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SIMULATION OF THE MANEUVERABILITY OF INLAND WATERWAY TOWS

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SIMULATION OF THE MANEUVERABILITY OF INLAND WATERWAY TOWS

by

George L. Petrie

A Thesis Submitted in Partial Fulfilment
of the Requirements for the
Professional Degree of NAVAL ARCHITECT

Department of Naval Architecture
and Marine Engineering
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CONTENTS

* APPENDICES bound separately, available upon written request.

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During the early stages of this research, the author was able to ride and observe the operations of an inland waterway barge tow on the Ohio and Cumberland Rivers as the guest of the American Commercial Barge Line. This opportunity for first-hand observation has been invaluable in the formulation of this simulation model. The visit was arranged by James E. Nivin, Vice President, American Commercial Barge Line. His personal interest and support plus that of the American Commercial Barge Line is greatly appreciated.

I. INTRODUCTION

A mathematical model has been formulated to compute the hydrodynamic forces acting on a barge and tugboat flotilla as it maneuvers around a curve of constant radius. The forces and moments considered include the longitudinal and transverse drag and yaw moment due to the flotilla's own motion through the water, as well as the applied forces of the propeller, rudder and bow thruster. These forces are then used to predict the motion of the flotilla, in surge, sway and yaw, under the prevailing conditions of rudder angle and propeller revolutions per minute. As the flotilla progresses through the curve, its proximity to the channel boundaries is computed, and the rudder angle and propeller speed are adjusted to keep the flotilla within the confines of the channel. The mathematical model thus makes possible a prediction of the time required for a flotilla to negotiate a series of one or more maneuvers through a restricted channel.

The analytical aspects of the mathematical model have been incorporated into a FORTRAN program to permit evaluation of the relationships by means of a high speed digital computer. The computer program has been structured to facilitate engineering evaluation and comparison of a variety of flotilla configurations, in a wide range of channel widths, bend radii and current speeds. The data input requirements have been kept as simple as possible, consistent with the goal of accomodating a broad range of conditions. A default option can be selected to assign values to certain input variables, and on multi-case runs, only those data values which differ from the preceding case must be re-entered. Three solution options are available, to suit differing user requirements, and the solution is printed out in both detailed and summary form along with the input data values, to facilitate review and analysis of the results. The FORTRAN programming is arranged in a rational, modular fashion, with each discrete portion of the solution handled by a separate subroutine.

In formulating this computer simulation, the objective was primarily to develop the overall structure of the solution. The degree to which the mathematical representation actually approximated the physical process was of a lesser priority. Thus, certain aspects of the mathematical modelling are acknowledged to be somewhat crude at this stage of development. However, the modular structure of the solution permits refinements to be easily incorporated into the model. Subroutines to supplement or replace the existing program modules can be developed and integrated, with a minimum amount of re-programming, into the present model.

II. BACKGROUND

One of the more troublesome aspects of operating a flotilla of river barges and a tugboat is the difficulty of maneuvering the long, wide flotilla around small radius bends in a narrow channel. This situation can be aggravated by adverse currents and, for a light-condition tow, strong winds. The traditional approach to this problem has been to reduce speed while maneuvering, thereby increasing the time required to execute a maneuver, and incurring an added penalty while decelerating before and accelerating after the maneuver.

With the availability of propulsive devices with improved efficiency, the use of towboats with increased shaft horse-power, and the introduction of auxiliary maneuvering devices such as bow thruster units integrated into the front of the tow, operators are now finding it possible to maintain higher forward speeds, and still keep the barge safely within the confines of the channel. This higher average speed results in shorter transit times for a given route, and a corresponding increase in productivity for each unit of labor and equipment. Clearly, this increased productivity can be expected to result in increased revenue to the barge operator. However, to realize this greater level of productivity, the owner/operator must first make an added capital investment to improve the maneuvering capability of his equipment.

How much should an owner invest to improve the maneuverability of his equipment? Certainly the expected yield from an incremental investment in new technology must be as great as the rate of return realized on equipment operated at the present level of technology; for if it is not, the owner is better advised to continue his operations at the present level of technology. The difficulty in implementing this decision philosophy arises from the inability to predict quantitatively the improvement in productivity that results from a specific technological change. The improvement in average speed along a route (and therefore increased productivity and revenue) must be balanced against the increased cost required to attain that speed to determine whether or not the additional investment is warranted.

The cost associated with introducing a particular technological change can usually be estimated with reasonable accuracy, and scale model tests are often undertaken to ensure that the change will result in a performance improvement. However, there is no presently available model that can be used to predict the net improvement to be realized by a specific change to a particular barge and tugboat operating over a specified route. Yet this is precisely the information needed to assess the economic impact of a technological improvement. Some managers[1]* within the barge industry have expressed interest in the development of a means by which an investigator can quantify the improvement that can be realized by implementing a technological change on a hardware system operating on a specified trade route. With this capability, an owner/operator would have a better basis for decisions involving development or acquisition of new equipment, and may also be able to improve the utilization of equipment already on hand. Furthermore, the same sort of analysis could be performed on a parametric basis, to identify general trends between certain hardware combinations and their level of performance in different types of maneuvers of varying severity. Such parametric analysis may give an investigator new insights into areas where greater performance improvement can be achieved.

*Superscripts refer to references at the end of the report.

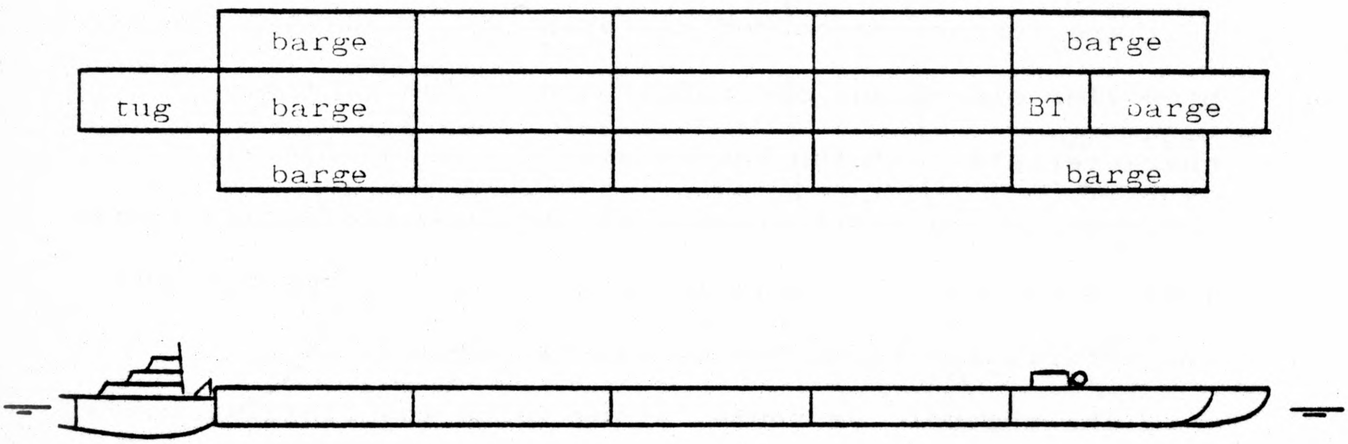
III. DISCUSSION

PROBLEM STATEMENT

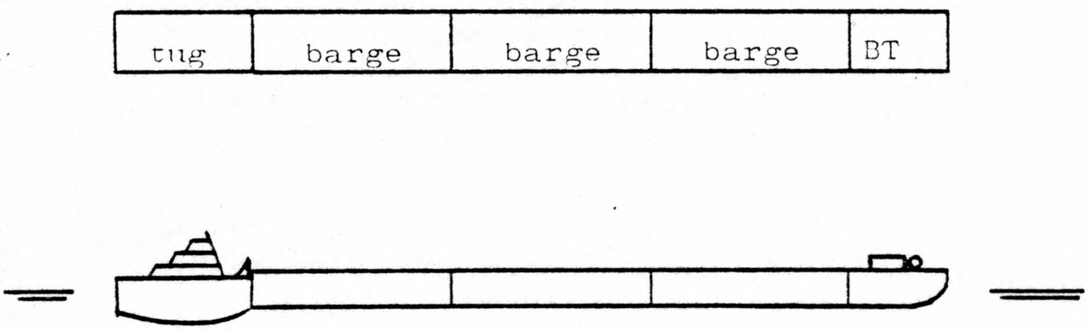
In response to the needs identified in the preceeding section, an investigation has been undertaken to formulate a general solution to the problem: What is the time required for a barge, tugboat and bowthruster combination to traverse the distance between two points along an inland waterway route?

In treating this problem a primary objective was to keep the solution applicable to as wide a range of configurations as possible. The motivation was to develop a method that would give reasonable treatment to a broad spectrum of cases, rather than rigorous treatment to a single restricted case. However, the final solution has been structured to permit subsequent investigators to readily adapt any portion of the model to treat the peculiarities of a particular application, while retaining the basic structure of the original solution.

It is assumed that the tugboat, barges and bowthruster are of rather conventional configuration; that is, with the tugboat fixed to the stern of a rectangular array of barges, and the (optional) bowthruster either integrated into the flotilla or fixed to the leading edge. Figure 1 illustrates two typical arrangements. The tugboat and barge



Top and side view of
15 barge flotilla,
with bowthruster (BT).



Top and side view of
3 barge flotilla,
with bowthruster (BT).

Figure 1. Typical barge, tugboat and bowthruster arrangements.

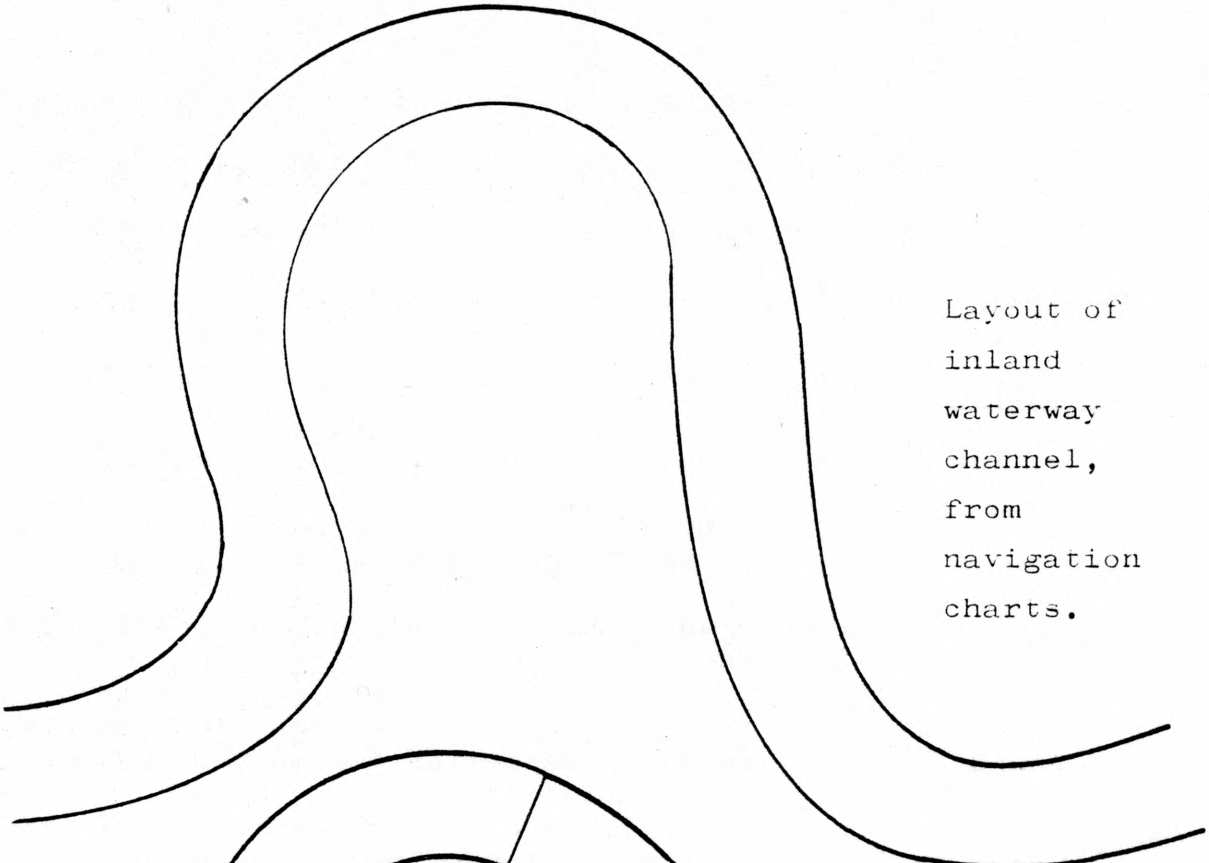
array are each described in terms of their gross characteristics; principal dimensions, block coefficient and mass distribution parameters. The characteristics of the tugboat are further described in terms of its shaft horsepower, propeller dimensions and rudder area. The physical characteristics of the bowthruster are assumed to be included in the gross properties of the barge array, therefore it is described only in terms of its net performance characteristics (i.e. net thrust vs. speed).

The channel, or route, along which the flotilla moves during the solution, is described by a piecewise-continuous series of segments. The solution recognizes three segment types, and describes each in terms of its appropriate characteristics. The three segment types are:

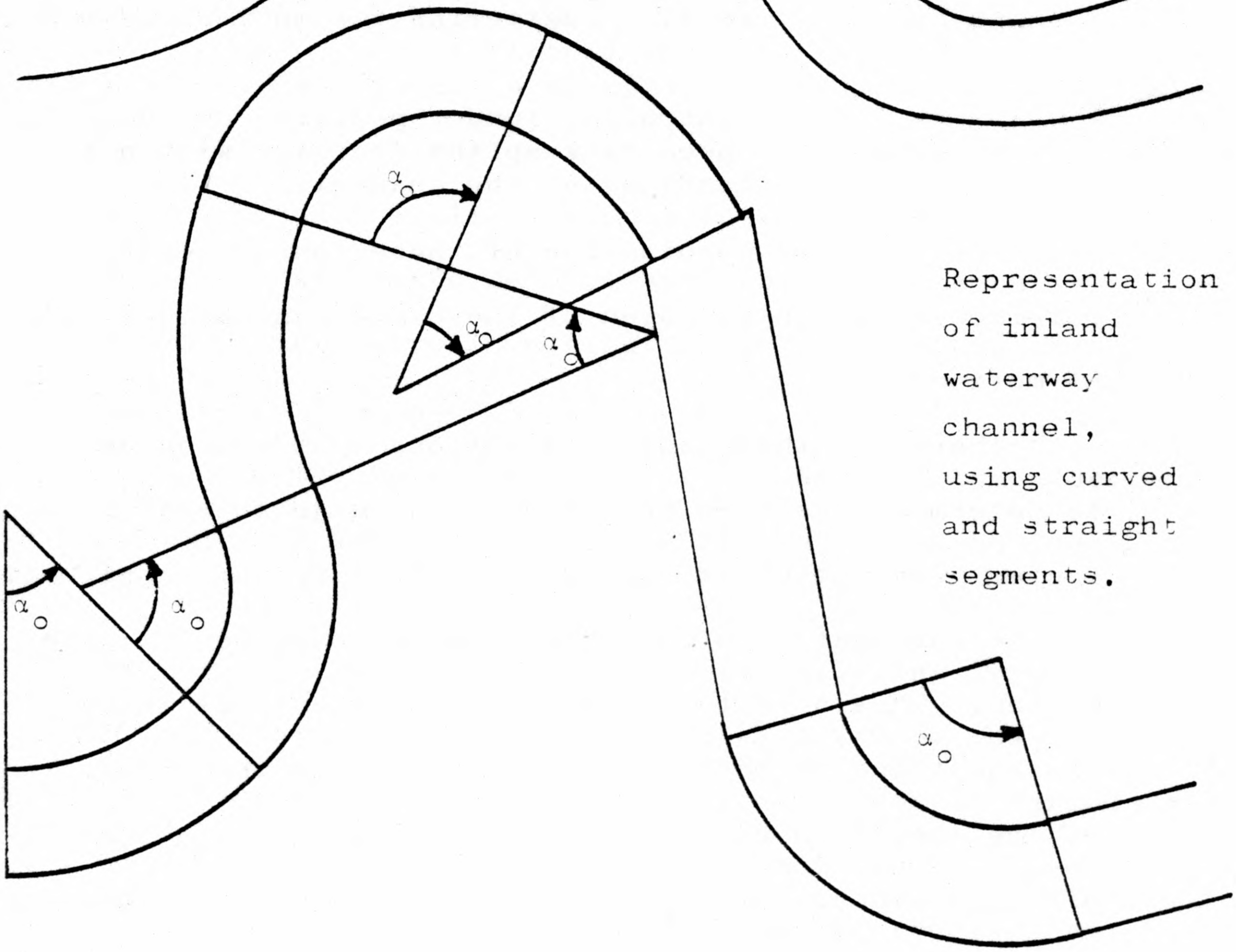
- curved; constant radius bend with uniform width and some specified angular extent,
- straight; section has negligible curvature and uniform width throughout some specified distance,
- delay; represents a fixed time delay such as encountered when passing through locks.

These segments may be combined serially in any order to approximate the characteristics of an inland waterway channel. Figure 2 shows one possible representation of a typical section along a barge route.

For any prescribed combination of flotilla and route, the model predicts the time required to move through each



Layout of inland waterway channel, from navigation charts.



Representation of inland waterway channel, using curved and straight segments.

Figure 2. Typical section of inland waterway and its equivalent sectional representation.

segment of the route, and to travel along the entire route, using a specified rudder and propeller control criteria. The various solution options available are discussed in the following section.

SOLUTION OPTIONS

The fundamental elements of the mathematical model are:

- computation of hydrodynamic forces acting on the barge flotilla,
- prediction of accelerations resulting from those forces,
- integration of those accelerations over time, to determine the motion and position of the flotilla,
- control of rudder position and propeller speed to keep the flotilla within the confines of the channel.

The result of the combination of these four elements is referred to as integration of the barge maneuvering equations of motion.

There are three solution options that can be selected to determine the time required for a barge to traverse a route. One option is simply a straight forward application of the integration of the barge maneuvering equations of motion. This direct integration option entails integrating the equations of motion continuously along the entire length of the specified route to determine the complete time history of the journey. At the start of each segment of the route, initial conditions compatible with the end of the preceeding segment are assumed, and the propeller speed is allowed to increase to maximum. These initial conditions will lead to a minimum elapsed time for each segment. If, during the solution, the barge violates the boundary constraints of a channel segment, the solution "back-tracks" and attempts to complete the maneuver at reduced initial forward speed and / or different propeller speed.

The direct integration option is valuable in that it provides a detailed time history of the barge motion throughout each maneuver. However, the execution time required (see Section VII) for the integration can be substantial, particularly if an iterative process is required to negotiate the more restricted channel segments. Furthermore, the results obtained are applicable only to the route over which the integration was performed. Therefore, this option is best suited to intensive investigation of performance through relatively short distances.

To improve the versatility and efficiency of the model, two other options are offered. They can be exercised independently or serially, and make it feasible to predict the time required to traverse a route of virtually any length. The first of these options involves integration of the equations of motion, but along a family of curves of various radii, width and angular extent, rather than along a specific route. This parametric integration option constructs a three dimensional array of elapsed time versus curve radius, channel width and angular extent. As in the direct integration option, initial forward speed and maximum propeller speed for each segment are controlled to determine the minimum elapsed time required for each segment. This parametric integration option facilitates the assessment of the maneuverability of a flotilla in curves of varying degrees of severity.

The final solution option is the interpolation option, and it is this feature that allows the mathematical model to be applied economically to routes with large numbers of curved segments. The interpolation option utilizes an array of elapsed time versus curve characteristics (radius, channel width, angular extent), such as produced by the parametric integration option. This array is unique to each barge flotilla; it may be generated internally by the parametric integration option, or it may be constructed externally with data from any source, then entered as an input. Then, for any specified route, the elapsed time for each segment of the route is determined by interpolation within the array, based on the characteristics of that segment. Having once determined the elapsed time versus curve characteristics array for a flotilla, the time required for that flotilla to traverse many different routes, with large numbers of segments, may be estimated with a reasonably small execution time (see Section VII). The combination of parametric integration and interpolation give the model great versatility and efficiency in investigating large numbers of alternative systems in different operating conditions.

RESULTS AND CONCLUSIONS

The mathematical model has been evaluated for a number of test cases. The numerical results indicate that the model predictions are consistent with the observed behavior of river tows. Inspection of the solution at intermediate stages of the integration of the equations of motion indicates that the rudder angle commands generated by the model are appropriate to steer the flotilla successfully along the channel. The algorithm for computing the propeller speed commands and varying the initial conditions to allow the flotilla to negotiate a small radius curve is likewise observed to function in the manner intended.

The parametric integration option has been exercised and the results obtained demonstrate the potential utility of this feature. The elapsed time for a flotilla to traverse a given route, as predicted by the interpolation option using the results of the parametric integration, agrees substantially with the results of direct integration of the equations of motion along the same route. However, having once performed the parametric integration for a particular flotilla, its transit time along any route can be estimated by interpolation at a cost of less than 2% of that required to integrate the equations of motion along the route.

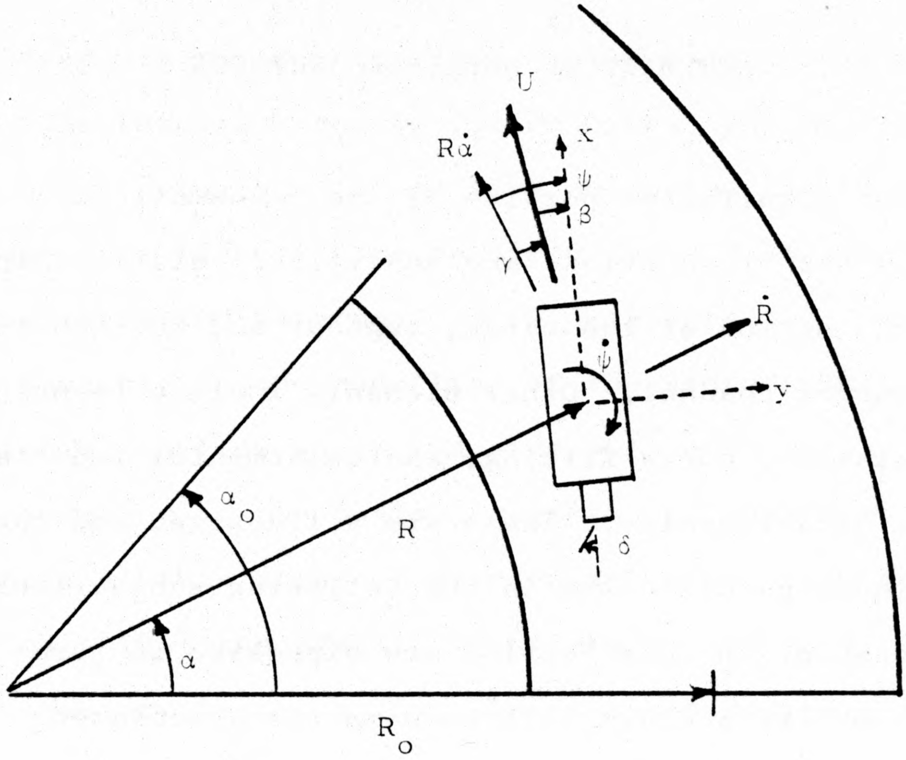
The results obtained indicate that the methods used in the maneuvering simulation have the potential to realistically predict the transit time required for a flotilla to proceed along an inland waterway. More development is required, particularly in the modelling of the hydrodynamic forces including shallow water and side wall effects, to improve the accuracy of the motion predictions. However, the interim results at this stage of development are encouraging and indicate that the overall structure of the model is satisfactory to accomplish the final objective.

IV. MATHEMATICAL MODEL DESCRIPTION

The most significant portion of the mathematical model is the set of equations and analytical relationships governing the motion of the barge, tugboat and bowthruster through a curved channel. Other elements include techniques for interpolation, curve fitting, and determining important geometrical relationships. These are all derived and their interrelationships discussed in the following subsections. All relationships in this Section are expressed in terms of algebraic symbols; a cross reference to the associated FORTRAN variables used in the computer program is included in APPENDIX H.

COORDINATE AXIS SYSTEM

The motion of a barge flotilla passing through a curved channel is simulated in three degrees of freedom in the horizontal plane. The systems of coordinate axes describing this motion are shown in Figure 3. The boundaries of the channel are fixed in a global reference system which has its origin at the center of curvature of the constant width segment. The position of the center of gravity of the flotilla within this global reference system is given in terms of its radial distance from the origin, R, and its angular progress, α, around the curve. The orientation of the barge flotilla is given by its yaw angle, ψ, relative



R_o = Radius of curve, to center of channel

α_o = Angular extent of curve, ↶+

R = Radius to center of gravity of flotilla

α = Angular position of center of gravity of flotilla

$\dot{R}$ = Radial velocity of flotilla

$R\dot{\alpha}$ = Tangential velocity of flotilla

U = Magnitude of flotilla velocity vector

ψ = Yaw angle, relative to tangential vector, ↷+ from tangent to heading

β = Orientation of heading, relative to velocity vector, ↷+ from velocity to heading

γ = Orientation of velocity vector, relative to tangent, ↷+ from tangent to velocity

δ = Rudder angle, ↷+

Note: In determing the hydrodynamic forces when a current is present, the flotilla velocity vector represents the velocity relative to the prevailing current.

Figure 3. Coordinate axis system.

to a vector in the tangential direction through the center of gravity.

The hydrodynamic forces acting on the barge are most conveniently determined in terms of a local coordinate system which moves with the flotilla. The origin of this local system is at the center of gravity of the flotilla. The longitudinal and transverse axes move and rotate with the barge. A detailed view of the local coordinate system is given in Figure 4, which shows the characteristic dimensions of the barge and tugboat.

The position of the center of gravity of the flotilla is computed from the individual characteristics of the barge array and the tugboat. The displacement of the barges and tugboat are given by,

$$\Delta_t = \rho g \; C_{bt} \; L_t \; B_t \; H_t \tag{1a}$$

and

$$\Delta_b = \rho g \; C_{bb} \; L_b \; B_b \; H_b \tag{1b}$$

where

ρ = fluid density, lb-sec^2/ft^4

g = acceleration of gravity, ft/sec^2

C_b = block coefficient, ∇/LBH

L = length overall, ft

B = beam, ft

H = draft, ft

and subscript 't' refers to the barge (or tow), subscript 'b' refers to the tugboat (or boat).

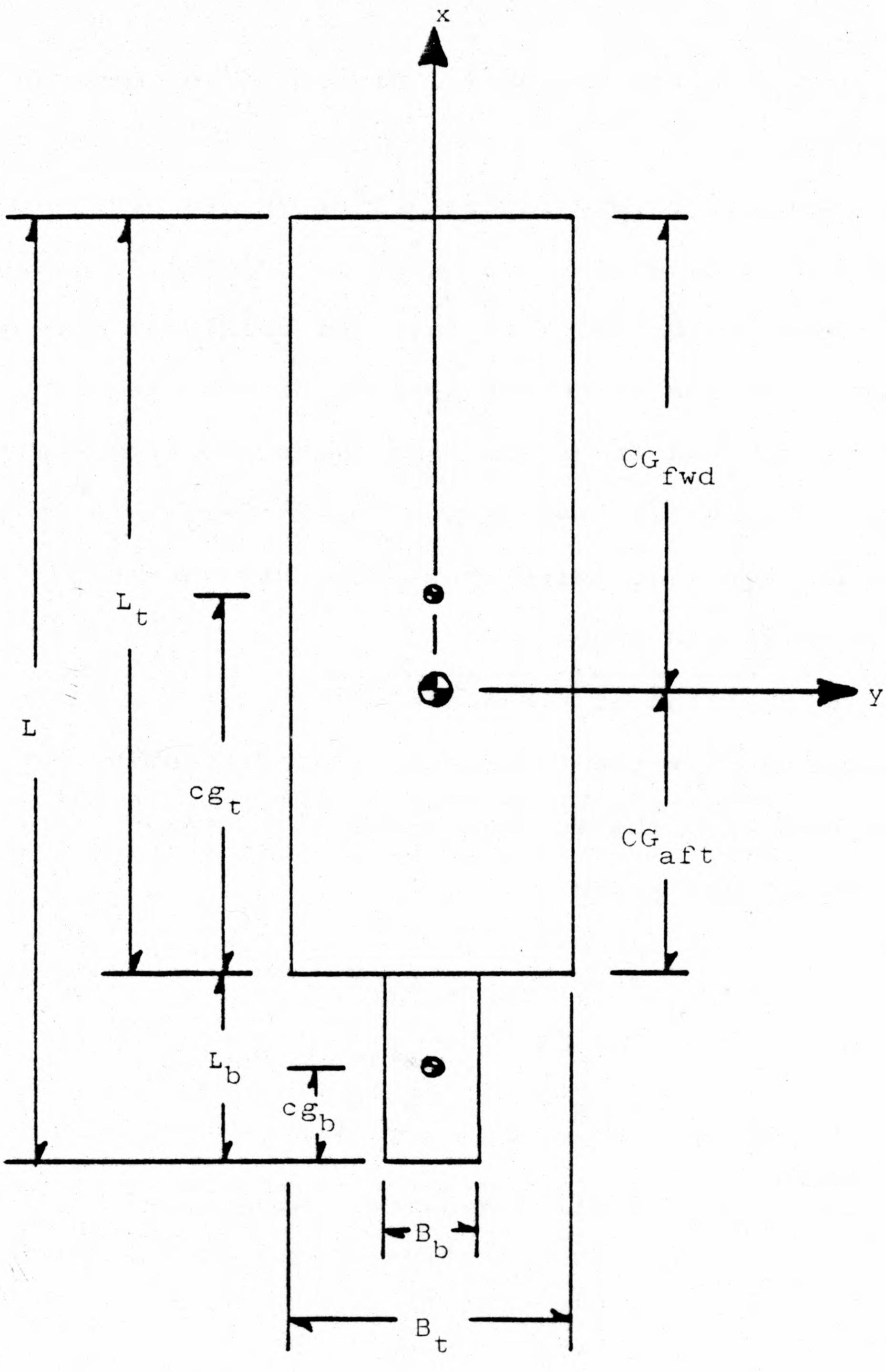


Figure 4. Local coordinate system and flotilla characteristics.

Then, taking moments about the stern of the barge array, the position of the center of gravity is given by,

$$CG_{aft} = \frac{(\Delta_t \; cg_t - \Delta_b \; (L_b - cg_b))}{(\Delta_t + \Delta_b)} \qquad (2)$$

where

cg_t = the distance from the stern of the barge array forward to the center of gravity of the array,

cg_b = the distance from the stern of the tugboat forward to the center of gravity of the boat,

CG_{aft} = the distance from the net flotilla center of gravity aft to the stern of the barge array.

Then,

$$CG_{fwd} = L_t - CG_{aft} \qquad (3)$$

where

CG_{fwd} = the distance from the net flotilla center of gravity forward to the bow of the barge array.

In a similar manner, the composite moment of inertia about the flotilla's center of gravity can be expressed, in terms of the inertia characteristics of the barge array and tugboat individually, as,

$$I_z = \frac{\Delta_b}{g} \; K_b^2 + (CG_{aft} + L_b - cg_b)^2 + \frac{\Delta_t}{g} \; K_t^2 + (cg_t - CG_{aft})^2 \qquad (4)$$

where K_b = the gyradius of the tugboat, ft,

K_t = the gyradius of the barge array, ft.

EQUATIONS OF MOTION

Referring to the global coordinate system on Figure 3, the kinetic energy of the barge flotilla may be written,

$$T = \tfrac{1}{2} I_z (\dot{\alpha} - \dot{\psi})^2 + \tfrac{1}{2} m \left\{ (R\,\dot{\alpha})^2 + (\dot{R})^2 \right\}. \qquad (5)$$

From Lagrange's Equations for a conservative system,[2]

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} = Q_i \qquad (6)$$

where

$i = \alpha,\ \psi, R$

and Q_i is a generalized force in the $i^{\underline{th}}$ direction. Then the equations governing the motion of the flotilla may be derived as follows:

for $i = \alpha$;

$$\frac{\partial T}{\partial \dot{\alpha}} = I_z(\dot{\alpha} - \dot{\psi}) + mR^2\dot{\alpha} \quad ; \quad \frac{\partial T}{\partial \alpha} = 0$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\alpha}}\right) = I_z(\ddot{\alpha} - \ddot{\psi}) + mR^2\ddot{\alpha} + 2mR\dot{R}\dot{\alpha} = Q_\alpha \qquad (7)$$

for $i = \psi$;

$$\frac{\partial T}{\partial \dot{\psi}} = I_z(-\dot{\alpha} + \dot{\psi}) \quad ; \quad \frac{\partial T}{\partial \psi} = 0$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\psi}}\right) = -I_z(\ddot{\alpha} - \ddot{\psi}) = Q_\psi \qquad (8)$$

for $i = R$;

$$\frac{\partial T}{\partial \dot{R}} = m\dot{R} \quad ; \quad \frac{\partial T}{\partial R} = mR\dot{\alpha}^2$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{R}}\right) - \frac{\partial T}{\partial R} = m\ddot{R} - mR\dot{\alpha}^2 = Q_R \qquad (9)$$

Rearranging equations (7), (8), and (9) gives:

$$\frac{d(\dot{\alpha})}{dt} = \ddot{\alpha} = \frac{Q_\alpha}{mR^2} + \frac{Q_\psi}{mR^2} - \frac{2\dot{R}\dot{\alpha}}{R} \tag{10a}$$

$$\frac{d(\alpha)}{dt} = \dot{\alpha} \tag{10b}$$

$$\frac{d(\dot{\psi})}{dt} = \ddot{\psi} = \frac{Q_\psi}{I_z} + \frac{Q_\alpha}{mR^2} + \frac{Q_\psi}{mR^2} - \frac{2\dot{R}\dot{\alpha}}{R} \tag{10c}$$

$$\frac{d(\psi)}{dt} = \dot{\psi} \tag{10d}$$

$$\frac{d(\dot{R})}{dt} = \ddot{R} = \frac{Q_R}{m} + R\dot{\alpha}^2 \tag{10e}$$

$$\frac{d(R)}{dt} = \dot{R} \tag{10f}$$

where

Q_α is a force acting tangent to the curve, at a distance 'R' from the center of curvature. It thus represents a moment moving the flotilla around the curve.

Q_ψ is a moment about the center of gravity of the flotilla, influencing the heading of the flotilla.

Q_R is a radial force acting through the center of gravity of the flotilla.

These six simultaneous first-order differential equations can be integrated from $\alpha = 0$ to $\alpha = \alpha_o$ to obtain the transit time through a curve. The method by which these equations are solved is discussed in APPENDIX A.

HYDRODYNAMIC FORCES

The three generalized hydrodynamic forces, Q_α, Q_ψ, Q_R, represent the forces and moments resulting from the action of the water against the hull, including its various control surfaces. These forces are the result of the relative motion between the hull and the water, and are thus assumed to be functions of parameters describing that motion. However, since the hydrodynamic forces are to be computed in the local coordinate system, and the generalized forces Q_α, Q_ψ, Q_R are in the global coordinate system, a simple transformation is required.
This is given by,

$$Q_\alpha = R(X \cos\psi - Y \sin\psi) \tag{11a}$$

$$Q_\psi = N \tag{11b}$$

$$Q_R = X \sin\psi + Y \cos\psi \tag{11c}$$

where

X = surge (longitudinal) force
Y = sway (lateral) force
N = yaw moment
R = radius
ψ = heading

The generalized forces can be expressed in terms of the conventional hydrodynamic effects of surge, sway and yaw. A convenient representation for the yaw moment, sway force and surge force respectively, in a manner similar to that used by Eda[3], is;

$$N' = a_1 + a_2\dot{y}' + a_3\dot{\Psi}' + a_4\dot{x}' + a_5\dot{y}'^2\dot{\Psi}' + a_6\dot{y}'\dot{\Psi}'^2 + a_7\dot{y}'^3 + a_8\ddot{x}' + a_9\ddot{\Psi}' + a_{10}\ddot{y}' + \frac{a_{11}(\delta) + a_{12}(J) + a_{13}(B)}{\frac{1}{2}\rho L^3U^2} \tag{12a}$$

$$Y' = b_1 + b_2\dot{y}' + b_3\dot{\Psi}' + b_4\dot{x}' + b_5\dot{y}'^2\dot{\Psi}' + b_6\dot{y}'\dot{\Psi}'^2 + b_7\dot{y}'^3 + b_8\ddot{x}' + b_9\ddot{\Psi}' + b_{10}\ddot{y}' + \frac{b_{11}(\delta) + b_{12}(J) + b_{13}(B)}{\frac{1}{2}\rho L^2U^2} \tag{12b}$$

$$X' = c_1 + c_2\dot{y}' + c_3\dot{\Psi}' + c_4\dot{x}' + c_5\dot{y}'^2\dot{\Psi}' + c_6\dot{y}'\dot{\Psi}'^2 + c_7\dot{y}'^3 + c_8\ddot{x}' + c_9\ddot{\Psi}' + c_{10}\ddot{y}' + \frac{c_{11}(\delta) + c_{12}(J) + c_{13}(B)}{\frac{1}{2}\rho L^2U^2} \tag{12c}$$

where

$\dot{\Psi} = (\dot{\psi} - \dot{\alpha})$, the true yaw rate in the global reference system,

$\ddot{\Psi} = (\ddot{\psi} - \ddot{\alpha})$, the true yaw acceleration in the global reference system,

and primes denote dimensionless forms of a quantity. Table 1 illustrates the conversion of the parameters to dimensionless form, and Table 2 defines each coefficient. Terms not listed in Table 2 are not considered, and their values are taken as zero in this model.

This formulation neglects the effects of side wall forces resulting from the proximity of the barge to the boundaries of the channel. In practice, the separation between the side of the barge and the channel wall may range from a few feet to several barge widths. Often in maneuvering situations, there is considerable variation in separation distance along the length of the barge, with the bow being ver close to one bank, and the stern very close

QUANTITY	SYMBOL	DIMENSIONLESS FORM
yaw moment	N	$N' = N/\frac{1}{2} L^3 U^2$
sway force	Y	$Y' = Y/\frac{1}{2} L^2 U^2$
surge force	X	$X' = X/\frac{1}{2} L^2 U^2$
yaw rate	$\dot{\psi}$	$\dot{\psi}' = \dot{\psi} L/U$
sway velocity	$\dot{y}$	$\dot{y}' = \dot{y}/U$
surge velocity	$\dot{x}$	$\dot{x}' = \dot{x}/U$
yaw acceleration	$\ddot{\psi}$	$\ddot{\psi}' = \ddot{\psi} L^2/U^2$
sway acceleration	$\ddot{y}$	$\ddot{y}' = \ddot{y} L/U^2$
surge acceleration	$\ddot{x}$	$\ddot{x}' = \ddot{x} L/U^2$

Table 1. Dimensionless Forms of Parameters

COEFFICIENT	VALUE	INTERPRETATION
a_1	$N_o' = N_o/\frac{1}{2}\rho L^3U^2$	constant, yaw moment
a_2	$N_{\dot{y}}' = N_{\dot{y}}/\frac{1}{2}\rho L^3U$	yaw moment due to sway
a_3	$N_{\dot{\psi}}' = N_{\dot{\psi}}/\frac{1}{2}\rho L^4U$	yaw damping
a_5	$N_{\dot{y}^2\dot{\psi}}' = N_{\dot{y}^2\dot{\psi}}\ U/\frac{1}{2}\rho L^4$	higher order terms
a_6	$N_{\dot{y}\dot{\psi}^2}' = N_{\dot{y}\dot{\psi}^2}\ U/\frac{1}{2}\rho L^5$	higher order terms
a_7	$N_{\dot{y}^3}' = N_{\dot{y}^3}\ U/\frac{1}{2}\rho L^3$	higher order terms
a_9	$N_{\ddot{\psi}}'-I_z'=(N_{\ddot{\psi}}-I_z)/\frac{1}{2}\rho L^5$	added moment of inertia
$a_{11}(\delta)$	function describing rudder induced yaw moment	
$a_{13}(B)$	function describing bowthruster induced yaw moment	
b_1	$Y_o' = Y_o/\frac{1}{2}\rho L^2U^2$	constant, sway force
b_2	$Y_{\dot{y}}' = Y_{\dot{y}}/\frac{1}{2}\rho L^2U$	sway damping
b_3	$Y_{\dot{\psi}}' = Y_{\dot{\psi}}/\frac{1}{2}\rho L^3U$	sway force due to yaw
b_5	$Y_{\dot{y}^2\dot{\psi}}' = Y_{\dot{y}^2\dot{\psi}}\ U/\frac{1}{2}\rho L^3$	higher order terms
b_6	$Y_{\dot{y}\dot{\psi}^2}' = Y_{\dot{y}\dot{\psi}^2}\ U/\frac{1}{2}\rho L^4$	higher order terms
b_7	$Y_{\dot{y}^3}' = Y_{\dot{y}^3}\ U/\frac{1}{2}\rho L^2$	higher order terms
b_{10}	$Y_{\ddot{y}}'-m'=(Y_{\ddot{y}}-m)/\frac{1}{2}\rho L^3$	added mass in sway
$b_{11}(\delta)$	function describing rudder induced sway force	
$b_{13}(B)$	function describing bowthruster induced sway force	
c_1	$X_o' = R_t/\frac{1}{2}\rho L^2U^2$	smooth water resistance
$c_{11}(\delta)$	function describing rudder induced drag force	
$c_{12}(J)$	function describing propeller thrust	
$c_{13}(B)$	function describing bowthruster thrust	

Table 2. Hydrodynamic Coefficients

to the opposite bank. In other cases, the channel boundary may not be a steep bank, but rather a long irregular slope, which may produce little or no side force on the barge. Because of the wide variety of situations involved, the consideration of sidewall forces is beyond the scope of this work. However, the model has been structured to permit sidewall forces to be incorporated with a minimum of difficulty if a suitable representation is developed in the future.

The values of the coefficients $a_1 - a_{13}$, $b_1 - b_{13}$ and $c_1 - c_{13}$ can be specified in several ways. By default, the model uses a first-order approximation of all terms, as given in equations (17) and (20), based on the gross characteris of the flotilla, neglecting shallow water effects. The two-dimensional damping and added mass properties of the barge and tug are used in this approximation. The sectional characteristics shown in Table 3 can be used in the approximation or other values for these sectional characteristics can be specified. As an alternative to this first-order approximation, values may be specified for the coefficients a_1, a_2, a_3, a_5, a_6, a_7 and a_9 and b_1, b_2, b_3, b_5, b_6, b_7 and b_{10}, based on independent computations or model tests, taking shallow water effects into consideration as appropriate.

CHARACTERISTIC	SYMBOL	VALUE
Sectional damping coefficient for barge array	C_{dt}	1.5
Sectional damping coefficient for tugboat	C_{db}	0.5
Sectional added mass coefficient for barge array	C_{at}	0.45
Sectional added mass coefficient for tugboat	C_{ab}	0.45
Surge added mass coefficient for flotilla	C_{as}	0.0

Table 3. Default Values for Sectional Characteristics

Constant, Yaw Moment and Sway Force

The coefficients a_1 and b_1 represent forces caused by port/starboard asymmetry. This asymmetry may be due to non-uniform draft within the barge flotilla, or propeller torque effects in the case of a single or triple screw tugboat. The default value of both coefficients is zero.

Damping Forces

The damping force on a body can be expressed in terms of a non-dimensional damping coefficient by the function,

$$C_d = \frac{D_f}{\frac{1}{2}\rho AU^2} \qquad \text{or} \qquad D_f = \tfrac{1}{2}\rho\, C_d AU^2 \tag{13}$$

where

C_d = a damping coefficient
D_f = the damping force
ρ = the fluid density
A = the projected area; draft x length, $H \cdot L$
U = speed.

Then the damping force per unit speed is

$$\frac{D_f}{U} = \tfrac{1}{2}\rho\, C_d AU = \tfrac{1}{2}\rho\, C_d UHL \tag{14}$$

and the damping force per unit speed per unit length, or the sectional damping factor, S_d, can be expressed

$$S_d = \tfrac{1}{2}\rho\, C_d UH \tag{15}$$

Referring to Figure 5, the damping force on a section at $x = x_1$ is the product of the damping factor, Sd, and the velocity of the section. The moment contribution of that section is merely the product of the force on the section and the distance of the section from the center of gravity. Then the damping forces and moments due to yaw and sway are

$$\dot{y}N_{\dot{y}} = -\int_s^b \dot{y}S_d x \, dx = -\dot{y} \int_s^b S_d x \, dx \tag{16a}$$

$$\dot{\psi}N_{\dot{\psi}} = -\int_s^b \dot{\psi}S_d x^2 dx = -\dot{\psi} \int_s^b S_d x^2 dx \tag{16b}$$

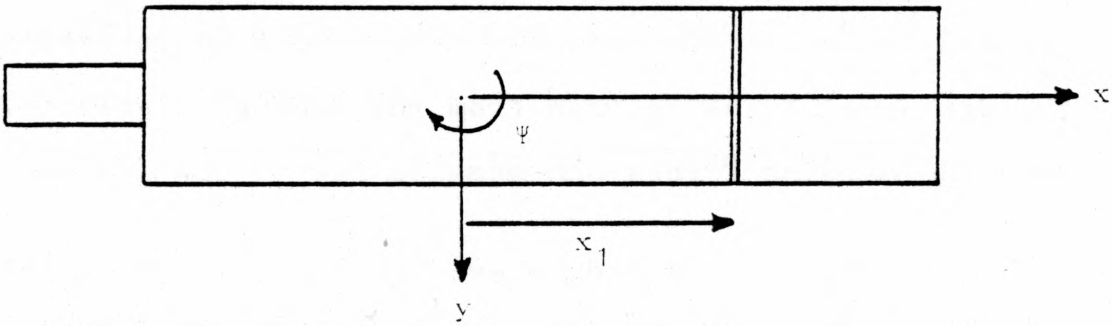
$$\dot{y}Y_{\dot{y}} = -\int_s^b \dot{y}S_d \, dx = -\dot{y} \int_s^b S_d \, dx \tag{16c}$$

$$\dot{\psi}Y_{\dot{\psi}} = -\int_s^b \dot{\psi}S_d x \, dx = -\dot{\psi} \int_s^b S_d x \, dx \tag{16d}$$

where the limits of integration are s=stern to b=bow. After evaluating the integrals over the entire length of the flotilla using proper values of S_d in each interval and substituting equation (15), the coefficients a_2, a_3, b_2, and b_3 can be written,

$$a_2 = \frac{N_{\dot{y}}}{\frac{1}{2}\rho L^3 U} = \frac{C_{db}H_b L_b(CG_{aft}+\frac{1}{2}L_b) - C_{dt}H_t L_t(CG_{fwd}-\frac{1}{2}L_t)}{(L_b + L_t)^3} \tag{17a}$$

$$a_3 = \frac{N_{\dot{\psi}}}{\frac{1}{2}\rho L^4 U} = \frac{-C_{db}H_b\left[(CG_{aft}+L_b)^3 - CG_{aft}^3\right]/3}{(L_b + L_t)^4} + \frac{C_{dt}H_t(CG_{aft}^3+CG_{fwd}^3)/3}{(L_b + L_t)^4} \tag{17b}$$



force on section is

$-S_d V_1$

where

S_d = damping factor

V_1 = transverse velocity

Note: for yaw, $V_1 = \dot{\psi} x_1$

for sway, $V_1 = \dot{y}$

Figure 5. Sectional damping force.

$$b_2 = \frac{Y_{\dot{y}}}{\frac{1}{2}\rho L^2 U} = \frac{-(C_{db}H_bL_b) + (C_{dt}H_tL_t)}{(L_b + L_t)^2} \quad (17c)$$

$$b_3 = \frac{Y_{\dot{\psi}}}{\frac{1}{2}\rho L^3 U} = a_2 \quad (17d)$$

where the symbols are as shown in Figure 4, and
the values of C_{db} and C_{dt} are given in Table 3.

Added Mass and Added Moment of Inertia

The added mass associated with a body accelerating in a fluid is generally expressed as a fraction of the mass of the fluid displaced by the body. That fraction is referred to as an added mass coefficient, C_a, where the total added mass is given by $\frac{1}{2}\rho C_a L BHC_b$. Then the added mass per unit length, or sectional added mass factor, S_a, can be written,

$$S_a = \frac{1}{2}\rho \; C_a \; B \; H \; C_b \quad (18)$$

where

C_a = added mass coefficient
ρ = fluid density
B = beam
H = draft
C_b = block coefficient

Then, in a manner similar to equations (16b) and (16c), the moment due to the added moment of inertia and the force due to the added mass in sway may be written

$$\ddot{\psi}(N_{\ddot{\psi}} - I_z) = -\int_s^b \ddot{\psi}\, S_a x^2 \, dx = -\ddot{\psi} \int_s^b S_a x^2 \, dx \quad (19a)$$

$$\ddot{y}(Y_{\ddot{y}} - m) = -\int_s^b \ddot{y}\, S_a \, dx = -\ddot{y} \int_s^b S_a \, dx \quad (19b)$$

where the limits of integration are s=stern to b=bow. After evaluating the integrals over the entire length of the flotilla using proper values of S_a in each interval and substituting equation (18), the coefficients a_9 and b_{10} may be written,

$$a_9 = \frac{N_{\ddot{\psi}} - I_z}{\frac{1}{2}\rho L^5} = \frac{-(C_{ab}V_b\ (CG_{aft}+L_b)^3 - CG_{aft}{}^3\ /3)}{(L_b + L_t)^5} + \frac{(C_{at}V_t(CG_{aft}{}^3 + CG_{fwd}{}^3)/3)}{(L_b + L_t)^5} \qquad (20a)$$

$$b_{10} = \frac{Y_{\ddot{y}} - m}{\frac{1}{2}\rho L^3} = \frac{-(C_{ab}V_b) + (C_{at}V_t)}{(L_b + L_t)^3} \qquad (20b)$$

where

$V_b = H_bB_bL_bC_{bb}$ = displaced volume of tugboat
$V_t = H_tB_tL_tC_{bt}$ = displaced volume of barge array
C_{bb} = block coefficient of tugboat
C_{bt} = block coefficient of barge array

and other symbols are as in Figure 4 and Table 3.

Higher Order Terms

The coefficients a_5, a_6, a_7, b_5, b_6, b_7 have no clear physical interpretation, and are most likely to arise during the course of fitting a polynomial to the results of a scale model test. Thus, the default values of these coefficients are taken to be zero in the model, but the coefficients are explicitly included to offer compatibility with conventional methods of presenting model test results.

Smooth Water Resistance and Longitudinal Motion Terms

The process that this mathematical model represents is the motion of a barge flotilla in a bounded waterway, and the objective is to determine the time required to execute a series of maneuvers. Small lateral and rotational motions (sway and yaw) are significant due to the constraints imposed by the necessity to remain within the channel. However, small perturbations of the longitudinal motion (surge) are essentially parallel to the centerline of the channel, and thus have negligible impact on the lateral position of the barge within the channel. The primary influence on the maneuvering problem of motion in the longitudinal direction is due to the quasi-steady-state forward speed. Furthermore, the transit time through a maneuver is determined almost entirely by the quasi-steady forward speed; small surge fluctuations tend to average out to no net effect when integrated over a long period of time. Thus, the mathematical model considers only the gross effects of longitudinal motion. Coefficients $c_2 - c_{10}$ are defined to be zero, and the terms corresponding to them are not included in the model. Similarly, due to the assumption of port/starboard symmetry the $\dot{x}$ and $\ddot{x}$ dependent coefficients a_4, a_8, b_4 and b_8 are also defined to be zero, and are therefore not included in the model.

The remaining hydrodynamic effect in the longitudinal direction is the resistance to forward motion of the hull. It is assumed that longitudinal accelerations will be small, and that the flow will be essentially steady at any time. Then the drag force (resistance) can be approximated by a representation of the steady-state smooth water resistance of a barge flotilla. The representation used for the mathematical model is adapted from Baier,[4] where the resistance of a barge train is given by;

$$R_T = \frac{184}{H^{2.49}} \left\{\frac{V_x}{B}\right\}^{1.86} \left\{\frac{T}{K_d}\right\}^{2.86} \qquad (21)$$

where

R_T = resistance in pounds
H = average barge draft, feet
V_x = forward speed, feet/second
B = overall beam of flotilla, feet
T = displacement, short tons
K_d = a constant, based on length.

The constant, K_d, is related to the effective length of the flotilla by the approximation,

$$K_d = \alpha_1 L_e^{\frac{1}{2}} + \alpha_2 L_e^{1.5} + \alpha_3 L_e^{2.5} \qquad (22)$$

where, for $L_e < 400$,

$\alpha_1 = 1.723578$
$\alpha_2 = 5.520842 \times 10^{-3}$
$\alpha_3 = -2.542 \times 10^{-6}$

and, for $L_e > 400$,

$$\alpha_1 = 2.63833$$
$$\alpha_2 = 2.21667 \times 10^{-3}$$
$$\alpha_3 = 0.0$$

and L_e is the effective length of the flotilla, in feet. The effective length of the flotilla is given by,

$$L_e = L_t + L_b \frac{B_b}{B_t} \qquad (23)$$

where

L_t = length of barge array, feet
L_b = length of tugboat, feet
B_t = beam of barge array, feet
B_b = beam of tugboat, feet.

Rudder Force

The rudder force term in equations (12a) (12b) and (12c) and in Table 2 is represented by the coefficients, $a_{11}(\delta)$, $b_{11}(\delta)$, $c_{11}(\delta)$. Unlike the coefficients discussed previously, however, the rudder force coefficients are not constant; they are influenced by the rudder angle, δ, and hence vary with time as does δ. Hence, $a_{11}(\delta)$, $b_{11}(\delta)$, $c_{11}(\delta)$ are represented as functions, rather than as constant coefficients.

To compute its lift and drag forces, the rudder(s) is modelled as a lifting surface inclined at an angle of attack in a flow field. The effect of the flanking rudders, positioned on each side ahead of the propeller(s) is ignored because they are used only when the direction of propeller rotation is reversed. This condition is not considered in the present model. The rudder(s) is assumed to be positioned directly behind the propeller(s), in line with the centerline of the shaft(s). The speed of the flow behind the propeller is the forward speed of the barge, modified by the speed of the propeller race. The net speed behind the propeller can be given by,[5]

$$V_p = \frac{2T_p}{\rho A_p} + V_x \qquad (24)$$

where

T_p = propeller thrust, pounds
ρ = fluid density
A_p = disc area of propeller, $\pi D_p^2/4$,
V_x = forward speed of flotilla, $U \cos\beta$ (see figure 3).

The lateral component of the flotilla's speed, $U \sin\beta$, is ignored, thus V_p is assumed to be directed straight aft. The lift and drag forces due to the rudder are, respectively,

$$b_{11}(\delta) = \tfrac{1}{2}\rho \left\{\varepsilon_1 \delta + \varepsilon_2\delta^2 + \varepsilon_3\delta^3\right\} A_r V_p^2 \qquad (25a)$$

and

$$c_{11}(\delta) = \tfrac{1}{2}\rho \left\{\varepsilon_1 \delta + \varepsilon_2\delta^2 + \varepsilon_3\delta^3\right\} A_r V_p^2 \qquad (25b)$$

and the rudder induced moment is,

$$a_{11}(\delta) = (CG_{aft} + L_b)\, b_{11}(\delta) \qquad (25c)$$

where

δ = rudder angle, radians
A_r = rudder area (chord length x height)
V_p = flow speed, as given in equation (24)
and the quantities in brackets, { }, represent lift and drag coefficients of the rudder.

While the terms $\varepsilon_1 - \varepsilon_6$ are all included in the model, the presently assumed values of ε_2 through ε_6 are all zero. The value assumed for ε_1 is 2.

Propeller Force

The coefficients describing the propeller forces, like those of the rudder forces, are actually functions whose values will change with time. The propeller force is assumed to be a simple thrust longitudinally, therefore $a_{12}(J)$ and $b_{12}(J)$ are identically zero for all time. The longitudinal thrust coefficient, $c_{12}(J)$, is determined from open water propeller characteristics, such as presented by VanLammeran,[6] based on propeller characteristics and the advance coefficient, J. The advance coefficient is defined as,

$$J = \frac{(1 - w)\ V_x}{n\ D_p} \tag{26}$$

where

w = wake fraction
V_x = forward speed, feet/second
n = propeller revolutions per second
D_p = propeller diameter, feet.

The propeller thrust and torque are then given by,

$$T_p = K_t\ \rho n^2\ D_p^{\ 4}\ (1-t) \tag{27a}$$

and

$$Q_p = K_q\ \rho n^2\ D_p^{\ 5} \tag{27b}$$

where

t = thrust deduction factor
and K_t and K_q are thrust and torque coefficients.

The thrust and torque coefficients are represented as higher order polynomial functions of the advance coefficient, the pitch/diameter ratio, and the blade area ratio. The value of $c_{12}(J)$ is given at any time by equation (27a).

Bowthruster Force

The bowthruster unit is a non-standard piece of equipment in river barge usage. The units in use are often one-of-a kind items that have been constructed for evaluation purposes. As such, they can not be assumed to have any particular physical attributes; therefore an attempt to model a bowthruster based on a description of its physical properties would be either severely restrictive in the variety of configurations treated, or prohibitively complex in order to accommodate a multitude of alternative physical arrangements.

Furthermore, since the bowthruster is often built for evaluation purposes, it is assumed that the builder/designer should have data regarding the predicted or measured performance of the unit. Alternatively, in evaluating an as-yet unbuilt bowthruster one might hope to determine either a minimum or optimum level of performance to use as a design goal.

Thus, the most expedient as well as potentially useful means of modelling a bowthruster is to model its net performance characteristics directly. To achieve this, it was assumed that the net thrust (or drag) of the bowthruster is known, as a function of forward speed. It was further assumed that the configuration allowed this thrust to be directed in any direction within $\pm 90°$ of dead aft.

The bowthruster performance is specified by a number of pairs of coordinates describing the shape of the thrust versus forward speed curve. At any stage of the solution, the forward speed of the flotilla is known and the maximum bowthruster thrust at that speed is found by interpolation. Using relationships developed in the next Section (Steering and Control Criteria), the maximum thrust to be directed laterally, T_{BLIM}, is determined. Then, for $T_{BT} < T_{BLIM}$,

$$a_{13}(B) = (CG_{fwd} + d_{BT})\, T_{BT} \frac{T_{BLIM}}{|T_{BLIM}|} \tag{28a}$$

$$b_{13}(B) = T_{BT} \frac{T_{BLIM}}{|T_{BLIM}|} \tag{28b}$$

$$c_{13}(B) = 0.0 \tag{28c}$$

and, for $T_{BT} > T_{BLIM}$,

$$a_{13}(B) = (CG_{fwd} + d_{BT})\, T_{BLIM} \tag{29a}$$

$$b_{13}(B) = T_{BLIM} \tag{29b}$$

$$c_{13}(B) = \sqrt{T_{BT}^2 - T_{BLIM}^2} \tag{29c}$$

where

T_{BT} = maximum bowthruster output at given speed
T_{BLIM} = lateral thrust command
d_{BT} = distance of bowthruster forward of the leading edge of the flotilla.

STEERING AND CONTROL CRITERIA

The objective of the steering and control elements of the mathematical model is to generate rudder position and propeller speed commands that will cause the barge flotilla to execute a given maneuver in a minimum period of time. Three functions must be coordinated to accomplish that objective;

1) determine at all times the position of the flotilla within the channel,

2) based on the present position and orientation, generate an appropriate rudder and bowthruster command,

3) based on previous unsuccessful attempts, restrict propeller speed and/or flotilla forward speed.

In formulating the steering and control elements of the mathematical model, an attempt was made to construct a mathematical analog of the decision criteria which seemed by personal observation to be used by tugboat helmsmen. The parameters used by the model are the same inputs that are available to the helmsman; lateral clearances along the length of the barge, yaw rate, and knowledge of the geometry of the upcoming section of the channel. Similarly, the variables controlled by the model are the same as those controlled by the helmsman; rudder angle, propeller speed, bowthruster angle, and speed upon entering a curve.

The primary difference, however, is that a helmsman (hopefully) knows from experience how fast to enter a curve, and at what speed the propellers should be set. The model has no such preconceived notion; and, because it seeks to minimize elapsed time, it always attempts to negotiate each curve with the maximum propeller speed, and the fastest initial forward speed compatible with the immediately preceeding maneuver. If the maneuver is successful under these conditions, minimum elapsed time will be realized. If not, the maneuver is restarted with different, successively slower, combinations of maximum propeller speed and initial forward speed.

When a maneuver through a curved segment is completed, the motions at the end of the curve become the initial conditions for the next segment. If a curved segment is preceeded by a straight or delay segment, the initial conditions for the curved segment are defined in accordance with arbitrary initial conditions which are input to the model. These initial conditions may be changed, as described above, to permit the flotilla to complete a maneuver. If the initial forward speed at which the flotilla enters a curve is reduced, then a penalty is added to the elapsed time for the preceeding segment, to correct for the increased time required to decelerate to the new initial speed for the curved segment. While decelerating, the only force assumed to act on the flotilla is the drag due to its resistance to forward motion.

The time and distance necessary to decelerate to the reduced initial speed is determined, and compared to the time required to traverse that distance at the (constant) former initial speed. The difference between these two times is the penalty assigned to the segment preceeding the curve.

Position of Flotilla within Channel

The position of the flotilla within the channel is determined by the lateral clearance between the channel boundary and the extremities of the barge and tugboat. Figure 6 illustrates a flotilla midway through a curve and the relationship between the various lateral clearances. Based on these clearances, four parameters are selected to express the position of the barge and tugboat relative to the channel boundaries. These four parameters are:

cx_i = the minimum of (d_1, d_3, d_5, d_7); the the minimum clearance to the inside wall of the channel,

Lx_i = the longitudinal position of the section corresponding to cx_i,

cx_o = the minimum of (d_2, d_4, d_6); the minimum clearance to the outside wall of the channel,

Lx_o = the longitudinal position of the section corresponding to cx_o.

The lateral clearances at the extremities, $d_1 - d_6$ are computed from the law of cosines as shown in Figure 7.

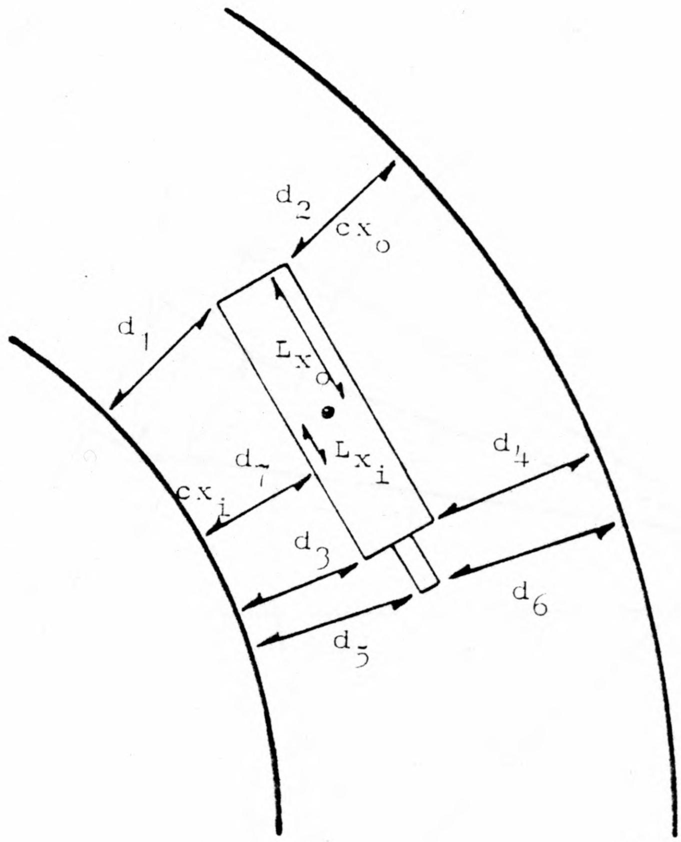


Figure 6. Lateral clearance parameters.

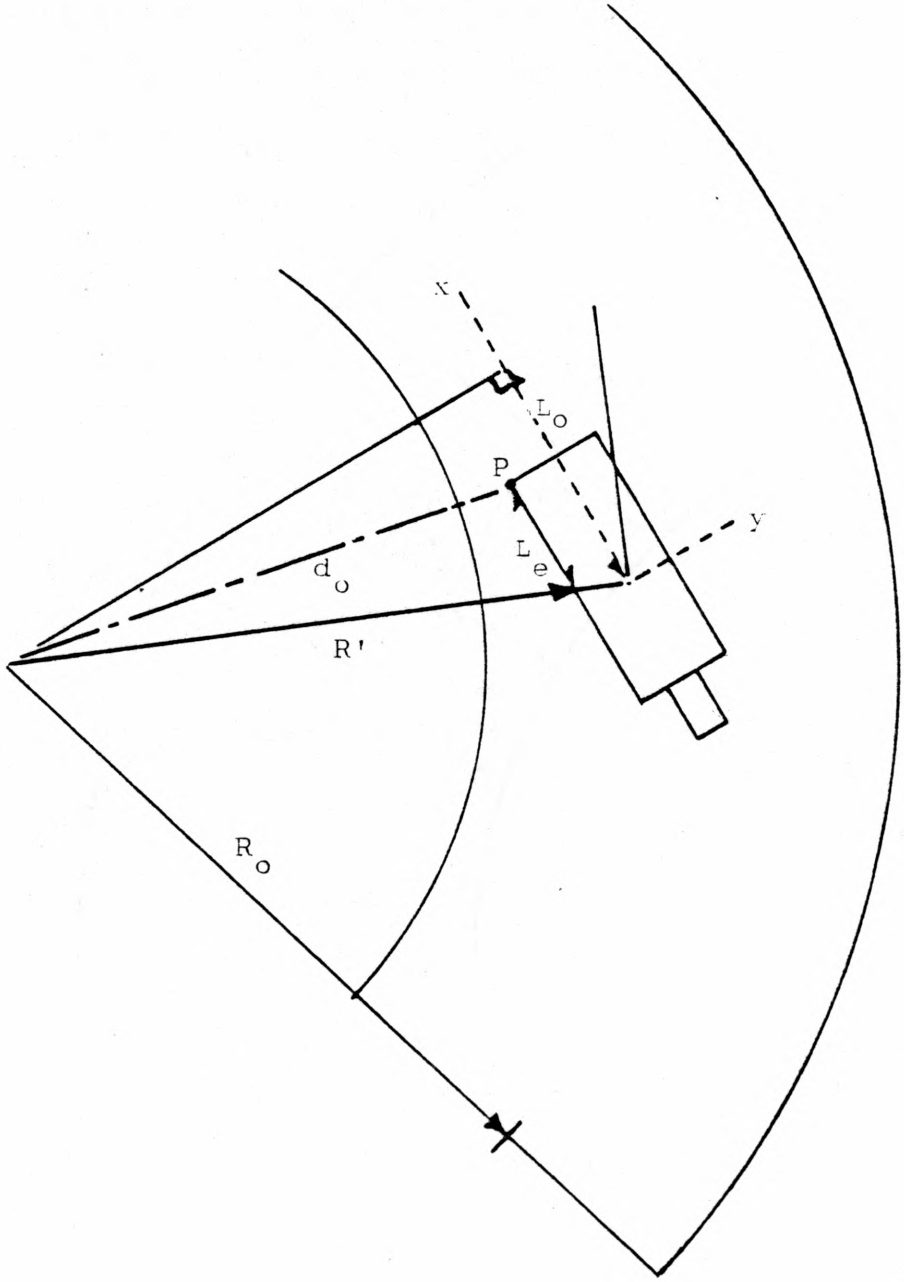


Figure 7. Distance from the origin to the extremities of the flotilla.

For any point, P, in the global coordinate system, a triangle can be constructed with one side along the line from the origin to the point P, the second side along the line from the origin to the center of gravity of the flotilla, and the third side along the line through P parallel to the center line of the flotilla. If P is at the point (a,b) in the local coordinate system, the triangle has sides d_o, R', and L_e as shown in Figure 7. The length of each side is given by,

$$R' = R + \frac{b}{\cos\psi} \qquad (30)$$

$$L_e = a - b \tan\psi \qquad (31)$$

$$d_o = \sqrt{R'^2 + L_e^2 - 2 R' L_e \cos(\pi+\psi)} \qquad (32)$$

where

a = the longitudinal position of P

b = the lateral offset of P from the center-line.

The radial clearance, d_i, of the point (a,b) is then simply,

$$d_i = \pm d_o \mp \left\{R_o \mp \tfrac{1}{2} W_o\right\} = \pm d_o \mp R_o + \tfrac{1}{2}W_o \qquad (33)$$

where

R_o = the radius of the curved segment

W_o = the width of the segment

d_i = d_1 through d_6.

and upper signs give clearance from inside wall, lower signs give clearance from outside wall.

Table 4 summarizes the values used in equations (30) through (33) for determining d_1 through d_6 from Figure 6.

	a	b	sign in equation (33)
d_1	CG_{fwd}	$-\frac{1}{2}B_t$	upper
d_2	CG_{fwd}	$\frac{1}{2}B_t$	lower
d_3	$-CG_{aft}$	$-\frac{1}{2}B_t$	upper
d_4	$-CG_{aft}$	$\frac{1}{2}B_t$	lower
d_5	$-CG_{aft}-L_b$	$-\frac{1}{2}B_b$	upper
d_6	$-CG_{aft}-L_b$	$\frac{1}{2}B_b$	lower

Note: signs for 'b' values are for counter clockwise (+) curve; for clockwise (-) curve, reverse signs for all 'b' values.

Table 4. Values Used in Computing Lateral Clearances

The distance d_7 represents the clearance between the inner wall of the channel and the inner edge of the barge, at the point where the edge of the barge is parallel to a tangent to the channel wall. The longitudinal position of the point of tangency is given by,

$$L_o = R \cos(\pi + \psi) = -R \sin \psi \qquad (34)$$

where

R = the radial position of the center of gravity of the flotilla.

ψ = the yaw angle.

Now if $L_o > CG_{fwd}$ or $L_o < (-CG_{aft} - L_b)$, the point of tangency is beyond the extremities of the flotilla and the minimum clearance will be at one of the extremities. If $CG_{fwd} > L_o > (-CG_{aft} - L_b)$, the point of tangency is along the length of the flotilla. The clearance at this point is then,

$$d_7 = R|\cos\psi| - B - R_o + \tfrac{1}{2}W_o \qquad (35)$$

where

$B = B_b$ for $(-CG_{aft}) > L_o > (-CG_{aft} - L_b)$

or $B = B_t$ for $CG_{fwd} > L_o > (-CG_{aft})$.

For the case shown in Figure 8, where the ends of the flotilla extend into the adjacent segment of the channel, the results of equations (30) - (35) must be modified. For each section of the flotilla corresponding to d_1 - d_7, the overlap of that section is computed by,

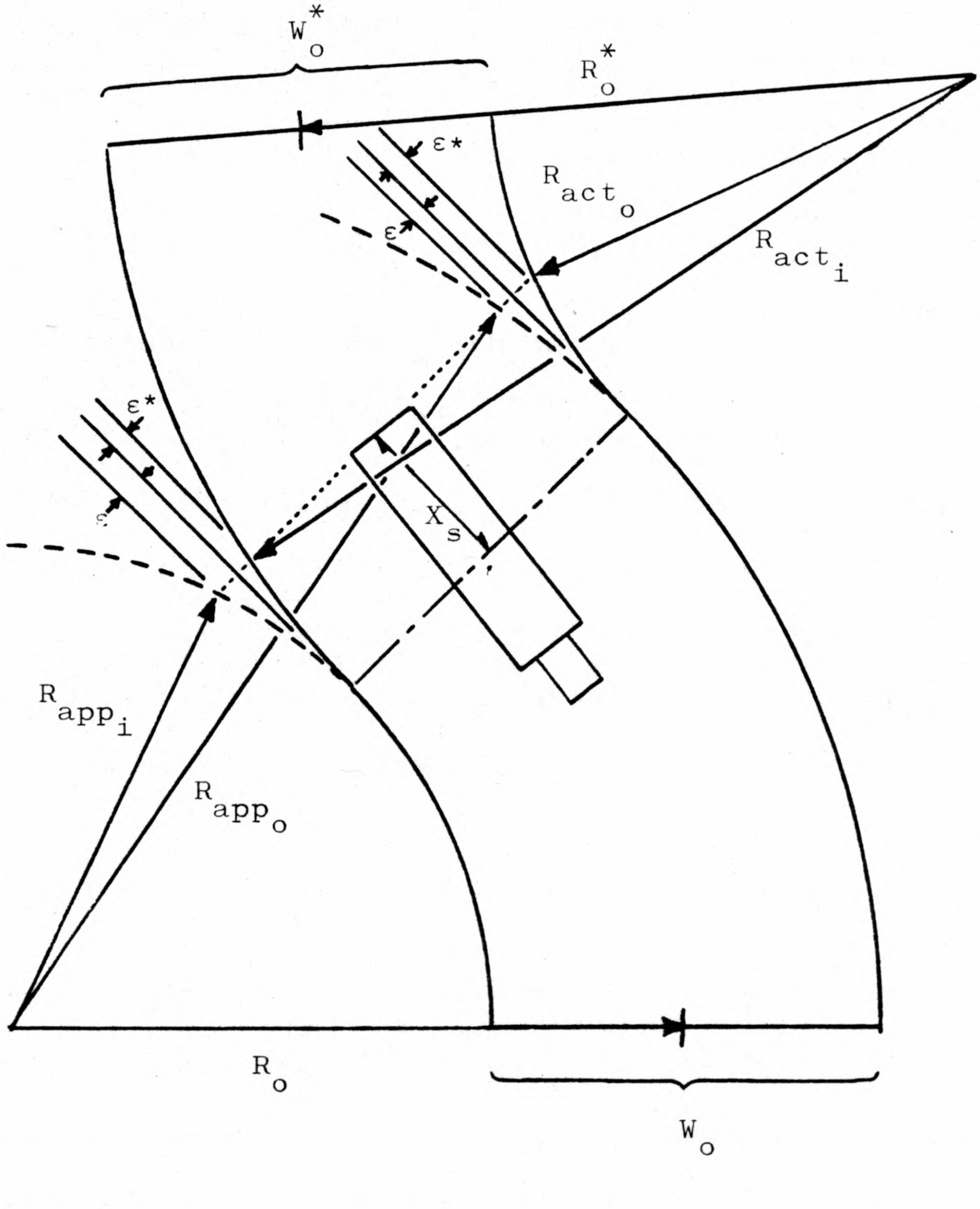


Figure 8. Lateral clearance at the junction of two segments.

$$X_s = (R\,\alpha + a\cos\psi - d_c)\frac{\alpha}{|\alpha|} \qquad (36)$$

where

R = radial position of the center of gravity of the flotilla

α = angular position of the center of gravity of the flotilla

a = as given in Table 4

ψ = the yaw angle

d_c = arc length of the curve (see below).

The arc length of the curve, d_c, refers to the distance from the beginning of the curve to the point where the flotilla begins to overlap the adjoining segment. Thus, if the overlap is into the next segment, d_c is R_oW_o. If the overlap is into the preceeding segment, d_c is 0. If, for any section along the flotilla forward of the center of gravity, the value of $X_s>0$, or aft of the center of gravity, the value of $X_s<0$, then that section does not overlap into another segment. Otherwise, the radial clearance of that section must be adjusted to account for the different radius, channel width or sign of the segment in which the section lies.

Referring to Figure 8, the apparent channel radius, R_{app}, at the leading edge of the barge is given by the radius, R_o, of the segment in which the center of gravity is positioned;

$$R_{app} = R_o \pm \tfrac{1}{2}W_o \qquad (37)$$

where

+ gives the radius of the outer wall of the channel

− gives the radius of the inner wall of the channel.

The actual radius, however, is,

$$R_{act} = R_o^* \pm \tfrac{1}{2}W_o^* \qquad (38)$$

where

R_o^* = the radius of the adjoining segment

W_o^* = the width of the adjoining segment

and the sign ($\pm$) is as in equation (37) if the adjoining segment curves in the same direction; if the curvature is in the opposite direction, + gives R_{act} of the inner wall, and - gives R_{act} of the outer wall of the channel.

The error in the radial clearance which results from applying equations (30) through (35) to the case shown in Fig. 8 is due to the difference between the actual and apparent radii. At a distance X_s away from the junction of the segments, the error due to the different curvature of the channel walls can be calculated from,

$$\varepsilon = R_{app} - \sqrt{R_{app}^2 - X_s^2} \qquad (39a)$$

and

$$\varepsilon^* = R_{act} - \sqrt{R_{act}^2 - X_s^2}\,. \qquad (39b)$$

There is also an error due to any difference in channel widths given by,

$$\Delta W = \frac{W_o^* - W_o}{2} \qquad (40)$$

The net correction to be added to the radial clearance, $d_1 \cdots d_7$, computed from equations (30) through (35) is,

$$\Delta d = \pm (\varepsilon + \varepsilon^*) + \Delta W \quad (41a)$$

when the adjoining curve has curvature in the opposite direction, and

$$\Delta d = \pm (\varepsilon - \varepsilon^*) + \Delta W \quad (41b)$$

when the adjoining curve has curvature in the same direction,

where

+ applies to d_2, d_4 and d_6

- applies to d_1, d_3, d_5 and d_7.

Rudder and Bow Thruster Commands

The rudder and bowthruster commands are based on the difference between the actual lateral clearance, cx_i, and a desired or target lateral clearance, ct_i, between the flotilla and the inner boundary of the channel. The target lateral clearance, as a function of α, describes a path that the flotilla seeks to follow as it progresses through a curve. It varies with α in a manner that will position the flotilla at the end of a curved segment in the proper orientation to begin the next segment. Therefore, the nature of the function defining ct_i depends on the relative characteristics (type, channel width, curve radius and angular extent) of the curved segment and the next segment of the route. The derivation of the functions describing ct_i is given in Appendix B.

Having the target lateral clearance at any time, the rudder command is then given by,

$$\delta = \delta_o + \delta_b + \delta_\psi + \delta_s \tag{42}$$

where

δ = the rudder command

δ_o = command based on clearance to inside wall of channel

δ_b = command based on longitudinal position of point closest to wall

δ_ψ = command based on yaw rate feedback

δ_s = command based on clearance to outside wall of channel.

Figure 9 illustrates the parameters considered in generating the four components of the rudder position command.

The difference between the actual lateral clearance, cx_i, and the target lateral clearance, ct_i, gives rise to the first component of the rudder command through the relationship

$$\delta_o = (cx_i - ct_i) \frac{S\alpha_o}{|\alpha_o|} \tag{43}$$

where

S = a specified command parameter, in radians/foot,

for $cx_i > ct_i$; $S = S_i$, the rate at which the rudder is applied to turn the bow of the barge toward the inside bank,

for $cx_i < c_{ti}$; $S = S_o$, the rate at which the rudder is applied to turn the bow of the barge away from the inside bank.

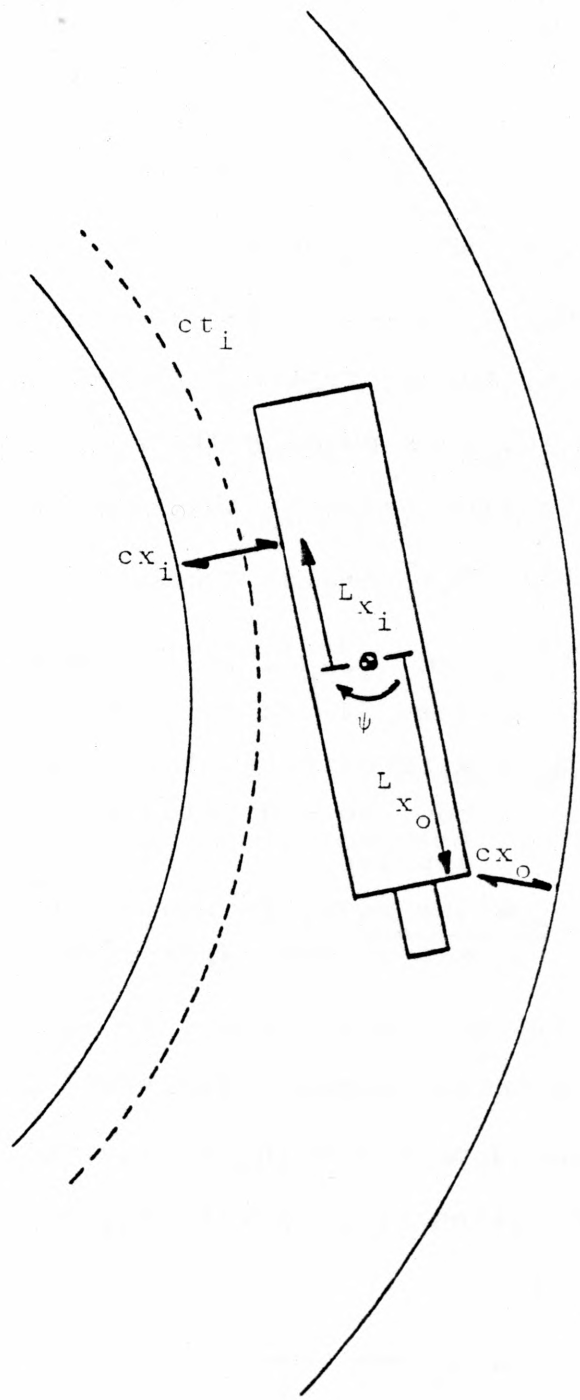


Figure 9. Parameters influencing rudder command.

A check is made to insure the rudder command, is within the maximum rudder angle limits,

$$|\delta_o| < |\delta_{max}| \tag{44}$$

A condition can exist such that the barge is close to the inside bank, so that δ_o indicates a rudder angle to steer the bow away from the bank. Yet if the closest point is near the stern, the appropriate rudder command would be to move the bow in, thus pushing the stern out away from the bank. The correction based on the longitudinal location of the closest point, Lx_i, is, for $Lx_i < (CG_{fwd} - S_b L)$,

$$\delta_b = (CG_{fwd} - S_b L - Lx_i)\ S_c \frac{\alpha_o}{|\alpha_o|} \tag{45}$$

where

S_b = a fraction of the length, aft from the bow, beyond which the correction is applied

S_c = the rate, in radians per foot, at which the rudder correction is applied.

Again, a test similar to equation (44) is applied.

Because the barge response lags the rudder command, a feedback loop based on yaw rate is required. The amount of feedback is controlled by a specified gain constant, R_G, in the following:

$$\delta_\psi = R_G \frac{\dot{\psi}}{\dot{\alpha}} \quad \frac{L_e}{R} \quad \frac{\alpha_o}{|\alpha_o|} \tag{46}$$

A test similar to equation (44) is again applied.

Finally, if the stern of the flotilla moves too near the edge of the outer bank, a correction is applied to move the stern away; for $(cx_o < ct_i)$ and $(Lx_o < 0)$,

$$\delta_s = (cx_o - ct_i) \frac{S_o \alpha_o}{2|\alpha_o|} \quad . \tag{47}$$

As before, a test similar to equation (44) is applied.

The bowthruster command is generated in a manner similar to that by which the rudder commands are developed. The lateral clearance between the leading edge of the flotilla and the inner wall of the channel, d_1 in Figure 6, is compared to the target lateral clearance, ct_i. The difference represents a lateral clearance error. This error generates a lateral thrust command which will orient the bowthruster force in a direction that tends to reduce the lateral clearance error. The lateral thrust command is given by,

$$T_{BLIM} = T_{BMAX} \, T_G \frac{(ct_i - d_1)\, \alpha_o}{ct_i |\alpha_o|} \tag{48}$$

where

T_G = the lateral thrust gain parameter

d_1 = the bow clearance, equations (30) through (33).

Using this command, the lateral and longitudinal components of the bowthruster force are determined from equations (28) and (29).

Propeller Speed Control

On the initial attempt to negotiate a curve, the propeller speed is allowed to increase without restriction. Thus, the shaft horsepower is the only limit on propeller speed. At each step of the solution, the required shaft horsepower, hp, is computed by

$$hp = \pi \; n \; Q_p \; / \; 275 \qquad (49)$$

where

n = revolutions per second

Q_p = propeller torque from equation (27b)

If the required horsepower is less than the available horsepower, shp, then the propeller speed is increased in increments of 5 revolutions per minute until the available shp is exceeded. The rpm is then reduced to the next lower 5 rpm increment.

If the barge speed is excessive for a curve (i.e. the barge hits the channel wall), the solution "steps back" in time and restricts the propeller speed by the relationship

$$rpm_{limit} = rpm - rpm_{max} \;/ 10. \qquad (50)$$

where

rpm_{limit} = maximum rpm, command

rpm = propeller speed when barge hit channel wall

rpm_{max} = maximum rpm possible for system.

The rpm is reduced in this manner each time the solution fails to complete a curve.

However, when

$$rpm'_{limit} < \frac{rpm_{max}}{5} \quad (51)$$

the solution restarts the curve from the beginning, with the initial forward speed reduced by 1.0 foot per second. The iteration of reduced rpm and reduced initial forward speed is continued until the solution is successful (or a finite number of tries is made).

OTHER RELATIONSHIPS

The relationships discussed in the preceeding parts of this Section are fundamental to the formulation of the mathematical model. In addition, there are several relationships that serve to enhance the utility of the mathematical model, particularly in the context of determining numerical results. The more significant of these, discussed in this Section, are the curve fitting and interpolation techniques and a convenient means of checking the input data describing a river channel. Discussion of other relationships is included in the Subroutine Descriptions in APPENDIX F.

Interpolation and Curve Fitting

To implement the third solution option (see Section III), solution by interpolation, a three dimensional interpolation technique using cubic curve fitting was developed. This permits the elapsed time for a given route segment to be interpolated from an array of data for segments of various radii, channel width and angular extent. The same interpolation technique is used to determine the net thrust of the bowthruster, at various speeds, from discrete data points of thrust versus forward speed.

The data used in the interpolation option is structured as an array of elapsed times, T, versus curve radius, R, channel width, W, and angular extent, α_o.

The size of the array is at least 3, but no more than 7, in each dimension. A value of T is assigned to each element in the array, and the array is arranged such that

$$R_1 < R_2 < R_3 \text{----} < R_7 \tag{52}$$

$$W_1 < W_2 < W_3 \text{----} < W_7 \tag{53}$$

$$\alpha_1 < \alpha_2 < \alpha_3 \text{----} < \alpha_7 \tag{54}$$

To determine the value of T corresponding to a particular segment, with characteristics $\overline{R}$, $\overline{W}$, $\overline{\alpha}$, it is first necessary to determine the elements bounding $\overline{R}$, $\overline{W}$, $\overline{\alpha}$, in the array. Starting with i=j=k=1, each parameter of the array is tested to determine the values of i, j, k which satisfy,

$$R_i \leq \overline{R} < R_{i+1} \tag{55}$$

$$W_j \leq \overline{W} < W_{j+1} \tag{56}$$

$$\alpha_k \leq \overline{\alpha} < \alpha_{k+1} \tag{57}$$

The eight elements, (R_i, W_j, α_k).......$(R_{i+1}, W_{j+1}, \alpha_{k+1})$ are the nearest in each direction to $\overline{R}\ \overline{W}\ \overline{\alpha}$. The next closest value for each parameter is then selected and satisfies either,

$$R_{i-1} < R_i < \overline{R} < R_{i+1} \tag{58}$$

or

$$R_i \leq \overline{R} < R_{i+1} \leq R_{i+2} \tag{59}$$

and similarly for $\overline{W}$ and $\overline{\alpha}$.

The low, intermediate and high values of each parameter $(R_1, R_m, R_n;\ W_1, W_m, W_n;\ \alpha_1, \alpha_m, \alpha_n)$ correspond to 27 elements of the array in the vicinity of $\bar{R}$, $\bar{W}$, $\bar{\alpha}$. For each combination of radius and width $(R_1\ W_1;\ R_1\ W_m;\ ---; R_n\ W_n)$ the elapsed time is represented as a function of α. These nine functions are denoted, respectively

$$\text{For } R_1\ W_1; \quad T = F_{11}\ (\alpha_1, \alpha_m, \alpha_n) \tag{60a}$$

$$\text{For } R_1\ W_m; \quad T = F_{1m}\ (\alpha_1, \alpha_m, \alpha_n) \tag{60b}$$

$$\vdots \qquad\qquad \vdots \qquad\qquad\qquad\qquad \vdots$$

$$\text{For } R_n\ W_n; \quad T = F_{nn}\ (\alpha_1, \alpha_m, \alpha_n) \tag{60i}$$

Then each function is evaluated at $\bar{\alpha}$, and for each value of W the values of F $(\bar{\alpha})$ are represented as a function of R,

$$\text{For } W_1; \quad T = G_1\ (R_1, R_m, R_n) \tag{61a}$$

$$\vdots \qquad\qquad \vdots \qquad\qquad\qquad\qquad \vdots$$

$$\text{For } W_n; \quad T = G_n\ (R_1, R_m, R_n) \tag{61c}$$

Then each function is evaluated at $\bar{R}$, and the values of G $(\bar{R})$ are represented as a function of W,

$$T = H\ (W_1, W_m, W_n) \tag{62}$$

Finally, H $(\bar{W})$ is evaluated to determine the value of T corresponding to T $(\bar{R}, \bar{W}, \bar{\alpha})$.

In each case above, equations (60) through (62), the three data points associated with each function are fit to a curve of the form

$$f(x) = ax + bx^2 + cx^3. \tag{63}$$

Three simultaneous equations in three unknowns (the coefficients a, b and c) are evaluated at the three data points. These equations are then solved by evaluating determinants to determine the values of the coefficients a, b and c.

This interpolation and curve fitting scheme is also used to determine the net thrust of the bowthruster at any given forward speed. The bowthruster performance curve, typically as shown in Figure 10, is described by a set of up to seven coordinates. To determine the thrust at speeds other than those corresponding to the given points, three data points are selected as in equations (58) and (59). The thrust, T_{BTi} at each point is transformed by subtracting from the zero speed thrust, T_{BMAX},

$$\overline{T}_{BTi} = T_{BMAX} - T_{BTi} \tag{64a}$$

Equation (63) is then fit through the three transformed points, and the transformed thrust, $\overline{T}_{BT}$, is obtained by evaluating the polynomial at the desired speed. The actual thrust is then found from the reverse transformation,

$$T_{BT} = T_{BMAX} - \overline{T}_{BT} \tag{64b}$$

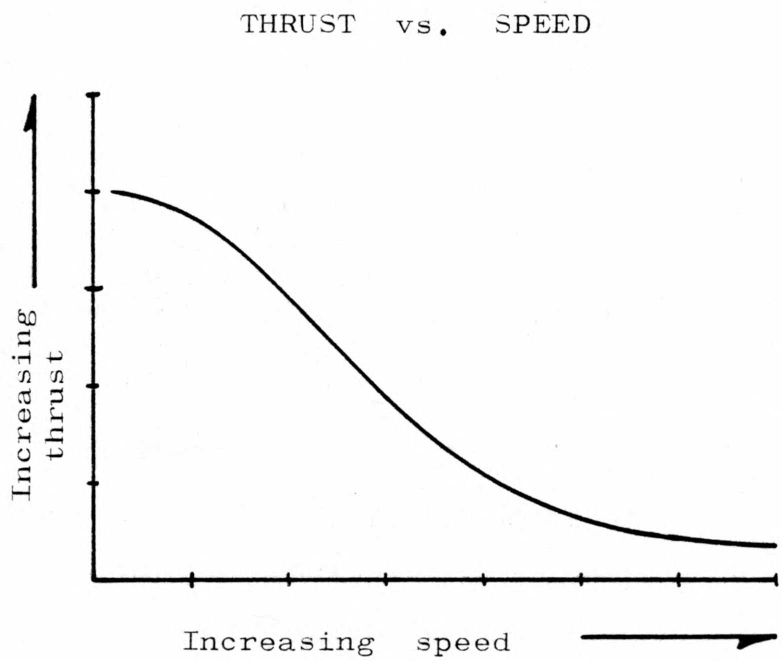


Figure 10. Typical bowthruster performance curve.

Input Data Checking

The route, along which the transit time for a flotilla is to be predicted, is described in terms of a finite number of segments. Each segment represents a distance along the channel, and possibly a change in its direction. For a route with a large number of segments, it may be difficult to check the input data, to verify that the radii and angles of all the segments correctly approximate a specific route. To aid in checking the overall length and direction specified by a series of segments, compared to the known length and direction of the route the segments are to represent, the following relationships are given.

A curved segment of radius R_o and angle α_o, in the auxiliary coordinate system X', Y' is shown in Figure 11a. Starting at the origin, and moving along the curve through the angle α_o, the distance travelled along the curve is $R_o \alpha_o$. The change in heading of a tangent to the curve is α_o as the point of tangency moves similarly along the curve. The coordinates of the point Q are (X'_Q, Y'_Q) with

$$X'_Q = R_o \frac{\alpha_o}{|\alpha_o|} \sin \alpha_o \tag{65a}$$

and

$$Y'_Q = R_o \frac{\alpha_o}{|\alpha_o|} (1 - \cos \alpha_o) \tag{65b}$$

where $\frac{\alpha_o}{|\alpha_o|}$ keeps the sign appropriate for the case of $\alpha_o < 0.0$, when the center of curvature then is at $(0, -R_o)$

Now, if the location and orientation of the X' Y' system is known with respect to a global system, X Y, then the coordinates of Q may be determined in the X Y system by the axis transformation

$$X_Q = X'_Q \cos\theta - Y'_Q \sin\theta + X_B \quad (66a)$$

and

$$Y_Q = Y'_Q \cos\theta + X'_Q \sin\theta + Y_B \quad (66b)$$

where

θ is the angle from the X to the X' axis and (X_B, Y_B) are the global coordinates of the origin of the X'Y' system.

The total distance along a route of N segments is

$$\sum_{i=1}^{N} R_{oi}\,\alpha_{oi}$$

and the angle of the tangent at the end of the $N^{\underline{th}}$ segment, relative to the X axis is

$$\sum_{i=1}^{N} \alpha_{oi} \quad .$$

Furthermore, the coordinates of the point at the end of the $N^{\underline{th}}$ segment can be determined by applying equations (65a)-(66b) sequentially to each segment of the route, as illustrated in Fugure 11b. The local end point of the n^{th} segment, $(X_Q{}', Y_Q{}')$, is transformed to its global equivalent, (X_Q, Y_Q), and becomes the base point, (X_B, Y_B), for the origin of the local axis system for the $(n+1)^{th}$ segment. The

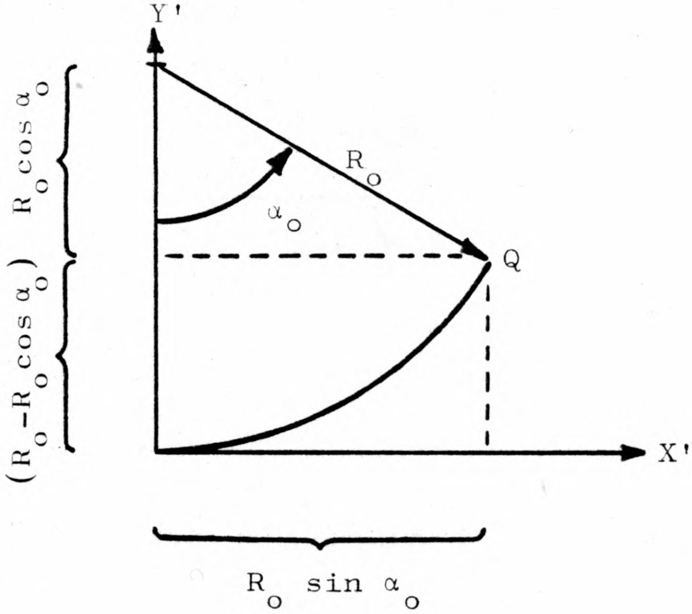


Figure 11a. Curved segment in moving coordinates.

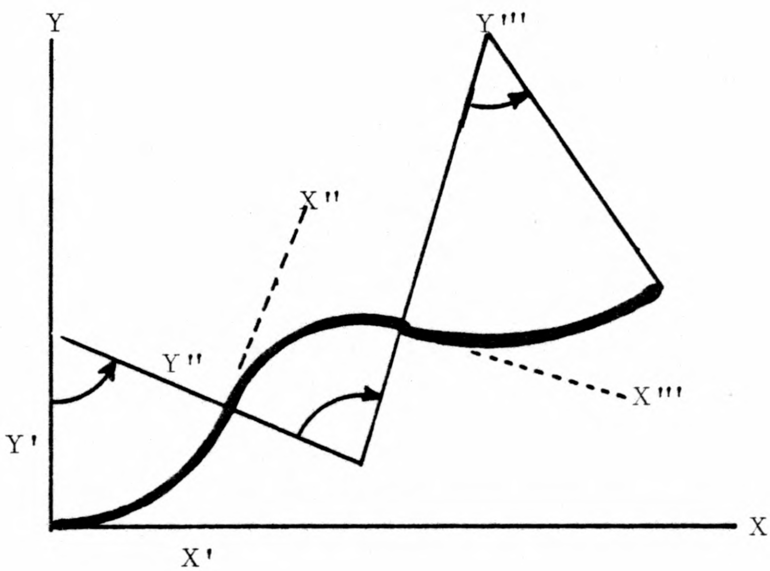


Figure 11b. Combination of curved segments along a route.

straight line distance from the global origin to the end of the N^{th} segment, where (X_B, Y_B) represents the base point for the $(N+1)^{th}$ segment, is given by,

$$\sqrt{X_B^2 + Y_B^2}$$

and the bearing, relative to the global X axis, is given by,

$$\tan^{-1}\left\{\frac{Y_B}{X_B}\right\}.$$

Thus, the input data describing a route can be checked for total distance and heading change along the route, as well as for the range and bearing from origin to destination.

V. COMPUTER PROGRAM DESCRIPTION

The mathematical model is implemented in a computer program written in standard FORTRAN IV code. A listing of the program is given in APPENDIX E. The structure of the program is modular, with separate SUBROUTINEs written to perform each function of the solution. The flow within each SUBROUTINE is generally straight forward; a description of each is given in APPENDIX F. The variables in each COMMON block are described in APPENDIX G, and a variable cross-reference list is given in APPENDIX H. A detailed description of the input data file format is given in APPENDIX D.

The numerical computations within the program are done in double precision, thus all real variables (those beginning with the letters A-H and O-$) are specified to have a length of 8-bytes. The remaining variables (I-N) are either 4-byte integers or logical variables. A definition of each variable is given in APPENDICES F and G.

VI VERIFICATION

The mathematical model is intended to represent a physical system. This representation involves simulation of the commands that control the rudder angle and propeller speed and prediction of the response of the flotilla to those commands. These functions are incorporated into the computer program and are supplemented by data handling routines, interpolation and curve fitting techniques, an algorithm for resetting the initial conditions of the flotilla in a curve and a host of other subordinate functions.

To ascertain the degree to which the simulation represents the physical system, two levels of program verification are used. The various functions performed by the computer program are each described by one or more SUBROUTINEs. Thus, the program functions can be checked out individually by verification at the SUBROUTINE level. The primary consideration, however, is to determine how well the performance predicted by the simulation agrees with the observed performance of an actual system. This verification is determined at the program level.

SUBROUTINE VERIFICATION

Each SUBROUTINE has associated with it a specific function; for a given set of input parameters, the SUBROUTINE should return a predictable result. Verification at this level is accomplished for each SUBROUTINE by writing a temporary test program to assign known values to the input parameters, call the SUBROUTINE, and print out the values returned by the SUBROUTINE. These values are then compared to results obtained independently, using the same values of the input parameters. Satisfactory agreement indicates that there are no programming errors in the FORTRAN code.

PROGRAM VERIFICATION

At the present stage of development of the simulation, correlation of the predicted performance of a flotilla with its observed behavior, on an absolute scale, is somewhat premature. Shallow water and side wall effects that influence the behavior of the physical system are presently neglected in the simulation. Furthermore, the predicted results depend on the numerical values assumed for the hydrodynamic coefficients, the rudder lift and drag characteristics and the propeller and bow thruster force terms. Thus, the absolute accuracy of the predicted results depends on factors that are beyond the scope of the present investigation.

Some degreee of verification, using reasonableness criteria, is still possible. The status of the flotilla at intermediate steps of the solution of the equations of motion, as shown in Appendix C, indicates how the flotilla is predicted to move around a curve. Analysis of the position and orientation of the flotilla as a function of time, and its response to changes in rudder angle and propeller speed, suggest that the predicted behavior is consistent with the expected behavior of a barge and tugboat in a similar maneuver. The rudder commands, as the flotilla progresses through a curve, are compatible with the position and orientation of the flotilla at any time and with the geometry of the next segment of the route. Finally, the predictions of elapsed time determined by direct integration of the equations of motion along a route agrees well with the prediction by interpolation for the same route. These results indicate that although the absolute accuracy of the model is limited to the degree of accuracy to which the forces are approximated, the overall structure of the solution is satisfactory, and the basic control functions of the model are performing as intended.

VII. SAMPLE CASES

Test cases have been run to evaluate the results obtained from the computer simulation. All of the basic solution options have been evaluated, and the results appear satisfactory (see Section VI). A sample input data file and the corresponding solution print out are shown in APPENDIX C, for a case using direct integration of the equations of motion along a route.

The execution time required for a case depends on several variables. The number of curved segments clearly influences the execution time required. For solutions involving integration of the equations of motion, the severity of the curves can increase the execution time if many iterations are required to successfully complete a maneuver. Furthermore, the values specified for the numerical integration error control limits (App. A) can significantly influence the program execution time. Typical values of these limits are in the range from 10^{-2} to 10^{-6}. Some testing is recommended to determine values appropraite to each case.

Representative execution times are expressed in terms of typical run costs for solutions done on the University of Michigan AMDAHL 470 computer system. For solutions involving integration of the equations of motion, solution costs are on the order of one dollar per mile of curved segments. For solutions by interpolation, typical costs are on the order of two cents per segment.

To illustrate one of the potential applications of the simulation program, a series of sample cases were run to predict the transit time for a flotilla maneuvering through a single bend, with a series of different bow thrusters. The characteristics assumed for the bowthruster in each case are plotted in Figure 12a. The parameters describing the curve are given in Figure 12b, which shows a plot of the elapsed time required to complete the maneuver as a function of the maximum thrust of the bowthruster in each case. Data of this sort will permit an investigator to quantitatively assess the benefits accrued from a given size bowthruster, compared to another size or none at all.

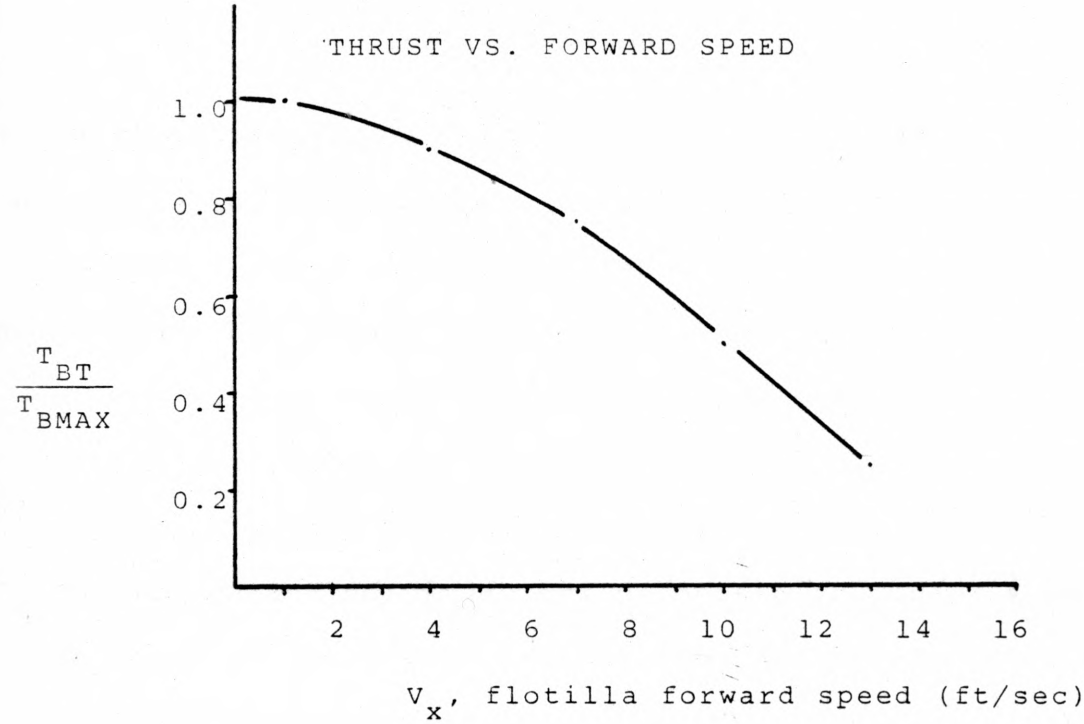


Figure 12a

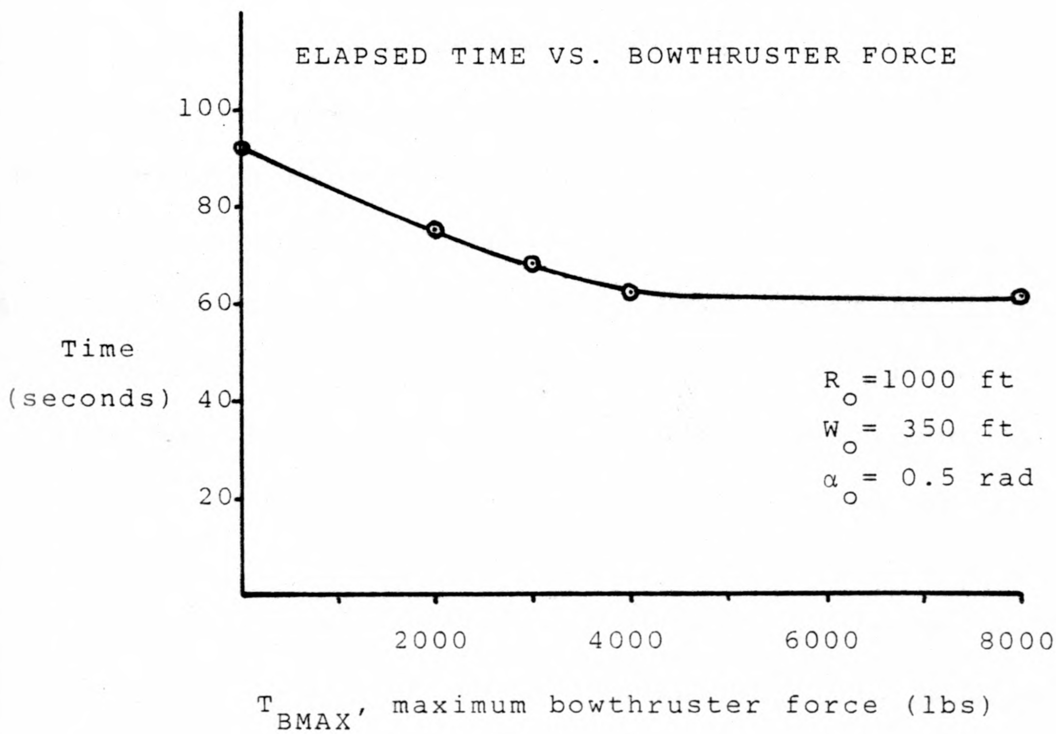


Figure 12b

VIII. RECOMMENDATIONS

The recommendations for further development stress three different phases. The first phase involves extensive testing of the existing model, to develop a data base for selecting values of the error control limits and the steering criteria parameters appropriate to a particular case. The second phase involves alteration of the model, to improve the execution time for simulating curved segments. Emphasis should be placed on improving the search method used to determine the best initial conditions to apply at the start of a curve. The present search method reduces propeller speed by several small increments, then reduces initial forward speed a small increment, and then repeats the sequence, until a maneuver is completed. A search method that varies the initial forward speed, and then searches for the best propeller speed at each forward speed tested, may substantially improve the convergence of the solution. The third phase of development involves improving the representation of the hydrodynamic forces and incorporating terms to account for shallow water and side wall effects. The model should also be modified to include a representation of the wind generated forces and moments acting on the flotilla.

Implementation of the preceeding recommendations will permit the simulation to more realistically represent a barge flotilla maneuvering in an inland waterway.

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APPENDIX A

NUMERICAL INTEGRATION

The differential equations of motion, equations (6a) - (6f), are simultaneously integrated to determine the velocity and position of the flotilla at any time. Because the representation of the forces, Q_α, Q_ψ , Q_R is not ammenable to analytic treatment, solution by numerical integration is required. A Kutta - Merson predictor-corrector technique is used, featuring a variable step size, Δt, to control the magnitude of the absolute and relative errors, ε_a and ε_r.

The integration along a curved segment is done in increments of time, ΔT. When error control limits are specified, Δt is successively halved or doubled from its initial value, Δt_o, until the maximum step size which will satisfy the error tolerances is achieved. To prevent the step size from becoming too small, a limit, N_c, is placed on the number of times the step size may be halved. Thus, the integration will terminate when,

$$|\Delta t| < |\Delta t|_o \left(\tfrac{1}{2}\right)^{N_c} \qquad \text{(A1)}$$

When $\varepsilon_a = \varepsilon_r = 0.0$, the error control feature is suppressed, and the step size is held constant at Δt_o throughout the entire interval ΔT.

At the end of each interval ΔT, the position of the barge is checked. If the angular position of the center of gravity is greater than the angular extent of the curve,

$$|\alpha| > |\alpha_{\bullet}| \qquad \text{(A2)}$$

the integration is terminated and the solution proceeds to the next segment. If the clearance between the flotilla and the channel boundary is negative,

$$cx_i < 0 \qquad \text{(A3a)}$$

or

$$cx_o < 0 \qquad \text{(A3b)}$$

the integration is assigned a new set of initial values, and is restarted. The integration continues until equation (A2) is satisfied, or until a specified number of integration steps have been evaluated. If equation (A2) is satisfied, the conditions at the end of the segment become the initial conditions for the next segment of the route. If the maximum number of integration steps is exceeded, an error message is printed, and the specified initial conditions are assumed for the next segment of the route.

APPENDIX B

TARGET LATERAL CLEARANCE FUNCTION

In specifying the target lateral clearance as a function of α, four cases are considered, as shown in Figure B1. In all four cases, when $|\alpha| < \frac{1}{2}|\alpha_d|$ the target lateral clearance is a constant specified as a fraction of the segment width,

$$ct_i = b_{cl} W_o \tag{B1}$$

where

b_{cl} = a constant, specified control parameter
W_o = the segment width.

The simplest case, Figure B1a, represents the last curve in a journey. In this case, there is no need to orient the flotilla to enter another segment so there is no problem in keeping the lateral clearance constant throughout the curve. Equation (B1) is applied throughout the entire segment.

In the second case, Figure B1b, a curve is followed by another curve in the same direction but with a different radius and/or channel width. If the width of the second curve is W_n, then the change in channel width from the first to the second segment is,

$$\Delta W = W_o - W_n \, . \tag{B2}$$

If

$$\Delta W < -b_{cl} W_o \tag{B3a}$$

then

$$\Delta W = -b_{cl} W_o \, . \tag{B3b}$$

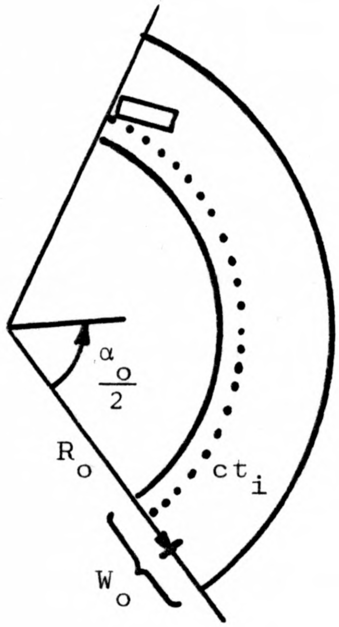

Figure B1a.

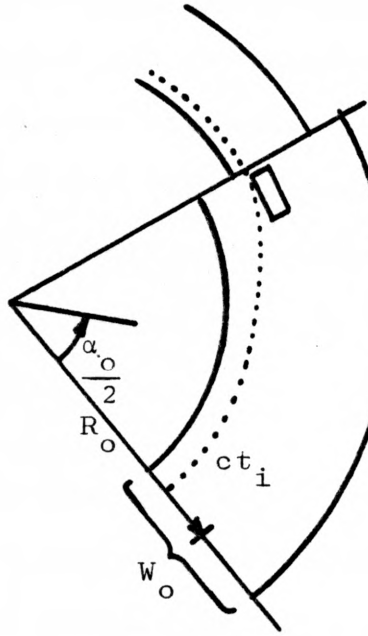

Figure B1b.

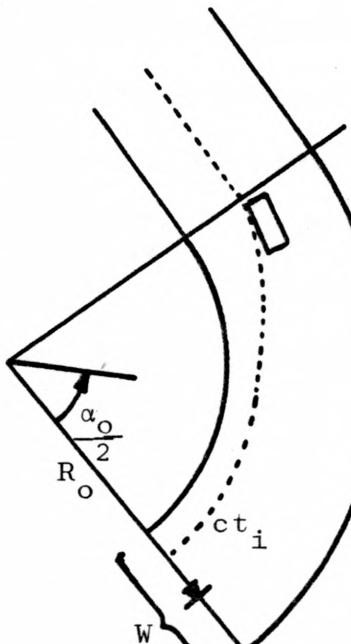

Figure B1c.

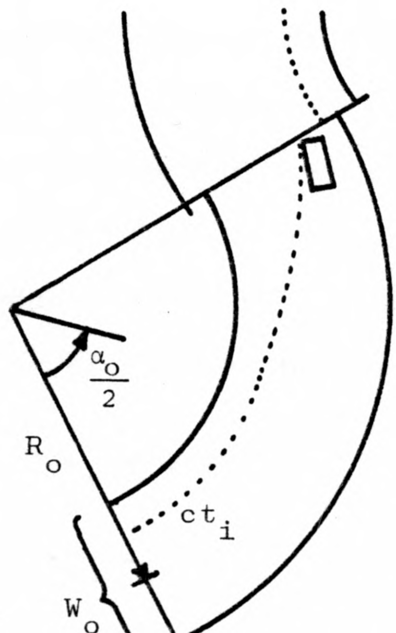

Figure B1d.

Figure B1. Target lateral clearance.

To position the flotilla at the desired lateral clearance for the start of the second curve, the target lateral clearance for $|\alpha| > \frac{1}{2}|\alpha_o|$ is given by,

$$ct_i = (b_{cl}\ W_o) + \frac{(2\alpha - \alpha_o)(\frac{1}{2}\Delta W)}{\alpha_o} \qquad \text{(B4)}$$

where

α = the angular position of the center of gravity of the flotilla

α_o = the angular extent of the curve.

Note that equations (B3a) and (B3b) are required to keep the barge clear of the inside wall of the channel when the second segment is much wider than the first.

In the third case, Figure B1c, a curve is followed by a straight (or delay) segment. The target lateral clearance for this case is adjusted to bring the flotilla to the center of the channel at the start of the next segment. Then for $|\alpha| > \frac{1}{2}|\alpha_o|$,

$$ct_i = (b_{cl}\ W_o) + \left\{\frac{\alpha - \frac{1}{2}\alpha_o}{\alpha_o}\right\} (W_o - 2b_{cl}W_o - B_t) \qquad \text{(B5)}$$

where

B_t = the beam of the barge array.

Finally, in the fourth case, Figure B1d, a curve is followed by another curve in the opposite direction. The target lateral clearance for this case is adjusted to bring the flotilla across the channel at the end of the segment to a proper distance from the inside wall of the next segment.

Then for $|\alpha| > \frac{1}{2}|\alpha_o|$,

$$ct_i = (b_{cl}\ W_o) + \left\{\frac{2\alpha - \alpha_o}{\alpha_o}\right\} (\overline{W} - 2b_{cl}W_o - B_t) \qquad \text{(B6)}$$

where

$$\overline{W} = \tfrac{1}{2}(W_o + W_n) \text{ for } W_n < W_o$$

or

$$\overline{W} = W_o \text{ for } W_n > W_o \ .$$

Note, as in the second case, $\overline{W}$ is defined to keep the barge from hitting the outside wall of the channel when the second segment is much wider than the first.

APPENDIX C

Sample Input and Output Data File

Sample Input Data File

```
TEST CASE USING EDA'S HYDRODYNAMIC COEFFICIENTS A1-A10/B1-B10
BARG 1  1      35.0        35.0
BARG 2  3     574.5       112.5       283.0        56.0       140.0       28.0
BARG 3  0      10.4        13.8        0.88        0.80
BARG 4  1     500.0       250.0        10.0       100.0         0.6       0.06
BARG 5  4       9.5         7.5        0.85         0.0         0.0
INIT 0  1     100.0         9.0         0.0         0.0         0.0        0.0    1.0
INIT 1 20       8.0         1.0        0.01        0.01
STER 0  0      0.15        0.06        0.06       0.001        0.05        4.0
ROUT 0  3
ROUT 1  1    5000.0       500.0         0.5
ROUT 1  2    5000.0       500.0        -0.5
ROUT 1  3    5000.0       500.0         0.5
COEF 1  1  -0.00058     -0.0011         0.0         0.0
COEF 2  1   -0.0033     -0.0011         0.0         0.0
COEF 1  2       0.0     -0.0072     0.00026     -0.0070
COEF 2  2       0.0      -0.012     -0.0081      -0.016
TRIP 2
QUIT
```

Sample Output

TEST CASE USING EDA'S HYDRODYNAMIC COEFFICIENTS A1-A10/B1-B10

CHARACTERISTICS OF TOWBOAT AND 3 LONG BY 1 WIDE BARGE TOW

	TOW	BOAT
LENGTH OVERALL	575.	113.
WIDTH	35.	35.
DRAFT	10.40	13.80
BLOCK COEFFICIENT	0.880	0.800
L C G (FORWARD)	283.0	56.0
GYRADIUS	140.0	28.0

PROPULSION AND RUDDER CHARACTERISTICS AND STEERING CRITERIA

NUMBER OF SHAFTS	1
HORSEPOWER PER SHAFT	500.
MAXIMUM RPM	250.
SHAFT OFFSET FROM CL	10.
BLADES PER PROPELLER	4
DIAMETER	9.5
PITCH	7.5
AREA RATIO	0.850
WAKE FRACTION	0.0
THRUST DEDUCTION	0.0
AREA PER RUDDER	100.
MAXIMUM RUDDER ANGLE	0.600
MAXIMUM RUDDER RATE	0.060

BOWCLR	STRSLO	STRSLI	STRCOR	STRBK	RGAIN
0.1500	0.0600	0.0600	0.0010	0.050	4.000

INITIAL BARGE/TOWBOAT VELOCITY AND ORIENTATION

RPM	SPEED	CURRENT ANGLE	GAMMA	YAW ANGLE	YAW RATE	RADIAL OFFSET
100.	9.00	0.0	0.0	0.0	0.0	1.000

INTEGRATION CONTROL PARAMETERS

NCUTS	FIRST-STEP	STEP-SIZE	REL-ERROR	ABS-ERROR
20	1.0000	8.0000	0.01000000	0.01000000

Sample Output

CURVE,HEAD,STRAIGHT,NET= 7500.00000 28.64790 7422.11878 14.32395

INTEGRATION THROUGH SEGMENT NUMBER 1
WITH 5000. FOOT RADIUS AND 500. FOOT WIDTH OVER 0.500 RADIANS

STEP	TIME	V(ANG)	ALPHA	V(HEAD)	PSI	V(RAD)	RADIUS	BETA	GAMA	DELTA	RPM	CLIN	XLIN	CLOUT	XLOUT
1	8.	0.183E-02	0.015	-0.474E-03	-0.0022	0.4511	4844.	-0.0531	0.0510	0.2764	130.	77.	11.	376.	356.
2	16.	0.185E-02	0.029	-0.102E-02	-0.0081	0.8481	4850.	-0.1025	0.0943	0.3131	130.	82.	40.	369.	-331.
3	24.	0.187E-02	0.044	-0.110E-02	-0.0167	1.1198	4858.	-0.1394	0.1228	0.2672	130.	89.	81.	358.	-331.
4	32.	0.189E-02	0.059	-0.115E-02	-0.0257	1.2921	4867.	-0.1656	0.1399	0.2536	130.	98.	125.	346.	-331.
5	40.	0.190E-02	0.074	-0.120E-02	-0.0350	1.3860	4878.	-0.1837	0.1486	0.2517	130.	108.	171.	332.	-331.
6	48.	0.191E-02	0.089	-0.120E-02	-0.0448	1.4227	4889.	-0.1957	0.1509	0.2227	130.	117.	218.	317.	-331.
7	56.	0.192E-02	0.105	-0.134E-02	-0.0544	1.4244	4901.	-0.2043	0.1499	0.2937	130.	126.	269.	303.	-331.
8	64.	0.193E-02	0.120	-0.943E-03	-0.0653	1.3637	4912.	-0.2078	0.1425	0.0368	130.	134.	311.	289.	-331.
9	72.	0.194E-02	0.136	-0.128E-02	-0.0749	1.3259	4923.	-0.2126	0.1377	0.2346	130.	141.	356.	274.	-331.
10	80.	0.195E-02	0.151	-0.129E-02	-0.0852	1.2549	4933.	-0.2148	0.1296	0.2315	130.	148.	356.	261.	-331.
11	88.	0.196E-02	0.167	-0.130E-02	-0.0955	1.1737	4943.	-0.2161	0.1206	0.2315	130.	154.	356.	247.	-331.
12	96.	0.197E-02	0.183	-0.130E-02	-0.1059	1.0855	4952.	-0.2170	0.1110	0.2323	130.	159.	356.	235.	-331.
13	104.	0.197E-02	0.198	-0.131E-02	-0.1164	0.9922	4960.	-0.2175	0.1011	0.2320	130.	164.	356.	223.	-331.
14	112.	0.198E-02	0.214	-0.131E-02	-0.1269	0.8958	4968.	-0.2178	0.0909	0.2361	130.	168.	356.	213.	-331.
15	120.	0.198E-02	0.230	-0.130E-02	-0.1374	0.7955	4975.	-0.2178	0.0804	0.2261	130.	171.	356.	202.	-331.
16	128.	0.199E-02	0.246	-0.136E-02	-0.1478	0.6975	4980.	-0.2181	0.0703	0.2622	130.	173.	356.	193.	-331.
17	136.	0.199E-02	0.262	-0.117E-02	-0.1587	0.5843	4986.	-0.2174	0.0587	0.1505	130.	175.	356.	185.	-331.
18	144.	0.200E-02	0.278	-0.132E-02	-0.1689	0.4662	4990.	-0.2179	0.0489	0.2382	130.	175.	356.	177.	-331.
19	152.	0.200E-02	0.294	-0.132E-02	-0.1795	0.3834	4993.	-0.2178	0.0383	0.2378	130.	175.	356.	170.	-331.
20	160.	0.201E-02	0.310	-0.132E-02	-0.1900	0.2778	4996.	-0.2177	0.0277	0.2385	130.	174.	356.	164.	-331.
21	168.	0.201E-02	0.326	-0.320E-03	-0.1989	0.0383	4998.	-0.2027	0.0038	-0.2244	130.	172.	356.	159.	-331.
22	176.	0.199E-02	0.342	0.366E-02	-0.1842	-0.8420	4995.	-0.0998	-0.0845	-0.6000	135.	174.	356.	168.	-331.
23	184.	0.199E-02	0.358	0.531E-02	-0.1466	-1.5614	4985.	0.0094	-0.1560	-0.4668	135.	178.	356.	189.	-331.
24	192.	0.201E-02	0.374	0.530E-02	-0.1032	-1.8793	4971.	0.0829	-0.1861	-0.2864	135.	179.	356.	217.	-331.
25	200.	0.204E-02	0.390	0.463E-02	-0.0634	-1.8282	4956.	0.1159	-0.1793	-0.1606	135.	178.	314.	245.	-331.
26	208.	0.206E-02	0.406	0.404E-02	-0.0289	-1.5610	4942.	0.1229	-0.1519	-0.0867	135.	172.	142.	270.	-331.
27	216.	0.209E-02	0.423	0.368E-02	0.0020	-1.2077	4931.	0.1188	-0.1167	-0.0306	135.	163.	-10.	288.	356.
28	224.	0.211E-02	0.440	0.345E-02	0.0306	-0.8236	4923.	0.1097	-0.0791	-0.0345	135.	153.	-149.	287.	356.
29	232.	0.213E-02	0.457	0.335E-02	0.0578	-0.4480	4918.	0.1006	-0.0428	-0.0408	135.	142.	-284.	286.	356.
30	240.	0.214E-02	0.474	0.325E-02	0.0842	-0.0825	4916.	0.0921	-0.0078	-0.0375	135.	131.	-331.	285.	356.
31	248.	0.215E-02	0.491	0.324E-02	0.1102	0.2597	4916.	0.0856	0.0246	-0.0495	135.	123.	-331.	284.	356.
32	256.	0.216E-02	0.508	0.322E-02	0.1360	0.5860	4920.	0.0808	0.0552	-0.0517	135.	118.	-331.	284.	356.
S,T,C,H,G,D			10.62342		3.85520	0.00322		3.26519		3.19678	82.63511				

INTEGRATION THROUGH SEGMENT NUMBER 2
WITH 5000. FOOT RADIUS AND 500. FOOT WIDTH OVER -0.500 RADIANS

STEP	TIME	V(ANG)	ALPHA	V(HEAD)	PSI	V(RAD)	RADIUS	BETA	GAMA	DELTA	RPM	CLIN	XLIN	CLOUT	XLOUT
1	8.	-0.214E-02	-0.017	0.108E-02	3.2776	-0.2614	5079.	0.1119	3.1656	-0.3259	135.	276.	356.	110.	-331.
2	16.	-0.215E-02	-0.034	0.114E-02	3.2864	0.0165	5078.	0.1463	3.1401	-0.3141	135.	272.	356.	101.	-331.
3	24.	-0.216E-02	-0.051	0.121E-02	3.2956	0.1700	5079.	0.1697	3.1261	-0.2968	135.	269.	356.	94.	-331.
4	32.	-0.216E-02	-0.069	0.126E-02	3.3057	0.2318	5081.	0.1852	3.1205	-0.2625	135.	267.	356.	88.	-331.
5	40.	-0.216E-02	-0.086	0.131E-02	3.3160	0.2306	5083.	0.1954	3.1206	-0.2779	135.	266.	356.	83.	-331.
6	48.	-0.216E-02	-0.103	0.131E-02	3.3266	0.1652	5084.	0.2018	3.1248	-0.2566	135.	264.	356.	78.	-331.
7	56.	-0.216E-02	-0.121	0.132E-02	3.3371	0.1077	5085.	0.2053	3.1318	-0.2410	135.	261.	356.	73.	-331.
8	64.	-0.216E-02	-0.138	0.957E-03	3.3465	-0.0392	5086.	0.2013	3.1452	-0.1548	135.	258.	356.	70.	-331.
9	72.	-0.216E-02	-0.155	0.104E-02	3.3536	-0.1766	5085.	0.1959	3.1577	-0.2738	135.	254.	356.	68.	-331.
10	80.	-0.215E-02	-0.173	0.827E-03	3.3606	-0.3108	5083.	0.1907	3.1700	-0.1924	135.	250.	356.	68.	-331.
11	88.	-0.215E-02	-0.190	0.857E-03	3.3673	-0.4068	5080.	0.1884	3.1789	-0.2096	135.	245.	356.	69.	-331.
12	96.	-0.215E-02	-0.207	0.943E-03	3.3745	-0.4817	5076.	0.1889	3.1856	-0.2278	135.	239.	356.	70.	-331.
13	104.	-0.216E-02	-0.224	0.106E-02	3.3825	-0.5374	5072.	0.1918	3.1907	-0.2463	135.	232.	356.	72.	-331.

14	112.	-0.216E-02	-0.241	0.119E-02	3.3915	-0.5847	5068.	0.1965	3.1950	-0.2628	135.	224.	356.	74.	-331.
15	120.	-0.216E-02	-0.259	0.811E-03	3.4015	-0.6853	5063.	0.1973	3.2042	0.0268	135.	216.	356.	75.	-331.
16	128.	-0.215E-02	-0.276	-0.228E-03	3.4027	-1.0165	5056.	0.1679	3.2349	0.0	135.	208.	356.	81.	-331.
17	136.	-0.214E-02	-0.293	-0.645E-03	3.3991	-1.2561	5047.	0.1420	3.2571	0.0	135.	201.	356.	92.	-331.
18	144.	-0.215E-02	-0.310	-0.920E-03	3.3928	-1.4270	5036.	0.1199	3.2729	0.0	135.	193.	356.	105.	-331.
19	152.	-0.215E-02	-0.328	-0.113E-02	3.3845	-1.5407	5024.	0.1013	3.2833	0.0	135.	184.	356.	119.	-331.
20	160.	-0.214E-02	-0.345	-0.412E-02	3.3654	-2.0543	5010.	0.0346	3.3308	0.6000	135.	176.	356.	139.	-331.
21	168.	-0.214E-02	-0.362	-0.562E-02	3.3247	-2.5855	4992.	-0.0546	3.3794	0.4776	135.	172.	356.	171.	-331.
22	176.	-0.215E-02	-0.379	-0.595E-02	3.2783	-2.8029	4970.	-0.1198	3.3981	0.4350	135.	167.	356.	207.	-331.
23	184.	-0.217E-02	-0.396	-0.585E-02	3.2298	-2.7600	4947.	-0.1630	3.3928	0.2321	135.	161.	356.	245.	-331.
24	192.	-0.220E-02	-0.414	-0.476E-02	3.1875	-2.3143	4927.	-0.1644	3.3518	0.0532	135.	154.	221.	280.	-331.
25	200.	-0.222E-02	-0.431	-0.387E-02	3.1526	-1.7302	4911.	-0.1461	3.2987	-0.0314	135.	143.	62.	306.	-331.
26	208.	-0.224E-02	-0.449	-0.333E-02	3.1218	-1.2077	4899.	-0.1293	3.2511	0.0574	135.	130.	-99.	316.	356.
27	216.	-0.225E-02	-0.467	-0.363E-02	3.0919	-0.7205	4891.	-0.1149	3.2068	0.0309	135.	118.	-242.	318.	356.
28	224.	-0.226E-02	-0.485	-0.349E-02	3.0635	-0.2667	4887.	-0.1022	3.1657	0.0387	135.	105.	-331.	321.	356.
29	232.	-0.226E-02	-0.503	-0.342E-02	3.0359	0.1464	4887.	-0.0924	3.1284	0.0429	135.	96.	-331.	323.	356.

S,T,C,H,G,D 11.06904 1.52564 -0.00342 -0.10044 -0.01323 113.29069

INTEGRATION THROUGH SEGMENT NUMBER 3
WITH 5000. FOOT RADIUS AND 500. FOOT WIDTH OVER 0.500 RADIANS

STEP	TIME	V(ANG)	ALPHA	V(HEAD)	PSI	V(RAD)	RADIUS	BETA	GAMA	DELTA	RPM	CLIN	XLIN	CLOUT	XLOUT
1	8.	0.222E-02	0.018	-0.110E-02	-0.1133	0.1331	5113.	-0.1250	0.0117	0.3341	135.	318.	356.	83.	-331.
2	16.	0.222E-02	0.036	-0.102E-02	-0.1221	0.3535	5115.	-0.1531	0.0311	0.2413	135.	317.	356.	71.	-331.
3	24.	0.222E-02	0.053	-0.414E-03	-0.1278	0.3402	5118.	-0.1577	0.0299	0.1334	135.	317.	356.	63.	-331.
4	32.	0.222E-02	0.071	-0.628E-04	-0.1295	0.2483	5120.	-0.1514	0.0219	0.1129	135.	319.	356.	59.	-331.
5	40.	0.221E-02	0.089	0.170E-03	-0.1291	0.1523	5122.	-0.1426	0.0134	0.0846	135.	321.	356.	58.	-331.
6	48.	0.221E-02	0.106	0.377E-03	-0.1268	0.0597	5123.	-0.1321	0.0053	0.0818	135.	323.	356.	58.	-331.
7	56.	0.220E-02	0.124	0.406E-03	-0.1236	0.0102	5123.	-0.1245	0.0009	0.1056	135.	324.	356.	59.	-331.
8	64.	0.220E-02	0.142	0.308E-03	-0.1207	0.0122	5123.	-0.1218	0.0011	0.1319	135.	325.	356.	59.	-331.
9	72.	0.220E-02	0.159	0.196E-03	-0.1187	0.0454	5123.	-0.1227	0.0040	0.1447	135.	326.	356.	60.	-331.
10	80.	0.220E-02	0.177	0.147E-03	-0.1174	0.0824	5124.	-0.1247	0.0073	0.1421	135.	327.	356.	60.	-331.
11	88.	0.220E-02	0.195	0.170E-03	-0.1161	0.1062	5125.	-0.1255	0.0094	0.1310	135.	328.	356.	59.	-331.
12	96.	0.220E-02	0.212	0.235E-03	-0.1145	0.1147	5126.	-0.1247	0.0102	0.1198	135.	330.	356.	59.	-331.
13	104.	0.220E-02	0.230	0.303E-03	-0.1124	0.1153	5126.	-0.1226	0.0102	0.1130	135.	331.	356.	59.	-331.
14	112.	0.220E-02	0.247	0.353E-03	-0.1097	0.1167	5127.	-0.1201	0.0103	0.1109	135.	333.	356.	59.	-331.
15	120.	0.220E-02	0.265	0.384E-03	-0.1068	0.1241	5128.	-0.1178	0.0110	0.1108	135.	335.	356.	59.	-331.
16	128.	0.220E-02	0.283	0.406E-03	-0.1036	0.1379	5129.	-0.1158	0.0122	0.1102	135.	337.	356.	59.	-331.
17	136.	0.220E-02	0.300	0.433E-03	-0.1003	0.1555	5131.	-0.1140	0.0138	0.1076	135.	340.	356.	59.	-331.
18	144.	0.220E-02	0.318	0.471E-03	-0.0967	0.1743	5132.	-0.1121	0.0154	0.1031	135.	342.	356.	58.	-331.
19	152.	0.220E-02	0.335	0.521E-03	-0.0927	0.1930	5133.	-0.1098	0.0171	0.0976	135.	345.	356.	58.	-331.
20	160.	0.220E-02	0.353	0.579E-03	-0.0883	0.2117	5135.	-0.1071	0.0188	0.0919	135.	348.	356.	58.	-331.
21	168.	0.220E-02	0.371	0.641E-03	-0.0834	0.2317	5137.	-0.1039	0.0205	0.0863	135.	352.	356.	58.	-331.
22	176.	0.220E-02	0.388	0.707E-03	-0.0780	0.2539	5139.	-0.1005	0.0225	0.0807	135.	356.	356.	58.	-331.
23	184.	0.220E-02	0.406	0.776E-03	-0.0721	0.2790	5141.	-0.0968	0.0247	0.0747	135.	360.	356.	57.	-331.
24	192.	0.220E-02	0.423	0.852E-03	-0.0656	0.3072	5143.	-0.0928	0.0272	0.0681	135.	365.	337.	57.	-331.
25	200.	0.219E-02	0.441	0.935E-03	-0.0585	0.3383	5146.	-0.0884	0.0299	0.0607	135.	369.	301.	57.	-331.
26	208.	0.219E-02	0.458	0.103E-02	-0.0506	0.3722	5149.	-0.0836	0.0329	0.0524	135.	375.	260.	57.	-331.
27	216.	0.219E-02	0.476	0.113E-02	-0.0420	0.4093	5152.	-0.0782	0.0362	0.0434	135.	380.	216.	56.	-331.
28	224.	0.219E-02	0.493	0.124E-02	-0.0325	0.4498	5155.	-0.0723	0.0398	0.0336	135.	385.	168.	56.	-331.
29	232.	0.219E-02	0.511	0.136E-02	-0.0221	0.4941	5159.	-0.0658	0.0437	0.0230	135.	390.	114.	56.	-331.

S,T,C,H,G,D 11.31749 5.02331 0.00136 -0.02895 0.04368 156.44610

Sample Output

TEST CASE USING EDA'S HYDRODYNAMIC COEFFICIENTS A1-A10/B1-B10

SUMMARY OF ELAPSED TIME FOR CASE 1

SEGMENT	RADIUS	WIDTH	ANGLE	TIME(SEG)	TIME(ACUM)
1	5000.	500.	0.5000	252.14	252.14
2	5000.	500.	-0.5000	230.47	482.62
3	5000.	500.	0.5000	226.98	709.60

*********1*********2*********3*********4*********5*********6*********7*********8*********9*