastic) model when the firm offers last-minute upgrades, and derive the consumer booking equilibrium and the firm's optimal revenue. To avoid too much repetition, we keep the description of the model elements that are same as the conditional upgrade model to a minimum, and we focus on explaining notations that are new to or different from the conditional upgrade model.

The firm offers upgrades and announces the upgrade price at the end of the booking period (e.g., during check-in) instead of in advance. Consistent with the conditional upgrade model, the firm offers upgrades to γ proportion of consumers (and consumers know whether they will be offered upgrades or not). Also consistent with the conditional upgrade model, the firm can overbook regular products during the booking period. However, if there are more consumers who do not accept the upgrade offers (i.e., they choose to consume the regular products) than the remaining capacity of regular products by the end of the booking period, the firm incurs a penalty cost c per consumer from "bumping" these consumers. At the end of the booking period, the firm chooses the upgrade price $p \leq p_H - p_R$ based on its belief about the probabilities that the consumers who have booked regular products will accept the upgrade offers.

During the booking period, consumers choose which product type to book (high-quality or regular) or not to book any product. When making booking decisions, strategic consumers anticipate the optimal upgrade price that is going to be chosen by the firm at the end of the booking period as well as the corresponding upgrade probability on every sample path of consumers' arrival and booking processes. More specifically, consumers' rational expectations take into account the following: 1) the probability that upgrades will be offered at the end of the booking period (because the firm has unsold high-quality products by then), 2) the probability the consumer will be willing to accept the upgrade offer (because the upgrade price that the firm charges is low enough), 3) the probability that the consumer will get upgraded if more consumers are willing to accept the upgrade offers than the remaining capacity of high-quality products (same as in our conditional upgrade model, we assume random rationing in this case).

Let $a_t(v_R, v_H)$ denote the consumer's utility-maximizing decision if she arrives at time t, has valuations (v_R, v_H) and will be offered an upgrade. $a_t(v_R, v_H) = H$ represents booking a high-quality product, $a_t(v_R, v_H) = R$ represents booking a regular product (the consumer later may or may not accept the upgrade offer), $a_t(v_R, v_H) = N$ represents not booking any product. We now use the fixed-point approach to derive the consumer booking equilibrium. Suppose all other consumers except the acting consumer are using strategy $a_t(v_R, v_H)$. For the acting consumer, given that both product types are still available by her arrival time t, her utility from booking a high-quality product is $v_H - p_H$ which does not depend on other consumers' strategies used in the consumer booking game. Let $u_R(a_t(v_R, v_H))$ denote the acting consumer's expected utility from booking a regular product upon arrival. $u_R(a_t(v_R, v_H))$ incorporates the potential utility gained from being upgraded at the end of the booking period. Let $b(a_t(v_R, v_H))$ denote the resulting optimal strategy for the acting consumer. Then,

$$b(a_t(v_R, v_H)) = \begin{cases} H & \text{if } v_H - p_H \geq u_R(a_t(v_R, v_H)) \text{ and } v_H \geq p_H; \\ R & \text{if } v_H - p_H < u_R(a_t(v_R, v_H)) \text{ and } u_R(a_t(v_R, v_H)) \geq 0; \\ N & \text{otherwise.} \end{cases}$$

The equilibrium condition is that for every t and every (v_R, v_H), we must have $b(a_t(v_R, v_H)) = a_t(v_R, v_H)$. The strategy space has three dimensions, namely, the arrival time dimension, and the two valuation dimensions. Note that different from the conditional upgrade model, we cannot reduce the strategy space to only the arrival time dimension by equivalently defining the anticipated upgrade probability as the strategy used by consumers in the booking game, because with last-minute upgrades, consumers' probabilities to actually get upgraded also depend on their valuations. If $v_H - v_R$ is lower than the upgrade price announced at the end of the booking period, the consumer will not accept the upgrade offer, and hence the upgrade probability is zero; another consumer with $v_H - v_R$ higher than the upgrade price will have a higher upgrade probability.

Given $a_t(v_R, v_H)$, the probabilities of any other consumer that will be offered an upgrade booking each type of product are as follows:

$$\begin{aligned} \xi_H^\gamma(t|a_t(v_R, v_H)) &= \iint\limits_{\Omega} \mathbb{1}\{a_t(v_R, v_H) = H\} f(v_R, v_H) \, \mathrm{d}v_R \, \mathrm{d}v_H, \\ \xi_R^\gamma(t|a_t(v_R, v_H)) &= \iint\limits_{\Omega} \mathbb{1}\{a_t(v_R, v_H) = R\} f(v_R, v_H) \, \mathrm{d}v_R \, \mathrm{d}v_H. \end{aligned}$$

The probabilities of any other consumer that will not be offered an upgrade booking each type of product are as follows:

$$\begin{aligned}
\xi'_H(t) &= \iint\limits_{\Omega} \mathbb{1}\{a'_t(v_R, v_H) = H\} f(v_R, v_H)\, \mathrm{d}v_R\, \mathrm{d}v_H, \\
\xi'_R(t) &= \iint\limits_{\Omega} \mathbb{1}\{a'_t(v_R, v_H) = R\} f(v_R, v_H)\, \mathrm{d}v_R\, \mathrm{d}v_H,
\end{aligned}$$

where $a'_t(v_R, v_H)$ denotes the utility-maximizing decision of a consumer that will not be offered an upgrade:

$$a'_t(v_R, v_H) = \begin{cases} H & \text{if } v_H - v_R \geq p_H - p_R \text{ and } v_H \geq p_H, \\ R & \text{if } v_H - v_R < p_H - p_R \text{ and } v_R \geq p_R, \\ N & \text{otherwise.} \end{cases}$$

The arrival processes of other consumers, $N_H(t|a_t(v_R, v_H))$ and $N_R(t|a_t(v_R, v_H))$, are Poisson processes with rates $\lambda_H(t|a_t(v_R, v_H)) = \lambda\gamma\xi^{\gamma}_H(t|a_t(v_R, v_H)) + \lambda(1-\gamma)\xi'_H(t)$ and $\lambda_R(t|a_t(v_R, v_H)) = \lambda\gamma\xi^{\gamma}_R(t|a_t(v_R, v_H)) + \lambda(1-\gamma)\xi'_R(t)$, respectively. The stopping time of the booking game is $\tau(a_t(v_R, v_H)) = \min\{\hat{\tau}(a_t(v_R, v_H)), T\}$, where $\hat{\tau}(a_t(v_R, v_H)) = \min\{\tau_H(a_t(v_R, v_H)), \tau_T(a_t(v_R, v_H))\}$, and $\tau_H(a_t(v_R, v_H)) = \inf\{t \geq 0 : N_H(t|a_t(v_R, v_H)) \geq K_H\}$, $\tau_T(a_t(v_R, v_H)) = \inf\{t \geq 0 : N_H(t|a_t(v_R, v_H)) + N_R(t|a_t(v_R, v_H)) + 1 \geq K_H + K_R\}$.

$u_R(a_t(v_R, v_H))$ is derived as follows:

$$
\begin{aligned}
& u_R(a_t(v_R, v_H)) \\
= \; & \mathop{\mathbb{E}}_{N_H(t|a_t(v_R,v_H)),N_R(t|a_t(v_R,v_H))|N_H(t|a_t(v_R,v_H))<K_H,N_H(t|a_t(v_R,v_H))+N_R(t|a_t(v_R,v_H))<K_H+K_R} \\
& \Bigg\{ \mathbb{1}\left\{t \le \tau(a_t(v_R, v_H))\right\} \cdot \Bigg\{ \mathbb{1}\left\{\tau(a_t(v_R, v_H)) = \tau_H(a_t(v_R, v_H))\right\} \cdot (v_R - p_R) + \\
& \mathbb{1}\left\{\tau(a_t(v_R, v_H)) \neq \tau_H(a_t(v_R, v_H))\right\} \\
& \cdot \Big[(1 - q(N_H(t|a_t(v_R, v_H)), N_R(t|a_t(v_R, v_H)) + 1)) \cdot (v_R - p_R) \\
& + q(N_H(t|a_t(v_R, v_H)), N_R(t|a_t(v_R, v_H)) + 1) \\
& \cdot (v_H - v_R - p^*(N_H(t|a_t(v_R, v_H)), N_R(t|a_t(v_R, v_H)) + 1)) \Big] \Bigg\} \Bigg\}.
\end{aligned}
$$

$q(N_H(t|a_t(v_R, v_H)), N_R(t|a_t(v_R, v_H)) + 1)$ is the probability that the acting consumer accepts the upgrade offer and gets upgraded on any sample path, $p^*(N_H(t|a_t(v_R, v_H)),$ $N_R(t|a_t(v_R, v_H))+1)$ is the optimal upgrade price chosen by the firm at the end of the booking period based on demand realizations on any sample path. The "+1" term represents the acting consumer. Note that consistent with the conditional upgrade model, in the above derivation, the expectation taken over each sample path is conditional expectation (i.e., conditional on that by the acting consumer's arrival time, both product types are still available).

Next, we derive $q(N_H(t|a_t(v_R, v_H)), N_R(t|a_t(v_R, v_H))+1)$ and $p^*(N_H(t|a_t(v_R, v_H)),$ $N_R(t|a_t(v_R, v_H)) + 1)$. We have

$$
\begin{aligned}
& q(N_H(t|a_t(v_R, v_H)), N_R(t|a_t(v_R, v_H)) + 1) = \\
& \mathbb{1}\left\{v_H - v_R \ge p^*(N_H(t|a_t(v_R, v_H)), N_R(t|a_t(v_R, v_H)) + 1)\right\} \\
& \cdot \sum_{i=0}^{N_R(\tau(a_t(v_R,v_H))|a_t(v_R,v_H))} \mathbb{P}(i \text{ other consumers accept upgrades}) \\
& \qquad \cdot \min\left\{ \frac{\left[K_H - N_H(\tau(a_t(v_R, v_H))|a_t(v_R, v_H))\right]^+}{i+1}, 1 \right\}.
\end{aligned}
$$

We still need to derive $\mathbb{P}(i$ other consumers accept upgrades). Let $\eta_t(p|a_t(v_R, v_H))$ denote the probability that a consumer who arrives at time t and books a regular product will accept the upgrade offer with upgrade price p. We have

$$\eta_t(p|a_t(v_R, v_H)) = \frac{\iint\limits_{\Omega} \mathbb{1}\{a_t(v_R, v_H) = R\}\mathbb{1}\{v_H - v_R \geq p\} f(v_R, v_H)\,\mathrm{d}v_R\,\mathrm{d}v_H}{\xi_R^{\gamma}(t|a_t(v_R, v_H))}.$$

Further, define $\eta(p|a_t(v_R, v_H)) = \mathop{\mathbb{E}}\limits_{t} \eta_t(p|a_t(v_R, v_H))$. We assume the acting consumer anticipate the other consumers' acceptance of the upgrade offers as a binomial distribution with probability $\eta(p|a_t(v_R, v_H))$.[11] Thus,

$$\begin{aligned}
&\mathbb{P}(i \text{ other consumers accept upgrades}) = \\
&\sum_{j=i}^{N_R(\tau(a_t(v_R,v_H))|a_t(v_R,v_H))} \binom{N_R(\tau(a_t(v_R, v_H))|a_t(v_R, v_H))}{j} \gamma^j (1-\gamma)^{N_R(\tau(a_t(v_R,v_H))|a_t(v_R,v_H))-j} \\
&\qquad \cdot \binom{j}{i} [\eta(p|a_t(v_R, v_H))]^i [1 - \eta(p|a_t(v_R, v_H))]^{j-i}.
\end{aligned}$$

$p^*(N_H(t|a_t(v_R, v_H)), N_R(t|a_t(v_R, v_H)) + 1)$ is the maximizer of the net revenue earned at check-in, which is the difference between the revenue from collecting upgrade fees and the cost from bumping consumers due to insufficient regular product capacity. When $\tau(a_t(v_R, v_H)) \neq \tau_H(a_t(v_R, v_H))$, let $\Pi_T(p|N_H(t|a_t(v_R, v_H)), N_R(t|a_t(v_R, v_H))+1)$ denote the firm's expected net revenue from selling upgrades at check-in on any sample

[11] An alternative way is to allow the acting consumer to form a heterogeneous binomial belief about the acceptance of the upgrade offer from each of the other consumers. For any other consumer with arrival time $t_j (j = 1, 2, ..., N_R(\tau(a_t(v_R, v_H))|a_t(v_R, v_H)))$, the probability of accepting the upgrade offer is $\eta_{t_j}(p^*(N_H(t|a_t(v_R, v_H)), N_R(t|a_t(v_R, v_H)) + 1)|a_t(v_R, v_H))$, where $t_1, t_2, ..., t_{N_R(\tau(a_t(v_R,v_H))|a_t(v_R,v_H))}$ denote the arrival times of other consumers who have booked regular products on any sample path. By using this approach, the computational burden of $\mathbb{P}(i$ other consumers accept upgrades) is significantly larger. The approach we take can be considered as an approximation by assuming that consumers have limited computational capability in the booking game. If the problem size is large enough, the equilibrium booking strategy of consumers becomes time-independent, in which case our approach produces the same result as this alternative approach (the examples we give in this essay have large enough problem sizes so that this occurs).

path. We have

$$
\begin{aligned}
& \Pi_T(p|N_H(t|a_t(v_R,v_H)),N_R(t|a_t(v_R,v_H))+1) \\
= & \sum_{i=0}^{N_R(t|a_t(v_R,v_H))+1} \mathbb{P}(i \text{ consumers accept upgrades}) \\
& \qquad \cdot \Big\{ p\cdot\min\left\{i,[K_H-N_H(\tau(a_t(v_R,v_H))|a_t(v_R,v_H))]^+\right\} \\
& \qquad -c\cdot\{N_R(\tau(a_t(v_R,v_H))|a_t(v_R,v_H))+1 \\
& \qquad -\min\left\{i,[K_H-N_H(\tau(a_t(v_R,v_H))|a_t(v_R,v_H))]^+\right\}-K_R\}^+ \Big\}.
\end{aligned}
$$

$\mathbb{P}(i$ other consumers accept upgrades) is calculated using the same approach when we derive $q(N_H(t|a_t(v_R,v_H)),N_R(t|a_t(v_R,v_H))+1)$. Note that when $\tau(a_t(v_R,v_H)) = \tau_H(a_t(v_R,v_H))$ (so $N_H(\tau(a_t(v_R,v_H))|a_t(v_R,v_H)) = K_H$), $p^*(N_H(t|a_t(v_R,v_H)),N_R(t|a_t(v_R,v_H))+1)$ and $\Pi_T(p|N_H(t|a_t(v_R,v_H)),N_R(t|a_t(v_R,v_H))+1)$ are irrelevant, because the firm does not earn any revenue from upgrades (also, the firm does not incur penalty cost, because $\tau(a_t(v_R,v_H)) = \tau_H(a_t(v_R,v_H))$ implies that the firm does not overbook regular products). Moreover, in this case, we naturally have $q(N_H(t|a_t(v_R,v_H)),N_R(t|a_t(v_R,v_H))+1) = 0$.

We have characterized the consumer book equilibrium. Then, we can calculate the firm's optimal expected revenue, $\Pi^*_{LM,s}$, as follows:

$$
\begin{aligned}
\Pi^*_{LM,s} = & \mathop{\mathbb{E}}_{N^*_H(t),N^*_R(t)} \Big\{ p_R N^*_R(\tau^*) + p_H N^*_H(\tau^*) \\
& +\mathbb{1}\{\tau^* \neq \tau^*_H\}\cdot \Pi_T(p^*(N^*_H(t),N^*_R(t))|N^*_H(t),N^*_R(t)) \\
& +\mathbb{1}\{\tau^* = \tau^*_H\}\cdot R \mathop{\mathbb{E}}_{N'_R(T-\tau^*)} \left[\min\left\{N'_R(T-\tau^*),K_R-N^*_R(\tau^*)\right\}\right] \Big\}.
\end{aligned}
$$

The revenue function can be expanded conditional on τ^* in the same way as the revenue function from conditional upgrades in Section 3.9.

CHAPTER IV

Strategic Pricing of Ancillary Services: To Bundle or to Unbundle?

4.1 Introduction

Many firms provide an ancillary service in addition to a main service to enhance the experience of consumers. When taking a flight, consumers may need to transport bags at the same time or have a meal during the flight. When staying at a hotel, consumers may need to have breakfast or use internet connection. In many service industries, such as travel industries, consumers book the main service in advance, at which time they may be uncertain about whether they are going to actually need the ancillary service from the firm or not and about their exact valuations for it. As the travel date approaches, consumers resolve the uncertainty. Different consumers can have different valuations for the ancillary service, some of them may end up not wanting the ancillary service at all.

Historically, most airline ancillary services were bundled with the main service, and the whole service was sold at a single price. In the last ten years, there has been a trend in the airline industry of unbundling several ancillary services from the main service and charging separate fees for them. Take the baggage service for example. After American Airlines started charging for checked bags in 2008, many other airlines have followed rapidly (*Maynard*, 2008). According to the Bureau of Transportation Statistics,[1] the 15 largest airlines in the U.S. combined collected \$3.35 billion in baggage fees in 2013. According to IdeaWorksCompany research,[2] the global airline non-ticket revenue in 2013 reached \$31.5 billion, which consisted of a la carte charges (e.g., baggage, food, seat preference), commissions on travel-oriented services (e.g., hotel or car rental bookings), and the sale of frequent flier points, etc. This revenue has increased 12-fold since 2007 (\$2.45 billion). In 2013, the global average airline ancillary service revenue per passenger was \$16.

However, even for the same type of ancillary services, different firms make different decisions regarding bundling or unbundling. For example, while many airlines have unbundled the checked baggage service, Southwest Airlines is still offering the first two checked bags for free, which means that the baggage fee is built into the ticket price. Also, unbundling certain ancillary services is more common compared to other ancillary services. For example, compared to checked bags, airlines are much more conservative in unbundling carry-on bags. The main questions we are going to study in this essay are whether a firm should unbundle the ancillary service, and what types of ancillary services a firm should unbundle. On the one hand, by unbundling the ancillary service, the firm gains pricing flexibility from being able to charge a separate price for the ancillary service and extracts more consumer surplus. On the other hand, the firm incurs inconvenience costs by unbundling the ancillary service, which may include the additional labor cost to process the ancillary service payments, the cost of congestion (e.g., if an airline has to process the payments for carry-on bags at the gate, its flights could be easily delayed, resulting in an undesirable on-time performance record), and the potential profit loss because of consumers' loss of goodwill from having to pay for the ancillary service. Thus, for unbundling to be the

[1] http://www.rita.dot.gov/bts/sites/rita.dot.gov.bts/files/subject_areas/airline_information/baggage_fees/html/2013.html

[2] http://www.ideaworkscompany.com/wp-content/uploads/2014/07/Press-Release-89-Ancillary-Revenue-Top-10.pdf

optimal strategy for the firm, the inconvenience cost that is incurred should not be significant. We answer the question of whether and when the firm should unbundle the ancillary service by analyzing models that capture the above fundamental trade-off.

In this essay, we consider a firm that sells a main service and an ancillary service to two types of consumers (e.g., business travelers and leisure travelers) with different valuations for the main service and different likelihoods of needing the ancillary service. We first study the optimal ancillary service strategy for a firm that can price-discriminate when selling the main service, that is, the firm can charge different main service prices to different consumer types. Main service price discrimination is common in the airline industry. It is common airline practice to charge business travelers, who usually book tickets closer to the travel date and have a higher willingness to pay, a higher price than leisure travelers, who usually book tickets well in advance and have a lower willingness to pay. We are going to examine the effects of several important factors on the firm's optimal ancillary service strategy. First, in Section 4.4, we study how the optimal strategy is determined by the firm's operating costs, demand portfolio (i.e., proportions of different types of consumers), and the consumer valuation structure (i.e., which consumer type is more likely to purchase the ancillary service). Second, due to the temporal separation between consumers' purchases of the main service and their purchases of the ancillary service, whether consumers make forward-looking decisions (i.e., accounting for future utilities from the ancillary service when purchasing the service bundle or main service in advance) or not affects the firm's decision of whether to bundle or unbundle the ancillary service. In Section 4.5, we study the effect of having myopic consumers who do not consider potential ancillary service utilities when making the main service purchasing decision. Third, we are interested in how the firm's bundling (unbundling) decision is affected by its supply chain structure. In particular, in Section 4.6, we study how using intermediaries such as online travel agencies affects the firm's optimal strategy for the ancillary service.

We find that it is optimal for a price-discriminating firm to unbundle the ancillary service if consumers' likelihoods of purchasing the ancillary service are low enough, and it is optimal to bundle if consumers' likelihoods of purchasing the ancillary service is high enough. The firm is more likely to benefit from unbundling an ancillary service with a higher marginal cost or with a lower inconvenience cost. Moreover, if the consumers that value the main service higher (i.e., high-type consumers) are more likely to purchase the ancillary service (which indicates that consumers' valuations for the main service and the ancillary service are positively correlated), a firm is more likely to bundle the ancillary service as its proportion of high-type consumers increases. If low-type consumers (i.e., consumers that do not value the main service highly) are more likely to purchase the ancillary service (which indicates that consumers' valuations for the main service and the ancillary service are negatively correlated), a firm is more likely to unbundle the ancillary service as its proportion of high-type consumers increases.

After the firm unbundles the ancillary service, the optimal main service price is lower than the optimal bundle price, but the total price to purchase both the main service and the ancillary service is higher than the optimal bundle price. The price reduction for purchasing the main service (i.e., the difference between the optimal bundle price when the firm bundles and the optimal main service price when the firm unbundles) is more significant for the consumer type with a higher likelihood of needing the ancillary service. For airlines, because leisure travelers are more likely to have bags to check compared to business travelers, the fare reduction after airlines start charging for checked bags should be more significant for leisure travelers. This phenomenon in the airline industry has been empirically observed by researchers (*Brueckner et al.*, 2014).

Moreover, we find that the existence of myopic consumers and the firm's use of intermediaries both make unbundling more profitable compared to bundling. It is optimal to unbundle the ancillary service if myopic consumers account for a large enough proportion of all consumers. Because consumers are more likely to ignore future purchases of small-item ancillary services (e.g., purchasing a can of coke during the flight) when purchasing the service bundle or main service in advance (i.e., consumers are more likely to be myopic), firms usually charge for small-item ancillary services. However, we show that regardless of whether the firm bundles or unbundles the ancillary service, its total profit increases with the proportion of forward-looking consumers. Unbundling the ancillary service also allows the firm to gain more profits back from the intermediaries. When the firm sells through intermediaries, unbundling becomes more profitable relative to bundling as the intermediaries' commission increases or the intermediaries' market share increases.

Although main service price discrimination is common in the airline industry, it is much less common in other travel industries. For example, many hotels, especially economy hotels, do not charge different room rates to different consumers. In Section 4.7, we study the optimal ancillary service strategy for firms that do not price-discriminate when selling the main service. In this case, the firm charges a uniform main service price to both consumer types. By comparing the results for a uniform-pricing firm to the results for a price-discriminating firm, we develop insights about how the firm's ability to price-discriminate when selling the main service affects its optimal ancillary service strategy.

We find a very interesting relationship between a firm's optimal ancillary service strategy and its ability to price-discriminate when selling the main service. For a uniform-pricing firm, it is optimal to unbundle the ancillary service if the consumers that value the main service higher have a high enough likelihood of purchasing the ancillary service. However, for a price-discriminating firm, it is optimal to unbundle the ancillary service if the consumers that value the main service higher have a low enough likelihood of purchasing the ancillary service. Compared to a uniform-pricing firm, bundling (unbundling) is more likely to be the optimal strategy for a price-discriminating firm if consumers' valuations for the main service and the ancillary service are positively (negatively) correlated. Moreover, if firms use uniform pricing for the main service, the way that the consumer valuation structure affects the differentiation of optimal ancillary service strategies across firms is reversed from the case of discriminatory pricing. In the case of uniform pricing, if consumers' valuations for the main service and the ancillary service are positively (negatively) correlated, unbundling (bundling) is more likely to be the optimal ancillary service strategy for firms with higher proportions of high-type consumers. This is exactly the opposite of the result for a price-discriminating firm.

4.2 Literature Review

Although there are not many papers that study ancillary pricing (also called add-on pricing in some papers), researchers have used both competition models and monopolistic models to address related issues. *Ellison* (2005), *Gabaix and Laibson* (2006), and *Shulman and Geng* (2013) study the competition between firms that sell the ancillary service with separate charges. Papers that study ancillary service pricing under monopolistic settings include *Allon et al.* (2011) and *Fruchter et al.* (2011). *Allon et al.* (2011) study airlines' baggage pricing problem and find that the firm should set the fee for the baggage service at the same level the social planner would. Their result also suggests that the way in which airlines have implemented baggage fees is more consistent with attempts to control consumers behavior (i.e., reduce baggage needs) than segmenting consumers based on their need to check a bag. *Fruchter et al.* (2011) consider a firm that charges the same price to different consumer segments and find that a free add-on (i.e., bundling the ancillary service) is more profitable than offering it for a fee (i.e., unbundling the ancillary service) if one consumer segment has a high valuation for the add-on but a relatively low valuation for the primary service, and another segment has a higher valuation for the primary service but places no value on the add-on. This essay is one of the first to study the question of whether the firm should unbundle the ancillary service in the first place, and this essay is the first to study this question for both a uniform-pricing firm and a price-discriminating firm.

A related stream of literature studies commodity bundling. By analyzing a bundling setting with two commodities, *Adams and Yellen* (1976), *McAfee et al.* (1989), and *Schmalensee* (1984) provide the insight that a higher degree of negative correlation between consumers' valuations for the two commodities makes bundling more profitable relative to unbundled sales. We find consistent results for a uniform-pricing firm. However, allowing for main service price discrimination fundamentally changes the previous finding from the bundling literature. We find that whereas it is optimal for a uniform-pricing firm to unbundle the ancillary service if the consumers that value the main service higher have a high enough likelihood of purchasing the ancillary service (which indicates a positive correlation between consumers' valuations for the main service and their valuations for the ancillary service), it is optimal for a price-discriminating firm to unbundle the ancillary service if the consumers that value the main service higher have a low enough likelihood of purchasing the ancillary service. Main service price discrimination makes unbundling less (more) likely to be the optimal ancillary service strategy if consumers' valuations for the main service and the ancillary service are positively (negatively) correlated. Thus, the correlation effect found by previous bundling literature becomes very different with firm's ability to charge discriminatory prices for the main service. Recently, researcher have explored more topics regarding bundling, such as bundling with channel interaction (e.g., *Bhargava*, 2012, *Chakravarty et al.*, 2013, *Girju et al.*, 2013, *Cao et al.*, 2015), bundling information goods (e.g., *Bakos and Brynjolfsson*, 1999, *Geng et al.*, 2005), bundling vertically differentiated products (e.g., *Banciu et al.*, 2010, *Honhon and Pan*, 2014), and the effect of bundling on firm's ordering decision (e.g., *Cao et al.*, 2014). Although our focus in studying ancillary pricing appears at first sight to have similarities to issues studied in commodity bundling, there are significant differences between the two. In the setting studied by the traditional commodity bundling literature, each commodity can be sold separately (e.g., a retailer that sells toothbrush-toothpaste bundles can sell the two products separately). In the ancillary pricing setting, the ancillary service cannot be sold by itself. Consumers can purchase the ancillary service only if they have already purchased the main service, and the purchase of the ancillary service often occurs later than the purchase of the main service.

There is also a related stream of literature on two-part pricing. Two-part pricing corresponds to the situation where the price of a service is composed of two parts – a lump-sum fee for the fixed part of the service (e.g., cover charge of a bar), and a per-unit charge for the variable part of the service (e.g., per-drink fee). Pioneered by *Oi* (1971) and *Schmalensee* (1981), the most important issue that the two-part pricing literature has focused on is when the optimal per-unit price should be above or below the marginal cost of providing the service. *Rosen and Rosenfield* (1997) find that whether the optimal per-unit price is above or below its marginal cost depends on whether the average consumer has higher or lower demand for the variable part of the service than the marginal consumer. If the average consumer has higher demand for the variable part of the service than the marginal consumer, the firm should set the per-unit price above the marginal cost; and vice versa. A more recent paper, *Png and Wang* (2010), finds that the result also depends on the correlation between marginal and total benefits from the service. The per-unit price should be set above the marginal cost if marginal and total benefits from the service are positively correlated; and vice versa. In the ancillary service pricing setting, we find that for a firm that does not price discriminate by charging different prices to different consumer types, the result depends on the underlying consumer valuation structure (i.e., the correlation between consumers' valuations for the main service and the ancillary service) in a way that appears to be consistent with the two-part pricing results. Moreover, we also find that the result becomes very different for a firm that price-discriminates when selling the main service. In this case, if consumers are forward-looking (i.e., they take future utilities from the ancillary service into consideration when purchasing the service bundle or main service in advance) as is the case studied by previous two-part pricing literature, then the optimal ancillary service price is always equal to the marginal cost. However, if there exists a significant proportion of myopic consumers (who do not take future utilities into consideration), the optimal ancillary service price is higher than the marginal cost.

Thus, as our literature review indicates, a key differentiator of this paper is that we study a price-discriminating firm's optimal bundling (unbundling) and pricing decisions for the ancillary service and how the results are changed compared to a uniform-pricing firm. Interestingly, we find that some key findings from the previous bundling and two-part pricing literature are reversed when one considers a price-discriminating firm instead of a uniform-pricing firm.

4.3 Model

The firm sells a main service and an ancillary service to two types of consumers that have different valuations for the service. There are λ_H consumers that value the main service at v_H and λ_L consumers that value the main service at v_L, where $v_H > v_L$. In travel industries, the λ_H consumers can be considered as business travelers and the λ_L consumers can be considered as leisure travelers. Throughout this chapter, we refer to consumers with main service valuation v_H as high-type consumers, and consumers with main service valuation v_L as low-type consumers. Consumers have random valuations for the ancillary service. Let u_H and u_L denote the (random) valuations for the ancillary service of high-type and low-type consumers, respectively.[3] The ancillary service valuations u_H and u_L have support $[\underline{u}, \bar{u}]$, where $\bar{u} > 0$ and $\underline{u} < 0$. We assume $\bar{u} \leq v_L$ (i.e., consumers' valuations for the ancillary service cannot exceed their valuations for the main service) and $v_L + \bar{u} \leq v_H$ (i.e., any low-type consumer's valuation for the whole service does not exceed any high-type consumer's valuation for the whole service). Note that we allow consumers' valuations for the ancillary service to be negative. A negative valuation for the ancillary service means that the consumer will not use the ancillary service even if it is offered for free. For example, some consumers do not have bags to check for the flight. Even if the firm does not charge for checked bags, these consumers still will not use the ancillary service. The cumulative distribution functions of u_H and u_L are denoted by $F_H(\cdot)$ and $F_L(\cdot)$, and the probability density functions are denoted by $f_H(\cdot)$ and $f_L(\cdot)$. For expositional simplicity, we will assume that u_H and u_L are both uniformly distributed over $[0, \bar{u}]$ but have different probability densities (we do not assume a specific functional form for the density over $[\underline{u}, 0)$). For $i = H, L$, the probability density function of u_i is given by $f_i(x) = \beta_i/\bar{u}$ for $0 \leq x \leq \bar{u}$. Furthermore, define $\beta_i = \bar{F}_i(0)$ for $i = H, L$. β_i measures type-i consumers' likelihood of purchasing the ancillary service. If $\beta_H \geq \beta_L$, high-type consumers are more likely to purchase the ancillary service than low-type consumers for any price that the firm charges for the ancillary service. Thus, in this case, consumers' valuations for the main service and their valuations for the ancillary service exhibit a positive correlation. If $\beta_H < \beta_L$, low-type consumers are more likely to purchase the ancillary service than high-type consumers for any price that the firm charges for the ancillary service. Thus, in this

[3] In most two-part pricing settings, the fixed part of the service is a permission for entry (e.g., entry to park as in *Oi*, 1971, entry to health club and bar as in *Hayes*, 1987). Thus, the models used by these two-part pricing papers usually assume that consumers have a single valuation for the whole service (or a budget to consume the fixed part as well as the variable part of the service). Under the ancillary service pricing setting, since the main service usually generates the primary utility for consumers, most previous papers allow consumers to have separate valuations for the main service and the ancillary service. Similarly, we also allow consumers to have two valuations, one for the main service and one for the ancillary service.

case, consumers' valuations for the main service and their valuations for the ancillary service exhibit a negative correlation. Therefore, the relationship between β_H and β_L defines the consumer valuation structure. As we will see, this relationship is an important factor in determining the firm's optimal strategy for the ancillary service.

Consumers make the purchasing decision in two stages. In the first stage, consumers decide whether to purchase the service bundle (in the bundling case), or whether to purchase the main service (in the unbundling case) before their valuations for the ancillary service is realized. Then, after their valuations for the ancillary service are realized, consumers decide whether to use the ancillary service (in the bundling case), or whether to purchase the ancillary service (in the bundling case). We first assume consumers are forward-looking, that is, when making the purchasing decision for the service bundle or main service in advance, they take future utilities from the ancillary service into consideration. In Section 4.5, we incorporate myopic consumers (who do not consider future utility from the ancillary service when making the purchasing decision for the service bundle or main service in advance) as well and study the effect of myopic consumers on the firm's optimal strategy for the ancillary service.

The firm's key decision is whether to sell the whole service as a bundle, or to unbundle the ancillary service from the main service and sell the two services separately. In the basic model, we assume that the firm can price-discriminate and charge different prices for the service bundle and main service to different types of consumers. For example, in the airline industry, leisure travelers usually plan their trip in advance and business travelers usually make reservations close to the travel date. Because of this demand characteristic, airlines have implemented price discrimination by changing prices over time (i.e., inter-temporal price discrimination). However, firms usually charge the same price for the ancillary service to consumers paying different prices for the main service. For example, if you buy a coach ticket, the price of a meal (most airlines in the U.S. charge for a meal in coach on domestic flights) does not depend on how much you paid for the ticket. Thus, consistent with industry practice, we assume that the firm charges a uniform ancillary service price to both types of consumers when the ancillary service is unbundled. In the bundling case, the firm charges price p_{bH} to high-type consumers and p_{bL} to low-type consumers for the service bundle. In the unbundling case, the firm charge prices p_{mH} and p_{mL} to two types of consumers for the main service and p_a for the ancillary service. We require $p_a > 0$ in the unbundling case because if $p_a = 0$, the unbundling case degenerates to the bundling case. In Section 4.7, we consider the case where the firm charges a uniform price for the main service and study how main service price discrimination affects the optimal strategy for the ancillary service.

The marginal cost of providing one unit of main service is c_m $(0 < c_m < v_L)$. The marginal cost of providing one unit of ancillary service is c_a $(0 < c_a < \bar{u})$. Moreover, the firm incurs an inconvenience cost $c(\cdot)$ due to consumers' separate purchases of the ancillary service when the firm unbundles it. Note that the marginal cost and the inconvenience cost are two different types of costs the firm incurs with the ancillary service. The marginal cost is incurred whenever a consumer uses the ancillary service, no matter whether the ancillary service is bundled or unbundled. For example, the marginal cost of airline baggage service would include the fuel cost and labor cost (e.g., loading and unloading the bag). On the other hand, the inconvenience cost is incurred because the ancillary service is purchased separately. If the ancillary service is unbundled, the inconvenience costs may include the additional labor cost to process the ancillary service payments and the cost of congestion. For example, passengers paying for carry-on bags at the gate can delay the boarding process and affect airlines' on-time performances. Finally, the inconvenience cost may include firm's potential profit loss because of consumers' loss of goodwill that is caused by unbundling. For example, by studying consumer perception at a travel resort, *Naylor* *and Frank* (2001) find that not receiving an all-inclusive package lessens perceptions of value for first-time guests. We define the inconvenience cost $c(\cdot)$ as a function of the number of consumers who purchase the ancillary service in the unbundling case. We assume $c(0) = c'(0) = 0$, $c'(\cdot) \geq 0$ and $c''(\cdot) \geq 0$. In practice, it would be difficult to significantly reduce the marginal cost, but it may be possible to significantly reduce the inconvenience cost (e.g., by using mechanisms that induce consumers to pay for the ancillary service in advance). The firm's goal is to choose the optimal bundling (unbundling) strategy and price the main service and the ancillary service optimally so that the total profit from selling the whole service is maximized.

4.4 Optimal Pricing Strategy

In this section, we derive and analyze the firm's optimal ancillary service pricing strategy. We analyze the bundling case and the unbundling case separately, and then compare these two cases to obtain the optimal strategy. First, consider the bundling case. For each consumer type $i = H, L$, given that a consumer purchases the service bundle, she uses the ancillary service if $u_i \geq 0$ after u_i is realized. Thus, type-i consumers' expected utility from purchasing the service bundle is $v_i + E(u_i)^+ - p_{bi}$. Therefore, the firm's optimal bundle prices are $p^*_{bH} = v_H + E(u_H)^+$ and $p^*_{bL} = v_L + E(u_L)^+$. Moreover, the firm incurs marginal costs for the ancillary service used by consumers who have non-negative valuations for the ancillary service. The firm does not incur inconvenience cost in the bundling case. Thus, the optimal profit in the bundling case is

$$\Pi^*_b = [v_H + E(u_H)^+ - c_m]\lambda_H + [v_L + E(u_L)^+ - c_m]\lambda_L - c_a[\lambda_H \bar{F}_H(0) + \lambda_L \bar{F}_L(0)].$$

Note that if $v_i + E(u_i)^+ - c_m - c_a\bar{F}_i(0) < 0$ for type-i consumers, the firm should not sell to this consumer type. We assume $v_L + E(u_L)^+ - c_m - c_a\bar{F}_L(0) \geq 0$, that is, the firm earns profits by selling to low-type consumers. Since this condition implies $v_H + E(u_H)^+ - c_m - c_a\bar{F}_H(0) > 0$, the firm also earns profits by selling to high-type consumers. Allowing the possibility that the firm may want to only sell to some consumer type does not result in different insights regarding the firm's optimal ancillary service strategy.

Second, consider the unbundling case. For each consumer type $i = H, L$, given that a consumer purchases the main service, she purchases the ancillary service if $u_i \geq p_a$ after u_i is realized. Thus, type-i consumers' expected utility from purchasing the main service is $v_i - p_{mi} + E(u_i - p_a)^+$. The firm should choose the optimal prices such that the individual rationality constraints for both consumer types are binding, i.e., $v_H - p_{mH} + E(u_H - p_a)^+ = 0$ and $v_L - p_{mL} + E(u_L - p_a)^+ = 0$. Moreover, the firm incurs marginal and inconvenience costs from those consumers who purchase the ancillary service (i.e., those who have $u_i \geq p_a$). Thus, the firm's profit maximization problem in the unbundling case can be reduced to a single-variable optimization problem of the ancillary service price $p_a > 0$ with the following profit function:

$$\begin{aligned}\Pi_u(p_a) &= (p_{mH} - c_m)\lambda_H + (p_{mL} - c_m)\lambda_L \\ &\quad +(p_a - c_a)[\lambda_H\bar{F}_H(p_a) + \lambda_L\bar{F}_L(p_a)] - c(\lambda_H\bar{F}_H(p_a) + \lambda_L\bar{F}_L(p_a)) \\ &= [v_H + E(u_H - p_a)^+ - c_m]\lambda_H + [v_L + E(u_L - p_a)^+ - c_m]\lambda_L \\ &\quad +(p_a - c_a)[\lambda_H\bar{F}_H(p_a) + \lambda_L\bar{F}_L(p_a)] - c(\lambda_H\bar{F}_H(p_a) + \lambda_L\bar{F}_L(p_a)).\end{aligned}$$

Note that the firm's optimization problem is well-defined, because as the following theorem indicates, the optimal ancillary service price p_a^* is always strictly positive.

Theorem IV.1. *(i) In the unbundling case, the optimal ancillary service price p_a^* is the solution to $p_a^* = c_a + c'(\lambda_H\bar{F}_H(p_a^*) + \lambda_L\bar{F}_L(p_a^*))$.*

(ii) For each consumer type $i = H, L$, the optimal prices satisfy $p_{mi}^* < p_{bi}^* < p_{mi}^* + p_a^*$.

(iii) The price reduction from the optimal bundle price to the optimal main service price when the firm unbundles is greater for the consumer type with a higher likelihood of purchasing the ancillary service (i.e., if $\beta_H \geq \beta_L$, $p^*_{bH} - p^*_{mH} \geq p^*_{bL} - p^*_{mL}$; if $\beta_H < \beta_L$, $p^*_{bH} - p^*_{mH} < p^*_{bL} - p^*_{mL}$).

Theorem IV.1(i) characterizes the optimal ancillary service price in the unbundling case, which is given by the condition that marginal benefit is equal to total marginal cost (p^*_a is the marginal benefit of selling the ancillary service, $c_a + c'(\lambda_H \bar{F}_H(p^*_a) + \lambda_L \bar{F}_L(p^*_a))$ is the total marginal cost of selling the ancillary service). In our basic model, consumers make forward-looking decisions when purchasing the main service, i.e., they take future utilities from the ancillary service into consideration. In Section 4.5, we consider a model that also has myopic consumers who do not make forward-looking purchasing decisions. As we will show, when myopic consumers exist, the firm's optimal ancillary service price can be higher than the total marginal cost. Theorem IV.1(ii) states that compared to the optimal bundle price, in the unbundling case, the firm should charge a lower main service price but a higher total price including the ancillary service to both types of consumers. Moreover, Theorem IV.1(iii) states that the consumer type with a higher likelihood of purchasing the ancillary service should see a more significant price reduction of the main service when the firm unbundles the ancillary service. For airlines, since business travelers usually check fewer bags than leisure travelers,[4] our result indicates that the fare reduction resulting from unbundling the baggage service should be more significant for leisure travelers. Our results in Theorem IV.1(ii) and (iii) are consistent with the empirical findings of *Brueckner et al.* (2014). *Brueckner et al.* (2014) find that after airlines started charging for baggage fees, leisure fares (as measured by the 25th percentile fare) fell by one-half to one-third of the baggage fee. Correspondingly, the full trip

[4]Checked bags per passenger at airport ticket counters increased to 0.677 in 2013, up from 0.668 in 2012; John Heimlich, A4A's chief economist, said one factor is that more leisure travelers were flying in 2013, and they tend to check more bags per person than business travelers (*Schaal*, 2014a).

price for a passenger paying the baggage fee rose by one-half to two-thirds of the baggage fee. Their empirical analysis also reveals that the fare impact of imposing a baggage fee is larger at the lower percentiles (i.e., leisure travelers) and smaller at the higher percentiles (i.e., business travelers), which is exactly what we find in Theorem IV.1(iii). Thus, our model explains the empirical findings of *Brueckner et al.* (2014). Next, we determine which strategy is more profitable for the firm, bundling or unbundling.

Theorem IV.2. *There exists a decreasing threshold function $\bar{\beta}_H(\beta_L)$ such that unbundling is more profitable than bundling if and only if $\beta_H \leq \bar{\beta}_H(\beta_L)$.*

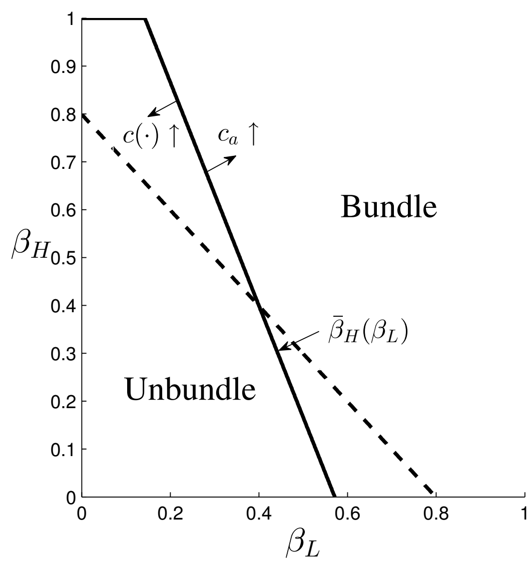


Figure 4.1: Optimal strategy and threshold $\bar{\beta}_H(\beta_L)$ ($v_H = 300$, $v_L = 200$, $\bar{u} = 50$, $\underline{u} = -20$, $c_m = 150$, $c_a = 20$, $c(x) = 0.5x^2$; solid curve: $\lambda_H = 30$, $\lambda_L = 70$; dashed curve: $\lambda_H = 50$, $\lambda_L = 50$)

The trade-off between bundling and unbundling the ancillary service is as follows. On the one hand, unbundling the ancillary service gives the firm more flexibility and allows the firm to extract more consumer surplus. On the other hand, to the extent that consumers postpone paying for the ancillary service till the last minute, unbundling may result in higher inconvenience costs. Theorem IV.2 states that it is optimal to unbundle the ancillary service when consumers' likelihoods of purchasing the ancillary service are low enough. A lower likelihood of consumers purchasing the ancillary service keeps the inconvenience cost less significant. For example, airlines usually charge for the ancillary services that are needed by very few consumers, such as pet fee. On the other hand, with a high enough likelihood of consumers purchasing the ancillary service, it is optimal for the firm to bundle the ancillary service into the main service. For example, since everyone needs to eat during long international flights, airlines usually offer "free" meals (i.e., meal price is included in flight ticket price) for international flights that are long enough (while they usually do not offer inclusive meals for domestic flights). Figure 4.1 illustrates when bundling or unbundling the ancillary service is the optimal strategy through one example and shows the threshold function $\bar{\beta}_H(\beta_L)$.

Theorem IV.3. *(i) Consider two scenarios with inconvenience costs $c_1(x)$ and $c_2(x)$, respectively. Suppose $c_1'(x) \leq c_2'(x)\ \forall x > 0$, then $\bar{\beta}_{H1}(\beta_L) \geq \bar{\beta}_{H2}(\beta_L)$. Moreover, if the inconvenience cost is negligible (i.e., $c(\cdot) = 0$), unbundling is always more profitable than bundling (i.e., $\bar{\beta}_H(\beta_L) = 1$).*

(ii) Consider two scenarios with ancillary service marginal costs c_{a1} and c_{a2}, respectively. Suppose $c_{a1} < c_{a2}$, then $\bar{\beta}_{H1}(\beta_L) \leq \bar{\beta}_{H2}(\beta_L)$.

(iii) Consider two scenarios. In the first scenario, the demand sizes are λ_{H1} and λ_{L1}. In the second scenario, the demand sizes are λ_{H2} and λ_{L2}. Suppose $\lambda_{H1} + \lambda_{L1} = \lambda_{H2} + \lambda_{L2} = \lambda$ and $\lambda_{H1} < \lambda_{H2}$ (hence $\lambda_{L1} > \lambda_{L2}$). Then, $\bar{\beta}_{H1}(\beta_L) \geq \bar{\beta}_{H2}(\beta_L)$ in the region of $\beta_H \geq \beta_L$ and $\bar{\beta}_{H1}(\beta_L) < \bar{\beta}_{H2}(\beta_L)$ in the region of $\beta_H < \beta_L$; $\bar{\beta}_{H1}(\beta_L)$ and $\bar{\beta}_{H2}(\beta_L)$ intersect on the $45°$ line $\beta_H = \beta_L$.

Theorem IV.3 describes how the optimal bundling (unbundling) decision is affected by the firm's operating costs and demand portfolio (Figure 4.1 illustrates the directions of change, for the threshold function $\bar{\beta}_H(\beta_L)$, that are caused by different factors). As expected, a higher inconvenience cost makes bundling more favorable (i.e., the threshold $\bar{\beta}_H(\beta_L)$ becomes smaller). In the extreme case where the firm does not incur any inconvenience cost, it is always more profitable to unbundle the ancillary service. On the contrary, a higher marginal cost of ancillary service makes unbundling more favorable (i.e., the threshold $\bar{\beta}_H(\beta_L)$ becomes larger), because unbundling the ancillary service results in fewer consumers using the ancillary service and reduces the marginal costs incurred compared to the bundling case. One could argue that an increase of fuel expenses increases the marginal cost of transporting bags. Interestingly, the airlines started unbundling the checked baggage service in 2008, when the fuel prices went up by 91.5% from the previous year (*Maynard*, 2008). Moreover, to reduce fuel expenses, airlines chose to unbundle checked bags instead of carry-on bags because the marginal cost of a carry-on bag is smaller than a checked bag. Carry-on bags are usually lighter and consume less fuel, and do not need labor for loading or unloading. Additionally, after airlines started charging for checked bags, some consumers may try to avoid checked bag fees by carrying more luggage on to the plane rather than checking it. Thus, the firm may have an increased consumer likelihood of having a carry-on bag. Theorem II.6 then indicates that it will become less likely that unbundling the carry-on baggage service is profitable.

Theorem IV.3(iii) characterizes how the optimal decision is affected by firm's demand portfolio. The threshold function $\bar{\beta}_H(\beta_L)$ is less steep for a firm with a higher proportion of high-type consumers ($\bar{\beta}_H(\beta_L)$ spins counterclockwise as the proportion of high-type consumers increases). If high-type consumers are more likely to purchase the ancillary service than low-type consumers (i.e., $\beta_H \geq \beta_L$), then increasing the proportion of high-type consumers expands the region in which bundling is optimal. If low-type consumers are more likely to purchase the ancillary service than high-type consumers (i.e., $\beta_H < \beta_L$), then increasing the proportion of high-type consumers expands the region in which unbundling is optimal. Therefore, if consumers' valuations for the main service and the ancillary service are positively correlated, bundling is more likely to be the optimal strategy for a firm with more high-type consumers than a firm with fewer high-type consumers; if consumers' valuations for the main service and the ancillary service are negatively correlated, unbundling is more likely to be the optimal strategy for a firm with more high-type consumers than a firm with fewer high-type consumers.

Consider airline baggage policies for example. Since business travelers are less likely to check bags than leisure travelers (i.e., $\beta_H < \beta_L$), Theorem IV.3(iii) indicates that unbundling is more likely to be optimal for airlines with higher proportions of business travelers (e.g., legacy airlines) than airlines with lower proportions of business travelers (e.g., low-cost airlines). As of 2014, legacy airlines charge for checked bags; the airline that stands firm on not charging for checked bags is Southwest (Southwest does not charge for the first or second checked bag) which is a low-cost carrier (some other low-cost airlines, including Spirit and Frontier, unbundle the baggage service under a different pricing structure; we discuss this case in the next paragraph). Additionally, after some firms unbundle the ancillary service, consumers with higher needs for the ancillary service may switch to the firms that are still bundling the ancillary service for their lower total prices, and consumers without ancillary service needs may switch to the unbundling firms for their lower main service prices. This would result in an increase in consumers' likelihood of using the ancillary service for the bundling firms, and a decrease in consumers' likelihood of purchasing the ancillary service for the unbundling firms. Thus, following from Theorem II.6, firms' different decisions regarding bundling (unbundling) the ancillary service will be consolidated. This type of consumer self-selection regarding airlines' checked bag fees (which is empirically supported by *Nicolae et al.*, 2013) provides another reason for Southwest to bundle the checked bags.[5] Moreover, the bundling firms can increase the bundle price due

[5] So far, there is not clear evidence that Southwest will unbundle the baggage service in short-term. Southwest's CEO Gary Kelly declared during the company's 2013 fourth quarter earnings call that Southwest Airlines won't begin to charge fees for first and second checked bags in 2014 (*Schaal*, 2014b).

to the increased consumer valuations for the ancillary service.[6] Another reason for Southwest to bundle the baggage service is its non-dependency on intermediary sales channels, which we will discuss in Section 4.6.

In Theorem IV.3(i), we have shown that in order to benefit from unbundling, the firm needs to reduce the inconvenience cost. One way to reduce the inconvenience cost is to induce consumers to pay for the ancillary service in advance. Spirit and Frontier Airlines have recently started to unbundle the baggage service while resorting to a new pricing structure for the ancillary service with late-payment penalty. Spirit and Frontier are now charging baggage fees contingent on when consumers pay for their bags. The later a consumer pays for the bag, the higher the fee is. For example, Spirit charges $100 for any bag (checked and carry-on) that is paid for at the gate, which is three to four times higher than the baggage fees other airlines normally charge and Spirit's advance baggage fee itself. The following explanation has been given by Spirit's spokesperson: "The fee is intentionally set high to encourage customers to reserve their bags in advance, and it is meant to deter customers from waiting until they get to the boarding gate. When customers wait until the boarding gate, this delays the boarding process for everyone." (*Brown*, 2012) Because the new pricing structure significantly reduces the inconvenience cost, Spirit and Frontier also charge for carry-on bags. Being recognized as the airline with the lowest fares, Spirit may not lose too many consumers even if its consumers are dissatisfied with the high late-payment penalty, because price-sensitive consumers are not very likely to get even lower ticket prices elsewhere if they refuse to accept the new baggage policy and pay in advance.[7] So far, Spirit's implementation of the new baggage policy appears to be

[6] *Henrickson and Scott* (2012) consider the top 150 domestic routes from 2007 to 2009, and find that a one dollar increase in baggage fees reduces airline ticket prices on the fee-charging airlines by $0.24 and increases Southwest Airlines' ticket prices on routes in which they compete with baggage-fee-charging airlines by $0.73.

[7] With the lowest fares, Spirit unbundles the ancillary services the most aggressively. Spirit charges separate fees for carry-on bags and other ancillary services including printing a boarding pass which are unconventional to consumers. In all, there are about 70 fees enumerated in detail on Spirit's website for consumers to navigate (*Mouawad*, 2013). In 2012, Spirit collected $19.99 per

a success. However, resorting to a pricing structure with the late-payment penalty may be riskier for other airlines.

4.5 Myopic Consumers

In this section, we investigate the effect of myopic consumers on the firm's optimal strategy for the ancillary service. In travel industries, consumers usually purchase the service bundle (when the firm bundles the ancillary service) or the main service (when the firm unbundles the ancillary service) in advance. Different from forward-looking consumers who take future utilities from the ancillary service into consideration when purchasing the service bundle or main service in advance, myopic consumers do not consider future utilities. Although our model in Section 4.4 assumed that all consumers were forward-looking, for many ancillary services that do not cost significant amounts of money, consumers are likely to be myopic. For example, it would be very unusual that a consumer takes the possible purchase of a can of coke during the flight (and the price of a can of coke) into consideration when booking the ticket several months in advance.[8]

To capture the effect of myopic consumers, we now introduce a model with a more general demand composition comprised of both forward-looking and myopic consumers. We assume α_H proportion of high-type consumers and α_L proportion of low-type consumers are forward-looking, the other consumers are myopic. In the bundling case, type-i ($i = H, L$) myopic consumers are willing to pay v_i for the service bundle when making purchasing decisions in advance, which is lower than forward-looking consumers' willingness to pay, $v_i + E(u_i)^+$. For each consumer type $i = H, L$,

---

passenger in baggage fees which is the highest in the industry, whereas Delta collected $7.44 per passenger which is about average for the industry (*Mutzabaugh*, 2013). Spirit's ancillary revenue makes up more than 30% of its total revenue (*Trejos*, 2012).

[8]Previous empirical research has looked into consumers' behavior of delaying purchases strategically, which is also a type of forward-looking behavior. For example, using airline data, *Li et al.* (2014) find that across markets, 5.2% to 19.2% of the population exhibits the behavior of delaying purchases strategically (measured by the first and third quartiles).

the firm can choose to price the service bundle at $p_{bi} = v_i + E(u_i)^+$ to induce only forward-looking consumers to purchase, or at $p_{bi} = v_i$ to induce both forward-looking and myopic consumers to purchase. Thus, the firm has four price combinations to choose from: "HH", "HL", "LH", "LL", where the former notation refers to the price for high-type consumers and the latter refers to the price for low-type consumers, "H" means pricing high and "L" means pricing low. The resulting profits are as follows, where we add a subscript "m" to represent the case with myopic consumers, and use the superscript to represent the price choice of the firm:

$$\begin{aligned}
\Pi_{b,m}^{HH*} &= [v_H + E(u_H)^+ - c_m]\alpha_H\lambda_H + [v_L + E(u_L)^+ - c_m]\alpha_L\lambda_L \\
&\quad -c_a[\alpha_H\lambda_H\bar{F}_H(0) + \alpha_L\lambda_L\bar{F}_L(0)], \\
\Pi_{b,m}^{HL*} &= [v_H + E(u_H)^+ - c_m]\alpha_H\lambda_H + (v_L - c_m)\lambda_L - c_a[\alpha_H\lambda_H\bar{F}_H(0) + \lambda_L\bar{F}_L(0)], \\
\Pi_{b,m}^{LH*} &= (v_H - c_m)\lambda_H + [v_L + E(u_L)^+ - c_m]\alpha_L\lambda_L - c_a[\lambda_H\bar{F}_H(0) + \alpha_L\lambda_L\bar{F}_L(0)], \\
\Pi_{b,m}^{LL*} &= (v_H - c_m)\lambda_H + (v_L - c_m)\lambda_L - c_a[\lambda_H\bar{F}_H(0) + \lambda_L\bar{F}_L(0)].
\end{aligned}$$

The optimal profit in the bundling case is $\Pi_{b,m}^* = \max(\Pi_{b,m}^{HH*}, \Pi_{b,m}^{HL*}, \Pi_{b,m}^{LH*}, \Pi_{b,m}^{LL*})$.

In the unbundling case, type-i ($i = H, L$) myopic consumers are willing to pay v_i for the main service when making purchasing decisions in advance, and forward-looking consumers have a higher willingness to pay, $v_i + E(u_i - p_a)^+$. For each consumer type $i = H, L$, the firm can choose to price the main service at $p_{mi} = v_i + E(u_i - p_a)^+$ to induce only forward-looking consumers to purchase, or at $p_{mi} = v_i$ to induce both forward-looking and myopic consumers to purchase, hence also leading to four price combinations. The resulting profits are as follows, as functions of the ancillary service price:

$$
\begin{aligned}
\Pi_{u,m}^{HH}(p_a) &= [v_H + E(u_H - p_a)^+ - c_m]\alpha_H\lambda_H + [v_L + E(u_L - p_a)^+ - c_m]\alpha_L\lambda_L \\
&\quad + (p_a - c_a)[\alpha_H\lambda_H\bar{F}_H(p_a) + \alpha_L\lambda_L\bar{F}_L(p_a)] - c(\alpha_H\lambda_H\bar{F}_H(p_a) + \alpha_L\lambda_L\bar{F}_L(p_a)), \\
\Pi_{u,m}^{HL}(p_a) &= [v_H + E(u_H - p_a)^+ - c_m]\alpha_H\lambda_H + (v_L - c_m)\lambda_L \\
&\quad + (p_a - c_a)[\alpha_H\lambda_H\bar{F}_H(p_a) + \lambda_L\bar{F}_L(p_a)] - c(\alpha_H\lambda_H\bar{F}_H(p_a) + \lambda_L\bar{F}_L(p_a)), \\
\Pi_{u,m}^{LH}(p_a) &= (v_H - c_m)\lambda_H + [v_L + E(u_L - p_a)^+ - c_m]\alpha_L\lambda_L \\
&\quad + (p_a - c_a)[\lambda_H\bar{F}_H(p_a) + \alpha_L\lambda_L\bar{F}_L(p_a)] - c(\lambda_H\bar{F}_H(p_a) + \alpha_L\lambda_L\bar{F}_L(p_a)), \\
\Pi_{u,m}^{LL}(p_a) &= (v_H - c_m)\lambda_H + (v_L - c_m)\lambda_L \\
&\quad + (p_a - c_a)[\lambda_H\bar{F}_H(p_a) + \lambda_L\bar{F}_L(p_a)] - c(\lambda_H\bar{F}_H(p_a) + \lambda_L\bar{F}_L(p_a)).
\end{aligned}
$$

The optimal profit in the unbundling case is $\Pi_{u,m}^* = \max\left(\Pi_{u,m}^{HH}(p_a^*), \Pi_{u,m}^{HL}(p_a^*), \Pi_{u,m}^{LH}(p_a^*), \Pi_{u,m}^{LL}(p_a^*)\right)$.

Theorem IV.4. *In the unbundling case, the optimal ancillary service price is strictly higher than the total marginal cost if the firm sells to both forward-looking and myopic consumers; the optimal ancillary service price is equal to the total marginal cost if the firm only sells to forward-looking consumers.*

In the unbundling case, Theorem IV.4 states that as long as the firm sells to myopic consumers (either high-type or low-type), it should price the ancillary service above the marginal cost.[9] Unlike forward-looking consumers, myopic consumers' decisions on purchasing the main service and purchasing the ancillary service are made independently. Thus, when selling to myopic consumers, the firm no longer wants to decrease the ancillary service price to the marginal cost so that it could extract more

[9]In fact, we can characterize the condition for the firm to price the ancillary service above or equal to the marginal cost. It can be shown that there exist two threshold functions, $\tilde{\alpha}_H(\alpha_L)$ and $\tilde{\alpha}_L(\alpha_H)$, such that $\Pi_{u,m}^* = \Pi_{u,m}^{HH*}$ (hence the optimal ancillary service price is equal to the marginal cost) if $\alpha_H > \tilde{\alpha}_H(\alpha_L)$ and $\alpha_L > \tilde{\alpha}_L(\alpha_H)$, and $\Pi_{u,m}^* \neq \Pi_{u,m}^{HH*}$ (hence the optimal ancillary service price is above the marginal cost) otherwise.

consumer surplus overall by increasing the main service price accordingly. In reality, "small-item" ancillary services are usually priced well-above their marginal costs, e.g., a can of coke is priced more than 10 times the cost of it if ordered during the flight. Since consumers are myopic, the firm extracts high margins from selling the ancillary service.

Theorem IV.5. *(i) If unbundling is more profitable when all consumers are forward-looking (i.e., $\alpha_H = \alpha_L = 1$), then it is more profitable for all α_H and α_L.*

(ii) If bundling is more profitable when all consumers are forward-looking (i.e., $\alpha_H = \alpha_L = 1$), then there exist two thresholds $\hat{\alpha}_H$, $\hat{\alpha}_L$ and a decreasing threshold function $\bar{\alpha}_H(\alpha_L)$ such that when $\alpha_H \leq \hat{\alpha}_H$, $\alpha_L \leq \hat{\alpha}_L$, and $\alpha_H \leq \bar{\alpha}_H(\alpha_L)$, unbundling is more profitable.

Now we compare the unbundling profit to the bundling profit when some of the firm's consumers are myopic. Theorem IV.5 states that as the firm's proportion of myopic consumers increases, it may become optimal for the firm to switch from bundling to unbundling but not the other way around. If it is optimal to unbundle the ancillary service when all consumers are forward-looking, then it is also optimal to unbundle the ancillary service with any proportion of myopic consumers. If it is optimal to bundle the ancillary service when all consumers are forward-looking, we find a sufficient condition so that the firm should actually unbundle the ancillary service when the proportion of myopic consumers is significant enough (i.e., α_H and α_L are small enough). When selling to myopic consumers, the firm does not capture any consumer surplus from the ancillary service in the bundling case, because myopic consumers' utilities from the ancillary service do not affect their willingness to pay for the service bundle. However, by unbundling the ancillary service, the firm is able to capture myopic consumers' surplus from the ancillary service, because the firm induces myopic consumers to actually pay for the ancillary service. Thus, the existence of myopic consumers may switch the firm's optimal ancillary service strategy from bundling to unbundling but not the other way around.

Theorem IV.6. *The optimal profits from bundling and unbundling are both increasing in the proportion of forward-looking consumers, α_H and α_L.*

Theorem IV.6 states that regardless of whether the firm bundles or unbundles the ancillary service, its profit becomes higher when more consumers are forward-looking. For a firm that sells an ancillary service in addition to a main service, having more forward-looking consumers is beneficial because by accounting for future utilities from the ancillary service, forward-looking consumers are willing to pay more for the service bundle and main service than myopic consumers when purchasing in advance. Thus, the firm benefits from providing guidance to consumers for their ancillary service needs and making the information of the ancillary services easily accessible to consumers. Notice that forward-looking (strategic) consumers play a different role in the ancillary service pricing setting than in the markdown pricing setting which has been extensively studied by previous literature. Although forward-looking consumers have been perceived as harmful to firms that salvage product leftovers at the end of the selling season, they actually benefit firms that manage the sales of a main service and an ancillary service simultaneously.

4.6 Selling Through Intermediaries

Online travel agencies (OTAs) constitute one sales channel of firms operating in travel industries. Travel firms' contracts with OTAs are usually subject to rate parity which requires that the firm charges the same price to consumers in all its sales channels. For historical reasons, hotels pay higher commissions to OTAs compared to airlines and car rental companies.[10] Industry research shows that hotel bookings

[10] One explanation is that many hoteliers surrendered to the temptations of the indirect channel during the recession, and have been accommodating the OTAs with bigger discounts; in Quarter 3 of 2010, OTA share of the online bookings for the top 30 hotel brands increased to 37.5%, from 25.4% in Quarter 3 of 2008 (*Starkov*, 2010).

constitute an average of 30% of OTA booking volume but generate over 60% of OTA booking revenue, whereas air tickets and car rental companies comprise 51% of OTA booking volume but generate 12% of OTA booking revenue (*Starkov*, 2010). Moreover, independent hotels are more OTA-dependent than branded hotel chains. According to *Starkov* (2013), more than 42% of roomnights for independent hotels are reserved online. Only 24% of these roomnights are reserved via hotel websites, whereas 76% are reserved via OTAs. In 2012, branded hotels received 38.7% of roomnight reservations online, with 68% via hotel websites and 32% via OTAs. While hotels are dependent on OTAs as an important sales channel, they are very much concerned about the high commissions paid to OTAs and are seeking ways to earn more profits back. Most of the solutions that have been proposed by industry analysts focus on enhancing brand loyalty and affinity with better marketing tactics (e.g., *Mayock*, 2011, *Weston*, 2013). However, it could be worthwhile to explore how firms could earn profits back from OTAs with better pricing strategies.

In this section, we investigate the effect of selling through intermediaries on the optimal selection of bundling or unbundling the ancillary service. We now analyze a model where γ_H proportion of high-type consumers and γ_L proportion of low-type consumers purchase directly from the firm, the other consumers purchase from the intermediaries. The firm pays a commission of τ (which is defined as a percentage of the revenue collected by the OTA) to the OTA for each unit of sale.[11] In the bundling

[11] There are two models that firms have been using to contract with OTAs: the merchant model and the agency model. In the merchant model, the OTAs purchase products from the firm at a negotiated wholesale rate (usually discounted by 20% to 30%). Then they mark the price back up and resell the products to consumers. Consumers pay up front to the OTAs. The merchant model has been used by Hotels.com, Orbitz and Travelocity. In the agency model, an OTA gets a commission from the firm for every product sold through its platform. Consumers pay to the firm and are allowed to pay at checkout. The agency model has been adopted by some OTAs recently. It has been used by Booking.com, and has been tested by Expedia with the Expedia Traveler Preference Program. Because of the more convenient payment time for consumers, the agency model is expected to be used more often. For our purpose of study, the equivalence of firm profits from these two models can be easily proved. Thus, we use a unified framework to study the effect of selling through intermediaries. τ can be interpreted as the commission that the firm pays to the OTA for each unit of sale (agency model), or alternatively, the discount rate that the firm offers to the OTA (merchant model).

case, the firm earns $1 - \tau$ fraction of the bundle price. In the unbundling case, the firm earns $1 - \tau$ fraction of the main service price and the full ancillary service price because consumers pay for the ancillary service directly to the firm.

Theorem IV.7. *The difference between the optimal unbundling profit and the optimal bundling profit is increasing in the intermediary commission τ, and decreasing in the proportion of direct sales, γ_H and γ_L.*

Theorem IV.7 states that as the OTA's commission increases, or as consumers shift from purchasing directly from the firm to purchasing from the OTA (i.e., γ_H and/or γ_L decrease), unbundling the ancillary service becomes more profitable relative to bundling. When the firm bundles the ancillary service into the main service, it has to pay commissions to the OTA for the whole service price. By unbundling the ancillary service, the firm only pays commissions to the OTA for the main service price and still collects the full price of the ancillary service. Thus, unbundling the ancillary service helps firms earn more revenues back from OTAs, which is what a lot of travel firms are trying to achieve now. For a firm that is facing a higher OTA commission or is more dependent on OTAs (i.e., OTAs account for a larger proportion of the firm's sales), unbundling the ancillary service is more valuable. Southwest Airlines is the only major U.S. carrier that does not use OTAs (i.e., Southwest's $\gamma_H = \gamma_L = 1$). Following our analysis in this section, bundling the baggage service is more likely to be optimal for Southwest than other airlines.

In Sections 4.4 - 4.6, we have analyzed when a price-discriminating firm benefits from unbundling the ancillary service and how a firm should choose the optimal ancillary service price. Our results provide some explanations for the interesting phenomenon in the airline industry that while most airlines charge for checked bags, Southwest Airlines provides this ancillary service for free. First, Southwest is the only major U.S. airline that does not use online travel agencies to sell tickets. Second, as a low-cost airline, Southwest has a larger proportion of leisure travelers who have higher checked baggage needs. Third, as other airlines started to charge for checked bags, consumers with higher baggage needs may switch to Southwest and result in an overall increase in Southwest's baggage demand. This would consolidate bundling as Southwest's optimal strategy.

So far, we have considered a firm that can price-discriminate when selling the main service. An interesting question is how the optimal strategy for the ancillary service is affected by the firm's ability to price-discriminate when selling the main service. Compared to airlines, price discrimination for the main service is much less used in other industries such as hotels. In the next section, we analyze a firm that does not price-discriminate (i.e., uses uniform pricing) when selling the main service and characterize how this firm would price ancillary services. This in turn enables us to contrast our results to the case where the firm *can* price discriminate and we find that the ability to price discriminate plays a key role in the decision whether it is optimal to bundle ancillary services. Since previous literature on commodity bundling has focused on the case where the firm uses uniform pricing when selling both commodities, our findings will also shed light on whether the insights for commodity bundling carry through to the ancillary service setting and how the insights become different if the firm uses or is unable to use discriminatory pricing.

4.7 Uniform Pricing of Main Service

In this section, we study the optimal strategy for the ancillary service for a firm that does not price-discriminate when selling the main service and compare the results to the case of a price-discriminating firm. We consider a firm that charges a uniform price for the main service to both types of consumers. Note that under discriminatory pricing, both types of consumers are served. Under uniform pricing, it may be optimal to serve only high-type consumers. However, to make a fair comparison, we consider a uniform-pricing firm that serves both types of consumers, that is, the firm charges the uniform price at low-type consumers' willingness to pay. In the bundling case, the firm sells the bundle at price $p_b^* = v_L + E(u_L)^+$.[12] We add a second subscript of "n" to denote the case where the firm does not price-discriminate. The optimal profit in the bundling case is

$$\Pi_{b,n}^* = [v_L + E(u_L)^+ - c_m](\lambda_H + \lambda_L) - c_a[\lambda_H \bar{F}_H(0) + \lambda_L \bar{F}_L(0)].$$

In the unbundling case, the firm sells the main service at price p_m which should satisfy $p_m = v_L + E(u_L - p_a)^+$. The profit function in the unbundling case is

$$\begin{aligned} \Pi_{u,n}(p_a) \quad &= \quad [v_L + E(u_L - p_a)^+ - c_m](\lambda_H + \lambda_L) \\ &\quad + (p_a - c_a)[\lambda_H \bar{F}_H(p_a) + \lambda_L \bar{F}_L(p_a)] - c(\lambda_H \bar{F}_H(p_a) + \lambda_L \bar{F}_L(p_a)), \end{aligned}$$

and the optimal ancillary service price is denoted by $p_{a,n}^*$. In this section, we assume $c'''(\cdot) \geq 0$ which holds at least if the inconvenience cost is polynomial with non-negative coefficients or exponential. This assumption is only needed to ensure the quasi-concavity of $\Pi_{u,n}(p_a)$ (hence to guarantee that the optimal solution $p_{a,n}^*$ is unique), and is not needed for the rest of the analysis in this section. The following theorem compares the optimal ancillary service price under uniform pricing, $p_{a,n}^*$, to the optimal ancillary service price under discriminatory pricing, p_a^*.

Theorem IV.8. *If high-type consumers are more likely to purchase the ancillary service than low-type consumers (i.e., $\beta_H \geq \beta_L$), the optimal ancillary service price under uniform pricing is greater than the optimal ancillary service price under discriminatory pricing (i.e., $p_{a,n}^* \geq p_a^*$); otherwise, the result reverses (i.e., if $\beta_H < \beta_L$, $p_{a,n}^* < p_a^*$).*

[12] Note that $\bar{u} \leq v_H - v_L$ ensures that when $p_b = v_L + E(u_L)^+$, high-type consumers purchase the service bundle as well, that is, charging at low-type consumers' willingness to pay induces high-type consumers to purchase as well even if high-type consumers may value the ancillary service lower. This also holds in the unbundling case.

Theorem IV.8 states that compared to the discriminatory-pricing case, the firm should charge a higher ancillary service price under uniform pricing when high-type consumers are more likely to purchase the ancillary service than low-type consumers, or equivalently, consumers' valuations for the main service and the ancillary service are positively correlated. The firm should charge a lower ancillary service price under uniform pricing when low-type consumers are more likely to purchase the ancillary service than high-type consumers, or equivalently, consumers' valuations for the main service and the ancillary service are negatively correlated. Recall from Section 4.4 that under discriminatory pricing, the optimal ancillary service price, p_a^*, is equal to the total marginal cost of providing the ancillary service. Thus, Theorem IV.8 indicates that under uniform pricing, the optimal ancillary service price should be above the total marginal cost in the case of a positive correlation and below the total marginal cost in the case of a negative correlation. This result appears to be consistent with the previous findings in two-part pricing literature (e.g., *Rosen and Rosenfield*, 1997, *Png and Wang*, 2010).

Under discriminatory pricing, the firm is able to extract full surplus ex ante from both types of consumers. Under uniform pricing, the firm only extracts full surplus from low-type consumers and leaves some surplus from high-type consumers un-captured. Different from a price-discriminating firm, a uniform-pricing firm needs to adjust the main service price and the ancillary service price to extract more surplus from high-type consumers, while keeping low-type consumers' individual rationality constraint binding. If high-type consumers are more likely to purchase the ancillary service than low-type consumers, to capture more of the high-type consumers' surplus from the ancillary service, the firm should increase the ancillary service price and decrease the main service price accordingly. Thus, compared to the discriminatory-pricing case, the firm's optimal ancillary service price is higher under uniform pricing. On the other hand, if low-type consumers are more likely to purchase the ancillary service than high-type consumers, the firm's optimal ancillary service price under uniform pricing is lower than the optimal ancillary service under discriminatory pricing. Therefore, the effect of main service price discrimination on the optimal ancillary service price depends on the correlation between consumers' valuations for the main service and the ancillary service in a way that is described in Theorem IV.8.

Theorem IV.9. *Under uniform pricing, there exists an increasing threshold function $\bar{\beta}_{H,n}(\beta_L)$ such that unbundling is more profitable than bundling if and only if $\beta_H \geq \bar{\beta}_{H,n}(\beta_L)$.*

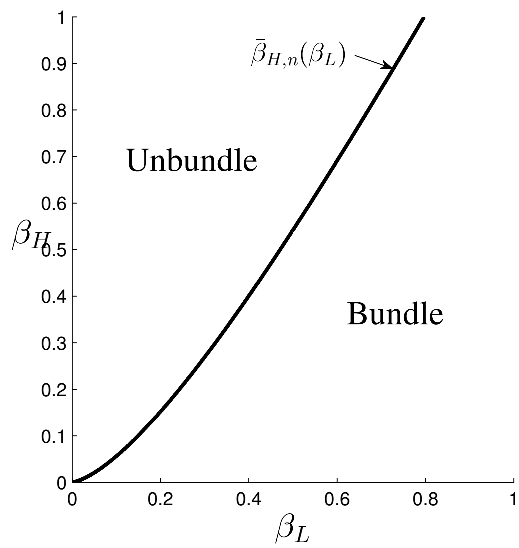


Figure 4.2: Optimal strategy and threshold $\bar{\beta}_{H,n}(\beta_L)$ in the uniform pricing case ($v_H = 300$, $v_L = 200$, $\bar{u} = 50$, $\underline{u} = -20$, $c_m = 150$, $c_a = 20$, $c(x) = 0.5x^2$, $\lambda_H = 30$, $\lambda_L = 70$)

Now we compare firm's optimal bundling (unbundling) decisions under uniform pricing and discriminatory pricing. Theorem IV.9 characterizes the firm's optimal strategy under uniform pricing. Recall that we characterized the regions where bundling and unbundling are optimal in Theorem II.6 and Figure 4.1 for the discriminatory-pricing case. It is easy to see the differences. First, under uniform pricing, unbundling is more profitable than bundling if high-type consumers' likelihood of purchasing the ancillary service is high enough; whereas under discriminatory pricing, unbundling is more profitable if high-type consumers' likelihood of purchasing the ancillary service is low enough. Second, the threshold function that separates the bundling region and the unbundling region is an increasing function under uniform pricing and a decreasing function under discriminatory pricing. Figure 4.2 illustrates when bundling or unbundling the ancillary service is optimal for a uniform-pricing firm through the same example used in Figure 4.1. By comparing Figure 4.2 to Figure 4.1, we can clearly see the above differences. Theorem IV.9 essentially states that for a uniform-pricing firm, it is optimal to unbundle the ancillary service if high-type consumers' likelihood of purchasing the ancillary service is high enough and low-type consumers' likelihood of purchasing the ancillary service is low enough, which is equivalent to requiring that the correlation between consumers' main service valuations and ancillary service valuations is positive enough. Thus, although the ancillary service setting is naturally different from the bundling setting that has been studied by previous literature (the ancillary service cannot be sold separately), our result in Theorem IV.9 for a uniform-pricing firm is consistent with the bundling literature finding. However, if the firm is able to price-discriminate when selling the main service, the above "correlation effect" goes away, and for unbundling to be the optimal strategy, both consumer types' likelihoods of purchasing the ancillary service should be low enough.

Under uniform pricing, if the firm bundles the ancillary service into the main service, it prices the bundle at low-type consumers' willingness to pay and both types of consumers purchase the bundle. If high-type consumers are very likely to purchase the ancillary service while low-type consumers are very unlikely to purchase the ancillary service, bundling the ancillary service would mean that the firm is leaving too much un-captured surplus to high-type consumers. In this case, the firm should unbundle and charge a high price for the ancillary service to capture more surplus from high-type consumers. On the other hand, if high-type consumers are very unlikely to purchase the ancillary service while low-type consumers are very likely to purchase the ancillary service, the firm benefits from bundling the ancillary service. If the firm unbundles in this case, it will charge a low price for the ancillary service (following from Theorem IV.8), which would not bring in a lot revenue but result in too much inconvenience cost. Overall, unbundling the ancillary service assists the firm in capturing more surplus from high-type consumers at the expense of distorting the prices charged to low-type consumers. A positive enough correlation between consumers' valuations for the main service and the ancillary service indicates that high-type consumers have significantly more surplus from the ancillary service compared to low-type consumers, and hence the firm should capture it by unbundling the ancillary service.

Under discriminatory pricing, by charging a different price to high-type consumers for the service bundle or main service, the firm can capture the surplus from high-type consumers without distorting the prices charged to low-type consumers. Thus, the above explanation for the uniform pricing case no longer holds. Even if high-type consumers are very likely to purchase the ancillary service and low-type consumers are very unlikely to purchase the ancillary service, by bundling the ancillary service, the firm can still capture the ancillary service surplus of high-type consumers by charging at their willingness to pay. Similarly, even if high-type consumers are very unlikely to purchase the ancillary service and low-type consumers are very likely to purchase the ancillary service, by unbundling the ancillary service, the firm can still avoid charging an ancillary service price that is too low (the optimal ancillary service price is always equal to the total marginal cost).

Theorem IV.10. *(i) If high-type consumers are more likely to purchase the ancillary service than low-type consumers (i.e., $\beta_H \geq \beta_L$), when unbundling is more profitable under discriminatory pricing, it is also more profitable under uniform pricing (i.e., when $\Pi_u^* \geq \Pi_b^*$, we also have $\Pi_{u,n}^* \geq \Pi_{b,n}^*$).*

(ii) If low-type consumers are more likely to purchase the ancillary service than high-type consumers (i.e., $\beta_H < \beta_L$), when unbundling is more profitable under uniform pricing, it is also more profitable under discriminatory pricing (i.e., when $\Pi^*_{u,n} \geq \Pi^*_{b,n}$, we also have $\Pi^*_u \geq \Pi^*_b$).

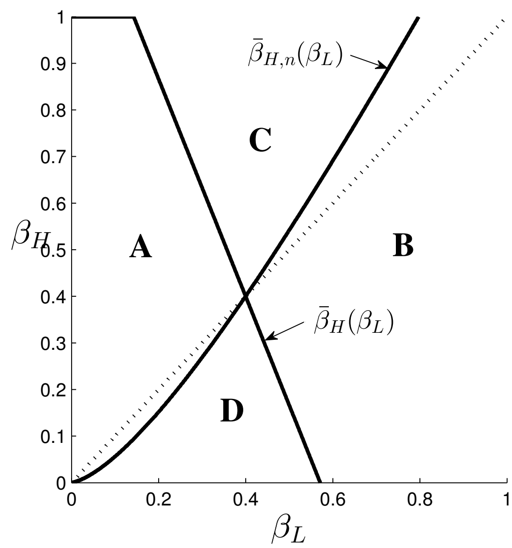


Figure 4.3: Comparison of optimal strategies under discriminatory pricing and uniform pricing ($v_H = 300$, $v_L = 200$, $\bar{u} = 50$, $\underline{u} = -20$, $c_m = 150$, $c_a = 20$, $c(x) = 0.5x^2$, $\lambda_H = 30$, $\lambda_L = 70$; Region A: unbundle in both cases; Region B: bundle in both cases; Region C: bundle under discriminatory pricing, unbundle under uniform pricing; Region D: unbundle under discriminatory pricing, bundle under uniform pricing)

Theorem IV.10 goes one step further and directly compares the optimal bundling (unbundling) strategies for a uniform-pricing firm and a price-discriminating firm. If high-type consumers are more likely to purchase the ancillary service than low-type consumers, or equivalently, if consumers' valuations for the main service and the ancillary service are positively correlated, bundling is more likely to be the optimal ancillary service strategy for a price-discriminating firm than a uniform pricing firm. If low-type consumers are more likely to purchase the ancillary service than high-type consumers, or equivalently, if consumers' valuations for the main service and the ancillary service are negatively correlated, unbundling is more likely to be the optimal ancillary service strategy for a price-discriminating firm than a uniform pricing firm. Figure 4.3 illustrates the result in Theorem IV.10 by plotting together the threshold functions under uniform pricing and discriminatory pricing. As Figure 4.3 shows, when consumers' valuations are positively correlated (i.e., in the region above the dotted line), the bundling region is larger for a price-discriminating firm; when consumers valuations are negatively correlated (i.e., in the region below the dotted line), the unbundling region is larger for a price-discriminating firm.

Therefore, when a firm switches from uniform pricing to discriminatory pricing for the main service, it should re-evaluate its policy for the ancillary service. For a firm managing an ancillary service that involves a positive consumer valuation correlation, a shift from unbundling to bundling may be needed; for a firm managing an ancillary service that involves a negative consumer valuation correlation, a shift from bundling to unbundling may be needed. Firms in several industries, such as sports game organizers and hotels, are currently trying to enforce inter-temporal price discrimination. Along with the adoption of main service price discrimination, it is important for these firms to identify which of their consumer segments values the ancillary service more and adjust the strategy for the ancillary service accordingly.

Theorem IV.11. *Consider two scenarios. In the first scenario, the demand sizes are λ_{H1} and λ_{L1}. In the second scenario, the demand sizes are λ_{H2} and λ_{L2}. Suppose $\lambda_{H1} + \lambda_{L1} = \lambda_{H2} + \lambda_{L2} = \lambda$ and $\lambda_{H1} < \lambda_{H2}$ (hence $\lambda_{L1} > \lambda_{L2}$). Then, $\bar{\beta}_{H,n1}(\beta_L) \geq \bar{\beta}_{H,n2}(\beta_L)$ in the region of $\beta_H \geq \beta_L$ and $\bar{\beta}_{H,n1}(\beta_L) < \bar{\beta}_{H,n2}(\beta_L)$ in the region of $\beta_H < \beta_L$; $\bar{\beta}_{H,n1}(\beta_L)$ and $\bar{\beta}_{H,n2}(\beta_L)$ intersect on the $45°$ line $\beta_H = \beta_L$.*

Finally, we study how the optimal ancillary service strategy differs for different firms and compare the result to our previous result in the discriminatory-pricing case. Theorem IV.11 characterizes how the optimal strategy is affected by the firm's demand portfolio in the uniform-pricing case. If high-type consumers are more likely to purchase the ancillary service than low-type consumers (i.e., $\beta_H \geq \beta_L$), then increasing the proportion of high-type consumers expands the region in which unbundling is optimal. If low-type consumers are more likely to purchase the ancillary service than high-type consumers (i.e., $\beta_H < \beta_L$), then increasing the proportion of high-type consumers expands the region in which bundling is optimal. If consumers' valuations for the main service and the ancillary service are positively correlated, compared to a firm with fewer high-type consumers, a firm with more high-type consumers has more incentive to capture the ancillary service surplus from high-type consumers, hence unbundling is more likely to be the optimal ancillary service strategy. On the other hand, if consumers' valuations for the main service and the ancillary service are negatively correlated, bundling is more likely to be the optimal ancillary service strategy for a firm with more high-type consumers.

Different from the airline industry where it is very common that consumers in different segments pay different prices for the same type of seats, discriminatory pricing of room rates is much less used in the hotel industry. Moreover, the most common ancillary services offered by hotels (e.g., breakfast, in-room internet connection) would usually involve a positive correlation between consumers' main service valuations and ancillary service valuations. Wealthier consumers are more likely to purchase these ancillary services from the hotel, whereas less wealthy consumers may seek cheaper outside options (e.g., having breakfast in a nearby fast-food store at a lower price). Thus, Theorem IV.11 indicates that unbundling is more likely to be optimal for hotels with higher proportions of wealthier consumers (e.g., luxury hotels) than hotels with lower proportions of wealthier consumers (e.g., economy hotels). The current industry practice is that luxury hotels usually charge for such ancillary services and economy hotels usually offer them for free. Our result here provides an explanation for this interesting phenomenon.

Interestingly, if we compare Theorem IV.11 to Theorem IV.3(iii) which characterizes how the optimal strategy is affected by the firm's demand portfolio in the discriminatory-pricing case, we see that the result is exactly reversed. Again, the fundamental reason is that the ancillary service price plays a different role in the uniform-pricing case than it does in the discriminatory-pricing case. Different from a discriminatory-pricing firm, a uniform-pricing firm uses the ancillary service price as a lever to capture more of the high-type consumers' surplus that is not captured by the uniform main service price. Table 4.1 summarizes the findings from this paper about the effect of the firm's demand portfolio on its optimal strategy for the ancillary service. It characterizes how the vertical differentiation of optimal ancillary service policies in an industry is jointly determined by firms' use of main service price discrimination as well as consumers' valuation structure.

	Uniform pricing	Discriminatory pricing
Positive consumer valuation correlation	Higher $\lambda_H\% \Rightarrow$ unbundle Lower $\lambda_H\% \Rightarrow$ bundle	Higher $\lambda_H\% \Rightarrow$ bundle Lower $\lambda_H\% \Rightarrow$ unbundle
Negative consumer valuation correlation	Higher $\lambda_H\% \Rightarrow$ bundle Lower $\lambda_H\% \Rightarrow$ unbundle	Higher $\lambda_H\% \Rightarrow$ unbundle Lower $\lambda_H\% \Rightarrow$ bundle

Table 4.1: Comparison of the effects of firm's demand portfolio on the optimal strategy for the ancillary service in the uniform-pricing case and in the discriminatory-pricing case

4.8 Conclusion

In this paper, we study whether and when a firm that sells a main service as well as an ancillary service benefits from charging separately for the ancillary service. We consider several important factors that affect this decision, including both firm characteristics (operating costs, demand portfolio, supply chain structure) and consumer characteristics (valuation structure, forward-looking or myopic). By analyzing a firm that price-discriminates when selling the main service, we find that it may be more profitable to unbundle the ancillary service because of the following three reasons: 1) consumers have low ancillary service needs, 2) a large proportion of consumers are myopic instead of forward-looking, 3) the firm is highly dependent on intermediaries to make sales. Our findings provide some explanations for airline ancillary service policies.

Moreover, we study how the firm's ability to price-discriminate when selling the main service affects its optimal strategy for the ancillary service. We find that some classic findings from the previous commodity bundling literature and two-part pricing literature actually do not carry through to the discriminatory-pricing case. Thus, our paper offers unique contributions to the existing literature. We find that whereas it is optimal for a uniform-pricing firm to unbundle the ancillary service if the consumers that value the main service higher have a high enough likelihood of purchasing the ancillary service, it is optimal for a price-discriminating firm to unbundle the ancillary service if the consumers that value the main service higher have a low enough likelihood of purchasing the ancillary service. Firm's ability to price-discriminate when selling the main service makes bundling (unbundling) more likely to be the optimal strategy for the ancillary service when consumers' valuations for the main service and the ancillary service are positively (negatively) correlated. This paper also provides the insight that firms' use of main service price discrimination and consumers' valuation structure jointly determine the optimal ancillary service policies in an industry.

CHAPTER V

Conclusion

This dissertation studies three topics in the arising fields of strategic-level revenue management and consumer-driven operations management. The results and findings provide managerial and policy insights to firms regarding the optimal selection of pricing policies.

In the first essay, I look into the event industry (i.e., sports games and concerts) and study a crucial issue in the industry – whether the existence of a resale market is beneficial or harmful to the event ticket providers. Three pricing policies are considered, including fixed pricing, multiperiod pricing, and option pricing. I find that contrary to what common wisdom may suggest, event ticket providers could benefit from the existence of a resale market in many cases under the currently used pricing policies (fixed and multiperiod pricing). Thus, efforts to move to paperless ticketing are likely to hurt not only consumers but also event ticket providers in many cases. I also find that by using a novel pricing policy with ticket options, an event ticket provider can eliminate consumer resale of tickets and reduce speculator resale of tickets. Option pricing not only results in the highest revenues for event ticket providers but also gives consumers greater choice. Therefore, my findings indicate that event organizers should not support paperless ticketing but instead consider novel pricing strategies such as ticket options.

In the second essay, I look into the hotel industry and study a type of conditional upgrades that are recently offered by many major hotels. I consider a model that incorporates consumers' strategic behavior of anticipating the upgrade probability when making booking decisions, and analyze the firm's optimal upgrade policy. My analysis reveals three benefits of conditional upgrades. First, conditional upgrades expand the firm's demand. Second, the upgrade price can correct the original suboptimal product price and reoptimize the firm's demand segmentation. Third, conditional upgrades give the firm more flexibility to match stochastic demand to supply and hence improve utilization. Conditional upgrades can effectively compensate for the firm's lack of ability in setting product prices optimally. For a firm that has the ability to set optimal static product prices, offering conditional upgrades can generate more revenue than using dynamic pricing.

In the third essay, I look into the airline and other travel industries and study whether a firm that sells a main service and also an ancillary service should separately charge for the ancillary service. First, I find that a firm that price-discriminates when selling the main service should unbundle the ancillary service because consumers have low ancillary service needs, or a large proportion of consumers are myopic instead of forward-looking, or the firm is highly dependent on intermediaries to make sales. I also study how the firm's ability to price-discriminate when selling the main service affects its optimal ancillary service policy. I find that whereas it is optimal for a uniform-pricing firm to unbundle the ancillary service if the consumers that value the main service higher have a high enough likelihood of purchasing the ancillary service, it is optimal for a price-discriminating firm to unbundle the ancillary service if the consumers that value the main service higher have a low enough likelihood of purchasing the ancillary service. In the case of uniform pricing, if consumers' valuations for the main service and the ancillary service are positively (negatively) correlated, unbundling (bundling) is more likely to be the optimal strategy for firms with higher proportions of high-type consumers. In the case of discriminatory pricing, if consumers' valuations for the main service and the ancillary service are positively (negatively) correlated, bundling (unbundling) is more likely to be the optimal strategy for firms with higher proportions of high-type consumers. This is exactly the opposite of the result for uniform-pricing firms.

The areas of strategic-level revenue management and consumer-driven operations management are rich with opportunities to explore the interface between operations management and other fields, such as economics and marketing. For example, one can use game theory models to incorporate consumer behaviors into operations management problems and study the interactions between firms and consumers. One can also use industrial organization models (e.g., price discrimination) to study firms' strategic-level pricing decisions. There are more interesting research questions in these areas that are worth studying analytically and empirically.

APPENDICES

APPENDIX A

Proofs of Theorems and Lemmas in Chapter II

Lemma A.1. *Under fixed pricing, given that the capacity provider's price is* p_f *and that* z *consumers and* y *speculators have purchased tickets in period 1, the equilibrium resale price in period 2 is*

$$r_f(z,y) = \begin{cases} \bar{r}(z,y) & \text{if } p_f > \bar{r}(z,y), \\ p_f & \text{if } \underline{r}(z,y) < p_f \le \bar{r}(z,y), \\ \underline{r}(z,y) & \text{if } p_f \le \underline{r}(z,y), \end{cases}$$

where $\bar{r}(z,y)$ *is the solution to* $(\lambda_1 - z + \lambda_2)\bar{F}(\bar{r}) = zF((1-\tau)\bar{r}) + y$ *and* $\underline{r}(z,y)$ *is the solution to* $(\lambda_1 - z + \lambda_2)\bar{F}(\underline{r}) = (C - z - y) + zF((1-\tau)\underline{r}) + y$.

Proof of Lemma A.1

In period 2, speculators resell tickets at the same price with consumers, because otherwise, the party with the lower price will raise it to gain more margin and if the party with the higher price cannot make sales, it will reduce the price to make sales. The provider has $C - z - y$ remaining capacity, and $\lambda_1 - z + \lambda_2$ consumers arrive, including the period 1 consumers that were not satisfied or decided to wait. $(\lambda_1 - z + \lambda_2)\bar{F}(p_f)$ is the number of consumers that are willing to buy the ticket if the ticket price is p_f. Also, $zF((1-\tau)r_f)$ is the number of period 1 consumers that would like to resell their tickets if the resale market price is r_f, because a period 1 consumer will want to resell her ticket if her valuation is smaller than the payoff from resale, $(1-\tau)r_f$.

If $p_f > \bar{r}(z,y)$, we have $(\lambda_1 - z + \lambda_2)\bar{F}(p_f) < zF((1-\tau)p_f) + y$. In this case, the equilibrium resale price is $r_f(z,y) = \bar{r}(z,y)$ which is lower than the capacity provider's price p_f. All demand is satisfied by the resale market. If $\underline{r}(z,y) < p_f \leq \bar{r}(z,y)$, we have $zF((1-\tau)p_f) + y \leq (\lambda_1 - z + \lambda_2)\bar{F}(p_f) < C - z + zF((1-\tau)p_f)$. In this case, resellers enter the resale market in the order of increasing valuations up to the one with valuation $(1-\tau)p_f$, because otherwise the resale price will be higher than p_f, hence the capacity provider will make sales first and the resellers with high valuations will not be able to make sales. Thus, the equilibrium resale price is $r_f(z,y) = p_f$ in this case. If $p_f \leq \underline{r}(z,y)$, we have $(\lambda_1 - z + \lambda_2)\bar{F}(p_f) \geq C - z + zF((1-\tau)p_f)$. In this case, the equilibrium resale price is $r_f(z,y) = \underline{r}(z,y)$ which is higher than or equal to the capacity provider's price p_f. The capacity provider sells tickets first and he sells out his capacity, the resale market captures the residual demand.

Proof of Theorem II.1

In the first part of the proof, we derive the period 1 consumers' purchasing decisions in equilibrium based on the equilibrium resale price given by Lemma A.1. Then, in the second part of the proof, we derive the capacity provider's optimal fixed price p_f^*. If a period 1 consumer buys a ticket, her payoff in period 2 is the maximum of her payoff from using the ticket, V, and her payoff from reselling the ticket, $(1-\tau)r_f(z,y)$. Thus, her payoff from buying a ticket in period 1 is $S_f^1(z,y) = -p_f + E[\max(V, (1-\tau)r_f(z,y))]$. If a period 1 consumer waits, then she can obtain a ticket in period 2 only if her valuation is high enough. As Lemma A.1 indicates, if $p_f > \bar{r}(z,y)$, she can buy a ticket from the resale market at price $\bar{r}(z,y)$ if $V > \bar{r}(z,y)$. If $\underline{r}(z,y) < p_f \leq \bar{r}(z,y)$, she can buy a ticket from either the resale market or the capacity provider at price p_f if $V > p_f$. If $p_f \le \underline{r}(z,y)$, she can buy a ticket from the capacity provider at price p_f if $V > \tilde{r}(z,y)$ where $\tilde{r}(z,y)$ is the solution to $(\lambda_1 - z + \lambda_2)\bar{F}(r) = C - z - y$, she can buy a ticket from the resale market at a higher price $\underline{r}(z,y)$ if $\underline{r}(z,y) < V \le \tilde{r}(z,y)$, and she may not obtain a ticket otherwise. Thus, a period 1 consumer's payoff from waiting is

$$S_f^2(z,y) = \begin{cases} E[V - \bar{r}(z,y)]^+ & \text{if } p_f > \bar{r}(z,y), \\ E(V - p_f)^+ & \text{if } \underline{r}(z,y) < p_f \le \bar{r}(z,y), \\ \int_{\tilde{r}(z,y)}^{\infty} (v - p_f)\,\mathrm{d}F(v) + \int_{\underline{r}(z,y)}^{\tilde{r}(z,y)} [v - \underline{r}(z,y)]\,\mathrm{d}F(v) & \text{if } p_f \le \underline{r}(z,y). \end{cases}$$

Note that the upper bound of z is $\bar{z} = \min(\lambda_1, C)$, as z cannot exceed the total number of period 1 consumers or the provider's capacity.

Next, we will show that any $z \in (0, \bar{z})$ cannot be an equilibrium. The Implicit Function Theorem gives that $\bar{r}(z,y)$ is decreasing in z, $\underline{r}(z,y)$ and $\tilde{r}(z,y)$ are increasing in z:

$$\begin{aligned} \frac{\partial \bar{r}}{\partial z} &= -\frac{\bar{F}(\bar{r}) + F((1-\tau)\bar{r})}{(\lambda_1 - z + \lambda_2) f(\bar{r}) + (1-\tau) z f((1-\tau)\bar{r})} \le 0, \\ \frac{\partial \underline{r}}{\partial z} &= \frac{\bar{F}((1-\tau)\underline{r}) - \bar{F}(\underline{r})}{(\lambda_1 - z + \lambda_2) f(\underline{r}) + (1-\tau) z f((1-\tau)\underline{r})} \ge 0, \\ \frac{\partial \tilde{r}}{\partial z} &= \frac{F(\tilde{r})}{(\lambda_1 - z + \lambda_2) f(\tilde{r})} > 0. \end{aligned}$$

First, consider the case of $p_f > \underline{r}(z,y)$. $y^* = 0$ in this case because speculators will incur a loss if entering the market. If $p_f > \bar{r}(z,0)$, $S_f^1(z,0) - S_f^2(z,0) = -p_f + E[\max(V, (1-\tau)\bar{r}(z,0))] - E[V - \bar{r}(z,0)]^+ < -\bar{r}(z,0) + E[\max(V, (1-\tau)\bar{r}(z,0))] - E[V - \bar{r}(z,0)]^+ = E[\max(V, (1-\tau)\bar{r}(z,0))] - E[\max(V, \bar{r}(z,0))] < 0$. Also, $S_f^1(z,0)$ is decreasing in z and $S_f^2(z,0)$ is increasing in z. Thus, we have $\sup_{0 \le z \le \bar{z}}(S_f^1(z,0)) < \inf_{0 \le z \le \bar{z}}(S_f^2(z,0))$, hence $z^* = 0$. Similarly, if $\underline{r}(z,y) < p_f \le \bar{r}(z,0)$, $S_f^1(z,0)$ and $S_f^2(z,0)$ stay constant with respect to z and $S_f^1(z,0) - S_f^2(z,0) = -p_f + E[\max(V, (1-\tau)p_f)] - E(V - p_f)^+ < 0$, hence we also have $z^* = 0$. Second, consider the case of

$p_f \le \underline{r}(z,y)$. Note that in this case, the equilibrium resale price is $\underline{r}(z,y)$ which is independent of y. We have $y^*(z) = C - z$ if $p_f < (1-\tau')\underline{r}(z,y)$ and $y^*(z) = 0$ otherwise. $S_f^1(z, y^*(z))$ is increasing in z and since

$$\begin{aligned}
\frac{\mathrm{d}S_f^2(z,y^*(z))}{\mathrm{d}z} &= [p_f - \underline{r}(z,y^*(z))]f(\tilde{r}(z,y^*(z)))\frac{\mathrm{d}\tilde{r}(z,y^*(z))}{\mathrm{d}z} \\
&\quad -[F(\tilde{r}(z,y^*(z))) - F(\underline{r}(z,y^*(z)))]\frac{\mathrm{d}\underline{r}(z,y^*(z))}{\mathrm{d}z} \\
&\le 0,
\end{aligned}$$

$S_f^2(z, y^*(z))$ is decreasing in z. Thus, if a small portion of period 1 consumers who are currently waiting deviate to buying tickets, more such deviations will occur; and vice versa. Therefore, $z^* = \bar{z}$ and $z^* = 0$ are the only possible equilibria. To induce $z^* = \bar{z}$, p_f needs to satisfy[1] $p_f \le \underline{r}(\bar{z}, y^*(\bar{z}))$ and $S_f^1(\bar{z}, y^*(\bar{z})) \ge S_f^2(0, y^*(0))$. The equilibrium resale price is $\underline{r}(\bar{z}, y^*(\bar{z})) = r_f^*$, hence Part (i) of the theorem is proved. Additionally, since the equilibrium resides in the case of $p_f \le \underline{r}(z,y)$, the proof of Lemma A.1 indicates that the provider sells out his capacity, hence his revenue is $p_f C$.

Now we derive p_f^*. Note that $\underline{r}(0, y^*(0)) = p_s$ and $\underline{r}(\bar{z}, y^*(\bar{z})) = r_f^*$. For $p_s < p_f \le r_f^*$, $S_f^2(0, y^*(0)) = E(V - p_f)^+$. In this case, $S_f^1(\bar{z}, y^*(\bar{z})) \ge S_f^2(0, y^*(0))$ becomes $-p_f + E[\max(V, (1-\tau)r_f^*)] \ge E(V - p_f)^+$, or equivalently, $E[\max(V, (1-\tau)r_f^*)] \ge E[\max(V, p_f)]$, which can be simplified to $p_f \le \max((1-\tau)r_f^*, v_{\min})$. Consider $[(\lambda_1 - C)^+ + \lambda_2]\bar{F}(r_f^*) = (C - \lambda_1)^+ + \min(\lambda_1, C)F((1-\tau)r_f^*)$ which defines r_f^*. As τ increases, the rhs decreases, so we need to increase r_f^* to maintain equality. Both the lhs and the rhs become smaller when the equality is reached again. Thus, as τ increases, r_f^* increases and $(1-\tau)r_f^*$ decreases. When $\tau = 0$, $(1-\tau)r_f^* = p_s$, hence $p_s > (1-\tau)r_f^*$ for any $\tau > 0$. Since $p_s > v_{\min}$, we have $p_f > p_s > \max((1-\tau)r_f^*, v_{\min})$ which contradicts

[1] We assume when period 1 consumers are indifferent between buying tickets and waiting, they buy immediately. The capacity provider can resolve the consumer indifference by reducing p_f by an infinitesimally small amount.

$S_f^1(\bar{z}, y^*(\bar{z})) > S_f^2(0, y^*(0))$. Therefore, $p_s < p_f \le r_f^*$ is not feasible.

For $p_f \le p_s$, $y^*(0) = C$ if $p_f < (1-\tau')p_s$ and $y^*(0) = 0$ otherwise; $y^*(\bar{z}) = (C-\lambda_1)^+$ if $p_f < (1-\tau')r_f^*$ and $y^*(\bar{z}) = 0$ otherwise. Since $p_s < r_f^*$, when $y^*(\bar{z}) = 0$, we must also have $y^*(0) = 0$. In this case, $S_f^2(0, y^*(0)) = \int_{p_s}^{\infty}(v - p_f)\,\mathrm{d}F(v)$, and we can rewrite $S_f^1(\bar{z}, y^*(\bar{z})) \ge S_f^2(0, y^*(0))$ as $p_f + \int_{p_s}^{\infty}(v - p_f)\,\mathrm{d}F(v) \le E[\max(V, (1-\tau)r_f^*)]$, the binding solution of which is p_f^n (the superscript of "n" refers to no speculators in equilibrium). Since the lhs of the above inequality is increasing in p_f, $p_f^* = p_f^n$ in this case. $y^*(\bar{z}) = 0$ indeed occurs if $p_f^n \ge (1-\tau')r_f^*$ or $\tau' \ge \bar{\tau}_f'(\tau)$. Thus, $y^*(\bar{z}) = 0$ if $\tau' \ge \bar{\tau}_f'(\tau)$ and $y^*(\bar{z}) = (C - \lambda_1)^+$ otherwise, Part (iii) of the theorem is proved. Finally, we consider the case of $\tau' < \bar{\tau}_f'(\tau)$ where $y^*(\bar{z}) = (C - \lambda_1)^+$. If $p_f^n \ge (1-\tau')p_s$, $y^*(0) = 0$ hence we still have $p_f^* = p_f^n$ in this case. If $p_f^n < (1-\tau')p_s$, $y^*(0) = C$ hence we have $S_f^2(0, y^*(0)) = E(V - p_s)^+$ and $S_f^1(\bar{z}, y^*(\bar{z})) \ge S_f^2(0, y^*(0))$ becomes $p_f \le E[\max(V, (1-\tau)r_f^*)] - E(V - p_s)^+$. Thus, we have $p_f^* = \min\left(E[\max(V, (1-\tau)r_f^*)] - E(V-p_s)^+, (1-\tau')p_s\right)$ when $\tau' < \bar{\tau}_f'(\tau)$. Combining all the cases above, we obtain the optimal fixed price given in Part (ii) of the theorem. Finally, we have shown that r_f^* is increasing in τ and we prove in Theorem II.2 that p_f^* is decreasing in τ. Then, since $p_f^* = r_f^* = p_s$ when $\tau = 0$, it follows that $p_f^* < r_f^*$. The proof is completed.

Proof of Theorem II.2

We showed in the proof of Theorem II.1 that $(1-\tau)r_f^*$ is decreasing in τ. Recall that p_f^n is the solution to $p_f^n + \int_{p_s}^{\infty}(v - p_f^n)\,\mathrm{d}F(v) = E[\max(V, (1-\tau)r_f^*)]$. Since the lhs is increasing in p_f^n, p_f^n is decreasing in τ. Thus, p_f^* is decreasing in τ. p_f^* is decreasing in τ' because p_f^* is decreasing in τ' when $p_f^* = (1-\tau')p_s$ and stays constant in τ' otherwise. The remaining results follow directly.

Lemma A.2. *Under multiperiod pricing, given that z consumers and y speculators have purchased tickets in period 1, in equilibrium, the capacity provider's period 2 price $p_2(z, y)$ and the resale price $r_m(z, y)$ are given by $p_2(z, y) = r_m(z, y) = \underline{r}(z, y)$.*

Proof of Lemma A.2

$p_2(z,y) = r_m(z,y)$ in equilibrium because otherwise, the party with the lower price will raise it to gain more margin, and if the party with the higher price cannot make sales, it will reduce the price to make sales. Moreover, the equilibrium prices are equal to $\underline{r}(z,y)$ which is the marketing clearing price in period 2 where the total supply comes from both the capacity provider and the resale market. If the prices are lower than $\underline{r}(z,y)$, both parties have the incentive to increase the price and earn more revenue. If the prices are higher than $\underline{r}(z,y)$ so that the provider has un-sold capacity, since he does not ration capacity in period 2, he will decrease $p_2(z,y)$ to sell more tickets. In this case, more resellers will enter the market and $r_m(z,y)$ is decreased to $p_2(z,y)$ until the market is cleared.

Proof of Theorem II.3

We follow the same approach of deriving the optimal pricing policy under fixed pricing. Period 1 consumers' payoffs from purchasing tickets in period 1 and waiting under multiperiod pricing are $S_m^1(z,y) = -p_1 + E[\max(V,(1-\tau)\underline{r}(z,y))]$ and $S_m^2(z,y) = E[V - \underline{r}(z,y)]^+$, respectively. $\underline{r}(z,y)$ is increasing in z as derived in the proof of Theorem II.1. Thus, $S_m^1(z,y)$ is increasing in z while $S_m^2(z,y)$ is decreasing in z, hence the only possible equilibria are $z^* = \bar{z}$ and $z^* = 0$. To induce $z^* = \bar{z}$, p_1 needs to satisfy $S_m^1(\bar{z}, y^*(\bar{z})) \geq S_m^2(0, y^*(0))$ or $p_1 \leq E[\max(V,(1-\tau)\underline{r}(\bar{z},y^*(\bar{z})))] - E[V - \underline{r}(0,y^*(0))]^+ = E[\max(V,(1-\tau)p_2^*)] - E(V-p_s)^+$ (note that $\underline{r}(z,y)$ is independent of y). Part (i) of the theorem follows from Lemma A.2. Moreover, $y^*(\bar{z}) = (C-\lambda_1)^+$ if $p_1 < (1-\tau')p_2^*$ and $y^*(\bar{z}) = 0$ otherwise. As the provider sells out the capacity with $z^* = \bar{z}$, his revenue is $\Pi_m(p_1) = p_1 C$ if $p_1 < (1-\tau')p_2^*$ and $\Pi_m(p_1) = p_1 \min(\lambda_1, C) + p_2^*(C-\lambda_1)^+$ otherwise. Since $\Pi_m(p_1)$ is increasing in p_1, $p_1^* = E[\max(V,(1-\tau)p_2^*] - E(V-p_s)^+$. In the proof of Theorem II.1, we showed that p_2^* is increasing in τ and $(1-\tau)p_2^*$ is decreasing in τ, hence p_1^* is decreasing in τ. Then, since $p_1^* = p_2^* = p_s$ when $\tau = 0$, we have $p_1^* < p_2^*$. Part (ii) of the theorem is proved. Finally, Part (iii) holds because $p_1^* < (1-\tau')p_2^*$ is equivalent to $\tau' < \bar{\tau}'_m(\tau)$.

Proof of Theorem II.4

Part (i) is proved in the proof of Theorem II.3. When $\tau > \bar{\tau}_m(\tau')$, the optimal revenue from multiperiod pricing is $\Pi_m^* = p_1^* C$, hence it is decreasing in τ. Part (ii) is proved.

Now we prove Part (iii). As $\tau \le \bar{\tau}_m(\tau')$, we have $y^*(\bar{z}) = 0$. If $C \le \lambda_1$, $\Pi_m^* = p_1^* C$ which is decreasing in τ. Next, consider $\lambda_1 < C < \lambda_1 + \lambda_2$. For $\tau \ge \hat{\tau}(C)$, $(1-\tau)p_2^* \le v_{\min}$ and p_2^* is the solution to $\lambda_2 \bar{F}(p_2^*) = C - \lambda_1$. In this case, p_2^* is independent of τ and so is $\Pi_m^* = E[\min(V, p_s)]\lambda_1 + p_2^*(C - \lambda_1)$. For $\tau < \hat{\tau}(C)$, $(1-\tau)p_2^* > v_{\min}$ and $\Pi_m^* = \{E[\max(V, (1-\tau)p_2^*)] - E(V - p_s)^+\}\, \lambda_1 + p_2^*(C - \lambda_1)$, where p_2^* is the solution to $\lambda_2 \bar{F}(p_2^*) = C - \lambda_1 \bar{F}((1-\tau)p_2^*)$. Taking derivative with respect to τ on both sides of this equation yields

$$\frac{\mathrm{d}[(1-\tau)p_2^*]}{\mathrm{d}\tau} = -\frac{\lambda_2 f(p_2^*)}{\lambda_1 f((1-\tau)p_2^*)} \frac{\mathrm{d}p_2^*}{\mathrm{d}\tau}.$$

Thus

$$\begin{aligned} \frac{\mathrm{d}\Pi_m^*}{\mathrm{d}\tau} &= \lambda_1 F((1-\tau)p_2^*) \frac{\mathrm{d}[(1-\tau)p_2^*]}{\mathrm{d}\tau} + (C - \lambda_1) \frac{\mathrm{d}p_2^*}{\mathrm{d}\tau} \\ &= \frac{\mathrm{d}p_2^*}{\mathrm{d}\tau} \left[C - \lambda_1 - \frac{\lambda_2 F((1-\tau)p_2^*) f(p_2^*)}{f((1-\tau)p_2^*)} \right]. \end{aligned} \tag{A.1}$$

Since $f(\cdot)$ is decreasing, as τ increases, $f(p_2^*)$ decreases, $F((1-\tau)p_2^*)$ decreases and $f((1-\tau)p_2^*)$ increases, hence the terms within the bracket in (A.1) are increasing in τ. Then, since $\mathrm{d}p_2^*/\mathrm{d}\tau > 0$, Π_m^* is quasi-convex in τ for $\tau \le \bar{\tau}_m(\tau')$. When $\tau = 0$, the terms in the bracket become $C - \lambda_1 - \lambda_2 F(p_s)$, as $p_2^* = p_s$ when $\tau = 0$. We consider $C - \lambda_1 - \lambda_2 F(p_s)$ as a function of C. Since p_s is decreasing in C, $C - \lambda_1 - \lambda_2 F(p_s)$ is increasing in C. If $C = \lambda_1$, $C - \lambda_1 - \lambda_2 F(p_s) = -\lambda_2 F(p_s) < 0$; if $C = \lambda_1 + \lambda_2$, $p_s = v_{\min}$ and $C - \lambda_1 - \lambda_2 F(p_s) = \lambda_2 > 0$. Thus, there exists a threshold $\bar{C}$ ($\lambda_1 < \bar{C} < \lambda_1 + \lambda_2$) such that, Π_m^* is decreasing in τ at $\tau = 0$ if $\lambda_1 < C < \bar{C}$ and Π_m^* is increasing in τ

at $\tau = 0$ if $\bar{C} \le C < \lambda_1 + \lambda_2$. Thus, due to quasi-convexity, we conclude that for $\tau \le \bar{\tau}_m(\tau')$, Π_m^* is decreasing in τ if $C \le \lambda_1$ and increasing in τ if $C \ge \bar{C}$; otherwise, Π_m^* may be decreasing or first decreasing then increasing in τ. The result in Part (iii) of the theorem regarding which τ gives the highest revenue follows from the monotonicity results we obtained above as well as Part (ii). The τ' that maximizes the revenue is $\tau' = \tau$ because we already know that the revenue decreases when speculators enter the market.

Lemma A.3. *Under option pricing, given that the strike price is p and that z consumers and y speculators have purchased options in period 1, in equilibrium, the capacity provider's period 2 price $p_o(z, y)$ and the resale price $r_o(z, y)$ are given by $p_o(z, y) = r_o(z, y) = \inf\{r \ge v_{\min} : (\lambda_1 - z + \lambda_2)\bar{F}(r) \le C - z\bar{F}(\max((1-\tau)r, p))\}$.*

Proof of Lemma A.3

Define $\bar{V} = \max(V, (1-\tau)r_o)$ and $\bar{F}_{\bar{V}}(t) = P(\bar{V} > t)$. In period 2, $z\bar{F}_{\bar{V}}(p)$ consumers exercise the options, $z\bar{F}_{\bar{V}}(p)P(V \le (1-\tau)r_o|\bar{V} > p)$ consumers resell the tickets, $y\mathbf{1}_{p<(1-\tau')r_o}$ speculators exercise the options then resell the tickets. Thus, the provider's remaining capacity to sell in period 2 is $C - z\bar{F}_{\bar{V}}(p) - y\mathbf{1}_{p<(1-\tau')r_o}$. Following the proof of Lemma A.2, we have $p_o(z, y) = r_o(z, y) = \inf\{r \ge v_{\min} : (\lambda_1 - z + \lambda_2)\bar{F}(r) \le [C - z\bar{F}_{\bar{V}}(p) - y\mathbf{1}_{p<(1-\tau')r_o}] + z\bar{F}_{\bar{V}}(p)P(V \le (1-\tau)r|\bar{V} > p)\} + y\mathbf{1}_{p<(1-\tau')r_o}$. For any r, $[C - z\bar{F}_{\bar{V}}(p) - y\mathbf{1}_{p<(1-\tau')r_o}] + z\bar{F}_{\bar{V}}(p)P(V \le (1-\tau)r|\bar{V} > p) + y\mathbf{1}_{p<(1-\tau')r_o} = C - z\bar{F}_{\bar{V}}(p)P(V > (1-\tau)r|\bar{V} > p) = C - zP(V > (1-\tau)r, \bar{V} > p) = C - z\bar{F}(\max((1-\tau)r, p))$.

Proof of Theorem II.5

To improve readability, we divide this long proof into four steps. In Step 1, we derive period 1 consumers' purchasing decisions in equilibrium. In Step 2, we show that consumers do not resell tickets in equilibrium. In Step 3, we derive the optimal strike price p^*. In Step 4, we show how the optimal prices and revenue change with respect to τ.

Step 1: Period 1 consumers' payoffs from buying options in period 1 and waiting are $S_o^1(z,y) = -x + E[\max(V,(1-\tau)p_o(z,y)) - p]^+$ and $S_o^2(z,y) = E[V - p_o(y,z)]^+$, respectively, where $p_o(z,y)$ is independent of y. Note that $p < (1-\tau)p_o(z,y)$ is equivalent to $p < (1-\tau)\underline{r}(z,y)$. If $p < (1-\tau)\underline{r}(z,y)$, as shown in the main text, option pricing is equivalent to multiperiod pricing, so the proof of Theorem II.3 implies that the only possible equilibria are $z^* = \bar{z}$ or $z^* = 0$. If $p \geq (1-\tau)\underline{r}(z,y)$, $S_o^1(z,y) = -x + E(V-p)^+$ and $p_o(z,y) = \inf\{r \geq v_{\min} : (\lambda_1 - z + \lambda_2)\bar{F}(r) \leq C - z\bar{F}(p)\}$. We have several subcases to discuss for $p \geq (1-\tau)\underline{r}(z,y)$. Define $\hat{p}(z) = \inf\{p \geq v_{\min} : \lambda_1 + \lambda_2 \leq C + zF(p)\}$. If $p > \hat{p}(z)$, $p_o(z,y) = v_{\min}$ and $S_o^1(z,y) < S_o^2(z,y)$, hence $z^* = 0$. If $p \leq \hat{p}(z)$, the provider sells out his remaining capacity in period 2. In this case, $p > p_o(z,y)$ if and only if $p > p_s$. If $p > p_s$, we still have $S_o^1(z,y) < S_o^2(z,y)$, hence $z^* = 0$. Otherwise, $p_o(z,y)$ is increasing in z, so $S_o^2(z,y)$ is decreasing in z. Thus, if a small portion of period 1 consumers who are currently buying options deviate to waiting, more such deviations will occur, then we know $z^* = \bar{z}$ and $z^* = 0$ are the only possible equilibria. Combining all the cases discussed above, we conclude that for any p, the only possible equilibria are $z^* = \bar{z}$ and $z^* = 0$. $z^* = 0$ is always a possible equilibrium. $z^* = \bar{z}$ is a possible equilibrium only if $p \leq \min(\hat{p}(\bar{z}), p_s) = p_s$ as we can easily prove $\hat{p}(\bar{z}) > p_s$; in this case, the provider's capacity is sold out.

To induce $z^* = \bar{z}$, x and p need to satisfy $p \leq p_s$ as well as $S_o^1(\bar{z}, y^*(\bar{z})) \geq S_o^2(0, y^*(0))$ or $-x + E[\max(V,(1-\tau)p_o(\bar{z}, y^*(\bar{z}))) - p]^+ \geq E(V - p_s)^+$. The capacity provider's revenue is $\Pi_o(x,p) = x[\bar{z} + y^*(\bar{z})] + p[\bar{z}\bar{F}_{\bar{V}}(p) + y^*(\bar{z})] + p_o(\bar{z}, y^*(\bar{z}))[C - \bar{z}\bar{F}_{\bar{V}}(p) - y^*(\bar{z})]$, where $y^*(\bar{z}) = (C - \lambda_1)^+$ if $x + p < (1-\tau')p_o(\bar{z}, y^*(\bar{z}))$ and $y^*(\bar{z}) = 0$ otherwise. Since $\Pi_o(x,p)$ is increasing in x, $x^*(p) = E[\max(V,(1-\tau)p_o(\bar{z}, y^*(\bar{z}))) - p]^+ - E(V - p_s)^+$. Note that $p \leq p_s$ ensures $x \geq 0$. We focus on $\Pi_o(p) = \Pi_o(x^*(p), p)$ from now on.

Step 2: Next, we show that in equilibrium, we must have $p^* \geq (1-\tau)p_o(\bar{z}, y^*(\bar{z}))$, or equivalently, $p^* \geq (1-\tau)p_2^*$, so that consumers do not resell tickets. When $p < (1-\tau)p_2^*$, $\Pi_o(p) = \{E[\max(V,(1-\tau)p_2^*)] - E(V-p_s)^+\}[\bar{z}+y^*(\bar{z})] + p_2^*[C-\bar{z}-y^*(\bar{z})]$. Since $x^*(p)+p$ is increasing in p and $p_o(\bar{z},y^*(\bar{z})) = p_2^*$ is independent of p in this case, as we increase p, $y^*(\bar{z})$ may shift from $(C-\lambda_1)^+$ to 0. When this occurs, $\Pi_o(p)$ becomes larger because $x^*(p)+p = E[\max(V,(1-\tau)p_2^*)] - E(V-p_s)^+ < E[\max(V,p_2^*)] - E(V-p_2^*)^+ = p_2^*$. Other than when $y^*(\bar{z})$ shifts from $(C-\lambda_1)^+$ to 0, $\Pi_o(p)$ is constant in p. Thus, $\Pi_o(p)$ is increasing in p for $p < (1-\tau)p_2^*$. On the other hand, when $p \geq (1-\tau)p_2^*$, $\Pi_o(p) = [E(V-p)^+ - E(V-p_s)^+][\bar{z}+y^*(\bar{z})] + p[\bar{z}\bar{F}(p) + y^*(\bar{z})] + p_o(\bar{z},y^*(\bar{z}))[C-\bar{z}\bar{F}(p)-y^*(\bar{z})]$. Then, we have

$$\lim_{p\downarrow(1-\tau)p_2^*} \Pi_o(p) - \lim_{p\uparrow(1-\tau)p_2^*} \Pi_o(p) = \tau p_2^* \bar{z} F((1-\tau)p_2^*) \geq 0.$$

Thus, $p \geq (1-\tau)p_2^*$ results in a higher revenue than $p < (1-\tau)p_2^*$, hence the optimal strike price satisfies $p^* \geq (1-\tau)p_o(\bar{z},y^*(\bar{z}))$, in which case $x^* = E(V-p^*)^+ - E(V-p_s)^+$ and $p_o(\bar{z},y^*(\bar{z}))$ is indeed given by Part (ii) of the theorem. The characterization of p_o^* and r_o^* then follows from Lemma A.3.

Step 3: Now we derive p^*. We first derive p^n which is the optimal strike price in the absence of speculators. If $p \leq v_{\min}$, p_o is independent of p, and so is $\Pi_o(p) = E[\min(V,p_s)]\min(\lambda_1,C) + p_o(C-\lambda_1)^+$. We then restrict p^n to be no less than $v_{\min}$, hence the feasible region of p becomes $\max((1-\tau)p_2^*, v_{\min}) \leq p \leq p_s$ and we have $\Pi_o(p) = [E(V-p)^+ - E(V-p_s)^+ + p\bar{F}(p)]\min(\lambda_1,C) + p_o(p)[C-\min(\lambda_1,C)\bar{F}(p)]$. When $p_o(p) = v_{\min}$ which occurs for larger enough p, it is easy to see that $\Pi_o(p)$ is decreasing in p, hence this case does not result in the optimal solution. We then know that at optimality, p_o is the solution to $[(\lambda_1-C)^+ + \lambda_2]\bar{F}(p_o) = C - \min(\lambda_1,C)\bar{F}(p)$. The Implicit Function Theorem gives

$$\frac{\mathrm{d}p_o}{\mathrm{d}p} = -\frac{\min(\lambda_1,C)f(p)}{[(\lambda_1-C)^+ + \lambda_2]f(p_o)}.$$

Taking derivative of $\Pi_o(p)$ with respect to p gives

$$\begin{aligned}\frac{\mathrm{d}\Pi_o}{\mathrm{d}p} &= \min(\lambda_1, C)f(p)(p_o - p) - \frac{\min(\lambda_1, C)f(p)[C - \min(\lambda_1, C)\bar{F}(p)]}{[(\lambda_1 - C)^+ + \lambda_2]f(p_o)} \\ &= \min(\lambda_1, C)f(p)\left[p_o - p - \frac{\bar{F}(p_o)}{f(p_o)}\right].\end{aligned}$$

Note that the second equality follows from $[(\lambda_1 - C)^+ + \lambda_2]\bar{F}(p_o) = C - \min(\lambda_1, C)\bar{F}(p)$. Since p_o is decreasing in p and $F(\cdot)$ has an increasing failure rate, $p_o - p - \bar{F}(p_o)/f(p_o)$ is decreasing in p, hence $\Pi_o(p)$ is quasi-concave. Then, if $\Pi_o(p)$ is decreasing at $p = \max((1-\tau)p_2^*, v_{\min})$, we have $p^n = \max((1-\tau)p_2^*, v_{\min})$; otherwise, we have $p^n > \max((1-\tau)p_2^*, v_{\min})$.

We need to determine the sign of $p_o - p - \bar{F}(p_o)/f(p_o)$ at $p = (1-\tau)p_2^*$. At $p = (1-\tau)p_2^*$, $p_o = p_2^*$ and $p_o - p - \bar{F}(p_o)/f(p_o) = \tau p_2^* - \bar{F}(p_2^*)/f(p_2^*)$. Consider $\tau p_2^* - \bar{F}(p_2^*)/f(p_2^*)$ as a function of τ. Since p_2^* is increasing in τ, $\tau p_2^* - \bar{F}(p_2^*)/f(p_2^*)$ is increasing in τ. When $\tau = 0$, $\tau p_2^* - \bar{F}(p_2^*)/f(p_2^*) = -\bar{F}(p_2^*)/f(p_2^*) < 0$. When $\tau = 1$, $\tau p_2^* - \bar{F}(p_2^*)/f(p_2^*) = p_2^* - \bar{F}(p_2^*)/f(p_2^*) > 0$ if $C \le \lambda_1$ because $p_2^* = \infty$ if $C \le \lambda_1$. However, if $\lambda_1 < C < \lambda_1 + \lambda_2$, p_2^* is given by $\bar{F}(p_2^*) = (C - \lambda_1)/\lambda_2$, hence p_2^* is finite. Then it may occur that $p_2^* - \bar{F}(p_2^*)/f(p_2^*) \le 0$. Define $\hat{r}$ as the solution to $\hat{r}f(\hat{r}) = \bar{F}(\hat{r})$ and define $\hat{C} \in (\lambda_1, \lambda_1 + \lambda_2)$ as the solution to $\bar{F}(\hat{r}) = (C - \lambda_1)/\lambda_2$. Then, when $\tau = 1$, if $\lambda_1 < C < \hat{C}$, we have $p_2^* > \hat{r}$ and $p_2^* - \bar{F}(p_2^*)/f(p_2^*) > 0$ which also occurs if $C \le \lambda_1$; if $\hat{C} \le C < \lambda_1 + \lambda_2$, we have $p_2^* \le \hat{r}$ and $p_2^* - \bar{F}(p_2^*)/f(p_2^*) \le 0$. Thus, if $\hat{C} \le C < \lambda_1 + \lambda_2$, $p_o - p - \bar{F}(p_o)/f(p_o) \le 0$ for any τ. If $C < \hat{C}$, define $\tilde{\tau}(C) \in (0, 1)$ as the solution to $\tau p_2^* = \bar{F}(p_2^*)/f(p_2^*)$. Then $p_o - p - \bar{F}(p_o)/f(p_o) \le 0$ for $\tau \le \tilde{\tau}(C)$ and $p_o - p - \bar{F}(p_o)/f(p_o) > 0$ for $\tau > \tilde{\tau}(C)$.

The above analysis implies that if $\hat{C} \le C < \lambda_1 + \lambda_2$ or if $C < \hat{C}$ and $\tau < \tilde{\tau}(C)$, $\mathrm{d}\Pi_o/\mathrm{d}p < 0$ for all $p > (1-\tau)p_2^*$. However, whether $p^n = (1-\tau)p_2^*$ or not depends on whether $(1-\tau)p_2^* \ge v_{\min}$ or not, so we need to determine whether $\tilde{\tau}(C)$ or $\hat{\tau}(C)$ is larger. We evaluate the sign of $\hat{\tau}(C)p_2^* - \bar{F}(p_2^*)/f(p_2^*)$ by varying C. If $C \ge \hat{C}$, $p_2^* \le$

$\bar{F}(p_2^*)/f(p_2^*)$ hence $\hat{\tau}(C)p_2^* < \bar{F}(p_2^*)/f(p_2^*)$, so $\hat{\tau}(C) < \tilde{\tau}(C)$. If $C \leq \lambda_1$, $p_2^* = \infty$ hence $\hat{\tau}(C)p_2^* > \bar{F}(p_2^*)/f(p_2^*)$, so $\hat{\tau}(C) > \tilde{\tau}(C)$. Moreover, as C increases, p_2^* decreases and $\hat{\tau}(C)$ decreases, hence $\hat{\tau}(C)p_2^* - \bar{F}(p_2^*)/f(p_2^*)$ decreases. Thus, there exists a threshold $\tilde{C} \in (\lambda_1, \hat{C})$ such that $\hat{\tau}(C) > \tilde{\tau}(C)$ if $C < \tilde{C}$ and $\hat{\tau}(C) \leq \tilde{\tau}(C)$ if $\tilde{C} \leq C < \lambda_1 + \lambda_2$.

Based on the above results, we can characterize p^n for different levels of C and τ. If $C < \tilde{C}$, $p^n = (1-\tau)p_2^*$ when $\tau \leq \tilde{\tau}(C)$. When $\tau > \tilde{\tau}(C)$, $\Pi_o(p)$ first increases then decreases in p for $p > (1-\tau)p_2^*$. Thus, $p^n > (1-\tau)p_2^*$ and p^n is the solution to the first-order condition, $p_o - p^n - \bar{F}(p_o)/f(p_o) = 0$. Note that in this case, p^n is independent of τ; also, p^n is indeed feasible (i.e., $p^n \leq p_s$), because when $\tau = \tilde{\tau}(C)$, $p^n = [1-\tilde{\tau}(C)]p_2^* < p_s$. On the other hand, if $\tilde{C} \leq C < \lambda_1 + \lambda_2$, since $\hat{\tau}(C) \leq \tilde{\tau}(C)$, when $\tau \leq \hat{\tau}(C)$, we have $p^n = (1-\tau)p_2^*$. When $\hat{\tau}(C) < \tau \leq \tilde{\tau}(C)$, since $(1-\tau)p_2^* < v_{\min}$, we have $p^n = v_{\min}$. When $\tau > \tilde{\tau}(C)$, p^n remains constant at $v_{\min}$. Therefore, if $\tilde{C} \leq C < \lambda_1 + \lambda_2$, $p^n = \max((1-\tau)p_2^*, v_{\min})$.

Now that we have found p^n, we proceed to characterize p^* by incorporating the case where speculators enter the market in equilibrium. Consider p_o as a function of p. If $x^*(p^n) + p^n = E[\max(V, p^n)] - E(V - p_s)^+ \geq (1-\tau')p_o(p^n)$ or $\tau' \geq 1 - \{E[\max(V, p^n)] - E(V-p_s)^+\}/p_o(p^n)$, we have $y^*(\bar{z}) = 0$ and $p^* = p^n$. If $\tau' < 1 - \{E[\max(V, p^n)] - E(V-p_s)^+\}/p_o(p^n)$, define $\bar{p}$ as the solution to $E[\max(V, p)] - E(V-p_s)^+ = (1-\tau')p_o(p)$. Since $E[\max(V, p)] - E(V-p_s)^+$ is increasing in p while $(1-\tau')p_o(p)$ is decreasing in p, $\bar{p} > p^n$. For $(1-\tau)p_2^* \leq p < \bar{p}$, we have $y^*(\bar{z}) = (C-\lambda_1)^+$ and $\Pi_o(p) = [E(V-p)^+ - E(V-p_s)^+]C + p[\min(\lambda_1, C)\bar{F}(p) + (C-\lambda_1)^+] + p_o(p)\min(\lambda_1, C)F(p)$. Denote $p^s = \arg\max_{(1-\tau)p_2^* \leq p < \bar{p}}(\Pi_o(p))$ as the optimal strike price when speculators exist in equilibrium. For $p \geq \bar{p}$, we have $y^*(\bar{z}) = 0$. Since $\Pi_o(p)$ is decreasing in p for $p \geq \bar{p}$, $\arg\max_{p \geq \bar{p}}(\Pi_o(p)) = \bar{p}$. Now we need to compare $\Pi_o(p^s)$ and $\Pi_o(\bar{p})$ to determine p^*. Since $\bar{p}$ is decreasing in τ', $\Pi_o(\bar{p})$ is increasing in τ' while $\Pi_o(p^s)$ is decreasing in τ'. Thus, there exists a threshold $\bar{\tau}_o'(\tau) \leq 1 - \{E[\max(V, p^n)] - E(V-p_s)^+\}/p_o(p^n)$ such that $p^* = \bar{p}$ and $y^*(\bar{z}) = 0$ if

$\bar{\tau}_o'(\tau) \le \tau' < 1 - \{E[\max(V, p^n)] - E(V - p_s)^+\}/p_o(p^n)$, $p^* = p^s$ and $y^*(\bar{z}) = (C - \lambda_1)^+$ if $\tau' < \bar{\tau}_o'(\tau)$. Part (iii) of the theorem is proved.

By now, we have fully characterized the optimal prices: 1) $p^* = p^n$ if $\tau' \ge 1 - \{E[\max(V, p^n)] - E(V - p_s)^+\}/p_o(p^n)$, $p^* = \bar{p}$ if $\bar{\tau}_o'(\tau) \le \tau' < 1 - \{E[\max(V, p^n)] - E(V - p_s)^+\}/p_o(p^n)$, $p^* = p^s$ if $\tau' < \bar{\tau}_o'(\tau)$; 2) $x^* = E(V - p^*)^+ - E(V - p_s)^+$; 3) $p_o^* = r_o^* = \inf\{r \ge v_{\min} : [(\lambda_1 - C)^+ + \lambda_2]\bar{F}(r) \le (C - \lambda_1)^+ + \min(\lambda_1, C)F(p^*)\}$. Moreover, $y^*(\bar{z}) = (C - \lambda_1)^+$ if $\tau' < \bar{\tau}_o'(\tau)$ and $y^*(\bar{z}) = 0$ otherwise.

Step 4: To complete the proof, we derive how the optimal prices and revenue change in τ. p^* is decreasing in τ because either $p^* = (1 - \tau)p_2^*$ which is decreasing in τ or p^* stays constant in τ. Thus, x^* and p_o^* are increasing in τ. Part (i) of the theorem is proved. Furthermore, $x^* + p^*$ is decreasing in τ. Since $x^* + p^* = p_o^*$ when $\tau = 0$, we have $x^* + p^* < p_o^*$, hence Part (ii) of the theorem is proved.

Finally, we derive how the optimal revenue Π_o^* changes in τ. First, consider the case of $p^* = p^n$. When $\tau > \tilde{\tau}(C)$ or $\tau > \hat{\tau}(C)$, p^n is independent of τ, hence so is Π_o^*. On the other hand, when $\tau \le \min(\tilde{\tau}(C), \hat{\tau}(C))$, $p^n = (1 - \tau)p_2^*$ and $p_o = p_2^*$, hence we have $\Pi_o^* = \{E[V - (1 - \tau)p_2^*]^+ - E(V - p_s)^+ + (1 - \tau)p_2^*\bar{F}((1 - \tau)p_2^*)\}\min(\lambda_1, C) + p_2^*[C - \min(\lambda_1, C)\bar{F}((1 - \tau)p_2^*)]$. Using a similar approach to derive $\mathrm{d}\Pi_m^*/\mathrm{d}\tau$ in the proof of Theorem II.4, we obtain

$$\frac{\mathrm{d}\Pi_o^*}{\mathrm{d}\tau} = \frac{\mathrm{d}p_2^*}{\mathrm{d}\tau} \cdot [(\lambda_1 - C)^+ + \lambda_2] \cdot \left[\bar{F}(p_2^*) - \tau p_2^* f(p_2^*)\right].$$

For $\tau \le \tilde{\tau}(C)$, we have $\tau p_2^* \le \bar{F}(p_2^*)/f(p_2^*)$, then $\mathrm{d}p_2^*/\mathrm{d}\tau > 0$ implies $\mathrm{d}\Pi_o^*/\mathrm{d}\tau \ge 0$. Second, if $p^* = \bar{p}$, p^* is independent of τ, hence so is Π_o^*. Third, if $p^* = p^s$, p^* and Π_o^* are independent of τ if $p^s > (1 - \tau)p_2^*$; if $p^s = (1 - \tau)p_2^*$, we have

$$\frac{\mathrm{d}\Pi_o^*}{\mathrm{d}\tau} = \left.\frac{\mathrm{d}\Pi_o}{\mathrm{d}p}\right|_{p=(1-\tau)p_2^*} \cdot \frac{\mathrm{d}[(1 - \tau)p_2^*]}{\mathrm{d}\tau} \ge 0,$$

because $p^s = (1 - \tau)p_2^*$ implies $(\mathrm{d}\Pi_o/\mathrm{d}p)|_{p=(1-\tau)p_2^*} \le 0$ and we already know $\mathrm{d}[(1 -$

$\tau)p_2^*]/\,\mathrm{d}\tau < 0$. Therefore, Π_o^* is increasing in τ overall. Furthermore, because the existence of speculators decreases Π_o^*, we conclude that Π_o^* is maximized when $\tau = \tau' = 1$. Part (iv) of the theorem is proved.

Proof of Theorem II.6

Since $p^n \geq (1-\tau)p_2^*$ and $p_o(p^n) \leq p_2^*$, we have $\bar{\tau}_o'(\tau) \leq 1 - \{E[\max(V, p^n)] - E(V-p_s)^+\}/p_o(p^n) \leq 1 - p_1^*/p_2^* = \bar{\tau}_m'(\tau)$. $\bar{\tau}_m'(\tau) < \bar{\tau}_f'(\tau)$ because $p_2^* = r_f^*$ and $p_1^* = E[\max(V, (1-\tau)p_2^*)] - E(V-p_s)^+ = p_f^n + \int_{p_s}^{\infty}(v - p_f^n)\,\mathrm{d}F(v) - E(V-p_s)^+ = p_f^n + \int_{p_s}^{\infty}(p_s - p_f^n)\,\mathrm{d}F(v) > p_f^n$.

Proof of Theorem II.7

First of all, we must have $b^* \leq \min(\lambda_1, C)$. If $C > \lambda_1$, the revenue does not change if speculators do not enter the market in equilibrium; if speculators enter the market in equilibrium, as we decrease b from C to λ_1, the capacity provider shifts sales from period 1 to period 2 where the price is higher, hence his revenue increases. The analysis in Section 2.5 implies that the revenue is $\Pi_m(b) = \{E[\max(V, (1-\tau)p_2^*(b))] - E(V-p_s)^+\}b + p_2^*(b)(C-b)$, where $p_2^*(b)$ is given by $(\lambda_1 - b + \lambda_2)\bar{F}(p_2^*) = C - b\bar{F}((1-\tau)p_2^*)$.

Next, we show that $\Pi_m(b)$ is concave in b. Taking derivative gives

$$\frac{\mathrm{d}\Pi_m}{\mathrm{d}b} = [(1-\tau)F((1-\tau)p_2^*)b + C - b] \cdot \frac{\mathrm{d}p_2^*}{\mathrm{d}b} + E[\max(V, (1-\tau)p_2^*)] - E(V-p_s)^+ - p_2^*,$$

where $\mathrm{d}p_2^*/\,\mathrm{d}b = \partial \underline{r}/\partial z$ is derived in the proof of Theorem II.1. Define $\Delta v = v_{\max} - v_{\min}$. With uniform distribution, we have $p_s = v_{\max} - C\Delta v/(\lambda_1 + \lambda_2)$ and

$$\begin{aligned}
\frac{\mathrm{d}\Pi_m}{\mathrm{d}b} \quad = \quad & \frac{(1-\tau)^2(\lambda_1+\lambda_2)}{[(\lambda_1+\lambda_2)v_{\max} - C\Delta v]\Delta v} \cdot (p_2^*)^3 \\
& + \left\{ \frac{\tau C\Delta v - (\lambda_1+\lambda_2)(v_{\max} - \tau v_{\min})}{[(\lambda_1+\lambda_2)v_{\max} - C\Delta v]\Delta v} - \frac{(1-\tau)^2}{2\Delta v} \right\} \cdot (p_2^*)^2 \\
& + \frac{1}{2\Delta v}\left[v_{\max}^2 - \frac{C^2\Delta v^2}{(\lambda_1+\lambda_2)^2} \right].
\end{aligned}$$

Then we have

$$\frac{\mathrm{d}}{\mathrm{d}p_2^*}\left(\frac{\mathrm{d}\Pi_m}{\mathrm{d}b}\right) = \frac{p_2^*}{[(\lambda_1+\lambda_2)v_{\max} - C\Delta v]\Delta v} \cdot B_0,$$

where $B_0 = 3(1-\tau)^2(\lambda_1+\lambda_2)p_2^* - (1-\tau)^2[(\lambda_1+\lambda_2)v_{\max} - C\Delta v] - 2(\lambda_1+\lambda_2)(v_{\max} - \tau v_{\min}) + 2\tau C\Delta v$. When $\tau = 0$, $B_0 = -2C\Delta v < 0$; when $\tau = 1$, $B_0 = 2(C - \lambda_1 - \lambda_2)\Delta v < 0$. Thus, if we can show B_0 is convex in τ, we know $B_0 < 0$ for all τ. Taking derivative of B_0 with respect to τ gives

$$\frac{\partial B_0}{\partial \tau} = \frac{(\lambda_1+\lambda_2)v_{\max} - C\Delta v}{(\lambda_1+\lambda_2-\tau b)^2} \cdot B_1,$$

where $B_1 = (1-\tau)[-6(\lambda_1+\lambda_2)(\lambda_1+\lambda_2-\tau b) + 3(1-\tau)(\lambda_1+\lambda_2)b + 2(\lambda_1+\lambda_2-\tau b)^2]$. Since $(\lambda_1+\lambda_2-\tau b)^2$ is decreasing in τ, it remains to show B_1 is increasing in τ, or equivalently,

$$\frac{\partial B_1}{\partial \tau} = -6b^2\tau^2 + 2[(\lambda_1+\lambda_2)b + 2b^2]\tau + 4(\lambda_1+\lambda_2)(\lambda_1+\lambda_2-b) \geq 0.$$

This is true because $\partial B_1/\partial\tau = 4(\lambda_1+\lambda_2)(\lambda_1+\lambda_2-b) > 0$ when $\tau = 0$, $\partial B_1/\partial\tau = 0$ when $\tau = 1$, and $\partial B_1/\partial\tau$ is concave in τ. By now we have shown that $\mathrm{d}\Pi_m/\mathrm{d}b$ is decreasing in p_2^*. Since $\mathrm{d}p_2^*/\mathrm{d}b \geq 0$, the chain rule then gives $\mathrm{d}^2\Pi_m/\mathrm{d}b^2 \leq 0$, hence we conclude that $\Pi_m(b)$ is concave in b.

Now that we have proved concavity, we derive the monotonicity of Π_m^* with respect to τ. First, if b^* is attained at an interior point, the Envelope Theorem gives

$$\frac{\mathrm{d}\Pi_m^*}{\mathrm{d}\tau} = \left.\frac{\partial \Pi_m}{\partial \tau}\right|_{b=b^*} = -\frac{\mathrm{d}p_2^*}{\mathrm{d}\tau} \cdot \frac{(\lambda_1+\lambda_2)v_{\max} - C\Delta v}{\tau(p_2^*)^2} \cdot [\alpha(p_2^*)^2 + \beta p_2^* + \gamma],$$

where

$$\alpha = \frac{1-\tau^2}{2\Delta v} \geq 0, \qquad \beta = -\frac{v_{\max}}{\Delta v} < 0, \qquad \gamma = \frac{1}{2\Delta v}\left[v_{\max}^2 - \frac{C^2\Delta v^2}{(\lambda_1+\lambda_2)^2}\right] > 0.$$

Note that in the above derivation, the first-order condition is used to simplify the algebra. We already know $\mathrm{d}p_2^*/\mathrm{d}\tau \geq 0$ and it is easy to see that $(\lambda_1+\lambda_2)v_{\max} > C\Delta v$, hence we only need to show $\alpha(p_2^*)^2 + \beta p_2^* + \gamma \leq 0$ to conclude that Π_m^* is increasing in τ. Recall that $p_2^* > p_s$. As the problem degenerates for $p_2^* > v_{\max}$, we restrict p_2^* to $p_2^* \leq v_{\max}$. Moreover, $-\beta/2\alpha = v_{\max}/(1-\tau^2) > v_{\max}$. Thus, it suffices to show $\alpha(p_2^*)^2 + \beta p_2^* + \gamma \leq 0$ at $p_2^* = p_s = v_{\max} - C\Delta v/(\lambda_1+\lambda_2)$. This is true as $\alpha p_s^2 + \beta p_s + \gamma = -\tau^2 p_s^2/(2\Delta v) < 0$. Second, if $b^* = \min(\lambda_1, C)$, we have the same optimal revenue function as in the basic model. For $\tau \geq \hat{\tau}(C)$, Π_m^* stays constant in τ. For $\tau < \hat{\tau}(C)$, to show Π_m^* is increasing in τ, (A.1) implies that with uniform distribution, we need to show $(C-\lambda_1)^+ - [(\lambda_1-C)^+ + \lambda_2]F((1-\tau)p_2^*) \geq 0$. Note that $b^* = \min(\lambda_1, C)$ implies $\mathrm{d}\Pi_m/\mathrm{d}b \geq 0$ at $b = \min(\lambda_1, C)$. With uniform distribution, this results in

$$
\begin{aligned}
& [(1-\tau)F((1-\tau)p_2^*)\min(\lambda_1, C) + (C-\lambda_1)^+] \cdot \frac{\tau p_2^*}{[(\lambda_1-C)^+ + \lambda_2] + (1-\tau)\min(\lambda_1, C)} \\
\geq \;& -E[\max(V, (1-\tau)p_2^*)] + E(V-p_s)^+ + p_2^* \\
> \;& -E[\max(V, (1-\tau)p_2^*)] + E[\max(V, p_2^*)] \\
\geq \;& \tau p_2^* F((1-\tau)p_2^*),
\end{aligned}
$$

where the second inequality follows from $p_s < p_2^*$ and the third inequality follows from the fact that the derivative of $E[\max(V,t)]$ is $F(t)$. Thus, $(C-\lambda_1)^+ - [(\lambda_1-C)^+ + \lambda_2]F((1-\tau)p_2^*) \geq 0$. Therefore, overall Π_m^* is increasing in τ[2].

[2] Note that the feasible region for b is $b > 0$. For any $\epsilon > 0$ where ϵ can be arbitrarily small, when b^* is attained at $b^* = \epsilon$ for the optimization problem over $\epsilon \leq b \leq \min(\lambda_1, C)$, Π_m^* is constant in τ, so overall Π_m^* is still increasing in τ.

APPENDIX B

Proofs of Theorems and Lemmas in Chapter III

Lemma B.1. *For any $a_1, a_2, a_3, b_1, b_2, b_3 \in [-1, 1]$, we have*

(i) $|a_1a_2 - b_1b_2| \leq |a_1 - b_1| + |a_2 - b_2|$.

(ii) $|a_1a_2a_3 - b_1b_2b_3| \leq |a_1 - b_1| + |a_2 - b_2| + |a_3 - b_3|$.

Proof of Lemma B.1

(i) $|a_1a_2 - b_1b_2| = |a_2(a_1 - b_1) + b_1(a_2 - b_2)| \leq |a_1 - b_1| + |a_2 - b_2|$.

(ii) $|a_1a_2a_3 - b_1b_2b_3| = |a_2a_3(a_1 - b_1) + a_3b_1(a_2 - b_2) + b_1b_2(a_3 - b_3)| \leq |a_1 - b_1| + |a_2 - b_2| + |a_3 - b_3|$.

Lemma B.2. *Let N_{λ_1} and N_{λ_2} be two Poisson variables with means λ_1 and λ_2, respectively. $n \geq 0$ is an integer. Then, for every n, there exist $\alpha_c(n), \alpha_p(n) \in (0, 1]$ such that*

(i) $\big|\mathbb{P}(N_{\lambda_1} \leq n) - \mathbb{P}(N_{\lambda_2} \leq n)\big| \leq \alpha_c(n)\,|\lambda_1 - \lambda_2|$.

(ii) $\big|\mathbb{P}(N_{\lambda_1} = n) - \mathbb{P}(N_{\lambda_2} = n)\big| \leq \alpha_p(n)\,|\lambda_1 - \lambda_2|$.

Moreover, $\alpha_c(n)$ is decreasing in n.

Proof of Lemma B.2

(i) The case of $n \geq 1$ is proved by *Caldentey and Vulcano* (2007) (Lemma A3 in online appendix). In particular, $\alpha_c(n) = \mathbb{P}(N_n = n)$. When $n = 0$, $\big|\mathbb{P}(N_{\lambda_1} \leq$

$0) - \mathbb{P}(N_{\lambda_2} \leq 0)\big| = \big|e^{-\lambda_1} - e^{-\lambda_2}\big| \leq \sup_{\lambda>0}\{e^{-\lambda}\}\,|\lambda_1 - \lambda_2| = |\lambda_1 - \lambda_2|$, hence $\alpha_c(0) = 1$. It is easy to see that $\alpha_c(n) \in (0, 1]$. $\alpha_c(n)$ is decreasing in n because $\alpha_c(n+1)/\alpha_c(n) = (1 + 1/n)^n e^{-1} < 1$.

(ii) When $n = 0$, $\alpha_p(n) = \alpha_c(n) = 1$. When $n \geq 1$,

$$\big|\,\mathbb{P}(N_{\lambda_1} = n) - \mathbb{P}(N_{\lambda_2} = n)\big| = \left|\frac{e^{-\lambda_1}\lambda_1^n}{n!} - \frac{e^{-\lambda_2}\lambda_2^n}{n!}\right| \leq \frac{|\lambda_1 - \lambda_2|}{n!}\sup_{\lambda>0}\left\{\left|\frac{\mathrm{d}(e^{-\lambda}\lambda^n)}{\mathrm{d}\lambda}\right|\right\}.$$

We have $\frac{\mathrm{d}(e^{-\lambda}\lambda^n)}{\mathrm{d}\lambda} = e^{-\lambda}\lambda^{n-1}(n-\lambda)$. Thus, $\frac{\mathrm{d}(e^{-\lambda}\lambda^n)}{\mathrm{d}\lambda} > 0$ for $0 < \lambda < n$ and $\frac{\mathrm{d}(e^{-\lambda}\lambda^n)}{\mathrm{d}\lambda} < 0$ for $\lambda > n$. Moreover, $\frac{\mathrm{d}^2(e^{-\lambda}\lambda^n)}{\mathrm{d}\lambda^2} = e^{-\lambda}\lambda^{n-2}[\lambda^2 - 2n\lambda + n(n-1)]$. Solving $\frac{\mathrm{d}^2(e^{-\lambda}\lambda^n)}{\mathrm{d}\lambda^2} = 0$ yields $\lambda = n - \sqrt{n}$ and $\lambda = n + \sqrt{n}$. Since $\left.\frac{\mathrm{d}(e^{-\lambda}\lambda^n)}{\mathrm{d}\lambda}\right|_{\lambda=0} = 0$ and it follows from L'Hospital's Rule that $\lim_{\lambda\to\infty}\frac{\mathrm{d}(e^{-\lambda}\lambda^n)}{\mathrm{d}\lambda} = 0$, we know that $\sup_{\lambda>0}\left\{\left|\frac{\mathrm{d}(e^{-\lambda}\lambda^n)}{\mathrm{d}\lambda}\right|\right\}$ is attained at either $\lambda = n - \sqrt{n}$ or $\lambda = n + \sqrt{n}$. Thus,

$$\sup_{\lambda>0}\left\{\left|\frac{\mathrm{d}(e^{-\lambda}\lambda^n)}{\mathrm{d}\lambda}\right|\right\} = \max\left\{e^{-(n-\sqrt{n})}(n - \sqrt{n})^{n-1}\sqrt{n}, e^{-(n+\sqrt{n})}(n + \sqrt{n})^{n-1}\sqrt{n}\right\},$$

and hence

$$\begin{aligned}\alpha_p(n) &= \max\left\{\frac{e^{-(n-\sqrt{n})}(n - \sqrt{n})^{n-1}\sqrt{n}}{n!}, \frac{e^{-(n+\sqrt{n})}(n + \sqrt{n})^{n-1}\sqrt{n}}{n!}\right\} \\ &= \frac{\max\left\{\mathbb{P}(N_{n-\sqrt{n}} = n), \mathbb{P}(N_{n+\sqrt{n}} = n)\right\}}{\sqrt{n}}.\end{aligned}$$

It is easy to see that $\alpha_p(n) \in (0, 1]$.

Lemma B.3. *For any $a, b \in [-1, 1]$ and integer $n \geq 0$, we have $|a^n - b^n| \leq n|a - b|$.*

Proof of Lemma B.3

$|a^n - b^n| = |a - b| \cdot \left|\sum_{i=0}^{n-1} a^i b^{n-1-i}\right| \leq |a - b| \cdot \sum_{i=0}^{n-1} |a^i b^{n-1-i}| \leq n|a - b|$.

Proof of Theorem III.2

In order to show the existence of $q^*(\cdot)$, we need to prove that the mapping $b(q(\cdot))$ from $\mathcal{Q}$ to $\mathcal{Q}$ has the fixed-point property. By the Schauder-Tychonoff Fixed-Point

Theorem, we need to prove: 1) $\mathcal{Q}$ is convex and compact, 2) $b(q(\cdot))$ is continuous. Convexity of $\mathcal{Q}$ is easy to verify. To prove compactness, by the Arzela-Ascoli Theorem, we need to prove that $\mathcal{Q}$ is closed, bounded, and equicontinuous. Closedness and boundedness of $\mathcal{Q}$ are easy to verify. To prove equicontinuity, first pick a $q(\cdot)$ from $\mathcal{Q}$. For any $t_1, t_2 \in [0, T]$, we have $|q(t_1) - q(t_2)| \leq \sup_{0 \leq t \leq T}\{|q'(t)|\}\,|t_1 - t_2|$. Next, let $\bar{q}' = \sup_{q(\cdot)\in\mathcal{Q}} \sup_{0\leq t\leq T}\{|q'(t)|\}$. Note that $\bar{q}'$ is finite because each $q(\cdot)$ is bounded. Then, for any $\epsilon > 0$, there exists $\delta = \epsilon/\bar{q}'$ such that if $|t_1 - t_2| < \delta$, then for all $q(\cdot) \in \mathcal{Q}$, $|q(t_1) - q(t_2)| \leq \bar{q}'|t_1 - t_2| < \epsilon$. Thus, we have proved equicontinuity of $\mathcal{Q}$.

Next, we prove that $b(q(\cdot))$ is a continuous mapping. In order to obtain a sufficient condition for the uniqueness of $q^*(\cdot)$, we will prove a stronger result that $b(q(\cdot))$ is Lipschitz continuous, that is, there exists a constant $\bar{\alpha} \geq 0$ such that for any $q_1(\cdot), q_2(\cdot) \in \mathcal{Q}$, $\left\|b(q_1(\cdot)) - b(q_2(\cdot))\right\|_\infty \leq \bar{\alpha}\left\|q_1(\cdot) - q_2(\cdot)\right\|_\infty$. For a given arrival time t, we start by bounding $\left|b(q_1(\cdot)) - b(q_2(\cdot))\right|$ from above as follows:

$$
\begin{aligned}
\left|b(q_1(\cdot)) - b(q_2(\cdot))\right| &= \frac{\left|g(q_1(\cdot))h(q_2(\cdot)) - g(q_2(\cdot))h(q_1(\cdot))\right|}{h(q_1(\cdot))h(q_2(\cdot))} \\
&\leq \frac{\left|g(q_1(\cdot)) - g(q_2(\cdot))\right| + \left|h(q_1(\cdot)) - h(q_2(\cdot))\right|}{h(q_1(\cdot))h(q_2(\cdot))}, \qquad \text{(B.1)}
\end{aligned}
$$

where the inequality follows from Lemma B.1(i).

We analyze (B.1) part by part. We first bound the denominator of (B.1) from below as follows:

$$
\begin{aligned}
h(q_1(\cdot)) &\geq \mathbb{P}(N_H(T|q_1(\cdot)) < K_H, N_R(T|q_1(\cdot)) < K_R, \\
&\qquad N_H(T|q_1(\cdot)) + N_U(T|q_1(\cdot)) + N_R(T|q_1(\cdot)) < K_H + K_R) \\
&\geq \mathbb{P}(N_\lambda(T) < K_H, N_\lambda(T) < K_R, N_\lambda(T) < K_H + K_R) \\
&= \mathbb{P}\left(N_\lambda(T) < \min\left\{K_H, K_R\right\}\right) \stackrel{\text{def}}{=\!=} \alpha_h,
\end{aligned}
$$

where $N_\lambda(t)$ denotes the Poisson process with rate λ. The above bound is also valid for $h(q_2(\cdot))$, hence

$$h(q_1(\cdot))h(q_2(\cdot)) \geq \alpha_h^2. \tag{B.2}$$

Now, consider the numerator of (B.1). To bound $|h(q_1(\cdot)) - h(q_2(\cdot))|$ from above, we can write $h(q(\cdot))$ as

$$\begin{aligned} h(q(\cdot)) &= \sum_{i_H=0}^{K_H-1} \sum_{i_R=0}^{K_R-1} \mathbb{P}(N_H(t|q(\cdot)) = i_H)\, \mathbb{P}(N_R(t|q(\cdot)) = i_R) \\ &\qquad \cdot \mathbb{P}(N_U(t|q(\cdot)) < K_H + K_R - i_H - i_R). \end{aligned}$$

Then, we have

$$\begin{aligned} &\big|h(q_1(\cdot)) - h(q_2(\cdot))\big| \\ \leq\ & \sum_{i_H=0}^{K_H-1} \sum_{i_R=0}^{K_R-1} \mathbb{P}(N_R(t|q_1(\cdot)) = i_R) \\ &\qquad \cdot \Big| \mathbb{P}(N_H(t|q_1(\cdot)) = i_H)\, \mathbb{P}(N_U(t|q_1(\cdot)) < K_H + K_R - i_H - i_R) \\ &\qquad - \mathbb{P}(N_H(t|q_2(\cdot)) = i_H)\, \mathbb{P}(N_U(t|q_2(\cdot)) < K_H + K_R - i_H - i_R)\Big| \\ \leq\ & \sum_{i_H=0}^{K_H-1} \sum_{i_R=0}^{K_R-1} \mathbb{P}(N_R(t|q_1(\cdot)) = i_R)\Big| \mathbb{P}(N_H(t|q_1(\cdot)) = i_H) - \mathbb{P}(N_H(t|q_2(\cdot)) = i_H)\Big| \\ &+ \sum_{i_H=0}^{K_H-1} \sum_{i_R=0}^{K_R-1} \mathbb{P}(N_R(t|q_1(\cdot)) = i_R)\Big| \mathbb{P}(N_U(t|q_1(\cdot)) < K_H + K_R - i_H - i_R) \\ &\qquad - \mathbb{P}(N_U(t|q_2(\cdot)) < K_H + K_R - i_H - i_R)\Big|, \end{aligned}$$

where the first step follows from the fact that $N_R(t|q(\cdot))$ does not depend on $q(\cdot)$, and the second step follows from Lemma B.1(i). Define $\xi_H(t|q(\cdot)) = \gamma\xi_H^\gamma(t|q(\cdot)) + (1-\gamma)\xi_H'(t|q(\cdot))$, $\xi_U(t|q(\cdot)) = \gamma\xi_U^\gamma(t|q(\cdot))$, $\xi_R(t|q(\cdot)) = \gamma\xi_R^\gamma(t|q(\cdot)) + (1-\gamma)\xi_R'(t|q(\cdot))$ as the proportions of total demand rate λ for $N_H(t|q(\cdot))$, $N_U(t|q(\cdot))$, $N_R(t|q(\cdot))$, respectively. We can bound $\Big|\mathbb{P}(N_H(t|q_1(\cdot)) = i_H) - \mathbb{P}(N_H(t|q_2(\cdot)) = i_H)\Big|$ as follows. Using Lemma

B.2(ii) yields

$$\begin{aligned}
& \left| \mathbb{P}(N_H(t|q_1(\cdot)) = i_H) - \mathbb{P}(N_H(t|q_2(\cdot)) = i_H) \right| \\
\leq \;& \alpha_p(i_H) \left| \int_0^t \lambda[\xi_H(s|q_1(\cdot)) - \xi_H(s|q_2(\cdot))] \, \mathrm{d}s \right| \\
\leq \;& \alpha_p(i_H)\lambda \int_0^t \left| \xi_H(s|q_1(\cdot)) - \xi_H(s|q_2(\cdot)) \right| \mathrm{d}s \\
\leq \;& \alpha_p(i_H)\lambda T \left\| \xi_H(t|q_1(\cdot)) - \xi_H(t|q_2(\cdot)) \right\|_\infty.
\end{aligned}$$

Similarly, We can use Lemma B.2(i) to obtain $\left| \mathbb{P}(N_U(t|q_1(\cdot)) < K_H + K_R - i_H - i_R) - \mathbb{P}(N_U(t|q_2(\cdot)) < K_H + K_R - i_H - i_R) \right| \leq \alpha_c(K_H + K_R - 1 - i_H - i_R)\lambda T \left\| \xi_U(t|q_1(\cdot)) - \xi_U(t|q_2(\cdot)) \right\|_\infty$. Combining these two inequalities leads to $\left| h(q_1(\cdot)) - h(q_2(\cdot)) \right| \leq \alpha_{H1}(t) \left\| \xi_H(t|q_1(\cdot)) - \xi_H(t|q_2(\cdot)) \right\|_\infty + \alpha_{U1}(t) \left\| \xi_U(t|q_1(\cdot)) - \xi_U(t|q_2(\cdot)) \right\|_\infty$, where

$$\begin{aligned}
\alpha_{H1}(t) &= \lambda T \sum_{i_H=0}^{K_H-1} \sum_{i_R=0}^{K_R-1} \mathbb{P}(N_R(t|q_1(\cdot)) = i_R)\alpha_p(i_H), \\
\alpha_{U1}(t) &= \lambda T \sum_{i_H=0}^{K_H-1} \sum_{i_R=0}^{K_R-1} \mathbb{P}(N_R(t|q_1(\cdot)) = i_R)\alpha_c(K_H + K_R - 1 - i_H - i_R).
\end{aligned}$$

We further bound $\alpha_{H1}(t)$ and $\alpha_{U1}(t)$ as follows:

$$\alpha_{H1}(t) = \lambda T \, \mathbb{P}(N_R(t|q_1(t)) < K_R) \sum_{i_H=0}^{K_H-1} \alpha_p(i_H) \leq \lambda T \sum_{i_H=0}^{K_H-1} \alpha_p(i_H) \stackrel{\text{def}}{=\!=} \alpha_{H1}.$$

Similarly,

$$
\begin{aligned}
\alpha_{U1}(t) &\leq \lambda T \sum_{i_H=0}^{K_H-1} \sum_{i_R=0}^{K_R-1} \mathbb{P}(N_R(t|q_1(\cdot)) = i_R)\alpha_c(K_H - i_H) \\
&= \lambda T \, \mathbb{P}(N_R(t|q_1(\cdot)) < K_R) \sum_{i_H=0}^{K_H-1} \alpha_c(K_H - i_H) \\
&\leq \lambda T \sum_{i_H=1}^{K_H} \alpha_c(i_H) \xlongequal{\text{def}} \alpha_{U1},
\end{aligned}
$$

where the first inequality follows from Lemma B.2(i) that $\alpha_c(n)$ is decreasing in n. Thus, we have obtained that

$$
\big|h(q_1(\cdot)) - h(q_2(\cdot))\big| \leq \alpha_{H1}\big\|\xi_H(t|q_1(\cdot)) - \xi_H(t|q_2(\cdot))\big\|_\infty + \alpha_{U1}\big\|\xi_U(t|q_1(\cdot)) - \xi_U(t|q_2(\cdot))\big\|_\infty. \tag{B.3}
$$

Next, we bound $|g(q_1(\cdot)) - g(q_2(\cdot))|$ from above. If $\tau_H(q(\cdot)) \leq \tau_T(q(\cdot))$ and $\tau_H(q(\cdot)) \leq T$, then $g(q(\cdot)) = 0$. Thus, we can write $g(q(\cdot))$ as $g(q(\cdot)) = g_1(q(\cdot)) + g_2(q(\cdot)) + g_3(q(\cdot))$, where

$$
\begin{aligned}
g_1(q(\cdot)) &= \mathbb{P}(\tau_R(q(\cdot)) \leq \tau_T(q(\cdot)), \tau_R(q(\cdot)) \leq T) g(q(\cdot)|\tau_R(q(\cdot)) \leq \tau_T(q(\cdot)), \tau_R(q(\cdot)) \leq T), \\
g_2(q(\cdot)) &= \mathbb{P}(\tau_H(q(\cdot)) > \tau_T(q(\cdot)), \tau_R(q(\cdot)) > \tau_T(q(\cdot)), \tau_T(q(\cdot)) \leq T) \\
&\quad \cdot g(q(\cdot)|\tau_H(q(\cdot)) > \tau_T(q(\cdot)), \tau_R(q(\cdot)) > \tau_T(q(\cdot)), \tau_T(q(\cdot)) \leq T), \\
g_3(q(\cdot)) &= \mathbb{P}(\tau_H(q(\cdot)) > T, \tau_R(q(\cdot)) > T, \tau_T(q(\cdot)) > T) \\
&\quad \cdot g(q(\cdot)|\tau_H(q(\cdot)) > T, \tau_R(q(\cdot)) > T, \tau_T(q(\cdot)) > T).
\end{aligned}
$$

Now we consider each term of $g(q(\cdot))$. Define $m_R(t|q(\cdot)) = \int_0^T \lambda \xi_R(t|q(\cdot))\,\mathrm{d}t$ as the mean value function of $N_R(t|q(\cdot))$. Define $f_{\tau_R(q(\cdot))}(t)$ as the probability density function of $\tau_R(q(\cdot))$, and $f_{\tau_T(q(\cdot))}(t)$ as the probability density function of $\tau_T(q(\cdot))$. We have

$$f_{\tau_R(q(\cdot))}(t) = \frac{e^{-m_R(t|q(\cdot))}[m_R(t|q(\cdot))]^{K_R-1}\lambda\xi_R(t|q(\cdot))}{(K_R-1)!} = \mathbb{P}(N_R(t|q(\cdot)) = K_R{-}1)\lambda\xi_R(t|q(\cdot)),$$

and similarly,

$$\begin{aligned} f_{\tau_T(q(\cdot))}(t) &= \mathbb{P}(N_H(t|q(\cdot)) + N_U(t|q(\cdot)) + N_R(t|q(\cdot)) = K_H + K_R - 2) \\ &\quad \cdot\lambda\left[\xi_H(t|q(\cdot)) + \xi_U(t|q(\cdot)) + \xi_R(t|q(\cdot))\right]. \end{aligned}$$

$g_1(q(\cdot))$ can be written as

$$\begin{aligned} g_1(q(\cdot)) &= \int_t^T f_{\tau_R(q(\cdot))}(s)\,\mathbb{P}(N_H(s|q(\cdot)) + N_U(s|q(\cdot)) \leq K_H - 1)\,\mathrm{d}s \\ &= \int_t^T \mathbb{P}(N_R(s|q(\cdot)) = K_R - 1)\lambda\xi_R(s|q(\cdot))\,\mathbb{P}(N_H(s|q(\cdot)) + N_U(s|q(\cdot)) \leq K_H - 1)\,\mathrm{d}s. \end{aligned}$$

$g_2(q(\cdot))$ can be written as

$$
\begin{aligned}
g_2(q(\cdot)) \quad = \quad & \int_t^T f_{\tau_T(q(\cdot))}(s) \cdot \Bigg[\sum_{i_H=0}^{K_H-1} \sum_{i_R=0}^{K_R-1} \frac{K_H - i_H}{K_H + K_R - i_H - i_R} \\
& \cdot \mathbb{P}(N_H(s|q(\cdot)) = i_H, N_R(s|q(\cdot)) = i_R, \\
& N_U(s|q(\cdot)) = K_H + K_R - 1 - i_H - i_R | \tau_T(q(\cdot)) = s) \Bigg] \mathrm{d}s \\
= \quad & \int_t^T \mathbb{P}(N_H(s|q(\cdot)) + N_U(s|q(\cdot)) + N_R(s|q(\cdot)) = K_H + K_R - 2) \\
& \cdot \lambda \left[\xi_H(s|q(\cdot)) + \xi_U(s|q(\cdot)) + \xi_R(s|q(\cdot)) \right] \\
& \cdot \Bigg\{ \sum_{i_H=0}^{K_H-1} \sum_{i_R=0}^{K_R-1} \frac{K_H - i_H}{K_H + K_R - i_H - i_R} \binom{K_H + K_R - 1}{i_H} \binom{K_H + K_R - 1 - i_H}{i_R} \\
& \cdot \left\{ \mathop{\mathbb{E}}_{0 \le r \le s} [\xi_H(r|q(\cdot))] \right\}^{i_H} \left\{ \mathop{\mathbb{E}}_{0 \le r \le s} [\xi_R(r|q(\cdot))] \right\}^{i_R} \\
& \cdot \left\{ \mathop{\mathbb{E}}_{0 \le r \le s} [\xi_U(r|q(\cdot))] \right\}^{K_H + K_R - 1 - i_H - i_R} \Bigg\} \mathrm{d}s.
\end{aligned}
$$

$g_3(q(\cdot))$ can be written as

$$
\begin{aligned}
g_3(q(\cdot)) \quad = \quad & \sum_{i_H=0}^{K_H-1} \sum_{i_R=0}^{K_R-1} \sum_{i_U=0}^{K_H+K_R-2-i_H-i_R} \mathbb{P}(N_H(T|q(\cdot)) = i_H) \, \mathbb{P}(N_R(T|q(\cdot)) = i_R) \\
& \cdot \mathbb{P}(N_U(T|q(\cdot)) = i_U) \min \left\{ \frac{K_H - i_H}{i_U + 1}, 1 \right\}.
\end{aligned}
$$

Next, notice that

$$
\big| g(q_1(\cdot)) - g(q_2(\cdot)) \big| \le \big| g_1(q_1(\cdot)) - g_1(q_2(\cdot)) \big| + \big| g_2(q_1(\cdot)) - g_2(q_2(\cdot)) \big| + \big| g_3(q_1(\cdot)) - g_3(q_2(\cdot)) \big|. \tag{B.4}
$$

By using the same approach that is used to bound $\big|h(q_1(\cdot)) - h(q_2(\cdot))\big|$, we can bound each term in the right-hand side (RHS) of (B.4) from above. Bounding the first term in the RHS of (B.4) results in

$$\left|g_1(q_1(\cdot))-g_1(q_2(\cdot))\right| \leq \alpha_{H2}\left\|\xi_H(t|q_1(\cdot))-\xi_H(t|q_2(\cdot))\right\|_\infty+\alpha_{U2}\left\|\xi_U(t|q_1(\cdot))-\xi_U(t|q_2(\cdot))\right\|_\infty, \tag{B.5}$$

where $\alpha_{H2} = \alpha_{U2} = (\lambda T)^2\alpha_c(K_H - 1)$. Bounding the second term in the RHS of (B.4) results in

$$\left|g_2(q_1(\cdot))-g_2(q_2(\cdot))\right| \leq \alpha_{H3}\left\|\xi_H(t|q_1(\cdot))-\xi_H(t|q_2(\cdot))\right\|_\infty+\alpha_{U3}\left\|\xi_U(t|q_1(\cdot))-\xi_U(t|q_2(\cdot))\right\|_\infty, \tag{B.6}$$

where

$$\begin{aligned}
\alpha_{H3} &= \lambda T\Bigg\{\lambda T\alpha_p(K_H+K_R-2)+1 \\
&\quad + \sum_{i_H=0}^{K_H-1}\sum_{i_R=0}^{K_R-1}\binom{K_H+K_R-1}{i_H}\binom{K_H+K_R-1-i_H}{i_R}\frac{i_H(K_H-i_H)}{K_H+K_R-i_H-i_R}\Bigg\}, \\
\alpha_{U3} &= \lambda T\Bigg\{\lambda T\alpha_p(K_H+K_R-2)+1 \\
&\quad + \sum_{i_H=0}^{K_H-1}\sum_{i_R=0}^{K_R-1}\binom{K_H+K_R-1}{i_H}\binom{K_H+K_R-1-i_H}{i_R} \\
&\quad\quad \cdot\frac{(K_H+K_R-1-i_H-i_R)(K_H-i_H)}{K_H+K_R-i_H-i_R}\Bigg\}.
\end{aligned}$$

Lemma B.3 is used in deriving α_{H3} and α_{U3}. Bounding the third term in the RHS of (B.4) results in $\left|g_3(q_1(\cdot)) - g_3(q_2(\cdot))\right| \leq \alpha_{H4}(t)\left\|\xi_H(t|q_1(\cdot)) - \xi_H(t|q_2(\cdot))\right\|_\infty + \alpha_{U4}(t)\left\|\xi_U(t|q_1(\cdot)) - \xi_U(t|q_2(\cdot))\right\|_\infty$, where

$$\begin{aligned}
\alpha_{H4}(t) &= \lambda T\sum_{i_H=0}^{K_H-1}\sum_{i_R=0}^{K_R-1}\sum_{i_U=0}^{K_H+K_R-2-i_H-i_R}\mathbb{P}(N_R(T|q_1(\cdot))=i_R)\alpha_p(i_H)\min\left\{\frac{K_H-i_H}{i_U+1},1\right\}, \\
\alpha_{U4}(t) &= \lambda T\sum_{i_H=0}^{K_H-1}\sum_{i_R=0}^{K_R-1}\sum_{i_U=0}^{K_H+K_R-2-i_H-i_R}\mathbb{P}(N_R(T|q_1(\cdot))=i_R)\alpha_p(i_U)\min\left\{\frac{K_H-i_H}{i_U+1},1\right\}.
\end{aligned}$$

We then have

$$\begin{aligned}
\alpha_{H4}(t) &\leq \lambda T \sum_{i_H=0}^{K_H-1} \sum_{i_U=0}^{K_H+K_R-2-i_H} \alpha_p(i_H) \min\left\{\frac{K_H - i_H}{i_U+1}, 1\right\} \stackrel{\text{def}}{=\!=} \alpha_{H4}, \\
\alpha_{U4}(t) &\leq \lambda T \sum_{i_H=0}^{K_H-1} \sum_{i_U=0}^{K_H+K_R-2-i_H} \alpha_p(i_U) \min\left\{\frac{K_H - i_H}{i_U+1}, 1\right\} \stackrel{\text{def}}{=\!=} \alpha_{U4}.
\end{aligned}$$

Thus, we have obtained that

$$\left|g_3(q_1(\cdot)) - g_3(q_2(\cdot))\right| \leq \alpha_{H4}\left\|\xi_H(t|q_1(\cdot)) - \xi_H(t|q_2(\cdot))\right\|_\infty + \alpha_{U4}\left\|\xi_U(t|q_1(\cdot)) - \xi_U(t|q_2(\cdot))\right\|_\infty, \tag{B.7}$$

Thus, by plugging (B.5), (B.6), (B.7) into (B.4) and then plugging (B.2), (B.3), (B.4) into (B.1), we obtain

$$\left|b(q_1(\cdot)) - b(q_2(\cdot))\right| \leq \alpha_H\left\|\xi_H(t|q_1(\cdot)) - \xi_H(t|q_2(\cdot))\right\|_\infty + \alpha_U\left\|\xi_U(t|q_1(\cdot)) - \xi_U(t|q_2(\cdot))\right\|_\infty, \tag{B.8}$$

where

$$\alpha_H = \frac{\alpha_{H1} + \alpha_{H2} + \alpha_{H3} + \alpha_{H4}}{\alpha_h^2}, \qquad \alpha_U = \frac{\alpha_{U1} + \alpha_{U2} + \alpha_{U3} + \alpha_{U4}}{\alpha_h^2}.$$

Note that α_H and α_U do not depend on t.

It remains to bound $\left\|\xi_H(t|q_1(\cdot)) - \xi_H(t|q_2(\cdot))\right\|_\infty$ and $\left\|\xi_U(t|q_1(\cdot)) - \xi_U(t|q_2(\cdot))\right\|_\infty$ from above. Define $\bar{v}_H = \sup\{v_H : (v_R, v_H) \in \Omega\}$, $\underline{v}_H = \inf\{v_H : (v_R, v_H) \in \Omega\}$, $\bar{v}_R = \sup\{v_R : (v_R, v_H) \in \Omega\}$, $\underline{v}_R = \inf\{v_R : (v_R, v_H) \in \Omega\}$. Fix t, and without loss of generality, assume $q_1(t) < q_2(t)$. First, consider $\left\|\xi_H(t|q_1(\cdot)) - \xi_H(t|q_2(\cdot))\right\|_\infty$. We have

$$\xi_H(t|q(t)) = \gamma \int_{p_H}^{\bar{v}_H} \left[\int_{\underline{v}_R}^{\max\left\{v_H - \frac{p_H - p_R - q(t)p}{1-q(t)}, \underline{v}_R\right\}} f(v_R, v_H)\, \mathrm{d}v_R \right] \mathrm{d}v_H + (1-\gamma)\xi'_H(t|q(t)).$$

Since $\max\left\{v_H - \frac{p_H - p_R - q(t)p}{1-q(t)}, \underline{v}_R\right\}$ is decreasing in $q(t)$, we have

$$
\begin{aligned}
& \left|\xi_H(t|q_1(t)) - \xi_H(t|q_2(t))\right| \\
= \ & \gamma \int_{p_H}^{\bar{v}_H} \left[\int_{\max\left\{v_H - \frac{p_H - p_R - q_2(t)p}{1-q_2(t)}, \underline{v}_R\right\}}^{\max\left\{v_H - \frac{p_H - p_R - q_1(t)p}{1-q_1(t)}, \underline{v}_R\right\}} f(v_R, v_H)\,\mathrm{d}v_R \right] \mathrm{d}v_H \\
\leq \ & \gamma \sup_{(v_R, v_H)\in\Omega} \{f(v_R, v_H)\} \int_{p_H}^{\bar{v}_H} \left[\max\left\{v_H - \frac{p_H - p_R - q_1(t)p}{1-q_1(t)}, \underline{v}_R\right\} \right. \\
& \left. - \max\left\{v_H - \frac{p_H - p_R - q_2(t)p}{1-q_2(t)}, \underline{v}_R\right\} \right] \mathrm{d}v_H.
\end{aligned}
$$

Define

$$
\begin{aligned}
\bar{q}_H &= \sup\left\{0 < q < 1 : v_H - \frac{p_H - p_R - qp}{1-q} \geq \underline{v}_R\right\} \\
&= \begin{cases} \frac{v_H - p_H + p_R - \underline{v}_R}{v_H - \underline{v}_R - p} & \text{if } v_H > p_H - p_R + \underline{v}_R, \\ 0 & \text{otherwise.} \end{cases}
\end{aligned}
$$

It is easy to see that $\bar{q}_H < 1$, and $v_H - \frac{p_H - p_R - q(t)p}{1-q(t)} \geq \underline{v}_R$ if and only if $q(t) \leq \bar{q}_H$. Then,

$$
\begin{aligned}
& \max\left\{v_H - \frac{p_H - p_R - q_1(t)p}{1-q_1(t)}, \underline{v}_R\right\} - \max\left\{v_H - \frac{p_H - p_R - q_2(t)p}{1-q_2(t)}, \underline{v}_R\right\} \\
= \ & \left[v_H - \frac{p_H - p_R - \min\{q_1(t), \bar{q}_H\}p}{1 - \min\{q_1(t), \bar{q}_H\}}\right] - \left[v_H - \frac{p_H - p_R - \min\{q_2(t), \bar{q}_H\}p}{1 - \min\{q_2(t), \bar{q}_H(t)\}}\right] \\
= \ & \frac{(p_H - p_R - p)\left[\min\{q_2(t), \bar{q}_H\} - \min\{q_1(t), \bar{q}_H\}\right]}{(1 - \min\{q_1(t), \bar{q}_H\})\,(1 - \min\{q_2(t), \bar{q}_H\})} \\
\leq \ & \frac{p_H - p_R - p}{(1 - \bar{q}_H)^2}\left[q_2(t) - q_1(t)\right].
\end{aligned}
$$

Thus,

$$\begin{aligned}\left|\xi_H(t|q_1(t))-\xi_H(t|q_2(t))\right| &\le \gamma \sup_{(v_R,v_H)\in\Omega}\{f(v_R,v_H)\}\,(p_H-p_R-p)\,[q_2(t)-q_1(t)]\\ &\quad \cdot\int_{p_H}^{\bar{v}_H}\frac{1}{(1-\bar{q}_H)^2}\,\mathrm{d}v_H\\ &= \alpha'_H\,[q_2(t)-q_1(t)]\,,\end{aligned}$$

where

$$\begin{aligned}\alpha'_H &= \gamma \sup_{(v_R,v_H)\in\Omega}\{f(v_R,v_H)\}\,(p_H-p_R-p)\\ &\quad\cdot\left[(\underline{v}_R-p_R)^+ + \frac{(\bar{v}_H-\underline{v}_R-p)^3-(p_H-\min\{\underline{v}_R,p_R\}-p)^3}{3(p_H-p_R-p)^2}\right].\end{aligned}$$

Note that α'_H is finite because Ω is finite. Then, we have

$$\left|\xi_H(t|q_1(t))-\xi_H(t|q_2(t))\right| \le \alpha'_H\left\|q_1(\cdot)-q_2(\cdot)\right\|_\infty. \tag{B.9}$$

Second, consider $\left\|\xi_U(t|q_1(\cdot))-\xi_U(t|q_2(\cdot))\right\|_\infty$. We have

$$\begin{aligned}\xi_U(t|q(t)) &= \gamma\int_{p_H}^{\bar{v}_H}\left[\int_{\max\left\{v_H-\frac{p_H-p_R-q(t)p}{1-q(t)},\underline{v}_R\right\}}^{v_H-p} f(v_R,v_H)\,\mathrm{d}v_R\right]\mathrm{d}v_H\\ &\quad+\gamma\int_{p_R+p}^{p_H}\left[\int_{\frac{p_R+q(t)p-q(t)v_H}{1-q(t)}}^{v_H-p} f(v_R,v_H)\,\mathrm{d}v_R\right]\mathrm{d}v_H.\end{aligned}$$

Note that we write the lower bound of the second integration as p_R+p instead of $\underline{v}_H$ because for $v_H<p_R+p$, we have $v_H-p<\frac{p_R+q(t)p-q(t)v_H}{1-q(t)}$ for all $q(t)$. Since $\max\left\{v_H-\frac{p_H-p_R-q(t)p}{1-q(t)},\underline{v}_R\right\}$ is decreasing in $q(t)$ and $\frac{p_R+q(t)p-q(t)v_H}{1-q(t)}$ is decreasing in

$q(t)$ for $v_H \geq p_R + p$, we have

$$\begin{aligned}
&\left|\xi_U(t|q_1(t)) - \xi_U(t|q_2(t))\right| \\
= \;& \gamma \int_{p_H}^{\bar{v}_H} \left[\int_{\max\left\{v_H - \frac{p_H - p_R - q_2(t)p}{1-q_2(t)}, \underline{v}_R\right\}}^{\max\left\{v_H - \frac{p_H - p_R - q_1(t)p}{1-q_1(t)}, \underline{v}_R\right\}} f(v_R, v_H)\, \mathrm{d}v_R \right] \mathrm{d}v_H \\
& + \gamma \int_{p_R+p}^{p_H} \left[\int_{\frac{p_R + q_2(t)p - q_2(t)v_H}{1-q_2(t)}}^{\frac{p_R + q_1(t)p - q_1(t)v_H}{1-q_1(t)}} f(v_R, v_H)\, \mathrm{d}v_R \right] \mathrm{d}v_H \\
\leq \;& \alpha'_H \left[q_2(t) - q_1(t)\right] + \gamma \int_{p_R+p}^{p_H} \left[\int_{\frac{p_R + q_2(t)p - q_2(t)v_H}{1-q_2(t)}}^{\frac{p_R + q_1(t)p - q_1(t)v_H}{1-q_1(t)}} f(v_R, v_H)\, \mathrm{d}v_R \right] \mathrm{d}v_H .
\end{aligned}$$

We can write the last integration equivalently as

$$\gamma \int_{p_R+p}^{p_H} \left[\int_{\frac{p_R + q_2(t)p - q_2(t)v_H}{1-q_2(t)}}^{\frac{p_R + q_1(t)p - q_1(t)v_H}{1-q_1(t)}} f(v_R, v_H)\, \mathrm{d}v_R \right] \mathrm{d}v_H = \gamma \int_{\underline{v}_R}^{p_R} \left[\int_{v_R + p + \frac{p_R - v_R}{q_2(t)}}^{v_R + p + \frac{p_R - v_R}{q_1(t)}} f(v_R, v_H)\, \mathrm{d}v_H \right] \mathrm{d}v_R .$$

Now we bound this integration from above case by case. Define $\bar{q}_U$ as the solution to $\frac{p_R + \bar{q}_U p - \bar{q}_U v_H}{1-\bar{q}_U} = \underline{v}_R$, so $\bar{q}_U = \frac{p_R - \underline{v}_R}{p_H - \underline{v}_R - p}$. At $q(t) = \bar{q}_U$, $\frac{p_R + q(t)p - q(t)v_H}{1-q(t)} = v_R$ becomes the negatively-sloped diagonal of the rectangle $\{(v_R, v_H) : \underline{v}_R \leq v_R \leq p_R, p_R + p \leq v_H \leq p_H\}$.

- If $q_1(t) < q_2(t) \le \bar{q}_U$,

$$
\begin{aligned}
&\gamma \int\limits_{p_R+p}^{p_H} \left[\int\limits_{\frac{p_R+q_2(t)p-q_2(t)v_H}{1-q_2(t)}}^{\frac{p_R+q_1(t)p-q_1(t)v_H}{1-q_1(t)}} f_t(v_R, v_H)\,\mathrm{d}v_R \right] \mathrm{d}v_H \\
\le\;& \gamma \sup_{(v_R,v_H)\in\Omega} \{f(v_R, v_H)\} \int\limits_{p_R+p}^{p_H} \frac{(v_H - p_R - p)\,[q_2(t) - q_1(t)]}{[1 - q_1(t)]\,[1 - q_2(t)]}\,\mathrm{d}v_H \\
\le\;& \gamma \sup_{(v_R,v_H)\in\Omega} \{f(v_R, v_H)\} \frac{q_2(t) - q_1(t)}{(1 - \bar{q}_U)^2} \int\limits_{p_R+p}^{p_H} (v_H - p_R - p)\,\mathrm{d}v_H \\
=\;& \alpha'_U\,[q_2(t) - q_1(t)]\,,
\end{aligned}
$$

  where

$$
\alpha'_U = \gamma \sup_{(v_R,v_H)\in\Omega} \{f(v_R, v_H)\} \frac{(p_H - \underline{v}_R - p)^2}{2}.
$$

- If $\bar{q}_U \le q_1(t) < q_2(t)$,

$$
\begin{aligned}
&\gamma \int\limits_{\underline{v}_R}^{p_R} \left[\int\limits_{v_R+p+\frac{p_R-v_R}{q_2(t)}}^{v_R+p+\frac{p_R-v_R}{q_1(t)}} f(v_R, v_H)\,\mathrm{d}v_H \right] \mathrm{d}v_R \\
\le\;& \gamma \sup_{(v_R,v_H)\in\Omega} \{f(v_R, v_H)\} \int\limits_{\underline{v}_R}^{p_R} \frac{(p_R - v_R)\,[q_2(t) - q_1(t)]}{q_1(t)q_2(t)}\,\mathrm{d}v_R \\
\le\;& \gamma \sup_{(v_R,v_H)\in\Omega} \{f(v_R, v_H)\} \frac{q_2(t) - q_1(t)}{\bar{q}_U^2} \int\limits_{\underline{v}_R}^{p_R} (p_R - v_R)\,\mathrm{d}v_R \\
=\;& \alpha'_U\,[q_2(t) - q_1(t)]\,.
\end{aligned}
$$

- If $q_1(t) < \bar{q}_U < q_2(t)$,

$$
\begin{aligned}
& \gamma \int_{p_R+p}^{p_H} \left[\int_{\frac{p_R+q_2(t)p-q_2(t)v_H}{1-q_2(t)}}^{\frac{p_R+q_1(t)p-q_1(t)v_H}{1-q_1(t)}} f(v_R, v_H)\, \mathrm{d}v_R \right] \mathrm{d}v_H \\
= \; & \gamma \int_{p_R+p}^{p_H} \left[\int_{\frac{p_R+\bar{q}_U p-\bar{q}_U v_H}{1-\bar{q}_U}}^{\frac{p_R+q_1(t)p-q_1(t)v_H}{1-q_1(t)}} f(v_R, v_H)\, \mathrm{d}v_R \right] \mathrm{d}v_H \\
& + \gamma \int_{\underline{v}_R}^{p_R} \left[\int_{v_R+p+\frac{p_R-v_R}{q_2(t)}}^{v_R+p+\frac{p_R-v_R}{\bar{q}_U}} f(v_R, v_H)\, \mathrm{d}v_H \right] \mathrm{d}v_R \\
\leq \; & \alpha'_U \left[\bar{q}_U - q_1(t)\right] + \alpha'_U \left[q_2(t) - \bar{q}_U\right] \\
= \; & \alpha'_U \left[q_2(t) - q_1(t)\right].
\end{aligned}
$$

Thus, we obtain that $\left|\xi_U(t|q_1(t)) - \xi_U(t|q_2(t))\right| \leq \alpha'_U \left[q_2(t) - q_1(t)\right]$, where α'_U is finite. Then, we have

$$
\left|\xi_U(t|q_1(t)) - \xi_U(t|q_2(t))\right| \leq \alpha'_U \left\|q_1(\cdot) - q_2(\cdot)\right\|_\infty, \tag{B.10}
$$

Finally, by plugging (B.9) and (B.10) into (B.8), we obtain that $\left|b(q_1(\cdot)) - b(q_2(\cdot))\right| \leq \bar{\alpha}\left\|q_1(\cdot) - q_2(\cdot)\right\|_\infty$ where $\bar{\alpha} = \alpha_H\alpha'_H + \alpha_U\alpha'_U$ and $\bar{\alpha}$ does not depend on t. Then, we have $\left\|b(q_1(\cdot)) - b(q_2(\cdot))\right\|_\infty \leq \bar{\alpha}\left\|q_1(\cdot) - q_2(\cdot)\right\|_\infty$. Therefore, we have proved the Lipschitz continuity of $b(q(\cdot))$, and hence the existence of $q^*(\cdot)$. If $\bar{\alpha} < 1$, $b(q(\cdot))$ is a contraction mapping from $\mathcal{Q}$ to $\mathcal{Q}$, hence $q^*(\cdot)$ is unique. $q^*(\cdot)$ is increasing in t because of the following. For every sample path, $g(q(\cdot))$ is constant in t for $t \leq \tau(q(\cdot))$ and is equal to zero for $t > \tau(q(\cdot))$. It is easy to see that $h(q(\cdot))$ is decreasing in t, hence after taking the average of $g(q(\cdot))$ for each sample path, we know that $b(q(\cdot))$ is increasing in t. Therefore, the solution to $b(q(\cdot)) = q(\cdot)$ must be increasing in t. The whole proof is complete.

Proof of Theorem III.3

(i) For any $q(\cdot)$ and any t, as $n \to \infty$, by the Strong Law of Large Numbers, we have

$$\begin{aligned}
\frac{N_H^n(t|q(\cdot))}{n} &\to \int_0^t \lambda_H(s|q(\cdot))\,\mathrm{d}s \qquad \text{a.s.,}\\
\frac{N_U^n(t|q(\cdot))}{n} &\to \int_0^t \lambda_U(s|q(\cdot))\,\mathrm{d}s \qquad \text{a.s.,}\\
\frac{N_R^n(t|q(\cdot))}{n} &\to \int_0^t \lambda_R(s|q(\cdot))\,\mathrm{d}s \qquad \text{a.s..}
\end{aligned}$$

Moreover, as $n \to \infty$, we have

$$\begin{aligned}
\tau_T^n(q(\cdot)) &= \inf\left\{t \geq 0 : N_H^n(t|q(\cdot)) + N_U^n(t|q(\cdot)) + 1 + N_R^n(t|q(\cdot)) \geq nK_H + nK_R\right\}\\
&= \inf\left\{t \geq 0 : \frac{N_H^n(t|q(\cdot))}{n} + \frac{N_U^n(t|q(\cdot))}{n} + \frac{1}{n} + \frac{N_R^n(t|q(\cdot))}{n} \geq K_H + K_R\right\}\\
&\to \inf\left\{t \geq 0 : \int_0^t \left[\lambda_H(s|q(\cdot)) + \lambda_U(s|q(\cdot)) + \lambda_R(s|q(\cdot))\right]\,\mathrm{d}s \geq K_H + K_R\right\} \qquad \text{a.s..}
\end{aligned}$$

The convergence of $\tau_H^n(q(\cdot))$ and $\tau_R^n(q(\cdot))$ follows from the same approach, then the convergence of $\tau^n(q(\cdot))$ is obtained.

(ii) To derive $q^{\infty *}(\cdot)$, we need to first derive $g^\infty(q(\cdot))$ and $h^\infty(q(\cdot))$ and then derive

$b^{\infty}(q(\cdot))$. First,

$$\begin{aligned}
g^{\infty}(q(\cdot)) &= \lim_{n\to\infty} \mathop{\mathbb{E}}_{N_H^n(t|q(\cdot)),N_U^n(t|q(\cdot)),N_R^n(t|q(\cdot))} \left\{ \min\left\{ \frac{[nK_H - N_H^n(\tau^n(q(\cdot))|q(\cdot))]^+}{N_U^n(\tau^n(q(\cdot))|q(\cdot))+1}, 1 \right\} \right. \\
&\qquad \left. \cdot \mathbb{1}\left\{t \le \tau^n(q(\cdot))\right\} \right\} \\
&= \lim_{n\to\infty} \mathop{\mathbb{E}}_{N_H^n(t|q(\cdot)),N_U^n(t|q(\cdot)),N_R^n(t|q(\cdot))} \left\{ \min\left\{ \frac{[nK_H - N_H^n(\tau^\infty(q(\cdot))|q(\cdot))]^+}{N_U^n(\tau^\infty(q(\cdot))|q(\cdot))+1}, 1 \right\} \right. \\
&\qquad \left. \cdot \mathbb{1}\left\{t \le \tau^\infty(q(\cdot))\right\} \right\} \\
&= \lim_{n\to\infty} \mathop{\mathbb{E}}_{N_H^n(t|q(\cdot)),N_U^n(t|q(\cdot)),N_R^n(t|q(\cdot))} \left\{ \min\left\{ \frac{\left[K_H - \frac{N_H^n(\tau^\infty(q(\cdot))|q(\cdot))}{n}\right]^+}{\frac{N_U^n(\tau^\infty(q(\cdot))|q(\cdot))}{n} + \frac{1}{n}}, 1 \right\} \right. \\
&\qquad \left. \cdot \mathbb{1}\left\{t \le \tau^\infty(q(\cdot))\right\} \right\} \\
&= \min\left\{ \frac{\left[K_H - \int_0^{\tau^\infty(q(\cdot))} \lambda_H(t|q(\cdot))\,\mathrm{d}t\right]^+}{\int_0^{\tau^\infty(q(\cdot))} \lambda_U(t|q(\cdot))\,\mathrm{d}t}, 1 \right\} \cdot \mathbb{1}\left\{t \le \tau^\infty(q(\cdot))\right\}.
\end{aligned}$$

Second,

$$\begin{aligned}
h^{\infty}(q(\cdot)) &= \lim_{n\to\infty} \mathbb{P}\left(\frac{N_H^n(t|q(\cdot))}{n} < K_H, \frac{N_R^n(t|q(\cdot))}{n} < K_R, \right. \\
&\qquad \left. \frac{N_H^n(t|q(\cdot))}{n} + \frac{N_U^n(t|q(\cdot))}{n} + \frac{N_R^n(t|q(\cdot))}{n} < K_H + K_R \right) \\
&= \mathbb{P}\left(\int_0^t \lambda_H(s|q(\cdot))\,\mathrm{d}s < K_H, \int_0^t \lambda_R(s|q(\cdot))\,\mathrm{d}s < K_R, \right. \\
&\qquad \left. \int_0^t \left[\lambda_H(s|q(\cdot)) + \lambda_U(s|q(\cdot)) + \lambda_R(s|q(\cdot))\right] \mathrm{d}s < K_H + K_R \right) \\
&= \mathbb{1}\left\{t \le \tau^\infty(q(\cdot))\right\}.
\end{aligned}$$

For $t \le \tau^\infty(q(\cdot))$, $b^\infty(q(\cdot))$ is constant in t, hence so is $q^{\infty *}(\cdot)$. Note that for $t > \tau^\infty(q(\cdot))$, $b^\infty(q(\cdot))$ is not defined. Since the upgrade probability is irrelevant in this case, without loss of generality, we let $b^\infty(q(\cdot))$ take the same value as $t \le \tau^\infty(q(\cdot))$ to preserve the differentiability of $b^\infty(q(\cdot))$. It then follows that $q^{\infty *}(\cdot)$ is given by Part (ii) of the theorem.

Proof of Theorem III.4

With uniform valuation distribution, the formula of $\hat{\tau}^\infty(q)$ in Theorem III.3 reduces to

$$\hat{\tau}^\infty(q) = \begin{cases} \frac{K_H}{\lambda_H(q)} & \text{if } \frac{K_H}{\lambda_H(q)} \le \frac{K_H+K_R}{\lambda_H(q)+\lambda_U(q)+\lambda_R}, \\ \frac{K_R}{\lambda_R} & \text{if } \frac{K_R}{\lambda_R} \le \frac{K_H+K_R}{\lambda_H(q)+\lambda_U(q)+\lambda_R}, \\ \frac{K_H+K_R}{\lambda_H(q)+\lambda_U(q)+\lambda_R} & \text{if } \frac{K_H}{\lambda_H(q)} > \frac{K_H+K_R}{\lambda_H(q)+\lambda_U(q)+\lambda_R} \text{ and } \frac{K_R}{\lambda_R} > \frac{K_H+K_R}{\lambda_H(q)+\lambda_U(q)+\lambda_R}. \end{cases} \tag{B.11}$$

(i) $q_f = 1$ corresponds to Case b, where $q_f = 1$ requires $K_H \ge (\lambda_H^b + \lambda_U^b)\tau^\infty(1)$. We will show that $K_H \ge (\lambda_H^b + \lambda_U^b)\tau^\infty(1)$ is equivalent to $K_H \ge (\lambda_H^b + \lambda_U^b)T$. First, if $\hat{\tau}^\infty(1) = K_H/\lambda_H^b$, since $K_H/\lambda_H^b \ge K_H/\lambda_H^a \ge T$, we have $\tau^\infty(1) = \min\{K_H/\lambda_H^b, T\} = T$. Second, if $\hat{\tau}^\infty(1) = K_R/\lambda_R^b$, since $K_R/\lambda_R^b \ge K_R/\lambda_R^a \ge T$, we have $\tau^\infty(1) = \min\{K_R/\lambda_R^b, T\} = T$. Third, if $\hat{\tau}^\infty(1) = (K_H + K_R)/(\lambda_H^b + \lambda_U^b + \lambda_R^b)$, suppose $\hat{\tau}^\infty(1) < T$, then it is easy to see that $K_H \ge (\lambda_H^b + \lambda_U^b)\tau^\infty(1)$ is equivalent to $K_R/\lambda_R^b \le (K_H+K_R)/(\lambda_H^b+\lambda_U^b+\lambda_R^b)$ which contradicts the second condition in (B.11) for $\hat{\tau}^\infty(1) = (K_H + K_R)/(\lambda_H^b + \lambda_U^b + \lambda_R^b)$ to occur. Thus, we also have $\tau^\infty(1) = T$ in this case. Overall, $K_H \ge (\lambda_H^b + \lambda_U^b)\tau^\infty(1)$ is equivalent to $K_H \ge (\lambda_H^b + \lambda_U^b)T$. Since $\lambda_H^b + \lambda_U^b$ is decreasing in p, if $K_H \ge (\lambda_H^b + \lambda_U^b)T$ at $p = 0$, that is, if $K_H \ge (\lambda T/u^2)[(u - p_H + 2p_R)(u - p_H) + \gamma(p_H - p_R)(2u - p_H + p_R)]$, then $q_f = 1$ for all $0 \le p < p_H - p_R$.

(ii) Next, consider the case of $K_H < (\lambda T/u^2)[(1 - \gamma)(u - p_H + 2p_R)(u - p_H) + \gamma(u^2 - p_R^2)]$. We first characterize when $q_f = 1$. In Case b, solving $K_H = (\lambda_H^b + \lambda_U^b)T$ yields $p = \bar{p}$. Since $(\lambda_H^b + \lambda_U^b)T$ is decreasing in p, $q_f = 1$ for $p \ge \bar{p}$.

Now we derive q_f for $0 \le p < \bar{p}$. We first derive q_f for the case of $\hat{\tau}^\infty(q_f) \ge T$ and then incorporate the case of $\hat{\tau}^\infty(q_f) < T$. When $\hat{\tau}^\infty(q_f) \ge T$, Cases c, d, e may occur in sequence as p decreases. Since $(p_H - p_R - q_f p)/(1-q_f) = \infty > u$ at $p = \bar{p}$, we are in Case c where (3.1) becomes $q = [K_H - \lambda_H^c T]/[\lambda_U^c(q)T]$. Solving (3.1) yields

$$q_f = \frac{\frac{K_H}{\lambda T}u^2 - (u - p_H + 2p_R)(u - p_H) + \gamma(u - p_H + p_R)^2}{\gamma(u-p)^2} \tag{B.12}$$

which is increasing in p. Then, solving the condition for Case c $(p_H - p_R - q_f p)/(1 - q_f) \geq u$ yields $p \geq \underline{p}$, hence Case c occurs where q_f is given by (B.12) for $\underline{p}^+ \leq p < \bar{p}$ (note that $\underline{p}$ can be negative). For $0 \leq p < \underline{p}^+$, Cases d and e may occur. In Case d, (3.1) becomes $q = [K_H - \lambda_H^d(q)T]/[\lambda_U^d(q)T]$. Solving (3.1) yields

$$q_f = \frac{\frac{K_H}{\lambda T}u^2 - (u - p_H + 2p_R)(u - p_H)}{\gamma(p_H - p_R - p)^2 + \frac{K_H}{\lambda T}u^2 - (u - p_H + 2p_R)(u - p_H)} \tag{B.13}$$

which is increasing in p. Then, solving the condition for Case d $(p_H - p_R - q_f p)/(1 - q_f) \geq p_H$ yields

$$p \geq p_H - p_R - \frac{1}{\gamma p_R}\left[\frac{K_H}{\lambda T}u^2 - (u - p_H + 2p_R)(u - p_H)\right] \stackrel{\text{def}}{=\!=} \tilde{p}.$$

So Case d occurs for $\tilde{p}^+ \leq p < \underline{p}^+$ and Case e occurs for $0 \leq p < \tilde{p}^+$. In Case e, (3.1) becomes $q = [K_H - \lambda_H^e(q)T]/[\lambda_U^e(q)T]$. Solving (3.1) also yields (B.13). Thus, q_f is given by (B.13) for $0 \leq p < \underline{p}^+$.

Now, we incorporate the case of $\hat{\tau}^\infty(q_f) < T$. Note that when $\hat{\tau}^\infty(q_f) < T$, we must have $\hat{\tau}^\infty(q_f) = (K_H + K_R)/[\lambda_H(q_f) + \lambda_U(q_f) + \lambda_R]$ which is the last case in (B.11), because $\hat{\tau}^\infty(q_f) = K_H/\lambda_H(q_f)$ implies $q_f = 0$ and $\hat{\tau}^\infty(q_f) = K_R/\lambda_R$ implies $q_f = 1$. To analyze the case of $\hat{\tau}^\infty(q_f) < T$, we first show that it may only occur for small enough p. In Cases c, d, e, $\hat{\tau}^\infty(q_f) < T$ if and only if $K_H + K_R < [\lambda_H(q_f) + \lambda_U(q_f) + \lambda_R]T$. We will show that when $\hat{\tau}^\infty(q_f) \geq T$, $K_H + K_R \geq [\lambda_H(q_f) + \lambda_U(q_f) + \lambda_R]T$ for large

p. Using the above derived q_f, in Case c when $\hat{\tau}^{\infty}(q_f) \geq T$, we have

$$
\begin{aligned}
& \frac{\mathrm{d}[\lambda_H^c + \lambda_U^c(q_f) + \lambda_R^c]}{\mathrm{d}p} \\
= \; & \frac{2\gamma\lambda p_R}{u^2}\left[\frac{\gamma p_R(u-p)}{\frac{K_H}{\lambda T}u^2 - (u-p_H+2p_R)(u-p_H) + \gamma(u-p_H+p_R)^2} - 1\right] \\
\leq \; & \frac{2\gamma\lambda p_R}{u^2}\left[\frac{\gamma p_R(u-\underline{p})}{\frac{K_H}{\lambda T}u^2 - (u-p_H+2p_R)(u-p_H) + \gamma(u-p_H+p_R)^2} - 1\right] \\
< \; & \frac{2\gamma\lambda p_R}{u^2}\left[\frac{\gamma(u-p_H+p_R)(u-\underline{p})}{\frac{K_H}{\lambda T}u^2 - (u-p_H+2p_R)(u-p_H) + \gamma(u-p_H+p_R)^2} - 1\right] \\
= \; & 0,
\end{aligned}
$$

hence $\lambda_H^c + \lambda_U^c(q_f) + \lambda_R^c$ is decreasing in p. Denote $\lambda_N(q) = \lambda - \lambda_H(q) - \lambda_U(q) - \lambda_R$ as the arrival rate of consumers who do not book any product. In Case d, we have

$$
\begin{aligned}
\lambda_N^d(q_f) \; = \; & \frac{\gamma\lambda p_R^2}{u^2}\left\{\frac{p^2 - 2\left\{p_H - p_R - \frac{1}{\gamma p_R}\left[\frac{K_H}{\lambda T}u^2 - (u-p_H+2p_R)(u-p_H)\right]\right\}p}{\frac{K_H}{\lambda T}u^2 - (u-p_H+2p_R)(u-p_H)}\right. \\
& \left. + \frac{\frac{1}{\gamma}\left[\frac{K_H}{\lambda T}u^2 - (u-p_H+2p_R)(u-p_H)\right] + (p_H-p_R)^2}{\frac{K_H}{\lambda T}u^2 - (u-p_H+2p_R)(u-p_H)}\right\},
\end{aligned}
$$

which is a parabola whose axis of symmetry is $\tilde{p}$. Thus, $\lambda_N^d(q_f)$ is increasing in p, hence $\lambda_H^d(q_f) + \lambda_U^d(q_f) + \lambda_R^d$ is decreasing in p. In Case e, we have

$$
\lambda_N^e(q_f) = \frac{\lambda}{\gamma u^2}\left[-\frac{K_H}{\lambda T}u^2 + (u-p_H+2p_R)(u-p_H) + 2\gamma p_H p_R - \gamma p_R^2\right]
$$

which is a constant, hence so is $\lambda_H^e(q_f) + \lambda_U^e(q_f) + \lambda_R^e$. Thus, combining the above analysis for Cases c, d, e, we know that $\lambda_H(q_f) + \lambda_U(q_f) + \lambda_R$ is decreasing in p for $0 \leq p < \bar{p}$. This means that $\hat{\tau}^{\infty}(q_f) < T$ may only occur for small enough p.

Next, we show that $\hat{\tau}^{\infty}(q_f) < T$ never occurs in Case d or e and may only occur in Case c. We will prove that $K_H + K_R \geq [\lambda - \lambda_N(q_f)]T$ at $p = 0$ in Cases d and e.

In Case d at $p = 0$, we have

$$\begin{aligned} & K_H + K_R - [\lambda - \lambda_N(q_f)]T \\ = \;& K_H + K_R - \lambda T + \lambda T(\frac{p_R}{u})^2 \left[1 + \frac{(p_H - p_R)^2}{\frac{K_H}{\lambda T}u^2 - (u - p_H + 2p_R)(u - p_H)}\right] \end{aligned} \quad \text{(B.14)}$$

The derivative of (B.14) with respect to K_H is

$$1 - \frac{p_R^2(p_H - p_R)^2}{\left[\frac{K_H}{\lambda T}u^2 - (u - p_H + 2p_R)(u - p_H)\right]^2}$$

which is negative for $\lambda_H^a T \leq K_H < (\lambda T/u^2)[u^2 - 2(p_H - p_R)u + p_H^2 - p_H p_R - p_R^2]$ and positive for $K_H > (\lambda T/u^2)[u^2 - 2(p_H - p_R)u + p_H^2 - p_H p_R - p_R^2]$. Thus, by taking $K_H = (\lambda T/u^2)[u^2 - 2(p_H - p_R)u + p_H^2 - p_H p_R - p_R^2]$, we obtain

$$\text{(B.14)} \geq K_R - \frac{\lambda T}{u^2}(p_H - p_R)(2u - p_H - p_R) = K_R - \lambda_R^a T \geq 0,$$

hence $K_H + K_R \geq [\lambda - \lambda_N^d(q_f)]T$ at $p = 0$ in Case d. In Case e, note that $\lambda_N^d(q_f) = \lambda_N^e(q_f)$ at $p = \tilde{p}$. Since $\lambda_N^e(q_f)$ stays constant in p and $\lambda_N^d(q_f)$ is increasing in p, $\lambda_N^e(q_f)$ at $p = 0$ is larger than $\lambda_N^d(q_f)$ at $p = 0$. Thus, our analysis for Case d implies that $K_H + K_R > [\lambda - \lambda_N^e(q_f)]T$ at $p = 0$ in Case e as well.

So far, we have known that $\hat{\tau}^\infty(q_f) < T$ may only occur in Case c for small enough p. Now we derive q_f in this case. (3.1) becomes

$$q = \frac{K_H - \lambda_H^c \frac{K_H + K_R}{\lambda_H^c + \lambda_U^c(q) + \lambda_R^c}}{\lambda_U^c(q) \frac{K_H + K_R}{\lambda_H^c + \lambda_U^c(q) + \lambda_R^c}},$$

and can be simplified to

$$\gamma(u - p)^2 q^2 + \beta q + \gamma k p_R^2 = 0, \quad \text{(B.15)}$$

where $k = K_H/(K_H + K_R)$. (B.15) is a quadratic equation. Now we show that the smaller root $q = [-\beta + \sqrt{\beta^2 - 4\gamma^2 k p_R^2 (u-p)^2}]/[2\gamma(u-p)^2]$ is infeasible because it yields $(p_H - p_R - qp)/(1-q) < u$ which contradicts the condition for Case c to occur. With $q = [-\beta + \sqrt{\beta^2 - 4\gamma^2 k p_R^2 (u-p)^2}]/[2\gamma(u-p)^2]$, we can simplify $(p_H - p_R - qp)/(1-q) < u$ to $\sqrt{\beta^2 - 4\gamma^2 k p_R^2 (u-p)^2} > -\beta - 2\gamma(u-p)(u - p_H + p_R)$. If $-\beta - 2\gamma(u-p)(u-p_H+p_R) < 0$, $\sqrt{\beta^2 - 4\gamma^2 k p_R^2 (u-p)^2} > -\beta - 2\gamma(u-p)(u-p_H+p_R)$ is trivially satisfied. If $-\beta - 2\gamma(u-p)(u-p_H+p_R) \geq 0$, by taking square on both sides of $\sqrt{\beta^2 - 4\gamma^2 k p_R^2 (u-p)^2} > -\beta - 2\gamma(u-p)(u-p_H+p_R)$ and rearranging terms, we obtain $-\beta(u-p_H+p_R) - \gamma(u-p_H+p_R)^2(u-p) - \gamma k p_R^2 (u-p) \geq 0$. Since $-\beta \geq 2\gamma(u-p)(u-p_H+p_R)$, we have $-\beta(u-p_H+p_R) - \gamma(u-p_H+p_R)^2(u-p) - \gamma k p_R^2(u-p) \geq \gamma(u-p)[(u-p_H+p_R)^2 - k p_R^2] > 0$. So, $q = [-\beta + \sqrt{\beta^2 - 4\gamma^2 k p_R^2 (u-p)^2}]/[2\gamma(u-p)^2]$ always leads to $(p_H - p_R - qp)/(1-q) < u$. Thus, in Case c when $\hat{\tau}^\infty(q_f) < T$, the equilibrium q_f is given by the larger root of (B.15):

$$q_f = \frac{-\beta + \sqrt{\beta^2 - 4\gamma^2 k p_R^2 (u-p)^2}}{2\gamma(u-p)^2} = \frac{2\gamma k p_R^2}{-\beta - \sqrt{\beta^2 - 4\gamma^2 k p_R^2 (u-p)^2}}. \tag{B.16}$$

To show that q_f is increasing in p, we need to show that $-\beta - \sqrt{\beta^2 - 4\gamma^2 k p_R^2 (u-p)^2}$ is decreasing in p. The derivative of $-\beta - \sqrt{\beta^2 - 4\gamma^2 k p_R^2 (u-p)^2}$ with respect to p is

$$\frac{-2\gamma k p_R}{\sqrt{\beta^2 - 4\gamma^2 k p_R^2 (u-p)^2}} \left[\sqrt{\beta^2 - 4\gamma^2 k p_R^2 (u-p)^2} + \beta + 2\gamma p_R (u-p) \right]. \tag{B.17}$$

If $\beta + 2\gamma p_R(u-p) \geq 0$, (B.17)$\leq 0$ trivially. If $\beta + 2\gamma p_R (u-p) < 0$, we have

$$\begin{aligned}
\text{(B.17)} &\leq \frac{-2\gamma k p_R}{\sqrt{\beta^2 - 4\gamma^2 k p_R^2 (u-p)^2}} \left[\sqrt{\beta^2 - 4\gamma^2 p_R^2 (u-p)^2} + \beta + 2\gamma p_R (u-p) \right] \\
&= \frac{-2\gamma k p_R}{\sqrt{\beta^2 - 4\gamma^2 k p_R^2 (u-p)^2}} \sqrt{-\beta - 2\gamma p_R (u-p)} \\
&\quad \cdot \left[\sqrt{-\beta + 2\gamma p_R (u-p)} - \sqrt{-\beta - 2\gamma p_R (u-p)} \right] \\
&\leq 0.
\end{aligned}$$

Thus, q_f is increasing in p in Case c when $\hat{\tau}^\infty(q_f) < T$.

Finally, we characterize the threshold p in Case c where $\hat{\tau}^\infty(q_f) \geq T$ switches to $\hat{\tau}^\infty(q_f) < T$. When $\hat{\tau}^\infty(q_f) \geq T$, $K_H + K_R = [\lambda_H^c + \lambda_U^c(q_f) + \lambda_R^c]T$ can be simplified to

$$\begin{aligned}\gamma p_R^2 p^2 + 2\gamma p_R \left[\frac{K_H}{\lambda T}u^2 - u^2 + 2(p_H - p_R)u - p_H^2 + p_H p_R + p_R^2\right] p \\ + \left[\frac{K_H + K_R}{\lambda T}u^2 - u^2 + 2(1-\gamma)p_H p_R + \gamma p_R^2\right] \\ \cdot \left[\frac{K_H}{\lambda T}u^2 - (u - p_H + 2p_R)(u - p_H)\right] + \gamma p_R^2 (p_H - p_R)^2 &= 0. \quad \text{(B.18)}\end{aligned}$$

(B.18) is a quadratic equation whose smaller root is always negative and larger root (which may also be negative) is $\underline{p}'$. Therefore, by combining all our analysis above, we conclude the following. If $\underline{p}^+ \geq \underline{p}'^+$, we always have $\hat{\tau}^\infty(q_f) \geq T$ for $0 \leq p < \bar{p}$; Case c occurs for $\underline{p}^+ \leq p < \bar{p}$ where q_f is given by (B.12), and Case d or e occurs for $0 \leq p < \underline{p}^+$ where q_f is given by (B.13). If $\underline{p}^+ < \underline{p}'^+$, Case c always occurs for $0 \leq p < \bar{p}$ and q_f is given by (B.12) for $\underline{p}'^+ \leq p < \bar{p}$ and (B.16) for $0 \leq p < \underline{p}'^+$. Finally, all previous analysis indicates that q_f is the unique solution to (3.1).

(iii) In the proof of Part (ii), we have shown that q_f is increasing in p in all cases.

Proof of Theorem III.5

If $K_H \geq (\lambda T/u^2)[(u - p_H + 2p_R)(u - p_H) + \gamma(p_H - p_R)(2u - p_H + p_R)]$, we have $q_f = 1$ for all $0 \leq p \leq p_H - p_R$, and as found in the proof of Theorem III.4(i), $\tau^\infty(1) = T$. Thus, the revenue function is $\Pi_f(p) = p_R\gamma\lambda(\xi_R^b + \xi_U^b)T + p\gamma\lambda\xi_U^b T + (1-\gamma)\Pi_{N,f} = (\gamma\lambda T/u^2)\left[p^3 - 2up^2 + (u^2 - 3p_R^2)p + u^2 p_R - p_R^3\right] + (1-\gamma)\Pi_{N,f}$. The first-order condition is $3p^2 - 4up + u^2 - 3p_R^2 = 0$. The larger root of this quadratic equation is $(2u + \sqrt{u^2 + 9p_R^2})/3$ which is larger than u; the smaller root is p_{foc}^b. Thus, $\Pi_f(p)$ is increasing in p for $p < p_{foc}^b$ and decreasing in p for $p > p_{foc}^b$; the optimal upgrade price is $\min\left\{(p_{foc}^b)^+, p_H - p_R\right\}$.

Next, consider the case of $K_H < (\lambda T/u^2)[(u - p_H + 2p_R)(u - p_H) + \gamma(p_H - p_R)(2u -$

$p_H + p_R)]$. For $\bar{p} \le p \le p_H - p_R$, we have $q_f = 1$, hence our proof above indicates that the local optimum is $\min\left\{\max\left\{(p_{foc}^b)^+, \bar{p}\right\}, p_H - p_R\right\}$. For $0 \le p < \bar{p}$, we have $0 < q_f < 1$ and

$$\Pi_f(p) = \left\{p_R[\lambda_U^i(q_f) + \lambda_R^i] + p\lambda_U^i(q_f)q_f + p_H\lambda_H^i(q_f)\right\}\min\left\{\frac{K_H + K_R}{\lambda_H^i(q_f) + \lambda_U^i(q_f) + \lambda_R^i}, T\right\},$$

where $i = c, d, e$ as we may be in Case c, d, or e. We will show that $\Pi_f(p)$ is increasing in p for $0 \le p < \bar{p}$, thus the global optimal upgrade price is also $p^* = \min\left\{\max\left\{(p_{foc}^b)^+, \bar{p}\right\}, p_H - p_R\right\}$.

First, consider Case c. If $(K_H + K_R)/[\lambda_H^c + \lambda_U^c(q_f) + \lambda_R^c] \ge T$, the revenue function becomes

$$\begin{aligned}
\Pi_f(p) &= p_R\gamma\lambda[\xi_U^c(q_f) + \xi_R^c]T + p[K_H - (1-\gamma)\lambda\xi_H^a T] + (1-\gamma)\Pi_{N,f} \\
&= \frac{\frac{\gamma\lambda T}{u^2}}{\frac{\tilde{K}_H}{\gamma\lambda T}u^2 + p_R^2}\left\{-p_R^3p^2 + \left[(\frac{\tilde{K}_H}{\gamma\lambda T})^2u^4 - \frac{\tilde{K}_H}{\gamma\lambda T}p_R^2u^2 + 2p_R^3u - 2p_R^4\right]p + \frac{\tilde{K}_H}{\gamma\lambda T}p_Ru^4\right\} \\
&\quad +(1-\gamma)\Pi_{N,f},
\end{aligned}$$

where $\tilde{K}_H = K_H - (1-\gamma)\lambda\xi_H^a T$. So, $\Pi_f(p)$ is concave in p. Solving the first-order condition

$$\frac{\mathrm{d}\Pi_f}{\mathrm{d}p} = \frac{\frac{\gamma\lambda T}{u^2}}{\frac{\tilde{K}_H}{\gamma\lambda T}u^2 + p_R^2}\left[-2p_R^3p + (\frac{\tilde{K}_H}{\gamma\lambda T})^2u^4 - \frac{\tilde{K}_H}{\gamma\lambda T}p_R^2u^2 + 2p_R^3u - 2p_R^4\right] = 0$$

yields

$$p = \frac{1}{2}(\frac{\tilde{K}_H}{\gamma\lambda T})^2\frac{u^4}{p_R^3} - \frac{1}{2}\frac{\tilde{K}_H}{\gamma\lambda T}\frac{u^2}{p_R} + u - p_R \stackrel{\text{def}}{=\!=} p_{foc}^c.$$

$\Pi_f(p)$ is increasing in p for $p < p_{foc}^c$ and decreasing in p for $p > p_{foc}^c$. Next, we show $p_{foc}^c > \bar{p}$ so that $\Pi_f(p)$ is always increasing in p in Case c. $p_{foc}^c > \bar{p}$ is equivalent to

$$\sqrt{\frac{\tilde{K}_H}{\gamma\lambda T}u^2 + p_R^2} > -\frac{1}{2}(\frac{\tilde{K}_H}{\gamma\lambda T})^2\frac{u^4}{p_R^3} + \frac{1}{2}\frac{\tilde{K}_H}{\gamma\lambda T}\frac{u^2}{p_R} + p_R. \tag{B.19}$$

If $\tilde{K}_H/(\gamma\lambda T) > 2(p_R/u)^2$, the RHS of (B.19) is negative so (B.19) holds. If $\tilde{K}_H/(\gamma\lambda T) \leq 2(p_R/u)^2$, after taking square on both sides, (B.19) can be simplified to $\tilde{K}_H/(\gamma\lambda T) < 3(p_R/u)^2$. Thus, (B.19) always holds.

If $(K_H + K_R)/[\lambda_H^c + \lambda_U^c(q_f) + \lambda_R^c] < T$ in Case c, the revenue function becomes

$$\begin{aligned}
\Pi_f(p) &= p_R\gamma\lambda[\xi_U^c(q_f) + \xi_R^c]\tau^\infty(q_f) + p\gamma\lambda_U^c(q_f)\tau^\infty(q_f)q_f \\
&\quad + p_H(1-\gamma)\lambda\xi_H^a\tau^\infty(q_f) + p_R(1-\gamma)\lambda\xi_R^a\tau^\infty(q_f) \\
&= p_R(K_H + K_R) + pK_H + (1-\gamma)(p_H - p_R - p)\lambda\xi_H^a\tau^\infty(q_f),
\end{aligned}$$

where the last equality follows from using $\tau^\infty(q_f) = (K_H+K_R)/\{\gamma\lambda[\xi_U^c(q_f)+\xi_R^c]+(1-\gamma)\lambda(\xi_H^a + \xi_R^a)\}$ and $q_f = [K_H - (1-\gamma)\lambda\xi_H^a\tau^\infty(q_f)]/[\gamma\lambda\xi_U^c(q_f)\tau^\infty(q_f)]$. The derivative of $\Pi_f(p)$ is

$$\frac{\mathrm{d}\Pi_f}{\mathrm{d}p} = K_H - (1-\gamma)\lambda\xi_H^a\tau^\infty(q_f) - \frac{(1-\gamma)\gamma(p_H - p_R - p)\lambda\xi_H^a\tau^\infty(q_f)\frac{\mathrm{d}[\xi_U^c(q_f)+\xi_R^c]}{\mathrm{d}p}}{\gamma[\xi_U^c(q_f) + \xi_R^c] + (1-\gamma)(\xi_H^a + \xi_R^a)}.$$

Now we show that $\xi_U^c(q_f) + \xi_R^c$ is decreasing in p and hence

$$\frac{\mathrm{d}\Pi_f}{\mathrm{d}p} \geq K_H - (1-\gamma)\lambda\xi_H^a\tau^\infty(q_f) \geq K_H - \lambda\xi_H^a T \geq 0.$$

Using the q_f in (B.16), we obtain

$$\xi_U^c(q_f) + \xi_R^c = \frac{1}{u^2}\left[u^2 - 2p_Rp + \frac{\beta + \sqrt{\beta^2 - 4\gamma^2kp_R^2(u-p)^2}}{2\gamma k}\right],$$

where $k = K_H/(K_H + K_R)$. Then,

$$\frac{\mathrm{d}[\xi_U^c(q_f) + \xi_R^c]}{\mathrm{d}p} = \frac{p_R}{u^2}\left[\frac{\beta + 2\gamma p_R(u-p)}{\sqrt{\beta^2 - 4\gamma^2kp_R^2(u-p)^2}} - 1\right],$$

and $\mathrm{d}[\xi_U^c(q_f) + \xi_R^c]/\mathrm{d}p \leq 0$ can be simplified to $-\beta + \sqrt{\beta^2 - 4\gamma^2kp_R^2(u-p)^2} \geq$

$2\gamma p_R(u-p)$. Note that the feasibility condition for Case c, $(p_H-p_R-q_fp)/(1-q_f) \geq u$, can be simplified to $-\beta + \sqrt{\beta^2 - 4\gamma^2 k p_R^2 (u-p)^2} \geq 2\gamma(u-p)(u-p_H+p_R)$. Since $2\gamma(u-p)(u-p_H+p_R) > 2\gamma p_R(u-p)$, $-\beta + \sqrt{\beta^2 - 4\gamma^2 k p_R^2 (u-p)^2} \geq 2\gamma p_R(u-p)$ is true in Case c. Thus, $\Pi_f(p)$ is increasing in p.

We have shown that $\Pi_f(p)$ is increasing in Case c. Second, consider Case d. As proved in Theorem III.4(ii), we must have $(K_H+K_R)/[\lambda_H^i(q_f)+\lambda_U^i(q_f)+\lambda_R^i] \geq T$ for $i=d,e$. The revenue function in Case d is

$$\begin{aligned}
\Pi_f(p) &= p_R\gamma\lambda[\xi_U^d(q_f)+\xi_R^d]T + p[K_H - \gamma\lambda\xi_H^d(q_f)T - (1-\gamma)\lambda\xi_H^a T] + p_H\gamma\lambda\xi_H^d(q_f)T \\
&\quad +(1-\gamma)\Pi_{N,f} \\
&= \gamma\lambda T p_R + (\tilde{K}_H - \frac{2\gamma\lambda T p_R^2}{u^2})p \\
&\quad + \frac{\gamma\lambda T}{u^2}\left\{ \frac{\left[-(u-p_H+p_R)p + (1-\frac{\tilde{K}_H}{\gamma\lambda T})u^2 - (p_H-p_R)u - p_R^2\right]^2}{p_H-p_R-p} \right. \\
&\quad \left. - \frac{p_R^3\left[p^2 - 2(p_H-p_R)p - (1-\frac{\tilde{K}_H}{\gamma\lambda T})u^2 + 2(p_H-p_R)u + p_R^2\right]}{-(1-\frac{\tilde{K}_H}{\gamma\lambda T})u^2 + 2(p_H-p_R)u - p_H^2 + 2p_Hp_R} \right\} \\
&\quad +(1-\gamma)\Pi_{N,f}.
\end{aligned}$$

Taking derivatives yields

$$\begin{aligned}
\frac{\mathrm{d}\Pi_f}{\mathrm{d}p} &= \tilde{K}_H - \frac{2\gamma\lambda T p_R^2}{u^2} \\
&\quad + \frac{\gamma\lambda T}{u^2}\Bigg\{ -(u - p_H + p_R)^2 + \frac{\Big[(1-\frac{\tilde{K}_H}{\gamma\lambda T})u^2 - 2(p_H - p_R)u + p_H^2 - 2p_H p_R\Big]^2}{(p_H - p_R - p)^2} \\
&\quad + \frac{2p_R^3(p_H - p_R - p)}{-(1-\frac{\tilde{K}_H}{\gamma\lambda T})u^2 + 2(p_H - p_R)u - p_H^2 + 2p_H p_R}\Bigg\}, \\
\frac{\mathrm{d}^2\Pi_f}{\mathrm{d}p^2} &= \frac{2\gamma\lambda T}{u^2}\Bigg\{ \frac{\Big[(1-\frac{\tilde{K}_H}{\gamma\lambda T})u^2 - 2(p_H - p_R)u + p_H^2 - 2p_H p_R\Big]^2}{(p_H - p_R - p)^3} \\
&\quad - \frac{p_R^3}{-(1-\frac{\tilde{K}_H}{\gamma\lambda T})u^2 + 2(p_H - p_R)u - p_H^2 + 2p_H p_R}\Bigg\}.
\end{aligned}$$

Since the second-order derivative is increasing in p and is equal to zero at $p = \tilde{p}$, $\mathrm{d}^2\Pi_f/\,\mathrm{d}p^2 \geq 0$ and $\Pi_f(p)$ is convex in p in Case d. Moreover, we have

$$\left.\frac{\mathrm{d}\Pi_f}{\mathrm{d}p}\right|_{p=\tilde{p}} = \frac{\gamma\lambda T}{u^2}\left[-(1-\frac{\tilde{K}_H}{\gamma\lambda T})u^2 + 2(p_H - p_R)u - p_H^2 + 2p_H p_R\right] = K_H - \lambda_H^a T \geq 0.$$

Thus, $\Pi_f(p)$ is increasing in p in Case d.

Third, consider Case e. The revenue function in Case e is

$$\begin{aligned}
\Pi_f(p) &= p_R\gamma\lambda[\xi_U^e(q_f) + \xi_R^e]T + p[K_H - \gamma\lambda\xi_H^e(q_f)T - (1-\gamma)\lambda\xi_H^a T] + p_H\gamma\lambda\xi_H^e(q_f)T \\
&\quad + (1-\gamma)\Pi_{N,f} \\
&= \frac{\gamma\lambda T}{u^2}\Bigg\{\left[-(1-\frac{\tilde{K}_H}{\gamma\lambda T})u^2 + 2(p_H - p_R)u - p_H^2 + 2p_H p_R\right]p \\
&\quad + p_H(u^2 - p_H^2) - (2u - 2p_H - p_R)\left[-(1-\frac{\tilde{K}_H}{\gamma\lambda T})u^2 + 2(p_H - p_R)u + p_R^2\right]\Bigg\} \\
&\quad + (1-\gamma)\Pi_{N,f}.
\end{aligned}$$

Since

$$\frac{\mathrm{d}\Pi_f}{\mathrm{d}p} = \frac{\gamma\lambda T}{u^2}\left[-(1-\frac{\tilde{K}_H}{\gamma\lambda T})u^2 + 2(p_H - p_R)u - p_H^2 + 2p_H p_R\right] \geq 0,$$

$\Pi_f(p)$ is increasing in p in Case e.

Therefore, we conclude that if $K_H < (\lambda T/u^2)[(1-\gamma)(u - p_H + 2p_R)(u - p_H) + \gamma(u^2 - p_R^2)]$, the optimal upgrade price is $p_f^* = \min\left\{\max\left\{(p_{foc}^b)^+, \bar{p}\right\}, p_H - p_R\right\}$ which induces $q_f = 1$. Finally, note that $\bar{p} \leq 0$ if $K_H \geq (\lambda T/u^2)[(1-\gamma)(u - p_H + 2p_R)(u - p_H) + \gamma(u^2 - p_R^2)]$, so we can always write the optimal upgrade price as $p_f^* = \min\left\{\max\left\{(p_{foc}^b)^+, \bar{p}\right\}, p_H - p_R\right\}$.

Proof of Theorem III.6

Offering upgrades increases the revenue if $p_f^* < p_H - p_R$ and decreases the revenue if $p_f^* = p_H - p_R$. $p_f^* = p_H - p_R$ if and only if $\max\left\{p_{foc}^b, \bar{p}\right\} \geq p_H - p_R$. Since $K_H \geq \lambda_H^a T$, we have $\bar{p} \leq p_H - p_R$. Thus, $p_f^* = p_H - p_R$ if and only if $p_{foc}^b \geq p_H - p_R$ or equivalently, $p_H \leq \left(2u + 3p_R - \sqrt{u^2 + 9p_R^2}\right)/3$.

Proof of Theorem III.7

First, consider the monotonicity of p_f^* in γ. Since p_{foc}^b and $p_H - p_R$ are independent of γ, we only need to show that $\bar{p}$ is increasing in γ. This is true because $K_H/(\lambda T) \cdot u^2 - (u - p_H + 2p_R)(u - p_H) = u^2/(\lambda T) \cdot (K_H - \lambda_H^a T) \geq 0$.

Second, consider the monotonicity of $\Pi_f(p_f^*)$ in γ. When $p_f^* = p_{foc}^b$, using the revenue function in Case b, the Envelope Theorem yields

$$\frac{\mathrm{d}\Pi_f(p_f^*)}{\mathrm{d}\gamma} = \frac{\lambda T}{u^2}\left[3(p_{foc}^b)^2 - 4up_{foc}^b + u^2 - 3p_R^2\right] - \Pi_{N,f} > 0,$$

because $p_f^* = p_{foc}^b$ (so $p_f^* \neq p_H - p_R$) implies $\Pi_f(p_{foc}^b) > \Pi_{N,f}$ which is equivalent to $(\lambda T/u^2)[3(p_{foc}^b)^2 - 4up_{foc}^b + u^2 - 3p_R^2] > \Pi_{N,f}$. Next, when $p_f^* = \bar{p}$, $\mathrm{d}\Pi_f(p_f^*)/\mathrm{d}\gamma \geq 0$

can be simplified to

$$2\gamma^2\sqrt{\frac{1}{\gamma}\left[\frac{K_H}{\lambda T}u^2-(u-p_H+2p_R)(u-p_H)\right]+(u-p_H+p_R)^2}$$
$$\cdot(u-p_H+p_R)(u^2-2p_Hu+2p_Ru+p_H^2-2p_Hp_R-2p_R^2)\geq a_1\gamma^2+b_1\gamma+c_1, \quad \text{(B.20)}$$

where

$$\begin{aligned}
a_1 &= 2(u-p_H+p_R)^2(u^2-2p_Hu+2p_Ru+p_H^2-2p_Hp_R-2p_R^2),\\
b_1 &= \left[\frac{K_H}{\lambda T}u^2-(u-p_H+2p_R)(u-p_H)\right](u^2-2p_Hu+2p_Ru+p_H^2-2p_Hp_R-2p_R^2),\\
c_1 &= -\left[\frac{K_H}{\lambda T}u^2-(u-p_H+2p_R)(u-p_H)\right]^2.
\end{aligned}$$

We show (B.20) indeed holds as follows. If $u^2-2p_Hu+2p_Ru+p_H^2-2p_Hp_R-2p_R^2\leq 0$, we have $a_1\leq 0$, $b_1\leq 0$, $c_1\leq 0$, and hence the RHS of (B.20) ≤ 0. Since the left-hand side (LHS) of (B.20) ≥ 0, (B.20) holds. If $u^2-2p_Hu+2p_Ru+p_H^2-2p_Hp_R-2p_R^2>0$, RHS can be both positive and negative. If RHS≤ 0, again (B.20) holds. If RHS>0, by taking square on both sides and rearranging terms, (B.20) is equivalent to

$$\left[\frac{K_H}{\lambda T}u^2-(u-p_H+2p_R)(u-p_H)\right]^2(a_2\gamma^2+b_2\gamma+c_2)\geq 0, \quad \text{(B.21)}$$

where

$$\begin{aligned}
a_2 &= 3(u^2-2p_Hu+2p_Ru+p_H^2-2p_Hp_R+2p_R^2)(u^2-2p_Hu+2p_Ru+p_H^2-2p_Hp_R-2p_R^2),\\
b_2 &= 2\left[\frac{K_H}{\lambda T}u^2-(u-p_H+2p_R)(u-p_H)\right](u^2-2p_Hu+2p_Ru+p_H^2-2p_Hp_R-2p_R^2),\\
c_2 &= -\left[\frac{K_H}{\lambda T}u^2-(u-p_H+2p_R)(u-p_H)\right]^2.
\end{aligned}$$

It is easy to see that $a_2>a_1$, $b_2\geq b_1$, $c_2=c_1$. Thus, RHS>0 implies that (B.21) is satisfied. We have proved that $\Pi_f(p_f^*)$ is increasing in γ when $p_f^*=\bar{p}$. Finally, when

$p_f^* = p_H - p_R$ or $p_f^* = 0$, $\Pi_f(p_f^*)$ is constant in γ. Therefore, we conclude that $\Pi_f(p_f^*)$ is increasing in γ overall.

Proof of Theorem III.8

If $K_H < (\lambda T/u^2)[(u-p_H+2p_R)(u-p_H)+\gamma(p_H-p_R)(2u-p_H+p_R)]$, we have $\bar{p} > 0$, hence $p_f^* > 0$. If $K_H \geq (\lambda T/u^2)[(u-p_H+2p_R)(u-p_H)+\gamma(p_H-p_R)(2u-p_H+p_R)]$, we have $\bar{p} \leq 0$, hence $p_f^* = 0$ if and only if $p_{foc}^b \leq 0$ which is simplified to $u \leq \sqrt{3}p_R$.

APPENDIX C

Proofs of Theorems in Chapter IV

Proof of Theorem IV.1

(i) The first-order derivative of $\Pi_u(p_a)$ is

$$\frac{\mathrm{d}\Pi_u}{\mathrm{d}p_a} = [c'(\lambda_H \bar{F}_H(p_a) + \lambda_L \bar{F}_L(p_a)) - (p_a - c_a)][\lambda_H f_H(p_a) + \lambda_L f_L(p_a)].$$

Since $c'(\lambda_H \bar{F}_H(p_a) + \lambda_L \bar{F}_L(p_a)) - (p_a - c_a)$ is decreasing in p_a, $\Pi_u(p_a)$ is quasi-concave in p_a. Thus, the optimal ancillary service price is the solution to the first-order condition, i.e., $p_a^* = c_a + c'(\lambda_H \bar{F}_H(p_a^*) + \lambda_L \bar{F}_L(p_a^*))$.

(ii) Since $p_a^* > 0$, for $i = H, L$, we have $p_{mi}^* = v_i + E(u_i - p_a^*)^+ < v_i + E(u_i)^+ = p_{bi}^*$. Since $p_{mi}^* + p_a^* = v_i + E[\max(u_i, p_a^*)]$, we have $p_{bi}^* < p_{mi}^* + p_a^*$.

(iii) Since $p_{bi}^* - p_{mi}^* = E(u_i)^+ - E(u_i - p_a^*)^+ = \int_0^{p_a^*} \bar{F}_i(x)\,\mathrm{d}x$ for $i = H, L$, the result follows.

Proof of Theorem IV.2

Since $E(u_i - x)^+ = \frac{\beta_i}{2\bar{u}}(\bar{u} - x)^2$ for $i = H, L$, taking derivatives of the optimal profit functions with respect to β_H and β_L yields

$$\begin{aligned}
\frac{\partial \Pi_u^*}{\partial \beta_H} &= \frac{(\bar{u}-p_a^*)^2}{2\bar{u}} \cdot \lambda_H + (p_a^* - c_a)\lambda_H \cdot \frac{\bar{u}-p_a^*}{\bar{u}} - c'(\lambda_H \bar{F}_H(p_a^*) + \lambda_L \bar{F}_L(p_a^*))\lambda_H \cdot \frac{\bar{u}-p_a^*}{\bar{u}} \\
&= \frac{(\bar{u}-p_a^*)^2}{2\bar{u}} \cdot \lambda_H, \\
\frac{\partial \Pi_b^*}{\partial \beta_H} &= \left(\frac{\bar{u}}{2} - c_a\right)\lambda_H, \\
\frac{\partial \Pi_u^*}{\partial \beta_L} &= \frac{(\bar{u}-p_a^*)^2}{2\bar{u}} \cdot \lambda_L + (p_a^* - c_a)\lambda_L \cdot \frac{\bar{u}-p_a^*}{\bar{u}} - c'(\lambda_H \bar{F}_H(p_a^*) + \lambda_L \bar{F}_L(p_a^*))\lambda_L \cdot \frac{\bar{u}-p_a^*}{\bar{u}} \\
&= \frac{(\bar{u}-p_a^*)^2}{2\bar{u}} \cdot \lambda_L, \\
\frac{\partial \Pi_b^*}{\partial \beta_L} &= \left(\frac{\bar{u}}{2} - c_a\right)\lambda_L,
\end{aligned}$$

where the derivation for derivatives of Π_u^* follows from the Envelope Theorem and the first-order condition. Thus,

$$\begin{aligned}
\frac{\partial(\Pi_u^* - \Pi_b^*)}{\partial \beta_H} &= \left[\frac{(\bar{u}-p_a^*)^2}{2\bar{u}} - \frac{\bar{u}}{2} + c_a\right]\lambda_H, \\
\frac{\partial(\Pi_u^* - \Pi_b^*)}{\partial \beta_L} &= \left[\frac{(\bar{u}-p_a^*)^2}{2\bar{u}} - \frac{\bar{u}}{2} + c_a\right]\lambda_L.
\end{aligned}$$

By applying the Implicit Function Theorem to the first-order condition, we obtain

$$\begin{aligned}
\frac{\partial p_a^*}{\partial \beta_H} &= \frac{c''(\lambda_H \bar{F}_H(p_a^*) + \lambda_L \bar{F}_L(p_a^*))\lambda_H \frac{\bar{u}-p_a^*}{\bar{u}}}{1 + c''(\lambda_H \bar{F}_H(p_a^*) + \lambda_L \bar{F}_L(p_a^*))[\lambda_H f_H(p_a^*) + \lambda_L f_L(p_a^*)]} > 0, \\
\frac{\partial p_a^*}{\partial \beta_L} &= \frac{c''(\lambda_H \bar{F}_H(p_a^*) + \lambda_L \bar{F}_L(p_a^*))\lambda_L \frac{\bar{u}-p_a^*}{\bar{u}}}{1 + c''(\lambda_H \bar{F}_H(p_a^*) + \lambda_L \bar{F}_L(p_a^*))[\lambda_H f_H(p_a^*) + \lambda_L f_L(p_a^*)]} > 0.
\end{aligned}$$

Thus p_a^* is increasing in β_H and β_L. Then, since $\frac{\partial(\Pi_u^* - \Pi_b^*)}{\partial \beta_H}$ is decreasing in p_a^*, it is decreasing in β_H, hence $\Pi_u^* - \Pi_b^*$ is concave in β_H. Similarly, $\Pi_u^* - \Pi_b^*$ is concave in β_L.

When $\beta_H = \beta_L = 0$, $\Pi_u^* - \Pi_b^* = 0$; also, $p_a^* = c_a$, hence $\left.\frac{\partial(\Pi_u^* - \Pi_b^*)}{\partial \beta_H}\right|_{\beta_H = \beta_L = 0} = \frac{c_a^2}{2\bar{u}} \cdot \lambda_H > 0$, $\left.\frac{\partial(\Pi_u^* - \Pi_b^*)}{\partial \beta_L}\right|_{\beta_H = \beta_L = 0} = \frac{c_a^2}{2\bar{u}} \cdot \lambda_L > 0$. Thus, when $\beta_L = 0$, there exists a threshold $\hat{\beta}_H$ such that $\Pi_u^* - \Pi_b^* \geq 0$ when $\beta_H \leq \hat{\beta}_H$, and $\Pi_u^* - \Pi_b^* < 0$ when $\beta_H > \hat{\beta}_H$.

Similarly, when $\beta_H = 0$, there exists a threshold $\hat{\beta}_L$ such that $\Pi_u^* - \Pi_b^* \geq 0$ when $\beta_L \leq \hat{\beta}_L$, and $\Pi_u^* - \Pi_b^* < 0$ when $\beta_L > \hat{\beta}_L$.

Next, notice that $\frac{\partial(\Pi_u^*-\Pi_b^*)}{\partial\beta_H}$ and $\frac{\partial(\Pi_u^*-\Pi_b^*)}{\partial\beta_L}$ have the same sign. If $\beta_L > \hat{\beta}_L$, we have $\left.\frac{\partial(\Pi_u^*-\Pi_b^*)}{\partial\beta_L}\right|_{\beta_H=0} < 0$, hence we also have $\left.\frac{\partial(\Pi_u^*-\Pi_b^*)}{\partial\beta_H}\right|_{\beta_H=0} < 0$. Then, since $\Pi_u^* - \Pi_b^*$ is concave in β_H, we have $\frac{\partial(\Pi_u^*-\Pi_b^*)}{\partial\beta_H} < 0$ for any $\beta_L > \hat{\beta}_L$. Thus, if $\beta_L > \hat{\beta}_L$, since $\Pi_u^* - \Pi_b^* < 0$ when $\beta_H = 0$, we have $\Pi_u^* - \Pi_b^* < 0$ for all β_H. Similarly, if $\beta_H > \hat{\beta}_H$, $\Pi_u^* - \Pi_b^* < 0$ for all β_L. Thus, the solution to $\Pi_u^* - \Pi_b^* = 0$ must satisfy $\beta_H \leq \hat{\beta}_H$ and $\beta_L \leq \hat{\beta}_L$. For any β_L, because $\Pi_u^* - \Pi_b^*$ is concave in β_H and $\Pi_u^* - \Pi_b^* \geq 0$ at $\beta_H = 0$, $\Pi_u^* - \Pi_b^*$ crosses the zero line once from positive to negative when varying β_H. Let $\bar{\beta}_H(\beta_L)$ denote this threshold. We must have $\left.\frac{\partial(\Pi_u^*-\Pi_b^*)}{\partial\beta_H}\right|_{\beta_H=\bar{\beta}_H(\beta_L)} < 0$ and $\left.\frac{\partial(\Pi_u^*-\Pi_b^*)}{\partial\beta_L}\right|_{\beta_H=\bar{\beta}_H(\beta_L)} < 0$. Thus, by applying the Implicit Function Theorem to $\Pi_u^* - \Pi_b^* = 0$ which is the equation that defines $\bar{\beta}_H(\beta_L)$, we know that $\bar{\beta}_H(\beta_L)$ is decreasing in β_L. Note that $\bar{\beta}_H(\beta_L)$ intersects with the β_H-axis and β_L-axis at $\hat{\beta}_H$ and $\hat{\beta}_L$, respectively.

Proof of Theorem IV.3

(i) When $c'(\cdot)$ increases, p_a^* increases, hence $\frac{\partial(\Pi_u^*-\Pi_b^*)}{\partial\beta_H}$ decreases. As a result, $\bar{\beta}_H(\beta_L)$ decreases. When $c(\cdot) = 0$, we have $\Pi_b^* = \Pi_u(0) \leq \Pi_u^*$ for any β_H and β_L.

(ii) Using the approach in Part (i), the result follows because p_a^* is decreasing in c_a.

(iii) When we increase λ_H and decrease λ_L such that $\lambda_H + \lambda_L = \lambda$, applying the Implicit Function Theorem to the equation $\Pi_u^* - \Pi_b^* = 0$ yields

$$
\begin{aligned}
\frac{\mathrm{d}\bar{\beta}_H}{\mathrm{d}\lambda_H} &= -\frac{\frac{\partial(\Pi_u^*-\Pi_b^*)}{\partial\lambda_H} - \frac{\partial(\Pi_u^*-\Pi_b^*)}{\partial\lambda_L}}{\frac{\partial(\Pi_u^*-\Pi_b^*)}{\partial\bar{\beta}_H}} \\
&= -\frac{\bar{\beta}_H\left[\frac{(\bar{u}-p_a^*)^2}{2\bar{u}} - \frac{\bar{u}}{2} + c_a\right] - \beta_L\left[\frac{(\bar{u}-p_a^*)^2}{2\bar{u}} - \frac{\bar{u}}{2} + c_a\right]}{\lambda_H\left[\frac{(\bar{u}-p_a^*)^2}{2\bar{u}} - \frac{\bar{u}}{2} + c_a\right]} \\
&= \frac{\beta_L - \bar{\beta}_H}{\lambda_H}.
\end{aligned}
$$

Thus, $\bar{\beta}_H(\beta_L)$ is decreasing in λ_H when $\bar{\beta}_H(\beta_L) \geq \beta_L$ and increasing in λ_H when $\bar{\beta}_H(\beta_L) < \beta_L$. Also, note that when $\bar{\beta}_H(\beta_L) = \beta_L$, $\bar{\beta}_H(\beta_L)$ does not change with λ_H if we keep $\lambda_H + \lambda_L = \lambda$. Thus, $\bar{\beta}_H(\beta_L)$ intersects at the same point on $\beta_H = \beta_L$ when we change λ_H and keep $\lambda_H + \lambda_L = \lambda$.

Proof of Theorem IV.4

For each of the four cases ("HH", "HL", "LH", "LL"), by using the same approach in the proof of Theorem III.5, we can prove the quasi-concavity of the profit function, and hence the optimal ancillary service price is given by the first-order condition as follows:

- "HH" case: The optimal ancillary service price $p_{a,m}^{HH*}$ is the solution to $p_{a,m}^{HH*} = c_a + c'(\alpha_H \lambda_H \bar{F}_H(p_{a,m}^{HH*}) + \alpha_L \lambda_L \bar{F}_L(p_{a,m}^{HH*}))$.
- "HL" case: The optimal ancillary service price $p_{a,m}^{HL*}$ is the solution to $p_{a,m}^{HL*} = c_a + c'(\alpha_H \lambda_H \bar{F}_H(p_{a,m}^{HL*}) + \lambda_L \bar{F}_L(p_{a,m}^{HL*})) + \frac{\lambda_L \bar{F}_L(p_{a,m}^{HL*})}{\alpha_H \lambda_H f_H(p_{a,m}^{HL*}) + \lambda_L f_L(p_{a,m}^{HL*})}$.
- "LH" case: The optimal ancillary service price $p_{a,m}^{LH*}$ is the solution to $p_{a,m}^{LH*} = c_a + c'(\lambda_H \bar{F}_H(p_{a,m}^{LH*}) + \alpha_L \lambda_L \bar{F}_L(p_{a,m}^{LH*})) + \frac{\lambda_H \bar{F}_H(p_{a,m}^{LH*})}{\lambda_H f_H(p_{a,m}^{LH*}) + \alpha_L \lambda_L f_L(p_{a,m}^{LH*})}$.
- "LL" case: The optimal ancillary service price $p_{a,m}^{LL*}$ is the solution to $p_{a,m}^{LL*} = c_a + c'(\lambda_H \bar{F}_H(p_{a,m}^{LL*}) + \lambda_L \bar{F}_L(p_{a,m}^{LL*})) + \frac{\lambda_H \bar{F}_H(p_{a,m}^{LL*}) + \lambda_L \bar{F}_L(p_{a,m}^{LL*})}{\lambda_H f_H(p_{a,m}^{LL*}) + \lambda_L f_L(p_{a,m}^{LL*})}$.

The result then follows.

Proof of Theorem IV.5

(i) We will show that when $\Pi_u^* \geq \Pi_b^*$, the following four results hold: 1) $\Pi_{u,m}^{HH*} \geq \Pi_{b,m}^{HH*}$, 2) $\Pi_{u,m}^{HL*} \geq \Pi_{b,m}^{HL*}$, 3) $\Pi_{u,m}^{LH*} \geq \Pi_{b,m}^{LH*}$, 4) $\Pi_{u,m}^{LL*} \geq \Pi_{b,m}^{LL*}$.

- $\Pi_{u,m}^{HH*} \geq \Pi_{b,m}^{HH*}$: Notice that $\Pi_{u,m}^{HH}(p_a)$ is equal to $\Pi_u(p_a)$ with λ_H replaced by $\alpha_H \lambda_H$ and λ_L replaced by $\alpha_L \lambda_L$. Theorem II.6 states that when $\Pi_u^* \geq \Pi_b^*$, we also have $\Pi_u^* \geq \Pi_b^*$ with smaller λ_H and λ_L. Thus, we know that when $\Pi_u^* \geq \Pi_b^*$, or equivalently, when $\Pi_{u,m}^{HH*} \geq \Pi_{b,m}^{HH*}$ with $\alpha_H = \alpha_L = 1$, we have $\Pi_{u,m}^{HH*} \geq \Pi_{b,m}^{HH*}$ for all α_H and α_L.

- $\Pi^{HL*}_{u,m} \geq \Pi^{HL*}_{b,m}$: Theorem II.6 indicates that by replacing λ_H with $\alpha_H\lambda_H$, we also have $\Pi^*_u \geq \Pi^*_b$, i.e.,

$$
\begin{aligned}
& [v_H + E(u_H - p^*_a)^+ - c_m]\alpha_H\lambda_H + [v_L + E(u_L - p^*_a)^+ - c_m]\lambda_L \\
& +(p^*_a - c_a)[\alpha_H\lambda_H\bar{F}_H(p^*_a) + \lambda_L\bar{F}_L(p^*_a)] - c(\alpha_H\lambda_H\bar{F}_H(p^*_a) + \lambda_L\bar{F}_L(p^*_a)) \\
\geq \; & [v_H + E(u_H)^+ - c_m]\alpha_H\lambda_H + [v_L + E(u_L)^+ - c_m]\lambda_L \\
& -c_a[\alpha_H\lambda_H\bar{F}_H(0) + \lambda_L\bar{F}_L(0)].
\end{aligned}
$$

Note that in the above inequality, p^*_a is the optimal ancillary service price in the basic model with λ_H replaced by $\alpha_H\lambda_H$. Next, subtracting the left-hand side of the above inequality by $E(u_L - p^*_a)^+\lambda_L$ and subtracting the right-hand side by a larger amount $E(u_L)^+\lambda_L$, we obtain

$$
\begin{aligned}
& [v_H + E(u_H - p^*_a)^+ - c_m]\alpha_H\lambda_H + (v_L - c_m)\lambda_L \\
& +(p^*_a - c_a)[\alpha_H\lambda_H\bar{F}_H(p^*_a) + \lambda_L\bar{F}_L(p^*_a)] - c(\alpha_H\lambda_H\bar{F}_H(p^*_a) + \lambda_L\bar{F}_L(p^*_a)) \\
> \; & [v_H + E(u_H)^+ - c_m]\alpha_H\lambda_H + (v_L - c_m)\lambda_L - c_a[\alpha_H\lambda_H\bar{F}_H(0) + \lambda_L\bar{F}_L(0)],
\end{aligned}
$$

which is equivalent to $\Pi^{HL}_{u,m}(p^*_a) > \Pi^{HL*}_{b,m}$. Since $\Pi^{HL*}_{u,m} \geq \Pi^{HL}_{u,m}(p^*_a)$, we have $\Pi^{HL*}_{u,m} > \Pi^{HL*}_{b,m}$ as well.

- $\Pi^{LH*}_{u,m} \geq \Pi^{LH*}_{b,m}$: It follows from the same approach we used above to prove $\Pi^{HL*}_{u,m} \geq \Pi^{HL*}_{b,m}$.

- $\Pi^{LL*}_{u,m} \geq \Pi^{LL*}_{b,m}$: This is true because $\Pi^{LL*}_{u,m} > \Pi^{LL}_{u,m}(\bar{u}) = v_H\lambda_H + v_L\lambda_L > \Pi^{LL*}_{b,m}$. Note that $\Pi^{LL*}_{u,m} \geq \Pi^{LL*}_{b,m}$ is actually always true and is not dependent on $\Pi^*_u \geq \Pi^*_b$.

Therefore, combining these four results, we conclude that when $\Pi^*_u \geq \Pi^*_b$, $\Pi^*_{u,m} \geq \Pi^*_{b,m}$ for all α_H and α_L.

(ii) In Part (i), we have proved that $\Pi^{LL*}_{u,m} \geq \Pi^{LL*}_{b,m}$ always holds. Thus, when

$\Pi^*_{u,m} = \Pi^{LL*}_{u,m}$ and $\Pi^*_{b,m} = \Pi^{LL*}_{b,m}$, we must have $\Pi^*_{u,m} \geq \Pi^*_{b,m}$. We first consider the unbundling case and characterize when $\Pi^*_{u,m} = \Pi^{LL*}_{u,m}$. $\Pi^*_{u,m} = \Pi^{LL*}_{u,m}$ requires 1) $\Pi^{LL*}_{u,m} \geq \Pi^{HH*}_{u,m}$, 2) $\Pi^{LL*}_{u,m} \geq \Pi^{HL*}_{u,m}$, 3) $\Pi^{LL*}_{u,m} \geq \Pi^{LH*}_{u,m}$.

- Condition for $\Pi^{LL*}_{u,m} \geq \Pi^{HH*}_{u,m}$: When $\alpha_H = \alpha_L = 0$, $\Pi^{LL*}_{u,m} > \Pi^{HH*}_{u,m}$ trivially because $\Pi^{HH*}_{u,m} = 0$. When $\alpha_H = \alpha_L = 1$,

$$\begin{aligned}
\Pi^{LL*}_{u,m} &= (v_H - c_m)\lambda_H + (v_L - c_m)\lambda_L \\
&\quad + (p^{LL*}_{a,m} - c_a)[\lambda_H \bar{F}_H(p^{LL*}_{a,m}) + \lambda_L \bar{F}_L(p^{LL*}_{a,m})] - c(\lambda_H \bar{F}_H(p^{LL*}_{a,m}) + \lambda_L \bar{F}_L(p^{LL*}_{a,m})) \\
&< [v_H + E(u_H - p^{LL*}_{a,m}) - c_m]\lambda_H + [v_L + E(u_L - p^{LL*}_{a,m}) - c_m]\lambda_L \\
&\quad + (p^{LL*}_{a,m} - c_a)[\lambda_H \bar{F}_H(p^{LL*}_{a,m}) + \lambda_L \bar{F}_L(p^{LL*}_{a,m})] - c(\lambda_H \bar{F}_H(p^{LL*}_{a,m}) + \lambda_L \bar{F}_L(p^{LL*}_{a,m})) \\
&= \Pi^{HH}_{u,m}(p^{LL*}_{a,m}) \\
&\leq \Pi^{HH*}_{u,m}.
\end{aligned}$$

  Moreover, we know from the proof of Theorem IV.6 that $\frac{\mathrm{d}(\Pi^{LL*}_{u,m} - \Pi^{HH*}_{u,m})}{\mathrm{d}\alpha_H} < 0$ and $\frac{\mathrm{d}(\Pi^{LL*}_{u,m} - \Pi^{HH*}_{u,m})}{\mathrm{d}\alpha_L} < 0$. Thus, there exists a threshold function $\bar{\alpha}_{H,u}(\alpha_L)$ such that $\Pi^{LL*}_{u,m} \geq \Pi^{HH*}_{u,m}$ when $\alpha_H \leq \bar{\alpha}_{H,u}(\alpha_L)$ Moreover, by applying the Implicit Function Theorem to the equation $\Pi^{LL*}_{u,m} - \Pi^{HH*}_{u,m} = 0$ which defines $\bar{\alpha}_{H,u}(\alpha_L)$, we obtain that $\bar{\alpha}_{H,u}(\alpha_L)$ is a decreasing function.

- Condition for $\Pi^{LL*}_{u,m} \geq \Pi^{HL*}_{u,m}$: When $\alpha_H = 0$, $\Pi^{HL*}_{u,m} = (v_L - c_m)\lambda_L + (p^{HL*}_{a,m} - c_a)\lambda_L \bar{F}_L(p^{HL*}_{a,m}) - c(\lambda_L \bar{F}_L(p^{HL*}_{a,m}))$ which is independent of λ_H. Consider $\Pi^{LL*}_{u,m}$ as a function of λ_H. At $\lambda_H = 0$, we have $\Pi^{LL*}_{u,m} = \Pi^{HL*}_{u,m}$. Moreover, by using the

Envelope Theorem and the first-order condition, we have

$$\begin{aligned}
\frac{\mathrm{d}\Pi_{u,m}^{LL*}}{\mathrm{d}\lambda_H} &= v_H - c_m \\
&\quad +(p_{a,m}^{LL*} - c_a)\bar{F}_H(p_{a,m}^{LL*}) - c'(\lambda_H \bar{F}_H(p_{a,m}^{LL*}) + \lambda_L \bar{F}_L(p_{a,m}^{HL*}))\bar{F}_H(p_{a,m}^{LL*}) \\
&= v_H - c_m + \bar{F}_H(p_{a,m}^{LL*}) \cdot \frac{\lambda_H \bar{F}_H(p_{a,m}^{LL*}) + \lambda_L \bar{F}_L(p_{a,m}^{LL*})}{\lambda_H f_H(p_{a,m}^{LL*}) + \lambda_L f_L(p_{a,m}^{LL*})} \\
&> 0.
\end{aligned}$$

Thus, when $\alpha_H = 0$, $\Pi_{u,m}^{LL*} > \Pi_{u,m}^{HH*}$ for any positive λ_H. When $\alpha_H = 1$,

$$\begin{aligned}
\Pi_{u,m}^{LL*} &= (v_H - c_m)\lambda_H + (v_L - c_m)\lambda_L \\
&\quad +(p_{a,m}^{LL*} - c_a)[\lambda_H \bar{F}_H(p_{a,m}^{LL*}) + \lambda_L \bar{F}_L(p_{a,m}^{LL*})] - c(\lambda_H \bar{F}_H(p_{a,m}^{LL*}) + \lambda_L \bar{F}_L(p_{a,m}^{LL*})) \\
&< [v_H + E(u_H - p_{a,m}^{LL*}) - c_m]\lambda_H + (v_L - c_m)\lambda_L \\
&\quad +(p_{a,m}^{LL*} - c_a)[\lambda_H \bar{F}_H(p_{a,m}^{LL*}) + \lambda_L \bar{F}_L(p_{a,m}^{LL*})] - c(\lambda_H \bar{F}_H(p_{a,m}^{LL*}) + \lambda_L \bar{F}_L(p_{a,m}^{LL*})) \\
&= \Pi_{u,m}^{HL}(p_{a,m}^{LL*}) \\
&\leq \Pi_{u,m}^{HL*}.
\end{aligned}$$

Moreover, we know from the proof of Theorem IV.6 that $\frac{\mathrm{d}(\Pi_{u,m}^{LL*} - \Pi_{u,m}^{HL*})}{\mathrm{d}\alpha_H} < 0$. Thus, there exists a threshold $\hat{\alpha}_{H,u}$ such that $\Pi_{u,m}^{LL*} \geq \Pi_{u,m}^{HL*}$ when $\alpha_H \leq \hat{\alpha}_{H,u}$.

- Condition for $\Pi_{u,m}^{LL*} \geq \Pi_{u,m}^{LH*}$: By using the same approach of deriving the condition for $\Pi_{u,m}^{LL*} \geq \Pi_{u,m}^{HL*}$, we can obtain that there exists a threshold $\hat{\alpha}_{L,u}$ such that $\Pi_{u,m}^{LL*} \geq \Pi_{u,m}^{LH*}$ when $\alpha_L \leq \hat{\alpha}_{L,u}$.

Therefore, we have obtained that $\Pi_{u,m}^{*} = \Pi_{u,m}^{LL*}$ when $\alpha_H \leq \hat{\alpha}_{H,u}$, $\alpha_L \leq \hat{\alpha}_{L,u}$, and $\alpha_H \leq \bar{\alpha}_{H,u}(\alpha_L)$.

Next, consider the bundling case. $\Pi_{b,m}^{*} = \Pi_{b,m}^{LL*}$ requires 1) $\Pi_{b,m}^{LL*} \geq \Pi_{b,m}^{HH*}$, 2) $\Pi_{b,m}^{LL*} \geq \Pi_{b,m}^{HL*}$, 3) $\Pi_{b,m}^{LL*} \geq \Pi_{b,m}^{LH*}$. We have the following:

- $\Pi_{b,m}^{LL*} \geq \Pi_{b,m}^{HH*}$ is equivalent to

$$\begin{aligned} \alpha_H \quad &\geq \quad \frac{(v_H - c_m)\lambda_H + (v_L - c_m)\lambda_L}{[v_H + E(u_H)^+ - c_m - c_a\bar{F}_H(0)]\lambda_H} \\ &\quad - \frac{c_a[\lambda_H\bar{F}_H(0) + \lambda_L\bar{F}_L(0)] + [v_L + E(u_L)^+ - c_m - c_a\bar{F}_L(0)]\lambda_L\alpha_L}{[v_H + E(u_H)^+ - c_m - c_a\bar{F}_H(0)]\lambda_H} \\ &\stackrel{\text{def}}{=\!=} \quad \bar{\alpha}_{H,b}(\alpha_L). \end{aligned}$$

- $\Pi_{b,m}^{LL*} \geq \Pi_{b,m}^{HL*}$ is equivalent to

$$\alpha_H \geq \frac{v_H - c_m - c_a\bar{F}_H(0)}{v_H + E(u_H)^+ - c_m - c_a\bar{F}_H(0)} \stackrel{\text{def}}{=\!=} \hat{\alpha}_{H,b}.$$

- $\Pi_{b,m}^{LL*} \geq \Pi_{b,m}^{LH*}$ is equivalent to

$$\alpha_L \geq \frac{v_L - c_m - c_a\bar{F}_L(0)}{v_L + E(u_L)^+ - c_m - c_a\bar{F}_L(0)} \stackrel{\text{def}}{=\!=} \hat{\alpha}_{L,b}.$$

Therefore, $\Pi_{b,m}^* = \Pi_{b,m}^{LL*}$ when $\alpha_H \leq \hat{\alpha}_{H,b}$, $\alpha_L \leq \hat{\alpha}_{L,b}$, and $\alpha_H \leq \bar{\alpha}_{H,b}(\alpha_L)$.

Finally, take $\bar{\alpha}_H(\alpha_L) = \min(\bar{\alpha}_{H,u}(\alpha_L), \bar{\alpha}_{H,b}(\alpha_L))$, $\hat{\alpha}_H = \min(\hat{\alpha}_{H,u}, \hat{\alpha}_{H,b})$, $\hat{\alpha}_L = \min(\hat{\alpha}_{L,u}, \hat{\alpha}_{L,b})$. Thus, when $\alpha_H \leq \hat{\alpha}_H$, $\alpha_L \leq \hat{\alpha}_L$, and $\alpha_H \leq \bar{\alpha}_H(\alpha_L)$, we have $\Pi_{u,m}^* = \Pi_{u,m}^{LL*}$ and $\Pi_{b,m}^* = \Pi_{b,m}^{LL*}$, and hence $\Pi_{u,m}^* \geq \Pi_{b,m}^*$.

Proof of Theorem IV.6

First, consider the monotonicity of $\Pi_{b,m}^*$. We need to show that each of $\Pi_{b,m}^{ij*}$, $i, j = H, L$, has a non-negative derivative with respect to α_H and α_L. This is true because

$$\begin{aligned}
\frac{\partial \Pi_{b,m}^{HH*}}{\partial \alpha_H} &= \frac{\partial \Pi_{b,m}^{HL*}}{\partial \alpha_H} = [v_H + E(u_H)^+ - c_m]\lambda_H - c_a \lambda_H \bar{F}_H(0) > 0, \\
\frac{\partial \Pi_{b,m}^{LH*}}{\partial \alpha_H} &= \frac{\partial \Pi_{b,m}^{LL*}}{\partial \alpha_H} = 0; \\
\frac{\partial \Pi_{b,m}^{HH*}}{\partial \alpha_L} &= \frac{\partial \Pi_{b,m}^{LH*}}{\partial \alpha_L} = [v_L + E(u_L)^+ - c_m]\lambda_L - c_a \lambda_L \bar{F}_L(0) \geq 0, \\
\frac{\partial \Pi_{b,m}^{HL*}}{\partial \alpha_L} &= \frac{\partial \Pi_{b,m}^{LL*}}{\partial \alpha_L} = 0.
\end{aligned}$$

Second, consider the monotonicity of $\Pi_{u,m}^*$. We have

$$\begin{aligned}
\frac{\partial \Pi_{u,m}^{HH*}}{\partial \alpha_H} &= [v_H + E(u_H - p_{a,m}^{HH*})^+ - c_m]\lambda_H \\
&\quad + (p_{a,m}^{HH*} - c_a)\lambda_H \bar{F}_H(p_{a,m}^{HH*}) - c'(\alpha_H \lambda_H \bar{F}_H(p_{a,m}^{HH*}) + \alpha_L \lambda_L \bar{F}_L(p_{a,m}^{HH*}))\lambda_H \bar{F}_H(p_{a,m}^{HH*}) \\
&= [v_H + E(u_H - p_{a,m}^{HH*})^+ - c_m]\lambda_H \\
&> 0,
\end{aligned}$$

where the first equality follows by using the Envelope Theorem and the second equality follows by using the first-order condition. Similarly, $\frac{\partial \Pi_{u,m}^{HH*}}{\partial \alpha_L} = [v_L + E(u_L - p_{a,m}^{HH*})^+ - c_m]\lambda_L > 0$. By applying the Envelope Theorem and the first-order condition, we also have

$$\begin{aligned}
\frac{\partial \Pi_{u,m}^{HL*}}{\partial \alpha_H} &= [v_H + E(u_H - p_{a,m}^{HL*})^+ - c_m]\lambda_H \\
&\quad + (p_{a,m}^{HL*} - c_a)\lambda_H \bar{F}_H(p_{a,m}^{HL*}) - c'(\alpha_H \lambda_H \bar{F}_H(p_{a,m}^{HL*}) + \lambda_L \bar{F}_L(p_{a,m}^{HL*}))\lambda_H \bar{F}_H(p_{a,m}^{HL*}) \\
&= [v_H + E(u_H - p_{a,m}^{HL*})^+ - c_m]\lambda_H + \lambda_H \bar{F}_H(p_{a,m}^{HL*}) \cdot \frac{\lambda_L \bar{F}_L(p_{a,m}^{HL*})}{\alpha_H \lambda_H f_H(p_{a,m}^{HL*}) + \lambda_L f_L(p_{a,m}^{HL*})} \\
&> 0.
\end{aligned}$$

Similarly,

$$\frac{\partial \Pi_{u,m}^{LH*}}{\partial \alpha_L} = [v_L + E(u_L - p_{a,m}^{LH*})^+ - c_m]\lambda_L + \lambda_L \bar{F}_L(p_{a,m}^{LH*}) \cdot \frac{\lambda_H \bar{F}_H(p_{a,m}^{LH*})}{\lambda_H f_H(p_{a,m}^{LH*}) + \alpha_L \lambda_L f_L(p_{a,m}^{LH*})} > 0.$$

Additionally, we have $\frac{\partial \Pi_{u,m}^{LH*}}{\partial \alpha_H} = \frac{\partial \Pi_{u,m}^{LL*}}{\partial \alpha_H} = \frac{\partial \Pi_{u,m}^{HL*}}{\partial \alpha_L} = \frac{\partial \Pi_{u,m}^{LL*}}{\partial \alpha_L} = 0$. Therefore, $\Pi_{u,m}^*$ is also increasing in α_H and α_L.

Proof of Theorem IV.7

We use a subscript of "o" to represent the case of selling through OTAs (intermediaries). In the bundling case, the optimal prices are $p_{bH,o}^* = v_H + E(u_H)^+$ and $p_{bL,o}^* = v_L + E(u_L)^+$. The optimal profit from bundling is

$$\begin{aligned}
\Pi_{b,o}^* &= [v_H + E(u_H)^+][\gamma_H + (1-\gamma_H)(1-\tau)]\lambda_H - c_m\lambda_H \\
&\quad + [v_L + E(u_L)^+][\gamma_L + (1-\gamma_L)(1-\tau)]\lambda_L - c_m\lambda_L \\
&\quad - c_a[\lambda_H \bar{F}_H(0) + \lambda_L \bar{F}_L(0)].
\end{aligned}$$

In the unbundling case, the optimal prices should satisfy $p_{mH} = v_H + E(u_H - p_a)^+$ and $p_{mL} = v_L + E(u_H - p_a)^+$. The profit function is

$$\begin{aligned}
\Pi_{u,o}(p_a) &= [v_H + E(u_H - p_a)^+][\gamma_H + (1-\gamma_H)(1-\tau)]\lambda_H - c_m\lambda_H \\
&\quad + [v_L + E(u_L - p_a)^+][\gamma_L + (1-\gamma_L)(1-\tau)]\lambda_L - c_m\lambda_L \\
&\quad + (p_a - c_a)[\lambda_H \bar{F}_H(p_a) + \lambda_L \bar{F}_L(p_a)] - c(\lambda_H \bar{F}_H(p_a) + \lambda_L \bar{F}_L(p_a)).
\end{aligned}$$

Taking derivative of $\Pi_{u,o}(p_a)$ yields

$$\begin{aligned}
\frac{\mathrm{d}\Pi_{u,o}}{\mathrm{d}p_a} &= (1-\gamma_H)\tau\lambda_H \bar{F}_H(p_a) + (1-\gamma_L)\tau\lambda_L \bar{F}_L(p_a) \\
&\quad + [c'(\lambda_H \bar{F}_H(p_a) + \lambda_L \bar{F}_L(p_a)) - (p_a - c_a)][\lambda_H f_H(p_a) + \lambda_L f_L(p_a)],
\end{aligned}$$

which is decreasing in p_a. Thus, $\Pi_{u,o}(p_a)$ is concave. Moreover, it is easy to see that

$\frac{\mathrm{d}\Pi_{u,o}}{\mathrm{d}p_a}\Big|_{p_a=c_a} > 0$ and $\frac{\mathrm{d}\Pi_{u,o}}{\mathrm{d}p_a}\Big|_{p_a=\bar{u}} < 0$. Thus, the optimal ancillary service price $p^*_{a,o}$ is the solution to the first-order condition $\frac{\mathrm{d}\Pi_{u,o}}{\mathrm{d}p_a} = 0$. Then, by using the Envelope Theorem, we have

$$\begin{aligned}
\frac{\mathrm{d}\Pi^*_{u,o}}{\mathrm{d}\tau} &= -[v_H + E(u_H - p^*_{a,o})^+](1-\gamma_H)\lambda_H - [v_L + E(u_L - p^*_{a,o})^+](1-\gamma_L)\lambda_L,\\
\frac{\mathrm{d}\Pi^*_{u,o}}{\mathrm{d}\gamma_H} &= [v_H + E(u_H - p^*_{a,o})^+]\tau\lambda_H,\\
\frac{\mathrm{d}\Pi^*_{u,o}}{\mathrm{d}\gamma_L} &= [v_L + E(u_L - p^*_{a,o})^+]\tau\lambda_L.
\end{aligned}$$

Moreover,

$$\begin{aligned}
\frac{\mathrm{d}\Pi^*_{b,o}}{\mathrm{d}\tau} &= -[v_H + E(u_H)^+](1-\gamma_H)\lambda_H - [v_L + E(u_L)^+](1-\gamma_L)\lambda_L,\\
\frac{\mathrm{d}\Pi^*_{b,o}}{\mathrm{d}\gamma_H} &= [v_H + E(u_H)^+]\tau\lambda_H,\\
\frac{\mathrm{d}\Pi^*_{b,o}}{\mathrm{d}\gamma_L} &= [v_L + E(u_L)^+]\tau\lambda_L.
\end{aligned}$$

Thus, $\frac{\mathrm{d}(\Pi^*_{u,o}-\Pi^*_{b,o})}{\mathrm{d}\tau} \geq 0$, $\frac{\mathrm{d}(\Pi^*_{u,o}-\Pi^*_{b,o})}{\mathrm{d}\gamma_H} \leq 0$, $\frac{\mathrm{d}(\Pi^*_{u,o}-\Pi^*_{b,o})}{\mathrm{d}\gamma_L} \leq 0$.

Proof of Theorem IV.8

Taking derivatives of the profit function yields

$$\begin{aligned}
\frac{\mathrm{d}\Pi_{u,n}}{\mathrm{d}p_a} &= \lambda_H[\bar{F}_H(p_a) - \bar{F}_L(p_a)]\\
&\quad +[c'(\lambda_H\bar{F}_H(p_a) + \lambda_L\bar{F}_L(p_a)) - (p_a - c_a)][\lambda_H f_H(p_a) + \lambda_L f_L(p_a)],\\
\frac{\mathrm{d}^2\Pi_{u,n}}{\mathrm{d}p_a^2} &= -2\lambda_H f_H(p_a) + \lambda_H f_L(p_a) - \lambda_L f_L(p_a)\\
&\quad -c''(\lambda_H\bar{F}_H(p_a) + \lambda_L\bar{F}_L(p_a))[\lambda_H f_H(p_a) + \lambda_L f_L(p_a)]^2,\\
\frac{\mathrm{d}^3\Pi_{u,n}}{\mathrm{d}p_a^3} &= c'''(\lambda_H\bar{F}_H(p_a) + \lambda_L\bar{F}_L(p_a))[\lambda_H f_H(p_a) + \lambda_L f_L(p_a)]^3.
\end{aligned}$$

If $\beta_H \geq \beta_L$, it is easy to see that $\frac{\mathrm{d}^2\Pi_{u,n}}{\mathrm{d}p_a^2} < 0$, so $\Pi_{u,n}$ is concave. If $\beta_H < \beta_L$,

$\frac{\mathrm{d}^3\Pi_{u,n}}{\mathrm{d}p_a^3} \geq 0$, hence $\frac{\mathrm{d}\Pi_{u,n}}{\mathrm{d}p_a}$ is convex. Moreover,

$$\left.\frac{\mathrm{d}\Pi_{u,n}}{\mathrm{d}p_a}\right|_{p_a=\bar{u}} = -(\bar{u} - c_a)[\lambda_H f_H(p_a) + \lambda_L f_L(p_a)] < 0.$$

Thus, $\frac{\mathrm{d}\Pi_{u,n}}{\mathrm{d}p_a}$ can cross the zero line at most once, from positive to negative, which means $\Pi_{u,n}$ is quasi-concave.

Therefore, $p^*_{a,n} = \inf\{0 < p_a < \bar{u} : \lambda_H[\bar{F}_H(p_a) - \bar{F}_L(p_a)] + [c'(\lambda_H \bar{F}_H(p_a) + \lambda_L \bar{F}_L(p_a)) - (p_a - c_a)][\lambda_H f_H(p_a) + \lambda_L f_L(p_a)] \leq 0\}$. Recall that p^*_a is the solution to $p^*_a = c_a + c'(\lambda_H \bar{F}_H(p^*_a) + \lambda_L \bar{F}_L(p^*_a))$. Then, if $\beta_H \geq \beta_L$, we have $\left.\frac{\mathrm{d}\Pi_{u,n}}{\mathrm{d}p_a}\right|_{p_a=p^*_a} \geq 0$, hence $p^*_{a,n} \geq p^*_a$; if $\beta_H < \beta_L$, we have $\left.\frac{\mathrm{d}\Pi_{u,n}}{\mathrm{d}p_a}\right|_{p_a=p^*_a} < 0$, hence $p^*_{a,n} < p^*_a$.

Proof of Theorem IV.9

Consider $\Pi^*_{u,n} - \Pi^*_{b,n}$ as a function of β_H. First consider the case of $p^*_{a,n} = 0$. $p^*_{a,n} = 0$ occurs when $\left.\frac{\mathrm{d}\Pi_{u,n}}{\mathrm{d}p_a}\right|_{p_a=0} = \lambda_H(\beta_H - \beta_L) + [c'(\lambda_H\beta_H + \lambda_L\beta_L) + c_a] \cdot \frac{\lambda_H\beta_H + \lambda_L\beta_L}{\bar{u}} \leq 0$, which requires β_H is small enough (if $p^*_{a,n} = 0$ ever occurs). When $p^*_{a,n} = 0$, we have $\Pi^*_{u,n} - \Pi^*_{b,n} = -c(\lambda_H\beta_H + \lambda_L\beta_L)$ which is negative and decreasing in β_H.

Second, consider the case of $p^*_{a,n} > 0$ which occurs when β_H is large enough. Taking derivatives of the optimal profit functions with respective to β_H yields:

$$\begin{aligned}
\frac{\partial \Pi^*_{u,n}}{\partial \beta_H} &= [p^*_{a,n} - c_a - c'(\lambda_H \bar{F}_H(p^*_{a,n}) + \lambda_L \bar{F}_L(p^*_{a,n}))]\lambda_H \cdot \frac{\bar{u} - p^*_{a,n}}{\bar{u}}, \\
\frac{\partial \Pi^*_{b,n}}{\partial \beta_H} &= -c_a\lambda_H,
\end{aligned}$$

where the derivative of $\Pi^*_{u,n}$ follows from the Envelope Theorem. Thus,

$$\frac{\partial(\Pi^*_{u,n} - \Pi^*_{b,n})}{\partial \beta_H} = \lambda_H \left\{[p^*_{a,n} - c_a - c'(\lambda_H \bar{F}_H(p^*_{a,n}) + \lambda_L \bar{F}_L(p^*_{a,n}))] \cdot \frac{\bar{u} - p^*_{a,n}}{\bar{u}} + c_a\right\}. \tag{C.1}$$

Recall from the proof of Theorem IV.8 that the first-order condition in the uniform pricing case is $p^*_{a,n} - c_a - c'(\lambda_H \bar{F}_H(p^*_{a,n}) + \lambda_L \bar{F}_L(p^*_{a,n})) = \frac{\lambda_H[\bar{F}_H(p^*_{a,n}) - \bar{F}_L(p^*_{a,n})]}{\lambda_H f_H(p^*_{a,n}) + \lambda_L f_L(p^*_{a,n})}$. Thus, if

$\beta_H \geq \beta_L$, $p^*_{a,n} - c_a - c'(\lambda_H \bar{F}_H(p^*_{a,n}) + \lambda_L \bar{F}_L(p^*_{a,n})) \geq 0$, and hence $\frac{\partial(\Pi^*_{u,n} - \Pi^*_{b,n})}{\partial \beta_H} > 0$. If $\beta_H < \beta_L$, by using the first-order condition, we can equivalently write $\frac{\partial(\Pi^*_{u,n} - \Pi^*_{b,n})}{\partial \beta_H}$ as

$$\frac{\partial(\Pi^*_{u,n} - \Pi^*_{b,n})}{\partial \beta_H} = \frac{\lambda_H}{\lambda_H \beta_H + \lambda_L \beta_L} \left\{ \lambda_H \cdot \frac{\beta_H - \beta_L}{\bar{u}} \cdot (\bar{u} - p^*_{a,n})^2 + c_a(\lambda_H \beta_H + \lambda_L \beta_L) \right\}.$$

If $p^*_{a,n}$ is increasing in β_H, then $\lambda_H \cdot \frac{\beta_H - \beta_L}{\bar{u}} \cdot (\bar{u} - p^*_{a,n})^2 + c_a(\lambda_H \beta_H + \lambda_L \beta_L)$ is increasing in β_H, and hence $\frac{\partial(\Pi^*_{u,n} - \Pi^*_{b,n})}{\partial \beta_H}$ is first decreasing then increasing in β_H. We now show that $p^*_{a,n}$ is increasing in β_H. By applying the Implicit Function Theorem to the first-order condition $\frac{\mathrm{d}\Pi_{u,n}}{\mathrm{d}p_a} = 0$, we obtain

$$\frac{\mathrm{d}p^*_{a,n}}{\mathrm{d}\beta_H} = -\frac{\frac{\partial}{\partial \beta_H}\left(\frac{\mathrm{d}\Pi_{u,n}}{\mathrm{d}p_a}\right)\Big|_{p_a = p^*_{a,n}}}{\frac{\partial}{\partial p_a}\left(\frac{\mathrm{d}\Pi_{u,n}}{\mathrm{d}p_a}\right)\Big|_{p_a = p^*_{a,n}}} = -\frac{\frac{\partial}{\partial \beta_H}\left(\frac{\mathrm{d}\Pi_{u,n}}{\mathrm{d}p_a}\right)\Big|_{p_a = p^*_{a,n}}}{\frac{\mathrm{d}^2\Pi_{u,n}}{\mathrm{d}p_a^2}\Big|_{p_a = p^*_{a,n}}}. \tag{C.2}$$

The numerator of (C.2) is

$$\begin{aligned} \frac{\partial}{\partial \beta_H}\left(\frac{\mathrm{d}\Pi_{u,n}}{\mathrm{d}p_a}\right)\Bigg|_{p_a = p^*_{a,n}} \quad &= \quad \lambda_H \cdot \frac{\bar{u} - p^*_{a,n}}{\bar{u}} \\ &\quad + c''(\lambda_H \bar{F}_H(p^*_{a,n}) + \lambda_L \bar{F}_L(p^*_{a,n})) \cdot \frac{\lambda_H(\bar{u} - p^*_{a,n})(\lambda_H \beta_H + \lambda_L \beta_L)}{\bar{u}^2} \\ &\quad - [p^*_{a,n} - c_a - c'(\lambda_H \bar{F}_H(p^*_{a,n}) + \lambda_L \bar{F}_L(p^*_{a,n}))] \cdot \frac{\lambda_H}{\bar{u}}. \end{aligned}$$

Since $\beta_H < \beta_L$, the first-order condition implies that $p^*_{a,n} - c_a - c'(\lambda_H \bar{F}_H(p^*_{a,n}) + \lambda_L \bar{F}_L(p^*_{a,n})) < 0$. Then, since $c(\cdot)$ is convex, we know that $\frac{\partial}{\partial \beta_H}\left(\frac{\mathrm{d}\Pi_{u,n}}{\mathrm{d}p_a}\right)\Big|_{p_a = p^*_{a,n}} > 0$. Moreover, in the proof of Theorem IV.8, we already know that $\frac{\mathrm{d}\Pi_{u,n}}{\mathrm{d}p_a}$ can only cross the zero line from positive to negative. Thus, $\frac{\mathrm{d}^2\Pi_{u,n}}{\mathrm{d}p_a^2}\Big|_{p_a = p^*_{a,n}} < 0$, and hence $\frac{\mathrm{d}p^*_{a,n}}{\mathrm{d}\beta_H} > 0$.

So far, we have obtained that 1) for small β_H (if $p^*_{a,n} = 0$ ever occurs), $\Pi^*_{u,n} - \Pi^*_{b,n}$ is negative and decreasing in β_H; 2) for large β_H (i.e., $\beta_H \geq \beta_L$), $\Pi^*_{u,n} - \Pi^*_{b,n}$ is increasing in β_H; 3) for medium β_H (i.e., $\beta_H < \beta_L$ and $p^*_{a,n} > 0$), $\Pi^*_{u,n} - \Pi^*_{b,n}$ is first decreasing then increasing in β_H. Thus, overall, $\Pi^*_{u,n} - \Pi^*_{b,n}$ is quasi-convex (i.e., first decreasing then increasing) in β_H. If $p^*_{a,n} = 0$ occurs for small β_H, $\Pi^*_{u,n} - \Pi^*_{b,n}$ first decreases from a negative value and then becomes increasing in β_H, thus it is negative for small β_H and positive for large β_H. If $p^*_{a,n} = 0$ never occurs, we may have two scenarios. First, if $\Pi^*_{u,n} - \Pi^*_{b,n}$ is increasing in β_H at $\beta_H = 0$, then it is always increasing in β_H, and hence $\Pi^*_{u,n} - \Pi^*_{b,n}$ can only be negative for small β_H and positive for large β_H. Second, if $\Pi^*_{u,n} - \Pi^*_{b,n}$ is decreasing in β_H at $\beta_H = 0$, we now show that we must have $\Pi^*_{u,n} - \Pi^*_{b,n} < 0$ at $\beta_H = 0$ in this case, so that $\Pi^*_{u,n} - \Pi^*_{b,n}$ is negative for small β_H and positive for large β_H. At $\beta_H = 0$, we have

$$\Pi^*_{u,n} - \Pi^*_{b,n} = -\lambda_H \cdot \frac{\beta_L}{2\bar{u}} \cdot p^*_{a,n}(2\bar{u} - p^*_{a,n}) + \lambda_L \cdot \frac{\beta_L}{2\bar{u}} \cdot p^*_{a,n}(2c_a - p^*_{a,n}) - c(\lambda_L \bar{F}_L(p^*_{a,n})).$$

$\frac{\partial(\Pi^*_{u,n} - \Pi^*_{b,n})}{\partial \beta_H}\Big|_{\beta_H = 0} < 0$ can be simplified to $-\lambda_H \cdot \frac{\beta_L}{\bar{u}} \leq -\frac{c_a \lambda_L \beta_L}{(\bar{u} - p^*_{a,n})^2}$. Thus, we have

$$\begin{aligned}
\Pi^*_{u,n} - \Pi^*_{b,n} &\leq -\frac{c_a \lambda_L \beta_L}{2(\bar{u} - p^*_{a,n})^2} \cdot p^*_{a,n}(2\bar{u} - p^*_{a,n}) + \lambda_L \cdot \frac{\beta_L}{2\bar{u}} \cdot p^*_{a,n}(2c_a - p^*_{a,n}) - c(\lambda_L \bar{F}_L(p^*_{a,n})) \\
&= \frac{\lambda_L \beta_L}{2\bar{u}} \cdot p^*_{a,n} \cdot \left[-c_a \cdot \frac{2\bar{u} - p^*_{a,n}}{\bar{u} - p^*_{a,n}} \cdot \frac{\bar{u}}{\bar{u} - p^*_{a,n}} + (2c_a - p^*_{a,n})\right] - c(\lambda_L \bar{F}_L(p^*_{a,n})) \\
&< -\frac{\lambda_L \beta_L}{2\bar{u}} \cdot (p^*_{a,n})^2 - c(\lambda_L \bar{F}_L(p^*_{a,n})) \\
&< 0,
\end{aligned}$$

where the first inequality follows from using $-\lambda_H \cdot \frac{\beta_L}{\bar{u}} \leq -\frac{c_a \lambda_L \beta_L}{(\bar{u} - p^*_{a,n})^2}$ and the second inequality follows from $\frac{2\bar{u} - p^*_{a,n}}{\bar{u} - p^*_{a,n}} > 2$ and $\frac{\bar{u}}{\bar{u} - p^*_{a,n}} > 1$. Therefore, combining all cases analyzed above, we obtain that there exists a threshold function $\bar{\beta}_{H,n}(\beta_L)$ such that $\Pi^*_{u,n} \geq \Pi^*_{b,n}$ if and only if $\beta_H \geq \bar{\beta}_{H,n}(\beta_L)$.

Next, we show that $\bar{\beta}_{H,n}(\beta_L)$ is an increasing function. By applying the Implicit Function Theorem to the equation $\Pi^*_{u,n} - \Pi^*_{b,n} = 0$ which defines $\bar{\beta}_{H,n}(\beta_L)$, we have

$$\frac{\mathrm{d}\bar{\beta}_{H,n}}{\mathrm{d}\beta_L} = -\frac{\frac{\partial(\Pi^*_{u,n} - \Pi^*_{b,n})}{\partial \beta_L}\Big|_{\beta_H = \bar{\beta}_{H,n}(\beta_L)}}{\frac{\partial(\Pi^*_{u,n} - \Pi^*_{b,n})}{\partial \beta_H}\Big|_{\beta_H = \bar{\beta}_{H,n}(\beta_L)}}.$$

We have shown that $\Pi^*_{u,n} - \Pi^*_{b,n}$ can only cross the zero line from negative to positive, thus $\frac{\partial(\Pi^*_{u,n} - \Pi^*_{b,n})}{\partial \beta_H}\Big|_{\beta_H = \bar{\beta}_{H,n}(\beta_L)} > 0$. It remains to show that $\frac{\partial(\Pi^*_{u,n} - \Pi^*_{b,n})}{\partial \beta_L}\Big|_{\beta_H = \bar{\beta}_{H,n}(\beta_L)} \leq 0$. Taking derivatives of the optimal profit functions with respective to β_L yields:

$$\begin{aligned}
\frac{\partial \Pi^*_{u,n}}{\partial \beta_L} &= \frac{(\bar{u} - p^*_{a,n})^2}{2\bar{u}} \cdot (\lambda_H + \lambda_L) + [p^*_{a,n} - c_a - c'(\lambda_H \bar{F}_H(p^*_{a,n}) + \lambda_L \bar{F}_L(p^*_{a,n}))]\lambda_L \cdot \frac{\bar{u} - p^*_{a,n}}{\bar{u}}, \\
\frac{\partial \Pi^*_{b,n}}{\partial \beta_L} &= \frac{\bar{u}}{2} \cdot (\lambda_H + \lambda_L) - c_a \lambda_H,
\end{aligned}$$

where the derivative of $\Pi^*_{u,n}$ follows from the Envelope Theorem. Thus,

$$\begin{aligned}
\frac{\partial(\Pi^*_{u,n} - \Pi^*_{b,n})}{\partial \beta_L} &= \frac{p^*_{a,n}(p^*_{a,n} - 2\bar{u})}{2\bar{u}} \cdot (\lambda_H + \lambda_L) \\
&\quad + \lambda_L \left\{ [p^*_{a,n} - c_a - c'(\lambda_H \bar{F}_H(p^*_{a,n}) + \lambda_L \bar{F}_L(p^*_{a,n}))] \cdot \frac{\bar{u} - p^*_{a,n}}{\bar{u}} + c_a \right\}.
\end{aligned}$$

At $\beta_H = \bar{\beta}_{H,n}(\beta_L)$, we have $\Pi^*_{u,n} = \Pi^*_{b,n}$ which is equivalent to the following equation after rearranging terms:

$$\begin{aligned}
\frac{\beta_L}{2\bar{u}} \cdot p^*_{a,n}(p^*_{a,n} - 2\bar{u})(\lambda_H + \lambda_L) &= -(p^*_{a,n} - c_a)(\lambda_H \beta_H + \lambda_L \beta_L) \cdot \frac{\bar{u} - p^*_{a,n}}{\bar{u}} \\
&\quad + c(\lambda_H \bar{F}_H(p^*_{a,n}) + \lambda_L \bar{F}_L(p^*_{a,n})) - c_a(\lambda_H \beta_H + \lambda_L \beta_L).
\end{aligned}$$

By using $c(x) \leq c'(x)x$, we obtain from the above equation that at $\beta_H = \bar{\beta}_{H,n}(\beta_L)$,

$$\begin{aligned}
\frac{\beta_L}{2\bar{u}} \cdot p^*_{a,n}(p^*_{a,n} - 2\bar{u})(\lambda_H + \lambda_L) &\leq -[p^*_{a,n} - c_a - c'(\lambda_H \bar{F}_H(p^*_{a,n}) + \lambda_L \bar{F}_L(p^*_{a,n}))](\lambda_H \beta_H + \lambda_L \beta_L) \\
&\quad \cdot \frac{\bar{u} - p^*_{a,n}}{\bar{u}} - c_a(\lambda_H \beta_H + \lambda_L \beta_L).
\end{aligned}$$

By using the above inequality, at $\beta_H = \bar{\beta}_{H,n}(\beta_L)$, we have

$$
\begin{aligned}
& \frac{\partial(\Pi^*_{u,n} - \Pi^*_{b,n})}{\partial \beta_L} \\
\leq \quad & -[p^*_{a,n} - c_a - c'(\lambda_H \bar{F}_H(p^*_{a,n}) + \lambda_L \bar{F}_L(p^*_{a,n}))] \cdot \frac{\lambda_H\beta_H + \lambda_L\beta_L}{\beta_L} \cdot \frac{\bar{u} - p^*_{a,n}}{\bar{u}} \\
& -c_a \cdot \frac{\lambda_H\beta_H + \lambda_L\beta_L}{\beta_L} \\
& +\lambda_L \left\{ [p^*_{a,n} - c_a - c'(\lambda_H \bar{F}_H(p^*_{a,n}) + \lambda_L \bar{F}_L(p^*_{a,n}))] \cdot \frac{\bar{u} - p^*_{a,n}}{\bar{u}} + c_a \right\} \\
= \quad & -\lambda_H \cdot \frac{\beta_H}{\beta_L} \cdot \left\{ [p^*_{a,n} - c_a - c'(\lambda_H \bar{F}_H(p^*_{a,n}) + \lambda_L \bar{F}_L(p^*_{a,n}))] \cdot \frac{\bar{u} - p^*_{a,n}}{\bar{u}} + c_a \right\} \quad \text{(C.3)}
\end{aligned}
$$

Thus, by comparing (C.1) and (C.3), we obtain that

$$
\frac{\partial(\Pi^*_{u,n} - \Pi^*_{b,n})}{\partial \beta_L}\Big|_{\beta_H = \bar{\beta}_{H,n}(\beta_L)} \leq \frac{\partial(\Pi^*_{u,n} - \Pi^*_{b,n})}{\partial \beta_H}\Big|_{\beta_H = \bar{\beta}_{H,n}(\beta_L)} \cdot \left[-\frac{\bar{\beta}_{H,n}(\beta_L)}{\beta_L} \right] \leq 0.
$$

Therefore, $\bar{\beta}_{H,n}(\beta_L)$ is increasing in β_L.

Proof of Theorem IV.10

(i) If $\beta_H \geq \beta_L$, we have

$$
\begin{aligned}
\Pi^*_{u,n} - \Pi^*_u \quad & \geq \quad \Pi_{u,n}(p^*_a) - \Pi^*_u \\
& = \quad -(v_H - v_L)\lambda_H - [E(u_H - p^*_a)^+ - E(u_L - p^*_a)^+]\lambda_H \\
& = \quad -(v_H - v_L)\lambda_H - \lambda_H \int_{p^*_a}^{\bar{u}} [\bar{F}_H(x) - \bar{F}_L(x)]\, \mathrm{d}x \\
& \geq \quad -(v_H - v_L)\lambda_H - \lambda_H \int_{0}^{\bar{u}} [\bar{F}_H(x) - \bar{F}_L(x)]\, \mathrm{d}x \\
& = \quad -(v_H - v_L)\lambda_H - [E(u_H)^+ - E(u_L)^+]\lambda_H \\
& = \quad \Pi^*_{b,n} - \Pi^*_b.
\end{aligned}
$$

Rearranging terms in the inequality obtained above yields $\Pi^*_{u,n} - \Pi^*_{b,n} \geq \Pi^*_u - \Pi^*_b$. Thus, when $\Pi^*_u \geq \Pi^*_b$, we also have $\Pi^*_{u,n} \geq \Pi^*_{b,n}$.

(ii) If $\beta_H < \beta_L$, we have

$$\begin{aligned}
\Pi^*_{u,n} - \Pi^*_u &< \Pi^*_{u,n} - \Pi_u(p^*_{a,n}) \\
&= -(v_H - v_L)\lambda_H + [E(u_L - p^*_{a,n})^+ - E(u_H - p^*_{a,n})^+]\lambda_H \\
&\leq -(v_H - v_L)\lambda_H + [E(u_L)^+ - E(u_H)^+]\lambda_H \\
&= \Pi^*_{b,n} - \Pi^*_b,
\end{aligned}$$

where the second inequality follows from the same approach used in Part (i). Thus, we have $\Pi^*_{u,n} - \Pi^*_{b,n} < \Pi^*_u - \Pi^*_b$; when $\Pi^*_{u,n} \geq \Pi^*_{b,n}$, we also have $\Pi^*_u \geq \Pi^*_b$.

Proof of Theorem IV.11

When we increasing λ_H and decreasing λ_L such that $\lambda_H + \lambda_L = \lambda$, applying the Implicit Function Theorem to the equation $\Pi^*_{u,n} - \Pi^*_{b,n} = 0$ yields

$$\begin{aligned}
\frac{\mathrm{d}\bar{\beta}_{H,n}}{\mathrm{d}\lambda_H} &= -\frac{\frac{\partial(\Pi^*_{u,n} - \Pi^*_{b,n})}{\partial\lambda_H} - \frac{\partial(\Pi^*_{u,n} - \Pi^*_{b,n})}{\partial\lambda_L}}{\frac{\partial(\Pi^*_{u,n} - \Pi^*_{b,n})}{\partial\bar{\beta}_H}} \\
&= -\frac{(\bar{\beta}_{H,n} - \beta_L)\left\{[p^*_{a,n} - c_a - c'(\lambda_H \bar{F}_H(p^*_{a,n}) + \lambda_L \bar{F}_L(p^*_{a,n}))] \cdot \frac{\bar{u} - p^*_{a,n}}{\bar{u}} + c_a\right\}}{\lambda_H \left\{[p^*_{a,n} - c_a - c'(\lambda_H \bar{F}_H(p^*_{a,n}) + \lambda_L \bar{F}_L(p^*_{a,n}))] \cdot \frac{\bar{u} - p^*_{a,n}}{\bar{u}} + c_a\right\}} \\
&= \frac{\beta_L - \bar{\beta}_{H,n}}{\lambda_H}.
\end{aligned}$$

Thus, $\bar{\beta}_{H,n}(\beta_L)$ is decreasing in λ_H when $\bar{\beta}_{H,n}(\beta_L) \geq \beta_L$ and increasing in λ_H when $\bar{\beta}_{H,n}(\beta_L) < \beta_L$. Also, note that when $\bar{\beta}_{H,n}(\beta_L) = \beta_L$, $\bar{\beta}_{H,n}(\beta_L)$ does not change with λ_H if we keep $\lambda_H + \lambda_L = \lambda$. Thus, $\bar{\beta}_{H,n}(\beta_L)$ intersects at the same point on $\beta_H = \beta_L$ when we change λ_H and keep $\lambda_H + \lambda_L = \lambda$.

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