Three Essays in Optimal Tax Enforcement Theory

by

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This thesis is dedicated to my parents Irina Paramonova and Andrei Paramonov.

All I have and will accomplish are only possible due to their love and support.

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ABSTRACT

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This dissertation advances the existing tax enforcement theory by determining the optimal use of third-party information reporting and by examining the role and effectiveness of collateral tax sanctions. Chapter 1 extends optimal tax-system theory by modeling information reporting as an additional enforcement instrument that allows the tax authority to acquire signals about taxpayers' incomes. Therefore, the model allows the tax authority to not only audit a taxpayer, but also to acquire a signal about taxpayers income. I rigorously characterize the optimal allocation of resources between audits and information reporting when the enforcement budget is limited. I show that the optimal level of information reporting is an inverse U-shaped function of the budget, which implies that at some point it may not be optimal to expand information reporting. Chapter 2 and 3 propose new rationales for the use collateral tax sanctions such as suspension of a driver's license or revocation of a passport – for tax enforcement. Chapter 2 proposes that by affecting consumption and providing enforcement targeted to a group, collateral sanctions may allow the government to impose punishment correlated with an individual's earning potential. Such punishment makes the effective tax rates correlated with the individuals earning

potential and enables achieving the redistribution of income more effectively. I show that using a collateral sanction increases social welfare when the earning potential of the poorest individual in the targeted group is sufficiently higher than the earning potential of the poorest individual in the rest of the population. Chapter 3 applies collateral sanctions to the context tax debt collection. I develop a dynamic model, where a debtor may not pay tax debt because of income constraints or because of high chances to escape from the collection process. I show that when debtors are heterogeneous in their ability to escape tax debt, it may be optimal to use a collateral sanction in addition to a monetary fine. In contrast to the monetary fine that can be delayed and paid only when the tax debt is collected, the collateral sanction applies and influences immediately. This shows the importance of the timing of a penalty.

CHAPTER I

The Optimal Deterrence of Tax Evasion: The Trade-off Between Information Reporting and Audits

1.1 Introduction

Information reporting is a highly valuable tool for improving tax compliance.[1] It serves this role by reducing the information asymmetry between the tax authority and taxpayers. It does this by allowing the tax authority to verify tax returns against reports about taxpayers' income information from third parties. This additional information, by signaling to the tax authority whom should be audited, makes audits more targeted and, thus, more effective.

But, at the same time, information reporting imposes costs and diverts resources from tax audits. This raises some serious questions. How many resources should the tax authority devote to information reporting? How many resources should it devote to tax audits? More generally, what is the optimal tax enforcement policy?

The existing tax enforcement literature does not answer these questions, because

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[1] According to the existing empirical literature (*Kleven et al.* (2011), *Gordon and Li* (2009), *Alm et al.* (2009), *Klepper and Nagin* (1989)), income sources that are subject to some information reporting are characterized by a higher tax compliance than income sources that are not: noncompliance is 56 percent for incomes without information reporting and withholding and 1 percent for income with both information reporting and withholding.

it has not modeled information reporting, presuming that tax audits are the only tool to prevent tax evasion. To address these limitations, this paper extends optimal tax enforcement theory by considering information reporting as an additional tax enforcement instrument and endogenizing it. The paper rigorously characterizes the optimal strategy to deter tax evasion when the tax authority maximizes tax revenue with limited enforcement resources.[2] In doing so, the paper determines the optimal allocation of resources between tax audits and information reporting. It also deepens the understanding of the link between audit policy and information reporting.

Based on this model, I find that the optimal level of information reporting depends on the tax authority's budget such that it first rises, but then declines, with the budget, while the optimal audit coverage always rises with the budget. This implies that information reporting helps making audits more effective but only up to a point, at which it is no longer optimal to extend information reporting. The reason is that the value of better targeted audits decreases with audit coverage.

This paper models information reporting as an instrument that provides the tax authority with signals about taxpayers' income. Based on a signal, the tax authority can construct a prediction about the taxpayer's income distribution and subdivide taxpayers into audit classes, which makes audits more targeted. To characterize signals, I introduce the notion of signal accuracy. A more accurate signal allows constructing a more accurate prediction of the income distribution and better target audits. The tax authority can increase signal accuracy by improving information reporting. Thus, by allowing the tax authority to control signal accuracy, I endogenize information reporting.

In practice, the tax authority can improve information reporting either by extending it to new income sources or by using collected information better. The extent of information reporting varies among countries. Most OECD countries have informa-

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[2]This paper leaves aside equity issues of taxation and enforcement.

tion reporting for dividend and interest incomes – some for incomes from independent personal services, royalties, and patents.[3] The United States has a substantial information reporting system and has expanded it recently. It now requires banks to report the gross amount of merchant payment card transactions and brokers to report the adjusted cost basis for certain securities. There is also scope for better use of the collected information reports. For instance, the IRS matching program does not pursue all the mismatches from the information returns it receives. Investments in better programs and data analysis would help to improve the match process and refine its case selection methodology.[4]

Certainly, information reporting is not free. It has its costs that are usually overlooked. It is costly to perform and improve matching of information returns with tax returns, as well as to publicize rules, to educate taxpayers, etc. For example, during the period 2009 through 2012 the IRS spent about $110 million just for developing the matching program needed to implement the new information reporting requirements.[5] Unless the tax agency is able to employ new resources, it has to divert resources from tax audits. This leads the tax authority to the trade-off of its resources between information reporting and audits.

To analyze the resource trade-off, the difference between audits and information reporting must be specified. I conceptualize this difference in the amount of information each of these tools reveals. Both audits and information reporting help the tax authority to access information about taxpayers' income, but they do this differently. An audit helps to reveal full information about one taxpayer, while information reporting helps to reveal partial information about many, or all, taxpayers. For example, information reporting on payments to independent contractors helps the IRS to

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[3]In some of these countries some of income sources are also subject to withholding. See *OECD* (2004).

[4]For more information, see *GAO* (2009).

[5]For more details, see *GAO* (2011).

discover cases when the recipients of these payments underreport their gross receipts.[6] However, it does not help to discover information about independent contractors' expenses, and, thus, their taxable income is still uncertain.[7] In general, information reporting helps to reduce only some uncertainty about true taxable income. This justifies modeling information reporting as signals that improve the prediction of the income distribution.

The model in this paper extends the model of *Sanchez and Sobel* (1993) by considering audit probability as a function of not only reported income, as they assume, but the signal as well. Based on the signals, the tax authority can subdivide taxpayers into audit classes and choose audit rules specific to each audit class. While *Scotchmer* (1987) also makes this point, in her model signals are exogenous. In contrast, this paper makes signals endogenous by allowing the tax authority to invest resources to improve signal accuracy.

The analysis of the model starts by deriving the optimal audit policy and analyzing how it depends on the accuracy of information about taxpayers. Because signals allow the tax authority to subdivide taxpayers into audit classes, the audit probability function can be chosen for each audit class separately. Within an audit class, the optimal audit strategy is to examine those taxpayers whose reported income is below the audit cutoff specific to this audit class. This means that within an audit class the tax authority should audit relatively low-income taxpayers. But, these taxpayers can actually be high-income if they belong to a high-income audit class. Note that, without signals, the tax authority would have to audit truly low-income taxpayers. Thus, receiving signals helps to target audits to some high-income taxpayers, which makes audits more effective and raises tax revenue.

Applying this model to the case of multiple taxpayers each characterized by their

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[6]In the US, information about payments to independent contractors is reported on 1099-MISC form.

[7]Of course, information reporting on different income sources is helpful to discover taxable income to different extents.

own signal accuracy, I find that the optimal audit policy requires that the marginal revenue of an audit should be the same for all signals. This means that the tax authority should focus audits on those taxpayers about whom it has less accurate signals; i.e., on those who have more opportunity to evade taxes. For example, if we consider two groups of taxpayers – wage earners and self-employed individuals – then audits should be focused on the self-employed, i.e., those who are characterized by less accurate information.

The analysis then moves to determining the *optimal* signal accuracy. The tax authority that has limited enforcement resources has to allocate them between conducting tax audits and improving signal accuracy. As we have learned that the optimal audit rule is the cutoff rule conditional on a signal, to characterize the audit policy we only need to specify audit coverage (i.e., the share of audited taxpayers). Therefore, the tax authority's problem is to allocate resources between signal accuracy and audit coverage.

The allocation of resources is determined by how effective in raising tax revenue is an increase in signal accuracy compared to an increase in audit coverage. Since an increase in signal accuracy raises the tax revenue by making tax audits more targeted, the value of signal accuracy itself depends on audit coverage. Two forces play a role here. First, the value of signal accuracy declines with audit coverage, because the value of better-targeted audits declines as more audits are conducted and less high-income taxpayers are left unaudited. At the same time, when audit coverage is low the value of an audit is very high, which makes the relative value of signal accuracy low (for low audit coverage). This is because there is no benefit of audit targeting when there are no audits.

This interaction between signal accuracy and audit coverage leads to the specific dependance of optimal signal accuracy on the enforcement budget. When a change in the tax authority's budget is considered, I find that the optimal signal accuracy

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depends on the tax agency's budget such that it first rises, but then declines with the budget. At the same time, the optimal audit coverage always rises with the budget. This implies that there is a level of budget at which it is no longer optimal to expand information reporting.

There are several implications of these results. The first implication is that investments in information reporting are especially critical for a tax authority with scarce resources. This suggests that developing countries could benefit by extending information reporting. The second implication concerns those tax authorities that have substantial resources for tax enforcement. These tax authorities might be close to the point at which it is no longer optimal to expand information reporting. This raises a question of whether some developed countries have reached their optimal limits of information reporting.

The rest of the paper is organized as follows. Section 2 reviews the related literature to show how this paper builds on existing theory and extends it. Section 3 presents the model that incorporates information reporting into the tax evasion framework by introducing the concept of signal accuracy. Section 4 first characterizes the optimal tax audit rule and how it depends on signal accuracy. Then, taking into account the dependence of the optimal audit rule on signal accuracy, the paper determines the optimal level of signal accuracy and the optimal audit coverage. Finally, the analysis of the comparative statics with respect to the tax authority's budget and costs of audits and information reporting reveals policy-relevant findings. Section 5 discusses policy implications of the results, while Section 6 concludes.

1.2   Related Literature

This section overviews the existing optimal tax enforcement literature in order to lay the groundwork for the model in this paper. Moreover, this section helps to ascertain a concept of the accuracy of information, which is a new notion to the

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literature. While existing literature apprehends that tax authorities can condition audit strategy on information beyond reported income, it does not reckon that the accuracy of information can be changed. It is this endogenizing of the accuracy of information that is implemented in the rest of this paper.

This paper extends game-theoretical models of tax enforcement. These models examine the strategic interaction between taxpayers and the tax authority and determine the tax authority's optimal strategy to enforce tax compliance.[8] Specifically, *Sanchez and Sobel* (1993) solve for the optimal audit strategy in a model where the tax authority observes only reported income and chooses the probability of audit based on that reported income. The optimal audit strategy in this case is a cutoff rule: to audit with high enough probability the taxpayers with reported income below a certain cutoff and do not audit those with a reported income above that cutoff.

Scotchmer (1987) notes that, in practice, the tax authority can observe both reported income and correlates like profession, age, and gross income as reported independently by the employer. Given that such information allows taxpayers to be segregated into audit classes, "the enforcement agency will find it lucrative to condition the probability of audit on audit class, as well as on reported income, particularly if the audit class is a good signal of income" (p. 229). Her model examines audit rules within an audit class. The other paper that allows conditioning tax audits on information about taxpayers is *Macho-Stadler and Perez-Castrillo* (2002). In contract to *Scotchmer* (1987), they assume only three discrete income levels and that signals are unobserved by the taxpayers. Nevertheless, those papers come to a similar conclusion that conditioning audits on information may lead to effective tax code being more progressive than the nominal one. However, both papers treat signal as free and exogenous, which precludes the possibility of addressing the optimal improvement in the accuracy of observed information.

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[8]A comprehensive literature review on optimal tax enforcement is provided in *McCubbin* (2004).

Similar to the model in this paper, *Menichini and Simmons* (2014) consider a costly state verification model where a principal entering a contract with an ex-post informed agent, as well as auditing, has the option of acquiring costly imperfect information about future revenues. In contrast to the current paper, their model has only two income realizations and two signal values.[9] The model in this paper has a continuous distribution of incomes and signals. Also, their optimization problem of maximizing of an agent expected income subject to the principal's participation constraint differs from the current paper optimization problem that solves for the maximum tax revenue collection given that agents maximize their expected income. Lastly, they do not allow the principal to choose the size of the correlation between the signal and state realization, while this paper allows the principal (tax authority) to choose signal accuracy.

To describe the accuracy with which the tax authority can observe true information (relevant for tax administration purposes), a new term, "observability" of the tax base, has been recently introduced by *Slemrod and Traxler* (2010). They define observability as the accuracy of the tax base measurement that the tax authority can obtain. In their model, the tax authority can influence observability by changing the amount of investment spent on tax enforcement. They determine the optimal level of observability that is defined by the trade-off between the social costs from allowing more inaccuracy and the net revenue gains from having a cheaper-to-administer, but more capricious, tax system. Their model is similar to the model in this paper in its idea to endogenize information observed by the tax authority; however, their model does not allow for tax evasion and, thus, tax enforcement is beyond its scope.

Kleven and Kopczuk (2011) note that the accuracy of information concerning applicants to social programs is an important factor affecting program design. They show that the government can influence accuracy of this information by setting a

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[9]The extension of the model has three income realizations.

complexity of screening tests. Their model characterizes the optimal social program in which policy makers can choose the rigor of screening, along with a benefit level and eligibility criterion. While their settings are sufficiently different from those considered in this paper, the idea that government can influence and, hence, optimally choose the accuracy of information is parallel.

Besley and Persson (2011) argue that tax bases like income taxes and value added taxes can only become effective through extensive government investments in tax compliance. This supports the importance to improve information reporting from a high level of generalization. According to them, in the process of administrative infrastructure development the countries are able to move from collecting around 10 percent of national income towards collecting around 40 percent, and to shift from trade taxes and excise taxes towards labor income and other broad bases. In their framework, a forward-looking government will decide to invest in fiscal capacity in order to build a more effective tax system. However, the ways to improve administrative infrastructure are beyond their scope.

Boserup and Pinje (2013) introduce information reporting into a model of tax evasion. However, there is no strategic interaction between taxpayers and the tax authority in their model, and, therefore, audit probability function is exogenous. A similar approach is used by *Phillips* (2014). Both papers incorporate information reporting by assuming that one part of taxpayer's income – income reported by third parties – is completely observed and that other part of income is completely unobserved by the tax authority. This is a reasonable assumption, but it precludes the general case when information reporting provides only partial information about taxable income. Also, in their models information reporting is exogenous and the tax authority cannot affect its extent.

As has been discussed so far, the importance of accuracy of observed information for the state policy and, in particular, for tax policy has been acknowledged in

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many studies. The next natural move is to consider information accuracy as a tax enforcement instrument and to determine its optimal level. I present this move in what follows.

1.3 Model

This section presents a model that describes taxpayers who evade their taxes and the tax authority that fights tax evasion. The key innovative part of the model is signals; they represent information collected by the tax authority through information reporting. By introducing signal accuracy, this model endogenizes information reporting within the existing tax evasion framework.

Consider an economy consisting of risk-neutral individuals characterized by their income, i. Income is exogenous and distributed on $\mathbb{R}$ according to c.d.f. $F_0(\cdot)$, which is assumed to be continuously differentiable with density $f_0(i) = F_0'(i)$. Each individual is subject to an income tax at rate t and is required to file a tax report.

The tax authority collects taxes and performs audits to enforce tax compliance. The tax authority does not observe taxpayers' true income, and a priori believes that taxpayers' true income is distributed according to $F_0(\cdot)$. However, the tax authority receives a signal about each taxpayer's income, $s \in \mathbb{R}$. Note that s denotes the realization of the random variable.[10] Given a signal, s, about a taxpayer's income, the tax authority forms an updated belief about the taxpayer's income distribution, which is the conditional distribution function $F(i \mid s)$. The associated conditional density function is denoted by $f(i \mid s)$. The tax authority relies on a signal when it chooses the probability of audit, $p(r, s)$. So, the probability of audit depends not only on taxpayer's reported income, r, but on the signal, s, as well.

In practice, signals about taxpayers' income are obtained by the tax authority from third-party information reporting of different income types. Additionally, the

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[10]Signal s is one-dimensional.

tax authority has information about a taxpayer's age, occupation, marital status, information from previous periods. I assume that the tax authority is able to aggregate this information to construct a univariate prediction of true income. This prediction is implied by the signal, s.

By improving information reporting, the tax authority can obtain more accurate signals that allow constracting a less dispersed conditional distribution of income given a signal. To formalize this, I introduce the notion of signal accuracy, a. Correspondingly, I use superscript a on $F^a(i \mid s)$ to indicate that the conditional distribution of income depends on signal accuracy. I define signal accuracy, a, as the inverse of the standard deviation of the conditional distribution of income, which allows expressing $F^a(i \mid s)$ as a transformation of a distribution function that has unit variance. Specifically, assume that for a given signal, s, the conditional distribution of income is

$$F^a(i|s) = G(a(i-s)), \tag{1.1}$$

where G is a symmetric distribution function with zero expectation and unit variance, and the expectation of the conditional distribution of income is equal to s. The conditional p.d.f., $f^a(i|s)$, is correspondingly equal to $ag(a(i-s))$.

Condition (1.1) implies that signals are characterized by the same accuracy, a. In reality, people with different income sources might be better described by different signal accuracies. However, this assumption is realistic, if we assume that the tax authority chooses enforcement strategy for a subgroup of taxpayers that have similar income sources. For example, these could be self-employed individuals. Given the tax authority knows the optimal strategy to enforce taxes within each existing subgroup, which I solve for in this paper, the allocation of resources among all subgroups can be determined. Note also that, in Section 4.2 where I treat signal accuracy as fixed, I will relax this assumption and analyze the case when signal accuracy is not uniform

across taxpayers.

To complete the description of signals, I need to introduce their distribution. I denote the c.d.f. of signals by $H^a(s)$ and p.d.f. of signals by $h^a(s)$. The distribution of signals itself depends on accuracy, a. Since I assume that the distribution of true income, $f_0(i)$, does not change with signal accuracy, both the distribution of signals, $h^a(s)$, and the conditional distribution of income given a signal should change in coordination with each other when signal accuracy changes. Specifically, the following condition should be satisfied:

$$f_0(i) = \int ag(a(i-s))h^a(s)ds. \tag{1.2}$$

For this condition to be satisfied under assumption (1.1) on the conditional distribution of income, the support of the distribution of income and correspondingly the support of the distribution of signals should be unbounded.[11] This permits a taxpayer to have a negative income, in which case she is entitled to receive a tax refund of $t|i|$.[12]

An example that satisfies both conditions (1.1) and (1.2) can be constructed when the conditional distribution of income is normal with expectation equaled to s and variance equaled to $\frac{1}{a^2}$. If $\Phi(\cdot)$ and $\varphi(\cdot)$ denote the c.d.f. and the p.d.f. of the standard normal distribution, then the conditional c.d.f. of income is $F^a(i|s) = \Phi(a(i-s))$ and the conditional p.d.f. is $f^a(i|s) = a\varphi(a(i-s))$. Additionally, the distribution of true income is also normally distributed with expectation μ and variance Ω^2. The distribution of signals that conforms to these assumptions is a normal distribution with expectation μ and variance $\Sigma^2 = \Omega^2 - \frac{1}{a^2}$ (i.e., the c.d.f. is $H(s) = \Phi(\frac{s-\mu}{\Sigma})$, the p.d.f. is $h(s) = \frac{1}{\Sigma}\varphi(\frac{s-\mu}{\Sigma})$). [13]

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[11]The proof of this assertion can be requested from the author.

[12]Having negative income does not change the incentives of a taxpayer; her incentives is still to understate the true income because understatement of true income allows to claim a higher tax refund.

[13]In the case of the standard normal distribution it can be shown that the notion of signal accu-

We are now ready to define the taxpayer's problem. A taxpayer is assumed to know the signal and its accuracy. The taxpayer also knows the probability of audit that is based on her reported income, r, and on the known signal, s. If a taxpayer's report is not audited, then the taxpayer owes the tax based on the reported income, tr. If her report is audited, then the taxpayer owes taxes based on her true income and also a penalty proportional to the concealed taxes at rate π, $tr + (1+\pi)t(i-r)$. Each taxpayer minimizes her expected tax payment:

$$\min_{r} \{tr + p(r,s)(1+\pi)t(i-r)\} \tag{1.3}$$

Define this minimum as $T(i, s, p(\cdot))$ and the optimal reported income that minimizes the above taxpayer's problem as $r(i, s)$. Note that the signal affects the optimal report only though the audit probability.

We turn now to the tax authority's problem. To deter tax evasion, the tax authority chooses the probability of audit, $p(r,s)$, and also signal accuracy, a. To conduct an audit, the tax authority has to incur cost c. To achieve signal accuracy, a, the tax authority has to investing $K(a)$. I assume that the cost function, $K(\cdot) > 0$, is increasing (i.e., $K'(\cdot) > 0$). The cost $K(a)$ represents that the tax authority can increase signal accuracy by improving information reporting. A detailed description of costs that improving signal accuracy incurs will follow the formulation of the tax authority's problem.

The tax authority aims to maximize the tax revenue given its limited resources to conduct tax audits and invest in signal accuracy.[14] While the tax authority does

racy agrees with the notion of informativeness introduced by Blackwell (i.e., more accurate signals are also more informative). According to one of the Blackwell's definitions of the more informative experiment, if the posteriors constructed after observing the outcome of experiment P are mean-preserving spread of the posteriors constructed after observing the outcome of experiment Q then experiment P is called to be more informative than experiment Q. More information about Blackwell's informativeness can be found in *Borgers* (2009).

[14]First, formulating the tax authority problem in this way rather than maximizing total welfare allows my analysis to focus on tax enforcement problem. Second, I consider constraint tax revenue maximization rather than *net* tax revenue maximization because, as the model of hierarchical policy

not observe the true income of each individual, it knows the conditional distribution of income given a signal, $F^a(i \mid s)$, and the distribution of signals over the entire population, $H^a(s)$. Thus, the tax authority problem is

$$
\max_{p(r,s),a} \left\{ \int \int T(i,s,p(\cdot))dF^a(i \mid s)dH^a(s) \right\}
$$
$$
s.t. \ \int \int c \cdot p(r(i,s),s)dF^a(i \mid s)dH^a(s) + K(a) \leq B, \tag{1.4}
$$

where c is the cost of an audit, and B is a budget of the tax authority. In what follows the tax authority is assumed to have a limited budget (i.e., $0 \leq B < \frac{c}{1+\pi}$) meaning that the tax authority is unable to audit every taxpayer with probability at least $\frac{1}{1+\pi}$. Otherwise (i.e., if $B = \frac{c}{1+\pi}$), the tax authority would be able to audit everybody and, therefore, nobody would evade taxes.

The costs of improving signal accuracy, $K(a)$, includes several components. First, if improving signal accuracy is achieved through expanding information reporting, then $K(a)$ includes administrative costs of information reporting expansion. Those costs arise because the tax authority has to publicize new rules in social media, create and publish new tax forms, etc. Second, the cost of improving signal accuracy includes the cost of processing the information returns collected from third parties. This is an important component of information reporting cost that is usually left in the shadow, but actually deserves a careful examination. Each year the IRS receives an enormous number of information returns. For example, in 2013 the number of information returns received was 2 billion.[15] To match all these information returns with tax

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described by *Sanchez and Sobel* (1993) shows, society benefits when the government controls the budget of the tax authority rather than allows the tax authority to maximize net tax revenue. In their model, the government first selects a tax function, a level of public good expenditure, and the tax authority's budget and then delegates the responsibility to collect taxes to the tax authority. The model shows that the budget provided by the government is always less than the net revenue-maximizing budget. An extra dollar in the tax authority's budget not just raises extra tax revenue, but also imposes heavier tax burden on society and hence decreases social welfare. Since I consider only tax authority's enforcement problem, I assume that the tax authority is subject to the budget constraint.

[15]See SOI Tax Stats - Information Reporting Program - IRS Data Book Table 14, available at http://www.irs.gov/uac/SOI-Tax-Stats-Information-Reporting-Program-IRS-Data-Book-Table-14.

returns, a sophisticated matching program had to be developed and used. In 2009 the IRS initiated the so-called Information Reporting and Document Matching (IRDM) program in order to implement the two new information reporting requirements - 1099-K and 1099-B information returns. To develop and support this program, IRS spent approximately $110 million during the period 2009 through 2012.[16]

Even without expanding information reporting, there are costs associated with processing and improving information reporting. An improvement in information reporting that increases signal accuracy can be achieving through a better examination of information reports, but this requires investments in computer, software, and programs. Additionally, the IRS has been modernizing its system in order to facilitate the information returns filing and submitting process.[17] Moreover, during the last decade the IRS has been constantly working to improve the matching process of existing programs and refining the methodology of selecting cases for taxpayer contact. However, the *GAO* (2009) report pointed out that the Automated Underreporter (AUR) program – a program responsible for matching 1099-MISC information returns –"currently has a narrow reach and pursues less than half of 1099-MISC-related cases in the AUR inventory" (p. 5). Thus, a scope for further improvement of information reporting remains.

The third important component of information reporting costs is the compliance costs imposed on third parties (firms/taxpayers). Compliance costs include the costs of getting taxpayer identification numbers, buying software, tracking reportable payments, filing returns, and mailing copies to taxpayers or paying to tax preparer. Contrary to common belief, the *GAO* (2007) report found that the size of existing compliance costs was relatively small. Specifically, "[o]ne small business employing under five people told GAO of possibly spending 3 to 5 hours per year filing Form 1099 information returns manually, using an accounting package to gather the in-

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[16]For more details, see *GAO* (2011).

[17]For further details, see *IRS* (2007).

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formation. An organization with more than 10,000 employees estimated spending less than .005 percent of its yearly staff time on preparing and filing Forms 1099, including record-keeping. Two external parties reported prices for preparing and filing Forms 1099 with IRS of about $10 per form for 5 forms to about $2 per form for 100 forms, with one of them charging about $.80 per form for 100,000 forms" (p. 3). Even though the size of the compliance costs seems to be moderate, it should not be excluded from consideration. The cost of improving signal accuracy, $K(a)$, may include a compensation for imposed compliance costs to third parties.

1.4 Analysis

This section describes the solution to problem (1.4). To facilitate the analysis, I subdivide the analysis into two steps. First, I treat signal accuracy as fixed and determine the optimal audit policy and how it depends on signal accuracy. Second, I allow signal accuracy to be chosen and, taking into account the dependence the optimal audit policy on signal accuracy, I derive the optimal level of signal accuracy and the optimal audit coverage. The analysis is concluded by comparative statics which examines how change in the budget and costs affect the solution. Finding from this subsection brings policy relevant implications.

1.4.1 The Optimal Audit Policy: How Does It Depend on Signal Accuracy?

In this subsection signal accuracy is treated as fixed and the optimal audit policy is determined. For notational simplicity, the superscript a is omitted. So, the tax authority problem is

$$
\max_{p(r,s)} \left\{ \int \int T(i, s, p(\cdot)) dF(i|s) dH(s) \right\}
$$
$$
s.t. \int \int c \cdot p(r(i, s), s) dF(i|s) dH(s) + K(a) \leq B,
$$

(1.5)

To identify a simple strategy to solve problem (1.5), we start by examining the nature of the problem. Recall that signals are observed by the tax authority and by taxpayers as well. Consequently, signals cannot be manipulated by taxpayers. Moreover, signals provide information about income distribution, which allows the tax authority to segregate taxpayers into audit classes. Thus, the tax authority can treat all taxpayers with signal, s, as belonging to one audit class. For each audit class (i.e., for each signal), the tax authority can determine the optimal audit function given the amount of resources assigned to conduct audits within this audit class. Let us denote this amount of resources by $B(s) = \int c \cdot p(r(i,s),s)dF(i|s)$. The sub-problem of choosing the optimal audit function within an audit class, given resources $B(s)$, can be analyzed using insights from *Sanchez and Sobel* (1993). Then, the allocation of resources among audit classes (i.e., the size of $B(s)$ for any s) can be chosen to maximize the total tax revenue subject to the constraint $\int B(s)dH(s) \leq B - K(a)$.

Pursuant to this strategy, a solution to problem (1.5), which is described in Proposition 1.1, can be found. While Proposition 1.1 relies on arguments employed by *Sanchez and Sobel* (1993), it extends them to the case when the support of the income distribution is unbounded.[18] Additionally, Proposition 1 does not require assumption (1.1) about conditional distribution of income $F(i|s)$. The conditional distribution function could have bounded or unbounded support, which w.l.o.g. is denoted by $[l(s), h(s)]$. Also, signals do not need to have the same accuracy, hence the variance of the conditional distribution of income could be different for different signals.

Proposition **1.1**. *Assume that for each signal, s, the conditional inverse hazard rate $\gamma(i \mid s) = \frac{1-F(i|s)}{f(i|s)}$ is strictly decreasing in i.[19] The optimal audit function that solves (1.5) satisfies:*

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[18] *Sanchez and Sobel* (1993) consider only the case when the support of the income distribution is bounded.

[19] The condition that the inverse hazard rate $\gamma(i \mid s) = \frac{1-F(i|s)}{f(i|s)}$ is strictly decreasing in i means that if a person has income greater than i, then for her the probability of having the lowest income in interval $[i, +\infty)$ increases in i.

$$p^*(r, s) = \begin{cases} \frac{1}{1+\pi}, & if \ r < \beta(s) \\ \\ 0, & if \ r \geq \beta(s) \end{cases}, \tag{1.6}$$

where the optimal audit cutoff $\beta(s)$ satisfies:

$$\gamma(\beta(s) \mid s) = \begin{cases} = \frac{\lambda c}{t(1+\pi)} & if \ l(s) < \beta(s) \leq h(s) \\ \\ < \frac{\lambda c}{t(1+\pi)} & if \ \beta(s) = l(s) \end{cases}, \tag{1.7}$$

and λ - Lagrange multiplier - is determined so that the budget constraint is satisfied:

$$\int F(\beta(s) \mid s)dH(s) = \frac{(1+\pi)[B - K(a)]}{c}. \tag{1.8}$$

Proof. See proof in Appendix.

Proposition 1.1 shows that the optimal audit rule depends both on reported income and the signal. For a given signal, s, the optimal audit policy is a cutoff rule: audit with probability $\frac{1}{1+\pi}$ if reported income is below the cutoff, $\beta(s)$, and do not audit if reported income is above the cutoff, $\beta(s)$. Note that with such an audit policy in place, a taxpayer's optimal strategy is to choose reported income equal to true income (i.e, $r(i, s) = i$), if her true income, i, is less than the cutoff, $\beta(s)$, and to choose reported income equal to the cutoff (i.e., $r(i, s) = \beta(s)$), if her true income i is greater or equal to the cutoff, $\beta(s)$.[20] [21]

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[20]The cutoff rule results in the solution of the model because this paper following mechanism design literature adopts a commitment principle assuming that the principal announces an audit strategy sufficient to induce truthful reports by the agent and then sticks to it despite it being costly.

[21]*Melumad and Mookherjee* (1989) point out that this type of solution has credibility problem. This rule is not credible because the tax authority has to audit those who do not evade. However, if we interpret this static model from a dynamic prospective - the tax authority has to audit those who not evade today in order that they do not evade tomorrow - this rule looks more reasonable.

The dependence of the audit cutoff, $\beta(s)$, on the signal is crucial. Because of this dependence, it is not just low-income reports, but low-income reports *conditional on signals* that are audited by the tax authority. This means that signals allow the tax authority to target audits to high-income taxpayers. Indeed, without signals, the tax authority would have to audit only truly low-income taxpayers. With signals, it chooses audit probability within each audit class separately. Although, within an audit class the tax authority still has to audit some relatively low-income taxpayers, these taxpayers might be high-income if they belong to a high-income audit class. So, receiving signals helps to redirect audits and reach by audits some high-income taxpayers.

Additionally, conditioning audits on signals makes the derived audit rule more realistic than the simple cutoff rule, like one in *Sanchez and Sobel* (1993) that was criticized for mismatching the reality.[22] Indeed, the derived audit rule, which conditions audit probability on signal/audit class, easily agrees with the experience that audit probability seems to increase with reported income. Even when the tax authority audits taxpayers with low-income reports within an audit class with higher probability than it audits high-report taxpayers, the observed probability of audit that pools audit classes could rise with reported income. Taxpayers with high reported income might have high probabilities of audit because they are in audit classes that signal high income.

Conditions (1.7) and (1.8), which determine the audit cutoff, $\beta(s)$, have intuitive interpretations. Condition (1.8) guarantees that resources spent on audits are equal to the available resources for audits. Condition (1.7) basically requires the marginal revenue of an audit to be the same for all signals (i.e., for all audit classes). This can be seen from considering a small hypothetical increase in the cutoff from $\beta(s)$ to $\beta(s)+\epsilon$, for a given signal, s. In the result of such an increase, those who have income

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[22]See *Scotchmer* (1987).

greater than $\beta(s)$, whose mass is equal to $(1 - F(\beta(s)|s))$, will increase the paid taxes on $t\epsilon$. But, this will require the tax authority to conduct $\frac{f(\beta(s)|s)\epsilon}{1+\pi}$ additional audits at cost c. Condition (1.7) requires that the ratio of marginal revenue to marginal cost of the hypothetical increase in the cutoff, $\frac{t\epsilon(1-F(\beta(s)|s))}{cf(\beta(s)|s)\epsilon(1+\pi)^{-1}} = \frac{t(1+\pi)}{c}\gamma(\beta(s)|s)$, equals λ (the Lagrange multiplier) for signals that require auditing at the optimum.[23] Because marginal cost of an audit, c, is constant, condition (1.7) implies that the marginal revenue of an audit is the same for all signals.

Condition (1.7) also covers the case when taxpayers with a certain signal (audit class) are not audited. It is the case when $\beta(s) = l(s)$. If for some signals the function $\gamma(i|s)$ is lower than $\frac{\lambda c}{t(1+\pi)}$, even for the lowest income, $l(s)$, then the audit cutoff, $\beta(s)$, is equal to $l(s)$. This situation can arise if $l(s)$ is finite and some signals are relatively more accurate than others, so that audits of the taxpayers with such signals cannot generate high enough tax revenue. In turn, this can happen when some taxpayers have a high degree of voluntary compliance because of higher information reporting, and, thus, less opportunity to evade. For instance, those taxpayers might be wage earners, and it might be optimal to concentrate audits only on self-employed people, leaving wage earners unaudited. A rigorous example of a situation when some audit classes are not audited is considered in Appendix A.

The result that optimal audit policy requires the marginal revenue of an audit to be the same for all signals has a valuable practical implication. Specifically, if we assume that there are multiple taxpayers, such that each taxpayer is characterized by her own accuracy, then this result implies that the tax authority should focus audits on those taxpayers about whom it has less accurate signals which means on those who have more opportunity to evade taxes. To see this, consider two groups of taxpayers such that the first group is characterized by signal accuracy, a_1, and second group is characterized by signal accuracy, a_2, which is greater than the first

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[23]$\gamma(i|s)$ shows the proportion of people with income greater than i to people with income exactly equal to i for a given signal s.

signal accuracy (i.e., $a_1 < a_2$). Assume that the conditional distributions of income are $F^{a_1}(i|s_1) = G(a_1(i - s_1))$ and $F^{a_2}(i|s) = G(a_2(i - s_1))$ for the first and second groups correspondingly. Then, the following corollary shows that audit coverage in the group with signal accuracy a_1 is higher than audit coverage in the group with signal accuracy, a_2, when $a_1 < a_2$.

Corollary 1.1. *If $a_1 < a_2$ then $G(a_1(\beta(s_1) - s_1)) > G(a_2(\beta(s_2) - s_2))$.*

Proof. See proof in Appendix.

This corollary shows that optimal audit policy prescribes the tax authority to conduct more audits among taxpayers with less accurate signals than among taxpayers with more accurate signals. Intuitively, prior to audits the expected audit revenue from taxpayers with a low accurate signal is higher then from taxpayers with a high accurate signal. Therefore, the tax authority should conduct more audits among the former ones in order for the marginal revenue of an audit to be the same for both groups. The result that the tax authority should focus audits on those taxpayers, about whom it has less accurate signals, closely matches reality. However, prior to this paper there has been no theory that provided solid ground for it.

1.4.2 The Optimal Signal Accuracy and the Optimal Audit Coverage

In this subsection I allow signal accuracy to be chosen and I determine the optimal level of signal accuracy as well as the optimal audit coverage. In deriving the optimal signal accuracy, I rely on the result from the previous subsection of how the optimal audit rule depends on signal accuracy. Additionally, I suppose that assumption (1.1) on the conditional distribution of income (i.e., $F(i|s) = G(a(i - s))$) is satisfied.

Given this assumption, we can explicitly derived the audit cutoff that is defined by (1.7) and (1.8) and characterizes the audit policy. As equation (1.7) indicates, the audit cutoff is characterized by the conditional inverse hazard rate function, $\gamma(i \mid s) = \frac{1 - G(a(i - s))}{ag(a(i - s))}$. As the distribution function itself, the conditional inverse hazard

rate function, $\frac{1-G(a(i-s))}{ag(a(i-s))}$, depends on i and s additively. Therefore, equation (1.7),

which is $\frac{1-G(a(\beta(s)-s))}{ag(a(\beta(s)-s))} = \frac{\lambda c}{t(1+\pi)}$, implies that $\beta(s) - s$ is a constant that does not

depend on s and depends only on a. By introducing notation $\delta \equiv \beta(s) - s$, equation

(1.8) – the budget constraint – can be simplified to $G(a\delta) = \frac{1+\pi}{c}(B - K(a))$. By

denoting the right-hand side in this equation by P (i.e., $P \equiv \frac{1+\pi}{c}(B - K(a))$) and

by noticing that P is the share of taxpayers audited (with probability $\frac{1}{1+\pi}$), we can

express the audit cutoff as

$$\beta(s) = s + \delta = s + \frac{1}{a}G^{-1}(P). \tag{1.9}$$

For the sake of brevity, I will refer to P as audit coverage. Note that $0 \leq P < 1$,

because $0 \leq B < \frac{c}{1+\pi}$.

As (1.9) shows, the audit cutoff increases in the magnitude of signal, s, and in

audit coverage, P. Intuitively, the higher is the magnitude of signal, s, the higher is

the expected true income, and, therefore, the higher is the cutoff. Also, the greater

is audit coverage, P, the higher is the cutoff, because the tax authority needs to have

a high audit cutoff in order to audit more people.

The cutoff (1.9) is also affected by signal accuracy, a. When signal accuracy in-

creases, for given P, the conditional density of income given signal, $G(a(i-s))$, shrinks

toward its expected value, s, and the cutoff income does the same.[24] Specifically, if

$P \leq \frac{1}{2}$ (i.e., $G^{-1}(P) \leq 0$), then an increase in a leads to an increase in the cutoff.

If $P > \frac{1}{2}$ (i.e., $G^{-1}(P) > 0$), then an increase in a leads to a decrease in the cutoff

. Intuitively, as signal accuracy increases, a signal provides more accurate prediction

of income, which is reflected by the conditional density of income becoming more

concentrated (lower variance) around that signal. This implies that, conditional on

the signal, the probability of income being in close proximity of the signal is higher,

therefore the audit cutoff moves toward the signal.

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[24]Here, I consider an exogenous change in a meaning that audit coverage, P, is fixed.

Importantly, when signal accuracy increases, shrinking of the conditional density of income occurs along with widening of the density of signals. For instance, when all distributions are normal, the distribution of signals, $H(s) = \Phi(\frac{s-\mu}{\sqrt{\Omega^2 - \frac{1}{a^2}}})$, has the variance, $\Omega^2 - \frac{1}{a^2}$, that increases in signal accuracy, a. Thus, as signal accuracy increases, the density of signals becomes more dispersed around the mean. This implies that signals become more distinct and better predict income. In their turn, more distinct signals allow the tax authority to target audits more effectively, which increases tax revenue.

Using expression (1.9) for the audit cutoff, we can calculate the tax revenue. We start by calculating the tax revenue collected from a taxpayer whose signal is s and then we calculate the total tax revenue by integrating over all signals. The tax revenue that is collected from a taxpayer with signal s is $T(s) = t \left[\int_{-\infty}^{\beta(s)} iag(a(i-s))di + \beta(s) \int_{\beta(s)}^{\infty} ag(a(i-s) \right]$ To simplify the integration, let us make the substitution $z = a(i-s)$, then $T(s) = t[\int_{-\infty}^{G^{-1}(P)} (z + as)g(z)\frac{dz}{a} + (s + \frac{1}{a}G^{-1}(P)) \int_{G^{-1}(P)}^{\infty} g(z)dz] = t[s + \frac{1}{a}P\mathbb{E}_G[z|z \leq G^{-1}(P)] + \frac{1}{a}(1 - P)G^{-1}(P)]$, where $\mathbb{E}_G[z|z \leq G^{-1}(P)]$ is an expectation of a random variable with the c.d.f. $G(\cdot)$, denoted by z, conditional on $z \leq G^{-1}(P)$, that is, $\mathbb{E}_G[z|z \leq G^{-1}(P)] = \int_{-\infty}^{G^{-1}(P)} z\frac{g(z)}{P}dz$. The total tax revenue is obtained by integrating $T(s)$ over all signals and is equal to

$$TR = \int_{-\infty}^{\infty} T(s)h(s)ds =$$
$$= t \left[\mu + \frac{1}{a} \left(P\mathbb{E}_G[z|z \leq G^{-1}(P)] + (1 - P)G^{-1}(P) \right) \right]. \tag{1.10}$$

Let us define $R(P)$ as $R(P) \equiv P\mathbb{E}_G[z|z \leq G^{-1}(P)] + (1 - P)G^{-1}(P)$. The expression $t \cdot R(P)$ has a simple interpretation and is equal to the expected tax collection from a taxpayer with signal $s = 0$ if $a = 1$ for whom the audit cutoff is equal to $G^{-1}(P)$. Specifically, with probability P, such a taxpayer has a true income lower than $G^{-1}(P)$, in which case she honestly reports her true income equal in expectation to $\mathbb{E}_G[z|z \leq G^{-1}(P)]$. With probability $(1 - P)$, her true income is greater

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than $G^{-1}(P)$, in which case she reports income equal to the cutoff, $G^{-1}(P)$. Because $R(P) \leq \mathbb{E}_G[z] = 0$, the tax revenue, $TR = t(\mu + \frac{1}{a}R(P))$, increases with a, for given P. Because $R'(P) = \frac{1-P}{g(G^{-1}(P))} > 0$, the tax revenue increases with P. Interestingly, $R'(P)$ is equal to the inverse hazard rate function evaluated at $i = G^{-1}(P)$ given $s = 0$ and $a = 1$ (i.e., $R'(P) = \gamma(G^{-1}(P)|s = 0, a = 1)$).

Finally, the tax revenue is

$$TR(a, P) = t(\mu + \frac{1}{a}R(P)). \tag{1.11}$$

As (1.11) shows, to raise tax revenue, the tax authority can use two tools: to increase signal accuracy and to increase audit coverage. First, by increasing signal accuracy, a, the tax authority makes each audit class more narrow and increases the number of audit classes. As we will see later, signal accuracy can be thought as a proxy for the number of audit classes. Given that in one audit class the tax revenue collected is $R(P)$, the tax revenue collected from all audit classes is $\frac{1}{a}R(P)$. Here it is important to remember that $R(P) < 0$. Second, by increasing audit coverage, P, the tax authority raises the revenue collected within each audit class, that is $R(P)$.

Recall that $P = \frac{1+\pi}{c}(B - K(a))$ and notice that this expression is exactly equivalent to the budget constraint $\frac{c}{1+\pi}P + K(a) = B$. Given this, we can state the tax authority problem as the problem of finding the optimal signal accuracy, a, and the optimal audit coverage, P, which maximize the tax revenue subject to the budget constraint, that is,

$$\max_{a_L \leq a,\, 0 \leq P} t(\mu + \frac{1}{a}R(P))$$
$$\text{s.t. } \frac{c}{1+\pi}P + K(a) = B, \tag{1.12}$$

where $a_L = \frac{1}{\Omega}$. To insure that accuracy, a, is not smaller than the inverse of the standard deviation of the unconditional income distribution, $\frac{1}{\Omega}$, I impose $a \geq a_L$ and assume $K(a) = 0$ for $a \leq a_L$.

Figure 1.1: Iso-revenue curve and the budget constraint
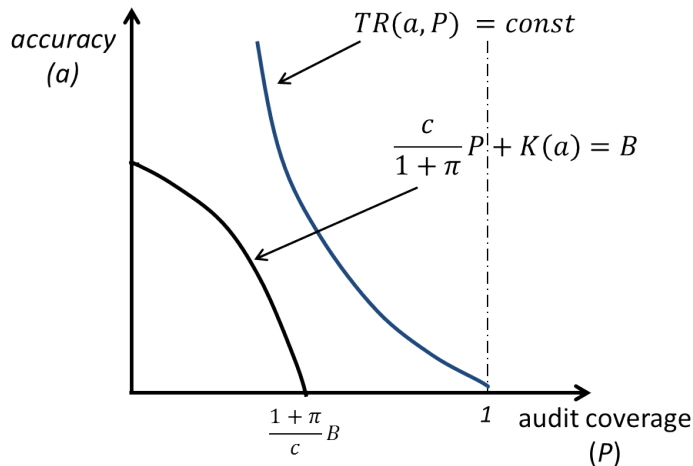
As Figure 1.1 shows, problem (1.12) can be presented graphically in (P, a) space in terms of finding the highest iso-revenue curve, $TR(a, P) = const$, given the budget constraint, $\frac{c}{1+\pi}P + K(a) = B$. An iso-revenue curve can be expressed as $a|_{TR=const} = \frac{-R(P)}{\mu - TR/t}$. From this expression it is clear that iso-revenue curves are strictly convex if $R(P)$ is strictly concave.[25] If $K(a)$ is convex, then the budget constraint curve is concave. The strict convexity of the iso-revenue curves and the concavity of the budget constraint curve ensure that there exists a unique optimal accuracy, a^*. If there is a point where an iso-revenue curve is tangent to the budget constraint curve, then it is the interior solution; otherwise, the solution is a corner solution. Formally, the FOCs for the maximization problem (1.12) can be written as follows:[26]

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[25]Note that, when $G(\cdot)$ is equal to the c.d.f. of the standard normal distribution, $\Phi(\cdot)$, as in the example above, $R(P)$ is indeed concave. It can be shown that $R''(P) = \frac{R(P)}{\varphi^2(\Phi^{-1}(P))} < 0$

[26]Strictly speaking, there are two FOCs for problem (1.12): w.r.t. a and w.r.t. P. Each of them includes a Lagrange multiplier. We can divide one by the other to eliminate the Lagrange multiplier.

$$\frac{\frac{\partial TR}{\partial a}}{\frac{\partial TR}{\partial P}} - \frac{1+\pi}{c}K'(a^*) = -\frac{R(P^*)}{a^*R'(P^*)} - \frac{1+\pi}{c}K'(a^*) = \begin{cases} = 0 & \text{if } a^* > a_L, \ P^* > 0 \\ \\ > 0 & \text{if } a^* > a_L, \ P^* = 0 \\ \\ < 0 & \text{if } a^* = a_L, P^* > 0. \end{cases}$$

$$(1.13)$$

The solution of (1.12) is determined by equation (1.13) and the budget constraint:

$$\frac{c}{1+\pi}P^* + K(a^*) = B. \tag{1.14}$$

Equation (1.13) has a simple interpretation. At an interior optimum, the ratio of the marginal tax revenue of increasing signal accuracy to the marginal tax revenue of increasing audit coverage is equal to the ratio of the marginal cost of increasing signal accuracy to the marginal cost of increasing audit coverage.

The following proposition characterizes the optimal solution.

Proposition 1.2. *i) If the inverse hazard rate $\gamma(z) = \frac{1-G(z)}{g(z)}$ is strictly decreasing and $K(a)$ is convex then the point – the accuracy a^* and audit coverage P^* – which satisfies to the condition (1.13) and (1.14) is the unique solution of (1.12);*

ii) If additionally $K'(a_L) = 0$ then the optimal solution is an interior solution characterized by

$$-\frac{R(P^*)}{a^*R'(P^*)} = \frac{1+\pi}{c}K'(a^*). \tag{1.15}$$

Proof. In the appendix.

Proposition 1.2 provides conditions that ensure the existence of a unique interior optimum. Relying on this result and imposing these assumptions hereafter we can examine the properties of the optimum.

Figure 1.2: The optimal solution (P^*, a^*) as intersection of the FOC condition and the budget constraint
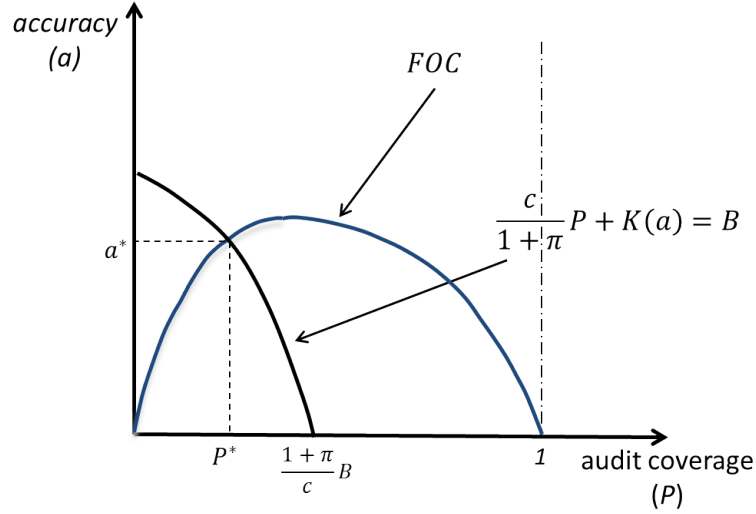
Figure 1.2 illustrates how the interior solution can be determined as the intersection of the FOC curve that represents solutions of (1.15) and budget constraint (1.14). As Figure (1.2) shows, the FOC curve has an inverse U-shape. This indicates a specific substitution pattern between signal accuracy and audit coverage. By the analogy to utility theory, let us define the marginal rate of substitution of signal accuracy, a, for audit coverage, P, as the ratio of the marginal tax revenue of increasing signal accuracy to the marginal tax revenue of increasing audit coverage, that is,

$$MRS_{a,P} = \frac{\frac{\partial TR}{\partial a}}{\frac{\partial TR}{\partial P}} = -\frac{R(P)}{aR'(P)}. \tag{1.16}$$

The marginal rate of substitution of signal accuracy, a, for audit coverage, P, shows how many audits could be saved through an increase in signal accuracy, keeping tax revenue the same. Based on an analogy with utility theory for two normal goods, one might expect that the marginal rate of substitution would decrease with signal accuracy and increase with audit coverage. $MRS_{a,P}$ indeed decreases with signal accuracy, a. However, $MRS_{a,P}$ first increases, but then decreases with audit coverage, P.

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This behavior is explained by two factors. First, the marginal tax revenue of increasing signal accuracy (i.e, the numerator in (1.16)) decreases with audit coverage. This occurs because the value of targeting audits declines with audit coverage. Intuitively, when audit coverage is high, there are not many high-income taxpayers that would be unaudited and need to be targeted. Second, when audit coverage is low, the marginal tax revenue of increasing audit coverage (i.e., the denominator in (1.16)) is very high, and consequently the relative value of signal accuracy is low. When audit coverage is low, the feasible number of audits is low and, thus, there are not many audits that can be targeted. As a result, the marginal rate of substitution of signal accuracy for audit coverage first increases, but then it decreases with audit coverage.

Intuition To build intuition for why the value of signal accuracy declines with audit coverage, let us consider an example. Recall first that in this model signals facilitate tax audits, because they allow the tax authority to subdivide taxpayers into audit classes and to conduct audits within each audit class separately. The example explains why the tax revenue is increased by subdividing people into audits classes and how this additional revenue depends on audit coverage.

The example compares two cases: when the tax authority conducts audits without obtaining any signals about taxpayers' income and when the tax authority does obtain signals allowing dividing taxpayers into audit classes. In each case, I describe how the audits are conducted and what the reported income is. For simplicity, the example assumes that taxpayer's true income is distributed uniformly on $[-1, 1]$. The rest of the assumptions about a taxpayer are the same.

First, consider a case when the tax authority does not have any signals about taxpayers' income and, therefore, all taxpayers belong to one audit class. According to Proposition 1.1, the audit rule in this case is to audit those taxpayers whose reported income is less than the audit cutoff, β, with probability $\frac{1}{1+\pi}$, and not audit

Figure 1.3: The example illustrating the value of dividing taxpayer into audit classes

(a)

(b)
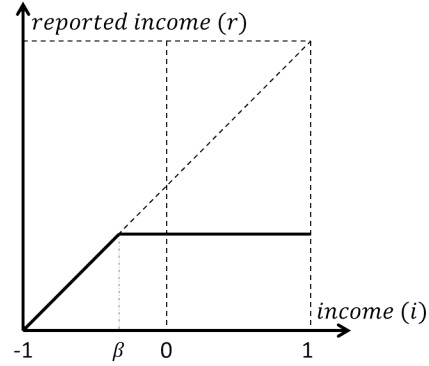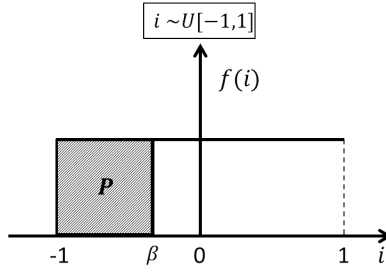
(c)
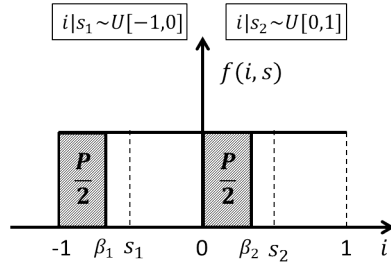
(d)
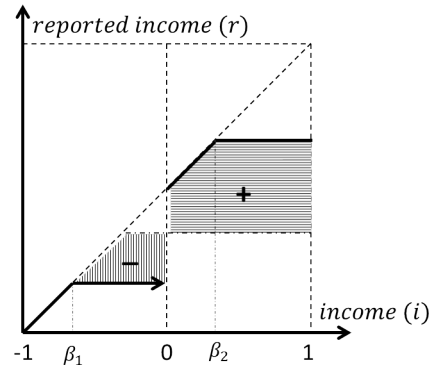
those whose reported income is above the audit cutoff, β. If audit coverage is P, then the audit cutoff, β, is equal to $2P - 1$. This is indicated in Figure 1.3(a). The next panel of the figure – Figure 1.3(b) – shows how reported income depends on true income. When true income is less than the audit cutoff, β, taxpayers' reported income equals to true income. When true income is above the audit cutoff, β, reported income is equal to the audit cutoff, β. Given reported income, we can calculate the tax revenue in this case. It is equal to $TR_1 = -t(1 - P)^2$. This case provides a baseline to which the following case is compared.

Consider the second case when the tax authority is able to obtain two signals such that conditional on the first signal, s_1, income is uniformly distributed on $[-1, 0]$ and conditional on the second signal, s_2, income is uniformly distributed on $[0, 1]$. The

probability of each signal is $\frac{1}{2}$. These signals allow the tax authority to subdivide taxpayers into two audit classes and conduct audits within each class separately. Figure 1.3(c) illustrates how the audits are allocated within each audit class. For an easy comparison with Figure 1.3(a), Figure 1.3(c) shows the joint distribution of income and signal (i.e., $f(i,s) = f(i|s)\frac{1}{2}$).

The optimal audit strategy in each audit class is calculated according to Proposition 1.1. If a taxpayer's signal is s_1, then she is audited if her reported income is less than the audit cutoff, β_1. If taxpayer's signal is s_2, then she is audited if her reported income is less than the audit cutoff, β_2. If audit coverage is P, then the audit cutoff, β_1, is equal to $P - 1$ and the audit cutoff, β_2, is equal to P. This implies that $\frac{P}{2} \cdot 100$ percent of taxpayers is audited in the first audit class and $\frac{P}{2} \cdot 100$ percent of taxpayers is audited in the second audit class. The share of audits conducted in each class is the same because the marginal revenue of an additional audit should be the same in both audit classes.[27]

This audit strategy induces reported income as shown in Figure 1.3(d). Reported income is equal to true income when true income i belongs to either $[-1, \beta_1)$ or $[0, \beta_2)$, reported income is constant and equal to β_1 when $i \in [\beta_1, 0]$, and reported income is constant and equal to β_2 when $i \in [\beta_2, 1]$.[28] As can be seen, reported income in this case differs from reported income in the baseline case shown in Figure 1.3(b). When audits are based on signals, reported income is substantially higher for $i \in [0, 1]$ and lower for $i \in [\beta_1, 0]$. So, dividing taxpayer into audit classes helps to deter evasion by some high-income taxpayers.

The tax revenue in this case is higher than tax revenue in the baseline case and is equal to $TR_2 = -\frac{t}{2}(1 - P)^2$. Interestingly, in the formula $TR_2 = -\frac{t}{2}(1 - P)^2$ the division of $-t(1 - P)^2$ by 2 resembles the division of $R(P)$ by accuracy a in the

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[27] Recall that the cost of an audit, c, is constant for all taxpayers.

[28] When true income is equal to 0, reported income could be equal to β_1 or 0 depending on whether the signal is s_1 or s_2.

Figure 1.4: The example illustrating the value of dividing taxpayer into audit classes: the case of high audit coverage

(a)

(b)

(c)

(d)
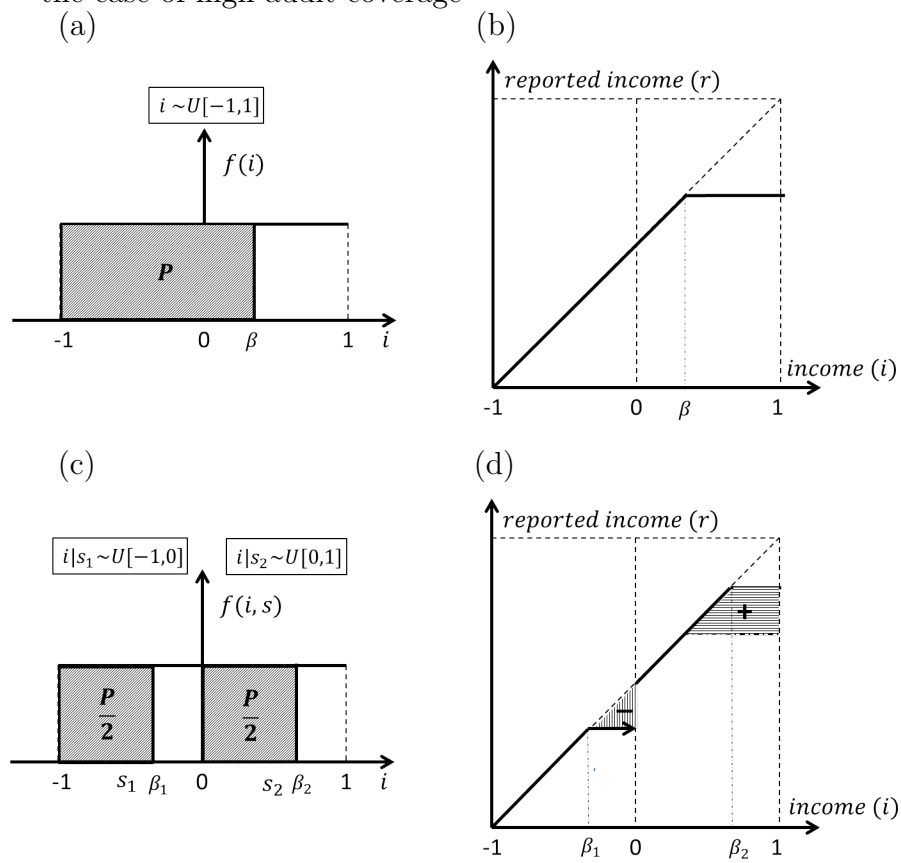
formula for tax revenue $TR(P) = t(\mu + \frac{1}{a}R(P))$ in general model. So, signal accuracy can be thought as a proxy for the number of audit classes. Overall, the comparison of the case when audits are based on signals with the baseline case shows that the ability to subdivide taxpayers into audit classes helps to increase tax revenue because audits can be redirected toward some high-income taxpayers.

The additional tax revenue obtained by dividing taxpayers into audit classes decreases with audit coverage, P. To show this, Figure 1.4 presents exactly the same example as Figure 1.3 but when audit coverage, P, is higher. As can be seen, there are more high-income taxpayers who are audited in the baseline case when audit coverage, P, is higher. As a result, the additional revenue from dividing taxpayers into audit classes is smaller. This also can be seen direct from the formula $TR_2 - TR_1 = \frac{t}{2}(1 - P)^2$. Indeed, $TR_2 - TR_1$ decreases with P.

What this example cannot illustrate is how the revenue of increasing the number of audit classes changes relative to the revenue of increasing audit coverage. This relative change is important for understanding the substitution pattern, because the substitution between signal accuracy and audit coverage is determined by the relative change in revenue. By definition, the substitution pattern depends on how marginal revenue from increasing signal accuracy changes relative to the marginal tax revenue from increasing audit coverage. To understand the behavior of the relative revenue change, we have to rely on the general model.

1.4.3 Comparative statics

In this section I analyze how the budget and costs affect the optimal signal accuracy and the optimal audit coverage. I examine first an effect of a change in the budget and second an effect of a change in the costs of audits as well as costs of improving signal accuracy. The finding of inverse an U-shaped dependence of the optimal signal accuracy on the budget leads to important policy implications.

1.4.3.1 Budget Change

The effect of the tax authority's budget change can be analyzed using Figure 1.2. An increase in the budget corresponds to a horizontal shift of the budget constraint curve to the right. As the budget constraint curve shifts to the right, the optimal solution moves along the FOC curve to the right. As a result, the optimal accuracy first increases and then decreases with the budget, B, while audit coverage always increases with the budget. The following proposition formally states this result.

> **Proposition 1.3**. *Assume that assumptions i) and ii) of Proposition 2 are satisfied. There exist a budget threshold, $\overline{B}$, such that when the budget is small, $B < \overline{B}$, the optimal signal accuracy, a^*, increases with the budget, B. However, when the budget is large, $B > \overline{B}$, the optimal signal accuracy, a^*, decreases with the budget. The optimal audit coverage, P^*, always increases with the budget, B.[29]*

Proof. In the appendix.

The results of Proposition 1.3 are illustrated in Figure 1.5. This dependance of the optimal solution on the size of the budget is the direct consequence of the substitution pattern between signal accuracy and audit coverage that has been explained in the previous subsection. Recall that the marginal rate of substitution first increases, but then decreases with audit coverage. Since the optimal audit coverage is itself rises with budget, the optimal signal accuracy first rises, but then declines with the budget.

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[29]To address a potential curiosity regarding whether the solution to the net revenue maximization problem lie to the left or to the right of the threshold budget, $\overline{B}$, I provide the answer. It could lie either to the left or to the right of the threshold budget, $\overline{B}$. The reason for this is the following. The solution to the net revenue maximization problem the point on the FOC curve depicted in Figure 1.2 for which the Lagrange multiplier from Proposition 1.1 is equal to one. This point can be determined as an intersection the FOC curve depicted in Figure 1.5 with the curve defined by equation $a = \frac{t(1+\pi)}{c} R'(P)$, which is a decreasing and convex curve. Note that the slope of $a = \frac{t(1+\pi)}{c} R'(P)$ depends on the value of t, π, and c. Therefore, the intersection is governed by the value of t, π, and c and can occur to the left or to right of the pick of the arc.

Figure 1.5: The effect of budget, B, change on the optimal solution (P^*, a^*)
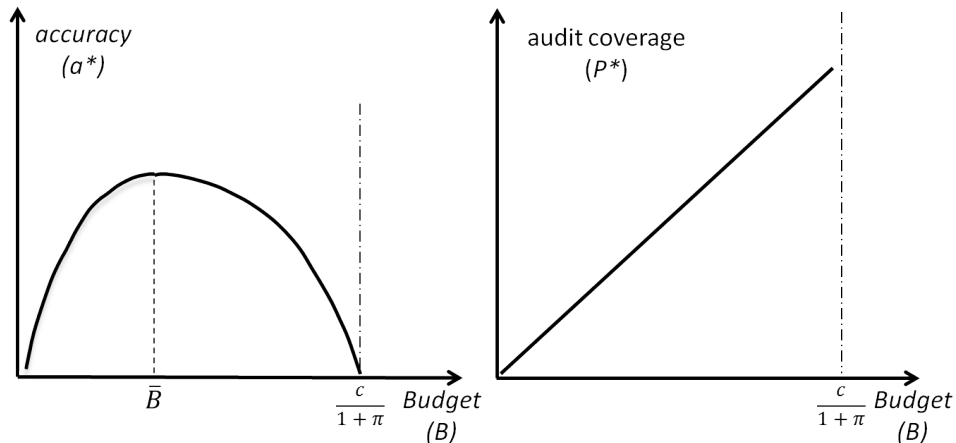
These results have important policy implications. To describe these implications, we step up from the model to real life by equating investments in signal accuracy with investments in information reporting. The first implication concerns the tax authorities that have relatively scarce resources to enforce tax compliance. For them, it is critical to invest in information reporting. As follows from Proposition 1.3, tax authorities with small budgets on average rely on information reporting relative to audits more than tax authorities with large budgets. The second implication concerns the tax authorities that have substantial resources. For them it may not be optimal to extend information reporting further.

1.4.3.2 Cost Increases

In order to examine the effect of an increase in the cost of signal accuracy on the optimal signal accuracy and the optimal audit coverage, we need to parametrize the cost of signal accuracy, because we cannot conduct comparative statics with respect to a function. I do this by introducing an additional parameter δ. I assume that the tax authority budget constraint is $\frac{c}{1+\pi}P + \delta K(a) = B$. The only difference of this budget constraint formula from the preceding one is in parameter δ that augments

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the cost function $K(a)$. When $\delta = 1$, this formula is identical to the preceding one. This addition of δ into the budget constraint allows us to model a raise in the cost of improving signal accuracy as an increase in parameter δ. The effect of an increase in the cost of improving signal accuracy is described in the following proposition. The proposition also characterizes how the optimal signal accuracy and the optimal audit coverage change when the cost of an audit, c, increases or the penalty rate, π, decreases.

Proposition 1.4. *Assume that assumptions i) and ii) of Proposition 1.2 are satisfied.*

i) When δ increases (i.e., the cost of improving signal accuracy increases), the optimal signal accuracy decreases, $\frac{da^*}{d\delta} < 0$, the optimal audit coverage may increase, decrease, or not change, $\frac{dP^*}{d\delta} \gtreqqless 0$, depending on the shape of the cost function $K(a)$.

ii) When $\frac{c}{1+\pi}$ increases (i.e., the cost of an audit increases or the penalty rate decreases), the optimal audit coverage decreases, $\frac{dP^*}{d\frac{c}{1+\pi}} < 0$, while the optimal signal accuracy may increase, decrease, or not change, $\frac{da^*}{d\frac{c}{1+\pi}} \gtreqqless 0$, when budget B is less than $\overline{B}$, and the optimal signal accuracy increases, $\frac{da^*}{d\frac{c}{1+\pi}} > 0$, when budget B is greater than $\overline{B}$.

Proof. In the appendix.

The results of this proposition are easy to interpret by considering two types of effects, analogous to the substitution and income effects from utility theory. An increase in δ can be interpreted as an increase in the "price" of signal accuracy. The substitution effect is the result of an increase in the *relative* "price" of signal accuracy and causes a decrease in signal accuracy and an increase in audit coverage. The income effect of the "price" raise is due to the decrease in real budget (income) and causes a decrease in both signal accuracy and audit coverage. Therefore, when the

"price" of signal accuracy increases, the optimal signal accuracy decreases, because both substitution and income effects are negative. However, the effect of an increase in the "price" of signal accuracy on the optimal audit coverage is uncertain, because the substitution effect is positive, but income effect is negative. Which effect dominates depends on the shape of cost function $K(a)$. For example, when $K(a)$ is equal to qa^m where $q > 0$ and $m \geq 1$, the optimal audit coverage does not change with δ (i.e., $\frac{dP^*}{d\delta} = 0$).

Similarly, an increase in $\frac{c}{1+\pi}$ can be interpreted as an increase in the "price" of audits. Therefore, when the "price" of audits increases, the optimal audit coverage decreases, because both substitution and income effects are negative. For the optimal signal accuracy, the substitution effect is positive, however, the sign of the income effect depends on the size of the budged. If $B < \overline{B}$ the income effect is negative, if $B > \overline{B}$ the income effect is positive. This is the consequence of Proposition 1.3. Hence, the change in the optimal signal accuracy is uncertain, $\frac{da^*}{d\frac{c}{1+\pi}} \gtreqless 0$, when the budget, B, is less than $\overline{B}$, and the optimal signal accuracy increases with $\frac{c}{1+\pi}$, $\frac{da^*}{d\frac{c}{1+\pi}} > 0$, when the budget, B, is greater than $\overline{B}$.

1.5   Discussion

In this section I discuss the results, their policy implications, and connections with the real world practice. I also critically examine assumptions of the model to access its applicability.

The model of this paper provides us new insights regarding the optimal enforcement of tax evasion. First, we learned how audit policy should be conducted when the extent of information reporting varies among people with various income sources such as wage earners and self-employed individuals. It is optimal for the tax authority to concentrate tax audits on those taxpayers about whom the tax authority has less accurate information. This result confirms our experience: audit probability is higher

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for self-employed people rather than for wage earners.

The second result of the model characterizes the optimal allocation of resources between tax audits and information when the tax authority has a limited budget. The optimal allocation of resources depends on the tax authority's budget such that the optimal level of information reporting initially increases, but then decreases when the budget increases, while the optimal audit coverage always increases with the budget. In other words, the optimal level of information reporting depends on the budget as an inverse U-shape function. This implies that there is a level of the budget at which it is no longer optimal to expand information reporting.

This result has two important implications. One implication is that investments in information reporting are especially important when the tax authority has scarce resources to conduct tax enforcement. Another implication is for a tax authority that already has substantial enforcement resources. Such a tax authority could be close to a point at which its no longer optimal to expand information reporting and therefore has to carefully examine any decision to enlarge their information reporting system.

Before we discuss these implications in the light of real world practice, it worth to evaluate the applicability of the model by recalling its main assumptions. First, when the trade-off between tax audits and signal accuracy is considered, it is assumed that all signals are characterized by one accuracy, which implies that the accuracy of each signal improves in a similar manner. Therefore, this part of the model is applicable to the case when optimal enforcement is determined among people with similar income sources. The model can be easily extended to the case when there are several groups of people with different income sources. The problem with several income source groups can be solved in two stages. The first stage would be to determine how to allocate resources *within* a group. The solution to this stage can be obtained by using the model in this paper. The second stage would be to determine how to allocate resources *among* the groups.

Two more assumptions are worth discussing. One assumption concerns the costs of improving information reporting. In practice, one part of these costs – compliance cost – is born by third parties (firms/taxpayers). In the model, I assume that all costs of improving information reporting are imposed on the tax authority. However, I still account for compliance cost because I assume that the total costs of improving information reporting include compensation to third parties for compliance cost.[30] Since the paper explores only the implication of information reporting of taxpayers' evasion behavior and no other aspects of behavior, this is a valid approach to account for compliance cost. Another assumption concerns the framework. I consider a one-period model, which does not allow accounting for "durable" property of information reporting. Once information reporting is put in place, it can be used in the next period at a smaller cost. However, in order to correctly model information reporting in dynamic settings, we need a better understanding of the size and shape of the cost of information reporting over time. Unfortunately, little is known about the costs of improving information reporting because of the absence of empirical studies that estimate this.

Keeping in mind these assumptions, we can return to the result implications and discuss their connection to real world practice. The first implication, that it is especially efficient to invest in information reporting when resources on enforcement are small, most likely applies to developing countries. While it is hard to estimate the use of information reporting by developing countries due to the lack of data, according to general observations, a number of developing countries has been used some information reporting. But, many developing countries seem to underestimate the benefits of information reporting. If so, the policy of adaptation of information reporting could be highly beneficial for the developing countries. The findings of *Gordon and Li* (2009) support this claim. Their study stresses that for developing countries the role

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[30]It can be interpreted as if the tax authority "buys" information from the third parties.

of financial institutions as third parties in tax enforcement is significant. Additionally, *Bird and Zolt* (2008) point out that recent technological progress significantly simplified the adaptation of information reporting in developing countries. Moreover, some developing countries have been successfully using tax withholding, which in many senses is similar to information reporting. In her study of the provision and proposals for withholding on business income in nine countries, *Soos* (1990) finds that developing countries seem to make greater use of withholding systems than those of industrialized countries.[31]

The second implication, which concerns countries that have substantial information reporting, is probably more relevant for developed countries. They might be close to the point where it is no longer optimal to expand an information reporting system. For them it becomes critical to perform a thorough cost-benefit analysis when an expansion of information reporting is considered. A decision whether this expansion is worthwhile should be made based on this analysis. Moreover, the cost calculation should definitely account for the indirect cost of information reporting that is diverting resources from audits.

The United States is an example of a developed country that has recently expanded its information reporting system. It introduced a requirement for banks to report the gross amount of merchant payment card transactions (1099-K Form), and brokers to report the adjusted cost basis for certain securities (1099-B Form). It also extended requirements for businesses regarding payments to vendors, subcontractors, and independent contractors (1099-MISC). To implement these new information reporting requirements, the IRS had to develop a new program that matches information returns with tax returns. The cost of this project constituted about $110 million during

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[31]Indonesia, Pakistan, and Egypt withhold on payments for goods and supplies, as well as on fees for services. The Philippines, Japan, and Ireland provide for withholding on fees for services. The United States has proposed withholding on fees for services. The United Kingdom, Ireland, and Australia have narrow provisions which limit withholding to fees for work or services in certain industries.

the period 2009 through 2012.[32] The exact revenue benefits of this expansion are still to be estimated. *Slemrod et al.* (2015) investigate some preliminary responses and find that the introduction of form 1099-K has led to a relatively small overall change in reported tax liability, while it has significantly increased tax reports within a small subgroup of sole proprietors.

1.6 Conclusion

The accuracy of information observed by the tax authority about taxpayers affects the extent of the asymmetry of information between them and therefore affects the level of tax evasion. To improve the accuracy of information and thus to deter tax evasion, the tax authority can use information reporting – a requirement on third parties to send reports to the tax authority with information about income payments to taxpayers that allows verifying taxpayers' tax returns.

Nevertheless, the optimal tax enforcement theory has not considered information reporting as an instrument to improve tax compliance, focusing primarily on tax audits. This paper extends tax theory by introducing a way to model and endogenize information reporting within a tax evasion framework. Moreover, this paper characterize the optimal allocation of resources resources between tax audits and information reporting, when both of these instruments are costly, and the tax authority has limited budget.

Specifically, in the model, information reporting provides the tax authority with signals that help to construct a prediction about each taxpayer's income distribution. By investing in information reporting, the tax authoriby can obtain more signals that help to produce a more accurate prediction of income distribution. However, an increase in signal accuracy is costly and diverts resources from tax audits. When the tax authority has a limited budget, it has to decide how to allocate resources

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[32]For more details see GAO (2011).

between tax audits and signal accuracy in order to maximize tax revenue. The model solves this trade-off by determining the optimal signal accuracy and the optimal audit coverage, and it also explains how the choice of one instrument depends on the choice of the other.

First, the model explains how the optimal tax audit policy depends on signal accuracy. In the model, the audit probability is a function of the taxpayer's reported income, as well as of the signal. In the optimum, the tax authority should concentrate tax audits on those taxpayers about whom it has less accurate signals, so that the marginal tax revenue is the same among all taxpayers. The practical flavor of this result is revealed when it is applied to people subject to information reporting to various extents. When two groups of taxpayers differ in the extent of information reporting, like wage earners and self-employed individuals, the optimal policy instructs to focus audits on the group with the lower extent of information reporting, like the self-employed.

Second, this paper derives the optimal allocation of resources between information reporting and audits. On the cost side, to raise signal accuracy the tax authority has to deprive resources from tax audits. On the benefit side, as signal accuracy rises, the taxpayers can be divided into more distinct audit classes, which allows the tax authority to target audits more effectively and, therefore, raise tax revenue. The model enables measuring the benefit of an increase in signal accuracy. I show that the value of an increase in signal accuracy depends on audit coverage and, specifically, it declines with audit coverage. It is because the value of audit targeting is lower as audit coverage is higher. At the same time, when audit coverage is low, the value of an audit is very high and, therefore, the relative value of signal accuracy is low. This interaction between signal accuracy and audit coverage leads to a particular dependence of the optimal solution on the tax authority's budget. The optimal signal accuracy depends on the budget as an inverse U-shaped function, while the optimal

audit coverage always increases with the budget. This indicates that for some budgets it becomes no longer optimal to increase signal accuracy.

This finding has two important implications. First, the investment in information reporting appears to be especially critical for developing countries. Second, for countries that have an extensive enforcement system, it may not be optimal to expand information reporting system further.

This paper clarifies that though both audits and information reporting help the tax authority to access information about taxpayers, their mechanisms of accessing information are different. As a result of this difference, the problem of optimal tax enforcement should be considered as a problem of *joint* choice of audit coverage and information reporting scope. This paper starts to address this question and highlights the necessity for further research in this area.

CHAPTER II

Collateral Tax Sanctions: A Way to Correlate Punishment with Ability

2.1   Introduction

Recently, to improve tax compliance, tax authorities have used a new punishment instrument – collateral tax sanction, which is a revocation of a privilege provided by the government, imposed for a failure to comply with tax obligations. An example of a collateral tax sanction is suspension of driver's licenses for tax noncompliance. Currently, three states – Louisiana,[1] California,[2] and New York state[3] – have established a driver's license suspension program which allows tax departments to suspend a driver's license from persons with delinquent tax liabilities.

Other examples of collateral tax sanctions used by some states are suspension of

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[1] See, e.g., Tax Topics, Louisiana Department of Revenue, Volume 24 Number 2 April 2004, available at http://www.revenue.louisiana.gov/forms/publications/tt(04_04).pdf. See also Suspension and Denial of Renewal of Drivers' Licenses (LAC 61:I.1355), Louisiana Department of Revenue, available at http://revenue.louisiana.gov/forms/lawspolicies/LAC61_I_1355.pdf

[2] See, e.g., California to Tax Scofflaws: Pay Up or Lose your Driver's (or CPA) License, AccountingWeb.com, Sept. 20, 2011, available at http://www.accountingweb.com/topic/tax/california-tax-scofflaws-pay-or-lose-your-drivers-license. See also Franchise Tax Board Meeting, September 5, 2012: Delinquent Taxpayer Accountability Act Informational Item available at https://www.ftb.ca.gov/law/meetings/attachments/090512/3.pdf

[3] See, e.g., States target tax scofflaws with incentives and shame, usatoday.com, Oct. 16, 2013 , available at http://www.usatoday.com/story/news/nation/2013/10/16/states-target-tax-scofflaws/2993447/. See also Summary of Budget Bill Personal Income Tax Changes Enacted in 2013, New York State Department of Taxation and Finance, Aug. 8, 2013, available at http://www.tax.ny.gov/pdf/memos/income/m13_4i.pdf

vehicle registration,[4] revocation of professional licenses,[5] and denial of hunting and gaming permits to residents who have failed to satisfy their tax obligations.[6] At the federal level under current and proposed laws, the failure to pay taxes owed may result in the loss of ability to obtain federal employment, apply for Federal Housing Authority mortgages, and enter contracts with the federal government. It also may result in revocation of passports, imprisonment, or deportation from the country.[7]

While collateral tax sanctions have become a popular tool of tax administration, they have not been extensively studied by economic scholars. The existing literature distinguishes monetary fines from non-monetary penalties, but finds monetary fines to be a preferable instrument. Specifically, *Becker* (1968), *Polinsky and Shavell* (1984), and *Andreoni* (1991, 1992) and others show that monetary fines should always be exhausted before non-monetary penalties are imposed, because non-monetary penalties are generally more costly to administer.

A recent law paper by *Blank* (2014) argues that collateral tax sanctions can promote voluntary tax compliance more effectively than monetary fines. To support the statement, he proposes three arguments. First, collateral tax sanctions may be more salient to individuals. Second and third, they may provoke feelings of reciprocity or fear of the stigma of tax noncompliance. These arguments are appealing, but mainly drawn on the behavioral aspects of decision making and do not exhaust all economic reasons for why collateral tax sanctions may be an attractive tax administration tool.

This paper proposes a new economic rationale for the use of collateral tax sanctions. By affecting consumption and providing enforcement targeted to a group, collateral tax sanctions may allow the government to impose punishment correlated

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[4]See Jay Soled, Using Driving Privilege to Solve States' Fiscal Crises, 60 STATE TAX NOTES 841 (June 13, 2011)

[5]See, e.g., Wis. Stat. 73.0301(d)(11) (revocation of law licenses); Min Stat. 270C.72 (revocation of medical licenses).

[6]See, e.g., Louisiana Dep't of Wildlife and Fisheries, Hunting Licenses, available at http://www.wlf.louisiana.gov/licenses/hunting-licenses (last visited Oct. 22, 2012).

[7]See Blank (2014) for more details.

with individual's earning potential. Such punishment makes the effective tax rates also correlated with individual's earning potential and, as a result, enables to achieve the redistribution of income more effectively. The mechanism, by which a collateral tax sanction improves the redistribution of income, resembles Akerlof's tagging. Similar to a tag, which indicates a taxpayer's category, a collateral tax sanction reduces the cost of income redistribution and, therefore, increases social welfare. The cost of redistribution arises when a tax schedule depends on income and not on ability, because such a tax system distorts labor supply decisions.

For the described mechanism, it is essential that collateral tax sanctions affect consumption directly, that is not through affecting income. Indeed, in contrast to a monetary fine that reduces taxpayer's income, a collateral tax sanction prohibits consumption of a specific good or terminates a specific activity. Given that consumption baskets differ among individuals, only a group of individuals who have that specific good in their baskets are affected by the collateral tax sanction. For example, only people who have the international passport are affected by revocation of one. Therefore, a collateral tax sanction provides enforcement targeted to a group of taxpayers.

The group of taxpayers that is targeted by a collateral tax sanction can differ in their skill distribution from the group that is not affected by the collateral tax sanction. For example, revocation of an international passport mainly affects those who have opportunities to travel abroad and likely have a higher earning potential on average than those who does not have an international passport. This illustrates that collateral sanctions may allow the tax authority to correlate punishment with taxpayer's ability.

The next important link in the mechanism of the collateral tax sanctions is the connection between punishment and effective tax. The model in this paper shows that punishment for tax avoidance affects effective tax rate. Moreover, by targeting

enforcement to a group of taxpayers, the tax authority rises the effective tax rate for that group. Thus, a collateral tax sanction leads to a higher effective tax rate for the targeted group of taxpayers and, as a result, helps to achieve a redistribution of income from the targeted group to the other group.

There is, however, an important difference between a collateral tax sanction and a tag. Unlike a tag that allows the government to set a separate statutory tax for the tagged group, a collateral tax sanction allows the government to influence the effective tax for the targeted group, but not the statutory tax. Because collateral tax sanctions affect only the effective tax, they are more restrictive than tags and, therefore, less efficient. However, in practice collateral tax sanctions might be more feasible than tags for political reasons. Note also that unlike a tag, a collateral tax sanction imposes some real cost on the taxpayers and, therefore, reduces social welfare. For example, suspension of an international passport restricts an individual's ability to travel, which likely decreases her utility.

The model in this paper aims to examine the welfare and redistribution consequence of the imposition of a collateral tax sanction for tax noncompliance. The model draws on the model in *Cremer et al.* (2010), which analyzes gains and losses as a consequence of tagging, and inherits its assumptions that the social welfare function is Rawlsian and preferences are quasi-linear and have a constant elasticity of labor supply. In the model, there is a continuum of individuals who are characterized by their skills. By imposing a collateral tax sanction, the government can rise the effective tax rate in the targeted group of taxpayers. I show that as a result of this, the new optimal statutory tax rate decreases, which allows to increase the utility of the rest of population at the cost of decreasing the utility of taxpayers in the targeted group. The social welfare increases only when the earning potential of the poorest individual in the targeted group is sufficiently higher than the earning potential of the poorest individual in the rest of the population. In contrast, a tag improves social

welfare even in the case when the supports of skill distributions for two groups are the same.

This paper proceeds as follows. Section 2 explains the mechanism through which a collateral tax sanction provides enforcement correlated with ability and as a result lead to a more efficient redistribution. Section 3 analyzes the welfare and redistribution consequence of the imposition of a collateral tax sanction. Section 4 discusses potential concerns and indirect effects of the use of collateral tax sanctions. Section 5 concludes.

2.2 From a Collateral Tax Sanction to a Tag

2.2.1 Collateral Tax Sanctions: In-kind Restriction that Affects a Subgroup of Taxpayers

By affecting consumption, collateral tax sanctions have a differential effect on taxpayers. In contrast to a monetary fine that affects income, a collateral tax sanction restricts consumption of a certain good or activity. For example, suspension of a driver's license causes a delinquent taxpayer to stop driving; revocation of an international passport restricts the ability to travel abroad. Not everybody, however, has a driver's license or an international passport. Consumption baskets differ among individuals. Therefore, only a group of people who have the restricted good or activity in their consumption baskets are influenced by the collateral tax sanction. For example, only people who have international passports are affected by revocation of an international passport.

The group affected by a collateral tax sanction could have different characteristics than the unaffected group. One important characteristic for taxation and redistribution purpose, on which I focus in this paper, is earning potential or ability. The distribution of ability within the affected group could be different than within the

unaffected group. For example, a revocation of boat registration or suspension of boating safety certificate as a sanction for tax evasion mostly affects wealthy people and fishermen. A revocation of an international passport mainly affects those who have opportunities to travel abroad. In these examples, the people with a boat certificate and people with an international passport are likely to have a higher earning potential than people without those documents.

To give more structure to this idea, assume that the government can use a collateral sanction that affects the consumption of a certain good. Refer to the group of individuals that do not have this good in there consumption basket as group 1. Refer to the group of individuals that do have this good in their consumption basket as group 2. Assume also that individuals are characterized by a skill level, w, (equal to the wage rate). Denote the distribution function of skill for the entire population by $F(w)$ and its corresponding density by $f(w)$. Denote also the distribution function for group i by $F_i(w)$ and the corresponding density by $f_i(w)$, where $i = 1, 2$. Assuming that these two groups of individuals have equal size, the distribution and the density for the entire population are related to the distribution and the density for the two groups according to:

$$F(w) = \frac{F_1(w) + F_2(w)}{2},$$

$$f(w) = \frac{f_1(w) + f_2(w)}{2}.$$

It is helpful to think about group 2 as a group with higher on average skills level than in group 1. But, at this point, I do not impose any assumption on the distribution for these two groups.

By providing a differential effect on individuals, collateral tax sanctions allows the government to target enforcement to group 2. In its turn, targeted enforcement to

group 2 enables rising the effective tax rate in group 2. In the following subsections, I explain in details how this works and why it is useful.

2.2.2 Collateral Sanctions and Effective Tax Rates

Here I show how collateral sanctions affect the size of effective tax rates. Effective tax rates usually differ from statutory tax rates. The taxpayers are able to avoid taxes and as result reduce the amount of taxes they pay, which makes effective tax rates lower than statutory tax rates. Thus, effective tax rates depends on statutory tax rates as well as on how costly it is to avoid taxes. The costs of avoidance depends, among other things, on the size of punishment for tax avoidance. Since a collateral tax sanction affects group 2 of taxpayers and does not effect the other group, it makes effective tax rate in group 2 higher.

To show this, I adapt the model used by *Kopczuk* (2001). Assume that individuals who are characterized by a skill level, w, enjoy leisure and consumption goods, C, which are financed from compensation received for providing labor, L, on the market. They can engage in avoidance that allows decreasing the amount of income, $I \equiv wL$, subject to taxation by A at the cost of $D(wL, A)$. The marginal tax rate is denoted by t and the lump-sum transfer is G. As a result, the budget constraint is

$$C = I + G - t(I - A) - D(I, A). \tag{2.1}$$

An individual chooses A to maximize consumption, that is

$$A^*(t, I) = \underset{A}{argmax}\, I - t(I - A) - D(I, A). \tag{2.2}$$

The optimal avoidance, $A^*(I, t)$ is determine by the FOC: $t = \frac{\partial D(I, A^*)}{\partial A}$. Having determined the optimal avoidance, we can rewrote the budget constraint as:

$$C = I + G - \rho(I,t)tI - D(I, A^*(I,t)), \tag{2.3}$$

where $\rho(I,t) \equiv 1 - \frac{A^*(I,t)}{I}$ is the share of statutory taxes that is paid to the government. I call ρ as the net effective tax factor. I also define $\theta(I,t) \equiv 1 - \frac{A^*(I,t)}{I} + \frac{D(I,A^*(I,t))}{tI}$. I call θ as the gross effective tax factor. It shows the share of paid taxes and cost associated with paying taxes out of statutory taxes.

For example, suppose that the cost of avoidance is proportional to the probability of being caught, $\alpha \frac{A}{I}$, which rises with avoided income and declines with true income, and proportional to the amount of avoided taxes, $t\pi A$, where $\pi > 1$ is gross penalty rate. That is, $D(I, A) = \alpha \pi t \frac{A^2}{I}$. In this case, the optimal avoidance is $A(I,t) = \frac{I}{2\alpha\pi}$, the net effective tax factor is $\rho = 1 - \frac{1}{2\alpha\pi}$, and the gross effective tax factor is $\theta = 1 - \frac{1}{4\alpha\pi}$.

Note that the difference between gross effective tax factor and net effective tax factor (i.e., $\theta - \rho$) shows the cost of avoidance as a share of statutory taxes. While a taxpayer spends θ dollars to pay her taxes, the tax authority receives only ρ dollars out of that amount.

Let us now consider the effect of the collateral sanction in this setting. Assume that the cost of avoidance is higher for taxpayers in group 2 that are effected by a collateral tax sanction. Their cost of avoidance is $D_2(I, A)$ and $\frac{\partial D_2(I,A)}{\partial A} > \frac{\partial D_1(I,A)}{\partial A}$, where $D_1(I, A)$ is the cost of avoidance for taxpayers in group 1 (i.e., those who are not affected by the collateral sanction). Under this assumption, both net and gross effective tax factors are higher for group 2 than for group 1. Proposition 2.1 states this formally.

Proposition 2.1. *Assume that $D_i(I, A)$ for $i = 1, 2$ is increasing and strictly convex in A, $D_i(I, 0) = 0$ for $i = 1, 2$, and $\frac{\partial D_2(I,A)}{\partial A} > \frac{\partial D_1(I,A)}{\partial A}$. Then $\rho_2(I,t) > \rho_1(I,t)$ and $\theta_2(I,t) > \theta_1(I,t)$.*

Proof. The FOCs the determines the optimal avoidance levels $A_1(I, t)$ and $A_2(I, t)$ are $t = \frac{\partial D_1(I, A_1^*)}{\partial A}$ and $t = \frac{\partial D_2(I, A_2^*)}{\partial A}$. Because $D_i(I, A)$ for $i = 1, 2$ strictly convex and $\frac{\partial D_2(I, A)}{\partial A} > \frac{\partial D_1(I, A)}{\partial A}$, these FOCs imply that $A_1^* > A_2^*$. Hence, $\rho_2 - \rho_1 = \frac{1}{I}(A_1^* - A_2^*) > 0$ and $\theta_2 - \theta_1 = \frac{1}{I}(A_1^* - A_2^*) - \frac{1}{tI}(D_1(A_1^*) - D_2(A_2^*)) = \frac{1}{I}[t(A_1^* - A_2^*) - (D_1(A_1^*) - D_1(A_2^*) + D_1(A_2^*) - D_2(A_2^*))] > \frac{1}{I}[t(A_1^* - A_2^*) - \frac{\partial D_1(A_1^*)}{\partial A}(A_1^* - A_2^*) + (D_2(A_2^*) - D_1(A_2^*)) = \frac{1}{I}[(D_2(A_2^*) - D_1(A_2^*)] > 0$, where $D_1(A_1^*) - D_1(A_2^*) < \frac{\partial D_1(A_1^*)}{\partial A}(A_1^* - A_2^*)$ because $D_1(\cdot)$ is strictly convex in A, and $(D_2(A_2^*) - D_1(A_2^*) > 0$ because $D_i(I, 0) = 0$ $i = 1, 2$, and $\frac{\partial D_2(I, A)}{\partial A} > \frac{\partial D_1(I, A)}{\partial A}$.

This proposition shows that a collateral tax sanction by making the enforcement targeted to a group of taxpayers rises the effective tax rate in this group. Because of this, collateral sanction resembles tagging. In the following subsection, I explain the connection between tagging and targeted enforcement.

2.2.3 Tagging and Targeted Enforcement

In his paper, *Akerlof* (1978) argues that conditioning taxes on a "tag" indicating the taxpayer's category increases social welfare, because this helps to mitigate the tradeoff between redistribution and efficiency. The redistribution achieved through income tax improves welfare. But, this gain in welfare comes at a cost: income tax creates inefficiency by distorting labor decisions. Tagging reduces the cost of income redistribution because it allows providing transfers only to tagged people and not to everybody, which in its turn allows reducing marginal tax rates. Thus, Akerlof's tagging is a way to improve the design of tax system by conditioning taxes based on some inherent characteristic correlated with earning potential.

The subsequent research on tagging has generalized Akerlof's model and explored who gains and loses as a result of tagging. In particular, *Cremer et al.* (2010) consider a model with a continuum of individuals who can be divided into two groups (referred to as l and h) with different ability distributions over the same support. They show

that tagging always improves social welfare when the two groups have different distributions of skills. When the skill distribution in group h first-order stohastically dominates the distribution of skills in group l, then tagging implies redistribution from group h to group l (under Rawlsian social welfare and a utilitarian social welfare function with decreasing weights in skills provided that preferences are quasi-linear). Additionally, they show that if the hazard rates (i.e., $\frac{f_i(w)}{1-F_i(w)}$ where $i = l, h$) in the two tagged groups do not cross, every individual in the group with lower average skills would benefit from tagging (assuming that preferences are quasi-linear and the social welfare function is Rawlsian).

The use of collateral tax sanctions is to some extent similar to tagging. By providing targeted enforcement to some groups of taxpayers, collateral tax sanctions allow conditioning enforcement on characteristic correlated with earning potential. The difference is that they affect punishment for noncompliance rather than to taxes directly. However, as we saw in the previous subsection, the punishment plays a role in determining the effective tax rate. Thus, collateral tax sanctions is another way to condition taxes based on ability. If a collateral tax sanction affects some taxpayers more the other, then the former taxpayers have a higher effective tax rate. If a collateral tax sanction is correlated with ability, then the effective tax rate is correlated with ability.

While both collateral tax sanctions and tagging help to make the redistribution of income among people with different earning potentials more efficient, they achieve this through different mechanisms. When a tag is available, it allows the government to subdivide people into those with and without the tag and to choose separate taxes for each group. When a collateral tax sanction is used, the taxpayers in the group targeted by the collateral tax sanction have a higher effective tax rate, but the government still has to choose the same statutory tax rate for both groups.

Note that when I refer to tagging I presume that the government observes the

tag and can condition on this tag all its income tax instruments. Specifically, in case of a linear tax system, it implies that both the marginal tax rate and the lump-sum transfers can be conditioned on the tag. If this is the case then tagging is a more efficient instrument than a collateral tax sanction. It is because tagging gives the government more flexibility in the sense that any equilibrium achieved with the collateral tax sanction can be replicated by using tagging. However, if the government can only condition on the tag the lump-sum transfers then it is not necessary that this type of a tagging dominates collateral tax sanction.

The fact that a collateral tax sanction enables to alter the effective tax rate but not all tax instruments (the statutory tax rate and the lump-sum transfers) reduces the effectiveness of the collateral tax sanctions. For example, within the group that is targeted by the collateral sanction (group 2) people with low earning potential would also experience high effective tax rate and thus pay higher taxes than their equally-skilled counterpart in group 1. The government, however, cannot adjust the lump-sum transfer for group 2 to compensate for this effect. This makes the redistribution from group 2 to group 1, which is achieved through higher effective tax rate, less efficient. Therefore, whether we want to have targeted enforcement or not depends on the skill distribution within each groups. In the following section, I investigate welfare application of imposing a collateral tax sanction and identify conditions when it is socially beneficial.

2.3   Welfare and Redistribution

Here I analyze the welfare and redistribution consequences of the imposition of a collateral tax sanction, which is modeled as an increase in effective tax rate in group 2, for reasons discussed in the previous section. To do this, I first derive the optimal income tax structure for the case when there is no collateral tax sanction and, thus, both groups of taxpayers have the same effective tax rates characterized by gross

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effective tax factor θ_0. Then, I consider the case when the government imposes the collateral sanction and raise the gross effective tax factor in group 2 from θ_0 to θ_2.

I also assume that the net effective tax factor is equal to share $\lambda \in (0,1)$ of the gross effective tax factor (i.e., $\rho = \lambda\theta$). Therefore, when the effective taxes rises (i.e., θ rises), the taxes received by the tax authority raises as well (i.e., ρ rises). At the same time, when θ rises, the cost associated with paying/avoiding taxes rises too (i.e., $\theta - \rho = (1 - \lambda)\theta$ rises).

Following the approach used by *Cremer et al.* (2010). I assume that individuals have identical preferences that depend on consumption, C, positively, and on labor supply, L, negatively. The wage rates, that represent the skill levels, are distributed on $[\underline{w}, \overline{w}]$ according to $F(w)$. Preferences are represented by the quasi-linear utility function

$$u = C - \varphi(L), \tag{2.4}$$

where φ is strictly convex. The social welfare criterion is Rawlsian (maxi-min), and it is being implemented through a purely redistributive income tax system.[8]

As before, I assume that tax system is linear, that is, it described by the marginal tax rate, t, and the lump-sum transfer, G. Based on the discussion in the previous section, I assume that an individual budget constraint is

$$C = I + G - \theta_0 tI, \tag{2.5}$$

where $\theta_0 < 1$ reflects that the effective tax rate is less than the statutory tax rate. Combining (2.4) and (2.5), we have $u = I + G - \theta_0 tI - \varphi(\frac{I}{w})$. Income, $I(w)$, that maximizes this utility is determined by the FOC:

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[8]The results can be generalized to a utilitarian social welfare function with decreasing weights in skills.

$$(1 - \theta_0 t) = \frac{1}{w} \varphi'(\frac{I(w)}{w}). \tag{2.6}$$

Integrating the local incentive compatibility constraint, $\frac{\partial u}{\partial w} = \frac{I(w)}{w^2} \varphi'(\frac{I(w)}{w})$, we have

$$u(w) = \underline{u} + \int_{\underline{w}}^{w} \frac{I(s)}{s^2} \varphi'(\frac{I(s)}{s}) ds, \tag{2.7}$$

where $\underline{u} = u(\underline{w})$ is the utility of the poorest individual. As *Cremer et al.* (2010) note, the second term on the right-hand side of equation (2.7) shows the "information rent" one has to leave for an individual with $w > \underline{w}$ to reveal her type. By using FOC (2.6), the "information rent" can be expressed as $\int_{\underline{w}}^{w} \frac{I(s)}{s} (1 - \theta_0 t) ds$, which shows that for a given tax rate, t, an increase in θ_0 reduces "information rent".

Because "information rent" is positive, an individual with the lowest skill, $\underline{w}$, receives the lowest utility. Therefore, the Rawlsian social welfare problem is to maximize $u(\underline{w}) = I(\underline{w}) + G - \theta_0 t I(\underline{w}) - \varphi(\frac{I(\underline{w})}{\underline{w}})$ by choosing t and G, subject to the revenue constraint:

$$\int_{\underline{w}}^{\overline{w}} \lambda \theta_0 t I(w) f(w) dw = G + R, \tag{2.8}$$

where R is external revenue requirement. By deriving G from revenue constraint (2.8) and plugging it into the above social welfare function, we can reduces the social welfare problem to choosing t to maximize

$$u(\underline{w}) = I(\underline{w}) - \theta_0 t I(\underline{w}) - \varphi(\frac{I(\underline{w})}{\underline{w}}) + \int_{\underline{w}}^{\overline{w}} \lambda \theta_0 t I(w) f(w) dw - R. \tag{2.9}$$

Maximize (2.9) and simplify the first-order condition to get

$$\frac{\theta_0 t_0}{1 - \theta_0 t_0} = \frac{\lambda \int_{\underline{w}}^{\overline{w}} I(w) f(w) dw - I(\underline{w})}{\lambda \int_{\underline{w}}^{\overline{w}} \epsilon(w) I(w) f(w) dw}, \tag{2.10}$$

where t_0 denotes the optimal tax rate for this case, and $\epsilon(w) = \frac{\varphi'(\frac{I}{w})}{\varphi''(\frac{I}{w})\frac{I}{w}}$ is equal to the inverse of the wage elasticity of labor supply for a w-type individual.

Equation (2.10) implies that the optimal marginal tax rate is proportional to the difference between average income multiplied by λ and the lowest income and inversely proportional to average weighted income when weights are equal to the corresponding elasticities. For the tax rate, t_0, to be positive, we need to assume that $\lambda > \frac{I(\underline{w})}{\int_{\underline{w}}^{\overline{w}} I(w) f(w) dw}$.

Note that the total tax revenue increases with the effective tax rate, $\frac{\partial TR}{\partial \theta_0 t_0} = \lambda \int_{\underline{w}}^{\overline{w}} (I(w) - \theta_0 t_0 \frac{\partial I(w)}{\partial \theta_0 t_0}) f(w) dw = I(\underline{w}) > 0$. However, the tax revenue collected from a w-type individual may not increase with the effective tax rate, because $\frac{\partial TR(w)}{\partial \theta_0 t_0} = \lambda \left(I(w) - \theta_0 t_0 \frac{\partial I(w)}{\partial \theta_0 t_0} \right) = \lambda \left(I(w) - \theta_0 t_0 \frac{w^2}{\varphi''(\frac{I}{w})} \right) = \lambda I(w) \left(1 - \epsilon(w) \frac{\lambda \int_{\underline{w}}^{\overline{w}} I(w) f(w) dw - I(\underline{w})}{\lambda \int_{\underline{w}}^{\overline{w}} \epsilon(w) I(w) f(w) dw} \right)$, which could be positive or negative depending on the size of $\epsilon(w)$. The reason for this is that an increase in the effective tax rate has two effects on the tax revenue. The first effect is a mechanical effect: an increase in the tax rate gives a higher tax revenue from each dollar of income. This effect increases the tax revenue. The second effect is labor supply effect: an increase in the tax rate reduces the labor supplied by an individual. This second effect decreases the tax revenue. The total effect of an increase in the tax rate on the tax revenue is determined by the size of those two effects. Note that if the inverse of the wage elasticity of labor supply is constant (i.e., $\epsilon(w) = \epsilon = const$) then the tax revenue collected from a w-type individual increases with the effective tax rate for all w, $\frac{\partial TR(w)}{\partial \theta_0 t_0} = \lambda I(w) \frac{I(\underline{w})}{\lambda \int_{\underline{w}}^{\overline{w}} \epsilon(w) I(w) f(w) dw}$.

2.3.1 Optimal tax with two groups

Consider now the case when the government imposes the collateral sanction and raise the effective tax factor in group 2 from θ_0 to θ_2. The effective tax factor in group 1 stays the same. For symmetry, I denote it by θ_1 ($\theta_1 = \theta_0$). The rest of the structure of the model is the same.

An individual in group i ($i = 1, 2$) maximizes now $u_i = I + G - \theta_i t I - \varphi(\frac{I}{w})$. Optimal income, $I_i(w)$, that maximizes group i's individual utility, u_i, is determined by the FOC:

$$1 - \theta_i t = \frac{1}{w} \varphi'(\frac{I_i(w)}{w}) \tag{2.11}$$

The government now maximizes $min\{\underline{u}_1, \underline{u}_2\}$, where $\underline{u}_1$ and $\underline{u}_2$ are the lowest utility in group 1 and group 2 correspondingly. Similar to equation (2.7), we can get $u_i(w) = \underline{u}_i + \int_{\underline{w}}^{w} \frac{I_i(s)}{s^2} \varphi'(\frac{I_i(s)}{s}) ds$, which shows that an individual with the lowest skill, $\underline{w}_i$, receives the lowest utility in group i, $\underline{u}_i$. Thus,

$$\underline{u}_i = u(\underline{w}_i) = I(\underline{w}_i) - \theta_i t I_i(\underline{w}_i) - \varphi(\frac{I_i(\underline{w}_i)}{\underline{w}_i}). \tag{2.12}$$

2.3.1.1 Skills Distributed Over the Same Support

When skills distributed over the same support (i.e., $\underline{w}_1 = \underline{w}_2 \equiv \underline{w}$ and $\overline{w}_1 = \overline{w}_2 \equiv \overline{w}$), an individual with the lowest skill in the second group (with $\theta_2 > \theta_0$) has lower utility than an individual with the lowest skill in the first group. The following lemma proves this result.

> **Lemma 2.1**. *Assume preferences are quasi-linear and φ is strictly convex. There are two groups of individuals of equal size, each with a continuum of skills distributed over the same support $[\underline{w}, \overline{w}]$. The effective tax factor in group 2 is greater than the effective tax factor in group 1 (i.e., $\theta_2 > \theta_1$).*

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Then, $\underline{u}_1 > \underline{u}_2$.

Proof. According to (2.11), the FOCs for $\underline{w}$ is $1 - \theta_i t = \frac{1}{\underline{w}}\varphi'(\frac{I_i(\underline{w})}{\underline{w}})$, for $i = 1, 2$. Because φ is strictly convex and $\theta_2 > \theta_1$, these FOCs imply $I_2(\underline{w}) < I_1(\underline{w})$. By using (2.12), get $\underline{u}_1 - \underline{u}_2 = I_1(\underline{w})(1 - \theta_1 t) - I_2(\underline{w})(1 - \theta_2 t) - [\varphi(\frac{I_1(\underline{w})}{\underline{w}}) - \varphi(\frac{I_2(\underline{w})}{\underline{w}})]$. Because $\varphi(\frac{I_1(\underline{w})}{\underline{w}}) - \varphi(\frac{I_2(\underline{w})}{\underline{w}}) < \varphi'(\frac{I_1(\underline{w})}{\underline{w}})(\frac{I_1(\underline{w})}{\underline{w}} - \frac{I_2(\underline{w})}{\underline{w}})$ due to strict convexity of φ,

$\underline{u}_1 - \underline{u}_2 > I_1(\underline{w})(1 - \theta_1 t) - I_2(\underline{w})(1 - \theta_2 t) - (1 - \theta_1 t)(\frac{I_1(\underline{w})}{\underline{w}} - \frac{I_2(\underline{w})}{\underline{w}}) = (\theta_2 - \theta_1)t\frac{I_2(\underline{w})}{\underline{w}}$.

This lemma implies that the Rawlsian social welfare problem reduces to maximization of $u_2(\underline{w}) = I_2(\underline{w}) + G - \theta_2 t I_2(\underline{w}) - \varphi(\frac{I_2(\underline{w})}{\underline{w}})$ by choosing t and G, subject to the revenue constraint:

$$\int_{\underline{w}}^{\overline{w}} \lambda\theta_1 t I_1(w)\frac{f_1(w)}{2}dw + \int_{\underline{w}}^{\overline{w}} \lambda\theta_2 t I_2(w)\frac{f_2(w)}{2}dw = G + R. \quad (2.13)$$

By deriving G from revenue constraint (2.13) and plugging it into the expression for $u_2(\underline{w})$, the social welfare problem reduces to choosing t to maximize

$$u(\underline{w}) = I_2(\underline{w}) - \theta_2 t I_2(\underline{w}) - \varphi(\frac{I_2(\underline{w})}{\underline{w}}) + \int_{\underline{w}}^{\overline{w}} \lambda\theta_1 t I_1(w)\frac{f_1(w)}{2}dw + \int_{\underline{w}}^{\overline{w}} \lambda\theta_2 t I_2(w)\frac{f_2(w)}{2}dw - R.$$

$$(2.14)$$

Maximize (2.14) and simplify the first-order condition to get

$$\frac{\theta_1 t}{1-\theta_1 t}\int_{\underline{w}}^{\overline{w}}\lambda\epsilon_1(w)I_1(w)\frac{f_1(w)}{2}dw + \frac{\theta_2 t}{1-\theta_2 t}\int_{\underline{w}}^{\overline{w}}\lambda\epsilon_2(w)I_2(w)\frac{f_2(w)}{2}dw =$$
$$= \theta_1\int_{\underline{w}}^{\overline{w}}\lambda I_1(w)\frac{f_1(w)}{2}dw + \theta_2\int_{\underline{w}}^{\overline{w}}\lambda I_2(w)\frac{f_2(w)}{2}dw - \theta_2 I_2(\underline{w}) \quad (2.15)$$

where $\epsilon_i(w) = \frac{\varphi'(\frac{I_i}{w})}{\varphi''(\frac{I_i}{w})\frac{I_i}{w}}$ is equal to the inverse of the wage elasticity of labor supply for a w-type individual in group i.

Intuitively, an increase in the effective tax factor in group 2 from θ_0 to θ_2 should redistribute income from group 2 to group 1, which is in this model is achieved through an adjustment in tax rate, t, and the lump-sum transfer, G. However, to be able to say how exactly they adjust, we need to impose additional assumptions.

To determine what happens to the welfare of individuals in each group, I assume that labor supply elasticity exhibits a constant wage elasticity. Thus, set $\varphi(L) = L^{1+1/\epsilon}$, where ϵ is the labor supply elasticity. Observe that the strict convexity of φ implies $\epsilon > 0$. This assumption leads to a closed-form solution for optimal incomes, for $i = 1, 2$, which are given by

$$I_i(w) = \left(\frac{1 - \theta_i t}{1 + 1/\epsilon}\right)^{\epsilon} w^{1+\epsilon}.$$

The closed-form solution for $I_i(w)$ allows us to simplify the equation determining the optimal tax rate:

$$
\begin{aligned}
\frac{\theta_1 t}{1 - \theta_1 t} \epsilon \theta_1 \left(\frac{1-\theta_1 t}{1+1/\epsilon}\right)^{\epsilon} \lambda \hat{W}_1 + \frac{\theta_2 t}{1 - \theta_2 t} \epsilon \theta_2 \left(\frac{1-\theta_2 t}{1+1/\epsilon}\right)^{\epsilon} \lambda \hat{W}_2 = \\
= \theta_1 \left(\frac{1-\theta_1 t}{1+1/\epsilon}\right)^{\epsilon} \lambda \hat{W}_1 + \theta_2 \left(\frac{1-\theta_2 t}{1+1/\epsilon}\right)^{\epsilon} \lambda \hat{W}_2 - \theta_2 \left(\frac{1-\theta_2 t}{1+1/\epsilon}\right)^{\epsilon} \underline{W},
\end{aligned}
\tag{2.16}
$$

where $\hat{W}_i = \int_{\underline{w}}^{\overline{w}} w^{1+\epsilon} \frac{f_i(w)}{2} dw$ and $\underline{W} = \underline{w}^{1+\epsilon}$.

Note that formula (2.10) for $\theta_0 t_0$ in the case of constant labor supply elasticity reduces to

$$\theta_0 t_0 = \frac{\lambda(\hat{W}_1 + \hat{W}_2) - \underline{W}}{(1 + \epsilon)\lambda(\hat{W}_1 + \hat{W}_2) - \underline{W}},$$

where I have used that $\int_{\underline{w}}^{\overline{w}} w^{1+\epsilon} f(w) dw = \int_{\underline{w}}^{\overline{w}} w^{1+\epsilon} \frac{f_1(w)}{2} dw + \int_{\underline{w}}^{\overline{w}} w^{1+\epsilon} \frac{f_2(w)}{2} dw$.

While we cannot derive the close form solution for tax rate, t, as can be seen from (2.16), we can determine the sign of its change. Differentiate (2.16) w.r.t. θ_2 and estimate the derivative $\frac{\partial t}{\partial \theta_2}$ at $\theta_2 = \theta_0$ to get

$$\frac{\partial t}{\partial \theta_2}\bigg|_{\theta_2=\theta_0} = \frac{t_0}{\theta_0(\hat{W}_1 + \hat{W}_2)}\left[-\hat{W}_2 - \frac{\theta_0 t_0 \epsilon \hat{W}_1 (\underline{W})^2}{(\lambda(\hat{W}_1 + \hat{W}_2) - \underline{W})^2}\right] < 0.$$

When the effective tax factor in group 2 increases, the statutory tax rate, t, decreases. This implies that taxes paid by taxpayers in group 1 decreases. However, the effective tax rate in group 2 increases, because $\frac{\partial \theta_2 t}{\partial \theta_2}\big|_{\theta_2=\theta_0} = \frac{t_0 \hat{W}_1 (\lambda(\hat{W}_1+\hat{W}_2)-(1-\theta_0 t_0)\underline{W})}{\hat{W}_1+\hat{W}_2} > 0$. Therefore, the taxes paid by taxpayers in group 2 increases, which is guaranteed by the facts that we consider a marginal increase in the effective tax rate factor and that at $\theta_0 t_0$ the tax revenue rises with the effective tax rate, $\frac{\partial TR}{\partial \theta_0 t_0} > 0$.[9] When the effective tax factor in group 2 increases, the lump-sum transfer decreases, because

$\frac{\partial G}{\partial \theta_2}\big|_{\theta_2=\theta_0} = -(1+\frac{1}{\epsilon})^{-\epsilon}(1-\theta_0 t)^\epsilon(1 - \epsilon\frac{\theta_0 t_0}{1-\theta_0 t_0})\frac{\epsilon\lambda\hat{W}_1 \underline{W}^2}{\theta_0(\lambda(1+\epsilon)(\hat{W}_1+\hat{W}_2)-\underline{W})^2} < 0$. This means that the increase in the taxes paid by taxpayers in group 2 does not outweigh the decrease in the taxes paid by taxpayers in group 1.

The closed-form solution for $I_i(w)$ allows us also to derive the expression for $\underline{u}_i(w)$ and $u_i(w)$ for $i = 1, 2$:

$$\underline{u}_i = (1+\frac{1}{\epsilon})^{-\epsilon}\left(\theta_1 t(1-\theta_1 t)^\epsilon \lambda \hat{W}_1 + \theta_2 t(1-\theta_2 t)^\epsilon \lambda \hat{W}_2 + \frac{1}{1+\epsilon}(1-\theta_i t)^{1+\epsilon}\underline{w}^{1+\epsilon}\right) - R,$$

(2.17)

$$u_i = \underline{u}_i + (1+\frac{1}{\epsilon})^{-\epsilon}\frac{1}{1+\epsilon}(1-\theta_i t)^{1+\epsilon}(w^{1+\epsilon} - \underline{w}^{1+\epsilon}).$$

(2.18)

By differentiating the above utilities w.r.t. θ_2, we can determine how an increase in the effective tax factor in group 2 affects the utility of individuals in each group. In doing this, remember that $\frac{\partial \underline{u}_2}{\partial t} = 0$, because t is chosen to maximize $\underline{u}_2$. The derivatives of the utilities w.r.t. θ_2 are

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[9]This means that the mechanical effect of an increase in the effective tax rates exceeds the labor supply effect.

$$\left.\frac{\partial \underline{u}_2}{\partial \theta_2}\right|_{\theta_2=\theta_0} = (1+\frac{1}{\epsilon})^{-\epsilon}t_0(1-\theta_0 t_0)^{\epsilon}\left[-\frac{\hat{W}_1 W}{\hat{W}_1+\hat{W}_2}\right] < 0,$$

$$\left.\frac{\partial u_2}{\partial \theta_2}\right|_{\theta_2=\theta_0} = \left.\frac{\partial \underline{u}_2}{\partial \theta_2}\right|_{\theta_2=\theta_0} + (1+\frac{1}{\epsilon})^{-\epsilon}(1-\theta_0 t_0)^{\epsilon}\left(-\left.\frac{\partial \theta_2 t}{\partial \theta_2}\right|_{\theta_2=\theta_0}\right)(w^{1+\epsilon}-\underline{w}^{1+\epsilon}) < 0,$$

$$\left.\frac{\partial \underline{u}_1}{\partial \theta_2}\right|_{\theta_2=\theta_0} = (1+\frac{1}{\epsilon})^{-\epsilon}t_0(1-\theta_0 t_0)^{\epsilon}\left[\frac{\hat{W}_2 W}{\hat{W}_1+\hat{W}_2}\right] > 0,$$

$$\left.\frac{\partial u_1}{\partial \theta_2}\right|_{\theta_2=\theta_0} = \left.\frac{\partial \underline{u}_1}{\partial \theta_2}\right|_{\theta_2=\theta_0} + (1+\frac{1}{\epsilon})^{-\epsilon}(1-\theta_0 t_0)^{\epsilon}\theta_0\left(-\left.\frac{\partial t}{\partial \theta_2}\right|_{\theta_2=\theta_0}\right)(w^{1+\epsilon}-\underline{w}^{1+\epsilon}) > 0.$$

These derivatives imply that everyone in group 2 receives a loss in their welfare and everyone in group 1 receives a gain in their welfare as a result of an increase in the effective tax factor in group 2. Because the Rawlsian social welfare function in this case is equal to $\underline{u}_2$, the social welfare decreases. The following proposition summarizes these results.

Proposition 2.2. *Assume preferences are quasi-linear and the social welfare function is Rawlsian. Assume that $\lambda > \frac{w^{1+\epsilon}}{\int_{\underline{w}}^{\overline{w}} w^{1+\epsilon} f(w)dw}$. There are two groups of individuals of equal size, 1 and 2, each with a continuum of skills distributed over the same support $[\underline{w}, \overline{w}]$. Assume that the wage elasticity of labor supply is constant and identical for the group 1 and 2. Then, an increase in the effective tax factor in group 2 leads to:*

i) a decrease in the tax rate, which allows the government to redistribute income from group 2 to group 1.

ii) an increase in the utility for each individual in group 1.

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iii) a decrease in the utility for each individual in group 2.

iv) a decrease in the Rawlsian social welfare function.

2.3.1.2   Skills Distributed Over Different Supports

Consider now the case when skills are distributed over the different supports: in group 1 over $[\underline{w}_1, \overline{w}_1]$ and in group 2 over $[\underline{w}_2, \overline{w}_2]$, where $\underline{w}_1 < \underline{w}_2$. In this case, it is not necessary that the utility of an individual with the lowest skill in group 2, $u_2(\underline{w}_2)$, is lower than the utility of an individual with the lowest skill in group 1, $u_1(\underline{w}_1)$. The following lemma shows that when $\underline{w}_2$ is sufficiently larger than $\underline{w}_1$, it is the opposite, $u_1(\underline{w}_1) < u_2(\underline{w}_2)$.

> **Lemma 2.2**. *Assume preferences are quasi-linear and φ is strictly convex. There are two groups of individuals of equal size, each with a continuum of skills distributed over different supports: in group 1 over $[\underline{w}_1, \overline{w}_1]$ and in group 2 over $[\underline{w}_2, \overline{w}_2]$, where $\underline{w}_2$ is sufficiently larger than $\underline{w}_1$. Specifically, $(1-\theta_2)\underline{w}_2 > (1-\theta_1)\underline{w}_1$. The effective tax factor in group 2 is greater than the effective tax factor in group 1 (i.e., $\theta_2 > \theta_1$). Then, $\underline{u}_1 < \underline{u}_2$.*

Proof. According to (2.11), the FOCs are $1 - \theta_i t = \frac{1}{w_i}\varphi'(\frac{I_i(w_i)}{w_i})$, for $i = 1, 2$. Because φ is strictly convex and $(1-\theta_2)\underline{w}_2 > (1-\theta_1)\underline{w}_1$, these FOCs imply $L_2(\underline{w}_2) < L_1(\underline{w}_1)$. Then, $u_2(\underline{w}_2) - u_1(\underline{w}_1) = I_2(\underline{w}_2)(1 - \theta_2 t) - I_1(\underline{w}_1)(1 - \theta_1 t) - [\varphi(L_2(\underline{w}_2)) - \varphi(L_1(\underline{w}_1))] > I_2(\underline{w}_2)(1 - \theta_2 t) - I_1(\underline{w}_1)(1 - \theta_1 t) - (1 - \theta_2 t)\underline{w}_2(\frac{I_2(\underline{w}_2)}{\underline{w}_2} - \frac{I_1(\underline{w}_1)}{\underline{w}_1}) = ((1 - \theta_2 t)\underline{w}_2 - (1-\theta_1)\underline{w}_1)I_1(\underline{w}_1) > ((1-\theta_2)\underline{w}_2 - (1-\theta_1)\underline{w}_1)I_1(\underline{w}_1)$, where I have used that $\varphi(L_2(\underline{w}_2)) - \varphi(L_1(\underline{w}_1)) < \varphi'(L_2(\underline{w}_2))(L_2(\underline{w}_2) - L_1(\underline{w}_1))$.

Note that condition $\underline{w}_1 < \underline{w}_2$ does not produce any problem for incentive compatibility to hold, that is an individual in group 2 with type $\underline{w}_2$ would not want pretend to be

type $w < \underline{w}_2$, because statutory tax rate in the same for both groups. Another words, t is chosen so that incentive compatibility is satisfied, and therefore there is no incentive to pretend to be another type.

Because $\underline{u}_1 < \underline{u}_2$, the Rawlsian social welfare function is equal now to $\underline{u}_1 = G + I_1(\underline{w}_1)(1 - \theta_1 t) - \varphi(\frac{I_1(\underline{w}_1)}{\underline{w}_1})$. Using the same calculation strategy as in the previous subsection and assuming that the wage elasticity of labor supply is constant, we can derive the equation determining the optimal tax rate, which is

$$\frac{\theta_1 t}{1 - \theta_1 t} \epsilon \theta_1 \left(\frac{1 - \theta_1 t}{1 + 1/\epsilon}\right)^\epsilon \lambda \hat{W}_1 + \frac{\theta_2 t}{1 - \theta_2 t} \epsilon \theta_2 \left(\frac{1 - \theta_2 t}{1 + 1/\epsilon}\right)^\epsilon \lambda \hat{W}_2 = $$
$$= \theta_1 \left(\frac{1 - \theta_1 t}{1 + 1/\epsilon}\right)^\epsilon \lambda \hat{W}_1 + \theta_2 \left(\frac{1 - \theta_2 t}{1 + 1/\epsilon}\right)^\epsilon \lambda \hat{W}_2 - \theta_1 \left(\frac{1 - \theta_1 t}{1 + 1/\epsilon}\right)^\epsilon \underline{W}_1, \tag{2.19}$$

where $\underline{W}_1 = \underline{w}_1^{1+\epsilon}$. Equation (2.19) differs from equation (2.16) only in the very last term in the formula.

To determine the sign of the change in the tax rate as a result of an increase in the effective tax factor, differentiate (2.19) w.r.t. θ_2 and estimate the derivative $\frac{\partial t}{\partial \theta_2}$ at $\theta_2 = \theta_0$ to get

$$\left.\frac{\partial t}{\partial \theta_2}\right|_{\theta_2 = \theta_0} = \frac{t_0 \hat{W}_2}{\theta_0(\hat{W}_1 + \hat{W}_2)} \left[\frac{-(\lambda(\hat{W}_1 + \hat{W}_2) - 2\underline{W}_1) - \theta_0 t_0 (1 + \epsilon)\underline{W}_1}{(\lambda(\hat{W}_1 + \hat{W}_2) - \underline{W}_1)}\right].$$

For $\left.\frac{\partial t}{\partial \theta_2}\right|_{\theta_2 = \theta_0}$ to be negative, it is sufficient to have $\lambda(\hat{W}_1 + \hat{W}_2) - 2\underline{W}_1 = \lambda \int_{\underline{w}_1}^{max\{\overline{w}_1, \overline{w}_2\}} w^{1+\epsilon} f(w) dw - 2\underline{w}_1^{1+\epsilon} > 0$.[10] This implies that the average skill level should be sufficiently higher than the lowest skill level. If so, the optimal tax rate decreases with an increase in the effective tax factor, implying that taxes paid by taxpayers in group 1 decreases. The effective tax rate in group 2, however, increases, because $\left.\frac{\partial \theta_2 t}{\partial \theta_2}\right|_{\theta_2 = \theta_0} = \frac{t_0}{\hat{W}_1 + \hat{W}_2} \left(\hat{W}_1 + \frac{\epsilon \theta_0 t_0 \hat{W}_2 \underline{W}_1^2}{(\hat{W}_1 + \hat{W}_2 - \underline{W})^2}\right) > 0$. Therefore, the taxes paid by taxpayers in group 2 increases. When the effective tax factor in group 2 increases, the lump-sum transfer also

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[10]Note that this condition is easily satisfied if $\underline{w}_1$ is close to zero.

increases, because $\frac{\partial G}{\partial \theta_2}\big|_{\theta_2=\theta_0} = (1+\frac{1}{\epsilon})^{-\epsilon}(1-\theta_0 t)^\epsilon(1-\epsilon\frac{\theta_0 t_0}{1-\theta_0 t_0})\frac{\epsilon\lambda\hat{W}_2 W_1^2}{(\lambda(1+\epsilon)(\hat{W}_1+\hat{W}_2)-\underline{W}_1)^2} > 0$.
This means that the increase in the taxes paid by taxpayers in group 2 outweigh the decrease in the taxes paid by taxpayers in group 1.

The expression for $\underline{u}_i(w)$ and $u_i(w)$ for $i = 1, 2$ are now:

$$\underline{u}_i = (1 + \frac{1}{\epsilon})^{-\epsilon}\left(\theta_1 t(1 - \theta_1 t)^\epsilon\lambda\hat{W}_1 + \theta_2 t(1 - \theta_2 t)^\epsilon\lambda\hat{W}_2 + \frac{1}{1+\epsilon}(1 - \theta_i t)^{1+\epsilon}\underline{w}_i^{1+\epsilon}\right) - R,$$

$$(2.20)$$

$$u_i = \underline{u}_i + (1 + \frac{1}{\epsilon})^{-\epsilon}\frac{1}{1+\epsilon}(1 - \theta_i t)^{1+\epsilon}(w^{1+\epsilon} - \underline{w}_i^{1+\epsilon}). \qquad (2.21)$$

By differentiating the above utilities w.r.t. θ_2, we can determine how an increase in the effective tax factor in group 2 affects the utility of individuals in each group. In doing this, remember that now $\frac{\partial u_1}{\partial t} = 0$, because t is chosen to maximize $\underline{u}_1$. The derivatives of the utilities w.r.t. θ_2 are

$$\frac{\partial \underline{u}_1}{\partial \theta_2}\bigg|_{\theta_2=\theta_0} = (1 + \frac{1}{\epsilon})^{-\epsilon}t_0(1 - \theta_0 t_0)^\epsilon\left[\frac{\hat{W}_2\underline{W}_1}{\hat{W}_1 + \hat{W}_2}\right] > 0,$$

$$\frac{\partial u_1}{\partial \theta_2}\bigg|_{\theta_2=\theta_0} = \frac{\partial \underline{u}_1}{\partial \theta_2}\bigg|_{\theta_2=\theta_0} + (1 + \frac{1}{\epsilon})^{-\epsilon}(1 - \theta_0 t_0)^\epsilon\theta_0\left(-\frac{\partial t}{\partial \theta_2}\bigg|_{\theta_2=\theta_0}\right)(w^{1+\epsilon} - \underline{w}_1^{1+\epsilon}) > 0,$$

$$\frac{\partial \underline{u}_2}{\partial \theta_2}\bigg|_{\theta_2=\theta_0} = (1+\frac{1}{\epsilon})^{-\epsilon}\frac{t_0(1 - \theta_0 t_0)^\epsilon}{\hat{W}_1 + \hat{W}_2}\left[-\hat{W}_1\underline{W}_2 - \frac{(1 - \theta_0 t_0)\hat{W}_2 W_1^2(\underline{W}_2 - \underline{W}_1)}{\lambda(\hat{W}_1 + \hat{W}_2)(\lambda(\hat{W}_1 + \hat{W}_2) - \underline{W}_1)}\right] < 0,$$

$$\frac{\partial u_2}{\partial \theta_2}\bigg|_{\theta_2=\theta_0} = \frac{\partial \underline{u}_2}{\partial \theta_2}\bigg|_{\theta_2=\theta_0} + (1 + \frac{1}{\epsilon})^{-\epsilon}(1 - \theta_0 t_0)^\epsilon\left(-\frac{\partial \theta_2 t}{\partial \theta_2}\bigg|_{\theta_2=\theta_0}\right)(w^{1+\epsilon} - \underline{w}_2^{1+\epsilon}) < 0.$$

These derivatives imply that everyone in group 2 receives a loss in their welfare and everyone in group 1 receives a gain in their welfare as a result of an increase in the effective tax factor in group 2. Because the Rawlsian social welfare function in this case is equal to $\underline{u}_1$, the social welfare increases. The following proposition summarizes these results.

> **Proposition 2.3**. *Assume preferences are quasi-linear and the social welfare function is Rawlsian. There are two groups of individuals of equal size, each with a continuum of skills distributed over different supports: in group 1 over $[\underline{w}_1, \overline{w}_1]$ and in group 2 over $[\underline{w}_2, \overline{w}_2]$, where $\underline{w}_2$ is sufficiently larger than $\underline{w}_1$. Specifically, $(1 - \theta_2)\underline{w}_2 > (1 - \theta_1)\underline{w}_1$. Assume that the wage elasticity of labor supply is constant and identical for the group 1 and 2. Assume the average skill level is sufficiently higher than the lowest skill level. Precisely, assume that $\lambda \int_{\underline{w}_1}^{max\{\overline{w}_1, \overline{w}_2\}} w^{1+\epsilon} f(w) dw - 2\underline{w}_1^{1+\epsilon} > 0$. Then, an increase in the effective tax factor in group 2 leads to:*
>
> *i) a decrease in the tax rate, which allows the government to redistribute income from group 2 to group 1.*
>
> *ii) an increase in the utility for each individual in group 1.*
>
> *iii) a decrease in the utility for each individual in group 2.*
>
> *iv) an increase in the Rawlsian social welfare function.*

In this case, an increase in the effective tax factor in group 2 increases social welfare and allows the government to redistribute income from group 2 to group 1. Recall that, in this case, group 2 is assumed to consist of individuals with relatively higher earning potentials than those in group 1 and that an increase in the effective tax factor in group 2 represents the imposition of collateral sanction. Putting these facts together, we see that the imposition of collateral sanction, similar to tagging,

enables to achieve a better redistribution. However, in contrast to a tag that improves social welfare even in the case when the supports of skill distributions for two groups are the same, a collateral tax sanction improves social welfare only when the poorest individual in the group targeted by the sanction has sufficiently higher earning potential than the poorest individual in the other group.

2.4 Some Concerns

One might wonder why we need collateral tax sanctions if we have tags. In practice, it might be easier to implement collateral tax sanctions than tags for political reasons. If we want to tax based on ability, then we might want to tax based on possession of an international passport. But, it might be impossible to do, because it is illegal to restrict your right to travel. However, in case when a person violates the law, it could be legal to use broader instruments and revoke the passport.

Not all collateral tax sanctions, however, work as a tag. That is, not all of them are correlated with taxpayers earning potential. For example, suspension of a hunting license may be not a good instrument for imposing punishment that is correlated with ability. Possession of a hunting license more likely reflects individual preferences.

As have been discussed, an important feature of collateral tax sanctions is that their imposition affects consumption directly. Moreover, it forces the consumption of a certain good/activity to be reduced to zero. How pronounced the effect of such a restriction is depends on the individual preferences. Certainly, some collateral tax sanctions could be very restrictive and could affect taxpayer's utility a lot.

A sanction that produces high utility cost has its benefits and downsides. On the one hand, it could be effective if it produces a high deterrence effect. That is, it creates strong incentives for a tax delinquent to pay the tax debt in order to avoid the sanction, so that many taxpayers would pay their debts and would not be subject to the sanction. On the other hand, if the produced deterrence effect is low, then many

taxpayers would be subject to the sanction. Hence, the sanction would substantially reduce the social welfare.

It is also necessary to acknowledge that this paper explores only one channel through which collateral tax sanctions affect people. There are many other channels. In addition to affecting avoidance behavior, some collateral tax sanctions might actually discourage labor supply. For example, suspension of a driver's or professional license may impose some restriction on people's ability to earn money. Second, whenever there exists a shadow economy, collateral sanction might stimulate some shadow consumption. The size of those effects is unclear and empirical work is needed to estimate it.

2.5   Conclusion

This paper analyzes a collateral tax sanction – a revocation of a privilege provided by the government, imposed for a failure to comply with tax obligations. The paper proposes a new rationale for why it may be beneficial to use collateral tax sanctions for the purpose of tax enforcement. Collateral tax sanctions might be a way to impose punishment correlated with taxpayer's ability and, as a result, increase social welfare by making the redistribution of income through tax system more efficient. In other words, a collateral tax sanction might work as a tag. It does this by affecting consumption rather than income, which makes the enforcement targeted to a group of taxpayers. When earning potentials in the targeted group is higher than in the rest of the population, the social welfare is raised by the imposition of the collateral tax sanction that helps to redistribute income from the former to the latter group.

The paper develops a model that explores the welfare and redistribution consequence of the imposition of a collateral tax sanction for tax noncompliance. In the model, individuals are heterogeneous in their skills. By imposing a collateral tax sanction, the government can rise the effective tax rate in the targeted group of tax-

payers whose skills are higher on average than in the rest of the population. I show that as a result of this, the new optimal statutory tax rate decreases, which allows to increase the utility of the rest of population at the cost of decreasing the utility of taxpayers in the targeted group. The social welfare increases only when the earning potential of the poorest individual in the targeted group is sufficiently higher than the earning potential of the poorest individual in the rest of the population.

<center>**CHAPTER III**</center>

Tax Debt Collection Enforcement: When Does Suspension of a Driver's License Help?

3.1 Introduction

Collection of tax debt is a critical and integral part of the tax enforcement process. It is the next step after tax evasion has been discovered and the due amount of taxes is assessed but failed to be paid. However, while tax evasion has received a lot of attention in the existing tax enforcement literature (see the surveys in *Cowell* (1990), *Andreoni et al.* (1998), *Slemrod and Yitzhaki* (2002)), enforcement of tax debt has not been well studied. The nearly sole study of tax debt enforcement, by *Perez-Truglia and Troiano* (2015), presents a theory and experimental evidence supporting that it may be optimal to use shaming penalty that involves publishing the names of tax delinquents online in addition to monetary fine. In this paper, I continue to explore what instruments are optimal to use for tax debt enforcement. Specifically, I examine when it is optimal to use collateral tax sanctions for the enforcement of tax delinquencies. An example of a collateral tax sanction that has been used for this purpose by several states is suspension of a driver's license.

In the United States, the size of tax debt and especially uncollected tax debt is significant and surprising. In 2013, tax debt, that consists of past-due tax liabilities,

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penalties, and interest, was roughly 60 billions of dollars, which is 5 percent of individual, estate, and trust net income tax collections. According to *Burman* (2003), in a statement before the US House of Representatives Committee on the Budget, "the IRS assesses almost \$30 billion of taxes that it will never collect. This is not theoretical tax evasion. The \$30 billion represents underpayments of tax that the IRS has identified but cannot collect because its staff is spread so thin. [. . .] According to IRS estimates, 60 percent of identified tax debts are never collected. These unclosed cases include: 75 % of identified nonfilers; 79 % of taxpayers who use 'known abusive devices' to avoid taxes; 78 % of taxpayers identified through document matching programs. It is possible that some of these people simply cannot afford to pay their tax debts, but more than half—56 %—of noncompliant taxpayers with incomes over \$100 000 get off scot-free."

The IRS has multiple instruments to enforce the collection of past-due tax liabilities. The common instrument is a monetary fine (i.e., an above-market interest rate on the debt amount). The tax authority has also collection tools that include liens (taking ownership of the taxpayer's property until a tax debt is paid) and levies (garnishing wages, seizing money from bank accounts or outright seizures of property). The tax authority starts the collection process by notifying a delinquent taxpayer about the tax amount owed and demanding the payment. Then, if a taxpayer does not take actions to pay the tax debt (like making a payment or entering an installment agreement), the IRS places a lien on the taxpayer's assets in order to secure payment of taxes. Lastly, if still no actions are taken, the IRS proceeds by implementing a tax levy, which, depending on the taxpayer's situation, could be wage garnishment, bank levy, or asset seizure.

However, even with these instruments the tax authority has limited ability to collect tax debt. One reason for this is explained by *Galmarini et al.* (2014) who focus on post-audit collection efforts in Italy. In the case of Italian tax system, a

critical step of the collection of past-due taxes is to notify the taxpayers with due amounts. Some taxpayers are, however, trying to escape the notice by "changing address". If the collection agency is not able to discover where the taxpayer hides, then the notice will not take place within the legally set time limit and the taxpayer will be cleared from the due tax amount.[1] Second, when a lien or levy is put in action, the collection process may be substantially delayed or impeded by lengthy court battles.

Another instrument that has been recently used for tax debt enforcement is suspension of a driver's license. For now, it has been implemented by three states – Louisiana, California, and New York State. This measure is an example of a collateral tax sanction, which is a revocation of a privilege provided by the government, imposed for a failure to comply with tax obligations. After a notice to a tax delinquent about tax debt with a request to pay it, the tax authority can proceed by instructing Department of Motor Vehicles to suspend a driver's license. The suspension remains in effect until the taxpayer has satisfied her past-due tax liabilities. The process of a driver's license suspension is easy and almost costless for the tax authority.[2]

According to New York State Governor Andrew M. Cuomo (March 17, 2014), initial results of tax scofflaw driver license suspension initiative show that as a result of the program tax collections increased nearly $56.4 million on a state and local basis, which is 5 percent of total delinquency collections in 2013.[3] Furthermore, 37 percent of tax debtors who were contacted beginning in August, 2013 have either paid in full or have been making payments on their debt.

One important difference between a collateral tax sanction and a monetary fine lies in the timing of their effects on tax debtors. Collateral tax sanctions can be

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[1] By using Italian data, *Galmarini et al.* (2014) found that the presence of a prior notice is associated with a reduced probability of changing address afterward.

[2] It is not, however, socially costless, which is discussed later.

[3] See press-release "Governor Cuomo Announces Initial Results of Tax Scofflaw Driver License Suspension Initiative", March 17, 2014, available at http://www.governor.ny.gov/press/03172014-drivers-license-suspension-initiative

imposed in a short time and start to influence tax debtors right away.[4] In contrast, monetary fines affect taxpayers only at the moment when they are collected, which occurs together with the tax debt collection. Because in reality a number of tax delinquents delay or escape paying their taxes and fines, monetary fines imposed on paper are delayed and often not collected at all. As a result, the actual time until the monetary fine is collected and, thus, influences the taxpayer may be longer than the time needed for the revocation of driver's license to affect the taxpayer.

This prompt influence of the collateral tax sanction is critical when debtors are heterogeneous in ability to delay tax debt collection actions. The heterogeneity could exist either because different collection tools are applicable or because people have different abilities to battle in courts, etc. When there is such a heterogeneity, the collateral tax sanction has some advantage over the monetary fine because the collateral tax sanction affects immediately even those debtors who can substantially delay their tax debt payment. Moreover, the longer the tax debt is postponed the longer the collateral tax sanction remains in effect.

The time when a punishment instrument becomes effective is an important factor that influences taxpayers' and the tax authority's decisions. However, the consequences of delays and variation in punishment effective time has not been well examined in the existing literature. The tax enforcement literature usually assumes that punishment happens immediately after evasion is conducted. One exception is the paper by *Andreoni* (1992) who considers settings when an audit and punishment (monetary fine) for tax evasion occur one period after evasion is undertaken. In these settings, tax evasion may be a high-risk substitute for a loan. As a result, when individuals face binding borrowing constraints, some evasion may be socially desirable. So, *Andreoni* (1992) argues that the government can increase welfare by playing the role of 'loan shark' to people whose borrowing is constrained on the private mar-

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[4]Of course, there is always a concern that some people may break the law and drive without their driver's license.

ket. Additionally, he shows that in his model it is not optimal to use non-monetary penalties (collateral sanctions).

In this paper, I develop a dynamic model where risk-averse individuals have a tax debt and decide whether to pay it in the first period or postpone paying until future. The individuals are heterogeneous in two different respects. First, they differ in their income in the first period. In the subsequent periods, they have identical incomes. Second, they differ in how likely the tax authority can collect the tax debt by using a collection tool in the second period (or any other subsequent period). This probability represents debtors' abilities to postpone their tax debts. So, the debtors are heterogeneous not only in how much they are income constrained, but also in how long they can delay their payments.

To enforce tax debt collection, the tax authority can impose a monetary fine and a collateral tax sanction (a driver's license suspension). The tax authority cares not only about tax revenue, but about the welfare of taxpayers. Therefore, when a taxpayer fails to pay her past-due tax liabilities because of being cash constrained, the tax authority may choose to wait for the repayment of the tax debt till the next period. There is, however, another reason for why the tax debt may not be paid in the first period. The taxpayer may expect to escape paying tax debt for a sufficiently long period of time. The tax authority does not want to dismiss from punishment those taxpayers who do not pay their tax debt because of the second reason. But, the monetary fine is favoring those who have a better ability to delay their debt payment because the monetary fine is collected at the moment when the debt is collected. In contrast, the collateral tax sanction can be applied immediately and remains in effect until the debt is paid. Therefore, it affects stronger those debtors who postpone paying their debt for a longer time.

I show that when individuals are homogeneous in ability to postpone paying tax debt, it is optimal to use only the monetary fine. In this case, while both the monetary

fine and the collateral tax sanction reduce private welfare, the monetary fine has an advantage over the collateral tax sanction because the monetary fine generates additional income for the tax authority. This resembles Andreoni's (1992) result.[5] However, when individuals are heterogeneous in ability to postpone paying the tax debt, it may be optimal to supplement the monetary fine with the collateral tax sanction. The collateral sanction helps to discourage deliberate procrastination of paying the tax debt by those who have higher abilities to delay tax debt payment.

To gain further insights, I consider the case when utility has constant relative risk aversion and the distribution of income in period 1 and the distribution of ability to postpone tax debt are uniform. In this case, I find that it is optimal to impose the collateral tax sanction when the upper bound of income in the first period is relatively large. This means that initially not all taxpayers are poor and income constrained; some taxpayers are relatively rich. Thus, when the population of tax debtors consists not just from people who are simply income constrained but also from high-income individuals, it is optimal to impose the collateral tax sanction.

The paper proceeds as follows. Section 2 explains how enforcement of tax evasion and tax debt collection is organized in practice and describes collateral tax sanctions. Section 3 presents a dynamic model of the tax debt repayment behavior when risk-averse individuals are heterogeneous to the extent of being income constrained and in ability to delay tax debt payment. Section 4 characterizes the optimal choice of a monetary fine and a collateral tax sanction for the tax debt enforcement purpose in the cases when taxpayers are homogeneous and heterogeneous in ability to delay tax debt payment. Section 5 concludes.

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[5]In *Andreoni* (1992), individuals are only characterized by the extent of being income constrained.

3.2 Enforcement in Practice

Tax enforcement can be considered as a two-stage process. The first stage is to discover tax evasion and avoidance and to assess the due amount of taxes. The next stage is to collect the assessed due amounts. To enforce tax evasion and avoidance, the tax authority conducts mail, office, and field audits as well as collects and cross-checks information reports. As a result of a tax examination, the IRS may discover additional tax liabilities and impose tax fines. The IRS notifies taxpayers about reassessed tax liabilities and imposed tax fines and asks the taxpayer to pay the owed amount. Tax evasion constitutes a serious problem for the IRS. Therefore, it is not surprising that a great body of the tax enforcement literature concentrates on tax evasion.[6] It is important to note, that an underlying assumption in this literature is that once evasion is discovered taxpayers fully pay their additional taxes and fines. However, people not only evade taxes, they also do not pay their tax debts.[7] According to the U.S. Department of Treasury (2012), delinquent taxes composed more than 20 percent of the total U.S. gross tax gap in 2006.[8]

3.2.1 Tax Debt Collection Tools

The tax debt collection procedure is multi-step. The first step taken by the tax authority is to notify a delinquent taxpayer by letter assessing the tax amount owed and demanding payment of that amount. If after that no actions are taken to pay the tax debt or to achieve a settlement agreement, then the second step is exerted, which is to place a lien on the taxpayer's assets. It is placed in order to secure payment of taxes. A tax lien is the government's claim to taxpayer assets. This means if the taxpayer tries to sell that asset the IRS will be able to take its cut of the funds before he does. If still no actions are taken to settle or pay the taxes owed, the third step

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[6]See the surveys in Cowell 1990, Andreoni et al. 1998, Slemrod and Yitzhaki 2002.

[7]See *Galmarini et al.* 2014.

[8]For more details, see *Perez-Truglia and Troiano* (2015).

will be taken. The IRS will proceed by implementing a tax levy. Depending upon the taxpayer's financial situation, the IRS may implement any of the following forms of levy: wage garnishment, bank levy, and asset seizure.

The duration of tax debt collection process substantially depends on the type of levy that can be applied to a tax delinquent. Usually, collection through wage garnishment takes a shorter time than through property seizure. The cases of property seizure involves court examinations which, depending on the situation, may take years. Because of this, 60 percent of identified tax debts are never collected (*Burman* (2003)). That is, half of tax debtors are able to escape imposed fines. This implies that in many cases fines could be postponed or not paid, which makes monetary fines less effective for enforcement of tax debt.

3.2.2 Collateral Tax Sanctions

The use of collateral sanctions for the enforcement of tax debt collection has grown over the last decade. One of the most recent examples of the introduction of a collateral sanction is a driver's license suspension program established by New York State Tax Department in August 2013. This program aids in the collection of past-due state tax liabilities by suspending the drivers' licenses of taxpayers with past-due tax liabilities of $10,000 or more. Other states, where similar programs have been introduced, are California and Louisiana. In California, the Delinquent Taxpayer Accountability Act (Assembly Bill 1424) signed in 2011 (effective in 2012) requires to suspend occupational, professional, and drivers' licenses from the top 500 debtors. It also prohibits taxpayers on the lists from entering into contracts to provide goods and services to state agencies. In Louisiana, the Department of Revenue has adopted Suspension and Denial of Renewal of Drivers' Licenses (LAC 61:I.1355) act in 2003 (effective in 2004), which initiated the suspension, revocation, or denial of a taxpayer's driver's license if the taxpayer owes more than $1,000 in Louisiana individual income

tax. The same act requires the Department to initiate the suspension, revocation, or denial of a taxpayer's hunting and fishing licenses when the taxpayer owes more than $500 in Louisiana individual income tax.

The implementation of a driver's license suspension is a quick and cheap procedure. If a delinquent taxpayer does not respond to the tax department notification to pay taxes, the tax department can notify the department of motor vehicles to proceed with the suspension of driver's license. The actual time needed for the revocation of driver's license to affect the taxpayer is short. The suspension will remain in effect until the department of motor vehicles receives the tax department notification that the taxpayer has satisfied his or her past-due tax liabilities. Therefore, suspension of a driver's license may be an effective instrument for tax debt collection enforcement.

3.3 Model of Tax Debt

This section considers a simple dynamic model with risk-averse individuals who differ in their ability and willingness to pay tax debt. The model introduces a collateral tax sanction in addition to a monetary fine as means to enforce tax debt collection. It draws attention to features that distinguish these two instruments.

Assume that there is an infinite number of periods. In period 1, individuals are characterized by their income, w, which is distributed on $[w_l, w_h]$ according to $F(w)$. In the subsequent periods, all individuals receive identical income, I. Each individual starts period 1 with debt equal to δ and decides whether to pay tax debt in period 1 or to postpone paying until future.

I assume that individuals cannot save or borrow, so that their consumption is equal to their income. The assumption that individuals cannot borrow is important because it enables me to model a situation when some individuals are income constrained in the first period. The assumption that individuals cannot save is imposed for simplicity; in the appendix, I show that relaxation of this assumption does not

substantially affect the analysis.

The preferences are assumed to be identical across individuals and in each period are characterized by a utility function $u(\cdot)$, which is an increasing and convex function. Individuals discount their utility is future periods by discount factor β. If a taxpayer chooses to pay her tax debt in period 1, then her lifetime utility is

$$u(w - \delta) + \sum_{t=1}^{\infty} \beta^t u(I) = u(w - \delta) + \frac{\beta}{1 - \beta} u(I). \tag{3.1}$$

3.3.1 Monetary Fine

If the tax debt is not paid in period 1, the tax authority starts collection actions. Depending on the collection tool that can be applied to a delinquent taxpayer, the duration of time until the tax debt is collected can differ among taxpayers. For example, it is much faster to collect tax debt through wage garnishment, rather than through property seizure. Assume that the probability that the tax debt is collected from a taxpayer in a given period of time, p, is distributed across the population with distribution $G(p)$. To discourage delaying of tax debt payment, the tax authority can impose monetary fine, $(\pi - 1) > 0$, that is proportional to tax debt amount. The monetary fine is paid by a delinquent taxpayer only at the time when the tax debt is collected.

When a tax debt is not paid in period 1 and a collection tool is applied, the expected utility of a tax debtor in period 2 is

$$E[u|\pi]_{t=2} = p \left[u(I - \pi\delta) + \frac{\beta}{1 - \beta} u(I) \right] + (1 - p) \left[u(I) + \beta E[u|\pi]_{t=2} \right]. \tag{3.2}$$

Use equation (3.2) to solve for $E[u|\pi]_{t=2}$, which is

$$E[u|\pi]_{t=2} = \frac{p}{1 - \beta + \beta p} u(I - \pi\delta) + \frac{\frac{\beta p}{1-\beta} + 1 - p}{1 - \beta + \beta p} u(I). \qquad (3.3)$$

Note that $\frac{p}{1-\beta+\beta p} + \frac{\frac{\beta p}{1-\beta}+1-p}{1-\beta+\beta p} = \frac{1}{1-\beta}$. Then, the tax debtor's lifetime utility, when she chooses not to pay her tax debt in period 1, is

$$u(w) + \beta E[u|\pi]_{t=2} = u(w) + qu(I - \pi\delta) + (\gamma - q)u(I), \qquad (3.4)$$

where for simplicity of notations I denote $q = \frac{\beta p}{1-\beta+\beta p}$ and $\gamma = \frac{\beta}{1-\beta}$.

As equation (3.4) suggests, q can be interpreted as the probability with which the tax debt and the monetary fine are collected. Given that $p \in [0, 1]$ is distributed across the population with probability $G(p)$, q is distributed on $[0, \beta]$ with probability $H(q) = G(\frac{1-\beta}{\beta} \frac{q}{1-q})$.

3.3.2 Collateral Tax Sanction

In addition to the monetary fine, the tax authority can impose a collateral tax sanction to enforce payment of the tax debt in period 1. In contrast to the monetary fine, a collateral tax sanction affects the tax debtor right away and remains in effect until the tax debt is paid. As a result, the longer the tax debt payment is postponed the longer the collateral tax sanction is applied. An imposition of the collateral tax sanction reduces the utility by c in each period when it is in effect. The lifetime utility, when the tax debt is not paid in period 1 and both the monetary fine and the collateral tax sanction are imposed, is $u(w) - c + \beta E[u|\pi, c]$, where $E[u|\pi, c]$ is determined by

$$E[u|\pi, c] = p \left[u(I - \pi\delta) + \frac{\beta}{1 - \beta} u(I) \right] + (1 - p) \left[u(I) - c + \beta E[u|\pi, c] \right]. \qquad (3.5)$$

Using equation (3.5), solve for $E[u|\pi, c]$ to obtain

$$E[u|\pi, c] = \frac{p}{1 - \beta + \beta p} u(I - \pi\delta) + \frac{\frac{\beta p}{1-\beta} + 1 - p}{1 - \beta + \beta p} u(I) - \frac{1 - p}{1 - \beta + \beta p} c. \qquad (3.6)$$

The tax debtor's lifetime utility, when she chooses not to pay her tax debt in period 1 and both the monetary fine and the collateral tax sanction are imposed, is

$$u(w) - c + \beta E[u|\pi, c] = u(w) + qu(I - \pi\delta) + (\gamma - q)u(I) - \theta(1 - q)c, \qquad (3.7)$$

where $\theta = \frac{1}{1-\beta} > 1$.

As equation (3.7) shows, the utility cost of the collateral tax sanction is proportional to $(1 - q)$, which reflects that the longer the tax debtor postpones paying her tax debt the longer she is subject to the collateral tax sanction. So, while those who have lower q are affected less by monetary fine, they are affected more by the collateral tax sanction.

3.3.3   The Tax Debtor's Problem

In this model, a tax debtor makes only one decision: she decides whether to pay tax debt today or not.[9] By combining equation (3.1) and equation (3.7), we can express the tax debtor's problem as

$$\max_{x \in \{0,1\}} u(w - x\delta) + qu(I - (1 - x)\pi\delta) + (\gamma - q)u(I) - (1 - x)\theta(1 - q)c.$$

A tax debtor chooses to pay tax debt $(x = 1)$ iff $u(w - \delta) + \gamma u(I) \geq u(w) + qu(I -$

---

[9]An extension of the model when individuals can make savings in period 1 is considered in the appendix.

$\pi\delta) + (\gamma - q)u(I) - \theta(1 - q)c$. This condition can be simplified and expressed as

$$u(w) - u(w - \delta) \leq q\left[u(I) - u(I - \pi\delta)\right] + \theta(1 - q)c. \qquad (3.8)$$

For a given q, let us define $\tilde{w}(q)$ such that

$$u(\tilde{w}(q)) - u(\tilde{w}(q) - \delta) = q\left[u(I) - u(I - \pi\delta)\right] + \theta(1 - q)c. \qquad (3.9)$$

To ensure that income does not exceed w_h, let us define $\hat{w}(q) = min\{\tilde{w}(q), w_h\}$.

Assumption 3.1. Assume also that the monetary fine is bounded from the above, so that $\pi \leq \frac{I}{\delta}$. This assumption guarantees that individuals do not have negative income (i.e, $I - \pi\delta \geq 0$).

As proposition 3.1 shows, individuals with income above $\hat{w}(q)$ choose to pay tax debt, and individuals with income below $\hat{w}(q)$ choose not to pay the tax debt.

Proposition 3.1. *Assume that $u(\cdot)$ is strictly convex. Let $\hat{w}(q)$ be defined by (3.9).*

i) If the tax debtor's income in period 1 is $w \in [w_l, \hat{w}(q))$, then the tax debtor chooses not to pay tax debt in period 1 ($x^* = 0$). If the tax debtor's income in period 1 is $w \in [\hat{w}(q)), w_h]$, then the tax debtor chooses to pay tax debt in period 1 ($x^* = 1$).

ii) If only the monetary fine is imposed ($c = 0$), then $\hat{w}(q)$ decreases with q (i.e., $\frac{\partial \hat{w}(q)}{\partial q} \leq 0$). If both the monetary fine and the collateral tax sanction are imposed, then $\hat{w}(q)$ may decrease or increase or be unchanged with q (i.e., $\frac{\partial \hat{w}(q)}{\partial q} \lesseqgtr 0$).

iii) The cut-off income $\hat{w}(q)$ decreases with π and c.

Proof. i) Because $u(\cdot)$ is strictly convex, the left-hand side of equation (3.8) is decreasing in w. The right-hand side of equation (3.8) is constant (w.r.t w). Therefore,

given the definition of $\hat{w}(q)$, when $w \in [\hat{w}(q)), w_h]$, condition (3.8) is satisfied and hence $x = 1$. When $w \in [w_l, \hat{w}(q))]$, the condition (3.8) is not satisfied and hence $x = 0$. ii) When $c = 0$, $\frac{\partial \hat{w}(q)}{\partial q} = -\frac{u(I) - u(I - \pi\delta)}{u'(\hat{w}(q) - \delta) - u'(\hat{w}(q))} < 0$, because $u(\cdot)$ is convex. When $c > 0$, $\frac{\partial \hat{w}(q)}{\partial q} = -\frac{[u(I) - u(I - \pi\delta)] - \theta c}{u'(\hat{w}(q) - \delta) - u'(\hat{w}(q))} \lessgtr 0$. iii) $\frac{\partial \hat{w}(q)}{\partial \pi} = -\frac{q\delta u'(I - \pi\delta)}{u'(\hat{w}(q) - \delta) - u'(\hat{w}(q))} < 0$, and $\frac{\partial \hat{w}(q)}{\partial c} = -\frac{\theta(1-q)}{u'(\hat{w}(q) - \delta) - u'(\hat{w}(q))} < 0$.

There are two reasons why a tax debtor may not pay the tax debt. First, she may be income constrained ($w < \hat{w}(q)$). Second, she may be able to postpone paying tax debt for a long period of time (has a low q). This second reason is responsible for the fact that $\frac{\partial \hat{w}(q)}{\partial q} < 0$ when only the monetary fine is imposed. The lower is q, the less likely the person will choose to pay the tax debt. An imposition of the collateral tax sanction reduces the motivation for people with low q to postpone paying the tax debt.

3.3.4   Tax Revenue

When the tax debt is paid in period 1, the tax authority receives δ. When the tax debt is not paid in period 1, the tax authority collects $\beta E[TR]$, which is determined by

$$E[TR] = p\pi\delta + (1 - p)\beta E[TR]. \tag{3.10}$$

From this equation, $E[TR] = \frac{p\pi\delta}{1 - \beta + \beta p} = \frac{q}{\beta}\pi\delta$. Hence, the tax revenue is equal to

$$
\begin{aligned}
TR &= \int_0^\beta \left[\int_{w_l}^{\hat{w}(q)} \delta dF(w) + \int_{\hat{w}(q)}^{w_h} q\pi\delta dF(w) \right] dH(q) \\
&= \int_0^\beta \left[\delta(1 - F(\hat{w}(q))) + q\pi\delta F(\hat{w}(q)) \right] h(q) dq.
\end{aligned}
\tag{3.11}
$$

The tax authority cares not only about tax revenue, but also about welfare of taxpayers, which is equal to

$$W = \int_0^\beta \int_{w_l}^{\hat{w}(q)} \left[u(w) + qu(I - \pi\delta) + (\gamma - q)u(I) - \theta(1-q)c \right] dF(w)dH(q) +$$
$$+ \int_0^\beta \int_{\hat{w}(q)}^{w_h} \left[u(w - \delta) + \gamma u(I) \right] dF(w)dH(q).$$

(3.12)

The tax authority maximizes the social welfare that is a weighted average of the tax revenue and private welfare:

$$\max_{1 \leq \pi \leq \frac{I}{\delta}, \, 0 \leq c} \alpha TR + (1 - \alpha)W.$$

(3.13)

As can be seen from (3.11), both the monetary fine and the collateral tax help to raise tax revenue by encouraging more people to pay their tax debt in the first period (i.e., by reducing the income cutoff, $\hat{w}(q)$). The monetary fine, however, generates additional revenue, because fines are added to the tax debt. As (3.12) reveals, both the monetary fine and the collateral tax sanction reduces private welfare of those individuals who choose not to pay the tax debt in the first period. But, these two instruments differ in how likely those welfare costs apply to a q-type individual.

3.4   Optimal policy

The analysis of the tax authority's problem proceeds by considering first the case when individuals are homogeneous in q and then when individuals are heterogeneous in q.

3.4.1   Optimal policy when taxpayer are homogeneous in ability to postpone paying tax debt

Assume that all individuals are characterized by the same q. Then, there is the same cut-off income $\hat{w}$ for everyone. Debtors with income above it choose to pay their tax debts. Debtors with income below it choose to delay paying.

The tax revenue, in this case, reduces to

$$TR = \delta(1 - F(\hat{w})) + q\pi\delta F(\hat{w}). \tag{3.14}$$

The formula (3.12), describing the welfare of taxpayers, reduces to

$$W = \int_{w_l}^{\hat{w}} [u(w) + qu(I - \pi\delta) + (\gamma - q)u(I) - \theta(1 - q)c] \, dF(w) + \tag{3.15}$$
$$+ \int_{\hat{w}}^{w_h} [u(w - \delta) + \gamma u(I)] \, dF(w).$$

The FOCs for the tax authority's maximization problem (3.13) w.r.t π and c are

$$FOC_\pi : \; \alpha \left[q\delta F(\hat{w}) + \delta(1 - q\pi)f(\hat{w}) \left(-\frac{\partial \hat{w}}{\partial \pi} \right) \right] - (1 - \alpha)q\delta u'(I - \pi\delta)F(\hat{w}) \lesseqgtr 0, \tag{3.16}$$

$$FOC_c : \; \alpha\delta(1 - q\pi)f(\hat{w}) \left(-\frac{\partial \hat{w}}{\partial c} \right) - (1 - \alpha)\theta(1 - q)F(\hat{w}) \leq 0, \tag{3.17}$$

where (3.16) holds with equality if $1 < \pi < \frac{I}{\delta}$, is negative if $\pi = 1$ and is positive if $\pi = \frac{I}{\delta}$, and (3.17) holds with equality if $c > 0$.

By substituting $\frac{\partial \hat{w}(q)}{\partial \pi} = -\frac{q\delta u'(I - \pi\delta)}{u'(\hat{w}(q) - \delta) - u'(\hat{w}(q))} < 0$ and $\frac{\partial \hat{w}(q)}{\partial c} = -\frac{\theta(1 - q)}{u'(\hat{w}(q) - \delta) - u'(\hat{w}(q))} < 0$ into equations (3.16) and (3.17), we can simplify those equations and express them as

$$FOC_\pi : \; \frac{F(\hat{w})}{u'(I - \pi\delta)} + \frac{\delta(1 - q\pi)f(\hat{w})}{u'(\hat{w} - \delta) - u'(\hat{w})} - \frac{(1 - \alpha)}{\alpha}F(\hat{w}) \lesseqgtr 0, \tag{3.18}$$

$$FOC_c : \; \frac{\delta(1 - q\pi)f(\hat{w})}{u'(\hat{w} - \delta) - u'(\hat{w})} - \frac{(1 - \alpha)}{\alpha}F(\hat{w}) \leq 0. \tag{3.19}$$

As can be seen, the expression for FOC_π differs from FOC_c only by the first

term, $\frac{F(\hat{w})}{u'(I-\pi\delta)}$, which arises because the monetary fine generates extra revenue for each dollar of delayed tax debt. Because this term is positive, whenever FOC_π is equal to zero, FOC_c is negative. Moreover, if FOC_c was zero, FOC_π would be positive. As a result, at the optimum $c^* = 0$ whenever $\pi^* < \frac{I}{\delta}$. That is, in this case, when it is optimal to use not the highest monetary fine (i.e., $\pi^* < \frac{I}{\delta}$), the tax authority does not use collateral tax sanction and relies only on the monetary fine. This is true for any $\alpha \in (0,1)$. The reason is the following. The only dimension, in which individuals are different, is the amount of income in the first period. Therefore, the only motivation for not paying tax debt is being income constrained. Hence, to maximize social welfare (i.e., to balance the tax revenue gain against private welfare loss), the tax authority only needs to select a uniform income cutoff, which can be done either by using the monetary fine or the collateral tax sanction. While both the monetary fine and the collateral tax sanction impose private welfare costs, the monetary fine, however, is preferable because it generates additional revenue.

The following proposition summarizes the result of this section.

Proposition 3.2. *Assume that all individuals are characterized by the same q and differ only in their income in period 1. Then, at the optimum $c^* = 0$ whenever $\pi^* < \frac{I}{\delta}$, where the optimal monetary fine, π^*, is the solution of FOC (3.18).*

3.4.2 Optimal policy when taxpayers are heterogeneous in q

Return now to the case when individuals differ in ability to postpone paying tax debt (i.e., heterogeneous in q). Recall that in this case debtors may have two motives for not paying the tax debt in period 1. While some debtors may not pay tax debt because they are income constrained, others may do this because they expect to delay in the tax debt collection for a long period of time.

The effect of the monetary fine and the collateral tax sanction depends on debtor's ability to postpone paying the tax debt. The later the tax debt is collected, the later the monetary fine is paid. However, the collateral tax sanction cannot be delayed, it affects the debtor immediately and is applied as long as the tax debt is not paid. Therefore, while a collateral tax sanction does not generate additional revenue (as a monetary fine does), it has an advantage over monetary fine as it allows a stronger punishment for those debtors who can delay paying taxes for longer.

To determine the conditions, when it is optimal to impose the collateral tax sanction, consider the tax authority's maximization problem (3.13). The FOCs for this problem w.r.t π and c are

$$
\begin{aligned}
FOC_\pi: \ & \alpha \int_0^\beta \left[qF(\hat{w}(q)) + (1 - q\pi)f(\hat{w}(q)) \left(-\frac{\partial \hat{w}(q)}{\partial \pi} \right) \right] dH(q) - \\
& -(1 - \alpha) \int_0^\beta qu'(I - \pi\delta)F(\hat{w}(q))dH(q) \lessgtr 0,
\end{aligned}
\tag{3.20}
$$

$$
\begin{aligned}
FOC_c: \ & \alpha \int_0^\beta \delta(1 - q\pi)f(\hat{w}(q)) \left(-\frac{\partial \hat{w}(q)}{\partial c} \right) dH(q) \\
& -(1 - \alpha) \int_0^\beta \theta(1 - q)F(\hat{w}(q))dH(q) \leq 0,
\end{aligned}
\tag{3.21}
$$

where (3.20) holds with equality if $1 < \pi < \frac{I}{\delta}$, is negative if $\pi = 1$ and is positive if $\pi = \frac{I}{\delta}$, and (3.21) holds with equality if $c > 0$. Let (π^*, c^*) represent the solution to (3.20) and (3.21).

By using $\frac{\partial \hat{w}(q)}{\partial \pi} = -\frac{q\delta u'(I - \pi\delta)}{u'(\hat{w}(q) - \delta) - u'(\hat{w}(q))} < 0$ and $\frac{\partial \hat{w}(q)}{\partial c} = -\frac{\theta(1 - q)}{u'(\hat{w}(q) - \delta) - u'(\hat{w}(q))} < 0$, we can simplify the above expressions:

$$
FOC_\pi: \ \int_0^\beta q\Psi(q, \pi)dH(q) + \frac{1}{u'(I - \pi\delta)} \int_0^\beta qF(\hat{w}(q))dH(q) \lessgtr 0,
\tag{3.22}
$$

$$FOC_c : \int_0^\beta (1-q)\Psi(q,\pi)dH(q) \le 0, \tag{3.23}$$

where $\Psi(q,\pi) = \frac{\delta(1-q\pi)f(\hat{w}(q))}{u'(\hat{w}(q)-\delta)-u'(\hat{w}(q))} - \frac{(1-\alpha)}{\alpha}F(\hat{w}(q))$.

Let $\tilde{\pi}$ denote the level of the monetary fine that solves (3.22) when c is constrained to be zero. Then the necessary and sufficient condition for $c^* > 0$ is that the left-hand side of (3.23) evaluated at $(\tilde{\pi}, c = 0)$ is positive. The following proposition summarizes this result.

> **Proposition 3.3**. *Assume that individuals are heterogeneous in q and as well as in their income in period 1. Then, the collateral tax sanction is used in the optimum if $\int_0^\beta (1-q)\Psi(q,\tilde{\pi})dH(q) > 0$, where $\Psi(q,\pi) = \frac{\delta(1-q\pi)f(\hat{w}(q))}{u'(\hat{w}(q)-\delta)-u'(\hat{w}(q))} - \frac{(1-\alpha)}{\alpha}F(\hat{w}(q))$ and $\tilde{\pi}$ the level of monetary fine that solves (3.22) when c is constrained to be zero.*

Let us discuss the meaning of $\Psi(q,\pi) = \frac{\delta(1-q\pi)f(\hat{w}(q))}{u'(\hat{w}(q)-\delta)-u'(\hat{w}(q))} - \frac{(1-\alpha)}{\alpha}F(\hat{w}(q))$. The numerator of the first term, $\delta(1-q\pi)f(\hat{w}(q))$, is the change in the tax revenue received from q-type taxpayers, given that the income cutoff, $\hat{w}(q)$, decreases by 1 (i.e., $\frac{\triangle TR(q)}{\triangle \hat{w}(q)}$). The denominator of the first term, $\frac{1}{u'(\hat{w}(q)-\delta)-u'(\hat{w}(q))}$, represents the change in the income cutoff when the monetary fine (or the collateral tax sanction) changes so that the corresponding decrease in utility, $q\delta u'(I - \pi\delta)(\triangle\pi)$ (or $\theta(1-q)(\triangle c)$), is equal to 1, which follows from the formulas $\frac{\partial \hat{w}(q)}{\partial \pi} = -\frac{q\delta u'(I-\pi\delta)}{u'(\hat{w}(q)-\delta)-u'(\hat{w}(q))}$ and $\frac{\partial \hat{w}(q)}{\partial c} = -\frac{\theta(1-q)}{u'(\hat{w}(q)-\delta)-u'(\hat{w}(q))}$. The second term, $\frac{(1-\alpha)}{\alpha}F(\hat{w}(q))$, is equal to the mass of those debtors, who chooses not to pay tax debt in period 1 and thus are exposed to the monetary fine (or the collateral tax sanction), multiplied by relative weight of private welfare, $\frac{1-\alpha}{\alpha}$. Given, the decrease in utility is equal to 1, this term represents the change in the private welfare. Thus, $\Psi(q,\pi)$ is the difference between the tax revenue gain and private welfare loss for a q-type taxpayer when the imposition of

the monetary fine or collateral tax sanction decreases the taxpayer's utility by 1.

Note that when $\tilde{\pi}$ is sufficiently large (but it is still true that $\tilde{\pi}\delta < I$) the value of $\frac{1}{u'(I-\pi\delta)}$ is close to zero and therefore the left-hand side of (3.22) is approximately zero (i.e., $\int_0^\beta q\Psi(q,\pi)dH(q) \simeq 0$). Then, the left-hand side of (3.23) evaluated at $(\tilde{\pi}, c = 0)$ is positive if $\int_0^\beta (1-q)\Psi(q,\tilde{\pi})dH(q) > \int_0^\beta q\Psi(q,\tilde{\pi})dH(q)$. This condition is likely satisfied given that $\Psi(q,\pi)$ is positive for small values of q and negative for large values of q.

The reason why an imposition of a collateral tax sanction may become optimal when tax debtors are heterogeneous in ability to postpone paying tax debt in addition to being heterogeneous in income is that the collateral tax sanction helps to segregate different types of tax debtors. That is, the tax debtors with low q from the tax debtors high q. Recall that when only a monetary fine is used, the income cutoff, $\hat{w}(q)$, declines with q (i.e., $\frac{\partial \hat{w}(q)}{\partial q} < 0$). So, those who have lower q are affected less by the monetary fine. When the collateral tax sanction is imposed in addition to the monetary fine, $\hat{w}(q)$ may become increasing with q. This means that an imposition of the collateral tax sanction reduces the motivation for people with low q to postpone paying their tax debt.

3.4.2.1 Special Case Example

To provide a better intuition of when it is optimal to use $c^* > 0$, I impose some additional assumptions. Assume that $u(c) = \frac{c^{0.5}-1}{0.5}$ and assume that w is uniformly distributed on $[0, w_h]$ and q is uniformly distributed on $[0, \beta]$. In this case, we can make some inferences about the sign of $\int_0^\beta \Psi(q,\pi)dH(q)$. Note also that for $\int_0^\beta (1-q)\Psi(q,\pi)dH(q)$ to be positive it is sufficient that $\int_0^\beta \Psi(q,\pi)dH(q)$ is positive given that $\tilde{\pi}$ is less that $\frac{I}{\delta}$ meaning that $\int_0^\beta q\Psi(q,\pi)dH(q)$ is negative as it follows from (3.22). As the following proposition shows, when w_h converges to the size of debt (i.e., δ), $\int_0^\beta \Psi(q,\pi)dH(q)$ is negative, while when w_h converges to infinity, $\int_0^\beta \Psi(q,\pi)dH(q)$ is

positive.

> **Proposition 3.4**. *Assume that $u(c) = \frac{c^{0.5}-1}{0.5}$ and assume that w is uniformly distributed on $[0, w_h]$ and q is uniformly distributed on $[0, \beta]$. Then,*
> $$\lim_{w_h \to \delta} \int_0^\beta \Psi(q, \pi)dH(q) = -\frac{1-\alpha}{\alpha}\frac{\sqrt{\delta}}{\sqrt{I}-\sqrt{I-\pi\delta}} < 0 \ and \ \lim_{w_h \to +\infty} \int_0^\beta \Psi(q, \pi)dH(q) = \frac{3\delta}{2\beta(\sqrt{I}-\sqrt{I-\pi\delta})} > 0.$$

Proof. In the appendix.

This proposition implies that for some high values of w_h the value of $\int_0^\beta \Psi(q, \pi)dH(q)$ is positive, which is (as discussed above) sufficient for the c^* to be positive at the optimum. Thus, when w_h is high and therefore the population of tax debtors includes some high-income individuals, it becomes optimal to impose the collateral tax sanction.

Numerical Calculation To illustrate the above result, I run a numerical calculation for the following case. I continue to assume that $u(c) = \frac{c^{0.5}-1}{0.5}$ and that w is uniformly distributed on $[0, w_h]$ and q is uniformly distributed on $[0, \beta]$. I assume also that $\beta = 0.8$, $I = 1$. Table 1 provides results for $\alpha = 0.4$, Table 2 for $\alpha = 0.5$, and Table 3 for $\alpha = 0.6$. The rows of the tables correspond to different values of tax debt, δ. The columns of the tables correspond to different values of upper bound of income, w_h. The pair (1.67;¡0) within a table indicates that $\tilde{\pi} = 1.67$, $FOC_c < 0$.

As Table 1 shows, when $w_h \leq 2.1$ the FOC_c is negative, when $w_h \geq 2.2$ the FOC_c is positive. This means that it becomes optimal to impose a collateral tax sanction ($c^* > 0$) once w_h is greater than some value that lies between 2.1 and 2.2. Note that small value of w_h (relative to $I = 1$) implies that in period 1 all taxpayers are relatively poor. So, when initially all taxpayers are relatively income constrained, it is non-optimal to use the collateral tax sanction. When the spread of income in period 1 is sufficiently large, it becomes optimal to impose the collateral tax sanction.

Table 3.1: Numerical Example for $\alpha = 0.4$

	$w_h = 1.8$	$w_h = 2.0$	$w_h = 2.1$	$w_h = 2.2$	$w_h = 2.3$	$w_h = 2.5$
$\delta = 0.1$	$(1.67; < 0)$	$(1.78; < 0)$	$(1.84; < 0)$	$(1.9; > 0)$	$(.96; > 0)$	$(2.07; > 0)$
$\delta = 0.3$	$(1.36; < 0)$	$(1.44; < 0)$	$(1.48; < 0)$	$(1.52; > 0)$	$(1.57; > 0)$	$(1.64; > 0)$
$\delta = 0.5$	$(1.12; < 0)$	$(1.18; < 0$	$(1.22; < 0)$	$(1.24; > 0)$	$(1.27; > 0)$	$(1.33; > 0)$

where pair $(1.67; ¡0)$ indicates that $\widetilde{\pi} = 1.67$, $FOC_c < 0$.

Table 3.2: Numerical Example for $\alpha = 0.5$

	$w_h = 1.2$	$w_h = 1.4$	$w_h = 1.5$	$w_h = 1.6$	$w_h = 1.8$	$w_h = 2.0$
$\delta = 0.1$	$(2.2; < 0)$	$(2.4; < 0)$	$(2.5; < 0)$	$(2.65; > 0)$	$(2.85; > 0)$	$(3.12; > 0)$
$\delta = 0.3$	$(1.71; < 0)$	$(1.44; < 0)$	$(1.9; > 0)$	$(1.95; > 0)$	$(2.1; > 0)$	$(2.22; > 0)$
$\delta = 0.5$	$(1.32; < 0)$	$(1.18; < 0$	$(1.42; < 0)$	$(1.5; > 0)$	$(1.58; > 0)$	$(1.64; > 0)$

where pair $(2.2; ¡0)$ indicates that $\widetilde{\pi} = 2.2$, $FOC_c < 0$.

Table 3.3: Numerical Example for $\alpha = 0.6$

	$w_h = 0.9$	$w_h = 1.1$	$w_h = 1.2$	$w_h = 1.3$	$w_h = 1.4$	$w_h = 1.6$
$\delta = 0.1$	$(2.94; < 0)$	$(3.35; < 0)$	$(3.56; < 0)$	$(3.78; > 0)$	$(4.0; > 0)$	$(4.5; > 0)$
$\delta = 0.3$	$(2.11; < 0)$	$(2.31; < 0)$	$(2.4; > 0)$	$(2.5; > 0)$	$(2.57; > 0)$	$(2.71; > 0)$
$\delta = 0.5$	$(1.52; < 0)$	$(1.64; < 0$	$(1.69; > 0)$	$(1.74; > 0)$	$(1.77; > 0)$	$(1.84; > 0)$

where pair $(2.94; ¡0)$ indicates that $\widetilde{\pi} = 2.94$, $FOC_c < 0$.

As can be seen from Table 2 and Table 3, similar pattern takes place for $\alpha = 0.5$ and $\alpha = 0.6$. When $\alpha = 0.5$, the value of w_h, at which it becomes optimal to use the collateral tax sanction, lies between 1.4 and 1.6. When $\alpha = 0.6$, the threshold value of w_h lies between 1.1 and 1.3.

Comparison of Tables 1, 2 and 3 gives two additional findings. First, the threshold value of w_h depends non-monotonically on the size of the debt, δ. Second, as α rises, that is as the tax revenue receives a higher weight in social welfare, the threshold value of w_h, at which it becomes optimal to use the collateral tax sanction, declines. This indicates that when the tax authority cares less about private welfare, it is willing to impose the collateral tax sanction at a lower spread of income in period 1.

3.5 Conclusion

In the United States, the size of unpaid tax debt is substantial: more than half of identified tax debts are never collected. Hence, it is critical to have the strategy to enforce tax debt collection that is effective and does not harm debtor's welfare. However, it has not been well studied in the existing literature. This paper explores under which conditions the optimal strategy to enforce tax debt collection involves using collateral tax sanctions (i.e., suspension of a driver's license).

The reasons for why people do not pay their tax debt could be different. It is possible that some people just cannot afford to pay their tax debts because they are income constrained. Other taxpayers do not pay their tax debt in the hope to escape from their liabilities by, for example, prolonging court hearings. This diversity in motivations for not paying the tax debt makes the enforcement of tax debt collection a complex task.

To discourage tax debt, the tax authority imposes monetary fines (i.e., an above-market interest rate on the debt amount). Additionally, to collect tax debt and monetary fines the tax authority uses such collection tools as wage garnishment,

bank levy, or asset seizure. Depending on which collection tool is applicable, the collection process may significantly vary in its length. As a result, it may take a long time until the tax debt and monetary fines are collected.

Another enforcement instrument that some state tax authorities have recently used is suspension of driver's license, which is an example of a collateral tax sanction. In contrast to the monetary fine, the collateral tax sanction can be quickly applied and starts to influence the tax debtor immediately. Moreover, it remains in effect until the tax debt is paid and, therefore, affects stronger those debtors who postpone paying tax debt for a longer period of time.

In this paper, I develop a simple dynamic model describing tax debtors' behavior when they are heterogeneous in income and ability to escape tax debt payment. I consider the tax authority that maximizes the weighted sum of the revenue from collecting tax debt and taxpayers' welfare. I characterize the optimal strategy of the tax authority that chooses the size of the monetary fine and the collateral tax sanction.

I show that when debtors are homogeneous in their ability to escape tax debt payment, it is optimal to use only monetary fine. However, when debtors differ in their ability to escape tax debt payment, it may be optimal to use the collateral tax sanction in addition to the monetary fine. Specifically, in the case when utility has constant relative risk aversion and the distribution of income and the distribution of ability to escape tax debt are uniform, it is optimal to use the collateral tax sanction if the upper bound of income distribution is sufficiently large. That is, if the population of taxpayers includes not only income constrained individuals, but also some high-income individuals.

It is important to note that there are, however, other properties of collateral tax sanctions that have not been considered in this paper. Some of these other properties are discussed by *Blank* (2014) and Paramonova (2015). When a policy implementing

a collateral sanction is designed, it is critical to take into consideration all features of the collateral tax sanctions.

APPENDICES

APPENDIX A

Appendix to Chapter 1

An example of an optimal audit strategy characterized by leaving some audit classes unaudited

Assume that income, i, is uniformly distributed on $[0, I]$. Also suppose that a signal is generated according to a process such that $s = i + \epsilon$, where noise ϵ is uniformly distributed on $[-\frac{1}{a}, \frac{1}{a}]$. In what follows, I assume that the lowest level of accuracy is $a_L = \frac{2}{I}$. Based on this, the density of signal, s, can be calculated and expressed as

$$
h(s) = \begin{cases} \frac{\min\{I, s+\frac{1}{a}\} - \max\{0, s-\frac{1}{a}\}}{2I\frac{1}{a}}, & if \ -\frac{1}{a} \leq s \leq I + \frac{1}{a} \\ 0, & otherwise \end{cases} = \begin{cases} \frac{s+\frac{1}{a}}{2I\frac{1}{a}}, & if \ -\frac{1}{a} \leq s \leq \frac{1}{a} \\ \frac{1}{I}, & if \ \frac{1}{a} \leq s \leq I - \frac{1}{a} \\ \frac{I-s+\frac{1}{a}}{2I\frac{1}{a}}, & if \ I - \frac{1}{a} \leq s \leq I + \frac{1}{a} \\ 0, & otherwise. \end{cases}
$$

(A.1)

The conditional density of income given signal, s, is

$$
f(i|s) = \begin{cases} \frac{1}{min\{I,s+\frac{1}{a}\}-max\{0,s-\frac{1}{a}\}}, & if\ max\{0, s - \frac{1}{a}\} < i < min\{I, s + \frac{1}{a}\} \\ 0, & otherwise \end{cases}, \quad (A.2)
$$

the conditional c.d.f. is

$$
F(i|s) = \begin{cases} \frac{i-max\{0,s-\frac{1}{a}\}}{min\{I,s+\frac{1}{a}\}-max\{0,s-\frac{1}{a}\}}, & if\ max\{0, s - \frac{1}{a}\} < i < min\{I, s + \frac{1}{a}\} \\ 0, & otherwise \end{cases}, \quad (A.3)
$$

and $\gamma(i|s) = min\{I, s + \frac{1}{a}\} - i, \quad for\ max\{0, s - \frac{1}{a}\} \leq i \leq min\{I, s + \frac{1}{a}\}$.

In this example, signal $s = -\frac{1}{a}$ can be generated only by income $i = 0$. More generally, very low (high) signals can be generated only by low (high) enough incomes. This is because each signal is generated according to the same process and both income and signal distributions have a bounded support. This relationship between signals and incomes allows the tax authority to infer that a taxpayer with low (high) signal has to have only relatively low (high) income. As a result, as seen from (A.2), low signals, $s \in [-\frac{1}{a}, \frac{1}{a}]$, and for high signals, $s \in [I - \frac{1}{a}, I + \frac{1}{a}]$, have a higher conditional density of income and are distributed on a shorter interval than the rest of the signals. This indicates that for a given a low signals, $s \in [-\frac{1}{a}, \frac{1}{a}]$, and high signals, $s \in [I - \frac{1}{a}, I + \frac{1}{a}]$ are relatively more accurate, (i.e., having a lower variance) than the rest of the signals. Moreover, the lower the signal within interval $[-\frac{1}{a}, \frac{1}{a}]$ (the higher the signal within interval $[I - \frac{1}{a}, I + \frac{1}{a}]$) the more accurate the signal is.

To describe the optimal audit policy for this example, let us calculate the audit cutoff function by using (1.7) and (1.8). The audit cutoff function can be expressed as

Figure A.1: The cutoff function
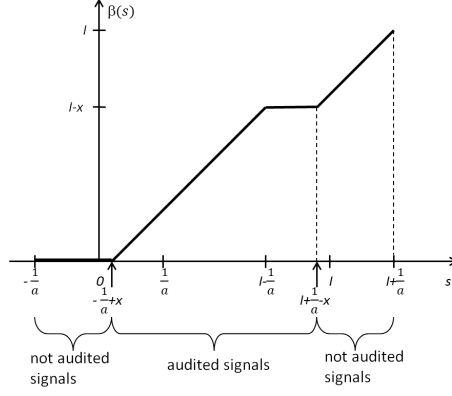
$$\beta(s) = \begin{cases} 0, & if \ -\frac{1}{a} \le s \le -\frac{1}{a} + x \\ min\{I, s + \frac{1}{a}\} - x, & if \ -\frac{1}{a} + x < s < I + \frac{1}{a} - x \\ s - \frac{1}{a}, & if \ I + \frac{1}{a} - x \le s \le I + \frac{1}{a}, \end{cases} \qquad (A.4)$$

where $x \equiv \frac{I + \frac{2}{a} - \sqrt{(I + \frac{2}{a})^2 - \frac{8I}{a}(1-P)}}{2}$ is the solution to the equation $x^2 - x(I + \frac{2}{a}) + \frac{2I}{a}(1 - P) = 0$, which satisfies $0 \le x \le \frac{2}{a}$, where $P = \frac{1+\pi}{c}[B - K(a)]$ is the share of taxpayers who are audited with probability $\frac{1}{1+\pi}$. For conciseness of notation, I write x instead of $x(a, P(a))$, but it is important to remember that x is a function of a.

As shown in Figure A.1, for low enough signals, $s \in [-\frac{1}{a}, -\frac{1}{a} + x]$, and high enough signals, $s \in [I + \frac{1}{a} - x, I + \frac{1}{a}]$, the audit cutoff, $\beta(s)$, is equal to the lowest income, $l(s)$. This means that taxpayers with those signals, $s \in [-\frac{1}{a}, -\frac{1}{a} + x]$ and $s \in [I + \frac{1}{a} - x, I + \frac{1}{a}]$, are not audited. A taxpayer with signal $s \in (-\frac{1}{a} + x; I + \frac{1}{a} - x)$ is audited if her reported income is below $min\{I, s + \frac{1}{a}\} - x$. The value of x is defined so that for a given level of signal accuracy, there are enough resources for audits. So, the optimal audit rule prescribes the tax authority not to audit those taxpayers about whom they have relatively more accurate signals and focus audits on those about whom the tax authority receives relatively less accurate signals.

Consider how the audit cutoff defined by (A.4) depends on signal accuracy. Specif-

ically, we are interested in the direct effect of signal accuracy: the effect of an exogenous change in a, assuming audit coverage, P, to be fixed.[1] Broadly speaking, when a increases and converges to infinity, the audit cutoff, $\beta(s)$, converges to the conditional expectation of the income, which is s. More specifically, this dependence for an interior signal $s \in [\frac{1}{a}; I - \frac{1}{a}]$ can be described as follows. When audit coverage is high enough, $P \geq \frac{1}{2}$, the audit cutoff, $\beta(s)$, is always greater than s, and it decreases as a increases.[2] When $\frac{1}{4} \leq P < \frac{1}{2}$, the audit cutoff, $\beta(s)$, is greater than s and decreases as a increases until $\frac{1}{a} > I(1 - 2P)$. After this, $\beta(s)$ becomes less than s, but continues to decrease as a increases while $\frac{1}{a} \geq \frac{I(1-2P)}{2}$. When $\frac{1}{a} < \frac{I(1-2P)}{2}$, $\beta(s)$ begins to increase as a increases staying below s. When $P < \frac{1}{4}$, the audit cutoff, $\beta(s)$, is less than s, it decreases as a increases until $\frac{1}{a} > I(1 - 2P)$, and after this increases as a increases.

Based on the described dependence of $\beta(s)$ on a, we can determine how the audit probability changes with a for each signal (i.e, each audit class). To do this, we need to take into account that the lowest (highest) conditional income $l(s)$ $(h(s))$ and the conditional density change with a as well. As a increases (assuming that audit coverage, P, is fixed), a reallocation of audits among audt classes (signals) takes place. First, the number of audit classes that are left unaudited, equaled to $\int_{-\frac{1}{a}}^{-\frac{1}{a}+x} h(s)ds + \int_{I+\frac{1}{a}-x}^{I+\frac{1}{a}} h(s)ds = \frac{x^2}{2\sigma I} = \frac{ax(a)^2}{2I}$, decreases as a increases. This means that a greater number of audit classes are exposed to audits. Second, the probability of being audited within an audit class decreases. Indeed, conditional on a signal, s $(s \in [\frac{1}{a}; I - \frac{1}{a}])$, the probability of being audited is $\int_{s-\frac{1}{a}}^{\beta(a)} f(i|s)di = 1 - \frac{x}{2\sigma} = 1 - \frac{ax(a)}{2}$, which decreases as a increases[3].

---

[1]Note that there is also a mechanical effect of changing accuracy a: an increase in a requires a higher investment in accuracy (i.e., a bigger $K(a)$) leaves fewer resources for audit, P, which results in a decrease of the audit cutoff for each signal.

[2]Note first that $x > \frac{1}{a}$ when $\frac{1}{a} < I(1 - 2P)$, that $\frac{\partial x}{\partial \sigma} = -a^2 \frac{\partial x}{\partial a} > 1$ when $\frac{1}{a} < \frac{I(1-2P)}{2}$, and that $\frac{\partial x}{\partial a} < 0$ always.

[3]For $y \in [-\frac{1}{a} + x, \frac{1}{a}]$ and $y \in [I - \frac{1}{a}, I + \frac{1}{a} - x]$, this dependence is more complicated. However, the share of such signals goes to zero as a goes to infinity.

To summarize the described changes in audit policy that accompany an increase in a, let us refer to a change in the number of audit classes that are exposed to audits as the extensive margin of audits, and to a change in the share of people being audited within an audit classes as the intensive margin of audits. Given that audit coverage is unchanged, the above observations demonstrate that when signal accuracy increases the extensive margin of audits increases, while the intensive margin of audits decreases.

The logic for this result is the following: The optimal auditing policy requires that, for a given audit class, the net marginal revenue of an audit is the same. If there exists a relatively more accurate set of signals (audit classes), so that audits of the taxpayers with such signals (within those audit classes) cannot generate high enough extra tax charges due to a high degree of voluntary compliance, than taxpayers with such signals (within those audit clases) are not audited. When accuracy of all signals is uniformly increased, the marginal revenue of an audit decreases. As a result, those signals, which initially were excluded from audits due to relatively low potential marginal revenue of an audit, might be included in the class of audited signals.

Proofs

Proof of Proposition 1.1

As discussed, the tax authority maximization problem (1.5) can be solved in two steps. First, for any given amount of resources assigned to conduct audits of taxpayers with signal s, $B(s)$, the audit probability is chosen to maximize $\int T(i, s, p(\cdot))dF(i \mid s)$ s.t. $\int c \cdot p(r(i, s), s)dF(i \mid s) = B(s)$. Second, optimal allocation of resources, $B(s)$, is chosen to maximize the tax authority's objective function s.t. $\int B(s)dH(s) \leq B - K(a)$.

To see this, let's denote $B(s) = \int c \cdot p(r(i, s), s)dF(i \mid s)$. Then, problem (1.5) can be rewritten as

$$\max_{p(r(i),s)} \left\{ \int \int T(i,s,p(\cdot))dF(i \mid s)dH(s) \right\}$$

$$\text{s.t. } \int c \cdot p(r(i,s),s)dF(i \mid s) = B(s) \; \forall s, \tag{A.5}$$

$$\int B(s)dH(s) \leq B - K(a).$$

The Lagrangian for this problem is

$$L = \int \left\{ \int T(i,s,p(\cdot))dF(i \mid s) + \mu(s)\left(B(s) - \int c \cdot p(r(i,s),s)dF(i \mid s)\right) \right\} dH(s) +$$

$$+ \lambda \left[B - K(a) - \int B(s)dH(s) \right].$$

As can be seen, the maximization of the Lagrangian w.r.t. $B(s)$ requires $\mu(s) = \lambda$ for all s if $B(s) > 0$.

Now, let us consider the problem of choosing optimal audit probability $p(r,s)$ for a given signal, s, to maximize $\int T(i,s,p(\cdot))dF(i \mid s)$ s.t. $\int c \cdot p(r(i,s),s)dF(i \mid s) = B(s)$.

If the conditional distribution of income has a bounded support, and hence the lower bound, $l(s)$, is finite, then this problem is identical to the one solved by *Sanchez and Sobel* (1993). It was shown that the optimal probability is $p^*(r,s) =$
$\begin{cases} \frac{1}{1+\pi}, & \textit{if } r < \beta(s) \\ 0, & \textit{if } r \geq \beta(s) \end{cases}$, where $\gamma(\beta(s) \mid s) = \frac{\mu(s)c}{t(1+\pi)}$. By combining this result with the requirement that $\mu(s) = \lambda$ for all s if $B(s) > 0$, we can express the optimal solution as described by (1.6), (1.7), and (1.8).

Now, let us consider the case when the support of the conditional distribution of income is unbounded. Let us divide the support of the conditional income distribution, $F(i|s)$, into two intervals: $(-\infty, l_n)$ and $[l_n, \infty)$, where l_n is some number. The total tax revenue can be calculated as a sum of two parts – the tax revenues corresponding each of two intervals, that is,

$$TR(s) = \int_{-\infty}^{+\infty} T(i, s, p(\cdot)) dF(i \mid s) = \int_{-\infty}^{l_n} T(i, s, p(\cdot)) dF(i \mid s) + \int_{l_n}^{+\infty} T(i, s, p(\cdot)) dF(i \mid s).$$

(A.6)

We can estimate the upper bound of the total tax revenue as follows. First, assume that taxpayers with income $i \in (-\infty, l_n)$ are honest and, therefore, report their income truthfully. So, the tax authority does not need to spend resources to conduct audits among those taxpayers, and at the same time can collect the maximal revenue, $\int_{-\infty}^{l_n} ti dF(i \mid s)$. This gives an estimate of $\int_{-\infty}^{l_n} T(i, s, p(\cdot)) dF(i \mid s)$ from above, which is $\int_{-\infty}^{l_n} ti dF(i \mid s)$.

Second, let assume that taxpayers with income $i \in [l_n, +\infty)$ cannot report income less than l_n. This assumption reduces the number of incentive compatibility constraints for the problem under consideration, and, therefore, makes our estimate of the upper bound for the tax revenue only higher. Also, this assumption allows us to apply results from Sanchez and Sobel (1993) to calculate the maximum for $\int_{l_n}^{+\infty} T(i, s, p(\cdot)) dF(i \mid s)$ given that budget constraint is $\int_{l_n}^{+\infty} c \cdot p(r(i, s), s) dF(i \mid s) = B(s)$. The optimal audit policy is characterized by $p^*(r, s) = \begin{cases} \frac{1}{1+\pi}, & \text{if } l_n \leq r < \beta_n(s) \\ 0, & \text{if } r \geq \beta_n(s) \end{cases}$,

$\gamma(\beta_n(s) \mid s) = \frac{\mu_n(s)c}{t(1+\pi)}$, and $\int_{l_n}^{\beta_n(s)} \frac{c}{1+\pi} dF(i \mid s) = B(s)$. The last equation defines $\beta_n(s)$. The solution provides the maximum for $\int_{l_n}^{+\infty} T(i, s, p(\cdot)) dF(i \mid s)$, which is equal to $\int_{l_n}^{\beta_n(s)} ti dF(i \mid s) + \int_{\beta_n(s)}^{+\infty} t\beta_n(s) dF(i \mid s)$. Thus, an upper bound for the total tax revenue is

$$\hat{TR}_n(s) = \int_{-\infty}^{l_n} ti dF(i \mid s) + \int_{l_n}^{\beta_n(s)} ti dF(i \mid s) + \int_{\beta_n(s)}^{+\infty} tb_n(s) dF(i \mid s) =$$
$$= \int_{-\infty}^{\beta_n(s)} ti dF(i \mid s) + \beta_n(s) \int_{\beta_n(s)}^{+\infty} td F(i \mid s).$$

(A.7)

Consider a sequence $\{l_n\}$ such that $\lim\limits_{n\to\infty} l_n = -\infty$. Let us calculate the limit of the upper bound of the tax revenue as l_n converge to $-\infty$. By the properties of limit

$$\lim_{l_n\to-\infty} T\hat{R}_n(s) = \int\limits_{-\infty}^{\beta(s)} ti\, dF(i\mid s) + \beta(s)\int\limits_{\beta(s)}^{+\infty} t\, dF(i\mid s),$$

where $\beta(s)$ denotes $\lim\limits_{l_n\to-\infty} \beta_n(s)$. Because $\lim\limits_{l_n\to-\infty}\int_{-\infty}^{l_n} ti\, dF(i\mid s) = 0$, $\beta(s)$ exists and is the solution for $\int_{-\infty}^{\beta(s)} \frac{c}{1+\pi} dF(i\mid s) = B(s)$.

Because $\lim\limits_{l_n\to-\infty} T\hat{R}_n(s)$ is still an upper bound, it is true that

$$TR(s) = \int\limits_{-\infty}^{+\infty} T(i,s,p(\cdot))dF(i\mid s) \leq \lim_{l_n\to-\infty} T\hat{R}_n(s).$$

If we verify that the policy function characterized by $p^*(r,s) = \begin{cases} \frac{1}{1+\pi}, & if\ r < \beta(s) \\ \\ 0, & if\ r \geq \beta(s) \end{cases}$

and $\int_{-\infty}^{\beta(s)} \frac{c}{1+\pi} dF(i\mid s) = B(s)$ allows the tax authority to obtain the amount of tax revenue equal to $\lim\limits_{l_n\to-\infty} T\hat{R}_n(s)$, then this will conclude our proof that this policy function is optimal. Indeed, this policy function provides the tax revenue equaled to $\int_{-\infty}^{b(s)} ti\, dF(i\mid s) + \int_{b(s)}^{+\infty} tb(s)dF(i\mid s) = \lim\limits_{l_n\to-\infty} T\hat{R}_n(s)$.

Therefore, even in the case when the support of income distribution is unbounded, the optimal audit policy is described by (1.6), (1.7), and (1.8). **Q.E.D.**

Proof of Corollary

Based on condition (1.7) from Proposition 1.1, it should be that

$$\frac{1 - G(a_1(\beta_1 - s_1))}{a_1 g(a_1(\beta_1 - s_1))} = \frac{1 - G(a_2(\beta_2 - s_2))}{a_2 g(a_2(\beta_2 - s_2))}, \tag{A.8}$$

where β_1 and β_2 are the audit cutoffs in the first group with accuracy a_1 and in the second group with accuracy a_2 correspondingly.

Let us define $\triangle_1 = a_1(\beta_1 - s_1)$ and $\triangle_2 = a_2(\beta_2 - s_2)$. Let us also define $\widetilde{\gamma}(\cdot) = \frac{1-G(\cdot)}{g(\cdot)}$. Equation (A.8) can be then rewritten as

$$\frac{\widetilde{\gamma}(\triangle_1)}{\widetilde{\gamma}(\triangle_2)} = \frac{a_1}{a_2}.$$

By assumption of Proposition 1.1, $\widetilde{\gamma}(\cdot)$ is strictly decreasing function. Hence, because $\frac{a_1}{a_2} < 1$, $\triangle_1 > \triangle_2$. Therefore, $G(\triangle_1) > G(\triangle_2)$. **Q.E.D.**

Proof of Proposition 1.2

i) Recall that $R'(P) = \gamma(G^{-1}(P))$, where $G^{-1}(P)$ is a strictly increasing function. Therefore, the condition that the inverse hazard rate, $\gamma(z) = \frac{1-G(z)}{g(z)}$, is strictly decreasing implies that $R'(P)$ is strictly decreasing and, thus, $R(P)$ is strictly concave. Moreover, strict concavity of $R(P)$ implies that $TR(a, P) = t(\mu + \frac{1}{a}R(P))$ is strictly quasi-concave.[4]

When function $K(a)$ is convex, the budget constraint set $\frac{c}{1+\pi}P + K(a) \leq B$ is convex. Thus, the conditions that $\gamma(z) = \frac{1-G(z)}{g(z)}$ is strictly decreasing and $K(a)$ is convex are sufficient to satisfy the assumptions of Theorem M.K.4 and Theorem M.K.2 from *Mas-Colell et al.* (1995), which guarantees that the accuracy a and audit coverage P which satisfy to the condition (1.13) and *(1.14)* constitute the unique solution of (1.12).

ii) When $K'(a_L) = 0$, a corner solution is impossible. Indeed, for a_L to be an optimum, it should be that $-\frac{R(\frac{1+\pi}{c}B)}{a_L R'(\frac{1+\pi}{c}B)} - \frac{1+\pi}{c}K'(a_L) < 0$, where I have used that $K(a_L) = 0$. Since $K'(a_L) = 0$, it follows that $-\frac{R(\frac{1+\pi}{c}B)}{a_L R'(\frac{1+\pi}{c}B)} - \frac{1+\pi}{c}K'(a_L) = -\frac{R(\frac{1+\pi}{c}B)}{a_L R'(\frac{1+\pi}{c}B)} > 0$ for any $0 < \frac{1+\pi}{c}B < 1$, which is a contradiction. Therefore, a_L cannot be a solution.

Consider now $P = 0$. Define a_H such that $K(a_H) = B$. For $P = 0$ to be an optimum, it should be that $-\frac{R(0)}{a_H R'(0)} - \frac{1+\pi}{c}K'(a_H) > 0$. Since $\frac{R(0)}{R'(0)} = 0$, as I will show below, it follows that $-\frac{R(0)}{a_H R'(0)} - \frac{1+\pi}{c}K'(a_H) = -\frac{1+\pi}{c}K'(a_H) < 0$. Therefore,

---
[4]This means that the better set $\{(a, P) : TR(a, P) \geq TR(a_0, P_0)\}$ is convex for any (a_0, P_0).

$(a_H, P = 0)$ cannot be a solution.

Let me now show that $\frac{R(0)}{R'(0)} = 0$.

$$
\begin{aligned}
\frac{R(0)}{R'(0)} &= \lim_{P\to 0}\frac{R(P)}{R'(P)} = \lim_{P\to 0}\frac{g(G^{-1}(P))}{(1-P)}\left[(1-P)G^{-1}(P) + \int_{-\infty}^{G^{-1}(P)} zg(z)dz\right] = \\
&= \lim_{P\to 0}g(G^{-1}(P))G^{-1}(P) + \frac{g(G^{-1}(P))}{(1-P)}\int_{-\infty}^{G^{-1}(P)} zg(z)dz = 0,
\end{aligned}
\tag{A.9}
$$

where the first term, $\lim_{P\to 0}g(G^{-1}(P))G^{-1}(P) = \lim_{\Delta\to -\infty} g(\Delta)\Delta$, converges to zero, because otherwise distribution $g(\cdot)$ would not have a finite first moment (mean) and the second term, $\lim_{P\to 0}\frac{g(G^{-1}(P))}{(1-P)}\int_{-\infty}^{G^{-1}(P)} zg(z)dz$, converges to zero, because $\lim_{P\to 0}\frac{g(G^{-1}(P))}{(1-P)} = 0$ and $\lim_{P\to 0}\int_{-\infty}^{G^{-1}(P)} zg(z)dz = 0$.

Thus, the solution can be only interior. **Q.E.D.**

Proof of Proposition 1.3

Under the assumptions of Proposition 1.2, the solution is unique and interior and it defined by $-\frac{R(P^*)}{a^*R'(P^*)} = \frac{c}{1+\pi}K'(a^*)$. By differentiating this expression w.r.t. B, we can obtain $\frac{da^*}{dB}$, which is equal to

$$
\frac{da^*}{dB} = \frac{1}{[-SOC(a^*)]}\frac{1}{a^*}\left[\frac{R(P^*)R''(P^*)}{R'(P^*)^2} - 1\right],
\tag{A.10}
$$

where $SOC(a^*) = -K''(a^*) + \frac{c}{1+\pi}\frac{R(P^*)^2R''(P^*)}{a^{*2}R'(P^*)^3} < 0$ is negative due to the assumptions.

From this expression it follows that the dependance of a^* on the budget, B, is defined by the sign of $\frac{R(P^*)R''(P^*)}{R'(P^*)^2} - 1$.

Let us show that there exists unique audit coverage that we denote by $\overline{P}$ such that $\frac{R(\overline{P})R''(\overline{P})}{R'(\overline{P})^2} = 1$. First, note that $\frac{R(P)R''(P)}{R'(P)^2} - 1$ is the derivative of $-\frac{R(P)}{R'(P)}$ w.r.t. P. This function, $-\frac{R(P)}{R'(P)}$, has the following properties. It is positive (i.e, $-\frac{R(P)}{R'(P)} > 0$) on $P \in (0,1)$ and $\lim_{P\to 0}\frac{R(P)}{R'(P)} = 0$ and $\lim_{P\to 1}\frac{R(P)}{R'(P)} = 0$. The fact that $\lim_{P\to 0}\frac{R(P)}{R'(P)} = 0$ has been proven above in the proof of Proposition 1.2. Let me now show that $\lim_{P\to 1}\frac{R(P)}{R'(P)} = 0$.

$$\lim_{P\to 1}\frac{R(P)}{R'(P)} = \lim_{P\to 1}\frac{g(G^{-1}(P))}{(1-P)}\left[(1-P)G^{-1}(P) + \int_{-\infty}^{G^{-1}(P)} zg(z)dz\right] =$$
$$= \lim_{P\to 1}g(G^{-1}(P))G^{-1}(P) + \lim_{P\to 1}\frac{g(G^{-1}(P))}{(1-P)}\int_{-\infty}^{G^{-1}(P)} zg(z)dz =$$
$$= \lim_{\Delta\to\infty} g(\Delta)\Delta + \lim_{P\to 1}g(G^{-1}(P))G^{-1}(P)\cdot \lim_{P\to 1}\frac{\int_{-\infty}^{G^{-1}(P)} zg(z)dz}{(1-P)G^{-1}(P)} =$$
$$= 0 + 0\cdot(-1) = 0$$

(A.11)

where the term, $\lim_{P\to 1}g(G^{-1}(P))G^{-1}(P) = \lim_{\Delta\to\infty} g(\Delta)\Delta$, converges to zero, because otherwise distribution $g(\cdot)$ would not have a finite first moment (mean). The term $\lim_{P\to 1}\frac{\int_{-\infty}^{G^{-1}(P)} zg(z)dz}{(1-P)G^{-1}(P)}$ converges to -1, because by L'hospital rule $\lim_{P\to 1}\frac{\int_{-\infty}^{G^{-1}(P)} zg(z)dz}{(1-P)G^{-1}(P)} = \lim_{P\to 1}\frac{G^{-1}(P)g(G^{-1}(P))\frac{1}{g(G^{-1}(P))}}{-G^{-1}(P)+\frac{(1-P)}{g(G^{-1}(P))}} = \lim_{P\to 1}\frac{G^{-1}(P)g(G^{-1}(P))}{-G^{-1}(P)g(G^{-1}(P))+(1-P)} = -1.$

Since $-\frac{R(P)}{R'(P)} > 0$ on $P \in (0,1)$ and $\lim_{P\to 0}\frac{R(P)}{R'(P)} = 0$ and $\lim_{P\to 1}\frac{R(P)}{R'(P)} = 0$, the derivative of of $-\frac{R(P)}{R'(P)}$ w.r.t. P has to be positive in neighborhood of zero and negative in the neighborhood of 1 and, therefore, has to change its sign at least once. If the derivative changed its sign more than once, that is minimum three times, then it would contradict to the strict quasi-convexity of the tax revenue function (1.11). It is because there would exist a convex cost function $K(a)$ such that the budget constraint curve would intersect the FOC curve in Figure 1.2 more than once which contradicts to the uniqueness of the optimal solution in Proposition 1.2 and therefore contradicts to the strict quasi-convexity of the tax revenue function (1.11). Thus, there exists unique $\overline{P}$ such that $\frac{R(\overline{P})R''(\overline{P})}{R'(\overline{P})^2} = 1$.

By differentiating $P^* = \frac{1+\pi}{c}(B - K(a^*))$ w.r.t. B and substituting $\frac{da^*}{dB}$ into the formula, we can obtain $\frac{dP^*}{dB}$, which is equal to

$$\frac{dP^*}{dB} = \frac{1}{[-SOC(a^*)]}\left[\frac{1+\pi}{c}K''(a^*) - \frac{R(P^*)}{a^{*2}R'(P^*)}\right] > 0.$$

(A.12)

This derivative shows that optimal P^* increases with the budget. Moreover, because the tax revenue function (1.11) is continuous in a and B, and the set of compacts

$a \in [a_L, a_H(B)]$, where $K(a_H) = B$, is continuous, the maximizer $a^*(B)$ is continuous in B by the Theorem of the Maximum.[5] Therefore, $P^* = \frac{1+\pi}{c}(B - K(a^*(B)))$ is continuous in B as well. Also P^* converges to 0 as B converges to 0, which follows from the fact that $P^* \leq \frac{1+\pi}{c}B$. And P^* converges to 1 as B converges to $\frac{c}{1+\pi}$, because when $B = \frac{c}{1+\pi}$, $P^* = 1$ gives the maximum value.

These four facts insure that there exists a value of B, which we define as $\overline{B}$, such that $\frac{1+\pi}{c}(\overline{B} - K(a^*(\overline{B}))) = \overline{P}$. Hence, $\frac{da^*}{dB} > 0$ when $B < \overline{B}$ and $\frac{da^*}{dB} < 0$ when $B > \overline{B}$. **Q.E.D.**

Proof of Proposition 1.4

i) Under the assumptions of Proposition 1.2, the solution is unique and interior and it defined by $-\frac{R(P)}{\delta a^* R'(P)} = \frac{1+\pi}{c}K'(a^*)$. Note that the FOC accounts for the inclusion of parameter δ. By differentiating this expression w.r.t. δ, we can obtain $\frac{da^*}{d\delta}$, which is equal to

$$\frac{da^*}{d\delta} = \frac{1}{[-SOC(a^*)]}\left\{-\frac{1}{\delta}\left[K'(a^*) - \frac{K(a^*)}{a^*}\right] - \frac{K(a^*)}{\delta a^*}\frac{R(P^*)R''(P^*)}{R'(P^*)^2}\right\} < 0, \quad \text{(A.13)}$$

where $K'(a^*) - \frac{K(a^*)}{a^*} > 0$ because $K(a)$ is convex, and, as before, $SOC(a^*) = -K''(a^*) + \frac{c}{1+\pi}\frac{R(P^*)^2 R''(P^*)}{a^{*2}R'(P^*)^3} < 0$. Therefore, $\frac{da^*}{d\delta}$ is negative.

An expression for $\frac{dP^*}{d\delta}$ can be obtained by differentiating $P^* = \frac{1+\pi}{c}(B - \delta K(a^*))$ w.r.t. δ and substituting $\frac{da^*}{d\delta}$ into the formula, which gives

$$\begin{aligned}
\frac{dP^*}{d\delta} &= -\frac{1+\pi}{c}K(a^*) + \frac{1+\pi}{c}\delta K'(a^*)(-\frac{da^*}{d\delta}) = \\
&= \frac{1+\pi}{c[-SOC(a^*)]}\left\{K'(a^*)\left[K'(a^*) - \frac{K(a^*)}{a^*}\right] - K(a^*)K''(a^*)\right\}.
\end{aligned} \quad \text{(A.14)}$$

The first term in (A.14) is positive, the second term in (A.14) is negative. As a

[5]Specifically, $a^*(B)$ is hemi-continuous, but because the maximizer, a^*, is unique for each B, it is continuous.

result, the sign of $\frac{dP^*}{d\delta}$ is uncertain. Intuitively, there are two effects of an increase in δ which can be referred to as substitution and income effects by analogy with the effects of price change on normal goods in standard consumption theory. An increase in δ can be interpreted as an increase in "price" of signal accuracy. First, it causes a decrease in the optimal signal accuracy, which leaves more resources for audits, and, therefore, increases P^*. This is the substitution effect which is positive. Second, it causes the income effect. which is negative. Specifically, an increase in δ requires greater resource expenditure to provide the same level of signal accuracy. This leaves less resources for audits, and, therefore, decreases P^*.

ii) By differentiating the FOC $-\frac{R(P)}{a^* R'(P)} = \frac{1+\pi}{c} K'(a^*)$ w.r.t. $\frac{c}{1+\pi}$, we can obtain

$$\frac{da^*}{d\frac{c}{1+\pi}} = \frac{1}{[-SOC(a^*)]} \frac{1}{a^*} \left[\frac{-R(P^*)}{R'(P^*)} + P^* \left(1 - \frac{R(P^*)R''(P^*)}{R'(P^*)^2} \right) \right]. \qquad \text{(A.15)}$$

The first term in (A.15) is positive, the second term in (A.15) is negative if $B < \overline{B}$ and positive if $B > \overline{B}$. As a result, the sign of $\frac{da^*}{d\frac{c}{1+\pi}}$ is uncertain if $B < \overline{B}$ and positive if $B > \overline{B}$. To provide the intuition for such a behavior, let us separate out the substitution and income effects. First, the substitution effect of an increase in $\frac{c}{1+\pi}$ increases the optimal signal accuracy, because an increase in $\frac{c}{1+\pi}$ by decreasing P^* releases additional resources for signal accuracy improvement. Second, since an increase in $\frac{c}{1+\pi}$ requires greater resources to conduct the same number of audits leaving less resources to invest in signal accuracy, an increase in $\frac{c}{1+\pi}$ produces the income effect on the optimal signal accuracy. The sign of the income effect depends on the size of the budget. If $B < \overline{B}$ the income effect is negative, if $B > \overline{B}$, the income effect is positive.

An expression for $\frac{dP^*}{d\frac{c}{1+\pi}}$ can be obtained by differentiating $P^* = \frac{1+\pi}{c} (B - K(a^*))$ w.r.t. $\frac{c}{1+\pi}$ and substituting $\frac{da^*}{d\frac{c}{1+\pi}}$ into the formula, which gives

$$\frac{dP^*}{d\frac{c}{1+\pi}} = \frac{1}{[-SOC(a^*)]}\frac{1+\pi}{c}\left\{-PK''(a^*) - \frac{K'(a^*)}{a^*}\frac{P^*R'(P^*) - R(P^*)}{R'(P^*)}\right\} < 0. \quad (A.16)$$

Both terms in (A.16) are negative, i.e., both the income and substitution effects are negative. Therefore $\frac{dP^*}{d\frac{c}{1+\pi}}$ is negative. So, an increase in the cost of audits causes a decrease in the optimal audit coverage. **Q.E.D.**

APPENDIX B

Appendix to Chapter 3

Model Extension with Saving Decisions

Consider a model when individuals in addition to deciding whether to pay or not to pay their tax debt can also choose how much to save in the first period. Then, the tax debtor's problem is

$$\max_{x\in\{0,1\},\, s\geq 0} u(w - x\delta - s) + qu(I - (1 - x)\pi\delta + s) + (\gamma - q)u(I + s). \qquad \text{(B.1)}$$

Let s_0 be the optimal savings when the debtor decides not to pay tax debt ($x = 0$) and s_1 be the optimal savings when the debtor decides to pay tax debt ($x = 1$). The FOCs for s_0 and s_1 are

$$u^{'}(w - s_0) - qu^{'}(I - \pi\delta + s_0) - (\gamma - q)u^{'}(I + s_0) \geq 0, \qquad \text{(B.2)}$$

$$u^{'}(w - \delta - s_1) - \gamma u^{'}(I + s_1) \geq 0, \qquad \text{(B.3)}$$

where (B.2) holds with equality if $s_0 > 0$ and (B.3) holds with equality if $s_1 > 0$.

Note that both $s_0(w)$ and $s_1(w)$ are increasing with w because $u()$ is concave.

Let us define $\hat{w}$ such that the tax debtor is indifferent between paying and not paying tax debt in period 1, that is

$$u(\hat{w} - \hat{s}_0) + qu(I - \pi\delta - \hat{s}_0) + (\gamma - q)u(I + \hat{s}_0) = u(w - \delta - \hat{s}_1) + \gamma u(I + \hat{s}_1), \quad \text{(B.4)}$$

where $\hat{s}_0 = s_0(\hat{w})$ and $\hat{s}_1 = s_1(\hat{w})$.

Note that if individuals would be able to borrow (i.e., s_i could be negative) it would be always better to pay tax debt in the first period and $\hat{w}$ would be undetermined.

To insure that for $w \geq \hat{w}$ we have $x^* = 1$ and for $w < \hat{w}$ we have $x^* = 0$, we need that the right-hand side of (B.4) ($RHS = u(w-\delta-\hat{s}_1)+\gamma u(I+\hat{s}_1)$) rises faster with w than the left-hand side of (B.4) ($LHS = u(\hat{w}-\hat{s}_0)+qu(I-\pi\delta-\hat{s}_0)+(\gamma-q)u(I+\hat{s}_0)$). That is, it is necessary that $\frac{\partial RHS}{\partial w}\big|_{w=\hat{w}} > \frac{\partial LHS}{\partial w}\big|_{w=\hat{w}}$, where

$$\frac{\partial RHS}{\partial w}\bigg|_{w=\hat{w}} = u'(\hat{w}-\delta-\hat{s}_1)(1 - \frac{\partial s_1(\hat{w})}{\partial w}) - \gamma u'(I+\hat{s}_1)\frac{\partial s_1(\hat{w})}{\partial w} = u'(\hat{w}-\delta-\hat{s}_1), \quad \text{(B.5)}$$

$$\frac{\partial LHS}{\partial w}\bigg|_{w=\hat{w}} = u'(\hat{w}-\hat{s}_0)(1 - \frac{\partial s_0(\hat{w})}{\partial w}) - (qu'(I-\pi\delta+\hat{s}_0)+(\gamma-q)u'(I+\hat{s}_0))\frac{\partial s_1(\hat{w})}{\partial w} = u'(\hat{w}-\hat{s}_0),$$
$$\text{(B.6)}$$

where the final expression for (B.5) is achieved because of (B.3) for $\hat{s}_1 > 0$ and because of $\frac{\partial s_1(\hat{w})}{\partial w} = 0$ for $\hat{s}_1 = 0$ and the final expression for (B.6) is achieved because of (B.2) for $\hat{s}_0 > 0$ and because of $\frac{\partial s_0(\hat{w})}{\partial w} = 0$ for $\hat{s}_0 = 0$.

The condition $u'(\hat{w} - \delta - \hat{s}_1) > u'(\hat{w} - \hat{s}_0)$ is true if $\hat{s}_0 < \delta + \hat{s}_1$. In its turn, $\hat{s}_0 < \delta + \hat{s}_1$ is true because $\hat{s}_0$ should be less than δ when $\frac{q\pi}{\gamma} \geq 1$. If we assume the opposite ($\hat{s}_0 \geq \delta$) then the debtor would be better off by paying tax debt and saving

$\widetilde{s} = \hat{s}_0 - \delta \geq 0$, because $\frac{q}{\gamma}u(I - \pi\delta + \hat{s}_0) + \frac{\gamma-q}{\gamma}u(I + \hat{s}_0) < u(I + \hat{s}_o - \delta) = u(I + \widetilde{s})$

by convexity of $u()$ when $\frac{q\pi}{\gamma} \geq 1$.

Thus, the behavior of a tax debtor in this model with savings is described by

$x^* = 1$ for $w \geq \hat{w}$ and $x^* = 0$ $w < \hat{w}$ and resembles the behavior of a tax debtor in the main model without saving.

Proof of Proposition 3.4

When $u(c) = \frac{c^{0.5}-1}{0.5}$, $\hat{w}(q) = \left(\frac{\delta+(q\sigma)^2}{2q\sigma}\right)^2$ for $q_l \leq q < q_h$, where $\sigma = \sqrt{I} - \sqrt{I - \pi\delta}$,

$q_l = \frac{\sqrt{w_h}-\sqrt{w_h-\delta}}{\sigma}$, and $q_h = min\{\beta, \frac{\sqrt{\delta}}{\sigma}\}$. Note that when $q < q_l$, $\hat{w}(q) = w_h$, and

thus everybody does not pay tax debt in period 1. When $q_h \leq q \leq \beta$, $\hat{w}(q) = 0$

and thus everybody pays tax debt in period 1. Then, $\int_0^\beta \Psi(q,\pi)\frac{dq}{\beta} = \int_0^{q_l} \Psi(q,\pi)\frac{dq}{\beta} + \int_{q_l}^{q_h} \Psi(q,\pi)\frac{dq}{\beta}$, where

$$\int_0^{q_l} \Psi(q,\pi)\frac{dq}{\beta} = \int_0^{q_l}\left[\frac{\delta}{w_h}\frac{1-q\pi}{\left(\frac{1}{\sqrt{w_h-\delta}} - \frac{1}{\sqrt{w_h}}\right)} - \frac{1-\alpha}{\alpha}\right]\frac{dq}{\beta} = \frac{\delta}{\beta\sigma}\frac{\sqrt{w_h-\delta}}{\sqrt{w_h}}\left(1 - \frac{q_l\pi}{2}\right) - \frac{1-\alpha}{\alpha}\frac{q_l}{\beta}, \quad \text{(B.7)}$$

$$\int_{q_l}^{q_h} \Psi(q,\pi)\frac{dq}{\beta} = \int_{q_l}^{q_h}\frac{(1-q\pi)(\delta^2-\sigma^4 q^4)}{4w_h\sigma^3 q^3}\frac{dq}{\beta} - \frac{1-\alpha}{\alpha}\int_{q_l}^{q_h}\frac{(\delta+\sigma^2 q^2)^2}{4w_h\sigma^2 q^2}\frac{dq}{\beta} =$$
$$= \frac{\delta}{4\beta w_h\sigma^3}\left[-\frac{\delta^2}{2}\left(\frac{1}{q_h^2} - \frac{1}{q_l^2}\right) - \frac{\sigma^4(q_h^2-q_l^2)}{2} + \pi\delta^2\left(\frac{1}{q_h} - \frac{1}{q_l}\right) + \frac{\pi\sigma^4(q_h^3-q_l^3)}{3}\right] - \quad \text{(B.8)}$$
$$- \frac{1-\alpha}{\alpha}\frac{1}{4\beta w_h\sigma^2}\left[-\delta^2\left(\frac{1}{q_h} - \frac{1}{q_l}\right) + 2\delta\sigma^2(q_h - q_l) + \frac{\sigma^4(q_h^3-q_l^3)}{3}\right].$$

When $w_h \to \delta$, then $q_l \to \frac{\sqrt{\delta}}{\sigma}$ (i.e., $q_l \to q_h$), and $\int_0^{q_l} \Psi(q,\pi)\frac{dq}{\beta} \to -\frac{1-\alpha}{\alpha}\frac{\sqrt{\delta}}{\beta\sigma} < 0$,

and $\int_{q_l}^{q_h} \Psi(q,\pi)\frac{dq}{\beta} \to 0$ (because $q_l \to q_h$). Thus, $\int_0^\beta \Psi(q,\pi)\frac{dq}{\beta} \to -\frac{1-\alpha}{\alpha}\frac{\sqrt{\delta}}{\beta\sigma} < 0$.

When $w_h \to +\infty$, then $q_l \simeq \frac{\delta}{2\sigma\sqrt{w_h}} \to 0$, and $\int_0^{q_l} \Psi(q,\pi)\frac{dq}{\beta} \to \frac{\delta}{\beta\sigma} > 0$, and

$\int_{q_l}^{q_h} \Psi(q,\pi)\frac{dq}{\beta} \simeq \frac{\delta}{4\beta w_h\sigma^3} \cdot \frac{\delta^2}{2q_l^2} \to \frac{\delta}{2\beta\sigma}$. Thus, $\int_0^\beta \Psi(q,\pi)\frac{dq}{\beta} \to \frac{3\delta}{2\beta\sigma} > 0$. **Q.E.D.**

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