through sequences of exercises, culminating in some substantial results. One can recommend as a taster the exercises leading to the triviality of K_0 of the Cuntz algebra O_2 in Chapter 4. Another notable feature (developed from a good idea in [**3**]) is the table of K_0 and K_1 groups, together with references to the relevant parts of the text. Also, the index of symbols certainly proved useful when I returned to the book after less pleasant duties had enforced a lengthy gap in my reading!

Although this book covers somewhat fewer topics than [**3**], and considerably fewer than Blackadar's book [**1**], it certainly succeeds in giving a very helpful and thorough introduction. It should give readers an ability to operate with confidence in the basic areas of C*-algebraic K-theory and leave them well-equipped to move onward to further important topics such as K-theory for crossed products and Kasparov's KK-theory.

References

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THEORY OF BERGMAN SPACES
(Graduate Texts in Mathematics 199)

By Haakan Hedenmalm, Boris Korenblum and Kehe Zhu: 286 pp., £37.50,
isbn 0-387-98791-6 (Springer, New York, 2000).
Bull. London Math. Soc. 34 (2002)
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A function $f(z)$ that is analytic in the unit disk $|z| < 1$ is in the *Bergman space* A^p if $|f|^p$ has finite area integral, where $0 < p < \infty$. It is in the *Hardy space* H^p if the integrals $\int |f(re^{i\theta})|^p \, d\theta$ over circles $|z| = r < 1$ remain bounded as $r \to 1$. Thus $H^p \subset A^p$.

The theory of Hardy spaces was well developed by 1960. Zero-sets are easily determined, Blaschke products serve as isometric zero-divisors, there is a canonical factorization into *inner* and *outer* functions, invariant subspaces (invariant under multiplication by polynomials) have a simple description, and there are elegant theories of extremal problems and interpolation. Corresponding problems for the Bergman spaces were long considered intractable, but the last decade has seen remarkable progress. Hedenmalm [**3**] constructed contractive zero-divisors in A^2, and the extension to A^p (by Khavinson, Shapiro, Sundberg, and the reviewer [**2**]) revealed an intimate connection with the biharmonic Green function. Seip [**4**] gave complete descriptions of interpolation and sampling sets for A^2 in terms of certain densities. The invariant subspaces of A^2 were known to be far more complicated than those of H^2, but Aleman, Richter, and Sundberg [**1**] found a general structural formula. Completing earlier work of Korenblum, they also obtained a credible analogue for A^p of inner-outer factorization, where the 'outer functions' are precisely the cyclic elements of A^p.

The book by Hedenmalm, Korenblum, and Zhu gives an authoritative account of these new developments and many other topics. Ambitious in scope, it culminates with technical accounts of cyclic vectors of the growth space $\mathscr{A}^{-\infty}$ and Korenblum's theory of premeasures, the Borichev–Hedenmalm construction of noncyclic invertible functions in A^p, and the Hedenmalm–Jacobsson–Shimorin theory of logarithmically subharmonic weights, which applies to generalize the contractive property of canonical divisors.

Since the book appears in a series called 'Graduate Texts in Mathematics' and contains a number of exercises, it may appear to be a book for beginners. It will not be easy reading, however, for the uninitiated. Basic principles are hastily dispatched, some proofs are incomplete, and many of the exercises are quite difficult. On the other hand, the book offers fresh perspectives on recent developments and older topics alike, making those results more accessible to workers in Bergman spaces and related fields. As a research monograph, the book is a valuable resource, and a very welcome addition to the literature.

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SOME NOVEL TYPES OF FRACTAL GEOMETRY
(Oxford Mathematical Monographs)

By STEPHEN SEMMES: 164 pp., £49.95, ISBN 0-19-850806-9
(Clarendon Press, Oxford, 2001).

Bull. London Math. Soc. 34 (2002)
DOI: 10.1112/S0024609301241009

The geometry of an arbitrary metric space can be strange and complicated. But what if some additional conditions are imposed? Suppose, for example, that there is a measure such that $\mu(B(x,r)) \approx r^a$. Such spaces are called *Ahlfors-regular*. Then, a theorem of Assouad [**1**] asserts that, for any such metric space (M,d) and any $s \in (0,1)$, there is an n such that (M,d^s) is bi-Lipschitz equivalent to a subset of the n-dimensional Euclidean space. One may want to postulate further properties capturing the idea that the space looks essentially the same at all scales and positions in the spirit of 'fractal geometry' (for example, the notion of BPI spaces in the book).

Another type of property which one may want to impose concerns the existence of many rectifiable curves and of some calculus-type inequality of the sort $|f(x) - f(y)| \leqslant d(x,y)\|\nabla f\|_\infty$, or its more sophisticated and more useful version, known as *Poincaré inequality*, which asserts that

$$\inf_{a\in\mathbb{R}} \int_B |f-a|\,d\mu \leqslant Cr \int_B |\nabla f|\,d\mu,$$