The Calculus Curriculum in the National Study of Calculus in the U.S.A.[1]

[a]Vilma Mesa - Helen Burn[b]

[a]University of Michigan, Ann Arbor - [b]Highline College, Des Moines, Washington

Abstract

We describe findings of an analysis of curricular aspects related to the teaching of first-year of university calculus in the US, based on data collected as part of the Mathematical Association of America's Characteristics of Successful Programs in College Calculus study (Bressoud, Rasmussen, Carlson, Mesa, & Pearson, 2010). The study was conducted in two phases, first via surveys and questionnaires on programs, teaching and learning of calculus, and student motivation and performance; and second via qualitative case studies of 18 institutions identified as "successful" in terms of student performance and motivation. Our analysis using Rico's conceptualization of curriculum illustrates the stability of the calculus curriculum in spite of the reform efforts of the 90s. It also gives departments tools to propose possible more viable interventions.

Keywords: curriculum, first-year university calculus, content, cognitive goals, features of teaching, assessment of student learning

First-year Calculus (Calculus I) provides the basic mathematical tools to study change. Different from other countries in the world, calculus is usually a first year university course in the United States and it is normally required for any science, technology, engineering, or mathematics (STEM) major. Indeed in Fall 2010, over 300,000 students nation wide were taking a calculus course at their college or university (Blair, Kirkman, & Maxwell, 2013). While these numbers are impressive, a persistent problem of the teaching of university calculus is the high failure rate in the course. Students' disengagement is usually a major reason: lectures are uninspiring or unimaginative, the curriculum is "over-stuffed" and taught at too fast a pace, and

---

[1] Working draft for a contribution to *Libro-homenaje a Luis Rico*. Granada: Editorial. The work reported in this article is supported in part by the National Science Foundation grant DRL REESE #0910240. Opinions expressed here are the sole responsibility of the authors and do not reflect the views of the Foundation. Parts of the article have appeared in Burn and Mesa (2015).

instructors show little concern for student understanding (Seymour & Hewitt, 1997). When students fail the course, their opportunities to pursue STEM fields are curtailed. A country with a high demand of technical jobs can't afford this situation (President's Council of Advisors on Science and Technology (PCAST), 2012). Added to this mix is the large influx of students interested in pursuing a college degree, a direct consequence of President Obama's initiative for increasing college graduation rates in the country.

This dramatic growth in incoming students aspiring to pursue STEM careers has been pitted against the drastic budget cuts that have forced departments to reduce the number of full-time faculty and to teach calculus in ever larger classes. Mathematics departments have also been slow to adapt to the changing demographics of college age students and who want to go into STEM fields. Success rates in calculus for women, students from underrepresented minorities, economically disadvantaged students, and first generation college students continue to be disappointingly low relative to those of Caucasian and Asian students (Linn & Kessel, 1996; Seymour, 2002; Wake, 2011).

Over 35 years ago several mathematicians, confronted with similar issues called for calculus to be a pump, not a filter (Steen, 1988). Such call for action resulted in an unprecedented effort to reform the calculus curriculum to bring more technology, more real work problems, and a more conceptual approach to the subject (e.g, using multiple representations, Harver, 1998). One wonders about the extent to which the initiatives proposed then are now part of the first year of Calculus curriculum in American universities: the "rule of four" (always using verbal, tabular, symbolic, and graphical representations), spread use of technology (notably graphing calculators and computer algebra systems), contextualized problems, and emphasis on conceptual understanding.

In this chapter, an extension of a chapter we wrote for a Mathematical Association of America (MAA) publication reporting major findings from the National Study of Calculus (Bressoud, Mesa, & Rasmussen, 2015), we start to answer that question. We do so by using Luis Rico's conceptualization of curriculum to analyse data that would allow us to describe the U.S. Calculus I curriculum and then discuss the description with what the reformers had in mind. We organize the chapter into four sections. We begin by a short description of the notion of curriculum followed by a brief description of the study and the data used to conduct the analysis presented in this chapter. We then present the major features of the Calculus I curriculum using Rico's (1997) four curricular dimensions: conceptual, cognitive, formative, and social. We conclude with a discussion of the findings and the affordances and challenges of studying curriculum in this way.

RICO'S CURRICULUM

From the Latin *currere*—to run—the word *curriculum* is a broad term used in education with multiple meanings (Stein, Remillard, & Smith, 2007). Curriculum can refer to the sequences of courses that a student can take, the topics that are covered in a given grade, or the content, skills, competencies, and habits of mind that a person needs to acquire through schooling in order participate successfully in the society (Lattuca & Stark, 2009). The classical distinction between intended, implemented, and attained curricula (Travers & Westbury, 1989) is useful to describe how the curriculum is transformed from intention to execution to assessment, but it is insufficient to define a curriculum.

In *Bases Teóricas del Currículo de Matemáticas en Educación Secundaria*, Luis Rico, using among others Stenhouse's (1975) work, proposes a definition of curriculum as an educative plan[2] that describes the group of people that will be educated or trained, the type of education or training that needs to be done, the institution where education or training takes place, the goals that need to be achieved, and the means of control and assessment (Rico, p. 27). Rico affirms that any educative plan needs to reconcile the theoretical and practical aspects that support it, in ways that acknowledges the professionalism of the teachers who are in charge of putting the plan into action. To this effect, he proposes a theoretical and practical conceptualization of curriculum that is well-suited for empirical studies of curriculum. This conceptualization responds to four core questions that the plan must address—what is the nature of knowledge, what is learning, what is teaching, and what constitutes useful knowledge?—each addressing the four dimensions of the curriculum: conceptual, cognitive, formative, and social (p. 381).

The *conceptual* dimension refers to content and topics that are specific to a given discipline; it defines those elements particular to a discipline (e.g., mathematics) that are a synthesis of historical and cultural traditions; this dimension is informed by epistemology and the history of

---

[2] In Spanish Rico uses "plan de formación." A direct translation would be "formative plan" yet this wording seems more restrictive than educative plan, the wording chosen here.

mathematics and defines larger cultural aims. This paper examined the content of Calculus 1. The *cognitive* dimension refers to learning and the learner, and deals with understanding what learning is, how it happens, and how do different people learn; it also has particular manifestations depending on a given discipline; it is directly informed by learning theories and defines specific expectations, development, and learning aims. We examine faculty's stated goals for the course. The *formative* dimension refers to teaching and the teacher; it deals with aspects such as what teaching is and what mathematics teaching is; it specifies practices that are believed to be useful for teaching (e.g., differentiating instruction) and it provides the basis for generating programs for preparing teachers; it is informed by pedagogical theories and defines formative aims. The formative dimension is explored by examining ways in which teaching was enacted in calculus lessons. The *social* dimension refers to the value that the society places on the utility and usefulness of the mathematical knowledge. This dimension addresses the mechanisms that need to be used in order to determine when an individual has useful knowledge or that he or she has mathematical capabilities, and the appropriateness of the curriculum itself (p. 385). We examined this dimension by analysing demands of graded and ungraded work in one college.

THE CSPCC STUDY

In 2010, the Mathematical Association of America launched a project funded by the National Science Foundation, the *Characteristics of Successful Programs in College Calculus*, with the purpose of studying how calculus contributed to students' academic goals fulfilment and to understand other features of calculus. The project had five goals:

- To improve our understanding of the demographics of students who enrol in calculus,
- To measure the impact of the various characteristics of calculus classes that are believed to influence student success,
- To conduct explanatory case study analysis of exemplary programs in order to identify why and how these programs succeed,
- To develop a theoretical framework that articulates the factors under which students are likely to succeed in calculus, and

- To use the results of these studies and the influence of the MAA to leverage improvements in calculus instruction across the United States.

The project was conducted in two phases. In Phase 1 surveys (Fall 2010) were sent to a stratified random sample of 521 institutions. At these institutions, all instructors teaching a calculus course designed to prepare students for the study of engineering, the mathematical, or physical sciences, and their students, were invited to participate. There were five surveys in total, pre- and post surveys of students and instructors administered at the beginning and the end of their Calculus I course, and one sent at the beginning of the course to the course administrator. The surveys were designed to gain an overview of the various calculus programs nationwide, and included questions on the department, the teaching, the learning, and the assessment of calculus; resource availability for students and instructors (e.g., technology, or tutoring and teaching centers), and student and instructor background. In addition, the surveys collected information on intention to take Calculus II (a proxy for persistence in a STEM major); affective aspects, including enjoyment of math, confidence in mathematical ability, interest to continue studying math; and self-reported performance (expected grade in course). We obtained full information for 3,103 students, 308 classrooms, and 168 calculus programs.

In Phase 2 of the project (Fall 2012), we conducted explanatory case studies at 18 institutions selected because they had more successful calculus programs. Success was defined by a combination of student variables: persistence in calculus and positive change in affective variables and program variables (e.g., passing rates). We conducted nearly 300 interviews with instructors and administrators, 45 focus groups with over 600 students, 76 classroom observations, and collected over 500 artifacts (syllabi, exams, quizzes, and institutional information).

For the purposes of this paper we relied on both the data from the surveys and the data from the case studies. In order to describe the four curricular elements (content, cognitive goals, features of teaching, and assessment) we worked with different sources. The content of Calculus I presented in this chapter derives from a synthesis of four sources, the course description for the Advanced Placement (AP) calculus (The College Board, 2012); Sofronas et al.'s (2011) "brain trust" study with 24 calculus experts who identified content and goals critical for student understanding of first-year calculus; the topics included in Khan Academy website (http://www.khanacademy.org/math/calculus), and Johnson, Ellis, and Rasmussen (2014) analysis of course content listed in master syllabi collected from the PhD-granting universities selected for case study. Data for the analysis of cognitive goals and assignments in Calculus I derive from the faculty surveys. The sample included all faculty who responded to pre- or post-course instructor survey (N = 503). Faculty response rates to survey questions analysed ranged from a minimum of 65% to a maximum of 84%, with an average response rate across the survey questions of 72%. For the analysis of features of teaching we used the field notes of observations made of 67 calculus lessons across the case study institutions. All the tasks recorded in the observation protocol (N = 497) were characterized according to four features: who was the main actor in solving the problem, which representations were used, what technology was used, and what other complexity characteristics were present (e.g., using multiple methods, requiring a proof). Finally, to characterize assessment we used the tasks assigned in graded and ungraded homework, quizzes, and exams assigned by five instructors in one college (N = 4,953). The analysis centred on the cognitive orientation of the tasks used in these contexts (White & Mesa, 2014).

As with any survey research, limitations due to construct validity need to be considered. For example, Calculus I faculty were not asked directly about cognitive goals for Calculus I; instead, selected survey questions served as proxies. A second limitation in any study using self-report data relates to potential differences between what faculty say they do and what they actually enact in classrooms—that is, between the intended and the enacted curriculum. However, there is no reason to believe that faculty would respond to questions without accuracy, given that there were no repercussions for the responses provided, and the case study data suggest that for the most part, instructors and students accurately described their practices. Thus, we have confidence that the self reported data accurately describe practice.

THE CALCULUS CURRICULUM IN THE CSPCC STUDY

We now present a synthesis of the analysis of the four dimensions defining curriculum as we interpreted them in the data collected in the study.

Conceptual Dimension: The Content of Calculus I

A course of calculus I includes four major content areas: limits and continuity, derivatives, integration and sequences and series (see Figure 1) with some courses starting with a review of the function concept. We note that although δ-ε proofs are no longer considered a standard part of the Calculus I curriculum, the "rule of four" (emphasizing the combination of graphical, numerical, algebraic, and verbal approaches) has had a lasting impact on Calculus I textbooks and that technology, particularly in the form of graphing calculators and computer algebra systems (e.g., Maple, Mathematica) that enables increased emphasis on visualizing graphs of functions and their derivatives and other graphical connections, are usually part of the labs offered to calculus students. Further, Calculus I courses now more often than in the past attend to employability or transfer skills, such as communication, teamwork, and technology skills (Houston, 2001).

Limits and Continuity

- Develop an intuitive understanding of the limiting process, calculate limits using algebra, estimate limits from graphs or tables, and limits at infinity
- The limit concept is necessary for the limit definition of the derivative and the integral
- Continuity (intuitive understanding and in terms of limits)

Derivatives

- The concept of the derivative as the instantaneous rate of change of a function, as the limit of the difference quotient, and as a measure of sensitivity of one variable to another when linked by an equation
- Computation of derivatives of basic functions (power, exponential, logarithmic, trigonometric, and inverse trigonometric), derivative rules (sum, power, product, quotient, chain), and implicit differentiation. Some Calculus I courses may include hyperbolic trigonometric functions
- Visualizing derivatives sand other graphical connections (e.g., using secant lines to approximate tangent slope, approximating rates of change from graphs or tables, analysis of $f, f'$and f'' and graphical connections)
- Applications of derivatives including analysis of curves, modelling, and interpreting rates of change, optimization, and related rates. The Mean Value Theorem (assumptions and geometric interpretations), L'Hôpital's Rule, and numerical solutions via Euler's method

(continues in next page)

Figure 1: Calculus I content identified in CSPCC study (Burn & Mesa, 2015).

Integrals

- Indefinite integral as antiderivative
- Definite integral as a limit of Riemann sums (Riemann sums are used to approximate the definite integral)
- Interpretation and application of the definite integral (e.g., integral as area, as net change/accumulated total change).
- Fundamental Theorem of Integral Calculus and techniques of integration (computing antiderivatives following from basic functions and substitution of variables).

Sequences and Series (Generated through the Advanced Placement Calculus and the Khan academy website)

- Series of constants (e.g., geometric series with applications) and Taylor series

Figure 1: Calculus I content identified in CSPCC study (Burn & Mesa, 2015) (cont.)

Cognitive Dimension: The Learning Goals of Calculus I

To discern the cognitive goals for students in Calculus I, we explored faculty expectations for student learning suggested by the types of problems they reported they assign students and by their expectations for students to explain their thinking. Maintaining higher levels of cognitive demand has been associated with higher levels of student achievement in mathematics (Boaler & Staples, 2008; Silver & Stein, 1996; Stigler, Givvin, & Thompson, 2010). Figure 2 shows the median response to the percentage of different types of problems faculty assign in exams and other assignments. The percentages were nearly identical for both.

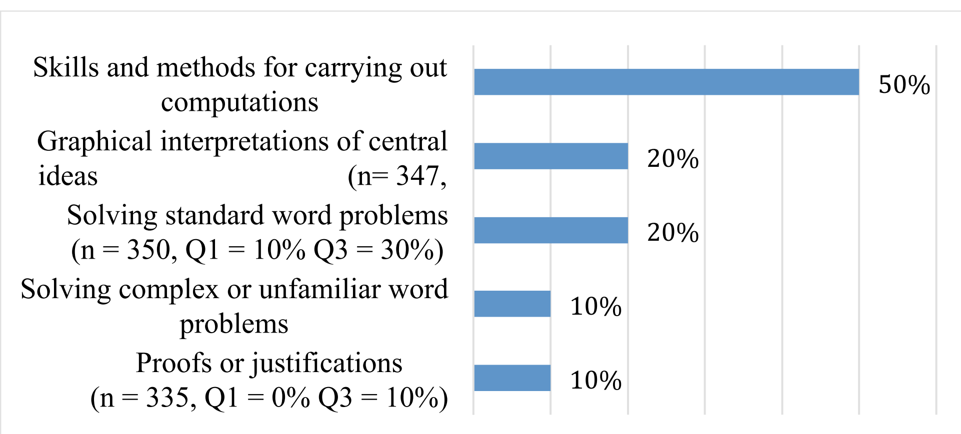

Figure 2: Median responses to the percentage (0% to 100%) of different problem types faculty include on exams and assignments. Q1 = 25th percentile; Q3 = 75th percentile.

Figure 2 suggests that instructors tend to include a larger number of problems focused on skills and computations (50%) over other types of problems (problems involving graphical interpretations of central ideas, standard word problems, complex or unfamiliar word problems, or proofs and justifications). This tendency might be partially explained by the fact that 66% of faculty agreed somewhat or strongly with the statement: *Understanding ideas in calculus typically comes after achieving procedural fluency.*

Faculty were asked to report how frequently they asked students to explain their thinking during class or required them to explain their thinking on exams or assignments on a scale that ranged from 1 (*Not at all*) to 6 (*Very often*). Overall, 44% of faculty reported that they *Often*[3] or *Very often* required students to explain their thinking on exams and 41% of faculty reported that they *Often* or *Very often* asked students to explain their thinking during class. In contrast, a smaller 34% of faculty reported that they *Often* or *Very often* required students to explain their thinking on assignments. These findings suggest that faculty privilege student practice with skills as a way to build understanding as a core feature of learning expectations in the course. Faculty do expect however that students explaining their thinking in class and in exams.

Formative Dimension: The Features of Calculus Teaching

The analysis of the ways in which the problems used during the classroom were enacted revealed several interesting patterns. First, we found that the instructor lectured or used direct instruction in 82% of the problems. In these lessons student presentations, group work, or students working in pairs was observed in less than 10% of the problems; work individually was observed in 15% of the problems. Symbolic representations were used in 65% of the problems and it was used in conjunction with other representations; a sizable proportion of problems, 43%, relied on other non-symbolic representations. The lessons observed did not appear as technologically adventurous as the reformers would have liked. Only 9% of the problems used some form of technology: basic calculator, graphing calculators, or computer algebra systems. Finally, 66% of problems sought to develop proficiency with procedures, 12% were contextualized in real world situations, 9% required a proof or a justification, 3% were open-

[3] A response of 5 is interpreted to mean *Often.*

ended, and only 2% showed different solution methods. Figure 3 shows an example of a coded entry in the observation protocol.

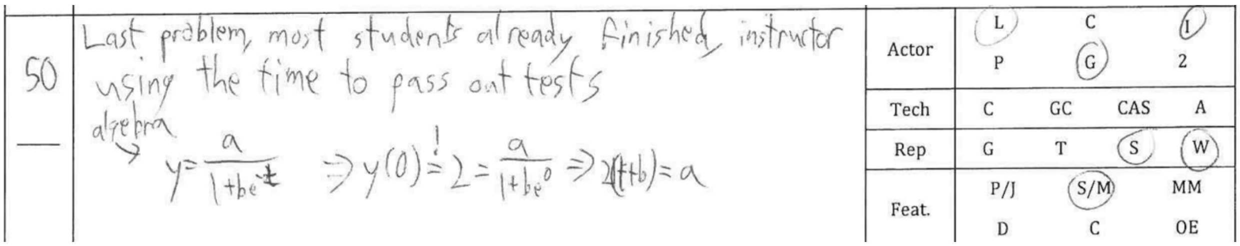

Figure 3: Problem log entry (White, Blum, & Mesa, 2013)

Social Dimension: The Assessment of Student Learning in Calculus I

The analysis of the problems that instructors in one college assigned suggest differences in the cognitive orientation by type of course work assigned (graded vs. ungraded) and differences by the instructors who assigned the course work. About half of the problems assigned were what we called simple procedure tasks[4], about 20% were complex procedures tasks[5], and about 30% were rich tasks[6]. The homework, which was ungraded, tended to include more simple procedure tasks than exams, which were graded (54% vs. 40% respectively), more complex procedures tasks than exams (21% vs. 11% respectively) but less rich tasks than exams (25% vs. 49% respectively). Thus exams tended to have a higher level of complexity than homework. In addition we found that while the instructors assigned homework with similar levels of complexity, some instructors assigned more rich tasks than others in the exams, which suggests that instructors had different values in deciding what counts as calculus work (White & Mesa, 2014).

[4] Students are prompted to retrieve information or carry out recalled procedures (e.g., Find the derivative of *f* at $x = a$).

[5] Students must recognize knowledge or procedures called for in a situation without being prompted (e.g., At what value of x does f attain a minimum?).

[6] Students are prompted to make interpretations, give explanations, recognize when to apply a procedure, analyse a situation, or judge based on certain criteria (e.g., Is it reasonable to use this model to predict the winning height [of the high jump] at the 2100 Olympics?).

DISCUSSION

Our analysis of the various dimensions of curriculum suggests that the calculus course is less reformed than what could have been anticipated. In terms of content, the topics included show remarkable stability (Steen, 1998) regardless of efforts to create a "lean and lively calculus" (Douglas, 1986), the reform movement of the 1990s (Hughes Hallett, n. d.), or general attempts to reduce the breadth and to increase the depth of topics included in mathematics courses in the United States (Hillel, 2001). Likewise, the cognitive goals espoused by the faculty seem to continue to privilege the development of proficiency with procedures under the assumption that one must master basic information before moving to more complex ideas, which is an implicit behaviourist assumption about how one learns. That said, the sizable interest in having students explain their thinking in class and in exams suggest a possible turn towards more discursive-based notions of learning. Moreover, small group work, use of multiple representations and technology and inclusion of contextualized problems that would emphasise the development of understanding of calculus notions over technical proficiency have not yet made it to all lessons in which calculus is taught. The dominant teaching paradigm still seems to be lecture and demonstration. Finally, we identified differences in emphasis instructors put on their exams. Instructors make the exams—the space where students show proficiency and their knowledge of the course content appraised—much harder than the un-graded homework. Overall the analyses along the four dimensions provide a richer picture of the calculus curriculum and supports that it has not changed much over time.

Curriculum usually gets described in terms of its content dimension, and this is clearly an insufficient description. In the case of a university course, one has to take into account institutional and societal demands. Universities in the United States are facing tremendous difficulties documenting their value, especially when they have a difficult time demonstrating that students learn (Arum & Roksa, 2011) and federal and state support for higher education is at an all-time low. Societal pressures on universities to demonstrate that they indeed make a difference have sparked inward reflection and examination of specific programs that might not support students in their goals to pursue STEM careers. Mathematics departments have been the major target of these critiques (PCAST, 2012) and, from the analyses reported here, some of these critiques might be well founded: the opportunities for students to experience a more lively calculus, one that uses more collaborative forms of learning and teaching, supported by technology, and valuing complex work are not uniformly offered. However, Rico's richer conceptualization of curriculum provides departments with specific dimensions to look at when thinking about possibilities for change. Leaving aside the potential need to evaluate the content in Table 1, a behaviourist approach to student learning with its accompanying pedagogy might be at odds with the potential of a course such as calculus to expand students' mathematical opportunities. Working to address how the visions of calculus learning and teaching might be made more current with recent research findings about the importance of inquiry might be a an important starting point. Likewise, attending to how students' calculus learning is appraised could be an area that has potential to boost the calculus curriculum in ways that might better satisfy the students' interests and the demands of the society.

REFERENCES

Arum, R., & Roksa, J. (2011). *Academically adrift: Limited learning on college campuses*. Chicago: University of Chicago Press.

Blair, R., Kirkman, E. E., & Maxwell, J. W. (2013). *Statistical abstract of undergraduate programs in the mathematical sciences in the United States. Fall 2010 CBMS Survey*. Washington D.C.: American Mathematical Society.

Boaler, J., & Staples, M. E. (2008). Creating mathematical futures through an equitable teaching approach: The case of Railside School. *Teachers College Record, 110*(3), 608-645.

Bressoud, D. M., Mesa, V., & Rasmussen, C. L. (2015). *Insights and recommendations from the MAA National Study of College Calculus*. Washington, DC: Mathematical Association of America.

Bressoud, D. M., Rasmussen, C. L., Carlson, M., Mesa, V., & Pearson, M. (2010). *Characteristics of successful programs in college calculus*. (NSF DRL REESE #0910240).

Burn, H. (2012). Factors that shape curricular reasoning about college algebra reform. *MathAmatyc Educator, 4*(1), 23-28.

Burn, H., & Mesa, V. (2015). The Calculus I curriculum. In D. M. Bressoud, V. Mesa, & C. L. Rasmussen (Eds.), *Insights and recommendations from the MAA National Study of College Calculus* (pp. 45-57). Washington, DC.

Douglas, R. G. (Ed.). (1986). *Toward a lean and lively calculus. MAA Notes #6*. Washington, D.C.: The Mathematical Association of America.

Harver, W. E. (Ed.). (1998). *Calculus: Catalyzing a national community for reform*. Washington, DC: Mathematical Association of America.

Hillel, J. (2001). Trends in curriculum: A working group report. In D. Holton (Ed.), *The teaching and learning of mathematics at university level* (pp. 407-422). Dordrecht: Kluwer.

Houston, K. (2001). Assessing undergraduate mathematics students: A working group report. In D. Holton (Ed.), *The teaching and learning of mathematics at university level* (pp. 59-69). Dordrecth: Kluwer.

Hughes Hallett, D. (Producer). (n. d.). What have we learned from calculus reform? The road to conceptual understanding. Retrieved from http://math.arizona.edu/~dhh/NOVA/calculus-conceptual-understanding.pdf

Johnson, E., Ellis, J., & Rasmussen, C. (2014). *How to make time: The relationship between concerns about coverage, material covered, instructional practices, and student success in college calculus*. Paper presented at the Annual Conference on Research in Undergraduate Education, Denver, CO.

Kaput, J. J. (1997). Rethinking calculus: learning and thinking. *The American Mathematical Monthly, 104*(8), 731-737.

Lattuca, L. R., & Stark, J. (2009). *Shaping the college curriculum: Academic plans in action*. San Francisco: Jossey-Bass.

Linn, M. C., & Kessel, C. (1996). Success in mathematics: Increasing talent and gender diversity among college majors. In J. J. Kaput, A. H. Schoenfeld, & E. Dubinsky (Eds.), *Research in Collegiate Mathematics Education* (Vol. II, pp. 101-144). Washington: American Mathematical Society and Mathematical Association of America.

President's Council of Advisors on Science and Technology (PCAST). (2012). *Engage to excel: Producing one million additional college graduates with degrees in science, technology, engineering, and mathematics*. Washington, DC: The White House.

Rico, L. (Ed.). (1997). *Bases teóricas del currículo de matemáticas en educación secundaria [Theoretical base for mathematics curriculum in secondary education]*. Madrid: Síntesis.

Seymour, E. (2002). Tracking the processes of change in US undergraduate education in science, mathematics, engineering, and technology. *Science Education, 79*, 79-105.

Seymour, E., & Hewitt, N. (1997). *Talking about leaving: Why undergraduates leave the sciences*. Boulder, CO: Westview.

Silver, E. A., & Stein, M. K. (1996). The QUASAR project: The "revolution of the possible" in mathematics instructional reform in urban middle schools. *Urban Education, 30*, 476-521.

Sofronas, K. S., DeFranco, T. C., Vinsonhaler, C., Gorgievski, N., Schroeder, L., & C., H. (2011). What does it mean for a student to understand the first-year calculus? Perspectives of 23 experts. *Journal of Mathematical Behavior, 30*(2), 131-148.

Steen, L. A. (1988). *Calculus for a new century: A pump, not a filter*. Washington, DC: Mathematical Association of America.

Steen, L. A. (1998). Core curriculum in context: History, goals, models, challenges. In J. Dossey (Ed.), *Confronting the core curriculum* (Vol. 45, MAA Notes, pp. 3-13). Washington, DC: Mathematical Association of America.

Stein, M. K., Remillard, J. T., & Smith, M. (2007). How curriculum influences student learning. In F. Lester (Ed.), *Second Handbook for Research in Mathematics Teaching and Learning* (pp. 319-369). Reston, Virginia: National Council of Teachers of Mathematics.

Stenhouse, L. (1975). *An introduction to curriculum research and development*. London: Heineman.

Stigler, J. W., Givvin, K. B., & Thompson, B. J. (2010). What community college developmental mathematics students understand about mathematics (Vol. Accessed March 19, 2010). Stanford, CA: Carnegie Foundation for the Advancement of Teaching (Accessed March 19, 2010, http://www.carnegiefoundation.org/elibrary/problem-solution-exploration-papers)

The College Board. (2012). *Calculus AB, Calculus BC course descriptions*. AP Central. http://apcentral.collegeboard.com/apc/public/courses/teachers_corner/2178.html.

Travers, K. J., & Westbury, I. (1989). *The IEA Study of mathematics I: Analysis of mathematics curricula*. Oxford: Pergamon Press.

Wake, G. (2011). Introduction to the Special Issue: Deepening engagement in mathematics in pre-university education. *Research in Mathematics Education, 13*(2), 109-118.

White, N. J., Blum, C., & Mesa, V. (2013). *A task analysis protocol for coursework for the Characteristics of Successful Programs in College Calculus*. [Manuscript in Preparation]. School of Education, University of Michigan, Ann Arbor.

White, N. J., & Mesa, V. (2014). Describing cognitive orientation of calculus I tasks across different types of coursework. *ZDM The International Journal on Mathematics Education, 46*(4), 675-690. doi: 10.1007/s1185801405889