Three Essays on the Economics of Changing Behavior With Costly Policy Instruments

by

Daniel Harrington Jaqua

A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
(Economics)
in the University of Michigan
2015

Doctoral Committee:

Professor James R Hines Jr, Chair
Assistant Professor Natalia Lazzatti
Professor James Jondall Prescott
Assistant Professor Ugo A Troiano

ACKNOWLEDGEMENTS

I am indebted to the chair of my dissertation committee, Jim Hines, for his counsel throughout my doctoral studies. I am also indebted to my dissertation committee as a whole, including JJ Prescott, Natalia Lazzatti and Ugo Troiano, for their feedback and support of my research. I also thank Tilman Börgers for his friendship and advice. Many other professors also taught me to be a better economist and offered helpful comments on this research. I specifically want to thank Jim Adams, Kfir Eliaz, Stephan Lauermann, Yusufcan Masatlioglu, Steve Salant, Dan Silverman, Joel Slemrod and Lones Smith.

Each chapter of this dissertation is based on a coauthored work with Danny Schaffa. His interest, skills and efforts have been a tremendous benefit to this research. Collaborating with him has been a tremendous benefit to me professionally and personally.

I also wish to thank seminar participants at the University of Michigan for helpful comments and suggestions.

I am also grateful for the assistance of the support staff at the University of Michigan during my doctoral studies, especially Mary Braun, Mary Magnum, Lauren Pulay, Vinnie Vinjmoor and Olga Mustata.

I appreciate the financial support I have received from the Rackham Graduate School and through the Economics Department at the University of Michigan in support of this dissertation and my doctoral studies.

TABLE OF CONTENTS

LIST OF TABLES

Table

LIST OF APPENDICES

Appendix

CHAPTER I

Pigouvian Taxation with Multiple Externalities and Costly Administration

From a work with Danny Schaffa

Abstract

This paper generalizes Pigouvian taxation to a setting with multiple externalities, multiple tax instruments, and costly administration of the tax system. Previous work studying costly administration with one external harm, previous work studying a presumptive tax, because one or more choice variables are untaxable, and previous work on multiple harmful activities with a costless tax system are all special cases of this paper's approach. When administrative costs limit the number of tax instruments, the optimal Pigouvian tax depends on the effect each tax has on all the externality causing activities, weighted by their relative harm. Surprisingly, even when all of the choice variables are taxed, the presence of administrative costs causes a similar result: the optimal Pigouvian taxes depend on the response of all activities which cause externalities, weighted by the size of the marginal external harm. This result contrasts with prior work because the optimal Pigouvian taxes are not equal to marginal harm of the taxed activity at the socially optimal level of the activity. In this setting, the additivity property of Pigouvian taxes no longer holds. Lastly, this paper quantifies the welfare loss associated with limited tax instruments or the presence of administrative costs.

1.1 Introduction

Consider a factory that produces, say, cars and emits many varieties of pollution. This takes little imagination. Most factories release a range of pollutants with sulfur, nitrogen, carbon or other bases. These pollutants may be released into the air, the water, or the earth. If there is some degree of substitutability in the production process, an activity tax could change the mix of activities used to produce cars. A sulfur-based process could be exchanged for a nitrogen-based process. One sulfur byproduct could be converted into another sulfur byproduct. A sulfur compound could be released into the air or the water. If these pollutants are differentially harmful, a single Pigouvian tax may not properly correct the externalities.

Left alone, the factory will choose whatever production technology is cheapest—taking into consideration only whatever portion of the externality directly affects it. Thus a familiar trade-off emerges: the total social harm of the externality weighed against the benefit of production. Taxing the output, cars, is likely to be one of the easiest taxes to administer. However, an output tax is a particularly blunt instrument. Although it will affect the level of output, the firm will still choose the privately cheapest production technology for any given level of output.

The optimal output tax is based on the marginal change in all types of pollution, as well as the possibility that the firm would chose a different production technology if the firm operated at a different scale.

The result that the output tax depends on the relationship among all the pollutants generalizes to any tax system with administrative costs. When some pollutants are untaxed, because taxing that pollutant is costly or detection is impossible, the optimal Pigouvian tax rates do take into account the effect of a tax on one pollutant on the amount of another pollutant. Even when it is optimal to tax all the choice variables, the existence of variable administrative costs causes the optimal tax rates to depend on these cross-pollution effects. The classic additivity result for Pigouvian taxation also depends on the tax system being costless. Below we formalize and expand on these ideas.

This chapter relaxes two assumptions present in the prior literature. Previous work allows the administrative costs to vary with the amount of the activity, ruling out administrative costs that vary with the tax liability or the tax rate. This paper allows the administrative costs to vary with the tax rate and the tax liability. This is important and realistic since administrative costs are, in this model, equivalent to many forms of excess burden which should vary with the tax liability. For example, if the tax authority hires an additional auditor, that is an administrative cost. If increasing tax liability causes a firm to hire an accountant in order to reduce their tax liability, the accountant's time is a real resource cost for the economy equivalent to the administrative cost in this model. We express this idea clearly in section 1.3.2, by contrasting this approach with the assumption that the administrative costs only vary with the quantity of the activity. The second innovation of this paper is to analyze a setting with than one activity. Some prior work has considered this possibility, but when all activities are taxed and administrative costs are ignored or assumed to vary only with the quantity of the activity, the tax on each activity is equivalent to the tax on that activity when considered in isolation. We show that this is a special case of our model in section 1.3.1 and 1.3.2.

In section 1.3 we present our model. Using this model we can analyze several possible tax regimes. We first explain the benchmark case in section 1.3.1, where all activities are taxed in a costless tax system. Section 1.3.2 presents the solution to this problem when all of the activities are taxed, but the tax system is costly. Section 1.3.3 explains how the solution changes when it is optimal to tax some subset of polluting activities. Then, in section 1.4, we present a modification of the model in which outputs are taxed. Section 1.4.2 presents the solution to the problem when output is taxed. In section 1.4.3, we propose a method for comparing tax systems which are constrained to have different tax bases and in section 1.4.4 we compute the welfare impact of being constrained to an output tax as opposed to a more direct tax on polluting activities. Section 1.5 concludes.

1.2 Literature Review

The relationship between externalities and taxes is well studied. Taxes used to change behaviors which generate externalities, named after Pigou (1920), improve welfare by aligning a notion of public well being with private incentives. This paper generalizes previous approaches to include administrative costs or other constraints on Pigouvian taxes. By including administrative costs, a wide class of government policies that influence private behavior can be analyzed in the framework of Pigouvian taxation. In the previous literature, first best Pigouvian taxes for one good do not depend on the cross price effects with other externality causing goods. The intuition is that each tax fully aligns public well being with the private incentive for each private choice. Since individual choice already accounts for the external harm, there is no net social benefit to further reducing the externality causing activities. This paper suggests that such a result depends on the tax system being costless. When Pigouvian taxes are not costless transfers, the optimal taxes do account for the relationships between activities and the optimal taxes are set at different levels than the previous literature suggests.

Within the literature on Pigouvian taxation a few papers have considered the significance of administrative costs for one activity in isolation. Polinsky and Shavell (1982) address the question of whether costly administration of a Pigouvian tax can affect the optimal number of firms. They also also make the point that for one commodity, the optimal level of output may be higher or lower when Pigouvian taxes are costly. A later paper by Kaplow (1990) makes this point for criminals punished by incarceration, a policy which has administrative costs and is also costly in that it fails to collect any of the 'tax revenue'. One of the main contributions of this paper is to generalize this line of research about administrative costs and the optimal output level to a model with more than one externality-causing activity.

Some other papers have considered limitations on the tax base, sometimes interpreted as high fixed costs of adding certain activities to the tax base, in relation to Pigouvian taxation. Fullerton and Wolverton (2005) demonstrate that a 'two part instrument', a subsidy on clean activities and a tax on a measurable activity, can mimic a tax on an unobservable dirty activity. The approach used in section 1.3.3, where an arbitrary combination of activities are prohibitively costly to tax, generalizes their two part instrument, however, Fullerton and Wolverton discuss practical considerations such as evasion and avoidance, which are not considered here. Fullerton and West (2002) compares several specific tax instruments, such as a tax on engine size or a tax on gasoline, with a tax on emissions. The models in section 1.3 generalize the approach used in Fullerton and West because section 1.3 does not restrict the relationships among the possible tax bases.

The interaction between the markets subject to Pigouvian taxation and other distorted markets is also studied in the literature on the 'double dividend'. This literature investigates whether tax revenue could be used to reduce the distortion of other taxes. The main insight from this literature is that Pigouvian taxes do raise revenue but they can also increase the distortion in other markets previously subject to taxation. For example, Bovenberg and De Mooij (1994) use a model with a clean consumption good, a dirty consumption good, leisure, and a preexisting tax on labor. They find that when leisure is separable in the utility function, the optimal tax on the dirty good is less than the marginal external harm because the Pigouvian tax tends to increase the excess burden associated with the distortion in the taxed labor market. The optimal Pigouvian tax is different when the government budget constraint and preexisting distortions in other markets are taken into account. In general equilibrium, if taxes were set to minimize excess burden in the absence of a Pigouvian tax, the addition of a Pigouvian tax can generate a welfare gain by reducing the harm caused by the externality, but does not create an additional improvement in welfare associated with the revenue generation of the Pigouvian tax.

The literature on optimal tax systems attempts to address similar questions about the simultaneous choice of several policy variables, especially the tax base and the tax rate, in the tax system. Yitzhaki (1979) has considered the optimal choice of the tax base and the tax rate for a commodity tax. This paper also characterizes the optimal tax base and tax rate, but does so in the presence of externalities. Mayshar (1991) is an example in this literature of relating the optimal policy to the cost of funds in a particular tax system, emphasizing that tax instruments generally should equate the welfare cost of funds with the welfare benefit of raising funds. Although it emphasizes tax evasion, Slemrod and Yitzhaki (2002) also provides an overview of the literature on administration and administrative costs.

1.3 Model of Multiple Externalities and Costly Administration

When applying a Pigouvian tax to more than one chosen activity or good, whether those activities or goods cause the externalities directly or through inputs, the classic result is that the Pigouvian tax does not depend on the relationships between those chosen activities or goods. The intuition is that each tax fully aligns public well being with the private incentive for each private choice. Since individual choice already accounts for the external harm, there is no net social benefit to further reducing the externality causing activities. We first present this result in our model, before demonstrating that this result does not hold when there are administrative costs associated with the taxes.

1.3.1 Costless, Complete Tax System

A factory produces output using n different activities, represented by an n-dimensional vector, $\mathbf{x}$. $q(\mathbf{x})$ maps that vector of activities to the maximum quantity of output that can be produced. $b(\mathbf{x})$ is the total surplus from the private market (i.e. consumer surplus + producer surplus + tax revenue) for this good. Social harm from production is a linear function of the activities, $\mathbf{e}^\top\mathbf{x}$.

If the social planner is free to set a linear tax for each externality, the private market must solve the following problem:

$$\max_{\mathbf{x}} b(\mathbf{x}) - \mathbf{t}^\top \mathbf{x}$$

which leads to the condition:

$$b'(\mathbf{y}(\mathbf{t})) - \mathbf{t}^\top = 0$$

where $\mathbf{y}(\mathbf{t})$ is the private market's optimal activity vector for tax vector $\mathbf{t}$. This leads to:

$$b'(\mathbf{y}(\mathbf{t})) = \mathbf{t}^\top$$

The social planner, too, must solve an optimization problem. Its problem is to select an n-dimensional tax vector that maximizes welfare:

$$\max_{\mathbf{t}} b(\mathbf{y}(\mathbf{t})) - \mathbf{e}^\top \mathbf{y}(\mathbf{t})$$

with first order condition:

$$b'(\mathbf{y}(\mathbf{t}))\mathbf{y}'(\mathbf{t}) - \mathbf{e}^\top \mathbf{y}'(\mathbf{t}) = 0$$

Thus the optimal tax is:

$$\mathbf{t} = \mathbf{e}$$

which is unique because $\mathbf{y}'(\mathbf{t})$ is invertible. This is not surprising–it simply generalizes the notion that an externality can be corrected by a tax equal to the marginal external harm. The following work will demonstrate that this result is sensitive to the assumption that all externality causing activities can be directly measured and taxed, and is also sensitive to the assumption that taxes are costless transfers. When some choices are not taxed, the optimal levels of the remaining Pigouvian taxes depend on the relationship among all the activities. Even more surprising is that even when all of the choice variables can be taxed, the existence of variable administrative costs causes the taxes to not only depend on the administrative costs themselves, but the entire set of externalities and the responses of each activity to taxes on other activities.

1.3.2 Costly, Complete Tax System

If implementing the Pigouvian tax incurs a fixed cost, but administrative costs do not otherwise vary with the amount of the tax or the amount of the externality generating activity, there are three possible cases. First, it may be that the fixed costs are sufficiently small and all of the taxes are used. In this case the fixed administrative costs will not alter the optimal tax rates. The second possibility is that some of the taxes are not used, in which case the optimal tax rates will differ from the optimal tax rates without administrative cost (see section 1.4.2). The third possibility is that the fixed costs are high enough that no Pigouvian taxation is optimal.

When administrative costs are variable with or without fixed costs, the particular form of the costs determines whether the optimal tax base and tax rates differ from the optimal Pigouvian taxes without administrative costs. The following model characterizes the optimal taxes for an arbitrary cost function, when all the taxes are used.

The agent's problem is still:

$$\max_{\mathbf{x}} b(\mathbf{x}) - \mathbf{t}^\top \mathbf{x}$$

which leads to the condition:

$$b'(\mathbf{y}(\mathbf{t})) - \mathbf{t}^\top = 0$$

where $\mathbf{y}(\mathbf{t})$ is the agent's optimal quantity of the activities for tax vector $\mathbf{t}$. This leads to:

$$b'(\mathbf{y}(\mathbf{t})) = \mathbf{t}^\top$$

The government must now solve a different problem. We assume that costs are paid by the government and are both a function of the taxes and the activity levels. Government's problem:

$$\max_{\mathbf{t}} b(\mathbf{y}(\mathbf{t})) - \mathbf{e}^\top \mathbf{y}(\mathbf{t}) - C(\mathbf{y}(\mathbf{t}), \mathbf{t})$$

with first order condition:

$$b'(\mathbf{y}(\mathbf{t}))\mathbf{y}'(\mathbf{t}) - \mathbf{e}^\top \mathbf{y}'(\mathbf{t}) - C_1(\mathbf{y}(\mathbf{t}), \mathbf{t})\mathbf{y}'(\mathbf{t}) - C_2(\mathbf{y}(\mathbf{t}), \mathbf{t}) = 0$$

If $\mathbf{y}'(\mathbf{t})$ is invertible, then:

$$\mathbf{t}^\top = \mathbf{e}^\top + C_1(\mathbf{y}(\mathbf{t}), \mathbf{t}) + C_2(\mathbf{y}(\mathbf{t}), \mathbf{t})\mathbf{y}'(\mathbf{t})^{-1}$$

Note that element i of the vector $C_2(\mathbf{y}(\mathbf{t}), \mathbf{t})\mathbf{y}'(\mathbf{t})^{-1}$ depends on $\frac{\partial y_j}{\partial t_k}$, the cross price effect between different activities.

$$\mathbf{t}^\top = \mathbf{e}^\top + C_1(\mathbf{y}(\mathbf{t}), \mathbf{t}) + C_2(\mathbf{y}(\mathbf{t}), \mathbf{t})\mathbf{y}'(\mathbf{t})^{-1}$$

A special case illuminates part of the relationship between the administrative costs and the resulting tax rates. Consider the above equation when variable administrative costs depend on the amount of $\mathbf{y}$, but do not directly depend on $\mathbf{t}$, so $C_2 = 0$. In this case, there is a solution where the tax on each activity equals the externality cost of that activity plus the marginal administrative cost associated with additional activity, $\mathbf{y}$. The optimal Pigouvian taxes in this case do not account for the cross price effects between elements of $\mathbf{y}$.

$$\mathbf{t}^\top = \mathbf{e}^\top + C_1(\mathbf{y}(\mathbf{t}), \mathbf{t})$$

1.3.3 Taxing m of n Activities

When there are no variable costs, but the fixed costs for some activities are sufficiently high, it may be optimal to only tax activities in a subset m of all of the activities. Because of the discrete nature of the problem, the social planner finds the optimal tax rates for 2^n different combinations of possible tax bases, and selects the tax base and rates which result in the highest social welfare. We do not propose a closed form solution describing which set of activities are optimal to tax. For any given set of activities m, the optimal tax rates are determined as follows.

Agent's problem:

$$\max_{\mathbf{x}} b(\mathbf{x}) - \sum_{i \in m} s_i x_i$$

Solution:

$$\frac{\partial b}{\partial x_j} = 0 \text{ for } j \notin m \text{ and } \frac{\partial b}{\partial x_i} = s_i \text{ for } i \in m$$

Government's problem:

$$\max_{\mathbf{s}} b(\mathbf{y}(\mathbf{s})) - \mathbf{e}^\top \mathbf{y}(\mathbf{s})$$

where $\mathbf{s}$ is the m-dimensional vector of taxes on the m taxed activities. Solution:

$$b'(\mathbf{y}(\mathbf{s}))\mathbf{y}'(\mathbf{s}) - \mathbf{e}^\top \mathbf{y}'(\mathbf{s}) = 0$$

Now let $\mathbf{y}'_*(s)$ be the matrix that is constructed by deleting rows $j \notin m$ of the Jacobian $\frac{\partial \mathbf{y}}{\partial \mathbf{s}}$. This is an $m \times m$ matrix. Then:

$$\mathbf{s}^\top \mathbf{y}'_*(\mathbf{s}) - \mathbf{e}^\top \mathbf{y}'(\mathbf{s}) = 0$$

If $\mathbf{y}'_*(s)$ is invertible, then we have:

$$\mathbf{s}^\top = \mathbf{e}^\top \mathbf{y}'(\mathbf{s})\mathbf{y}'_*(\mathbf{s})^{-1}$$

There are two special cases. $m = n$ and $m = 1$. For the first note that $\mathbf{y}'(\mathbf{s}) = \mathbf{y}'_*(\mathbf{s})^{-1}$, so we have $\mathbf{s} = \mathbf{e}$ as before. For the second case, note that $\mathbf{y}'_*(\mathbf{s}) = (\frac{\partial y}{\partial s_i})^{-1}$, so $s_i = \frac{\sum_j e_j \frac{\partial y_j}{\partial s_i}}{\frac{\partial y_i}{\partial s_i}}$.

When there are variable administrative costs:

$$\max_{\mathbf{s}} b(\mathbf{y}(\mathbf{s})) - \mathbf{e}^\top \mathbf{y}(\mathbf{s}) - C(\mathbf{y}(\mathbf{s}), \mathbf{s})$$

$$b'(\mathbf{y}(\mathbf{s}))\mathbf{y}'(\mathbf{s}) - \mathbf{e}^\top \mathbf{y}'(\mathbf{s}) - C_1(\mathbf{y}(\mathbf{s}), \mathbf{s})\mathbf{y}'(\mathbf{s}) - C_2(\mathbf{y}(\mathbf{s}), \mathbf{s}) = 0$$

$$\mathbf{s}\mathbf{y}'_*(s)) - \mathbf{e}^\top \mathbf{y}'(\mathbf{s}) - C_1(\mathbf{y}(\mathbf{s}), \mathbf{s})\mathbf{y}'(\mathbf{s}) - C_2(\mathbf{y}(\mathbf{s}), \mathbf{s}) = 0$$

$$\mathbf{s} = \left[\mathbf{e}^\top \mathbf{y}'(\mathbf{s}) + C_1(\mathbf{y}(\mathbf{s}), \mathbf{s})\mathbf{y}'(\mathbf{s}) + C_2(\mathbf{y}(\mathbf{s}), \mathbf{s})\right] \mathbf{y}'_*(s))^{-1}$$

The special case of $n = m$ corresponds to the result for variable administrative costs above.

1.4 Taxing Output or Activities

When activities are impossible to measure or a tax on those activities incurs a sufficiently large fixed cost, the policy which maximizes social welfare will not tax those activities. Output may be particularly inexpensive to measure because of the record keeping associated with market transactions.

In many elementary economic textbooks, the tax used to correct an externality is a constant per unit tax on output (or consumption). Under the simplifying assumptions that there is only one pollution-creating activity and that production is a function of this activity, this is the correct Pigouvian tax. An output tax is also the correct Pigouvian tax in a variety of special cases. For example, when all the pollution-creating production activities are perfect complements in production. In that case, because some set proportion of the activities is required to increase output, the optimal tax can be set considering only the marginal externality associated with a marginal change in output.

If there are multiple externality producing activities which are chosen by the firm an output tax will generally be unable to induce the factory to select the least socially costly activities to produce a given level of output. Consider, for example, a production process that requires activity A and either activity B or C. Assume B is privately cheaper (by an arbitrarily small amount) but also has a larger externality. An output tax will not discourage activity B, but an activity tax on activity B will. In this particular case, since B is only barely cheaper than C and since they are perfect substitutes, an infinite tax (or really any tax that makes C preferable to B) on activity B will be optimal.

While these examples are insightful, they are limited. The remainder of this section is devoted to formalizing this above intuition.

1.4.1 Model

A factory produces output using n different activities, represented by an n-dimensional vector, $\mathbf{x}$. $q(\mathbf{x})$ maps that vector of activities to the maximum quantity of output that can be produced. $b(x)$ is the total surplus from the private market (i.e. consumer surplus + producer surplus + tax revenue) for this good. Social harm from production is a linear function of the activities, $\mathbf{e}^\top \mathbf{x}$.

1.4.2 Optimal Output Tax

As described above, the canonical Pigouvian tax is (or can be thought of as) a per unit tax on output. When there are multiple activities, this tax may be optimized as follows, beginning with the private market's problem:

$$\max_{\mathbf{x}} b(\mathbf{x}) - sq(\mathbf{x})$$

First order condition:

$$b'(\mathbf{z}(s)) - sq'(\mathbf{z}(s)) = 0$$

where $\mathbf{z}(s)$ maps the tax rate to the optimal vector of activities. The planner's problem is then:

$$\max_{s} b(\mathbf{z}(s)) - \mathbf{e}^\top \mathbf{z}(s)$$

With first order condition:

$$b'(\mathbf{z}(s))\mathbf{z}'(s) - \mathbf{e}^\top \mathbf{z}'(s) = 0$$

which leads to the classic Pigouvian tax result, that the increase in tax burden associated with a marginal increase in the choice variable being taxed, equals the increase in the externality burden of a marginal increase in the choice variable:

$$sq'(\mathbf{z}(s))\mathbf{z}'(s) = \mathbf{e}^\top \mathbf{z}'(s)$$

which we rewrite to define the optimal production tax:

$$s = \frac{\mathbf{e}^\top \mathbf{z}'(s)}{q'(\mathbf{z}(s))\mathbf{z}'(s)}$$

Intuitively, higher externalities associated with the activities which vary most with changes in the output tax will tend to increase the tax while higher marginal production associated with the activities which vary most with changes in the output tax will tend to decrease the tax.

1.4.3 Comparing Two Tax Policies

In order to make a welfare comparison between the output tax and various taxes levied on activities, we will convert the output tax into an equivalent tax on activities.

We can also find the vector of activity taxes that will induce the same level of activities and thus the same level of social welfare as the optimal output tax. In essence we want to find $\mathbf{t}$ such that $\mathbf{y}(\mathbf{t}) = \mathbf{z}(s)$. Recall that $b'(\mathbf{y}(\mathbf{t})) = \mathbf{t}^\top$, so:

$$b'(\mathbf{z}(s)) - sq'(\mathbf{z}(s)) = 0 \implies \mathbf{t}^\top - sq'(\mathbf{z}(s)) = 0$$

thus:

$$\mathbf{t}^\top = \frac{\mathbf{e}^\top \mathbf{z}'(s)}{q'(\mathbf{z}(s))\mathbf{z}'(s)} q'(\mathbf{z}(s))$$

rewriting this result using summations may make this result more transparent:

$$t_i = \frac{\sum_j e_j \frac{\partial z_j}{\partial s}}{\sum_j \frac{\partial q}{\partial z_j} \frac{\partial z_j}{\partial s}} \frac{\partial q}{\partial z_i}$$

Note that the higher the marginal product of an activity the higher the tax on that activity. This relationship makes sense because with a tax on output, an increase in an activity causes an increase the tax burden that is proportional to the activity's marginal product.

We now have two tax vectors, the output optimum and the activity optimum, that we would like to compare. Note that the welfare change from moving from one tax vector to another is the is the sum of the change in the private market surplus (DWL) and the change in the harm from the externality. The change in the harm is not difficult to compute because the harm is a linear function of the change in the activity vectors. The change in the DWL is somewhat more involved.

Consider two arbitrary tax vectors, $\mathbf{u}$ and $\mathbf{v}$. We can find the DWL of moving from $\mathbf{u}$ and $\mathbf{v}$:

$$\Delta \text{DWL} = b(\mathbf{y}(\mathbf{v})) - b(\mathbf{y}(\mathbf{u})) =$$

$$b(\mathbf{y}(\gamma(1))) - b(\mathbf{y}(\gamma(0))) =$$

where $\gamma : \mathbb{R} \to \mathbb{R}^n,\ \gamma(r) = \mathbf{u} + r(\mathbf{v} - \mathbf{u})$

$$\int_0^1 b'(\mathbf{y}(\gamma(r)))\ \mathbf{y}'(\gamma(r))\ \gamma'(r)\ dr =$$

by the fundamental theorem of calculus

$$\int_0^1 \gamma(r)^\top\ \mathbf{y}'(\gamma(r))\ \gamma'(r)\ dr =$$

integrating by parts:

$$\gamma^\top(r)\ \mathbf{y}(\gamma(r))\Big|_{r=0}^{1} \ - \ \int_0^1 \gamma'(r)^\top\ \mathbf{y}(\gamma(r))\ dr =$$

$$\mathbf{v}^\top \mathbf{y}(\mathbf{v}) - \ \mathbf{u}^\top \mathbf{y}(\mathbf{u}) \ - \ (\mathbf{v}-\mathbf{u})^\top \ \int_0^1 \mathbf{y}(\gamma(r))\ dr =$$

Assuming $\int_0^1 \mathbf{y}(\gamma(r))\ dr \approx \frac{1}{2}(\mathbf{y}(\mathbf{v})+\mathbf{y}(\mathbf{u}))$, which is analogous to Harberger's first-order approximation, we have:

$$\mathbf{v}^\top \mathbf{y}(\mathbf{v}) - \ \mathbf{u}^\top \mathbf{y}(\mathbf{u}) \ - \ \frac{1}{2}(\mathbf{v}-\mathbf{u})^\top(\mathbf{y}(\mathbf{v})+\mathbf{y}(\mathbf{u})) =$$

$$\frac{1}{2}\left[(\mathbf{v}+\mathbf{u})^\top(\mathbf{y}(\mathbf{v})-\mathbf{y}(\mathbf{u}))\right]$$

As an aside, this is a generalization of the Harberger triangle to a setting with more than one taxed good and an initial set of taxes (when $\mathbf{u} \neq \mathbf{0}$). This equation corresponds to the Harberger 'trapezoid' associated with a change from an existing set of taxes to another set of taxes. Setting $\mathbf{u} = \mathbf{0}$ simplifies this equation to the Harberger 'triangle' associated with a change from no taxes to a new set of taxes, in a setting with more than one good and more than one tax:

$$\frac{1}{2}\mathbf{v}^\top(\mathbf{y}(\mathbf{v})-\mathbf{y}(\mathbf{0}))$$

If we add in the externalities, then moving from $\mathbf{u}$ and $\mathbf{v}$ causes a change in total welfare:

$$\frac{1}{2}\left[(\mathbf{v}+\mathbf{u})^\top(\mathbf{y}(\mathbf{v})-\mathbf{y}(\mathbf{u}))\right] - \mathbf{e}^\top\mathbf{y}(\mathbf{v}) + \mathbf{e}^\top\mathbf{y}(\mathbf{u}) =$$

$$\frac{1}{2}\left[(\mathbf{v}+\mathbf{u}-\mathbf{2e})^\top(\mathbf{y}(\mathbf{v})-\mathbf{y}(\mathbf{u}))\right]$$

1.4.4 Comparing the Output and Activity Taxes

Recall that the optimal tax on activities is $\mathbf{t} = \mathbf{e}$ and the optimal tax on output is $\mathbf{t}^\top = \frac{\mathbf{e}^\top\mathbf{z}'(s)}{q'(\mathbf{z}(s))\mathbf{z}'(s)}q'(\mathbf{z}(s))$. Using the comparison formula derived above, substituting $\mathbf{e}$ for the optimal activity tax and representing the optimal tax on output with $\mathbf{t}$:

$$\frac{1}{2}\left[(\mathbf{e}+\mathbf{t}-\mathbf{2e})^\top(\mathbf{y}(\mathbf{e})-\mathbf{y}(\mathbf{t}))\right] =$$

$$\frac{1}{2}\left[(\mathbf{t}-\mathbf{e})^\top(\mathbf{y}(\mathbf{e})-\mathbf{y}(\mathbf{t}))\right]$$

This is the Harberger triangle approximation of the welfare lost by limiting the tax to a tax on output. At the optimal activity tax there are no uncorrected externalities. At the optimal product tax the uncorrected externality is $(\mathbf{t}-\mathbf{e})$. The uncorrected externality for each activity equals the difference between the social value and the private value of one unit of each activity. Because of the linear approximation, the average uncorrected externality for each unit of activity is half of the uncorrected externality under the product tax, $\frac{1}{2}(\mathbf{t}-\mathbf{e})$. Multiplying the uncorrected externality by the 'excess' amount of the activity that occurs under the product tax, $(\mathbf{y}(\mathbf{e})-\mathbf{y}(\mathbf{t}))$ yields the total welfare loss.

We can write the optimal product tax in terms of more primitive elements of the model and let $\mathbf{y}(\mathbf{e})-\mathbf{y}(\mathbf{t})=\mathbf{w}$:

$$\frac{1}{2}\left[\left(\frac{\mathbf{e}^\top\mathbf{z}'(s)}{q'(\mathbf{z}(s))\mathbf{z}'(s)}q'(\mathbf{z}(s))-\mathbf{e}\right)^\top\mathbf{w}\right]=$$

$$\frac{1}{2}\left[\frac{(\mathbf{e}^\top\mathbf{z}'(s))(q'(\mathbf{z}(s))\mathbf{w})}{q'(\mathbf{z}(s))\mathbf{z}'(s)}-\mathbf{e}^\top\mathbf{w}\right]$$

which we can also rewrite entirely in term's of the $\mathbf{y}$ function:

$$\frac{1}{2}\left[\frac{(\mathbf{e}^\top\mathbf{y}'(\mathbf{t})\mathbf{t}'(s))(q'(\mathbf{y}(\mathbf{t}))\mathbf{w})}{q'(\mathbf{y}(\mathbf{t}))\mathbf{y}'(\mathbf{t})\mathbf{t}'(s)}-\mathbf{e}^\top\mathbf{w}\right]$$

Because the optimal activity tax weakly dominates the optimal output tax, this quantity is non-negative. It follows that uniformly increasing $\mathbf{e}$ and $\mathbf{w}$ will make this quantity larger. Increasing only some elements will have an ambiguous effect.

A pigouvian tax on either tax base will tend to decrease output and externalities associated with activities, but comparing output and activitiy taxes distinguishes the way in which taxing these bases affects the mix of activities and the associated externalitites. First, if it is possible to achieve the same output with different combinations of inputs or processes, taxing can encourage one set of processes over another. An activity tax is necessary to affect choice in this dimension. Second, if the optimal production process does not scale (i.e. producing a large quantity requires different inputs than producing a small quantity), then taxes can change the processes used by encouraging large or small scale production. An output tax can cause change along this dimension without an activity tax.

1.5 Conclusion

There are many reasons to believe that administering a tax on a good which is exchanged between parties, such as a car, is simpler than administering a tax on a pollutant byproduct of car production. Especially in developed economies, businesses have their own reasons for tracking the number of cars sold, while the same company is unlikely to track sulfur dioxide production for any business purpose. Naturally, tax systems prioritize excise taxes, sales taxes, income taxes, and the like, to reduce the administrative and compliance burdens, as well as evasion. When considering a Pigouvian tax, it is easy to think of taxes on similarly observable tax bases and it is often presented in this way. In many cases a tax on the observable tax base is actually a problem of the 'second best' which fails to fully correct the externalities.

Considering Pigouvian taxation in this 'second best' framework leads to several important conclusions. First, when taxing an observable tax base, such as cars, rather than taxing the choice variables associated with the externalities, the loss in welfare relative to the optimal tax levied directly on each choice variable can be quantified in terms of the taxes and observed behavior. Second, when any of the externality causing activities are not subject to taxation, the optimal tax rate associated with each activity no longer equals the marginal harm of that activity. In contrast to prior literature, each tax rate depends on the externality of all other activities and the response of all other activities to each tax rate. Third, even when all of the externality causing activities are taxed, the existence of administrative cost in the tax system changes the optimal tax rates in complex ways. Although certain kinds of administrative cost can be an additive term in the expression for the tax rate, in general administrative costs cause each tax rate to be a function of all of the externalities and the response of all of activities to each tax rate.

Taken together, these three conclusions describe the changes in the optimal Pigouvian taxes which result from relaxing assumptions about how many activities are taxed, how directly they are measured and the administrative costs of the taxes. Despite the additional complexity of the problem, the welfare consequences can be approximated with tax rates and observed behavioral responses.

Only when taxes are a set of costless transfers, and only when a complete set of taxes is applied directly to all the choice variables which cause externalities, is each Pigouvian tax equal to the marginal external harm of the taxed activity, without regard to the effect on other externality generating activities. When there are limitations on the number or type of taxes or when there are administrative costs, Pigouvian taxes are much more complicated and depend on all of the agent's interrelated choices.

CHAPTER II

How to Catch Capone: The Optimal Punishment of Interrelated Crimes

From a work with Danny Schaffa

Abstract

This paper characterizes optimal criminal punishments when there are multiple interrelated crimes. Optimal punishments are functions of the extent to which related crimes are complements or substitutes weighted by their relative harms to society. This is a specific application of Pigouvian taxation with costly administration: in a second-best setting, the optimal Pigouvian tax is partly a function of spillovers to other externality-generating activities. The available empirical evidence on the relationship between index crimes in the United States suggests that tailoring criminal punishments properly to incorporate relationships between crime could reduce the aggregate harm to victims by 3%, or about $8 billion dollars annually, for a given level of enforcement resources. The actual harm reduction of a marginal increase in arrests for an index crime is about 1.5-3 times greater than the harm reduction calculated without these effects.

2.1 Introduction

Crime places a heavy burden on society. A partial list of costs includes the direct harm from crime, decreased property values, policing, arresting, trying, convicting, sentencing, and incarcerating criminals, the lost contributions of those incarcerated, and the onus of wrongful convictions. Governments in the United States spent more than $260 billion in 2010 to administer criminal justice,[1] and Anderson (1999) estimates the total burden of crime to be $1.7 trillion. Sentencing crimes optimally should decrease these burdens.

Most crimes are not committed in isolation. Al Capone headed a criminal organization that profited from bootlegging, prostitution, racketeering, and murder. He was tried and served a prison term, but his convictions were not for any of these crimes. He served time for tax evasion and contempt of court. In Capone's case, the rules underlying the criminal justice system were certainly stretched (and possibly broken) to ensure that he ended up behind bars. Nonetheless his experience illustrates that (1) criminals often undertake many criminal actions; (2) the returns to these actions are a function of other criminal acts; (3) these actions vary in their social harm; and (4) these actions also vary in how costly they are to detect and punish.[2] Thanks to the relationship between Capone's more socially harmful crimes and tax evasion, it was probably optimal to convict him for tax evasion and give him the most severe sentence possible. More generally, basing part of the enforcement of crimes on related criminal activities can reduce the social burden of criminal activities.

A criminal may commit several different crimes contemporaneously, the same crime serially, or different crimes non-contemporaneously. This is not random chance; in many cases criminals can increase the profit of a crime by committing other crimes in conjunction or in series. Previous work has touched on this, suggesting for example that economies of scale or learning might increase the payoff from serial repetition (Aizer and Doyle, 2015). While both of these models suggest specialization, surveys of prisoners and records of criminals re-arrested for different crimes demonstrate that many criminals are generalists (Beck and Shipley, 1989). Empirical work supports this claim, finding that changing the punishment for one crime may have large and significant effects on the commission of other crimes (Levitt, 1998*b*; Kuziemko and Levitt, 2004; Benson, Rasmussen and Kim, 1998; Shepherd, 2002).

[1] U.S. Bureau of Justice Statistics.

[2] A cost may be monetary but need not be. It may have been possible to convict Capone for his more socially harmful crimes, but doing so would have required an infringement on his rights that was socially too costly to bear.

If increasing the punishment of a crime leads to decreased commission of other crimes, then there is additional motivation to increase that punishment. If increasing the punishment of a crime leads to increased commission of other crimes, then there is cause to doubt that the punishment should be increased. Taking these cross-crime effects into consideration could reduce the total burden of crime. Our preliminary estimates suggest that reallocating existing enforcement resources could reduce the harm to victims of index crimes by about \$8 billion annually, which is about 3% of the total annual harm to victims of index crimes.[3]

Our paper analyzes interrelated crimes. However, our model's implications extend beyond criminal justice in two important ways. First, there is no need for the actions analyzed to be socially harmful acts. Second, there is no need for the punishment to be administered through the criminal justice system.

We illustrate the first point with the example of money laundering. In isolation, money laundering is not socially harmful–the launderer is simply obscuring her source of income. The benefit of obscuring the source of income is high when that income is criminally derived or deployed, and this is why money laundering is criminalized. Stated differently, the punishment of money laundering is aimed at decreasing the commission of other acts, not money laundering itself. Similar logic applies to many other activities including many possessory and conspiratorial crimes.

The second extension of the model is to sanctions outside of the criminal justice system. The two most obvious bodies under which sanctions are issued are civil law courts and government regulatory agencies. Tort law affects the propensity to commit litigable acts, but it surely encourages and discourages other acts too. Government regulation, for example through corrective taxation, can be analyzed in the same model. Chapter 1 of this dissertation analyzes a related model which can be applied to a variety of other settings such as the control of pollution, taxation of unhealthy foods, or subsidies for public health projects. This approach has purchase even outside of government structures–parents may set curfews (and corresponding punishments) for several reasons and likely not only because they object to their children being out late. Curfews may increase children's sleep and homework completed or decrease their drug-use and hooliganism.

[3] The FBI designates certain crimes as index crimes and uses these crimes to produce its annual report. These crimes have a standardized definition across states. In this paper, we consider the following index crimes: murder and non-negligent manslaughter; forcible rape; robbery, assault; burglary; larceny; and auto-theft.

Legislatures, judiciaries, and prosecutors already informally take into consideration crime interdependence. Famously, Al Capone was convicted of tax evasion because enforcement agencies were unable to convict him for any of his other, more socially harmful, criminal acts. When statutory punishments are enacted, legislative records suggest that interrelation is sometimes taken into consideration. For example, legislators have made claims about the effect of illegal immigration on criminal activity in support of a particular sanction on illegal immigration, and similar arguments are made concerning illegal drug use. The legislators' analysis underlying these claims, however, may not be rigorous. We aim to provide machinery to improve this analysis.

In the following section we present a model of criminal activity and characterize the social planner's optimal policy response. Sections 2.2.2 and 2.2.3 explain the difference between this result and other analyses of criminal activity. Section 2.2.4 relates this result to an abstract Pigouvian tax setting. Section 2.3 relates insights about the optimal policy to observations of the criminal justice system. Section 2.4 discusses prior estimates of the own-price elasticity of crime and the few studies which attempt to measure some type of cross price elasticity. Section 2.4.1 through 2.4.4 convert existing estimates into a welfare measure of the relative importance of our paper's contribution. Section 2.5 concludes.

2.2 A Model of Interrelated Crimes

Our model considers individuals who allocate their time to various activities under a temporal budget constraint.[4] These activities generate utility either directly or by generating income that can be used for consumption thus indirectly increasing utility. Some activities are socially harmful and some are socially harmless.[5] We call the socially harmful activities crimes. This is in contrast to the common definition of crime as an activity which the law criminalizes–a definition we cannot use since punishment is endogenously determined.

A social planner may choose to punish activities. A punishment is an additional cost, borne by the criminal, of committing a crime. An obvious intuitive interpretation of punishment is an additional expected time cost of committing a crime, equal to the probability of conviction multiplied by the length of prison sentence or community service for an activity. Formally, these are the punishments in the model, but the insights are valid for other sanctions. For example, rehabilitation, fines, or a job training program could each be prescribed

[4] Heineke (1978); Ehrlich (1973) also uses this approach to individual behavior

[5] A more general version of the model would allow socially beneficial activities and describe the optimal policies governing them. The intuition of such a model is the same, but some activities would be subsidized.

responses to conviction. We model the behavior of the individual as responding to a time cost, but the model could be extended to explicitly analyze more complex inputs into the individual's decision making process.

2.2.1 Representative Agent Model

Becker (1968) models the decision to commit crime as the result of the maximization of a Von Neuman Morgenstern utility function $EU = pU(Y - f) + (1 - p)U(Y)$.[6] Where Y is the wealth of the individual, inclusive of psychic gain, f is the fine, and p is the probability of punishment. Becker's contribution focuses on the insight that criminals respond to incentives in the form of the probability p and severity f of punishment.

According to Becker's famous result, high sanctions that completely deter are costless and optimal. As previous research has shown, this result does not hold when complete deterrence is infeasible. Since some crime will occur, high or maximal sanctions are not optimal because they must still be administered. This result also does not hold if threatening sanctions is costly. The costliness of threatened sanctions is a realistic assumption for a variety of reasons, the most obvious being that both criminals and the criminal justice system are not completely deterministic. Mistakes, a lack of information, or criminal behavior that is not influenced by sanctions could all result in the threats being carried out, even if only infrequently. Even if complete deterrence of some acts is feasible, severe punishments may have additional costs to administer or monitor. Crimes that have severe punishments, and crimes that might be mistaken for a crime with a severe punishment (e.g. manslaughter may be difficult to distinguish from murder), should be more carefully investigated and more intensely litigated. As long as threats themselves have a potential cost, complete deterrence of offenders who are responsive to sanctions may be possible but not optimal.

We extend Becker's work by modeling an individual who chooses how much of each of n different activities to engage in. The individual's behavior will be described by demand functions. If one simply aggregated heterogeneous individual's demand functions then the calculations below do not account for the distributional consequences of policy changes. If individuals are the same, or if there is redistribution in the background so that the value of a dollar is the same to all individuals, then these demand functions can be interpreted

[6] In a footnote, Becker recognizes that the payoffs for a given crime may depend on other aspects of the criminal justice system, particularly the punishments for other crimes because of substitution, but his analysis considers only one crime.

as aggregate demand and minimizing the dollar value of crime and enforcement on society determines the optimal policy from a social welfare perspective. When discussing aggregates, we have this, admittedly strong, assumption in mind.

The individual is restricted by a temporal budget constraint with total time T. The activities are indexed $\{1, 2, ...n\}$. The individual is a utility maximizer with preferences represented by a strictly quasi-concave, differentiable utility function. For each activity, j, she faces a time cost of t_j and must decide what quantity of activity, x_j, to undertake. In addition to the time cost t_j, the government may impose sanctions for some of the activities. Following Becker, the punishments chosen by the government are described by two parameters for each activity j. s_j is the sanction per unit of activity j that the government detects, and p_j is the probability of detection for activity j. The individual is risk neutral and thus responds only to changes in the expected time cost, including punishment, of each activity.[7] The individual's problem can be written:

$$\max_{x_j \geq 0} U(x_1, ..., x_n) \text{ subject to } \sum_j (p_j s_j + t_j) x_j \leq T$$

The solution to the individual's problem can be characterized by a set of demand functions $\mathbf{x}^* = \{x_1^*, x_2^*, ...x_n^*\}'$. Each demand function depends on the total time the individual has and the time prices she faces: $x_j^*(\sigma_1, ..., \sigma_n, T)$ where $\sigma_k = p_k s_k + t_k$. We omit T from the demand function because we assume that it is constant.

The government, too, must solve an optimization problem. Its problem is to select a set of $2n$ parameters such that total social cost is minimized. The government faces three varieties of cost that combine to generate a cost function, C: (1) the direct cost of crime; (2) the cost associated with the vector of sanctions; and (3) the cost associated with detection. C is increasing in all of its arguments. The government's problem is:[8]

[7] Because we model a representative agent and mean sanctions, the representative activity choices will not violate the budget constraint in expectation. Any given individual may violate her budget constraint depending on how often her criminal activity is detected. This is analogous to models of lifetime consumption decisions under uncertainty, in which a person could die with unspent wealth or unpaid debts.

[8] An analogous problem exists using Hicksian demand functions. For a finite population of individuals, each with Hicksian demand functions $\mathbf{h}_i(\sigma, u)$ and a level of utility z_i for each individual i it is easy to define an aggregate function $\mathbf{f}(\sigma) = \Sigma_i \mathbf{h}_i(\sigma, z_i)$. The government solves $\min_{p_j, s_j \geq 0} C(\mathbf{f}(\sigma_1, ..., \sigma_n), \mathbf{p}, \mathbf{s})$, which, when $\mathbf{f}(\sigma)$ is continuous and differentiable, results in first order conditions $0 = \frac{\partial C}{\partial p_k} + \sum_j \frac{\partial C}{\partial f_j} \frac{\partial f_j}{\partial \sigma_k} \frac{\partial \sigma_k}{\partial p_k}$ and $0 = \frac{\partial C}{\partial s_k} + \sum_j \frac{\partial C}{\partial f_j} \frac{\partial f_j}{\partial \sigma_k} \frac{\partial \sigma_k}{\partial s_k}$. With appropriate second order conditions, these equations define the sanctions which minimize the cost of crime at the given levels of utility for each individual. These sanctions are a pareto improvement over any other sanction for the given levels of utility.

$$\min_{p_j, s_j \geq 0} C(\mathbf{x}^*(\sigma_1, ..., \sigma_n), \mathbf{p}, \mathbf{s})$$

We assume that the cost function has second order conditions such that the local optimum is the unique global optimum. The government's optimal policy is fully characterized by the first order conditions. The first order conditions are:[9]

$$\begin{aligned} 0 =& \frac{\partial C}{\partial p_k} + \sum_j \frac{\partial C}{\partial x_j^*} \frac{\partial x_j^*}{\partial \sigma_k} \frac{\partial \sigma_k}{\partial p_k} \\ 0 =& \frac{\partial C}{\partial s_k} + \sum_j \frac{\partial C}{\partial x_j^*} \frac{\partial x_j^*}{\partial \sigma_k} \frac{\partial \sigma_k}{\partial s_k} \end{aligned}$$

The first order conditions carry the intuition that the 'cost' of increasing s_k or p_k must equal the 'benefit' which is the fall in costs due to the decrease in all crime. The conditions can be used to solve for the optimal s_k and p_k as follows:[10]

$$\begin{aligned} s_k &= -\frac{\frac{\partial C}{\partial p_k}}{\sum_j \frac{\partial C}{\partial x_j^*} \frac{\partial x_j^*}{\partial \sigma_k}} \\ p_k &= -\frac{\frac{\partial C}{\partial s_k}}{\sum_j \frac{\partial C}{\partial x_j^*} \frac{\partial x_j^*}{\partial \sigma_k}} \end{aligned}$$

We define the elasticity of a crime rate with respect to enforcement against that crime as an *own price elasticity* and the elasticity of a crime with respect to another crime's enforcement as a *cross price elasticity*.

[9] We write $\frac{\partial C}{\partial p_k}$ to denote the partial derivative of C with respect to the second argument; we write $\frac{\partial C}{\partial s_k}$ to denote the partial derivative of C with respect to the third argument.

[10] This holds if the solution is interior. Otherwise:

$$\begin{aligned} s_k &= \max\left\{0, -\frac{\frac{\partial C}{\partial p_k}}{\sum_j \frac{\partial C}{\partial x_j^*} \frac{\partial x_j^*}{\partial \sigma_k}}\right\} \\ p_k &= \max\left\{0, -\frac{\frac{\partial C}{\partial s_k}}{\sum_j \frac{\partial C}{\partial x_j^*} \frac{\partial x_j^*}{\partial \sigma_k}}\right\} \end{aligned}$$

2.2.2 Complements and Substitutes

We can distinguish activities that are complements, $\frac{\partial x_j^*}{\partial \sigma_k} < 0$, and substitutes, $\frac{\partial x_j^*}{\partial \sigma_k} > 0$, with activity k and separate the above summand. We use the notation $\sum_+$ to refer to the sum over all activities j that are gross substitutes with activity k, that is $\sum_+ \frac{\partial C}{\partial x_j^*}\frac{\partial x_j^*}{\partial \sigma_k} = \sum_{j \text{ s.t. } \frac{\partial x_j^*}{\partial \sigma_k}>0} \frac{\partial C}{\partial x_j^*}\frac{\partial x_j^*}{\partial \sigma_k}$. The notation $\sum_-$ refers to the analogous sum over all of the activities that are gross complements with activity k.

$$\sum_j \frac{\partial C}{\partial x_j^*}\frac{\partial x_j^*}{\partial \sigma_k} = \sum_+ \frac{\partial C}{\partial x_j^*}\frac{\partial x_j^*}{\partial \sigma_k} + \sum_- \frac{\partial C}{\partial x_j^*}\frac{\partial x_j^*}{\partial \sigma_k} + \frac{\partial C}{\partial x_k^*}\frac{\partial x_k^*}{\partial \sigma_k}$$

Activity k should be punished whenever:

$$\sum_- \frac{\partial C}{\partial x_j^*}\frac{\partial x_j^*}{\partial \sigma_k} + \frac{\partial C}{\partial x_k^*}\frac{\partial x_k^*}{\partial \sigma_k} < -\sum_+ \frac{\partial C}{\partial x_j^*}\frac{\partial x_j^*}{\partial \sigma_k}$$

even if k has no social cost (i.e. $\frac{\partial C}{\partial x_k^*} \leq 0$). Similarly, the optimal punishments sometimes leave socially harmful activities unpunished. Activity k should go unpunished whenever:

$$\sum_- \frac{\partial C}{\partial x_j^*}\frac{\partial x_j^*}{\partial \sigma_k} + \frac{\partial C}{\partial x_k^*}\frac{\partial x_k^*}{\partial \sigma_k} > -\sum_+ \frac{\partial C}{\partial x_j^*}\frac{\partial x_j^*}{\partial \sigma_k}$$

even if k is socially harmful (i.e. $\frac{\partial C}{\partial x_k^*} > 0$). In fact, when the model is modified to allow for subsidies, the optimal sanctions may include a subsidy for certain criminal activities.

2.2.3 Comparison to the Naive Social Planner

The difference between our model and prior literature is the harm weighted sum of the response of related crimes: $\sum_+ \frac{\partial C}{\partial x_j^*}\frac{\partial x_j^*}{\partial \sigma_k} + \sum_- \frac{\partial C}{\partial x_j^*}\frac{\partial x_j^*}{\partial \sigma_k}$. We refer to this term as the correction for related crimes. When this term is positive (negative), activity k is a harm weighted gross substitute (complement) to other activities. Consider the behavior of a policymaker who does not recognize that crimes are interrelated. This 'naive policymaker' and the associated 'naive punishments' do not correctly maximize the social welfare function. The optimal punishments are more (less) harsh than the naive punishment when an activity has a positive (negative) correction for related crimes.

2.2.4 Relationship to Pigouvian Taxation

These punishments bear some resemblance to a Pigouvian tax. We model incarceration as a time tax designed to disincentivize socially harmful behaviors. The classic Pigouvian taxation result states that the tax is equal to the marginal social harm at the optimal activity level. In our model the first-best activity levels are generally not optimal because administering punishment is costly (See Chapter 1, as well as Kaplow (1990); Polinsky and Shavell (1982)). Within this second best environment the optimal punishments are partly a function of spillovers from other externality producing activities.

By assumption every crime has a net social cost. Thus it is not optimal to set punishments sufficiently high to achieve first best levels. Instead the optimal activity levels are higher than their first best alternatives. Any optimal 'tax' in a setting with second best activity levels will be based on a weighted sum of other activities' cross price elasticities and the externalities of those activities.

The features of our model are prominent features of the criminal justice system but are also present in any environment in which there is regulation of externality generating activities. For example, administrative cost, measurement error, and political power will likely make it suboptimal to induce the first-best activity levels for polluting activities. If this is the case, regulations and taxes should be set taking into account the spillover effects to other externality causing activities.

2.3 Applications of the Model

As mentioned in the introduction, legislatures are aware that crimes are interrelated.

"Simply put, prescription drug abuse is one of the fastest growing drug problems in the nation, resulting in ever increasing rates of robberies and other attendant crimes." - US Senator Sherrod Brown advocating for enforcement against theft of prescription drugs

"Illegal aliens commit horrendous crimes against American citizens, crimes that strain State and Federal judicial systems, police and sheriff departments, and prisons that are already overcrowded and in a financial crisis." - US Representative Mo Brooks discussing immigration reform

However, there is no evidence that interrelation is taken into consideration carefully or consistently. Although it is probably true that some immigrants commit other socially costly crimes, it is also likely that some undocumented immigrants avoid crimes with small payoffs because those crimes carry the additional risk of being caught for undocumented immigration. While punishing undocumented immigration may incapacitate some individuals who are committing a variety of crimes, strict enforcement against undocumented immigration will decrease the marginal deterrence of increased participation in other criminal activities. This is an application of marginal deterrence, originally discussed by Stigler (1970), to our model of multiple crimes. Whether enforcement of immigration law would increase undocumented immigrant's participation in criminal activities is ultimately an empirical question. For a discussion of the existing evidence, see Bell and Machin (2013).

More generally, both unpunished substitutes and harshly punished complements exist in the US criminal justice system. For example, drug possession by itself, as opposed to drug use, causes no social harm. Drug possession is punished only because it is a complement with other harmful activities. Similarly, the punishments for carrying a concealed weapon are best explained by the argument that carrying a weapon is related to using it in a harmful way. The controversy around concealed carry laws is at least in part a controversy about whether concealed carry increases or decreases other socially harmful activities. There are numerous studies on both sides of this issue. For example, Lott and Landes (2000) consider whether more relaxed carry laws might increase the chance that others have guns and thereby deter shootings. On the other hand Duggan (2001) links higher rates of gun ownership to additional gun homicides.

In many cases our model suggests a logic underlying existing punishment schemes. For example, if drugs tend to be substitutes, our model explains why it may be optimal to criminalize most drugs, while leaving a few drugs, such as alcohol and tobacco, uncriminalized. This policy is optimal if for relatively low enforcement cost it steers users towards a select few drugs. The more harmful a drug, the better a candidate it makes to be criminalized. The more inelastic its response to its own punishments, and the larger the cross price elasticities with costly activities, the better a candidate a drug makes to remain legal. Free needle exchanges take this reasoning one step further. If using a clean needle is a near perfect but less socially harmful substitute for sharing needles, then subsidizing a needle exchange is socially optimal.

Looking to medieval English law we find another interesting case. Pollock and Maitland

(1899) state that burglary could be excused if committed by a hungry man whose aim was to steal a small amount of food. A sufficiently hungry man's criminal acts to acquire food might be unresponsive to sanctions. The distinction between 'theft for food when hungry' and 'other theft' carves out an opportunity for the hungry man to commit a crime for food without society being forced to incur the costs of sanctioning the theft while retaining strong disincentives for other acts which cause more net social harm.

The examples in this section suggest that legislatures, judiciaries, and prosecutors already take into consideration crime interrelation. Nonetheless, some might find it odd that a person be punished for a socially harmless activity or that the punishment of one activity be based on the commission of another. This, however, is not as striking as it may initially seem. Note first that the model does not suggest a violation of due process. The criminal must perform an action to be sanctioned and a consistent sanction is applied to each punishable activity. Prior work by legal scholars discusses the significance of prosecutorial discretion when there is information about criminal activities other than the charged offense. Richman and Stuntz (2005) explain that such 'pretextual prosecution' is typically allowed under the argument that tax evasion, for example, is a legitimate crime and criminals should not be exempted from prosecution for tax evasion because they engage in more harmful illicit activities. Richman and Stuntz cite Wayte v. United States, 470 U.S. 598 (1985) as the leading case addressing this argument.

Second, the model does not require an expansion of punishment setting powers. Legislatures are free to map an activity to any sanction they like, subject to the constraint that the punishment not be cruel or unusual. If the sanction is a prison sentence and the criminal is a mentally competent adult, the legislature is given nearly free reign.

Third, the model accepts the utilitarian premise that punishment should be socially optimal. It follows that the effect of punishing activity A on the commission of crime B should be included in the optimal sanction. Outside of a utilitarian structure a punishment may be considered a moral construct that depends only on the act it punishes, but a utilitarian framework cannot accept an a priori punishment–fair, right, or just punishments make sense only insofar as they affect individuals' utility. It is inconsistent with utilitarianism to accept that a punishment should be designed to deter the crime that it punishes, but no other crime. The optimal utilitarian punishment must take into consideration all of its effects.

2.4 Empirical Evidence

A large literature attempts to relate policy variables, such as the death penalty, police force size, clearance rate, even gun ownership regulations and the existence of a state lottery, to crime rates. Some of these studies measure responsiveness in a way that can be interpreted as an own price elasticity. Very few studies measure anything like a cross price elasticity. Some studies use control variables which could proxy for a price of a related activity, but they do not systematically investigate the relationship. For example Mikesell and Pirog-Good (1990) examine the effect on crime rates of the existence of a state lottery. Corman and Mocan (2000) use Assistance for Families with Dependent Children as a control variable. One reason to include AFDC as a control variable is because it can proxy for the labor market opportunities of the individual. In this sense it is related to the 'price' of work in the mainstream labor market.

Kuziemko and Levitt (2004) calculate the the effect on the aggregate crime rate of imprisoning drug, property, or violent offenders. The effect of switching one type of offender for another, at the margin, is not statistically distinct from zero. This is interpreted as suggestive evidence that some agents in the criminal justice system are making adjustments in order to minimize the total crime rate subject to the constraint of a fixed prison system size. Minimizing the aggregate crime rate is unlikely to coincide with optimal policy. As described earlier in the paper, the optimal policy does not treat each type of crime as equivalently costly.

One challenge in measuring cross price elasticities in the United States is the discretion of various agents in the criminal justice system. It is well documented that prosecutors use some discretion in choosing who to prosecute, what crimes to charge and what plea deals to offer. Judges also can differ significantly in the sentences they assign to all crimes, to different crimes or to different individuals. This discretion could bias the estimates of cross price elasticities, but will not necessarily do so. Using the response of crime rates to observed punishments as in Kuziemko and Levitt, rather than statutory punishments, will minimize the bias resulting from discretion on the part of agents in the criminal justice system. The elasticity will then be the average marginal response of crime to a marginal change in the average observed punishment across the unit of observation, across counties for example.

The following two studies calculate cross price elasticities for a group of crimes. Hakim, Spiegel and Weinblatt (1984) calculate cross price elasticities for different property crimes using a set of simultaneous equations. Levitt (1998*b*) calculates the elasticities of index crimes with respect to arrest rates for the crime itself as well as the elasticity with respect to an aggregate of other violent crime arrest rates and an aggregate of other property crime arrest rates.

Table 1: Studies Which Calculate Cross Price Elasticity of Crime Rates

Study	Crimes studied	'Prices'	Result
Levitt (1998*b*)	Index crimes	Arrest rates, arrest rates for groups of other index crimes	Cross price elasticities negative and about half the magnitude of own price elasticities; negative elasticities $\in [-0.012, -0.298]$, robbery $+0.062$ with respect to pecuniary crime arrest rate; larceny $+0.048$ with respect to violent crime arrest rate
Hakim, Spiegel and Weinblatt (1984)	Auto theft; burglary; larceny; robbery	Clearance rates	Negative own price elasticities $\in [-0.171, -0.891]$; most cross price elasticities are positive $\in [-0.329, 1.444]$

The cross price elasticities calculated by Levitt (1998*b*) are about half as large as own price elasticities. An important question to ask is how significant these cross price elasticities are. A simple interpretation would be that cross price effects should account for about half of punishments. Below we detail more precise approaches to determining the importance of cross price elasticities.

First, we estimate the potential gains from reallocating existing police resources across crimes. The computational method uses existing data to find the arrest rates which minimize the aggregate harm to victims of index crimes while keeping enforcement costs constant. Our approach is motivated in part by the availability of information about the cross price effects of crime.

2.4.1 Sources of Data

We consider only index crimes and use data about the number of each type of offense and the number of arrests for each type of offense as reported in the FBI Uniform Crime Reports

(UCR). The elasticities of crime rates with respect to arrest rates come from Levitt (1998*b*), and the reported costs for each additional crime come from McCollister, French and Fang (2010). These costs include per crime cost to the criminal justice system, lost productivity of offenders, tangible costs to the victim and intangible costs to the victim. The estimates reported in McCollister are \$8.98 million per murder, \$241 thousand per rape, \$42 thousand per robbery, and \$107 thousand per assault. The harm for each burglary is \$6462. The harm per incident of larceny is \$3532 and the harm of auto theft is \$10772 per offense.

2.4.2 Computational Method

Levitt (1998*b*) reports elasticities of index crimes with respect to index crime arrest rates. We convert the elasticities reported by Levitt to an elasticity with respect to arrests. Arrests for crime i, a_i, can be interpreted as the 'price' of crime to the individual[11], and the reported number of each crime, x_i can be interpreted as a 'demand' for crime. The demand for each crime has an elasticity with respect to each category of arrest. Assuming that these elasticities are constant over the relevant range of arrests, these are the demand functions describing the amount of each crime that occurs for any given combination of arrests:

$$x_j(\mathbf{a}) = K_j \prod_i a_i^{\varepsilon_{ji}}$$

where K_j is a constant determined by the observed values of x_i and a_i. ε_{ij} is the price elasticity of demand for crime j with respect to price i.

We assume that the costs per offense, B_j, reported in McCollister, French and Fang (2010) are constant in the relevant region, so the total harm to victims is $\sum_j B_j x_j(\mathbf{a})$. We compute the cost for an arrest of each type of crime, and assuming that this cost A_i is constant in the relevant region, the problem of finding the arrest rates that minimize the harm to victims while holding enforcement costs constant is:

$$\min \sum_j B_j x_j(\mathbf{a}) \text{ subject to } \sum_i A_i a_i = \bar{A}$$

To compute the optimal arrest rates, we test combinations of arrests which satisfy the constraint $\sum_i A_i a_i = \bar{A}$ and are within a certain range of the 2010 arrest rates. We then use

[11] For these computations we ignore the length of sanctions.

the demand functions and per crime costs to compute the harm to victims associated with each of those combinations of arrests. These arrest rates map to crime levels which then map to total victim harm. We record the combination of arrests that yield the lowest total victim harm and then repeat the computation for arrests that are similar to that combination of arrests. This algorithm terminates when no tested vector of arrests lowers total victim harm.

2.4.3 Estimates of the Harm Reduction

The 2010 crime levels correspond to a total victim harm of approximately \$296 billion. Assuming that the current arrest rates are optimized using only own price elasticities (i.e. assuming the naive optimum) and keeping enforcement cost constant, using cross price elasticities reduces the harm to \$288 billion. This represents a 3% reduction in harm to victims by reallocating existing enforcement resources.

A similar exercise minimizes total victim harm allowing enforcement expenditures to vary. Because the cross price elasticities reported in Levitt are mostly negative, this approach causes enforcement expenditure to rise. The reduction of harm to victims is about \$34 billion but is offset by a \$19 billion increase in enforcement costs resulting in a net reduction in total cost of crime of \$14 billion. These values are associated with large changes in levels of enforcement. Assumptions about the constant marginal harm of crime, the constant marginal cost of enforcement, and the constant Slutsky matrix are less tenable for large changes. Nonetheless, this suggests that changing enforcement levels based on cross-crime elasticities can have significant benefits in addition to the benefits of reallocating existing enforcement resources.

These estimates are conservative. These estimates only account for the seven index crimes and one aspect of expenditure on criminal justice. The harm from non-index crimes, wrongful convictions and other social burdens are not factored into these estimates. Additionally, Levitt groups crimes when making his estimates and his econometric specification suffers from division bias. We expect both of these factors to artificially reduce the variance in responsiveness across activities. The welfare impact of reallocating existing resources is driven by the variation in cross price elasticities, relative to own price elasticities. There may be a larger effect if the elasticities were computed for each crime arrest pair.

2.4.4 Estimates of Marginal Effects

An alternative calculation of the significance of the cross price elasticities considers the effects of a small change from current enforcement levels. For example, a small increase in the enforcement against assault will decrease assaults. It will also affect other crime rates. The elasticities of each crime with respect to the arrest rate for each other crime allows us to calculate the change in the number of crimes that would occur in response to a change in each crime's arrest rate.

Using the UCR data and Levitt's elasticities we can calculate the change in the number of each index crime that would result from a 1% increase in the arrest rate for assault. Such an increase would lead to approximately 1306 fewer assaults according to Levitt (1998*b*).[12] Using Levitt's estimates the increase in assault arrest rates would decrease the number of murders by 14.3 and decrease the number of rapes by 6.6 but the change would increase the number of robberies by 213. The changes in property crimes would include 193 fewer burglaries, 1331 fewer larcenies and 301 fewer auto thefts.

Using per crime cost estimates from McCollister, French and Fang (2010), we find that the assault arrest rate change described above would reduce the harm to victims by about $270 million. The decrease in assaults alone would be valued at $140 million, while the decreases in the number of murders and rapes would correspond to a decreased burden of $129 million and $1.5 million. The increase in robberies would increase the harm to victims by $9.0 million. The decrease in burglaries larcenies and auto thefts would correspond to reduced harms valued at $1.2 million $4.7 million and $3.2 million.

The significant insight from this calculation is that approximately half of the harm reduction is due to a reduction in crimes other than assault. In this example, the decline in murder represents a large fraction of the harm reduction from a marginal increase in arrests for assault. The actual benefits of raising the arrest rate for assault is almost 2 times as large as would be calculated by a policymaker who did not include the effect on other crimes. When we use a small negative value for the own price elasticity of murder, we find

[12] The cross price elasticities reported by Levitt are for groups of crimes. To use the example of murder and nonnegligent manslaughter, Levitt reports the percentage change in the number of murders associated with a 1% change in the 'arrest rate for violent index crimes excluding murder'. To calculate the change in the number of murders we first calculate the change in the 'arrest rate for violent index crimes excluding murder' which occurs when the arrest rate for assault rises 1%. In this example, the change in assault arrest rates causes the 'arrest rate for violent index crimes excluding murder' to rise by roughly 0.623%. Given the reported cross price elasticity of -0.129 and 14,722 murders in 2010, we calculate the decrease in murders reported above.

that cross crime effects only account for 10% of the marginal harm reduction. However, if we use Levitt's measured, positive own price elasticity for murder, then increasing the arrest rate for murder actually increases the net harm to victims. If increasing arrests for murder increases the number of murders, then the cross crime effects are the only gain from increased arrests for murder. Repeating the calculations for the other arrest rates we find that cross crime effects account for between 29% and 67% of the harm reduction at the margin. Refer to the table in appendix VI.3 for more details.

2.5 Conclusion

This paper generalizes prior work on criminal punishments by allowing multiple interrelated crimes. The insights extend beyond crime to any environment with spillovers between socially harmful or beneficial activities. When a friction makes first best activity levels suboptimal, the optimal policy intervention accounts for spillover effects of other externality generating activities. Crime is an excellent application since the most common sanction, incarceration, is sufficiently costly to ensure that the activities are not set to their first best levels.

The main intuitive contribution is that complementary (substitutable) crimes should be punished more (less) harshly than they would be if punishments were set for each crime in isolation. Moreover, making few assumptions, we provide a precise formulation for how to find the optimal punishments. The optimal punishment is proportional to the sum of the responsiveness of each crime with respect to that punishment, weighted by the measure of the costliness of that crime. A corollary of this insight is that social harm is neither sufficient nor necessary for an activity to be punished under the optimal criminal justice policy. The optimal punishments are based on the relationships with all activities, whether they are criminalized or not and whether they are harmful, beneficial, or benign. Some harmful activities should not be punished at all because of their substitutability, while other benign or beneficial activities should be punished because they are complements with harmful activities.

Applying this insight to criminal punishments promises significant benefits to society. For index crimes alone and using conservative assumptions, current enforcement spending could be reallocated to reduce the harm to victims by approximately $8 billion, or about 3% of the total harms to victims.

Other possible practivcal applications include the taxation of pollution, hiring based on big data (e.g. using zip code as a signal for future employee retention), taxation of unhealthy foods, subsidies for public health projects, and other principal agent problems with multiple activities.

CHAPTER III

Deterrence and Incapacitation

From a work with Danny Schaffa

Abstract

This paper carefully defines deterrence and incapacitation. Existing literature uses these terms inconsistently and does not consider how wealth effects change behavior. For incapacitation, it is important to know how wealth affects individual behavior when considering how much crime the individual would have committed if they were not incarcerated. For deterrence, it is important to correctly compensate for changes in the scarcity of the criminals time associated with changing sanctions. Deterrence is a substitution effect while incapacitation is an income effect. This definition can be applied in many different economic models.

3.1 Introduction

The study of crime admits four theories of punishment: deterrence, incapacitation, rehabilitation, and retribution. Under a deterrence theory crime is punished because doing so makes crime less attractive and thus reduces the amount of crime committed. Under an incapacitation theory crime is punished because doing so makes it impossible (or at least unlikely) for criminals to commit crime. Under a rehabilitation theory crime is punished because it lessens criminals' interest in committing crime. And under a retribution theory crime is punished because the criminal deserves it under some moral code.

Unsurprisingly, when legislators justify a statute codifying a punishment or jurists justify a punishment, they typically refer to more than one theory of punishment. Surprisingly, when economists study punishment–with few exceptions–they examine only one theory of punishment.

Of these four theories deterrence and incapacitation fit most neatly into a utilitarian framework and thus most easily submit to economic analysis. The economic literature on deterrence is prolific–its most modern incarnation can be traced to Becker (1968), but similar conceptions have been espoused by Beccaria, Bentham, Plato, and likely even earlier authors. Incapacitation is less studied but has a niche in the law and economics literature.

Our contribution to the literature is to model incapacitation and deterrence in the same framework and offer a clear definition of both, a definition which can be interpreted across the variety of models which exist in the literature. Two theoretical papers, Miceli (2010) and Shavell (2015), have considered deterrence and incapacitation in the same model. Miceli (2010) uses a model in which each individual makes a choice about whether to commit a crime and, eventually, suffer punishment. In Miceli's model, each individual either commits a crime at every opportunity or never commits a crime. Therefore deterrence occurs at the extensive margin, that is, when individuals with less preference for crime decide not to engage in crime. Incapacitation happens at the intensive margin, that is, when a criminal who commits crime at every opportunity, is incarcerated. The assumption that for any marginal incentives, the individual will always commit crime or commit no crime is an extreme assumption about individual behavior. Shavell (2015) uses a two period model. In Shavell's model, sentences can be set differently for different individuals, and there are no wealth effects, so changing the sentence for 'young' individuals in the first period does not change the incentives for 'old' individuals in the second period. Although age may not be a protected class for this purpose, and judicial discretion would allow for different sentencing for different individuals, in practice we think it is still important to be able to distinguish between the incapacitation and deterrence effects of a sanction that changes for all individuals at the same time. It is also important to be able to analyze the wealth effect on behavior.

We use a model, analogous to traditional consumer theory, in which an agent allocates time to activities subject to a temporal budget constraint. The main result is that when the criminal allocates their time among different activities, then the substitution effect of a change in punishment is the deterrent effect of that punishment, while the wealth effect of that change in punishment is the incapacitation effect. The compensated demand function of the criminal describes what the criminal would have done if the criminal was not incapacitated.

A clear definition of deterrence and incapacitation is important for three reasons. First, whether jurists or policymakers are seeking to define the punishments which maximize deterrence or incapacitation separately, or whether they seek to maximize social welfare, a clear definition of these effects will help determine which policies achieve their ends. Second, deterrence and incapacitation are relevant to the question, posed by Becker, of whether criminal behavior can be accurately described by models similar to those describing other economic behavior. Third, deterrence and incapacitation can be used to better understand the welfare impact of punishment. These contributions are discussed in more detail in section 3.3.3 after the model and definitions are explained.

One additional contribution of this paper is to demonstrate an application of substitution and income effects in a dynamic setting. The Slutsky decomposition is a prominent tool in consumer choice, but we are not aware of prior applications in a dynamic setting.

3.2 Defining Deterrence and Incapacitation

The deterrence effect is the change in the amount of crime committed because the potential criminal wishes to avoid a sanction. For intuition, consider cases in which the behavioral change is entirely driven by deterrence and not incapacitation. An imprisonment sanction so fearsome that it completely deters crime, regardless of how much time the individual has to engage in activities, has an effect on behavior entirely attributable to deterrence. The amount of crime deterred is simply the difference between the amount of crime the criminal chooses under the two different sanction regimes. The deterrence effect always reduces the amount of crime committed.

The incapacitation effect is the change in the amount of crime committed because the criminal has less time. Consider incarcerating all individuals for a fraction of their life independent of their conduct. We would expect to see a decrease in the aggregate level of activities and we might see a decrease in crime. Since the decrease in the time available to engage in activities does not depend on the individual's choices, it cannot have a deterrent effect. Any change in criminal activity is an incapacitation effect. In an extreme case, an individual who would not commit crimes in the absence of the incarceration might become a criminal with what free time they do have and spend additional time in prison when they are caught. The incapacitation effect for such an individual is positive.

In calculating the amount of deterrence and incapacitation, the key question is what actions the criminal would have chosen in a hypothetical situation. The counter factual choice problem wherein the individual is not incarcerated for a period of time while punishments, therefore marginal incentives, remain unchanged, is difficult to precisely describe since the marginal incentives depend on the individual actually being incarcerated. The very fact that the individual is not incarcerated for a particular period of time is likely to change their behavior later in life. This is both consistent with empirical observation (see especially Aizer and Doyle) and rational behavior under consumer choice theory. By considering the behavioral change over the lifetime of the individual, it becomes easy to define the hypothetical choice which precisely defines deterrence and incapacitation. The hypothetical choice which corresponds to the intuitive notions of deterrence and incapacitation is the criminal's Hicksian demand for activities.

The incapacitation effect is the income effect of a change in the expected sanction for an activity. The deterrent effect of a change in the expected sanction is the substitution effect of that change in sanction.

3.3 Lifetime Choice Model

We model the decision to commit crime as an activity consumption problem. This is the simplest way to define deterrence and incapacitation, but we will present a dynamic version in order to make comparisons with prior literature.

A risk neutral agent makes a single choice of how much of each activity, in expectation, to engage in over their entire lifetime. These activities generate utility either directly or by generating income that can be used for consumption c, thus indirectly increasing utility. The individual derives utility and income from N different activities x_i and is subject to a temporal budget constraint. We write the budget constraint in terms of the amount of time the individual is endowed with, T, and the expected amount of time it takes, inclusive of expected prison time, to engage in each activity. The expected amount of time it takes to engage in an activity i is the time-price σ_i which consists of the actual time required to engage in the activity t_i in addition to the expected prison sentence, which is a product of the probability of apprehension and the severity of sanction $p_i s_i$, for the activity.

The individual behaves as if they are solving:

$$\max_{x_1,x_2,...x_N} V(c, x_1, x_2, ...x_N) \text{ s.t. } T = \Sigma_{i\in N}\ \sigma_i x_i \text{ and } c = f(x_1, x_2, ...x_N)$$

In most of the analysis we intend to abstract from the fact that some activities, such as leisure or drug use, may directly affect utility, while others, such as working for a wage or stealing a car, affect utility directly and through consumption. We simply substitute the financial budget constraint into the utility function and relabel the objects of interest $U(x_1, x_2, ...x_N) = V(f(x_1, x_2, ...x_N), x_1, x_2, ...x_N)$. The following maximization problem describes the same individual behavior.

$$\max_{x_1,x_2,...x_N} U(x_1, x_2, ...x_N) \text{ s.t. } T = \Sigma_{i\in N}\ \sigma_i x_i$$

If the utility maximization problem has associated demand functions $D_i(\sigma, T)$ for $i \in N$ where σ is the vector of time prices $\{\sigma_1, \sigma_2, ...\}$ and a Hicksian demand function $H_i(\sigma, U)$, then the deterrent effect on crime i of an increase in the expected prison time for committing that crime, $\sigma_i \to \sigma_i'$, is the usual[1] substitution effect:

$$SE = D_i(\sigma, T) - H_i(\sigma', U(D_i(\sigma, T), D_j(\sigma, T), ..., D_N(\sigma, T)))$$

While the incapacitation effect is equivalent to the income effect:

$$IE = H_i(\sigma', U(D_i(\sigma, T), D_j(\sigma, T), ..., D_N(\sigma, T))) - D_i(\sigma', T)$$

Additionally there are deterrence and incapacitation effects of this price change that affect the quantity of other crimes. For example, the deterrent effect on crime j due to the increase in expected prison time for committing crime i is:

$$SE = D_j(\sigma, T) - H_j(\sigma', U(D_i(\sigma, T), D_j(\sigma, T), ..., D_N(\sigma, T)))$$

3.3.1 Implications and Intuition of This Approach

The model's predictions are sensitive to the choice of utility function. For example, the model suggests that even rational criminals might increase criminal activity in response to increased enforcement. Such an increase in crime would correspond to a 'Giffen crime' in this model. The incapacitation effect of a sentence increase would be so large and positive as to cause the total effect to be positive. Alternatively, a rational criminal might switch their criminal behavior to a different crime or to an activity which is not a crime.

Two types of individual behavior which are not accounted for in this model are risk preferring behavior and risk aversion. The agent in the model is risk neutral. When there is more than one activity with risky sanctions, the choice problem describing behavior of an individual who is not risk neutral would be very complicated. Such a problem is analogous to the 'portfolio problem' of deciding how much of several risky assets to hold in a portfolio.

When there is one crime, which is risky, and one activity which does not carry the risk of sanction, then the individual's behavior can be described, but it cannot be represented by rational preferences as in the model in the previous section. An increase in probability

[1] Despite the multiplicity of definitions of the Slutsky decomposition for discrete changes, we think this particular approach corresponds most closely to the intuition that previous authors have about deterrence and incapacitation. For a change in sanctions, this substitution effect is the change in what the individual would do when the relative severity of sanctions changed but the total amount, as measured by the individual's utility, of activities the individual is engages in is not changed. We present this substitution and income effect for a discrete change rather than the unique definition associated with a differential change in price. In the context of crime and punishment, we think understanding discrete changes, and this shortcoming of the Slutsky decomposition, is more important.

or an increase in severity, which lead to the same increase in σ, will no longer have the same effect on individual behavior. Because the individual will make different choices when different probabilities and severities result in the same expected sanction, their behavior cannot be represented by rational preferences over the expected outcomes. In general terms it is obvious that for any given set of risky sanctions, an increase in risk aversion will lead to less criminal activity. Increasing relative risk aversion is generally defined over wealth measured in a single dimension. Consider the case where legitimate and criminal activities are simply two types of labor, each with diminishing financial returns, and the individual has decreasing absolute risk aversion over the resulting wealth. The decreasing absolute risk aversion will tend to make incapacitation effects more negative. An increase in the expected sanction attributable to an increase in severity will be more 'risky' and therefore have the effect akin to a larger increase in the 'price' of crime relative to the same increase in expected sanction achieved through an increase in probability. Either method of increasing the sanction will tend to increase the individual's risk aversion as they become less wealthy, but the larger effect would occur with the increase in severity. In this example, the incapacitation effect, and the total effect, is larger for the change in severity.

Previous work on law and economics has noted that criminals may substitute between crimes - and that crimes can be complimentary goods with other crimes, or legal work. This model gives us a precise language to articulate the response across all activities to a change in expected incarceration for a particular activity. Activities which are gross compliments will decline in response to an increase in expected incarceration for any one of those activities. Activities which are gross substitutes for a particular activity will increase in response to an increase in expected incarceration. Similar relationships exist for changes in the wages for legal work.

In this model, increasing the wage $\frac{\partial f}{\partial x_1}$ for honest work (without loss of generality, x_1) will increase the absolute value of the Marginal Rate of Substitution $\frac{dx_1}{dx_i}$ of honest work with each other activity. Without further restrictions on the shape of f, V or U, we believe that increasing the financial return of honest work can have an ambiguous effect on the total effect, the incapacitation effect, and the substitution effect for any given change in sanctions $\sigma_i \to \sigma_i'$. In other words, a individual who is wealthier, in terms of receiving higher pay for legal work, may have a larger or smaller change in behavior in response to a change in sanction, relative to an individual who is poorer. It is possible that more or less of the wealthier individual's response will be attributable to deterrence (or incapacitation).

The nature of deterrence is that it shifts behavior to other activities. Incapacitation reduces the overall amount of activities.[2] Previous authors have noted that, if a crime is not committed because of deterrence, the sanction is never imposed. The definition offered in this paper clarifies that when a crime is not committed because of incapacitation, the sanction is not imposed for that uncommitted crime either. Deterrence has been argued to increase equitable treatment; would-be criminals no longer face different outcomes depending on whether they are caught and would be victims are not subject to risky payoffs. In fact, had the same change in behavior been a result of incapacitation, it would have the same effect on equitable treatment. A crime not committed due to incapacitation cannot lead to disparate outcomes. Avoiding costly sanctions and increasing equity are two main reasons why deterrence has been argued to be desirable relative to incapacitation. However, when understood to be two portions of a total change in behavior, the difference between deterrence and incapacitation is how the change in behavior has affected the individual's utility. If a change in sanction leads to a certain decrease in crime and most of the effect is deterrence, one possible intuitive explanation is that the criminal's outside option was almost as good as the crime. If the same sanction and change in behavior is attributable to incapacitation, that suggests that the individual's outside option was relatively poor. A policy changing the expected amount of incarceration does not control how the behavior change in response to increased incarceration is divided between deterrence and incapacitation, since the division depends on the preferences of the individual.

When a policy change leads to an incapacitation effect, the policy change does decrease the utility of certain individuals. The change in utility associated with incapacitation suggests a link between retribution and incapacitation which is different than the link between retribution and deterrence. Consider a change in policy where an activity which was unpunished becomes punished. If the behavior changes are attributable to deterrence individuals are not worse off, in expectation, under the punishment. If the behavior changes are attributable to incapacitation, the individuals are worse off under the punishment. Definitions of retribution often include complex ideas of moral codes and the cost of mistakenly incarcerating innocent individuals. In this case, we are only talking about the aspect of retribution that relates to the idea of decreasing a criminal's well being. That aspect is linked to incapacitation, which is associated with the individual's well being, differently than it is linked to deterrence, which is not linked to an individual's well being.

[2] Although the 'count' of activities undertaken may go up or down (substituting to an activity that 'takes less time'), the 'overall amount of activities' in this context refers to the total time cost of non-prison activities or the utility from activities undertaken, both of which are reduced

Clearly defining deterrence and incapacitation contributes to our understanding of the theories of justice, but is also important for three more practical reasons.

First, whether jurists or policymakers are seeking to define the punishments which maximize deterrence or incapacitation separately, or whether they seek to maximize social welfare, a clear definition of these effects will help distinguish which policies achieve their ends. Many authors seek to describe the policies which achieve deterrence or incapacitation. The definition in this paper suggests that the optimal criminal justice policy from a utilitarian perspective depends on the total change in behavior of criminals, rather than the division between deterrence and incapacitation. Moreover, since the amount of deterrence and incapacitation associated with a particular policy is based on the utility function of the criminal, it is not always under the control of a policymaker. In this context, a clear definition of deterrence and incapacitation serves the dual purpose of assisting policymakers who seek to identify policies which achieve a particular theory of justice, while also encouraging those policymakers to instead identify policies which maximize total welfare.

Second, deterrence and incapacitation are relevant to the question posed by Becker, of whether criminal behavior can be accurately described by models similar to those describing other economic behavior. Becker's proposition that criminals respond to incentives sparked empirical investigations to determine whether the deterrent effect had the correct sign. Becker claimed that "a useful theory of criminal behavior can dispense with special theories of anomie, psychological inadequacies, or inheritance of special traits and simply extend the economist's usual analysis of choice." Becker suggested that if the amount of crime declines in response to increased sanctions, that was evidence of deterrence, and therefore evidence of rational behavior on the part of criminals. When incorporating wealth effects, it is consistent with rational behavior for crime to increase in response to an increase in sanctions. When criminals allocate their time rationally, in other words, when criminal's behavior is consistent with complete, transitive and reflexive preferences over activities, the deterrent effect necessarily has the correct sign. A negative substitution effect is not a sufficient condition for rational preferences however, a negative substitution effect would be suggestive evidence that the analysis of crime can "simply extend the economist's usual analysis of choice."

When Criminals are not rational, and an old policy is changed to some new policy, the distinction between deterrence and incapacitation can still be defined so long as the criminal's behavior can be described for a hypothetical choice problem. The hypothetical choice problem is the problem for which the individual is indifferent between their behavior in the hypothetical problem and their behavior under the old policy, but their marginal incentives are the same as they are under the new policy. Little is intuitive about these theories of justice, especially deterrence, for an irrational individual.

Third, deterrence and incapacitation can be used to better understand the welfare impact of punishment. Deterrence and incapacitation affect welfare differently, especially when individuals are heterogeneous. Both incapacitation and deterrence are associated with a decrease in the harm to victims, but each has a different effect on the criminal and on the deadweight loss of the criminal justice system.

Deterrence does not create a welfare burden for criminals. To the extent that punishments deter criminal behavior without creating a wealth effect it is easy to analyze the welfare consequences of the policy for the criminal without reference to a particular set of welfare weights or social welfare function. This holds only when the size of the externalities does not affect the criminal's welfare and, therefore, choices. Incapacitation is associated with the welfare burden on criminals of an increase in punishment. Correctly analyzing the welfare effects of a punishment which has an incapacitation effect requires selecting welfare weights or knowing the compensation, and the compensated demand, of individuals.

3.4 Comparing This Definition with Other Definitions

The intuitive definition of deterrence used by many authors is very similar to the precise mathematical definition offered here. To quote Shavell 2015, "The deterrence of crime - its discouragement by means of threat of imprisonment" is a fairly precise description of a substitution effect. In spite of the similarity in language, we cannot say definitively that prior authors intended for this interpretation. We do not know of prior work which explicitly considers wealth effects, nor any work which explains how to deal with wealth effects when computing deterrence.

The intuitive definition of incapacitation used by many authors differs from the definition we use in two key ways. First, our definition includes wealth effects in individual behavior while these effects have been omitted from previous definitions of incapacitation. Second, other authors generally refer to a behavior which the individual would have done during the period of incarceration, we refer to this as the 'direct' portion of incapacitation but, for reasons explained below, we suggest that it is important to include changes in behavior which occur over the entire lifetime of the individual, which we refer to as 'indirect' incapacitation.

Previous authors suggest that the incapacitation effect is the amount of crime not committed because the criminal is in prison and is unable to commit crime. A common approach to compute this effect is to multiply a crime rate by the length of time a prisoner is incarcerated. Marvell and Moody explain their approach as follows. "The impact of state prison populations on crime is typically estimated by applying, the lambda, the individual crime rate, of prisoners or arrestees. We... use lambdas for estimating the incapacitation impact." Marvell and Moody explain several methods of calculating the crime rate, including surveys of active offenders, surveys about what incarcerated individual's actions had been prior to incarceration, and other methods. Using any of these crime rates as a proxy for what an imprisoned individual would have done may approximate the incapacitation effect. This approximation is not 'what the prisoner would have done.' By assumption, an individual who is not incarcerated is better off than an individual who is incarcerated. According to consumer choice theory, this can affect choices. When the wealth effects are properly incorporated, we refer to this notion of 'the crimes the criminal was prevented from doing during the period of incarceration' as the direct incapacitation effect.

When considering the counter factual of what the individual would have done, it is important to specify what choice problem we have in mind. For some situations, such as an individual who commits a crime at every opportunity, the crimes they do not commit because they are in prison is relatively easy to define (although we still have to specify whether the individual would be re-incarcerated during this time period, and for any individual whose behavior is sensitive to punishments, whether the individual behaves as if they believe that they would be re-incarcerated). This is a very restrictive model of behavior. As soon as we consider more complex models of behavior, it becomes difficult to carefully define the individual's behavior. Some issues which seem relevant include whether we imagine the individual choosing their behavior as if they had never been caught and convicted, or whether they are caught and released. Either case might change the individual's perception of the risks of criminal activity. For example, does being caught change the probability, or the perception of the probability, of being caught in the future? Does being released affect the individual's perception of the credibility of threatened punishments in the future? If a particular version of this choice problem were formulated and agreed on as a benchmark in the literature, such a choice problem would precisely define direct incapacitation. As it stands, the choice problem corresponding to direct incapacitation is often defined ad hoc, based on the model under consideration.

One effect which is not accounted for in direct incapacitation is how the individual's behavior over the rest of their life will be affected because they were able to engage in other activities, legitimate or criminal, rather than being incarcerated. For example, consider an individual who has legitimate employment opportunities available, conditional on having reliable transportation. We could imagine the case where the individual is incarcerated and cannot keep up with their car payments, so that upon release they frequently pursue criminal sources of income. If the individual were not incarcerated and were able to keep their vehicle, it may be the case that they never pursue those criminal sources of income. We refer to all the effects on behavior of this kind as the indirect incapacitation effect.

So long as the particular notion of incapacitation is defined, and the wealth effects are not ignored, the particular label applied to these effects is a relatively unimportant semantic distinction. The belief that a fine has an entirely deterrent effect illustrates this point. Some of the effect on the individual's decision will be due to a wealth effect. Taking the case where crime is a 'giffen good', the positive income effect is larger than the negative substitution effect. The total change in behavior will be positive, and if one believes that fines can only have a deterrent effect, then the deterrent effect as defined includes the wealth effect and is positive. As long as one understands the role of wealth effects in any analysis, which label is applied is relatively unimportant. For example, if it is defined, as it has sometimes been in the previous literature, to be equal to the total effect of a fine, this "deterrent effect" being positive is consistent with rational decision making.

We think that considering deterrence and incapacitation in their entirety is a particularly appealing, unifying definition of these concepts. The compensated demand function very precisely describes what we mean by 'what the criminal would have done'. Using this counter factual answers questions about how the individual's marginal incentives can be adjusted separately from the amount of time they have to commit crimes. Furthermore, this definition accounts for all of the behavioral changes which would occur if the criminal were not incarcerated, rather than focusing only on direct incapacitation. We also find Direct and Indirect incapacitation more difficult to define and use when wealth effects are appropriately incorporated.

3.5 Substitution and Wealth Effects in a Dynamic Model

Other authors usually use a dynamic model, because they are addressing direct incapacitation, which is especially difficult to define over the lifetime of the individual. We present dynamic models in order to clarify the relationship between our work and the existing literature. This also demonstrates that substitution and wealth effects are interpretable across a variety of models. Although we do not know a 'standard' approach to income and substitution effects in a dynamic setting, it is straightforward to define them.

In each period t the agent selects an act $X_t \in \{n, c\}$. Act c is a crime. If the individual commits the criminal act, there is a probability p that the individual will be apprehended and incarcerated for the following S periods. In any period when the agent is incarcerated, they cannot commit any act and they receive a payoff of 0. The individual's payoff in a period in which they choose act n is a constant, normalized to be 1. The payoff from crime in each period is larger, and varies based on a draw from a distribution. We will use a payoff of $1 + \pi_t$ where $\pi_t \sim U[0, 1]$. The individual observes the realization of the payoff π_t before choosing X_t, but chooses X_t before knowing the realization of π_{t+1} or subsequent payoffs. The individual lives for T periods.

We consider a behavior pattern where in each period i that the individual is not incarcerated, the individual uses a cutoff value $\overline{\pi}_t$. If the draw of π_t exceeds this value, the agent chooses $X_t = c$. If π_t does not exceed this value the agent chooses $X_t = n$. The cutoff value in each period maximizes the expected value of the stream of payoffs the agent receives. The probability that the individual commits a crime, conditional on not being incarcerated in a particular period, is simply $\overline{\pi}$. Based on this decision rule we can calculate the expected number of crimes per period which the individual would commit.

We will calculate the substitution and income effect of moving from no punishment, $S = 0$, to a punishment of incarceration for one period $S = 1$. When $S = 1$ and $p > \frac{2}{3}$ the agent only commits crime in the last period. The following assumes that $p < \frac{2}{3}$.

Because there is no punishment after T, the agent always commits crime in the last period and $\overline{\pi}_T = 0$. The expected value of not being incarcerated in period T is then $V_T = 1.5$.

When $S = 0$, the individual commits crime in every period, for a total of T crimes and the expected total payoff to the criminal is $1.5T$.

When there is a punishment we can describe the individual's behavior using backward induction, $\overline{\pi}_{T-1} = \frac{3}{2}p$ and the total expected payoff from period $T-1$ and period T is $V_{T-1} = \frac{9}{8}p^2 - \frac{3}{2}p + 3$. By induction we can solve for $\overline{\pi}_i = p(V_{i+1} - V_{i+2})$ and the total expected payoff from period i and all subsequent periods $V_t = V_{i+1} + \frac{3}{2} - p(V_{i+1} - V_{i+2}) + \frac{[p(V_{i+1}-V_{i+2})]^2}{2}$. Define $V'(t)$ as the payoff V_1 associated with an agent who has t periods while facing sanctions of p and $S = 1$.

The expected amount of crime the individual engages in can be computed by forward induction. In the first period, the individual commits a crime with probability $1 - \overline{\pi}_1$. We will denote the expected number of crimes the individual will commit in each period t by c_t and since the individual is never incarcerated in the first period, $c_1 = 1 - \overline{\pi}_1$. In the second period, the individual commits a crime with probability $1 - \overline{\pi}_2$ if they are not incarcerated. The expected number of crimes the individual commits in period 2 is then $c_2 = (1 - \overline{\pi}_2)(\overline{\pi}_1 + (1 - \overline{\pi}_1)(1 - p))$. Using forward induction, the expected number of crimes the individual will commit in period t is $c_t = (1 - \overline{\pi}_t)(1 - c_{t-1}p)$.

In order to calculate the Substitution and income effect, we have to define a compensated demand function. In this case, we define W by solving the following: $W = \text{argmin}_t V'(t) - 1.5T$. Using analogous methods, we can define $\overline{\pi}'_t$ and c'_t. The income effect is $\sum_t^T c_t - \sum_t^W c'_t$. The substitution effect is $T - \sum_t^W c'_t$

In this model, what the criminal would have done is clearly defined and does not depend on whether or not the criminal was caught in the previous period.

3.6 Literature Review

Becker (1968) poses a form of the following question: does the amount of crime decrease when the probability or severity of punishment increase? Becker intended for the answer to this question to be evidence that criminal decision making was similar to the decision making of individuals who do not commit crimes. This is in contrast to the idea that criminals behave because of some "anomie, psychological inadequacies, or inheritance of special traits." Becker argued that finding a deterrent effect of sanctions would be evidence that criminals are maximizing their perceived benefits subject to the constraints they face.

Ehrlich (1973) uses a risky single period model similar to our own to model the decision to commit crime. The paper focuses on the total effect of changing probability or severity of sanction, but the paper discusses deterrence and incapacitation in order to consider the idea that, when criminal activity behaves like a market for crime, incapacitation may be offset by other individuals becoming criminals. Ehrlich points out that because the total effect includes incapacitation, deterrence is only part of the total effect. This insight bears on the debate about the rationality of criminal behavior, but because Ehrlich is thinking of direct incapacitation without wealth effects, he argues that both deterrence and incapacitation decrease the total amount of crime. As we have argued, the presence of wealth effects means that even when offenders are rational, the amount of crime can increase in response to an increase in sanctions.

Differentiating between incapacitation and deterrence is one way to corroborate Becker's claim that criminal's motivations are not different from the motivations of non-criminals. If criminals are rational in allocating their time among activities, then the deterrent effect is necessarily negative. Therefore, a correctly calculated substitution effect of the correct sign would suggest rationality of ofenders, while a substitution effect of the incorrect sign would suggest that modeling criminals as rational agents does not match observed behavior.

As mentioned in the introduction, the economic literature on deterrence is prolific. We mentioned Becker, Beccaria, Bentham and Plato, but there are discussions of deterrence in Polinsky and Shavell (1984), Kaplow (1990), Andreoni (1991) and the survey Polinsky and Shavell (2000), to name only a few.

Shavell (1987) studies incapacitation separate from deterrence. Shavell calculates the optimal sentence when individuals cause known but heterogeneous harms in each time period and can be incapacitated. This paper generalizes Shavell's approach to include deterrence and disputes one of the main conclusions of the paper. The conclusion that sentence length is independent of probability of apprehension when incapacitation is the goal of criminal justice policy does not hold when the individual's behavior is responsive to the change in sanction because the harm associated with each individual depends on the effect of deterrence on individual behavior.

Two papers, Miceli (2010) and Shavell (2015), have considered deterrence and incapacitation together. Miceli (2010) uses a model in which each individual makes a choice about whether to commit a crime and, eventually, suffer punishment. In Miceli's model, each individual either commits a crime at every opportunity or never commits a crime. Therefore deterrence occurs at the extensive margin, that is, when individual's with less preference for crime decide not to engage in crime. Incapacitation happens at the intensive margin, that is, when a criminal, who commits crime at every opportunity, is incarcerated. Using the substitution and income effect is a generalization of Miceli's definition. If the income effect and substitution effect were calculated for an individual who always or never commits crime, the result would match the effects described in Miceli. Miceli's assumption about individual behavior is analogous to restricting crime and honest activities to be perfect substitutes in the lifetime utility function. The effect on an individual is either entirely deterrence or entirely incapacitation.

Shavell (2015) allows for the behavior of each individual to exhibit both a deterrence or an incapacitation effect. Because sentences can be set differently for different individuals, individual's incentives in the second period are unchanged whether or not individuals in the first period are threatened with incarceration. Since there are no wealth effects, it is possible to use individual behavior in the second period as a proxy for what an incarcerated individual would have done. Although age may not be a protected class, and judicial discretion would allow for different sentencing for different groups, in practice we think it is still important to be able to distinguish the incapacitation and deterrence effects when sentences are changing for all individuals at the same time. It is also interesting to note that even though Shavell's punishments are discussed in terms of incarceration, the punishments can affect how much time the individual has to commit crime separately from their effect on utility and incentives. Because of this distinction, the punishments in Shavell's model would be equivalent to a sanction combining incarceration and a fine in this paper.

In addition to different theoretical conclusions, correctly identifying deterrence and incapacitation is important to discuss the variety of empirical approaches to estimates of deterrence and incapacitation.

Empirical attempts to separate deterrence and incapacitation use a variety of definitions of these ideas. For example, Levitt (1998*b*), claims that incapacitation will tend to decrease all crimes, while deterrence will tend to increase other crimes. His logic ignores the fact that an individual who is able to commit fewer crimes, because they are incarcerated more often, may choose to commit crimes more often when not incarcerated. The empirical approach used categorizes index crimes as violent or pecuniary crimes and assumes that crimes withing these categories are more substitutable than other crimes. Comparing the response of these categories of crime to arrest rates for crimes in their own category to the response of these crime categories to the arrest rates for crimes in the other category, Levitt claims to be identifying something about the difference between incapacitation, which always decreases all crimes, and substitution, which tends to increase substitute crimes. The assumptions in the empirical analysis amount to a strong restriction on the utility function of a representative agent. It is likely that at least some crimes, perhaps murder and the illegal possession of a firearm, are complementary. An increase in the arrest rate for one of them will decrease the other through substitution, also known as deterrence. It is worth noting that Levitt is analyzing seven of the index crimes, and it is hard to argue for strong complementarity of the aggregate crime rate for these crimes. It is, nonetheless, a strong assumption to treat the difference in cross price elasticities across groups of crimes as a measure of deterrence or incapacitation.

Marvell and Moody (1994) discuss a number of studies which calculate the incapacitation effect associated with an incarcerated prisoner by computing the number of crimes 'active offenders' commit. This approach is reasonable if incarcerated offenders are similar to active offenders and, critically, if offender behavior does not exhibit wealth effects. Marvell and Moody also perform a regression of the total change in crime rates associated with an increase in prison population which, as the authors note, does not identify how much of the total effect is attributable to deterrence and how much to incapacitation.

Kessler and Levitt (1999) use the transition path between steady states in a multi period model to identify deterrence and incapacitation. This study is consistent with our definition.

Recent empirical work has used random assignment of judges to better identify the relationship between incarceration and later outcomes. Aizer and Doyle (2015) is an example of this approach, which focuses on the effect of juvenile incarceration on later outcomes, such as high school completion and recidivism. The effect of being assigned to a judge who tends to incarcerate offenders for longer periods is an incapacitation effect. The effects of incarceration measured in Aizer and Doyle, including capital accumulation and the effect on future criminal activity are a part of the incapacitation effect of changes in incarceration. In Aizer and Doyle, monitoring programs and other incapacitating sanctions are part of the counter factual outcome. This almost, though not completely, separates the portion of incapacitation attributable to a temporary inability to perform criminal acts from the portion of incapacitation attributable to a wealth effect. Since the empirical approach used in Aizer and Doyle is particularly close to a controlled experiment, it is appealing to use theirs, and related work, to compute the total effect of a change in incarceration. Such a calculation requires understanding the relationship between incapacitation and deterrence.

3.7 Concluding Remarks

When calculating deterrence and incapacitation, it is important to take into consideration a variety of factors which affect individual behavior. For a rational individual, the impact of all of these factors which are not captured by relative prices, are conveniently described by one aspect of observed behavior, and that is the wealth effect.

This paper contributes to the law and economics literature on deterrence and incapacitation in three main ways. First, this paper provides a definition of deterrence and incapacitation which is precise and interpretable across a variety of models. Second, this definition is the first to incorporate wealth effects. Third, because this definition includes wealth effects, it makes clear that rational criminals may increase the amount of crime they commit in response to an increase in sanctions.

We argue that direct and indirect incapacitation are less well defined, in that there is not a well defined choice problem which precisely defines these effects. If these effects can be defined clearly, then the distinction between direct, indirect, and total incapacitation becomes an issue of semantics, and we have no preference for one label over another. Including wealth effects and indirect incapacitation when considering the effects of policy is not an issue of semantics. Empirical evidence suggests that incarceration can affect criminal behavior for the life of the individual.

In contrast to prior work which determines the optimal punishments when deterrence or incapacitation are the goal in isolation, this definition suggests that utilitarian punishments are determined by the total effect on behavior, without reference to the portion of the effect attributable to deterrence or to incapacitation. Future work will attempt to determine what, if any, policy significance there is in distinguishing between deterrence and incapacitation.

APPENDIX A

Appendix: Pigouvian Taxation with Multiple Externalities and Costly Administration

A.1 Definitions

- $\mathbf{x}$ is the n-dimensional vector of activities
- $b(\mathbf{x})$ is a function, $b : [0, \infty]^n \to \mathbb{R}$, that maps an activity vector to total private benefit
- $\mathbf{e}$ is the n-dimensional vector of activity externalities
- $\mathbf{t}$ is the n-dimensional vector of activity taxes
- $c(\mathbf{x}, \mathbf{t})$ is a function, $c : \mathbb{R}^{2n} \to \mathbb{R}$, that maps an activity vector and a tax vector to total administrative cost

A.2 Assumptions

A.2.1 b function

- $b(\mathbf{x})$ is twice continuously differentiable
- $b(\mathbf{x})$ is strictly concave
- $\lim_{x_i \to 0} \frac{\partial b}{\partial x_i} = \infty$
- $b(\mathbf{x})$ achieves its maximum value with all $0 < x_i < \infty$

A.2.2 c function

- $c(\mathbf{x}, \mathbf{t})$ is continuously differentiable
- $c(\mathbf{x}, \mathbf{t})$ is weakly convex
- $\arg\min(c) = (\mathbf{0}, \mathbf{0})$

A.3 Propositions

Proposition 1. *When administration is free and every activity may be taxed, the optimal tax vector is equal to the externality vector.*

The private market solves the following problem:

$$\max_{\mathbf{x}} b(\mathbf{x}) - t^{\top}\mathbf{x}$$

which leads to the first order condition $b'(\mathbf{x}) - \mathbf{t}^{\top} = \mathbf{0}$. Because b is strictly concave is has a negative definite hessian which means that it is invertible. We can, therefore, apply the implicit function theorem, to find a continuously differentiable function, $\mathbf{y}(\mathbf{t}) : \mathbb{R}^n \to \mathbb{R}^n$, such that $b'(\mathbf{y}(\mathbf{t})) - \mathbf{t}' = \mathbf{0}$. Note that $b''(\mathbf{y}(\mathbf{t}))\mathbf{y}'(\mathbf{t}) = \mathbf{I}$, so $\mathbf{y}'(\mathbf{t})$ is also invertible.

The social planner solves the following problem:

$$\max_{\mathbf{t}} b(\mathbf{y}(\mathbf{t})) - \mathbf{e}^{\top}\mathbf{y}'(\mathbf{t})$$

which leads to the first order condition $b'(\mathbf{y}(\mathbf{t}))\mathbf{y}'(\mathbf{t}) - \mathbf{e}^{\top}\mathbf{y}'(\mathbf{t}) = \mathbf{0}$. Substituting in $\mathbf{t}$ and applying $\mathbf{y}'(\mathbf{t})$'s invertibility, we have $\mathbf{t} = \mathbf{e}$. The other assumptions ensure that all activity levels are positive at this optimum.

For this result b need only be quasi-concave with an invertible hessian.

The intuition underlying this result is that the tax corrects the externality by making private cost equal to social cost.

Proposition 2. *When administration is costly and the cost is a function of the activity levels, each activity tax includes the marginal administrative cost.*

The private market problem is the same as before. The social planner now solves:

$$\max_{\mathbf{t}} b(\mathbf{y}(\mathbf{t})) - \mathbf{e}^{\top}\mathbf{y}'(\mathbf{t}) - c(\mathbf{y}(\mathbf{t}))$$

which leads to the first order condition $b'(\mathbf{y}(\mathbf{t}))\mathbf{y}'(\mathbf{t}) - \mathbf{e}^\top\mathbf{y}'(\mathbf{t}) - c'(\mathbf{y}(\mathbf{t}))\mathbf{y}'(\mathbf{t}) = \mathbf{0}$. Substituting in $\mathbf{t}$ and applying $\mathbf{y}'(\mathbf{t})$'s invertibility, we have $\mathbf{t}^\top = \mathbf{e}^\top + c'(\mathbf{y}(\mathbf{t}))$. The other assumptions ensure that all activity levels are positive at this optimum.

The intuition underlying this result is that the marginal cost of an activity increases when administration of an activity tax is costly. The tax incorporate this marginal cost to ensure that marginal

Proposition 3. *When administration is costly and the cost is a function of the tax levels, each activity tax includes the marginal administrative cost. However, since the administrative cost increases with the tax, but the firm's decision variable is the activity, the optimal tax distributes the marginal cost of administration across the taxes according to how each individual tax affects each individual activity level.*

The private market problem is the same as before. The social planner now solves:

$$\max_{\mathbf{t}} b(\mathbf{y}(\mathbf{t})) - \mathbf{e}^\top\mathbf{y}'(\mathbf{t}) - c(\mathbf{t})$$

which leads to the first order condition $b'(\mathbf{y}(\mathbf{t}))\mathbf{y}'(\mathbf{t}) - \mathbf{e}^\top\mathbf{y}'(\mathbf{t}) - c'(\mathbf{t}) = \mathbf{0}$. Substituting in $\mathbf{t}$ and applying $\mathbf{y}'(\mathbf{t})$'s invertibility, we have $\mathbf{t}^\top = \mathbf{e}^\top + c'(\mathbf{t})[\mathbf{y}'(\mathbf{t})]^{-1}$ or $\mathbf{t}^\top = \mathbf{e}^\top + c'(\mathbf{t})b''(\mathbf{y}(\mathbf{t}))$. The matrix $\mathbf{y}'(\mathbf{t})$ describes the effect of a change in the tax vector on the activity levels. Because b is concave, this matrix is negative definite, which means that the diagonals are all negative. This means that an increase in t_i will certainly reduce x_i although it may increase or have no effect on x_j. This implies that $\mathbf{t}$ may have negative entries—a subsidy may be optimal for some activities. c' will take negative values whenever there is a subsidy because the cost of administration will decrease the less negative the subsidy becomes.

Propositions 1 and 2 may be combined to show a general form. In that case, the social planner will solve $\max_{\mathbf{t}} b(\mathbf{y}(\mathbf{t})) - \mathbf{e}^\top\mathbf{y}'(\mathbf{t}) - c(\mathbf{y}(\mathbf{t}), \mathbf{t})$ and the optimal tax is $\mathbf{t}^\top = \mathbf{e}^\top + c_1 + c_2 b''(\mathbf{y}(\mathbf{t}))$.

Proposition 4. *When each activity tax has its own fixed cost, the social planner must compute 2^n possible net benefits and select the optimal one. At the optimum, the social planner will choose m of n taxes to administer. If $m < n$ the cross activity effects will enter into the optimality condition similarly to proposition 2.*

Let θ be a subset of $\{1, ..., n\}$ and let m be the dimension of θ. The private market solves the following problem:

$$\max_{\mathbf{x}} b(\mathbf{x}) - \sum_{i \in \theta} t_i x_i$$

with solution $\frac{\partial b}{\partial x_j} = 0$ for $j \notin \theta$ and $\frac{\partial b}{\partial x_i} = t_i$ for $i \in \theta$. $\mathbf{t}_\theta$ is the m-dimensional vector of taxes on the m taxed activities. As before $\mathbf{y}(\mathbf{t}_\theta)$ exists (does it?). The social planner then solves the following problem for each possible θ:

$$\max_{\mathbf{t}_\theta} b(\mathbf{y}(\mathbf{t}_\theta)) - \mathbf{e}^\top \mathbf{y}(\mathbf{t}_\theta)$$

which leads to the first order condition $b'(\mathbf{y}(\mathbf{t}_\theta))\mathbf{y}'(\mathbf{t}_\theta) - \mathbf{e}^\top \mathbf{y}'(\mathbf{t}_\theta) = \mathbf{0}$. Now let $\mathbf{y}'_\theta(t_\theta)$ be the matrix that is constructed by deleting rows $j \notin \theta$ of the Jacobian $\mathbf{y}'(t_\theta)$. This is an $m \times m$ matrix. Then we can substitute in t_θ and use the invertibility of $\mathbf{y}'_\theta(\mathbf{t}_\theta)$ to arrive at $\mathbf{t}_\theta^\top = \mathbf{e}^\top \mathbf{y}'(\mathbf{t}_\theta)\mathbf{y}'_\theta(\mathbf{t}_\theta)^{-\mathbf{1}}$. There are two special cases. $n = n$ and $n = 1$. For the first note that $\mathbf{y}'(\mathbf{t}_\theta) = \mathbf{y}'_\theta(\mathbf{t}_\theta)^{-1}$, so we have $\mathbf{t}_\theta = \mathbf{e}$ as before. For the second case, note that $\mathbf{y}'_\theta(\mathbf{t}_\theta) = (\frac{\partial y}{\partial t_i})^{-1}$, so $t_i = \frac{\sum_j e_j \frac{\partial y_j}{\partial t_i}}{\frac{\partial y_i}{\partial t_i}}$.

Let d_i be the fixed cost associate with implementing tax i. The social planner's second stage problem can be written:

$$\max_\theta \left\{ \max_{\mathbf{t}_\theta} b(\mathbf{y}(\mathbf{t}_\theta)) - \mathbf{e}^\top \mathbf{y}(\mathbf{t}_\theta) - \sum_{i \in \theta} d_i \right\} \text{ where } \mathbf{t}_\theta^\top = \mathbf{e}^\top \mathbf{y}'(\mathbf{t}_\theta)\mathbf{y}'_\theta(\mathbf{t}_\theta)^{-\mathbf{1}}$$

When there are also variable administrative costs we can combine propositions 2, 3, and 4 to arrive at:

$$\mathbf{t}_\theta{}^\top = \left[\mathbf{e}^\top \mathbf{y}'(\mathbf{t}_\theta) + c_1 \mathbf{y}'(\mathbf{t}_\theta) + c_2\right] \mathbf{y}'_\theta(t_\theta)^{-1}$$

APPENDIX B

Appendix: How to Catch Capone: The Optimal Punishment of Interrelated Crimes

B.1 Appendix

B.1.1 Estimate of the Cost

Let a_i be the arrests for crime i, x_j be the quantity of crime j, and $C(\mathbf{x}(\mathbf{a}), \mathbf{q}(\mathbf{a}))$ be the total cost of crime. We define $q_i(a_i) = a_i \ \forall i$ for notational convenience. Each index crime j is assumed to be a function of arrests for each index crime. We assume that $\frac{\partial C}{\partial q_i}$ and $\frac{\partial C}{\partial x_i}$ are constant. Thus the total cost of crime is:

$$\sum_i \frac{\partial C}{\partial q_i} a_i + \sum_j \frac{\partial C}{\partial x_j} x_j(a_1, ..., a_n) = \sum_i A_i a_i + \sum_j B_j x_j(a_1, ..., a_n)$$

where $A_i = \frac{\partial C}{\partial q_i}$ and $B_j = \frac{\partial C}{\partial x_j}$. The total differential of $x_j(\mathbf{a})$ is:

$$dx_j = \sum_i \frac{\partial x_j}{\partial a_i} da_i = \sum_i \varepsilon(x_j, a_i) \frac{x_j}{a_i} da_i \implies \frac{dx_j}{x_j} = \sum_i \varepsilon(x_j, a_i) \frac{da_i}{a_i}$$

where $\varepsilon(y, z)$ is the elasticity of y with respect to z. Assuming that the elasticities are constant:

$$\int \frac{1}{x_j} dx_j = \sum_i \left[\varepsilon(x_j, a_i) \int \frac{1}{a_i} da_i \right] \implies \ln(x_j) = \sum_i \varepsilon(x_j, a_i) \ln(a_i) + k_j$$

where k_j is a constant of integration. We have:

$$x_j = \exp\left[\sum_i \varepsilon(x_j, a_i) \ln(a_i) + k_j \right] = e^{k_j} \prod_i \exp\left[\varepsilon(x_j, a_i) \ln(a_i)\right] = e^{k_j} \prod_i a_i^{\varepsilon(x_j, a_i)}$$

We then return to the cost function as a function of $\mathbf{a}$ only:

$$C(\mathbf{a}) = \sum_i A_i a_i + \sum_j \left[B_j e^{k_j} \prod_i a_i^{\varepsilon(x_j, a_i)} \right]$$

Our aim is:

$$\min_{a_i \geq 0} C(\mathbf{a}) \text{ such that } \sum_i A_i a_i = \bar{A}$$

where $\bar{A}$ is the total cost of arrest at the current level of arrests.

Solving this problem for index crimes requires estimates of A_i, B_j, and $\varepsilon(x_j, a_i)$. McCollister et. al. provides B_j. We use the reported information in McCollister et. al., Levitt, and the UCR to find A_i and $\varepsilon(x_j, a_i)$.

Levitt reports elasticities of crimes with respect to arrest rates for own price elasticities and average arrest rates for cross price elasticities. For the own price elasticities, the elasticity with respect to arrests, $\varepsilon(x_i, a_i)$, is the same as Levitt's reported elasticity with respect to arrest rates, $\varepsilon(x_i, \frac{a_i}{x_i})$,:

$$\varepsilon(x_i, \frac{a_i}{x_i}) = \frac{\partial x_i}{\partial(a_i/x_i)} \frac{a_i/x_i}{x_i} \implies$$

$$\varepsilon(x_i, a_i) = \frac{\partial x_i}{\partial a_i} \frac{a_i}{x_i} = \frac{\partial x_i}{\partial(a_i/x_i)} \frac{\partial a_i/x_i}{\partial a_i} \frac{a_i}{x_i} = \varepsilon(x_i, \frac{a_i}{x_i}) \frac{x_i}{a_i/x_i} \frac{1}{x_i} \frac{a_i}{x_i} = \varepsilon(x_i, \frac{a_i}{x_i})$$

For cross price elasticities we must modify Levitt's estimates. To convert Levitt's elasticities to $\varepsilon(x_j, a_i)$, we use the 2010 nationwide counts of crime and arrests from the UCR. Levitt reports the elasticity with respect to the arrest rate for substitute and non-substitute crimes, where violent crimes are assumed to be substitutes with each other and non-substitutes with pecuniary crimes and vice-versa. The equation for computing the cross price elasticity is the same for substitutes and non-substitutes. For example, if Levitt's categories imply that crime j is not a substitute for crime i, then using k as an index for all of the crimes that Levitt categorizes as non-substitute crimes including i:

$$\varepsilon(x_j, \frac{\sum a_k}{\sum x_k}) = \frac{\partial x_j}{\partial(\frac{\sum a_k}{\sum x_k})} \frac{(\frac{\sum a_k}{\sum x_k})}{x_i} \implies$$

$$\varepsilon(x_j, a_i) = \frac{\partial x_i}{\partial(\frac{\sum a_k}{\sum x_k})} \frac{\partial(\frac{\sum a_k}{\sum x_k})}{\partial a_i} \frac{a_i}{x_i} = \varepsilon(x_j, \frac{\sum a_k}{\sum x_k}) \frac{x_i}{(\frac{\sum a_k}{\sum x_k})} \frac{1}{\sum x_k} \frac{a_i}{x_i} = \varepsilon(x_j, \frac{\sum a_k}{\sum x_k}) \frac{a_i}{\sum a_k}$$

If Levitt considers crime j a substitute for crime i, then using h as an index for all of the crimes Levitt considers substitutes (j is not included in this set of crimes):

$$\varepsilon(x_j, \frac{\sum a_h}{\sum x_h}) = \frac{\partial x_j}{\partial(\frac{\sum a_h}{\sum x_h})} \frac{(\frac{\sum a_h}{\sum x_h})}{x_i} \implies$$

$$\varepsilon(x_j, a_i) = \frac{\partial x_i}{\partial(\frac{\sum a_h}{\sum x_h})} \frac{\partial(\frac{\sum a_h}{\sum x_h})}{\partial a_i} \frac{a_i}{x_i} = \varepsilon(x_j, \frac{\sum a_h}{\sum x_h}) \frac{x_i}{(\frac{\sum a_h}{\sum x_h})} \frac{1}{\sum x_h} \frac{a_i}{x_i} = \varepsilon(x_j, \frac{\sum a_h}{\sum x_h}) \frac{a_i}{\sum a_h}$$

Since Levitt reports a small positive own price elasticity for murder, this approach would imply that additional arrests for murder reduce costs, even in the absence of an effect on crime levels. To resolve this we replace the entry for murder's own-price elasticity with a small negative value.

The Slutsky matrix is made up of these elasticities of demand for each crime with respect to the number of each type of arrest. The Slutsky matrix is not symmetric which may be because there are income effects and heterogenous agents.

We can make the Slutsky matrix symmetric by averaging each pair of off diagonal terms, computing $\frac{\varepsilon(x_j,a_i)+\varepsilon(x_i,a_j)}{2}$ and using that value for both $\varepsilon(x_j, a_i)$ and $\varepsilon(x_j, a_i)$. Leaving the Slutsky matrix asymmetric leads to similar results.

A_i is a measure of the costliness of arrests of type i computed as follows. If the arrest rates are set independently in order to control each crime separately, the own-price-elasticities and the victim harms imply a marginal cost A_i of additional arrests of type i. This is equivalent to assuming that current punishments correspond to the naive punishments and computing the cost per arrest which satisfies the naive first order conditions. Equating the marginal cost of additional arrests for crime i with the marginal decrease in harm to victims associated with crime i and rewriting the expression in terms of the information from McCollister et. al., Levitt, and the UCR:

$$A_i = \frac{\partial C}{\partial x_i}\frac{\partial x_i}{\partial a_i} \implies A_i = B_i \varepsilon(x_i, \frac{a_i}{x_i})\frac{x_i}{a_i}$$

The vectors of arrests which satisfy the constraint $\sum_i A_i a_i = \bar{A}$ can be constructed by creating random vectors in the null space of **A** and adding them to any vector which satisfies the constraint. By constructing many random combinations of arrests which satisfy the constraint, we can simply check which of those combinations of arrests has the lowest cost. We restrict the search to a region around a starting value, initially set at 2010 arrest rates, and repeat the process starting with the combination of arrests associated with the lowest cost from the previous iteration.

B.1.2 Tables of Previous Empirical Work

Table 2: Previous Empirical Estimates of Crime Rate Elasticity

Study	Crimes studied	'prices'	Result
Marvell and Moody (1994)	Index crimes	Total prison population	Murder -0.065; Agg Assault 0.056; Robbery -0.260; Rape -0.113; Burglary -0.253; Larceny -0.138; Auto theft -0.200
Marvell and Moody (1996)	Index crimes	Number of police officers	Murder -0.36; Agg Assault -0.35; Robbery -0.63; Rape -0.20; Burglary -0.33; Larceny -0.22; Auto theft -0.85
Evans and Owens (2007)	Index crimes	Number of police officers	Elasticities: Murder -0.84; Rape -0.42; Agg Assault -0.96; Robbery -1.34; Burglary -0.59; Larceny -0.08; Auto theft -0.85
Levitt (1997)	Index crimes	Number of police officers	2SLS results Murder $\in [-1.18, -3.05]$; Rape $\in [0.67, -0.27]$; Agg Assault $\in [-0.36, -1.09]$; Robbery $\in [-0.38, -1.20]$; Burglary $\in [-0.05, -0.58]$; Larceny $\in [0.26, -0.43]$; Auto theft $\in [0.14, -0.61]$
Corman and Mocan (2000)	Murder, robbery, burglary, auto theft	Arrests; police	Elasticity with respect to arrests: Murder -0.34; Robbery -0.94; Burglary -0.36; Auto Theft -0.40
Shepherd (2002)	Index crimes	Truth In Sentencing legislation (increases punishment for violent felonies)	Murder: -1.178; Agg Assault -44.809; Robbery -39.615; Rape -4.226; Burglary 174.721; Larceny -89.486; Auto theft 70.252
Benson et al. (1992)	Aggregate Property	Probability of arrest	-0.826
Levitt (1998a)	Aggregate violent; aggregate property	Differences in juvenile and adult punitiveness	Violent -0.121 Property -0.050
Kessler and Levitt (1999)	Aggregate crime	Sentence enhancement legislation in California	After legislation, crime rates for crimes eligible for sentence enhancement diverge from ineligible crimes in California, relative to the national rates

Study	Crimes studied	'prices'	Result
Shepherd (2004)	Types of murder	Probability of sentence; probability of execution	Elasticities for seven subcategories of murder By intimates: -0.04; Acquaintance -0.012; Stranger -0.002; Crime of Passion -0.04; Felonies -0.001; White -0.017; African American -0.02
Ehrlich (1975)	Murder	Probability of arrest, conditional probability of conviction, conditional probability of execution	Variety of specifications, reporting point estimates: $P_a \in [-1.182, -2.225]$; $P_{c\|a} \in [-0.374, -0.850]$; $P_{e\|c} \in [-0.039, -0.068]$
Dezhbakhsh et al. (2003)	Murder	Probability of arrest, conditional probability of conviction, conditional probability of execution	$P_a \in [-2.184, -10.096]$; $P_{c\|a} \in [-3.597, -47.661]$; $P_{e\|c} \in [-2.715, -7.396]$
Durlauf et al. (2010)	Murder	Probability of Arrest, Death Sentence and/or Execution	Net lives saved can vary greatly based on specification
Duggan (2001)	Gun and nongun homicide; all index crimes	One year lagged gun ownership (may be a proxy for victim precaution or a proxy for ease of committing violent crime)	Gun Homicide: 0.306; 0.223; nongun homicide: 0.020; 0.040; homicide: 0.180; 0.210; Agg Assault -0.007; -0.013; Robbery -0.016; 0.069; Rape -0.052; -0.092; Burglary -0.002; 0.094; Larceny 0.081; 0.032; Auto theft 0.043; 0.019
Lott and Mustard (1997)	Index crimes	'shall issue' laws (gun ownership - may be a proxy for victim precaution or a proxy for ease of committing violent crime)	Murder: -0.049; Agg Assault -0.0701; Robbery -0.0221; Rape -0.0527; Burglary 0.00048; Larceny 0.03342; Auto theft 0.0714
Kuziemko and Levitt (2004)	Aggregate crime	Share of prisoners incarcerated for violent, pecuniary, and drug crime	Cannot reject equal impact at the margin of incarcerating different types of prisoners

Table 3: Table of Computed Elasticities Using Levitt (1998)

	Changes in each type of offense due to a 1% change in each arrest rate							Total harm	Harm reduction from
1% increase in	Murder	Rape	Robbery	Assault	Burglary	Larceny	Auto Theft	reduction	cross crime effects
Murder arrests	-1.47	-0.18	5.83	-14.64	-5.28	-36.51	-8.25	$14m	$1.6m
Rape arrests	-0.71	-92.44	10.45	-26.25	-9.47	-65.47	-14.80	$31m	$9.2m
Robbery arrests	-3.94	-1.81	-1225.38	-146.75	-52.93	-366.01	-82.74	$106m	$54m
Assault arrests	-14.34	-6.57	212.55	-1305.68	-192.54	-1331.35	-300.97	$270m	$131m
Burglary arrests	-1.05	-25.22	-102.19	-27.75	-6765.59	2388.87	-409.06	$62m	$19m
Larceny arrests	-4.59	-110.65	-448.38	-121.77	-1272.88	-17869.25	-1794.84	$191m	$127m
Auto theft arrests	-0.26	-6.22	-25.21	-6.85	-71.57	589.34	-547.28	$9.9m	$4.0m

BIBLIOGRAPHY

Aizer, Anna, and Joseph J Doyle. 2015. "Juvenile Incarceration, Human Capital and Future Crime: Evidence from Randomly-Assigned Judges." *The Quarterly Journal of Economics*, qjv003.

Anderson, David A. 1999. "The Aggregate Burden of Crime." *The Journal of Law and Economics*, 42(2): 611–642.

Andreoni, James. 1991. "Reasonable doubt and the optimal magnitude of fines: should the penalty fit the crime?" *The RAND Journal of Economics*, 385–395.

Beck, Allen J, and Bernard E Shipley. 1989. *Recidivism of prisoners released in 1983.* US Department of Justice, Office of Justice Programs, Bureau of Justice Statistics Washington, DC.

Becker, Gary. 1968. "Punishment: An Economic Approach." *Journal of Political Economy*, 169: 169–217.

Bell, Brian, and Stephen Machin. 2013. "Immigration and crime." *A. Constant and K. Zimmermann, International Handbook on the Economics of Migration, Edward Elgar Publishing*, 353–372.

Benson, Bruce L, David W Rasmussen, and Iljoong Kim. 1998. "Deterrence and public policy: trade-offs in the allocation of police resources." *International Review of Law and Economics*, 18(1): 77–100.

Benson, Bruce L, Iljoong Kim, David W Rasmussen, and Thomas W Zhehlke. 1992. "Is property crime caused by drug use or by drug enforcement policy?" *Applied Economics*, 24(7): 679–692.

Bovenberg, A Lans, and Ruud A De Mooij. 1994. "Environmental levies and distortionary taxation." *The American Economic Review*, 1085–1089.

Corman, Hope, and H Naci Mocan. 2000. "A Time-Series Analysis of Crime, Deterrence, and Drug Abuse in New York City." *American Economic Review*, 584–604.

Dezhbakhsh, Hashem, Paul H Rubin, and Joanna M Shepherd. 2003. "Does capital punishment have a deterrent effect? New evidence from postmoratorium panel data." *American Law and Economics Review*, 5(2): 344–376.

Duggan, Mark. 2001. "More Guns, More Crime." *Journal of Political Economy*, 109(5).

Durlauf, Steven N, Salvador Navarro, and David A Rivers. 2010. "Understanding aggregate crime regressions." *Journal of Econometrics*, 158(2): 306–317.

Ehrlich, Isaac. 1973. "Participation in illegitimate activities: A theoretical and empirical investigation." *Journal of political Economy*, 81(3): 521–565.

Ehrlich, Isaac. 1975. "The Deterrent Effect of Capital Punishment: A Question of Life and Death." *The American Economic Review*, 65(3): 397–417.

Evans, William N, and Emily G Owens. 2007. "COPS and Crime." *Journal of Public Economics*, 91(1): 181–201.

Fullerton, Don, and Ann Wolverton. 2005. "The two-part instrument in a second-best world." *Journal of Public Economics*, 89(9): 1961–1975.

Fullerton, Don, and Sarah E West. 2002. "Can taxes on cars and on gasoline mimic an unavailable tax on emissions?" *Journal of Environmental Economics and Management*, 43(1): 135–157.

Hakim, Simon, Uriel Spiegel, and Jimmy Weinblatt. 1984. "Substitution, size effects, and the composition of property crime." *Social Science Quarterly*, 65(3): 719–34.

Heineke, John. 1978. "Economic models of criminal behavior: An overview." *North-Holland Publishing Company.*

Kaplow, Louis. 1990. "A note on the optimal use of nonmonetary sanctions." *Journal of Public Economics*, 42(2): 245–247.

Kessler, D, and SD Levitt. 1999. "Using Sentence Enhancements to Distinguish Between Deterrence and Incapacitation." *Journal of law and economics*, 42(1): 343–363.

Kuziemko, Ilyana, and Steven D Levitt. 2004. "An empirical analysis of imprisoning drug offenders." *Journal of Public Economics*, 88(9): 2043–2066.

Levitt, Steven D. 1997. "Using electoral cycles in police hiring to estimate the effect of police on crime." *The American Economic Review*, 270–290.

Levitt, Steven D. 1998*a*. "Juvenile Crime and Punishment." *Journal of Political Economy*, 106(6): 1156–1185.

Levitt, Steven D. 1998*b*. "Why do increased arrest rates appear to reduce crime: deterrence, incapacitation, or measurement error?" *Economic inquiry*, 36(3): 353–372.

Lott, Jr, John R, and David B Mustard. 1997. "Crime, deterrence, and right-to-carry concealed handguns." *The Journal of Legal Studies*, 26(1): 1–68.

Lott, Jr., John R, and William M Landes. 2000. "Multiple victim public shootings." *University of Chicago Law and Economics Working Paper.*

Marvell, Thomas B, and Carlisle E Moody. 1996. "Specification Problems, Police Levels, And Crime Rates." *Criminology*, 34(4): 609–646.

Marvell, Thomas B, and Carlisle E Moody, Jr. 1994. "Prison population growth and crime reduction." *Journal of Quantitative Criminology*, 10(2): 109–140.

Mayshar, Joram. 1991. "Taxation with costly administration." *The Scandinavian Journal of Economics*, 75–88.

McCollister, Kathryn E, Michael T French, and Hai Fang. 2010. "The cost of crime to society: New crime-specific estimates for policy and program evaluation." *Drug and alcohol dependence*, 108(1): 98–109.

Miceli, Thomas J. 2010. "A model of criminal sanctions that incorporate both deterrence and incapacitation." *Economics Letters*, 107(2): 205–207.

Mikesell, John, and Maureen A Pirog-Good. 1990. "State lotteries and crime." *American Journal of Economics and Sociology*, 49(1): 7–20.

Pigou, Arthur C. 1920. "The economics of welfare, 1920." *McMillan&Co., London.*

Polinsky, A Mitchell, and Steven Shavell. 1984. "The optimal use of fines and imprisonment." *Journal of Public Economics*, 24(1): 89–99.

Polinsky, A Mitchell, and Steven Shavell. 2000. "The economic theory of public enforcement of law." *Journal of Economic Literature*, 38(1): 45.

Polinsky, Mitchell, and Steven Shavell. 1982. "Pigouvian taxation with administrative costs." *Journal of Public Economics*, 19(3): 385–394.

Pollock, Frederick, and Frederic William Maitland. 1899. *The history of English law before the time of Edward I.* Vol. 2, University Press.

Richman, Daniel C, and William J Stuntz. 2005. "Al Capone's Revenge: An Essay on the Political Economy of Pretextual Prosecution." *Columbia Law Review*, 583–640.

Shavell, Steven. 1987. "A model of optimal incapacitation." *The American Economic Review*, 107–110.

Shavell, Steven. 2015. "A simple model of optimal deterrence and incapacitation." *International Review of Law and Economics*, 42: 13–19.

Shepherd, Joanna M. 2002. "Police, Prosecutors, Criminals, and Determinate Sentencing: The Truth about Truth-in-Sentencing Laws." *Journal of Law and Economics*, 45(2): 509–533.

Shepherd, Joanna M. 2004. "Murders of passion, execution delays, and the deterrence of capital punishment." *The Journal of Legal Studies*, 33(2): 283–321.

Slemrod, Joel, and Shlomo Yitzhaki. 2002. "Tax avoidance, evasion, and administration." *Handbook of public economics*, 3: 1423–1470.

Stigler, George J. 1970. "The Optimum Enforcement of Laws." *Journal of Political Economy*, 78(3): pp. 526–536.

Yitzhaki, Shlomo. 1979. "A note on optimal taxation and administrative costs." *The American Economic Review*, 475–480.