RADIO SCIENCE, Vol. 1 (New Series), No. 9, September 1966

A Modified Fock Function for the Distribution of Currents in the Penumbra Region With Discontinuity in Curvature

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(Received March 23, 1966; revised April 25, 1966)

A modified Fock function is obtained to describe the current distribution on a smooth convex boundary, composed of a flat plane smoothly joined to a parabolic cylinder with the join in the penumbra region. Both analytical and numerical methods are used to obtain the modified Fock function which now depends on the distance between the shadow boundary and the flat plane-parabolic cylinder join. The modified Fock function is applied to estimate the backscattering cross section of a cone-sphere.

1. Introduction

When a high-frequency plane electromagnetic wave is tangentially incident upon a locally parabolic convex surface, the distribution of the induced current near the shadow boundary is described by the Fock function [Fock, 1946]. However, if a portion of the scattering surface near the shadow boundary is no longer parabolic but flat (e.g., wedge-cylinder or cone-sphere), the effect of the surface discontinuity at the join of the flat plane and parabolic cylinder must be taken into account. Weston [1965] has, in a previous paper, discussed an extension of the Fock theory when the position of the surface discontinuity coincides exactly with the shadow boundary.

It is the purpose of this paper to present a modified Fock function which provides the current distribution in the penumbra and shadow regions when the surface discontinuity is in the penumbra region. This new modified Fock function describes the current distribution as a function of two variables: one is the distance between the shadow boundary and the observation point, and the other the distance between the shadow boundary and the flat plane-parabolic cylinder join.

The method to be used is as follows: an exact integral equation governing the total magnetic field on the boundary is formulated by Maue's method [Maue, 1949]. The high-frequency asymptotic expression of the exact integral equation is a Volterra type, and both analytic and numerical solutions of the Volterra equation are obtained. These solutions may be called the modified Fock functions. In section 5, this modified Fock function is applied to estimate the backscattering cross section of a cone-sphere.

2. Integral Equation Governing the Surface Fields

Consider a plane electromagnetic wave

$$\mathbf{H}_{\text{in}} = \hat{z} e^{iky - i\omega t} \tag{1}$$

incident upon a perfectly conducting convex cylinder whose boundary is composed of a smoothly joined flat plane and a parabolic cylinder (fig. 1). In terms of a Cartesian coordinate system whose origin is taken at the shadow boundary, the surface of the scattering body is represented by the following equations:

$$x = -\frac{y^2}{2R} \qquad \text{for } y \geqslant -R \tan \alpha \tag{2}$$

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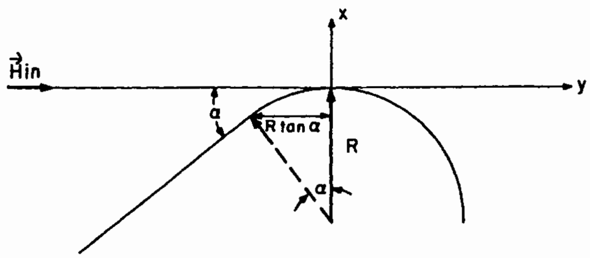

FIGURE 1. *Geometry of the scattering surface.*

$$x = \tan\alpha\left(y + \frac{R}{2}\tan\alpha\right) \qquad \text{for } y \leq -R\tan\alpha, \tag{3}$$

where R is the radius of curvature of the parabolic section near the origin and is assumed large in comparison with wavelength.

The total magnetic field on the boundary is defined as

$$\mathbf{H}_{\text{total}} = \hat{\mathbf{z}} u e^{-i\omega t}. \tag{4}$$

Then the scalar function u is the solution of the following equations [Maue, 1949]:

$$\frac{\partial u}{\partial n} = 0, \qquad u(P) = 2e^{iky} - \frac{ik}{2}\int_{-\infty}^{\infty} ds\, \frac{\hat{\mathbf{n}}_Q \cdot \mathbf{r}}{r} H_1^{(1)}(kr) u(Q), \tag{5}$$

where r is the distance between two points P and Q, and $\hat{\mathbf{n}}$ is a unit vector normal to the boundary.

It was shown by Weston [1965] that on the flat section the field reflected from the surface discontinuity is of the order $1/kR$. Therefore, for the high-frequency region, the reflected field contributes to higher-order corrections, and u on the flat section becomes equal to the geometrical optics term $2e^{iky}$.

On the parabolic section, let us define

$$u(y) \equiv I(y) e^{iks} \qquad \text{for } y > -R\tan\alpha, \tag{6}$$

where s is the distance along the surface between the shadow boundary and the observation point:

$$s = \int_0^y \sqrt{1 + (y/R)^2}\, dy. \tag{7}$$

Using the relationships (5), (6), and (7), we obtain an integral equation governing $I(y)$:

$$I(y) = 2e^{iky - iks(y)} - ik\int_{-\infty}^{-R\tan\alpha} \frac{dt}{\cos\alpha} \frac{\hat{\mathbf{n}}(t)\cdot\mathbf{r}_1}{r_1} H_1^{(1)}(kr_1)[\exp\{ikt - iks(y)\} + O(1/kR)]$$

$$- \frac{ik}{2}\int_{-R\tan\alpha}^{\infty} dt I(t)\left(1 + \frac{t^2}{R^2}\right)^{1/2} \frac{\hat{\mathbf{n}}(t)\cdot\mathbf{r}_2}{r_2} H_1^{(1)}(kr_2)\exp\{iks(t) - iks(y)\}, \tag{8}$$

with

$$r_1^2 = (y-t)^2 + \left\{\frac{y^2}{2R} + \tan\alpha\left(t + \frac{R}{2}\tan\alpha\right)\right\}^2 \tag{9}$$

and

$$r_2^2 = (y-t)^2 + \frac{1}{4R^2}(y^2 - t^2). \tag{10}$$

In order to obtain an appropriate high-frequency asymptotic form of $I(y)$ near the shadow boundary, we shall set

$$(kR)^{1/3}=m,$$

$$ky=m^2\xi,$$

$$kt=m^2\zeta, \tag{11}$$

and

$$I(y)=J(\xi).$$

When the surface discontinuity lies in the penumbra region, $m \tan \alpha$ is of the order of unity or less, and can be replaced by $m\alpha$ for large m. The asymptotic expression of (9) becomes

$$J(\xi)=2e^{-i\frac{\xi^3}{6}}-\frac{e^{-i\frac{\pi}{4}}}{2}\sqrt{2/\pi}\,(\xi+m\alpha)^2\int_{-\infty}^{-m\alpha}\frac{d\zeta}{(\xi-\zeta)^{3/2}}\cdot\exp\left\{i\frac{(\xi+m\alpha)^4}{8(\xi-\zeta)}\right.$$

$$\left.+i\frac{m^2\alpha^2}{2}(\xi-\zeta)-i\frac{m\alpha}{2}(\xi+m\alpha)^2-i\frac{\xi^3}{6}\right\}-\frac{e^{-i\frac{\pi}{4}}}{4}\sqrt{2/\pi}\int_{-m\alpha}^{\xi}d\zeta J(\zeta)(\xi-\zeta)^{1/2}e^{-i\frac{(\xi-\zeta)^3}{24}}+O(m^{-2}). \tag{12}$$

When $m\alpha$ goes to infinity, the second term in (12) disappears, and the solution becomes the ordinary Fock function [Cullen, 1958]. This means that when the position of the flat plane-parabolic cylinder join is far away from the shadow boundary, the current distribution is given by the Fock function. Otherwise, the Fock function has to be modified. The solution $J(\xi)$ of (12) will be called the modified Fock function. When α is identically zero, the solution of (12) is that obtained by Weston [1965].

In the following two sections, the solution of (12) is obtained when $m\alpha$ is finite but is not identically zero.

3. Analytic Solution

In this section, the solution of (12) is derived when the distance between the shadow boundary and the position of the surface discontinuity is small, so that $m^3\alpha^3$ is negligible.

In order to solve (12) by the Laplace transform method, it is convenient to modify (12) by setting

$$\xi=\theta-m\alpha,$$

$$\zeta=\phi-m\alpha,$$

and

$$J(\theta-m\alpha)=j(\theta). \tag{13}$$

Substituting these new variables into (12) and taking the Laplace transform of both sides of the integral equation, we obtain

$$\bar{j}(p)=\frac{2N(p)}{1+\frac{e^{-i\frac{\pi}{4}}}{4}\sqrt{2/\pi}M(p)}, \tag{14}$$

where

$$\bar{j}(p)=\int_0^\infty d\theta e^{-p\theta}j(\theta), \tag{15}$$

$$M(p)=\int_0^\infty d\theta e^{-p\theta-i\frac{\theta^3}{24}}\theta^{1/2}, \quad \text{and} \tag{16}$$

$$N(p)=\int_0^\infty d\theta e^{-p\theta-i\frac{(\theta-m\alpha)^3}{6}}\left[1-\frac{e^{-i\frac{\pi}{4}}}{4}\sqrt{2/\pi}\,\theta^2\int_{-\infty}^0 \frac{d\phi}{(\theta-\phi)^{3/2}}\right.$$
$$\left.\cdot\exp\left\{i\frac{\theta^4}{8(\theta-\phi)}+i\frac{m^2\alpha^2}{2}(\theta-\phi)-i\frac{m\alpha}{2}\theta^2\right\}\right]. \tag{17}$$

The integral $M(p)$ has been evaluated by Weston [1965] as

$$M(p)=4\pi\sqrt{2\pi}\left[Ai(q)\{Ai'(q)-iBi'(q)\}+\frac{i}{2\pi}\right]e^{i\frac{3\pi}{4}}, \tag{18}$$

with $q=-ip2^{1/3}$, where Ai and Bi are Airy functions of the first and second kind, respectively, and the prime denotes differentiation with respect to the argument. The denominator of (14) becomes

$$2\pi i Ai(q)\frac{d}{dq}[Ai(q)-iBi(q)]. \tag{19}$$

The integral $N(p)$ is evaluated in the appendix under the assumption that $(m\alpha)^3$ is negligible. We obtain

$$\bar{j}(p)=-\frac{2i}{[Ai'(q)-iBi'(q)]}\left[2^{4/3}\left\{\frac{1}{3}+\frac{1}{2}\int_0^q [Ai(x)-iBi(x)]dx\right\}\right.$$
$$\left.\cdot\{1-q^2m^2\alpha^2 2^{-5/3}\}+2^{-4/3}m^2\alpha^2\{[Ai(q)-iBi(q)]+q[Ai'(q)-iBi'(q)]\}\right]. \tag{20}$$

The desired solution $J(\xi)$ is obtained by taking the inverse transform of (20) and using the relationships $\xi=\theta-m\alpha$ and $j(\theta)=J(\theta-m\alpha)$:

$$J(\xi)=\frac{2^{-1/3}}{\pi}\int_{-\infty-ic}^{\infty-ic} dq\,\frac{\exp\{iq2^{-1/3}(\xi+m\alpha)\}}{w_1'(q)}\left[\{1-q^2m^2\alpha^2 2^{-5/3}\}\right.$$
$$\left.\cdot\left\{\frac{2^{4/3}}{3}\sqrt{\pi}-i2^{1/3}\int_0^q w_1(x)dx\right\}-i2^{-4/3}m^2\alpha^2\{w_1(q)+qw_1'(q)\}\right], \tag{21}$$

with

$$w_1'(q)=i\sqrt{\pi}\,[Ai'(q)-iBi'(q)].$$

When ξ is sufficiently large and positive, the contour integral may be evaluated in terms of the residues at the zeros of $w_1'(q)$, resulting in the expression of the field as a sum of the creeping waves. For numerical calculation of the residue series, it is convenient to substitute the following relationship:

$$w_1(q)=e^{i\frac{\pi}{6}}2\sqrt{\pi}Ai(qe^{i\frac{2\pi}{3}}), \qquad q=e^{i\frac{\pi}{3}}\beta. \tag{22}$$

The high-frequency asymptotic expression of the total field in the shadow region for small $m\alpha$ is

$$u(y)=e^{iks}\sum_{l=1}^{\infty}\frac{\exp\{i\beta_l e^{i\frac{\pi}{3}}y/d\}}{\beta_l Ai(-\beta_l)}\exp\{im\alpha\beta_l 2^{-1/3}e^{i\frac{\pi}{3}}\}\left[\left\{\frac{2}{3}+2\int_0^{\beta_l}Ai(-x)dx\right\}\right.$$
$$\left.\cdot\{1-\beta_l^2 m^2\alpha^2 2^{-5/3}e^{i\frac{2\pi}{3}}\}+e^{-i\frac{\pi}{3}}m^2\alpha^2 2^{-2/3}Ai(-\beta_l)\right], \qquad (23)$$

with $d=(\lambda R^2/\pi)^{1/3}$. When α goes to zero, (23) becomes equal to the solution given by Weston [1965] except for a factor 2. The reason for this is that when α is identically zero, the amplitude of the total field on the flat portion was taken by Weston [1965] to be unity, while it is taken to be 2 in this paper. Numerical values of β_l, $Ai(-\beta_l)$, and $\int_0^{\beta_l}Ai(-x)dx$ are given by Weston [1965].

Therefore, when $kR\alpha^3$ is negligible, each mode of the creeping waves given by (23) is different from the Fock solution [Goodrich, 1959] by the factor

$$\exp\{i\beta_l 2^{-1/3}m\alpha e^{i\frac{\pi}{3}}\}\left[\left\{\frac{2}{3}+2\int_0^{\beta_l}Ai(-x)dx\right\}\{1-\beta_l^2 m^2\alpha^2 2^{-5/3}e^{i\frac{2\pi}{3}}\}+e^{-i\frac{\pi}{3}}m^2\alpha^2 2^{-2/3}Ai(-\beta_l)\right]. \qquad (24)$$

4. Numerical Solution

In this section, a numerical method is described to obtain the solution of (12) without the assumption that $m^3\alpha^3$ is negligible.

The high-frequency asymptotic expansion reduces Maue's integral equation from that of the Fredholm class to that of the Volterra type, which is much easier to handle numerically. Several methods are available for the solution of the Volterra integral equation. When a high-speed digital computer is available, the simplest approach is to expand the unknown in a set of algebraic functions $J_1, \ldots, J_n$, and to require the integral equation be satisfied at n different points. The solution of (12) may be approximated by setting

$$J_n=J(\xi=n\Delta-m\alpha), \qquad (25)$$

where Δ is the distance between two adjacent points on the contour of integration.

Insertion of this expression in (12) gives

$$J_n=H_n+\frac{\Delta}{3}\left[K_{n0}J_0+4K_{n1}J_1+2K_{n2}J_2+\ldots+2K_{n,n-2}J_{n-2}+4K_{n,n-1}J_{n-1}\right], \qquad (26)$$

with

$$H_n=2e^{-i\frac{(n\Delta-m\alpha)^3}{6}}\left[1-\frac{e^{-i\frac{\pi}{4}}}{4}\sqrt{2/\pi}\int_{-\infty}^{-m\alpha}d\zeta\frac{(n\Delta)^2}{(n\Delta-m\alpha-\zeta)^{3/2}}\right.$$
$$\left.\cdot\exp\left\{i\frac{(n\Delta)^4}{8(n\Delta-m\alpha-\zeta)}+i\frac{m^2\alpha^2}{2}(n\Delta-m\alpha-\zeta)-i\frac{m\alpha}{2}(n\Delta)^2\right\}\right] \qquad (27)$$

and

$$K_{n,m}=-\frac{e^{-i\frac{\pi}{4}}}{4}\sqrt{2/\pi}\,(n-m)^{1/2}\exp\left\{-i\frac{(n-m)^3\Delta^3}{24}\right\}. \qquad (28)$$

As shown in (26), the current at the nth point ($\xi=n\Delta-m\alpha$) can be obtained by simply substituting the previously known currents between the 0th to the $(n-1)$th points. This is the reason why, for high-frequency scattering, the asymptotic expression of the integral equation governing the surface is simpler to solve numerically than the exact (Maue's) equation.

In figures 2 and 3, numerical solutions of the modified Fock function are compared with the regular Fock function.

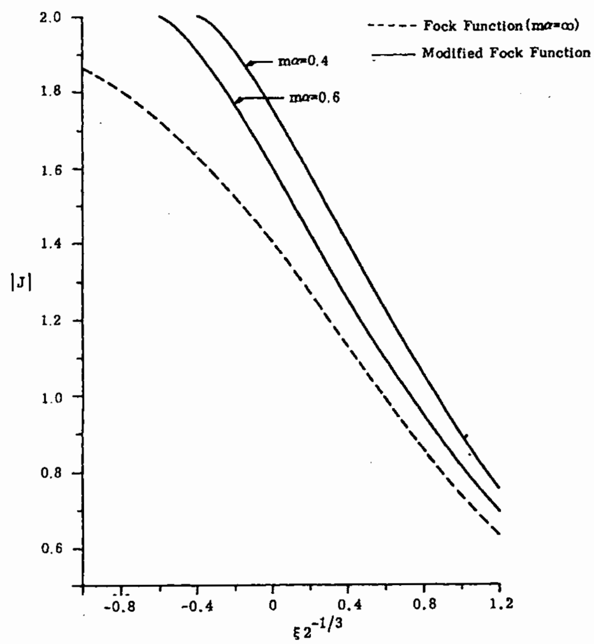

FIGURE 2. *Amplitude of* J *in the penumbra region.*

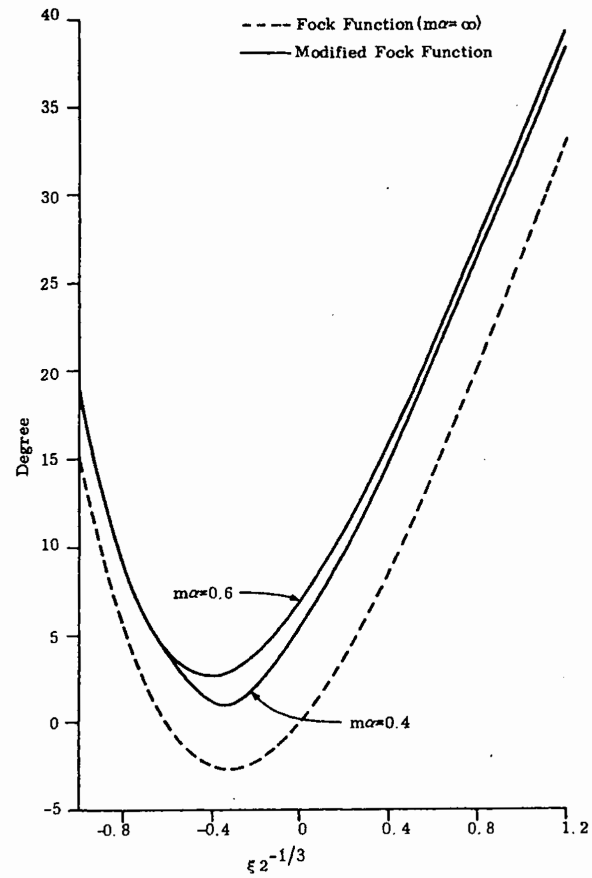

FIGURE 3. *Phase of* J *in the penumbra region.*

5. Application: Estimation of the Backscattering Cross Section of a Cone-Sphere

It is known that a good approximation to the nose-on backscattering cross section of a cone-sphere can be had by adding the contribution of a sphere creeping wave to the physical optics estimate of the scattering from the cone-sphere join [Senior, 1965]. Nevertheless, Senior reported that there is evidence of a small but systematic discrepancy between the amplitudes of the creeping-wave contributions from a cone-sphere and a sphere alone. This phenomenon can be explained by results obtained in the previous sections. When a plane wave is incident upon a cone-sphere at nose-on direction, the cone-sphere join lies near the shadow boundary. We have to take into account that the geometry of the scattering surface near the shadow boundary is no longer entirely spherical, but that a portion of it is conical.

Fock has shown that when a high-frequency plane electromagnetic wave is incident upon a three-dimensional conducting body, the dominant mode of the creeping waves may be described by the two-dimensional solution [Goodrich, 1959]. Therefore, we may apply the results of the previous sections in analyzing the diffraction problems of a cone-sphere. At nose-on incidence, the distance between the cone-sphere join and the shadow boundary is very small in many practical cases, and the result given by (23) may be applied.

For example, when the cone angle (2α) is 25°, Senior [1965] gives the formula for the back-scattering cross section of a cone-sphere at nose-on incidence as

$$\sigma/\lambda^2 = 0.02190|A(1.916-0.05593kR) + \exp\{i\pi(1.45410-1.16335kR\}|^2. \tag{29}$$

Here A is the ratio between the amplitudes of the two dominant creeping waves; one is supported on a spherical portion of a cone-sphere, and the other on a sphere alone. Senior [1965] has obtained an approximate expression for A on the basis of physical reasoning.

We may obtain A from (24) as

$$A = \left[\left\{ \frac{2}{3} + 2\int_0^{\beta_1} Ai(-x)dx \right\} \left\{ 1 - \beta_1^2 \alpha^2 (kR/2)^{2/3} \frac{e^{i\frac{2\pi}{3}}}{2} \right\} + e^{-i\frac{\pi}{3}}(kR/2)^{2/3}\alpha^2 Ai(-\beta_1) \right] \exp\{i\beta_1 (kR/2)^{1/3}\alpha e^{i\frac{\pi}{3}}\}. \tag{30}$$

Insertion of this expression in (29) gives the backscattering cross section of a cone-sphere with the cone angle $2\alpha = 25°$. The result is shown in figure 4. In order to assist the computation of the backscattering cross section of either a cone-sphere or a wedge-cylinder at nose-on incidence, the numerical value of A as a function of both kR and α is shown in figure 5. This figure provides a reasonably accurate means of estimating the creeping-wave contribution when kR is of order 5 or greater. For smaller kR, a more refined evaluation of A is necessary. This may be achieved by including higher-order terms in the asymptotic representation, or more important, by employing the three-dimensional vectorial form of Maue's equation [1949] which is necessary to account for curvature in the direction transverse to the geodesic.

When a plane wave is not incident along the nose-on direction, the distance between the shadow boundary and the cone-sphere join is not necessarily small. In this case we may use the numerical method described in the previous section to obtain the creeping-wave contribution.

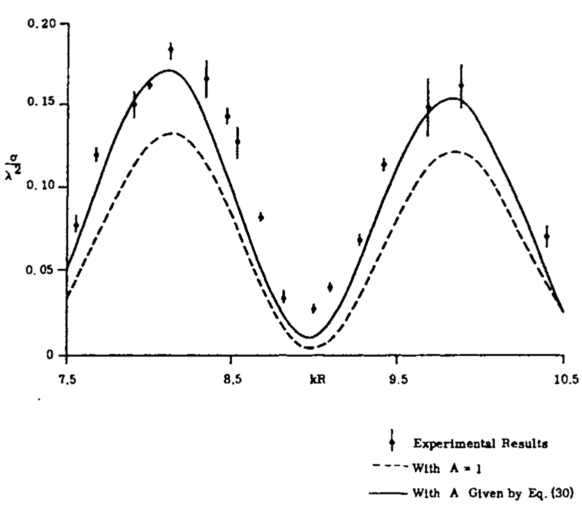

FIGURE 4. *Nose-on Backscattering cross section of a 25° cone-sphere.*

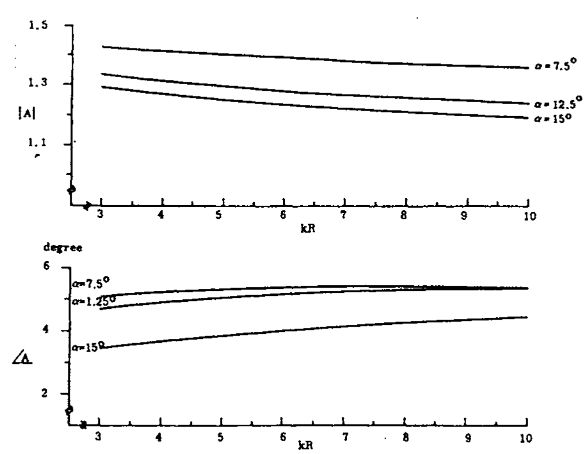

FIGURE 5. *Absolute value and phase of* A *given by (30): 2α = cone angle.*

6. Appendix: Evaluation of $N(p)$

In this appendix the integral

$$N(p)=\int_0^{\infty} d\theta e^{-p(\theta)-i\frac{(\theta-m\alpha)}{6}}\left[1-\frac{e^{-i\pi/4}}{4}\sqrt{2/\pi}\,\theta^2\int_{-\infty}^{0}\frac{d\phi}{(\theta-\phi)^{3/2}}\cdot\exp\left\{i\frac{\theta^4}{8(\theta-\phi)}-i\frac{m\alpha}{2}\theta^2+i\frac{m^2\alpha^2}{2}(\theta-\phi)\right\}\right] \tag{A1}$$

is evaluated when $m\alpha$ is small, so that $(m\alpha)^3$ and higher-order terms are negligible. The integrand of (A1) may be further simplified by making the change of variable $\theta-\phi=x$ and using the relationship

$$\theta^2\int_0^{\infty}\frac{dx}{x^{3/2}}\exp\left\{i\frac{\theta^4}{8x}+i\frac{m^2\alpha^2}{2}x\right\}=2\sqrt{2\pi}\,e^{i\frac{\pi}{4}}e^{i\frac{m\alpha}{2}\theta^2}. \tag{A2}$$

We obtain

$$e^{-i\frac{(\theta-m\alpha)^3}{6}}\left[1-\frac{e^{-i\frac{\pi}{4}}}{4}\sqrt{2/\pi}\,\theta^2\int_{-\infty}^{0}\frac{d\theta}{(\theta-\phi)^{3/2}}\exp\left\{i\frac{\theta^4}{8(\theta-\phi)}-i\frac{m\alpha}{2}\theta^2+i\frac{m^2\alpha^2}{2}(\theta-\phi)\right\}\right]$$

$$=\frac{e^{i\frac{m^3\alpha^3}{6}-i\frac{\pi}{4}}}{2\sqrt{2\pi}}\theta^2\int_0^{\theta}\frac{dx}{x^{3/2}}\exp\left\{i\frac{\theta^4}{8x}-i\frac{\theta^3}{6}+i\frac{m^2\alpha^2}{2}(x-\theta)\right\}. \tag{A3}$$

Neglecting terms involving $(m\alpha)^3$ and higher order in (A3), $N(p)$ may be written as a sum of the two functions:

$$N(p)=\frac{e^{-i\frac{\pi}{4}}}{2\sqrt{2\pi}}N_1(p)+m^2\alpha^2\frac{e^{i\frac{\pi}{4}}}{4\sqrt{2\pi}}N_2(p)+O(m^3\alpha^3), \tag{A4}$$

with

$$N_1(p)=\int_0^{\infty} d\theta e^{-p\theta}\theta^2\int_0^{\theta}\frac{dx}{x^{3/2}}e^{i\frac{\theta^4}{8x}-i\frac{\theta^3}{6}} \tag{A5}$$

and

$$N_2(p)=\int_0^{\infty} d\theta e^{-p\theta}\theta^2\int_0^{\theta}dx\frac{(x-\theta)}{x^{3/2}}e^{i\frac{\theta^4}{8x}-i\frac{\theta^3}{6}}. \tag{A6}$$

Weston [1965] evaluated $N_1(p)$ as [$N_1(p)$ is equal to $4\sqrt{2}\,F(p)$, where $F(p)$ was evaluated in the appendix]

$$N_1(p)=e^{i\frac{\pi}{4}}8\;2^{5/6}\pi^{3/2}Ai(q)\left[\frac{1}{3}+\frac{1}{2}\int_0^{q}\{Ai(x)-iBi(x)\}\,dx\right], \tag{A7}$$

with $q=-ip2^{1/3}$.

The $N_2(p)$ can be evaluated by integration by parts:

$$N_2(p)=\frac{d}{dp}N_1(p)+\frac{i}{4}\frac{d^4}{dp^4}N_1(p)+2\frac{d^2}{dp^2}M(p), \tag{A8}$$

where $M(p)$ is given by (18).

Using (A7) and (A8), we obtain

$$N(p)=\pi 4 2^{1/3} Ai(q)\left[\frac{1}{3}+\frac{1}{2}\int_0^q \{Ai(x)-iBi(x)\}dx\right][1-m^2\alpha^2 a^{-5/3}q^2]$$
$$+\pi 2^{-1/3}m^2\alpha^2 Ai(q)[\{Ai(q)-iBi(q)\}+q\{Ai'(q)-iBi'(q)\}]. \qquad \text{(A9)}$$

The authors wish to thank Dr. T. B. A. Senior for assistance with the manuscript and Messrs. J. Ducmanis and H. Hunter for assistance with the computation.

The work reported in this paper was sponsored by Ballistic Systems Division, USAF, Norton Air Force Base, Calif., under Contract AF 04(694)–834.

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(Paper 1–9–112)