# ON A PROBLEM OF SIDON IN ADDITIVE NUMBER THEORY AND ON SOME RELATED PROBLEMS

## ADDENDUM

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In a note in this *Journal* [16 (1941), 212–215], Turan and I proved, among other results, the following: Let $a_1 < a_2 < \ldots < a_x \leqslant n$ be a sequence of positive integers such that the sums $a_i + a_j$ are all different. Then $x < n^{\frac{1}{2}} + O(n^{\frac{1}{4}})$. On the other hand, there exist such sequences with $x > n^{\frac{1}{2}}(2^{-\frac{1}{2}} - \epsilon)$, for any $\epsilon > 0$.

Recently I noticed that J. Singer, in his paper "A theorem in finite projective geometry and some applications to number theory" [*Trans. Amer. Math. Soc.*, 43 (1938), 377–385], proves, among other results, that, if $m$ is a power of a prime, then there exist $m+1$ numbers $a_1 < a_2 < \ldots < a_{m+1} < m^2+m+1$ such that the differences $a_i - a_j$ are congruent, mod $(m^2+m+1)$, to the integers 1, 2, ..., $m^2+m$. Clearly the sums $a_i + a_j$ are all different, and since the quotient of two successive primes tends to 1, Singer's construction gives, for any large $n$, a set with $x > n^{\frac{1}{2}}(1-\epsilon)$, for any $\epsilon > 0$. Singer's method is quite different from ours. His result shows that the above upper bound for $x$ is best possible, except perhaps for the error term $O(n^{\frac{1}{4}})$.

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# NOTE ON $H_2$ SUMMABILITY OF FOURIER SERIES

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1. Let $s_n(t)$ denote a partial sum of the Fourier series of an integrable function $f(t)$, periodic with period $2\pi$, and let $\phi(t) = \frac{1}{2}\{f(x+t)+f(x-t)-2s\}$. Recently I proved the following result of Hardy and Littlewood†:

*If* $\int_0^t |\phi(u)| \{1+\log^+|\phi(u)|\} du = o(t)$ *as* $t \to 0$, *then the Fourier series of* $f(t)$ *is summable* $H_2$ *to sum* $s$ *for* $t = x$, *i.e.*

$$\sum_{\nu=0}^{n} |s_\nu(x) - s|^2 = o(n).$$

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† Hardy and Littlewood (1), *Fund. Math.*, 25 (1935), 162–189; Wang (2), *Duke Math. Journal* (in the press).