RADIO SCIENCE Vol. 1 (New Series), No. 1, January 1966

Slots in Dielectrically Loaded Waveguide[1]

R. W. Larson and V. M. Powers

Radiation Laboratory, University of Michigan, Ann Arbor, Mich.

(Received March 22, 1965; revised July 14, 1965)

Simple modifications of formulas presented by A. A. Oliner are found to allow the determination of the impedance and resonant length of a slot in rectangular waveguide when the guide is filled with dielectric material. The modifications are justified by reference to earlier work by Oliner and others, and are verified by experimental results.

(It has been recently learned that B. R. Cheo of New York University and H. Jones have collaborated on a very similar study. Their work is in agreement and will be reported in the near future.)

1. Introduction

This note describes a theoretical technique for the design of slots in rectangular waveguide loaded with dielectric material. It is found that simple modifications of previous design formulas suffice to describe the slot characteristics. Experimental verification is included.

It is well known in the field of array design that slot spacings greater than one free-space wavelength between adjacent slots lead to grating lobes [Eaton, Eyges, and Macfarlane, 1949, pp. 318 ff.]. It is common practice in array design to avoid grating lobe problems by using half-guide wavelength spacing. Equal phase of the elements is then obtained by means of a 180° phase reversal between adjacent elements, introduced by alternating their asymmetry. The alternative approach, reducing the guide wavelength below a free-space wavelength, has several drawbacks [Eaton, Eyges, and Macfarlane, 1949, p. 319]. However, in the course of a design study for a dual-polarized slot array, where it was found impossible to employ phase reversal, dielectric loading seemed the best alternative. Since slot design formulas are universally presented for air-filled waveguides only, it is believed that presentation of formulas modified for dielectric-filled guides may prove useful to others for the above application, as well as to those interested in:

(1) prevention of breakdown (especially multipactor)

(2) reduction of waveguide size

(3) increased flexibility in slot location

(4) reduction of slot length (perhaps to allow simple edge shunt slots to remain entirely in the narrow wall)

(5) modification of the resonant resistance.

2. Modified "Stevenson" Theory

Two approaches have been developed for the study of slotted waveguides. The first is that of Stevenson [1948] which is presented in MKS units by Eaton, Eyges, and Macfarlane [1949]. The second, discussed in the next section, is that of Oliner [1957]. The Stevenson approach gives information on the magnitude of the resonant radiation resistance or conductance only. The basic ideas of the Stevenson approach are to equate the radiated, reflected, and transmitted power to the incident power and to obtain the radiated power from a knowledge of the radiation characteristics of an equivalent dipole through application of Babinet's principle [Booker, 1946]. It is apparent from a consideration of the fields of the slot and the complementary dipole that one must study a dipole entirely in air (*not* at an air-dielectric interface) in order to analyze the slot radiating into an air half-space. The magnetic field of the dipole can be the complement of the slot's radiated electric field only if the dielectric is ignored. We must, therefore, evaluate the dipole resistance, R_{dip}, for a length a' not equal to $\lambda_0/2$; R_{dip} can be obtained by the Poynting vector method [Booker, 1946].

The far field of a dipole can be obtained by assuming a sinusoidal current. The radiated power is obtained from a spatial integration of the Poynting vector and can be expressed in terms of sine and cosine integrals, by using partial fraction expansions and integration by parts. When this power is set equal to I_0^2R, the radiation resistance can be put in terms of powers of a'/λ_0. The value obtained in this way is so close to a similar value given by Oliner that the coefficients of his expansion will be incorporated into this result [Oliner, 1957, eq (14)]:

$$R_{\mathrm{dip}}=80\left(\frac{2a'}{\lambda_0}\right)^2\left[1-0.374\left(\frac{a'}{\lambda_0}\right)^2+0.130\left(\frac{a'}{\lambda_0}\right)^4\right]. \tag{1}$$

[1] The research reported in this paper was sponsored by the Aeronautical Systems Division, AFSC, Wright-Patterson AFB, Ohio, under Contract AF33(615)–1452.

For a half-wavelength dipole, this reduces to the familiar 73.2 Ω.

The Stevenson approach assumes that the slot is "resonant," allowing the simplifications that follow from having a real reflection coefficient. The reader is referred to Eaton, Eyges, and Macfarlane [1949] for that discussion [2] so that the subject here may be confined to the modifications that are required when using an interior dielectric. Then using Babinet's principle [Eaton, Eyges, and Macfarlane, 1949, p. 295; or Booker, 1946, eq (22)],

$$R_{\text{slot}} = \frac{1}{R_{\text{dip}}} \cdot \frac{\mu_0}{4\epsilon_0}, \tag{2}$$

(note use of ϵ_0 here), and otherwise following the presentation of Eaton, Eyges, and Macfarlane, we eventually obtain the modifications given in table 1, where

$$Y_0 = \left(\frac{\epsilon_0}{\mu_0}\right)^{1/2} \cdot \frac{\lambda_0}{\lambda_g} \tag{3}$$

[2] An error that should be noted occurs in eq (51b) on page 293, which should read (replacing κ by k):

$$S_{10} = Y_{10}^{(0)} \frac{ab}{2} \left(\frac{\beta_{10}}{k}\right)^2.$$

$$\lambda_{g\epsilon} = \frac{\lambda_0}{\sqrt{\epsilon_r - (\lambda_0/2a)^2}}. \tag{4}$$

It should be noted that the poles of f_1 and f_4 are canceled by corresponding zeros in the cosine terms. These results have been presented in a somewhat redundant manner to facilitate comparison with the Stevenson forms [Eaton, Eyges, and Macfarlane, 1949; Ehrlich, 1961]. They reduce to the air-filled guide results where $\epsilon_r = 1$.

3. Modified "Oliner" Theory

It is only from the more complete analysis presented by Oliner (table 2) for broadwall slots that the resonant length and reactance can be found. The Oliner results reduce to the Stevenson results for the special case when the slot is a half wavelength long ($a' = \lambda_0/2$) and the guide is air-filled. The resonant length is defined in this study as the length for which the reactance is zero. Because all of the Oliner cases depend on the results for the admittance of a centered broadwall series slot, it is enough to consider in detail only the

TABLE 1. *Resonant resistance or conductance*

Slot type	Equation	Definitions
Broadwall shunt	$g = f_1 \left\{\left[2.09 \frac{\lambda_{g_\epsilon}}{\lambda_0} \left(\frac{a}{b}\right) \cos^2\left(\frac{2\pi a'}{\lambda_{g_\epsilon}}\right)\right] \sin^2\left(\frac{\pi x}{a}\right)\right\}$	$f_1 = f_0 \left[\frac{\epsilon_r - \left(\frac{\lambda_0}{\lambda_{g_\epsilon}}\right)^2}{1 - \left(\frac{2a'}{\lambda_{g_\epsilon}}\right)^2}\right]^2$ $f_0 = \frac{73.2}{80\left[1 - 0.374\left(\frac{a'}{\lambda_0}\right)^2 + 0.130\left(\frac{a'}{\lambda_0}\right)^4\right]}$
Broadwall inclined series	$r = f_2 \left\{0.131 \left(\frac{\lambda_0}{\lambda_{g_\epsilon}}\right) \frac{\lambda_0^2}{ab} \left[I(\theta) \sin\theta + \frac{\lambda_{g_\epsilon}}{2a} J(\theta) \cos\theta\right]^2\right\}$	$f_2 = f_0$ $I(\theta) = \frac{\cos\left(\frac{\pi\xi}{2}\right)}{1-\xi^2} + \frac{\cos\left(\frac{\pi\eta}{2}\right)}{1-\eta^2}$ $J(\theta) = \frac{\cos\left(\frac{\pi\xi}{2}\right)}{1-\xi^2} - \frac{\cos\left(\frac{\pi\eta}{2}\right)}{1-\eta^2}$ $\xi = \frac{2a'}{\lambda_{g_\epsilon}} \cos\theta - \frac{a'}{2a} \sin\theta$ $\eta = \frac{2a'}{\lambda_{g_\epsilon}} \cos\theta + \frac{\lambda_0}{2a} \sin\theta$
Edge shunt	$g = f_3 \left\{0.131 \left(\frac{\lambda_{g\epsilon}}{\lambda_0}\right) \frac{\lambda_0^4}{a^3 b} \left[\frac{\sin\theta \cos\left(\frac{\pi a'}{\lambda_{g_\epsilon}} \sin\theta\right)}{1 - \left(\frac{2a'}{\lambda_{g_\epsilon}} \sin\theta\right)^2}\right]^2\right\}$	$f_3 = f_0$
Broadwall displaced series	$r = f_4 \left\{\left[0.523 \left(\frac{\lambda_{g_\epsilon}}{\lambda_0}\right)^3 \left(\frac{\lambda_0^2}{ab}\right) \cos^2\left(\frac{\pi a'}{2a}\right)\right] \cdot \cos^2\left(\frac{\pi d}{a'}\right)\right]$ *	$f_4 = f_0 \left[\frac{\epsilon_r - \left(\frac{\lambda_0}{2a}\right)^2}{1 - \left(\frac{a'}{a}\right)^2}\right]^2$

*Note typographical error in Ehrlich [1961], eq (9-7).

TABLE 2. *Modifications of Oliner formulas*

Equations	Definitions
$\frac{G_r}{Y_0}=\frac{\frac{1}{n_j^2}\left(\frac{Y'_0}{Y_0}\right)^2\frac{G_{rj}}{Y_0}(1+\tan^2\kappa'_0 t)}{\left(\frac{G_{rj}}{Y_0}\right)^2\tan^2\kappa'_0 t+\left[\frac{Y'_0}{Y_0}-\frac{B_{rj}}{Y_0}\tan\kappa'_0 t\right]^2}$	$\kappa'_0=\sqrt{k_0^2-(\pi/a')^2}$ (when κ'_0 is imaginary: $\kappa'_0=-i\sqrt{\left(\frac{\pi}{a'}\right)^2-k_0^2}=-i\lvert\kappa'_0\rvert$. $\cot(\kappa'_0 t)=i\coth(\lvert\kappa'_0\rvert t))$
$\frac{G_{rj}}{Y_0}=\frac{\lambda_{g\epsilon}a'b'}{\lambda_0^3}\frac{32}{3\pi}\left[1-0.374\left(\frac{a'}{\lambda_0}\right)^2+0.130\left(\frac{a'}{\lambda_0}\right)^4\right]$	$k_0=2\pi/\lambda_0$
$\frac{1}{n_j^2}=\frac{ab}{a'b'}\left[\frac{\pi}{4}\frac{1-(a'/a)^2}{\cos(\pi a'/2a)}\right]^2$	$\kappa_\epsilon=2\pi/\lambda_{g_\epsilon}=\sqrt{\epsilon_r k_0^2-\left(\frac{\pi}{a}\right)^2}$
$\frac{B_r}{Y_0}=\frac{B_j}{Y_0}+\frac{\frac{1}{n_j^2}\frac{Y'_0}{Y_0}\left\{\frac{B_{rj}}{Y_0}\frac{Y'_0}{Y_0}(1-\tan^2\kappa'_0 t)+\left[\left(\frac{Y'_0}{Y_0}\right)^2-\left(\frac{B_{rj}}{Y_0}\right)^2-\left(\frac{G_{rj}}{Y_0}\right)^2\right]\tan\kappa'_0 t\right\}}{\left(\frac{G_{rj}}{Y_0}\right)^2\tan^2\kappa'_0 t+\left[\frac{Y'_0}{Y_0}-\frac{B_{rj}}{Y_0}\tan\kappa'_0 t\right]^2}$	$\frac{Y'_0}{Y_0}=\frac{\kappa'_0}{\kappa_\epsilon}$
$\frac{B_{rj}}{Y_0}=\frac{2b'\lambda_{g_\epsilon}}{\lambda_0^2}\left\{\left(\frac{\kappa'_0}{k_0}\right)^2\left[C+\frac{3}{2}-\ln\frac{\gamma\lvert\kappa'_0\rvert b'}{2}\right]+\frac{\sin k_0a'}{k_0a'}+\left[1+\left(\frac{\lambda_0}{2a'}\right)^2\right](S_-)-\frac{2b'}{3a'}\left(\frac{\lambda_0}{2a'}\right)^2\right\}$	$C=\frac{Ci(k_0a'+\pi)+Ci\lvert k_0a'-\pi\rvert}{2}$
$\frac{B_j}{Y_0}=\frac{1}{2}\frac{B_t}{Y_0}+\frac{2b}{\lambda_{g_\epsilon}}\left[\ln 2+\frac{\pi}{6}\frac{b'}{b}+\frac{3}{2}\left(\frac{b}{\lambda_{g_\epsilon}}\right)^2\right]$	$S_-=\frac{Si(k_0a'+\pi)-Si(k_0a'-\pi)}{2\pi}$
$\frac{B_t}{Y_0}=\frac{4b}{\lambda_{g_\epsilon}}\left[\ln\csc\frac{\pi b'}{2b}+\frac{1}{2}\left(\frac{b}{\lambda_{g_\epsilon}}\right)^2\cos^4\left(\frac{\pi b'}{2b}\right)\right]$	$Si(x)=\int_0^x\frac{\sin t}{t}dt,\ Ci(x)=\int_\infty^x\frac{\cos t}{t}dt$
$-\frac{4b}{\lambda_{g_\epsilon}}\left(\frac{\lambda_{g_\epsilon}}{\lambda_{g3_\epsilon}}\right)^2\left[\frac{\cos\frac{3\pi a'}{2a}}{\cos\frac{\pi a'}{2a}}\cdot\frac{1-\left(\frac{a'}{a}\right)^2}{1-9\left(\frac{a'}{a}\right)^2}\right]^2\left[1+\left(\frac{\pi b'}{2\lambda_{g3_\epsilon}}\right)^2\right]\ln\left(\frac{4}{\pi\gamma}\frac{\lambda_{g3_\epsilon}}{b'}\right)$	$\ln\gamma=0.5772,\quad\gamma=1.781$; $\lambda_{g3_\epsilon}=\left\lvert\frac{\lambda_0}{\sqrt{\epsilon_r-\left(\frac{3\lambda_0}{2a}\right)^2}}\right\rvert$

modifications of those formulas for which experimental results are also given below. The formulas corresponding to Oliner's other cases follow directly.

The basic Oliner approach is to obtain an equivalent circuit for the slot, as shown in figure 1. The radiating centered series slot is characterized as an *E*-plane *T* junction (series susceptance B_j and transformer ratio n_j: 1), a short section of small waveguide (characteristic admittance Y'_0), and a radiating junction (admittance $G_{rj}+jB_{rj}$).

It is obvious in this circuit that the parameters associated with the radiating junction (fig. 1*a*) are unaffected by the material inside the waveguide. This corresponds to the use, in the preceding section, of Babinet's principle with a dipole radiating in free space rather than one radiating at an air-dielectric interface. Thus, the values of B_{rj} and G_{rj} are unchanged by the presence of the dielectric. Similarly, the section of transmission line characterized in figure 1*c* by Y'_0 is unchanged. However, the parameters of the *E*-plane *T* junction shown in figure 1*b* must be carefully re-evaluated to determine how they are altered. This method gives the results in the preceding section on conductance at resonance as a byproduct, but our main interest is the effect on the susceptance.

Oliner [1957] does not derive formulas for n_j and B_j, but notes the results of an earlier study [1951]; this report in turn draws upon still earlier material by Marcuvitz [1949, 1951]. We shall only briefly describe how the results are obtained from all of these sources and how they are modified by the presence of a dielectric.

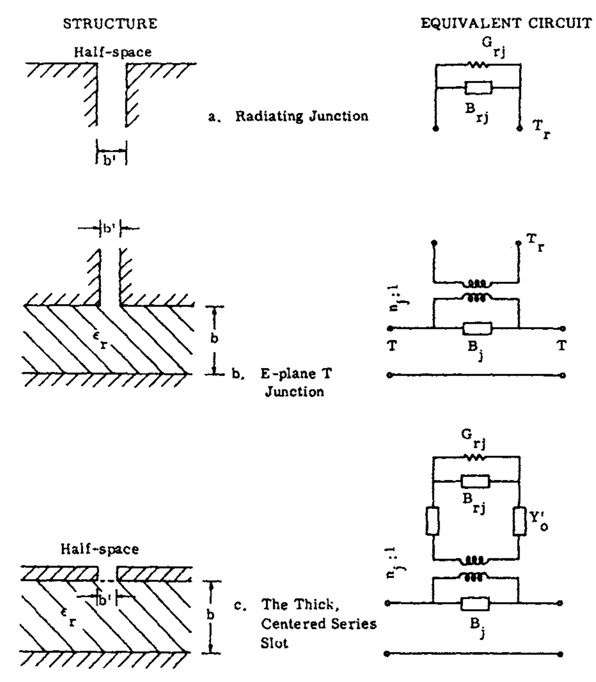

FIGURE 1. *Slot structures and their equivalent circuits.*

Several assumptions can be made that greatly simplify the work. First is that of ignoring the shunt elements (not shown in fig. 1) in the more general pi-equivalent circuit. This was done by Oliner because the experimental results showed it to be a valid approximation. Also (again following Oliner) we shall drop X_c and X_d of the T representation given by Marcuvitz [1951, p. 336]. For further justification we can refer to the theoretical-experimental agreement discussed in the sixth section.

The derivation of n_j as given by Oliner follows that of Marcuvitz [1951, pp. VI–14 ff.]; it is easily shown that the value of n_j obtained from a variational formulation is independent of the guide dielectric. In fact, with their normalization, ϵ_r does not appear in the derivation of n_j.

Oliner obtained an expression for B_j by using the assumptions given above and by further assuming that the rate of change of energy storage in the main guide is unaffected by the stub-guide dimensions (although it is a strong function of the slot dimensions). It also follows that the main guide energy storage can be assumed independent of the stub-guide dielectric (in this case, air). The susceptance value for the longitudinal broadwall slot is thus obtained from the value for an E-plane junction of identical guides, which is in turn obtained from the theory of a transverse slot in a uniform guide, which in turn is obtained from the theory for a capacitive iris. The longitudinal slot differs from the transverse slot only by inclusion of image terms. This difference is obtained by comparison of longitudinal and transverse values given by Marcuvitz [1951, p. 219, p. 351]. Since these results are given in terms of λ_g, it is only necessary to use

$$\lambda_{gm_\epsilon}=\frac{\lambda_0}{\sqrt{\epsilon_r-\left(\frac{m\lambda_0}{2a}\right)^2}}. \tag{5}$$

It is thus found that the expressions given by Oliner can be used directly in the analysis of dielectrically loaded guides as long as (denoting by $\rightarrow$ the replacement of a symbol by the one to be used when $\epsilon_r \neq 1$ inside the guide)

$$\lambda_g \Rightarrow \lambda_{g_\epsilon}$$

$$\lambda_{g3} \Rightarrow \lambda_{g3_\epsilon}$$

$$\lambda \rightarrow \lambda_0 \tag{6}$$

κ stays the same (not so if the dielectric extends up into the slot).

4. Resonant Length

There does not seem to be a more direct method of obtaining the resonant length of a slot in a dielectrically filled waveguide than that of obtaining the slot length a' which gives a total susceptance $B_c=0$ for each ϵ_r. For RG–52/U waveguide ($a=0.900$ in., $b=0.400$ in.) and a square-ended broadwall series slot that is resonant near $f=10.70$ Gc ($a'=0.521$ in.) when $\epsilon_r=1.0$, a curve is given in figure 2 showing the variation of the normalized resonant length, $2a'/\lambda_0$, with ϵ_r. This variation was plotted from calculations at a number of frequencies with $\epsilon_r=1.0$, 1.6, 2.25, and 3.0, using the Oliner formulas as modified by (6). An empirical fit to this curve is given by

$$2a'/\lambda_0=0.945-0.16(\epsilon_r-1)+.03(\epsilon_r-1)^2. \tag{7}$$

5. Experimental Verification

Two types of verification were attempted. Using $\epsilon_r=1.0$ and dielectric slugs with nominal values of $\epsilon_r=1.6$ and 2.55, the impedance characteristics were measured near resonance at several frequencies. The measured resonant values of both length and resistance are seen to compare favorably with the theoretical values in figure 2. In accordance with Oliner's suggestion for comparing round- and square-ended slots, these measurements used a round-ended slot that was 2 percent longer ($a'=0.532$ in.) than the value (0.521 in.), used in the calculations.

The second type of verification consisted of comparing the theoretical and experimental variation with frequency for the case, $\epsilon_r=1.6$. This is shown in figure 3; the agreement is reasonably good and seems to exhibit the same variations observed by Oliner for an air-filled guide.

The measurement technique consisted of measuring the shift in null positions in an air-filled, slotted section when the slot was covered and when it was radiating. A probe under the slot was monitored to allow positioning of the short-circuit guide termination to give a current maximum at the slot. Knowing the length of dielectric between the generator and the slot (typically about 0.5 in.) allowed a calculation of the slot characteristics taking into account the impedance transformation at the dielectric interface. Nominal rather than experimental values of ϵ_r were used in these calculations.

6. Conclusions

It has been shown that unexpectedly simple modifications of existing slot formulas suffice to predict slot characteristics in dielectrically loaded waveguides. Although the slot length results presented are only for the broadwall series slot, their use is undoubtedly justified for other cases. The use of the modified Stevenson formulas (given in sec. 2) in conjunction with (7) for the proper slot length should allow the solution of most design problems for slots in rectangular guide.

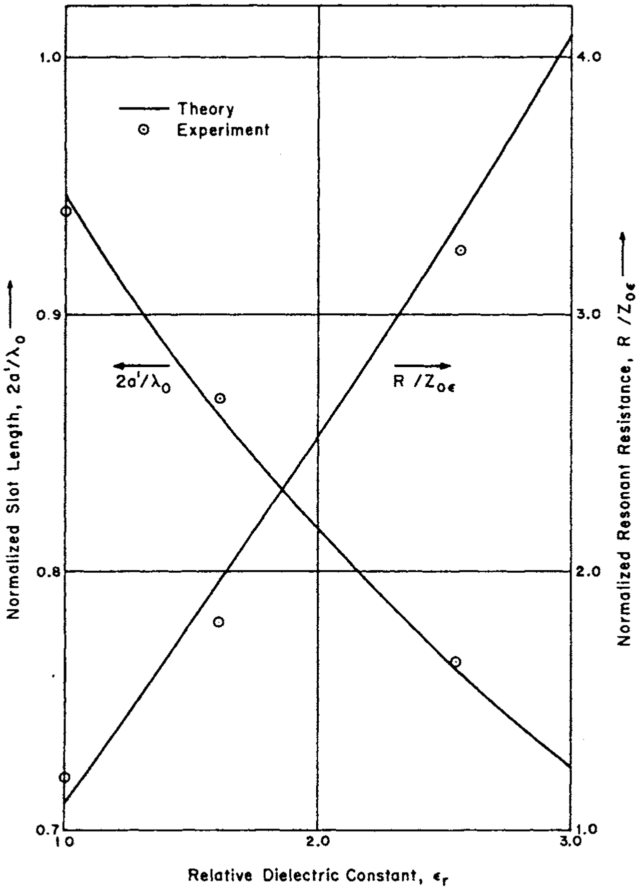

FIGURE 2. *Normalized resonant slot length and resistance versus dielectric constant.*

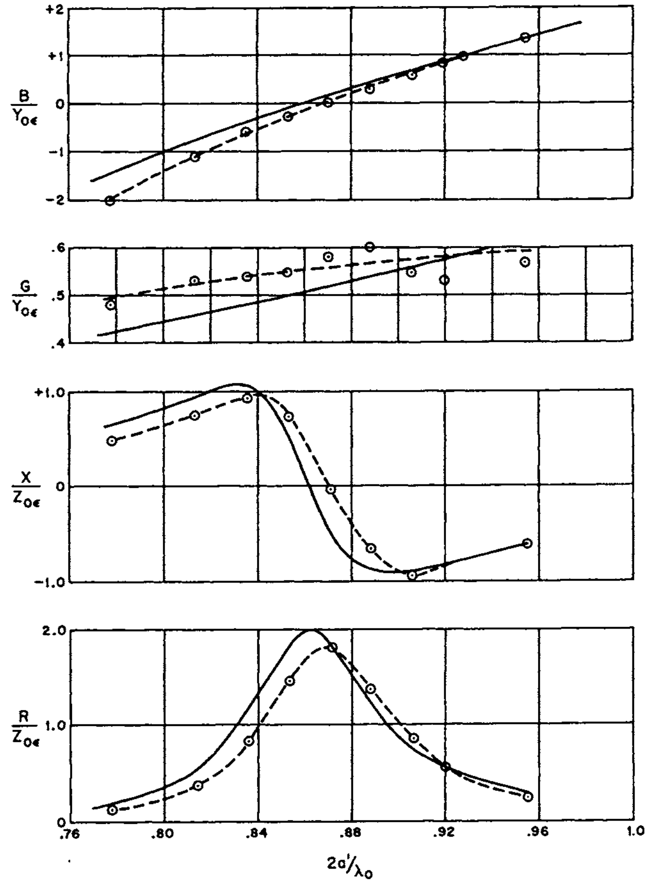

FIGURE 3. *Impedance and admittance parameters versus normalized slot length.*

The authors thank R. E. Hiatt for suggesting the problem and for several helpful discussions, H. Hunter and M. Larudee for assistance in computation, and J. Ferris and K. Young for assistance in performing the experimental measurements. The authors also acknowledge the reviewers' suggestion to present the results in tabular form.

7. References

Booker, H. G. (1946), Slot aerials and their relation to complementary wire aerials (Babinet's principle), Proc. Inst. Elec. Engrs. (London) **93**, Pt. 3A, 620–626.

Eaton, J. E., L. J. Eyges, and G. G. Macfarlane (1949), Linear array antennas and feeds, Microwave Antenna Theory and Design, MIT Radiation Lab. Series, **12**, ed. S. Silver, ch. 9 (McGraw-Hill Book Co., Inc., New York).

Ehrlich, M. J. (1961), Slot-antenna arrays, Antenna Engineering Handbook, ed. H. Jasik, ch. 9 (McGraw-Hill Book Co., Inc., New York).

Marcuvitz, N. (1949), The representation, measurement, and calculation of equivalent circuitry for waveguide discontinuities with application to rectangular slots, Report No. R–193–49, PIB–137, Microwave Res. Inst., Brooklyn, N.Y.

Marcuvitz, N. (1951), Waveguide Handbook, MIT Radiation Lab. Series, **10** (McGraw-Hill Book Co., Inc., New York).

Oliner, A. A. (Aug. 1951), Equivalent circuits for slots in rectangular waveguide, Report No. 4.11–216, 11212, Microwave Res. Inst., Brooklyn, N.Y.

Oliner, A. A. (1957), The impedance properties of narrow radiating slots in the broad face of rectangular waveguide, I, Theory; II, Comparison with measurement, IRE Trans. Ant. Prop. **AP–5**, No. 1, 4–20.

Stevenson, A. F. (Jan. 1948), Theory of slots in rectangular waveguides, J. Appl. Phys. **19**, No. 1, 24–38.

(Paper 1–1–6)