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Thermal Radiation Fields and Antenna Parameters in Magnetoplasma

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Spectral densities of components of the electromagnetic fields of thermal radiation in a uniform magnetoplasma are shown to be related to the radiation resistances of electrically small antennas radiating in the magnetoplasma.

For antennas of arbitrary size and shape the conventional concepts used to describe antenna performance in isotropic media, i.e., gain, effective area and effective length are generalized to anisotropic media and relationships between them and radiation resistance established. A thermodynamic method is used in the derivation of the relationships.

A new derivation of the radiation resistance for an electrically small dipole in magnetoplasma is given based only on the far-zone radiated fields.

1. Introduction

In recent years there has been considerable interest in the measurement of field strengths or radiation intensities of radio waves in magnetoplasmas, for example for radio astronomy measurements from vehicles in the topside ionosphere. Two related aspects of this problem are treated in this paper. First, several papers have been published which deal either with the strengths of thermal fields or with the radiation resistances of small dipole antennas in magnetoplasma. Simple relationships exist between these quantities in an isotropic plasma (Hugill, 1961). In this paper, these quantities are discussed for magnetoplasma and general relationships between them are established. The last section is concerned with properties of antennas of arbitrary size and form in magnetoplasma, with electrically small antennas entering only as an explicit, special case.

In section 2, we consider infinitesimal electric and magnetic dipoles radiating into an embedding magnetoplasma. By equating the power fed to each antenna to the power fed by the antenna into the medium directly surrounding the antenna, Kogelnik (1960) and Kogelnik and Motz (1963) obtained integral expressions for the respective radiation resistances R_e and R_m. Using these formulas, the ratios of R_e or R_m to the corresponding free-space values can be written in the form:

$$R_p/R_{p0}=\rho_{1p}\sin^2\psi+\rho_{3p}\cos^2\psi=\tfrac{1}{2}(\rho_{3p}+\rho_{1p})+\tfrac{1}{2}(\rho_{3p}-\rho_{1p})\cos 2\psi, \tag{1.1}$$

where ψ is the angle between the vector dipole moment and the static magnetic field, subscript p stands for either e or m, and subscript 0 stands for free space. The ρ_{1p} and ρ_{3p} are functions of the usual ionospheric parameters: $X=[(\text{plasma frequency})/(\text{operating frequency})]^2$, $Y=(\text{gyro frequency})/(\text{operating frequency})$. Extensive numerical values are available (Weil and Walsh, 1964, 1965) for the quantities $\tfrac{1}{2}(\rho_{3p}\pm\rho_{1p})$ computed on the assumption of a cold, collisionless plasma. In this case there are separate sets of curves, one for radiation in each of the two modes of propagation, ordinary and extraordinary, from which may be obtained the net radiation resistance which is the sum of the contributions due to propagation of energy from the antenna in each of these modes.

By a general argument, ρ_{1p} and ρ_{3p} are shown in section 2 to be proportional to the spectral densities of certain components of the thermal electromagnetic fields in an isothermal enclosure filled with a uniform magnetoplasma. Such radiation is equivalent, as far as local radiation intensity is concerned, to radiation within the topside of the ionosphere (a non-uniform plasma) due to a cosmic source of uniform brightness temperature across the sky. The latter problem has been treated by Budden and Hugill (1964), who obtained expressions for the spectral densities of those components of the time-varying electric and magnetic fields within the ionosphere which are respectively perpendicular and parallel to the static magnetic field. They also presented numerical results for the ratios of these densities relative to the corresponding free-space quantities. These ratios are shown in section 2 to be respectively identical to ρ_{1p} and ρ_{3p}. The various formulas representing the same quantities are each integrals over an angle θ from 0 to $\pi/2$ of integrands which do not look at all alike. This is due in part to the introduction of various non-independent variables to prevent the formulas from becoming unwieldy. As a check on the equivalences established by general argument, the integrands were expressed in the same set of parameters, X, Y and θ, and by algebraic manipulation shown to be identical. The algebra is straightforward but tedious and will not be given.

In section 3, formulas for intensity of thermal radiation in anisotropic media are summarized in preparation for section 4, where generalizations to anisotropic media are made of the customary descriptors of antenna performance, i.e., gain, effective area and effective length. Relationships between these quantities and radiation resistance are found by thermodynamic argument for antennas of arbitrary type in magnetoplasma. The simple formula for effective length of a dipole used in section 2 is a special case of the general results found in this section. The generalized effective length is related to far-zone fields of the antenna when used as a radiator. From this result, a new derivation of formulas for ρ_{1e} and ρ_{3e} is carried out using only the far zone field. These new formulas are again integrals over θ from 0 to $\pi/2$. All three forms give identical numerical results in spite of the very different explicit forms of the formulas. Hence a useful by-product of the different forms is that they provide means of verifying the computational results presented by Weil and Walsh and by Budden and Hugill in their respective publications.

The following notations will be used. Bold face type represents a vector, while underlining any scalar or vector quantity, as $\underline{V}$ or $\underline{\mathbf{V}}$ indicates that the quantity is complex. Thus $\mathbf{V}$ may be written in terms of two real vectors $\mathbf{V}_1$ and $\mathbf{V}_2$ as $\underline{\mathbf{V}} = \mathbf{V}_1 + j\mathbf{V}_2$; $\underline{\mathbf{V}}^*$ is its complex conjugate and its magnitude squared will be written

$$|V|^2 = \underline{\mathbf{V}} \cdot \underline{\mathbf{V}}^* = \underline{V}\underline{V}^* = V_1^2 + V_2^2.$$

A unit vector associated with $\underline{\mathbf{V}}$ is indicated by a circumflex, thus $\hat{\underline{\mathbf{V}}} = \underline{\mathbf{V}}/|V|$. For the most part we shall be representing thermal radiation by quasi-periodic electromagnetic fields; that is, by vector fields of the form $\underline{\mathbf{A}}(t) = \underline{\mathbf{a}}(t) \exp(j\omega t)$, where the amplitude and phase of $\underline{\mathbf{a}}$ are random but vary only slightly over a period $2\pi/\omega$ of the exponential factor. Accordingly, two types of averaging enter. First, averaging over a period $2\pi/\omega$ which yields a mean square value of $\underline{\mathbf{A}}(t)$ which is essentially $\frac{1}{2}|a(t)|^2$, and second, a long term averaging of $|A(t)|^2$ over a time interval much greater than the reciprocal of the bandwidth of $a(t)$. This latter average we denote as $<|A|^2>$; it is equal to $<|a|^2>$ and is twice the long term mean square value of $\underline{\mathbf{A}}(t)$.

2. Equality of Relative Spectral Densities and Relative Radiation Resistances

Following Dicke (1946), consider a lossless antenna with impedance $R + jX$ in a large enclosure and connected to a load, the whole being at a constant absolute temperature, T. The second law of thermodynamics requires that the net radiation exchange between walls and load via the antenna, integrated over all frequencies and all directions, must be zero. To study energy exchange within a limited frequency range, the principle of detailed balancing may be applied. In an isotropic medium, this principle asserts that the net exchange of radiation between the load and the walls is zero within any frequency interval. Hence, in frequency interval df, the available noise power $KTdf$ generated by thermal fluctuations of electrons in the load resistance (Johnson, 1928; Nyquist, 1928) must equal the noise power available from the antenna due to incident radiation. (Throughout this paper the electrical path between antenna and load is assumed to be reciprocal, i.e., the same fraction of the available power is transferred from antenna to load as from load to antenna, hence nonreciprocal elements such as isolators are excluded.)

For a magnetoplasma, the argument must be modified. In this case, the principle of detailed balancing as applied to radiation (Vlieger, Mazur and Degroot, 1961) is applicable if, for radiation from one body A to a second body B the actual medium is used, while for radiation from B to A the radiation is assumed to take place in a plasma differing from the original by having the static magnetic field reversed. This reversal causes the dielectric tensor, (A.1), to be replaced by its transpose; in effect, the original medium, as described by its dielectric tensor, is replaced by a fictitious medium which will be referred to as the transpose medium. The modified principle no longer implies that in a given system in thermal equilibrium there is a detailed balancing of physical processes, but it does provide an aid to analysis.

This modified principle may now be applied to the exchange of energy between the cavity walls and the load of the antenna in the case where the cavity is filled with a homogeneous magnetoplasma. Quantities associated with one of the two characteristic waves that may propagate will be distinguished by subscripts I and II; the total radiation resistance is $R = R_{\mathrm{I}} + R_{\mathrm{II}}$. Then the power radiated in characteristic wave $q(q=\mathrm{I}$ or $\mathrm{II})$ is

$$P_{q\mathrm{rad}} = (R_q/R)KTdf, \tag{2.1}$$

assuming a power match between load and antenna at the frequency of interest.

If radiation incident on the antenna from all directions in characteristic wave q and in frequency range df produces mean square open-circuit voltage $\frac{1}{2} < |V_q|^2 >$, then the power dissipated in the load is

$$P_{q\mathrm{rec}} = < |V_q|^2 > /8R. \tag{2.2}$$

Applying the modified principle of detailed balancing, and indicating quantities evaluated in the transpose medium by a superscript T, it follows that

$$< |V_q|^2 > = 8KTR_q^T(R/R^T)df. \tag{2.3}$$

For antennas symmetrical about a point it is clear that $R_q^T = R_q$, since reversal of the static magnetic field does not affect the geometry of the antenna with respect to the field. For this class of antennas, which includes symmetrical linear dipoles and plane circular loops, it follows that

$$< |V_q|^2 > = 8KTR_q df. \tag{2.4}$$

Suppose now that the antenna is a center-fed, linear antenna, "electrically small" in that the dimensions of the antenna are much less than the shortest wavelength incident on it. If the antenna is electrically small in free space it will remain so in a medium in which the index of refraction for propagation in any direction is not too large compared with unity. For a collisionless magnetoplasma this requirement is not satisfied under either of the conditions $1 - Y^2 \leqslant X \leqslant 1$ with $Y \leqslant 1$, or $X \geqslant 1$ with $Y \geqslant 1$ (one of the indices of refraction, as determined by the Appleton-Hartree formula, becomes infinite for propagation directions forming the generators of a cone about the direction of the static magnetic field). However, if these conditions are excluded from consideration, antennas of sufficiently small dimension may safely be considered electrically small in free space and in the plasma.

The electric field $\underline{\mathbf{E}}$ at the antenna may be regarded as the summation of elementary plane waves with all possible directions of propagation, any one of which is uncorrelated with the others. Applying the result (4.27), derived later, to each elementary wave and summing over them, yields for the open-circuit voltage due to the total electric field $\underline{\mathbf{E}}$,

$$|V|^2 = |L|^2 |\hat{\mathbf{p}} \cdot \underline{\mathbf{E}}|^2, \tag{2.5}$$

where $\hat{\mathbf{p}}$ is a unit vector along the antenna; $|L|$ is a constant of proportionality sometimes referred to as the (scalar) effective length of the antenna. It is assumed to be independent of the presence of the magnetoplasma on the basis of the discussion following (4.27).

Combining (2.4) and (2.5) yields for characteristic wave q,

$$|L|^2 < |\hat{\mathbf{p}} \cdot \underline{\mathbf{E}}_q|^2 > = 8KTR_{eq}df, \tag{2.6}$$

where the suffix e indicates that a short electric dipole is considered. For the special case of vacuum, free from static magnetic field,

$$|L|^2 < |E_0|^2 > = 8KTR_{e0}df, \tag{2.7}$$

where $\underline{E}_0 = \hat{\mathbf{p}} \cdot \underline{\mathbf{E}}$ is the projection of the total field along an arbitrary direction. The electric field in the plasma may be resolved into components $\underline{\mathbf{E}}_{||}$ and $\underline{\mathbf{E}}_{\perp}$, respectively parallel and perpendicular to the static magnetic field. Evaluating (2.6) for the cases where $\hat{\mathbf{p}}$ is parallel and perpendicular to the static magnetic field, and dividing by (2.7), gives

$$< |E_{q||}|^2 > / < |E_0|^2 > = \rho_{3eq}, \tag{2.8}$$

$$\tfrac{1}{2} < |E_{q\perp}|^2 > / < |E_0|^2 > = \rho_{1eq}. \tag{2.9}$$

The factor $\frac{1}{2}$ enters (2.9) because $E_{\perp}$ is the sum of two perpendicular components, each of which has (because of symmetry) the same mean square value.

An analogous treatment may be applied to an electrically small, plane loop antenna in a magnetic induction field, $\underline{\mathbf{B}}$. The open-circuit voltage is $|V|^2 = \omega^2 A^2 |\hat{\mathbf{p}} \cdot \underline{\mathbf{B}}|^2$, where $\hat{\mathbf{p}}$ is parallel to the magnetic moment and A is a function of the loop geometry only (not to be confused with the effective area used later in this paper). Reasoning similar to that for the electric dipole then yields, with obvious notation,

$$< |B_{q||}|^2 > / < |B_0|^2 > = \rho_{3mq}, \tag{2.10}$$

$$\tfrac{1}{2} < |B_{q\perp}|^2 > / < |B_0|^2 > = \rho_{1mq}. \tag{2.11}$$

The numerical results of Weil and Walsh (1964, 1965) for the quantities ρ were obtained assuming the current distribution in the antenna used as a radiator is unaffected by the medium. This is equivalent to the assumption that $\underline{L}$ is similarly unaffected, so that their results and the results of Budden and Hugill (1964) for the left sides of (2.8)–(2.11) should agree regardless of the physical validity of the assumption. As stated in the introduction, this agreement has been verified by direct algebraic manipulation of the integrands of the appropriate expressions in the respective papers.[1]

3. Intensity of Thermal Radiation

In section 4, directional properties of antennas in magnetoplasma will be considered, as opposed to section 2 where the quantities studied represented the effects of integrations over all directions. Use will be made of some properties of thermal radiation which will be reviewed in this section.

Consider the intensity I_f for a given ray direction, i.e., direction of the Poynting vector, S (which in general does not coincide with the direction $\hat{\mathbf{k}}$ of the normal to the wavefront). I_f is defined for radiation at frequencies in the range $(f, f+df)$ and ray directions within a range of solid angle $d\Omega_g$ about the given ray direction, in terms of the long term average magnitude of the Poynting vector:

$$<dS>=I_f\, d\Omega_g df. \tag{3.1}$$

At radio frequencies, I_f for a vacuum-filled isothermal enclosure is given, to sufficient accuracy for most purposes, by the Rayleigh-Jeans approximation to Planck's law, i.e., $I_f=2KT/\lambda_0^2$. For an isothermal enclosure filled with an anisotropic medium, a similar approximation leads to

$$I_f=KT/(\lambda_0^2\,|\,K_G\,|\,), \tag{3.2}$$

where K_G is the local Gaussian curvature of the refractive index surface, i.e., the surface which is given, for each direction of the wave normal, by a radial distance equal to the refractive index. For an isotropic medium, $K_G=n^{-2}$. For the anisotropic case,

$$K_G=\frac{\cos\alpha}{n^2}\cdot\frac{d\Omega_g}{d\Omega}, \tag{3.3}$$

where α is the angle between the ray and wave normal, and $d\Omega$ is the range of solid angle of wave normals corresponding to $d\Omega_g$. Formula (3.3) facilitates the transformation from ray to wave normal directions for integrations such as that needed for (4.29).

Equation (3.2) is the low frequency approximation to the generalization of Planck's law to anisotropic media derived by Mercier (1964). Mercier also showed that even under conditions where $K_G=0$, the intensity formula integrated over all solid angles and all frequencies yields a finite energy density. Rytov (1953, 1959) had previously derived a form for I_f equivalent to (3.2) and (3.3) combined, but which does not exhibit K_G explicitly. Budden and Hugill (1964) demonstrated (3.2) for the case of radiation in a horizontally-stratified ionosphere due to an external source with brightness temperature T, and used (3.3) in their integrations to determine the spectral densities which entered in section 2.

4. Relationship Between Antenna Gain, Effective Area, Effective Length, and Radiation Resistance

The concepts of gain, effective area, effective length, radiation resistance, and their relationships, have proven useful in specifying the behavior of arbitrary antennas in isotropic media. With suitable modification, they may be extended to anisotropic media. To provide a basis for

[1] A minus sign should appear before the right-hand side of the second of equations (62) of Budden and Hugill's paper and was used in verifying algebraically the agreement with the Kogelnik forms. The correct sign was used in their numerical work; that is, their numbers are consistent with their formulas when the minus is used.

these extensions, the definitions and relationships of these quantities in the lossless, isotropic case will first be summarized in a manner emphasizing the properties which are modified upon extension to anisotropic media.

4.1. Isotropic Case

In (2.5) we used a scalar effective length for a short, linear dipole. Sinclair (1950) extended the concept to arbitrary antennas in vacuum by defining a complex vector effective length, $\underline{\mathbf{h}}(\mathbf{k})$. His results are readily generalized to isotropic, homogeneous media. If a radiating antenna, excited by current $\underline{I}\exp j\omega t$, produces a radiation field $\underline{\mathbf{E}}_r(\mathbf{k})\exp j(\omega t-\mathbf{k}\cdot\mathbf{r})$, then $\underline{\mathbf{h}}(\mathbf{k})$ is defined by

$$\underline{\mathbf{E}}_r(\mathbf{k})=j\frac{\zeta_0\underline{I}}{2\lambda_0 r}\underline{\mathbf{h}}(\mathbf{k}), \tag{4.1}$$

where λ_0 is the wavelength in vacuum and ζ is the impedance of free space. Any effect of the medium is included in $\underline{\mathbf{h}}(\mathbf{k})$.

By use of a reciprocity theorem between two antennas in an isotropic medium, Sinclair showed that, when used in reception, an incident monochromatic, plane wave with wave vector $-\mathbf{k}$ and amplitude $\underline{\mathbf{E}}_i(-\mathbf{k})$ produces an open-circuit voltage with amplitude

$$\underline{V}=\underline{\mathbf{h}}(\mathbf{k})\cdot\underline{\mathbf{E}}_i^*(-\mathbf{k}). \tag{4.2}$$

It is assumed here, as in the appendix, that the convention by which the phase of any one cartesian component of $\underline{\mathbf{E}}(\mathbf{k})$ is defined relative to any other is independent of the direction of propagation, that is, the same set of coordinate axes are used to described the fields in transmission and reception. This differs from the convention adopted by Sinclair.

It should be pointed out that Sinclair arbitrarily assumed one component of $\underline{\mathbf{h}}(\mathbf{k})$ to be real, the other components then being complex; this arbitrary choice of a real component is in fact not possible, since $\underline{\mathbf{h}}(\mathbf{k})$ expressed the relative phases of $\underline{\mathbf{E}}_r(\mathbf{k})$ and $\underline{I}$ in (4.1), and of $\underline{V}$ and $\underline{\mathbf{E}}_i(-\mathbf{k})$ in (4.2).

From the definition (4.1), one may write

$$\underline{\mathbf{h}}(\mathbf{k})=\underline{h}(\mathbf{k})\hat{\underline{\mathbf{E}}}_r(\mathbf{k}). \tag{4.3}$$

Thus, while the incident wave may have any polarization, (4.2) shows that the receiving antenna selects a component with polarization determined by that of the radiation field when the antenna is used as a radiator. The opposite polarization produces no response. The polarization of the radiation field depends on the geometrical characteristics of the antenna, the refractive index and the direction, $\hat{\mathbf{k}}$.

If the antenna has radiation resistance R, it radiates power $\frac{1}{2}|I|^2R$ and the power gain may be defined as

$$G(\mathbf{k})=\frac{8\pi}{|I|^2R}\lim_{r\to\infty}[r^2\tilde{S}_r(\mathbf{k})], \tag{4.4}$$

where $\tilde{S}_r(\mathbf{k})$ is the flux of the radiation field at distance r, and the tilde indicates that it is averaged over a complete cycle. The average value of $G(\mathbf{k})$ over 4π steradians is unity.

If, in an incident wave, the component with the polarization to which the antenna is sensitive has flux $(\tilde{S}_i - \mathbf{k})$, since the available power from the antenna is $|V|^2/8R$, an effective antenna area may be defined by

$$A_e(-\mathbf{k}) = |V|^2/[8R\tilde{S}_i(-\mathbf{k})]. \tag{4.5}$$

A number of simple relationships exist between the quantities $\underline{\mathbf{h}}(\mathbf{k})$, $G(\mathbf{k})$, $A_e(-\mathbf{k})$ and R. Using the fact that, in an isotropic medium, a plane wave with electric field $\underline{\mathbf{E}}$ has flux $\tilde{S} = n|E|^2 2\zeta_0$, one may show directly from the above definitions that

$$A_e(-\mathbf{k}) = \zeta_0 |h(\mathbf{k})|^2/4nR \tag{4.6}$$

and

$$G(\mathbf{k}) = \pi\zeta_0 n|h(\mathbf{k})|^2/\lambda_0^2 R. \tag{4.7}$$

Combining (4.6) and (4.7) yields

$$G(\mathbf{k}) = 4\pi n^2 A_e(-\mathbf{k})/\lambda_0^2. \tag{4.8}$$

Also, by integrating the flux of the radiation field over a closed surface around the antenna, and equating to the radiated power, $\frac{1}{2}|I|^2 R$,

$$R = \frac{\zeta_0 n}{4\lambda_0^2} \int_{4\pi} |h(\mathbf{k})|^2 d\Omega. \tag{4.9}$$

The relation (4.8) may also be derived without the intermediary of (4.6) and (4.7) by thermodynamic arguments involving detailed balancing. In effect, this replaces the reciprocity theorem used by Sinclair to derive (4.2) from (4.1), giving an alternate method to relate the antenna properties in transmission to those in reception. This is done, for example, by Pawsey and Bracewell (1955) for vacuum, and may readily be extended to an isotropic, refracting medium to yield (4.8).

It is our objective to generalize the definitions of power gain, effective area and vector effective length, as well as the relations (4.6)–(4.9), to an anisotropic plasma.

4.2. Gain and Effective Area in Magnetoplasma

The plasma will again be assumed lossless. It will, however, not be necessary to exclude the ranges $1 - Y^2 \leqslant X \leqslant 1$, $Y \leqslant 1$ and $X \geqslant 1$, $Y \geqslant 1$ except for parts of subsections 4.2 and 4.4. It will be required only that the radiation resistance R be finite. That R will be finite for large classes of current distributions, even when $n \to \infty$ and $K_G \to 0$, has been demonstrated by Arbel and Felsen (1963), Staras (1964), Seshadri (1965), and Lafon (1965).

Several properties of an anisotropic plasma must be borne in mind. For a given wave normal, $\hat{\mathbf{k}}$, there are two possible wave vectors representing the two characteristic waves, one or both of which may be nonpropagating under certain conditions. A particular characteristic wave with wave normal $\hat{\mathbf{k}}$, having direction (θ, ϕ), has a single associated ray direction (Θ, Φ), where $\Phi = \phi$ and $\theta - \Theta = \alpha$. However, a given ray direction and given index surface may have more than one associated wave vector, in general with different directions and magnitudes. In the following discussion of directional effects, unless otherwise stated, it is to be understood that a single characteristic wave (ordinary or extraordinary) and a single wave-normal $\hat{\mathbf{k}}$, together with its associated ray direction, is under consideration. This will be emphasized by explicit association of various quantities with a wave vector, $\mathbf{k}$. Thus the symbolism $G(\mathbf{k})$, $A_e(-\mathbf{k})$, $S(\pm\mathbf{k})$ is retained even though they are associated with energy flow along the ray direction $\Theta(\mathbf{k})$, $\Phi(\mathbf{k})$.

A particular wave vector has an associated polarization which, in contrast to the isotropic case, is dependent only on the medium and θ, and is independent of the method by which the wave is launched. In particular, the polarization of the radiation field of an antenna is independent of the antenna structure. Furthermore, from (A.4) it is readily shown that

$$\hat{\underline{\mathbf{E}}}(-\mathbf{k}) = \hat{\underline{\mathbf{E}}}(\mathbf{k}). \tag{4.10}$$

It is now clear that the definitions (4.4) and (4.5) for power gain and effective area may be retained for an anisotropic plasma, where the logical qualification is made in (4.5) that the flux $S_i(-\mathbf{k})$ is total flux with wave vector $-\mathbf{k}$, rather than a particular component of the flux determined by the polarization characteristics of the antenna. In addition, one has an option in generalizing (4.4) and (4.5):

(i) leave the formulas unchanged, R being total radiation resistance, or

(ii) replace R by R_q.

The second option gives a more complete separation of the effects of the two modes but leads, in general, to less elegant formulas relating $G(\mathbf{k})$ and $A_e(-\mathbf{k})$. Note that $|V|^2/8R_q$ is not the power available to the antenna load; as before, the available power is $|V|^2/8R$. For simplicity of notation, separate symbols will not be assigned to $G(\mathbf{k})$ and $A_e(-\mathbf{k})$ for each of the two options but the appropriate option will be indicated with the formulas.

For both options, the value of gain averaged over 4π steradians is unity (by Poynting's theorem) if it is understood that for option (i) the average includes both characteristic waves, but for option (ii) the average is for characteristic wave q only. Thus, option (i) for $G(\mathbf{k})$ gives the power radiated per unit solid angle in a particular characteristic wave as a fraction of the total power radiated, whereas option (ii) gives it as a fraction of the power radiated in characteristic wave q only.

The generalization of (4.8) will be carried out by modifying appropriately the thermodynamic proof given by Pawsey and Bracewell (1955). Thus, consider an enclosure filled with magnetoplasma and inside it the antenna matched at the frequency of interest to a load. Because our interest is in a directional effect (as opposed to the situation in section 2), a small black body, subtending solid angle $d\Omega_b$ at the antenna, is inserted. Let the whole be in thermal equilibrium at temperature T.

Consider first the power originating in the load and radiated by the antenna. The power in the elementary frequency range df, in characteristic wave q with wave vector $\mathbf{k}$, radiated into the solid angle $d\Omega_b$ and absorbed by the black body is (cf. (2.1)):

$$< dP_{\text{rad}}(\mathbf{k}) > = \begin{cases} KTdfd\Omega_b G(\mathbf{k})/4\pi, & \text{option (i)} \\ (R_q/R)KTdfd\Omega_b G(\mathbf{k})/4\pi, & \text{option (ii).} \end{cases} \tag{4.11}$$

The incident flux at the antenna in frequency range df, in characteristic wave q with wave vector $-\mathbf{k}$, due to radiation with ray directions in the solid angle $d\Omega_b$, is given by (3.1). Using definition (4.5) extended to the anisotropic case, the open-circuit voltage due to this flux is

$$< d|V|^2 > = \begin{cases} 8RA_e(-\mathbf{k})I_f(-\mathbf{k})d\Omega_b df, & \text{option (i)} \\ 8R_q A_e(-\mathbf{k})I_f(-\mathbf{k})d\Omega_b df, & \text{option (ii).} \end{cases} \tag{4.12}$$

The resulting power dissipated in the matched load is $< d|V|^2 > /8R$, so using (3.2), the power originating in the black body and absorbed by the load is

$$< dP_{\text{rec}}(-\mathbf{k}) > = \begin{cases} KTdfd\Omega_b A_e(-\mathbf{k})/\lambda_0^2|K_G|, & \text{option (i)} \\ (R_q/R)KTdfd\Omega_b A_e(-\mathbf{k})/\lambda_0^2|K_G|, & \text{option (ii).} \end{cases} \tag{4.13}$$

A reversal of the static magnetic field clearly does not affect $|K_G|$, since the refractive index is unaffected. Thus, applying the modified principle of detailed balancing to assert that the net energy exchange is zero, not only in each frequency interval but in the cone of rays subtended at the antenna by the small body, and vice versa, (4.11) and (4.13) yield the relations

$$G(\mathbf{k})=\begin{cases}(4\pi/\lambda_0^2|K_G|)A_e^T(-\mathbf{k}), & \text{option (i)}\\ (R_q^T R/R_q R^T)(4\pi/\lambda_0^2|K_G|)A_e^T(-\mathbf{k}), & \text{option (ii).}\end{cases} \tag{4.14}$$

These expressions are the generalization of (4.8) to a magnetoplasma and include (4.8) as a degenerate case. If the factor $R_q^T R/R_q R^T$ is unity, the formulas for both options are the same. This is so in two obvious cases. One was pointed out in section 2, namely antennas symmetrical about a point. The other is when conditions are such that only one characteristic wave propagates, so that $R_q=R$.

Although properties of random, thermal fields were used in the derivation of (4.14), $G(\mathbf{k})$, and $A_e(-\mathbf{k})$ were defined for CW propagation and it is clear that (4.14) must be valid for the CW case.

4.3. Effective Length and Radiation Field in Magnetoplasma

If the definition (4.1) of vector effective length is retained for a magnetoplasma it will be shown that the result (4.2) is not obtained. In view of the relatively simple physical interpretation of (4.2), $\underline{\mathbf{h}}(\mathbf{k})$ will instead be defined to satisfy (4.2). This condition by itself leaves $\underline{\mathbf{h}}(\mathbf{k})$ indeterminate, since one may add an arbitrary vector whose scalar product with $\underline{\mathbf{E}}_i^*(-\mathbf{k})$ is zero. However, (4.10) enables one to write

$$\underline{\mathbf{E}}_i^*(-\mathbf{k})=\underline{E}_i^*(-\mathbf{k})\hat{\underline{\mathbf{E}}}^*(\mathbf{k}), \tag{4.15}$$

and it is reasonable to impose the condition

$$\underline{\mathbf{h}}(\mathbf{k})=\underline{h}(\mathbf{k})\hat{\underline{\mathbf{E}}}(\mathbf{k}), \tag{4.16}$$

so that (4.2) reduces to a purely scalar expression,

$$\underline{V}=\underline{h}(\mathbf{k})\underline{E}_i^*(-\mathbf{k}). \tag{4.17}$$

The expressions (4.16) and (4.17) together may be taken as the definition of $\underline{\mathbf{h}}(\mathbf{k})$. For calculation of voltages in receiving antennas (4.17), involving only scalars, is sufficient. Note that $\hat{\underline{\mathbf{h}}}(\mathbf{k})$ is independent of the physical structure of the antenna. However, this structure influences $\underline{h}(\mathbf{k})$, which depends also on the medium and on θ.

Now $A_e(-\mathbf{k})$ may be related to $\underline{\mathbf{h}}(\mathbf{k})$ by substitution of $\tilde{S}$ from (A.8), and of (4.17) in definition (4.5) (modified to the anisotropic case):

$$A_e(-\mathbf{k})=\begin{cases}\zeta_0|h(\mathbf{k})|^2/4mR, & \text{option (i)}\\ \zeta_0|h(\mathbf{k})|^2/4mR_q, & \text{option (ii)}\end{cases} \tag{4.18}$$

(cf. (4.6)). The factor m is given by (A.9) and reduces to n in the isotropic case.

Next the radiating properties of the antenna will be investigated. Since conditions for which R can be infinite are excluded, the dominant field component at large distances varies inversely with distance. A quantity $\underline{\ell}(\mathbf{k})$ will be defined such that the radiation field is given by

$$\underline{\mathbf{E}}_r(\mathbf{k}) = j\frac{\zeta_0 \underline{I}}{2\lambda_0 r}\underline{\ell}(\mathbf{k}); \tag{4.19}$$

clearly,

$$\underline{\ell}(\mathbf{k}) = \underline{\ell}(\mathbf{k})\hat{\underline{\mathbf{E}}}(\mathbf{k}). \tag{4.20}$$

Combining (A.8) and (4.19) with the definition (4.4), again modified to the anisotropic case, yields

$$G(\mathbf{k}) = \begin{cases} \pi\zeta_0 m|\ell(\mathbf{k})|^2/\lambda_0^2 R, & \text{option (i)} \\ \pi\zeta_0 m|\ell(\mathbf{k})|^2/\lambda_0^2 R_q, & \text{option (ii).} \end{cases} \tag{4.21}$$

This may be compared with (4.7), although $\underline{\ell}(\mathbf{k})$ is not the effective length. The relationship between $|\ell(\mathbf{k})|$ and $|h(\mathbf{k})|$ is obtained by inserting (4.18) and (4.21) in (4.14), giving the result

$$|h^T(\mathbf{k})|^2 = m^2|K_G| \ |\ell(\mathbf{k})|^2, \text{ or } |h(\mathbf{k})|^2 = m^2|K_G| \ |\ell^T(\mathbf{k})|^2. \tag{4.22}$$

The relation between gain and effective length is now given by inserting this result in (4.21),

$$G(\mathbf{k}) = \begin{cases} \pi\zeta_0|h^T(\mathbf{k})|^2/m|K_G|\lambda_0^2 R, & \text{option (i)} \\ \pi\zeta_0|h^T(\mathbf{k})|^2/m|K_G|\lambda_0^2 R_q, & \text{option (ii).} \end{cases} \tag{4.23}$$

This is the generalization of (4.7). The complete relationship between $\underline{\ell}(\mathbf{k})$ and $\underline{\mathbf{h}}(\mathbf{k})$, which determines the radiation field in terms of the vector effective length, has not been established. From (4.22) it is clear that each of these quantities is related to the other one measured in the transpose medium. However, the relative phase of $\underline{h}(\mathbf{k})$ and $\underline{\ell}^T(\mathbf{k})$ has not been found because the link between radiation and reception characteristics was through (4.14), which is essentially a power relationship and cannot reveal phase information. To obtain this information, it would be necessary to appeal to a principle other than that of detailed balancing of energy exchange. Formulas equivalent to the option (i) forms (4.14), (4.18), and (4.22), but with only brief indications of the proofs, have been given by Weil (1965).

A receiving antenna may have several coherent waves with different wave vectors incident on it. In such a case the voltage $\underline{V}$ associated with each $\mathbf{k}$ must be calculated separately and the resultant voltage obtained by adding these contributions with due regard to phase. An example would be radiation from another antenna in the homogeneous plasma, in which case there is a single ray direction, namely the line joining the antennas, but there may be several different values of $\mathbf{k}$. There is no way of compounding the several values of $\underline{\mathbf{h}}(\mathbf{k})$ associated with a given ray direction without invoking the relative phases of the incident waves. The resultant available power is $|\Sigma\underline{V}|^2/8R$, and again there is no way of compounding the several values of $A_e(-\mathbf{k})$ associated with a given ray direction into a "total" effective area. Similarly, in the far-zone field of a radiating antenna, the total field associated with a given ray direction must be calculated with due regard to the relative phases of the various associated waves. Because of the different phase factors $\exp(-j\mathbf{k}\cdot\mathbf{r})$, it is not useful, for a given ray direction, to define a "total" $\underline{\ell}$ equal to $\Sigma\underline{\ell}(\mathbf{k})$. On the other hand, the total time average flux is simply the sum of the fluxes associated with the various values of $\mathbf{k}$. Thus it is possible to define a "total gain" for a particular ray direction as $\Sigma G(\mathbf{k})$, so that the power radiated per unit solid angle in all waves associated with this ray direction is proportional to this total gain.

With the help of (4.22), the quantity $|L|$ used in (2.5) may be derived. Consider a radiating, electrically short, center-fed, linear dipole of length $2a$ with current distribution $\underline{I}(p)\hat{\mathbf{p}}$. The radiation field may be found by direct application of the formula for far-zone fields of arbitrary antennas in magnetoplasma derived by Deschamps and Kessler (1964). The result is that

$$\underline{\ell}(\mathbf{k}) = \underline{L}\hat{\mathbf{p}} \cdot \hat{\underline{\mathbf{E}}}(\mathbf{k})/m|K_G|^{1/2}, \tag{4.24}$$

where

$$\underline{L} = \frac{1}{\underline{I}(0)} \int_{-a}^{a} \underline{I}(p)dp. \tag{4.25}$$

Using (4.22), together with the fact that the dipole is symmetrical, yields

$$|h^T(\mathbf{k})|^2 = |h(\mathbf{k})|^2 = |L|^2 |\hat{\mathbf{p}} \cdot \hat{\underline{\mathbf{E}}}(\mathbf{k})|^2. \tag{4.26}$$

If now the dipole is used to receive an incident plane wave $\underline{\mathbf{E}}_i(-\mathbf{k})$, then application of (4.10) and (4.17) yields the open-circuit voltage,

$$|V|^2 = |L|^2 |\hat{\mathbf{p}} \cdot \underline{\mathbf{E}}_i(-\mathbf{k})|^2. \tag{4.27}$$

A sufficient condition for $|L|$ to be independent of the presence of the magnetoplasma is that the relative current distribution, $\underline{I}(p)/\underline{I}(0)$, of the dipole when used as a radiator be similarly independent. It is reasonable to assume that this is so for a dipole which is electrically short in the sense discussed in section 2, which requires the ranges $1-Y^2 \leqslant X \leqslant 1$, $Y \leqslant 1$ and $X \geqslant 1$, $Y \geqslant 1$ to be excluded.

4.4. Radiation Resistance and Radiation Field in Magnetoplasma

Radiation resistance may also be related to effective length by using the fact that by application of Poynting's theorem the average value of gain over 4π steradians is unity. Hence, integrating (4.21) or (4.23), option (ii), over all ray directions of characteristic wave q and setting the result equal to 4π, yields for an arbitrary antenna,

$$R_q = \frac{\zeta_0}{4\lambda_0^2} \int_{4\pi} m|\underline{\ell}(\mathbf{k})|^2 d\Omega_g$$

or

$$R_q = \frac{\zeta_0}{4\lambda_0^2} \int_{4\pi} \frac{|h^T(\mathbf{k})|^2}{m|K_G|} d\Omega_g \tag{4.28}$$

(cf. (4.9)). This completes our objective of generalizing the relations (4.6)–(4.9) to magnetoplasma.

With this result, a new form may be obtained for the radiation resistance of a short electric dipole or current element in a magnetoplasma. Again, to ensure that the dipole is electrically short and may be approximated by a current element, $X-Y$ regions in which there may be refractive index infinities will be excluded. Then the radiation field will be free of singularity, and Poynting's theorem is clearly applicable. Substituting (3.3), (4.26), and (A.10) into (4.28) (it may be shown that in this transformation, the modulus signs around K_G should be removed), gives

$$R_{eq} = \frac{\zeta_0 |L|^2}{4\lambda_0^2} \int_{4\pi} n\Gamma^{-2} |\hat{\mathbf{p}} \cdot \hat{\underline{\mathbf{E}}}(\mathbf{k})|^2 d\Omega, \tag{4.29}$$

where the integration now is over all wave normal directions rather than over all ray directions, and Γ is defined by (A.11). The total free space resistance is readily evaluated by setting $\Gamma^2 = 1$, $n = 1$, $|\hat{\mathbf{p}} \cdot \hat{\mathbf{E}}(\mathbf{k})|^2 = \sin^2\theta$ and $d\Omega = 2\pi \sin\theta d\theta$, giving

$$R_{e0} = (8\pi/3)\zeta_0 |L|^2/4\lambda_0^2. \tag{4.30}$$

For the antenna in the magnetoplasma, setting $d\Omega = \sin\theta d\theta d\Phi$ and noting that Γ and n are independent of ϕ, the ratio of (4.29) to (4.30) is

$$\frac{R_{eq}}{R_{e0}} = \frac{3}{4\pi} \int_0^{\pi/2} d\theta\, n\Gamma^{-2} \sin\theta \int_0^{2\pi} d\phi |\hat{\mathbf{p}} \cdot \underline{\hat{\mathbf{E}}}(\mathbf{k})|^2. \tag{4.31}$$

The integral over ϕ is obtained in (A.15), and using (A.12) for Γ^2, the result is of the form (1.1) with

$$\rho_{3e} = \frac{3}{2} \int_0^{\pi/2} \frac{nw^2 \sin\theta}{x^2 \cot^2\theta + w^2 - v^2 \cos^2\theta}\, d\theta,$$

$$\tag{4.32}$$

and

$$\rho_{1e} = \frac{3}{4} \int_0^{\pi/2} \frac{nx^2 \sin\theta}{x^2 + w^2 \tan^2\theta - v^2 \sin^2\theta}\, d\theta.$$

It may be verified that the integrands of (4.32) are identical to those of Kogelnik's (1960) corresponding expressions, and to those of the appropriate expressions for relative spectral densities derived by Budden and Hugill (1964).

This new derivation of the radiation resistance of a short electric dipole in a magnetoplasma is particularly interesting in that, in view of the definition of gain, (4.4) it is obtained by integrating the radiation flux over a large closed surface around the antenna in contrast to the method used by Kogelnik (1960), who obtained the power radiated in the form of an expression for

$$-\tfrac{1}{2} \operatorname{Re} \int \underline{\mathbf{j}}^* \cdot \underline{\mathbf{E}} dV$$

over the antenna. Lee and Papas (1965) have criticized this latter "conventional" method as not yielding solely the true radiated power, i.e., the "time average real power absorbed by the sphere at infinity." Since the derivation in this section depends only on outward traveling wave fields varying as r^{-1} with an associated Poynting vector varying as r^{-2}, it clearly can represent only true radiated power. The agreement with the result of Kogelnik confirms that his form represents only true radiated power as has been argued on other grounds by Staras (1966) and by Walsh and Weil (1966). This conclusion, based on the purely electromagnetic derivation as given, is also confirmed by the thermodynamic arguments of section 2, where the resistance discussed is evidently the true radiation resistance since it is associated with radiation exchange between the terminated antenna and the walls of a cavity of arbitrarily large radius. The equality of relative radiation resistances with relative spectral densities proved on thermodynamic grounds in section 2, combined with the fact that the relative spectral densities derived by Budden and Hugill (1964) agree with Kogelnik's result for relative radiation resistances, indicates quite clearly that, at least with the frequency restrictions previously noted, Kogelnik deals with true radiation resistance and that his formulas are appropriate, for example, to the analysis of observations of cosmic noise by antennas in the topside ionosphere (Walsh, Haddock and Schulte, 1964).

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5. Appendix

This appendix is devoted to obtaining expressions for m, $\Gamma(\mathbf{k})$ and related quantities appearing in the text. All field quantities are understood to vary as exp $[j(\omega t-\mathbf{k}\cdot\mathbf{r})]$, and this factor will be suppressed so that we are dealing with complex amplitudes. The dielectric tensor will be used as it occurs for the static field parallel to the z axis of a rectangular coordinate system:

$$\epsilon=\begin{pmatrix}\epsilon_1 & -j\epsilon_2 & 0\\ j\epsilon_2 & \epsilon_1 & 0\\ 0 & 0 & \epsilon_3\end{pmatrix} \tag{A.1}$$

For a cold, collisionless plasma, $\epsilon_1=1-X/(1-Y^2)$, $\epsilon_2=XY/(1-Y^2)$ and $\epsilon_3=1-X$. The refractive indices for the two characteristic waves with wave normals at angle θ to the z direction are then

$$n^2=\frac{(\epsilon_1^2-\epsilon_2^2)\sin^2\theta+\epsilon_1\epsilon_3(1+\cos^2\theta)\pm|[(\epsilon_1^2-\epsilon_2^2-\epsilon_1\epsilon_3)^2\sin^4\theta+4\epsilon_2^2\epsilon_3^2\cos^2\theta]^{1/2}|}{2(\epsilon_1\sin^2\theta+\epsilon_3\cos^2\theta)}. \tag{A.2}$$

In the following, certain combinations of n and the ϵ_i occur naturally:

$$\begin{aligned} v&=(\epsilon_1-\epsilon_3)(n^2-\epsilon_1)+\epsilon_2^2\\ w&=\epsilon_1(n^2-\epsilon_1)+\epsilon_2^2\\ x&=\epsilon_3[(n^2-\epsilon_1)^2+\epsilon_2^2]^{1/2}. \end{aligned} \tag{A.3}$$

The electric field of a plane wave with normal $\hat{\mathbf{k}}$ is

$$\underline{\mathbf{E}}\equiv|E|\hat{\underline{\mathbf{E}}}=E_z\mathscr{E}\begin{pmatrix}k_1(\epsilon_1-n^2)+jk_2\epsilon_2\\ k_2(\epsilon_1-n^2)-jk_1\epsilon_2\\ \mathscr{E}^{-1}\end{pmatrix}, \tag{A.4}$$

where
$$\mathscr{E}=\frac{k_3\epsilon_3}{(k_1^2+k_2^2)w}=\frac{\epsilon_3\cos\theta}{w\sin^2\theta},$$

and k_1, k_2, and k_3 are the cartesian components of $\hat{\mathbf{k}}$, while E_z is an arbitrary scale factor which is taken here to be real, so that $\underline{\mathbf{E}}\cdot\hat{\mathbf{k}}$ is real.

From Maxwell's equations,

$$\underline{\mathbf{H}}=(n/\zeta_0)\hat{\mathbf{k}}\times\underline{\mathbf{E}}, \tag{A.5}$$

so that Poynting's vector averaged over one cycle, i.e.,

$$\tilde{\mathbf{S}}=\tfrac{1}{2}\mathrm{Re}\,(\underline{\mathbf{E}}\times\underline{\mathbf{H}}^*), \tag{A.6}$$

becomes, for real n,
$$\tilde{\mathbf{S}}=(n/2\zeta_0)|E|^2[\hat{\mathbf{k}}-(\hat{\mathbf{k}}\cdot\hat{\underline{\mathbf{E}}})\,\mathrm{Re}\,\hat{\underline{\mathbf{E}}}]. \tag{A.7}$$

Thus, the flux may be written

$$\widetilde{S}=m|E|^2/2\zeta_0, \tag{A.8}$$

where

$$m=n|\hat{\mathbf{k}}-(\hat{\mathbf{k}}\cdot\hat{\underline{\mathbf{E}}})\text{ Re }\hat{\underline{\mathbf{E}}}| \tag{A.9}$$

may be determined explicitly from (A.4).

The form for m used in deriving (4.29) is obtained from the fact that $\cos\alpha=\hat{\mathbf{k}}\cdot\widetilde{\mathbf{S}}/\widetilde{S}$, together with (A.7) and (A.8):

$$m=n\Gamma^2(\hat{\mathbf{k}})/\cos\alpha, \tag{A.10}$$

where

$$\Gamma^2(\mathbf{k})\hat{=}1-(\hat{\mathbf{k}}\cdot\hat{\underline{\mathbf{E}}})^2. \tag{A.11}$$

Use of (A.4) gives explicitly

$$\Gamma^2(\mathbf{k})=1-\frac{v^2\cos^2\theta}{w^2+x^2\cot^2\theta}. \tag{A.12}$$

To evaluate the integral over ϕ in (4.31), the coordinate system may be chosen so that $\hat{\mathbf{p}}$, which makes angle ψ with the z axis, lies in the xz plane, i.e.,

$$\hat{\mathbf{p}}=\hat{\mathbf{x}}\sin\psi+\hat{\mathbf{z}}\cos\psi. \tag{A.13}$$

$\hat{\mathbf{E}}(\mathbf{k})$ may be obtained from (A.4), leading to

$$\hat{\mathbf{p}}\cdot\hat{\underline{\mathbf{E}}}(k)=\frac{\epsilon_3\cot\theta}{(x^2\cos^2\theta+w^2\sin^2\theta)^{1/2}}\left\{[k_1(\epsilon_1-n^2)+jk_2\epsilon_2]\sin\psi+\frac{w\sin^2\theta}{\epsilon_3\cos\theta}\cos\psi\right\}, \tag{A.14}$$

where $k_1=\sin\theta\sin\Phi$ and $k_2=\sin\theta\cos\phi$. Elementary algebra and integration then gives the result

$$\int_0^{2\pi}d\phi|\hat{\mathbf{p}}\cdot\hat{\underline{\mathbf{E}}}(\mathbf{k})|^2=\frac{\pi(x^2\cos^2\theta\sin^2\psi+2w^2\sin^2\theta\cos^2\psi)}{(x^2\cos^2\theta+w^2\sin^2\theta)}. \tag{A.15}$$

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