A Two-Stage Dynamic Programming Model for Nurse Rostering Problem Under Uncertainty

by

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DEDICATION

This edition of the Master's Thesis by Wenjie Wang, "A TWO-STAGE DYNAMIC PROGRAMMING MODEL FOR NURSE ROSTERING PROBLEM UNDER UNCERTAINTY", is dedicated to all the professionals, students and people who are working on or interested in nurse scheduling problem.

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CHAPTER I: Introduction

1.1 Motivation

The Nurse Rostering Problem (NRP) is to find the policies of assigning nurses to shifts for both fulfilling the objectives of hospital and respecting the welfare of nurses. Hospitals should maintain safe nurse-to-patient ratios in nurse units to guarantee a safe patient care. A crucial problem worthy of consideration is uncertainty on a daily basis due to emergency responses for patients and absenteeism among registered nurses (*Zboril-Benson* (2016)). The uncertainty leads to an unforeseen shortage of nursing workload, and as a result, decreases standards of care. To meet the required workload, hospitals often call off-duty nurses who voluntarily work additional hours. If there is no one available for overtime, hospitals have to resort to expensive external nursing resources (e.g. hospital reserve pool, external interim nurses, etc.) (*Maenhout and Vanhoucke* (2011)).

The uncertainty of Nurse Rostering Problem can be viewed from three perspectives: (i) patient demand uncertainty; (ii) nurse absenteeism; and (iii) nurses' inclination to work overtime.

Note that absenteeism is also a major concern for hospitals because it leads to diminished quality of patient care (*Borda and Norman* (1997); *Shamian and Villeneuve* (2009)). Data has shown that 8% of the nursing workforce is absent in any given week due to sickness (2009building). Absenteeism also increases pressure on those nurses who remain working their job, resulting in decreased morale and job satisfaction, and possibly turnover. (*Zboril-Benson* (2016); *Carayon and Gurses* (2008)). Since nurses are overtime voluntarily, it's important to respect the inclination of nurses.

This research aims to design a resilient nurse scheduling plan and on-call policy, which is capable of withstanding the unpredictable shortage of nursing workloads while protecting nurses' welfare. To achieve this goal, a two-stage dynamic programming model is proposed to generate a roster that not only meets the scheduling rules, but also maximizes flexibility for finding appropriate overtime nurses; a substitute list would also be created to help the manager find the best overtime nurses in respect to all schedule rules.

1.2 Contribution

The proposed model distinguishes itself from models in the following aspects: the proposed model accounts for the uncertainty of demand changes, nurse absenteeism and nurses' inclination to work overtime, which are the three top issue among nurse units.It extends the stochastic programming method into nurse rostering that helps to against uncertainties. This resilient model makes the roster more flexible and also gives the nurse unit manager a guideline to find the appropriate overtime nurse under the overtime policy. With the proposed method, hospitals can save time and money on creating the roster in the mean time improving the quality of patient care and job satisfaction of nurses.

1.3 Organization

The remainder of the paper is constructed as follows: Section 2 reviews the related literature. In section 3 describes the mathematical fulmination of the resilient two-stage model. In section 4, results of the simulation tests and properties analysis are reported. Finally, conclusions are given in section 5.

CHAPTER II: Literature Review

Considerable research efforts have been devoted to nurse rostering and rerostering problems. Extensive reviews of nurse rostering problems are provided in articles by *Cheang et al.* (2003) and *Burke et al.* (2004), which highlighted a summary of methodologies used in the nurse rostering problems and utilization statistics of commonly occurring constraints among models. A summary list of constraints used in different papers is listed by *Burke et al.* (2004). Several surveys among nurse units are made to understand the requirements, concerns and constrains of nurse rostering problems in hospitals. *Siferd and Benton* (1992) conducted a survey of 348 first-line managers of nursing units in 31 Acute-care hospitals to understand managers' internal concerns and to show the staffing and scheduling characteristic facts among these hospitals. 400 questionnaire surveys are collected by *Azaiez and Al Sharif* (2005) to better understand the nurses' preferences.

Most research studies on nurse rostering problems (e.g.,*Burke et al.* (2006); *Warner and Prawda* (1972); *Berrada et al.* (1996); *Burke et al.* (2001); *Ozkarahan and Bailey* (1988); *Bard and Purnomo* (2007); *Chun et al.* (2000); *Warner* (1976); *Meyer aufm Hofe* (2001); *Moz and Pato* (2004); *Miller et al.* (1976); *Cheng et al.* (1996)) have only focused on meeting the workload demand while complying with constrains. Only a few studies paid attention to the demand uncertainty. *Davis et al.* (2014) studied the problem of permanent nurse staffing level estimation under demand uncertainty as a newsvendor model, which was based on limited moment information of the demand distribution.

Rerostering problems are also being widely discussed under NRP. *Moz and Pato* (2007) who proposed several versions of genetic algorithms based on specific encoding and operators for sequencing problems , which applied to the nurse rerostering problems with hard constraints.A fitness value was assigned to individuals and maximized to make the new roster as similar as possible to the current one (similar idea in *Moz and Pato* (2003), *Moz and Pato* (2004), *Pato and Moz* (2008)). *Maenhout and Vanhoucke* (2011) proposed a genetic algorithm to solve nurse rerostering problems considering hospital's policies and the nurses' preferences.

A handful of studies focused on using a Two-Stage Stochastic Programming (TSSP) approach to deal with workforce planning. *Kao and Queyranne* (1985) present a TSSP model that deals with the sizing of regular-time workforce in the first-stage and use second-stage decisions to determine the extent of use of recourse in each period of overtime. *Solos et al.* (2013) presented a two-phase stochastic variable neighborhood approach that dealt with the assignment of nurses to working days in the first stage and dealt with the assignment of nurses to shift types in the second stage. In a recent paper, *Kim and Mehrotra* (2015) presented a two-stage stochastic inter programming model for staffing and scheduling under demand uncertainty, where the first-stage was to find initial staffing schedules and the second-stage was to adjust these schedules at a time closer to the actual date of demand realization. *Zhong et al.* (2017) proposed a two-stage heuristic algorithm considering the fairness and weekend workload in the first stage.The second-stage generates nurse rosters that allocate the weekend shifts as even as possible over a given planning horizon based on hospital's requests.

CHAPTER III: Resilient Two-Stage Dynamic Programming Model

3.1 Model Description and Assumptions

In this paper, assuming the nurse unit operates 24 hours a day and all nurses follow 12 hour shifts: night shift(7pm - 7am) and day shift(7am - 7pm). Before making the rostering decision, a forecast demand for the planning period should be known. The planning horizon is a month and at the end of each month a new roster should be made. This model also assumes that no reserve pool of nurses exists.

In addition, the following assumptions are made:

- No minimum workload requirement for each nurse.
- All nurses are working full-time.
- No reserved pool of nurses exists.
- Day shift nurses and night shift nurses are scheduled separately.
- Nurses' working preferences are collected before planning.
- Extra workforce demand follows a normal distribution with zero mean.

Note that nurses' working preference is collected by the head nurse, which shows the day or the shift that one nurse does not want to be assigned. Adding the nurses' preference will be beneficial on work-life balance for each nurse as a result of improving the quality of the personnel roster. Giving the right to choose the preference does not mean the nurse can choose any shift without any limits. Some limitations should be built depending upon the situation of each hospital.

In this model demand uncertainty, nurse absenteeism and nurses' inclination to work overtime are taken into account.

The resilient two-stage dynamic programming model is composed of two stages: planning stage and operation stage. The objective of this research project is to help nurse units develop a resilient approach to prepare the nurse schedule and find an optimal on-call policy. This initial schedule will be easily, but cheaply, adjusted with the on-call policy to fulfill the actual need of the nursing workload while respecting the nurses' legal rights and welfare.

3.2 Notations

The notation is described in Table 3.1. A set of nurses N (i.e. set of nurses, index i ($i = 1,,n$)) are scheduled on a pre-defined period T. A forecast demand R_t for the planning period is known before planning. The decision variables x in the planning stage are used to generate the first-stage rostering, while decision variables a are used to find out the alternative nurse to work overtime.

3.3 Problem Formulation

A two-stage decision making model is developed, where the here-and-now decision in the Planning Stage makes an initial schedule, and the wait-and-see decision in the Operation Stage finds the optimal on-call policy adjusting the schedule for the nurse workload realization. The two-stage model will be formulated as

$$min_{x \in X} f(x) + E[J_1(x, F_1)] \tag{1}$$

In the above model, X is a decision region including all valid nurse schedules, which obey all hard constraints. These hard constraints protect nurses' legal rights. For example, a nurse's total workload per week should be restricted, there should be a limited number of consecutive working days and also should respect nurses' individual preferences. The function $f(x)$ is the total penalty cost of the schedule that violates soft constraints. These soft constraints are specified for promoting patient safety and nurses job satisfaction. For example, a penalty may result from a shortage of nurses. During the operation stage, the random scenarios including bed occupancy rate (patient demand), nurse absenteeism, and nurse's inclination to volunteer to work additional hours are taken into consideration. Let w_t represent the random scenario at the t^{th} shift, and $F_t = (w_1, ..., w_t)$

Table 3.1: Notation Summary

Indices and Sets	
I	Index of nurses $i \in I, I = \{1, 2, ..., N\}$
T	Index of planning time periods $t \in T, T = \{1, 2, ..., T\}$
The first-stage problem:	
Parameters	
N	Total number of nurses scheduled
R_t	Forecasted workforce demand for each shift
L_1	Max. number of shifts per week
C	Max. number of consecutive days
	Nurse's working preference
P_{it}	1, if nurse i refuse to work on shift t
	0, otherwise
A	Set of scheduling rules
Decision Variables	
x_{it}	1, if nurse i assigned to shift t 0, otherwise
The second-stage problem:	
Parameters	
L_2	Max. workload when making overtime decision
A_t	Set of rerostering rules at time t
ω_t	Demand fluctuation for shift t
	Nurse's overtime preference
ξ_{it}	1, if nurse i refuse to work overtime on shift t
	0, otherwise
	Nurse absenteeism
η_{it}	1, if nurse i absent on shift t
	0, otherwise
Stage Variables	
l_{it}	Total working assigned to nurse i for the planning horizon at shift t
c_{it}	Number of consecutive working days previous to shift t of nurse i
S_t	State vector at time t
Decision Variables	
	Action took in the operation stage
a_{it}	1, if nurse i assigned to shift t in new roster
	0, otherwise

be the nature filtration recording to the past knowledge up to the t^{th} shift. The function $J_1(x, F_1)$ is the total cost of adjusting the schedule during the operation stage due to the filtration F_1. Since the schedule has a planning horizon of a month, the operation stage may be depicted as the Bellman equation of stochastic dynamic programming:

$$J_t(S_t,F_t) = \min_{a_t \in A_t(S_t,F_t)} g_t(a_t,S_t,F_t) + E[J_{t+1}(S_{t+1},F_{t+1}|F_t)], \quad t = 1,...,T-1. \tag{2}$$

where $A_t(S_t,F_t)$ is the set of available actions at the t^{th} shift under the current situation and the function g_t is the cost of adjusting the schedule at the t_{th} shift. Also the state $S_1 = x$ and S_{t+1} is a function of (a_t,S_t,F_t). More detail will be discussed in Chapter 3.3.1 and 3.1.2.

3.3.1 Planning Stage

The planning stage generates a roster that minimizes the total shortage of nurses while satisfying labor contract rules, scheduling rules of the hospital and nurses' preference. Time-related rules are widely used in nurse scheduling (*Siferd and Benton* (1992)). Time-related constraints used in this study are:

Constraint 1: Each nurse can only work one shift in 24 hours

Constraint 2: Maximum number of assignments per week

Constraint 3: Nurse's preference

Constraint 4: Number of consecutive working days

Note that the threshold for Constraint 2 and Constraint 4 are determined by the hospitals according to their scheduling rules. The planning stage can be formulated as follows:

$$\min_{x \in X} \sum_{t=1}^{T} (R_t - \sum_{i=1}^{I} x_{it})_+ + E[J_1(x,F_1)] \tag{3}$$

s.t.

$$\sum_{t=7(wk-1)+1}^{wk} x_{it} \leq L_1, wk = mod(T,7),\ i \in I \tag{3a}$$

$$x_{it} \leq (1 - P_{it}),\ i \in I,\ t \in T \tag{3b}$$

$$\sum_{k=1}^{T-C} \sum_{t=k}^{k+C} x_{it} \leq C \tag{3c}$$

The function $f(x) = \sum_{t=1}^{T}(R_t - \sum_{i=1}^{I} x_{it})_+$ in equation (3) is the total shortage of nurses in the planning stage. The plus function here indicates that only the shortage of the nurse workforce will be counted and the surplus workforce will not contribute to the objective function. The function $J_1(x, F_1)$ is the total cost of adjusting the schedule x during the Operation Stage due to the filtration F_1. Four rules are used as constraints in the planning stage:

• Inequality (3a) sets the workload limit where each nurse cannot work more than L_1 shifts in a calender week.

• Inequality (3b) represents the preferences of nurses.

• Inequality (3c) limits the number of consecutive working days for each nurse cannot exceed C days.

An eligible roster will be generated by using 0-1 integer programming approach as an initial schedule.

3.3.2 Operation Stage

The Operation Stage aims to find out the flexibility of a certain roster. Flexibility of a roster is defined by the total number of shortages that the current roster is unable to fulfill by overtime when unexpected demand occurs. As mentioned in the model assumption, no reserved pool of nurses exists, which means when an unexpected demand occurs and they require extra hands, and the nurse unit has to solve the workforce shortage by finding overtime nurses. Similar to the planning stage, the new roster has to satisfy the constraints to which the current one is subjected to. However, for the particularity of nurse overtime, some of the constrains in the planning stage can be relaxed according to hospital overtime policies. For example, since no federal (and only a few state) regulations restrict the number of hours a nurse can work in a 24-hour period or over a period of 7 days (*Page et al.* (2004)), the threshold for Constraint 2 can be different from the threshold in the planning stage. Also, Constraint 3 could be ignored. Moreover, the new roster must be as similar as possible to the current one. A new constraint is set to ensure the similarity:

Constraint 5: If a nurse was assigned a job on certain roster he/she could not miss that shift unless he/she requests for an absence.

The decision variables a_{it} in the operation stage are actions that can be made under the current roster against demand uncertainty. A forward dynamic programming approach is used to find out the flexibility of a roster:

$$J_t(S_t,F_t) = \min_{a_t \in A_t(S_t,F_t)} (\sum_{i=1}^{n} x_{it} + \omega_t - \sum_{i=1}^{n} a_{it})_+ + E[J_{t+1}(S_{t+1},F_{t+1}|F_t)], t = 1,...,T-1 \tag{4}$$

s.t.

$$\sum_{t=1}^{T} x_{it} + (1-\eta_{it})(a_{it} - x_{it}) \le L_2,\ i \in I \tag{4a}$$

$$a_{it} \le (1 - P_{it}),\ i \in I \tag{4b}$$

$$a_{it} \le \frac{3C - c_{it} - x_{i(t+1)} - x_{i(t+1)}x_{i(t+2)} - x_{i(t+1)}x_{i(t+2)}x_{i(t+3)} - 2}{2C-1}, i \in I \tag{4c}$$

$$a_{it} \le (1 - \xi_{it}),\ i \in I \tag{4d}$$

$$a_{it} \le 1 - \eta_{it}$$

$$-a_{it} \le x_{it}(\eta_{it} - 1), i \in I \tag{4e}$$

The state $S_1 = x$ and S_{t+1} is a function of (a_t, S_t, F_t), the transaction function is:

$$S_t = a_t \tag{5}$$

.

• Inequality (4a) sets the workload limit where each nurse cannot work more than L_2 shifts during the planning horizon.

• Inequality (4b) represents the preferences of nurses.

• Inequality (4c) limits the number of consecutive working days.

• Inequality (4d) represents the nurses' inclination to work overtime.

• Inequality (4e) indicates on-duty nurses can only leave their shift because of absenteeism.

The roster generated from the planning stage will be used as the initial roster in the operation stage. A random demand is generated base on the forecast demand and estimated volatility. Appendix A shows the algorithm used to solve the dynamic programming problem.

3.4 Solution Approach

The formulated optimization problem is a two-stage dynamic programming problem. The formulation presented in the previous section was implemented with Matlab. The main algorithm structure is shown in Figure 3.1.

At the beginning, algorithms parameters are set, namely, number of nurses, planning period length, threshold for every constraints and number of iterations in the Operation Stage. A initial roster is generated by using the mix-integer programming approach(ILP). This initial roster will be sent to Operation Stage and return a cost value by the approximate dynamic programming approach (ADP).

The basic idea of approximate dynamic programming approach is shown in Figure 3.2. To explain it with the nurse rostering problem: S_t is the schedule for all nurses at the time of shift t; $a_t \in A_t(S_t, F_t)$ is the set of available actions at the t^{th} shift under the overtime constraints; S_t^a is the schedule after applying the best action a_t at the t^{th} shift; $g_t(a_t, S_t, F_t)$ is the total number of unsatisfied demand right after applying action a_t; $J(S_{t+1})$, the value function of S_{t+1}, which represents the future cost if applying action a_t; $\overline{J}_k(S_{t+1})$ is the approximate value of $J(S_{t+1})$ in k^{th} iteration. A detail algorithm that generate a approximate value function is shown in Figure 3.3 (by *Powell* (2007)).

The roster generated from the Planning Stage will be used as the initial state S_0^1. We are going to run the algorithm iteratively. Assuming we are in iteration k and at the shift t, which means we are in state S_t^k. The best action could be found by solving:

$$\hat{j}_t^k = \min_{a_t \in A_t(S_t, F_t)} (\sum_{i=1}^{n} x_{it} + \omega_t - \sum_{i=1}^{n} a_{it})_+ + \bar{J}_t^{k-1}(S_t^{a,k}(S_t^k, a_t^k)) \tag{6}$$

here use the notation S_t^k to indicate that we are at a particular state rather than at a set of all possible states. Let a_t^k be the solution solves equation (6). We next need to update the value function approximation. $\hat{j}_t^k$ is a sample realization of the value being in state S_t^k so that the value function approximation will updated by using:

$$\bar{J}_{t-1}^k(S_{t-1}^{a,k}) = (1-\alpha)\bar{J}_{t-1}^{k-1}(S_{t-1}^{a,k}) + \alpha \hat{j}_t^k \tag{7}$$

with the action a_t^k, the post-decision state variable will be found with:

$$S_t^{a,k} = a_t^k \tag{8}$$

After recalculate the total workload and number of consecutive working days for each nurse, it will go to the next iteration. After K^{th} iteration, the value table $(\bar{J}_t^K)_{t=0}^T$ will be generated.

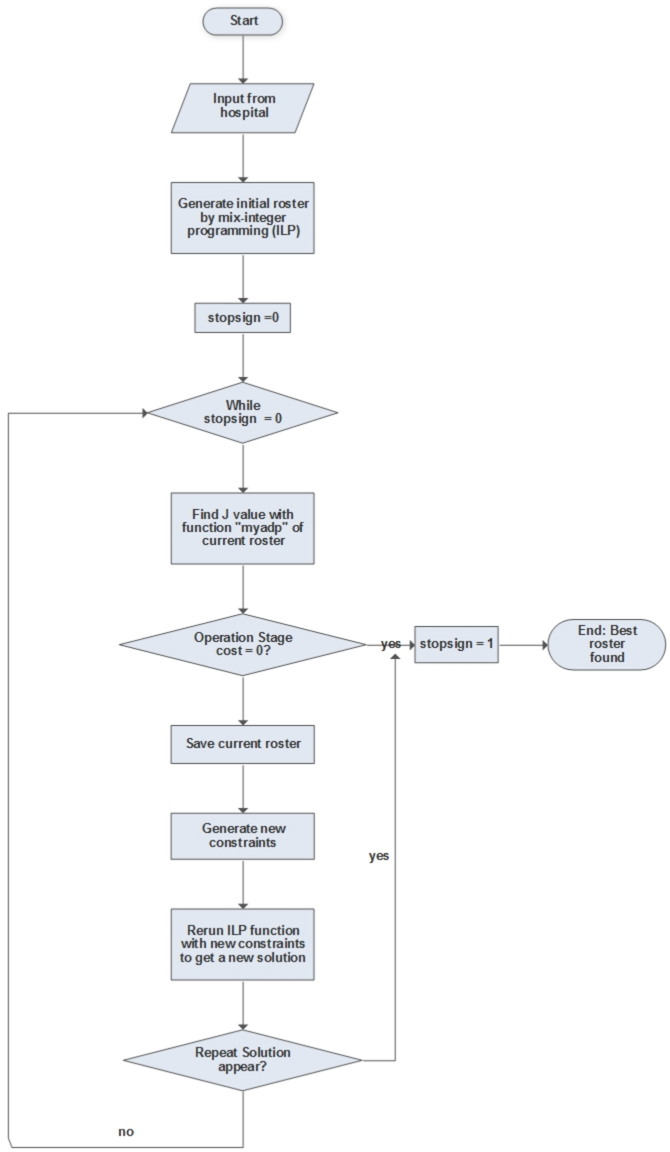

Figure 3.1: The structure of the proposed two-stage dynamic programming model

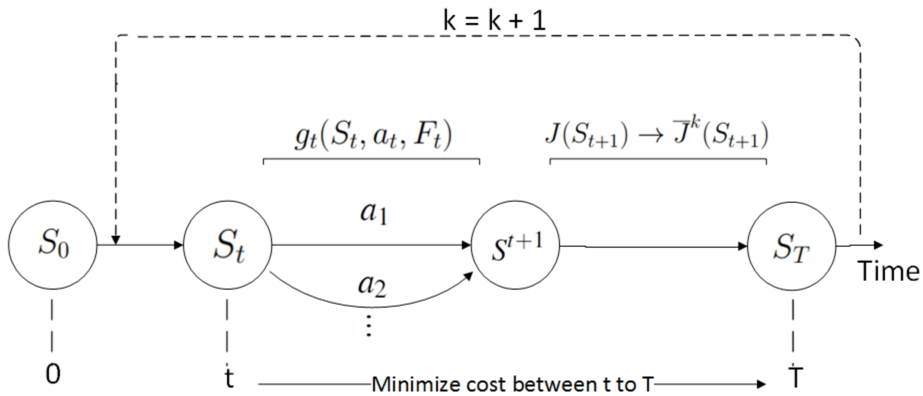

Figure 3.2: Theory diagram of typical ADP solution

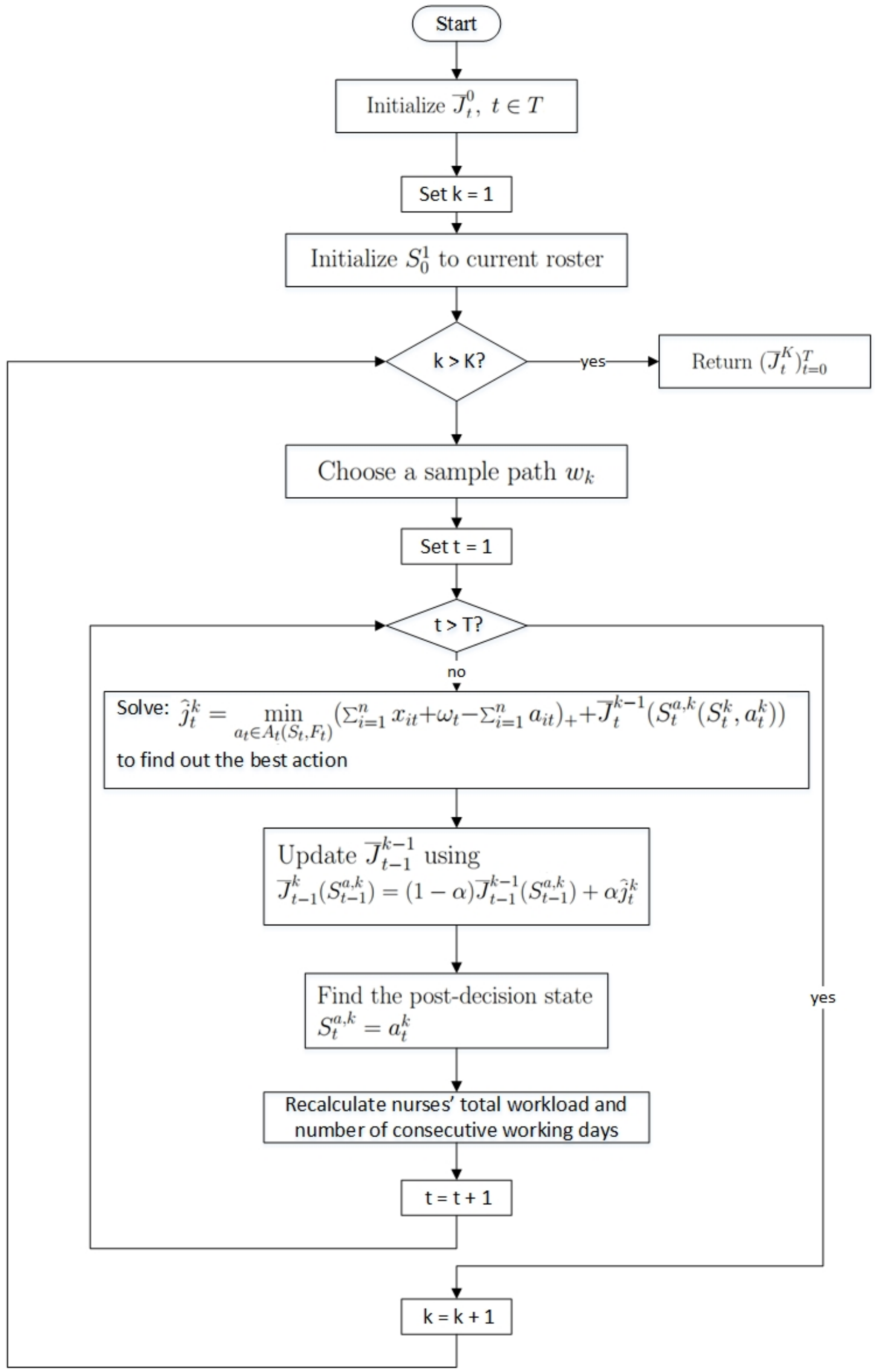

Figure 3.3: The structure of the approximate dynamic programming

This Operation Stage cost $J_0(\hat{x})$ will be found from the value table and put back to the Planning Stage. Since the cost is associated with the current roster, new constraints are added to the Planning Stage. Let $x = (x_1, x_2, ..., x_N)$ represents the set of decision variables and $\hat{x} = (\hat{x}_1, \hat{x}_2, ..., \hat{x}_N)$

represents the current roster. A dummy variable y is added to the object function of the Planning Stage as coefficient to the Operation Stage cost of current roster:

$$y = \begin{cases} 1 & , if\ x = \hat{x} \\ 0 & , otherwise \end{cases}$$

so the objective function of the Planning Stage will become:

$$\min_{x \in X} \sum_{t=1}^{T} (R_t - \sum_{i=1}^{I} x_{it})_+ + y \cdot J_0(\hat{x}) \tag{9}$$

where $J_0(\hat{x})$ is the cost found in the value table for current roster.

In order to make sure the dummy variable y will equal to one only when every decision variable is the same to the current roster, another set of dummy variables z^+ and z^- are added to the constraints of the Planning Stage as assistance to dummy variable y. The relationship between decision variable x, current roster $\hat{x}$ and dummy variables z^+, z^- are:

$$|x_i - \hat{x}_i| = z_i^+ + z_i^-,\ i \in I \tag{10}$$

$$x_i - \hat{x}_i = z_i^+ - z_i^-,\ i \in I \tag{11}$$

when $x = \hat{x}$, value of y should satisfy equation (12):

$$1 - y \leq |x_1 - \hat{x}_1| + ... + |x_N - \hat{x}_N| \tag{12}$$

by combining equation(10) to (12), the following new constraints will be added to the Planning Stage:

$$z_1^+ + z_1^- \leq 1$$

$$\vdots$$

$$z_N^+ + z_N^- \leq 1 \tag{13}$$

$$-y - z_1^+ - z_1^- - ... - z_N^+ - z_N^- \leq -1 \tag{14}$$

After adding the new constrains to the Planning Stage, rerun the ILP model to find next solution and repeat until best solution found.

CHAPTER IV: Case Study: Nurse Stuffing at Nurse Unit 31 of Henry Ford Macomb

Currently, the roster is created by the unit manager manually (see Figure 4.1), which takes 8hrs for creating one roster. In this section the proposed two-stage dynamic programming model was applied to nurse unit 31 in Henry Ford Macomb hospital. This case study aims to (1) evaluate the result from the two-stage dynamic programming model as compared with the hand made schedule and (2) use the empirical data to verify the performance of two-stage dynamic programming model under different degree of uncertainty.

Payroll -10/29/17 to 11/11/17

RN - DAYS	S	M	T	W	T	F	S	S	M	T	W	T	F	S
	29	30	31	1	2	3	4	5	6	7	8	9	10	11
N1	7A	7A		BC	7A						7A	7A	7A	
N2		7A	7A				7A	7A				7A	7A	
N3			7A	7A	7A					7A	7A			7A
N4	7A			7A	7A				7A	7A				7A
N5	7A			7A	7A				7A	R	7A		R	7A
N6		7A	7A	R	R	7A			7A	R	R	7A	7A	
N7		7A	7A		7A				7A			7A		7A
N8			R	7A	7A		7A	7A		7A		7A		
N9	7A		7A		R	7A			R	R	7A	CTO		7A
N10	7A		R	7A	7A				7A	R	7A			7A
N11		7A	R	7A			7A	7A		M		7A	7A	
N12	7A	7A	R			7A				7A	7A		7A	
N13	7A	E			7A	7A			7A	7A			7A	R
N14		7A	R	R	R	7A	7A	7A	7A	7A	R	R	R	

7A	7:00A-7:00P	M	Meeting
R	Request Rest	CTO	Vacation
E	Education	BC	Committee work

Figure 4.1: Manual schedule of Unit 31 at Henry Ford Macomb

4.1 Model Instances and Input Data

The total number of full-time nurses working in nurse Unit 31 is 28: 14 of them are day time nurses and the rest are night time nurses. In this nurse unit, nurses are scheduled for full, which

Table 4.1: Probability of Absence

Rated Level	Probability
Extremely Likely	0.2
Likely	0.1
Neutral	0.05
Unlikely	0

means nurse workload requirements are known before scheduling. A fixed nurse-to-patient ratios of 7.05 was provided by the unit manager. Day time and night time nurses are scheduled separately. A day time nurse could not work at night time and vice versa, which means no nurse will work continually for 24 hours.

Based on the employment types, full time nurses at Henry Ford Macomb work 12 hours a shift. Each 12-hour shift starts from 7 A.M. or 7 P.M. Nurses' preferences are collected by the head nurse two months before the planning period. The preference table (see Appendix A) shows the days that a nurse does not want to work. Unit managers will try their best to satisfy the preference request from each nurse. Nurse workforce demands for each shift and each nurse's preference are given as inputs to the two-stage dynamic programming model. Nurses' preferences are collected by the unit manager two months before scheduling. Specific planning rules are also given:

- No more than three shifts in one calendar week
- Respect nurses' preferences
- No more than three consecutive workdays

Constraint 1 mentioned in the Planning Stage is always met because day shift and night shift nurses are scheduled separately. Unit 31 does not have certain overtime rules to follow. Currently the nurse manager will call off-duty nurse based on his/her knowledge of his/her employees.

A nurses' absenteeism rates are collected by a quick survey. The unit manger should evaluate each nurse's probability of absence into four levels based on their historical performance. These four levels will be transferred into probabilities(see table 4.1).

Nurses' inclination to work overtime rates are also collected by a quick survey. Unit manager evaluates the degree of willingness to work overtime for each nurse based on his/her knowledge of his/her employees.

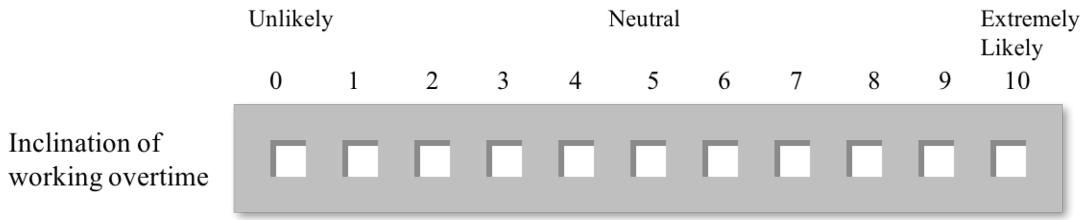

Figure 4.2: Quick survey to collect degree of willingness to work overtime

4.2 Solving nurse rostering problem for Henry Ford Macomb

With the input from Henry Ford Macomb, a roster was generated with the proposed two-stage dynamic programming model (see Figure 4.3). The mix integer linear programming model (ILP) results were generated by running the Planning Stage only, which means setting the cost of Operation Stage to zero.

Two - Stage Dynamic Programing

Payroll -10/29/17 to 11/11/17

S	M	T	W	T	F	S	Unit 31			S	M	T	W	T	F	S
29	30	31	1	2	3	4	Nurse #	Inclination of overtime score	Probability of Absence	5	6	7	8	9	10	11
1	0	0	0	1	1	0	N1	10	0	0	0	1	0	1	0	1
1	0	0	0	1	1	0	N2	10	0.2	0	0	1	1	0	0	1
1	0	0	1	1	0	0	N3	10	0.1	0	1	0	0	0	1	1
0	0	1	1	1	0	0	N4	10	0.05	0	0	1	1	0	1	0
0	0	0	1	1	0	1	N5	9	0	0	1	0	1	1	0	0
0	1	1	0	0	1	0	N6	5	0	1	1	0	0	0	1	0
0	1	1	0	1	0	0	N7	3	0.1	0	1	1	0	0	1	0
0	1	0	0	1	0	1	N8	3	0.05	0	1	1	0	0	1	0
0	1	1	1	0	0	0	N9	5	0.05	0	0	0	1	0	1	0
1	1	0	0	0	1	0	N10	6	0	1	1	0	0	1	0	0
0	1	0	0	1	1	0	N11	3	0	0	0	0	1	1	0	1
1	0	0	1	0	0	1	N12	3	0	0	0	1	0	1	0	1
1	0	1	1	0	0	0	N13	1	0	1	0	0	1	1	0	0
1	1	0	0	0	0	1	N14	2	0.1	1	1	0	0	0	0	1
4	4	4	3	3	5	7	# Able Overtime			8	6	1	5	4	4	4

Nurse's Preference
Able to work overtime
Nurse absenteeism
Nurse refuse to work overtime

Figure 4.3: Roster created by using two-stage dynamic programming model.

ILP

Payroll -10/29/17 to 11/11/17

S	M	T	W	T	F	S	Unit 31			S	M	T	W	T	F	S
29	30	31	1	2	3	4	Nurse #	Inclination of overtime score	Probability of Absence	5	6	7	8	9	10	11
0	0	1	0	1	1	0	N1	10	0	0	0	1	0	0	1	1
1	1	0	0	0	1	0	N2	10	0.2	0	0	0	0	0	1	1
1	1	0	0	1	0	0	N3	10	0.1	0	0	0	1	0	1	1
1	1	1	0	0	0	0	N4	10	0.05	0	0	1	0	1	0	1
1	0	0	0	1	0	1	N5	9	0	0	1	0	1	1	0	0
0	1	1	0	0	0	1	N6	5	0	1	1	0	0	1	0	0
1	1	1	0	0	0	0	N7	3	0.1	0	0	1	1	0	0	1
1	0	0	1	1	0	0	N8	3	0.05	0	0	1	0	0	1	1
0	0	0	1	0	1	1	N9	5	0.05	1	0	0	1	0	1	0
0	0	0	1	1	1	0	N10	6	0	1	1	0	0	1	0	0
0	1	0	1	1	0	0	N11	3	0	0	1	0	1	1	0	0
0	0	0	1	1	0	1	N12	3	0	0	1	1	1	0	0	0
0	0	1	1	1	0	0	N13	1	0	0	1	0	0	1	1	0
1	1	0	0	0	1	0	N14	2	0.1	1	1	1	0	0	0	0
4	4	4	3	3	4	7	# Able Overtime			7	6	1	6	4	4	4

Nurse's Preference
Able to work overtime
Nurse absenteeism
Nurse refuse to work overtime

Figure 4.4: Roster created by ILP model

Manual Work

Payroll -10/29/17 to 11/11/17

S	M	T	W	T	F	S	Unit 31			S	M	T	W	T	F	S
29	30	31	1	2	3	4	Nurse #	Inclination of overtime score	Probability of Absence	5	6	7	8	9	10	11
1	1	0	0	1	0	0	N1	10	0	0	0	0	1	1	1	0
0	1	1	0	0	0	1	N2	10	0.2	1	0	0	0	1	1	0
0	0	1	1	1	0	0	N3	10	0.1	0	0	1	1	0	0	1
1	0	0	1	1	0	0	N4	10	0.05	0	1	1	0	0	0	1
1	0	0	1	1	0	0	N5	9	0	0	1	0	1	0	0	1
0	1	1	0	0	1	0	N6	5	0	0	1	0	0	1	1	0
0	1	1	0	1	0	0	N7	3	0.1	0	1	0	0	1	0	1
0	0	0	1	1	0	1	N8	3	0.05	1	0	1	0	1	0	0
1	0	1	0	0	1	0	N9	5	0.05	0	0	0	1	0	0	1
1	0	0	1	1	0	0	N10	6	0	0	1	0	1	0	0	1
0	1	0	1	0	0	1	N11	3	0	1	0	0	0	1	1	0
1	1	0	0	0	1	0	N12	3	0	0	0	1	1	0	1	0
1	0	0	0	1	1	0	N13	1	0	0	1	1	0	0	1	0
0	1	0	0	0	1	1	N14	2	0.1	1	1	1	0	0	0	0
4	4	4	4	3	4	8	# Able Overtime			8	5	0	6	5	5	3

Nurse's Preference
Able to work overtime
Nurse absenteeism
Nurse refuse to work overtime

Figure 4.5: Roster created by manual work

Different colors were used to mark the nurses' preferences, date of absenteeism, the nurses' willingness to work overtime and nurses that are able to work overtime under the overtime policy. The nurses' absenteeism and inclination to work overtime were generated based on probabilities. The absenteeism score and inclination to work overtime score were shown in the middle of the table. With the same random set, the total number of nurses that can work overtime on each shift will reflect the flexibility of the roster. In this case, on Nov. 17^{th} nobody can work overtime under the ILP roster nor the manual roster. While in the two-stage dynamic programming roster, since all three aspects of uncertainty were counted in the Operation Stage there will still be one nurse that is available to work overtime on Nov. 17^{th}.

4.3 Simulation results and schedule performance

In order to test and compare the performance of the two-stage dynamic programming model, mix integer liner programming model (use Planning Stage as model only) and the manual work from the unit manager, computational experiments1 were performed with 500 Monte Carlo simulation samples. These simulation instances were generated by randomly creating a number of new coming demands, absences and inclination to work overtime table by their distribution or probability. Experiments are used to verify the performance of the proposed model under different levels of demand uncertainty and absenteeism uncertainty.

4.3.1 Impact of demand uncertainty

By increasing the standard deviation of the extra demand ω, the probability of having extra demand become higher, which indicates the increase of demand uncertainty.Experiments results presented in Figure 4.6 show that the average number of unsatisfied demand among 500 Monte Carlo samples with different models. Since the nature of the proposed two-stage algorithm is stochastic, different computational results may be obtained with different random set. So, in order to demonstrate its efficiency, we also present a comparison of standard deviations(see Figure 4.7). Note that for each different input instance, one set of random tables were generated for demand uncertainty, absenteeism uncertainty and inclination to work overtime and used the same set of random tables to compare the performance of three models, and repeating this process 500 times to generate 500 Monte Carlo samples.

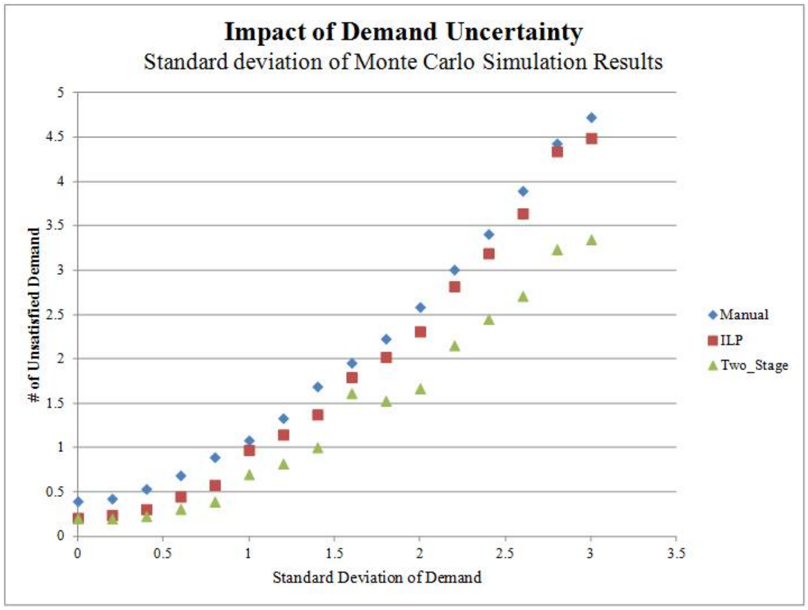

Figure 4.7: Impact of demand uncertainty - Standard deviation of Monte Carlo Simulation Results

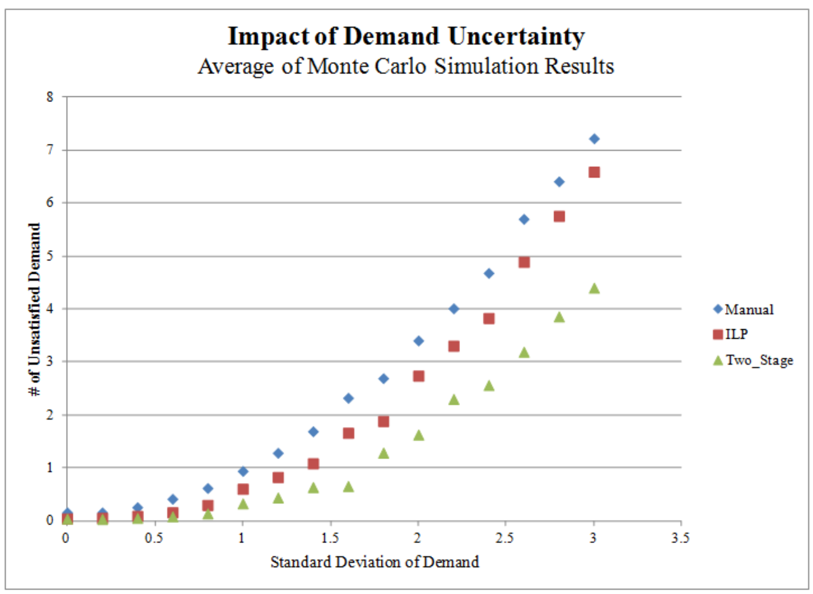

Figure 4.6: Impact of demand uncertainty - Average of Monte Carlo Simulation Results

• In Figure 4.6, the two-stage dynamic programming model is always better than manual work when standard deviation of demand is less than 3. The performance is heightened for the two-stage dynamic programming model with the increase of demand uncertainty. Conversely, the difference between the ILP model and manual work is less obvious.

• Figure 4.7, demonstrates that the two-stage dynamic programming model also gives the smallest variance among these three methods.

4.3.2 Impact of absenteeism uncertainty

Another 500 Monte Carlo simulations are made to test the impact of absenteeism uncertainty. In order to show the variation of uncertainty of absenteeism, a scale factor k was applied to the probability of the the four-lever survey on absenteeism so the probabilities will change base upon the change of the scale factor k. Similar to previous experiments, a mean value (see Figure 4.8) and standard deviation (see Figure 4.9) figures were generated based on the Monte Carlo simulation results. These two figures demonstrate that the two-stage dynamic programming model have the best performance among these three methods. Since the two-stage dynamic programming model takes the uncertainty into consideration, the performance is heightened when increasing the uncertainty of absenteeism.

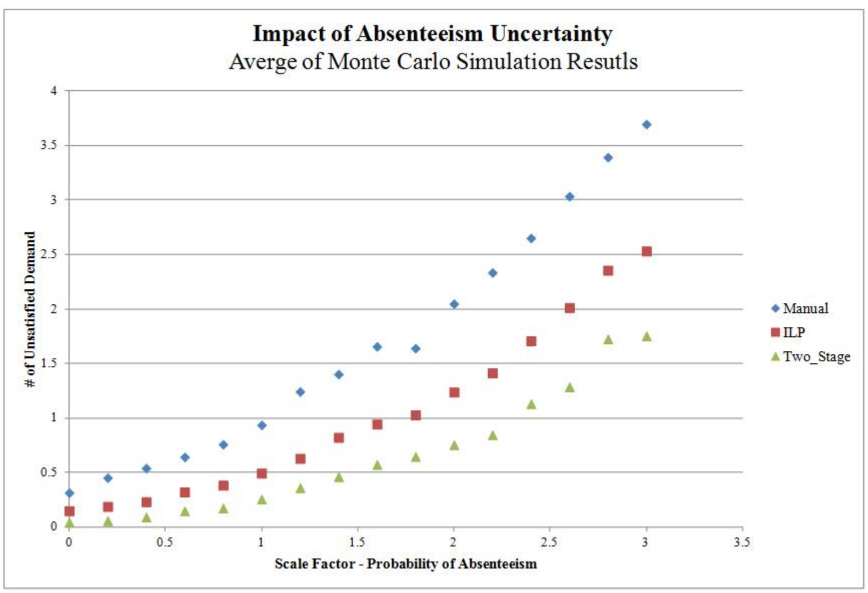

Figure 4.8: Impact of absenteeism uncertainty - Average of Monte Carlo Simulation Results

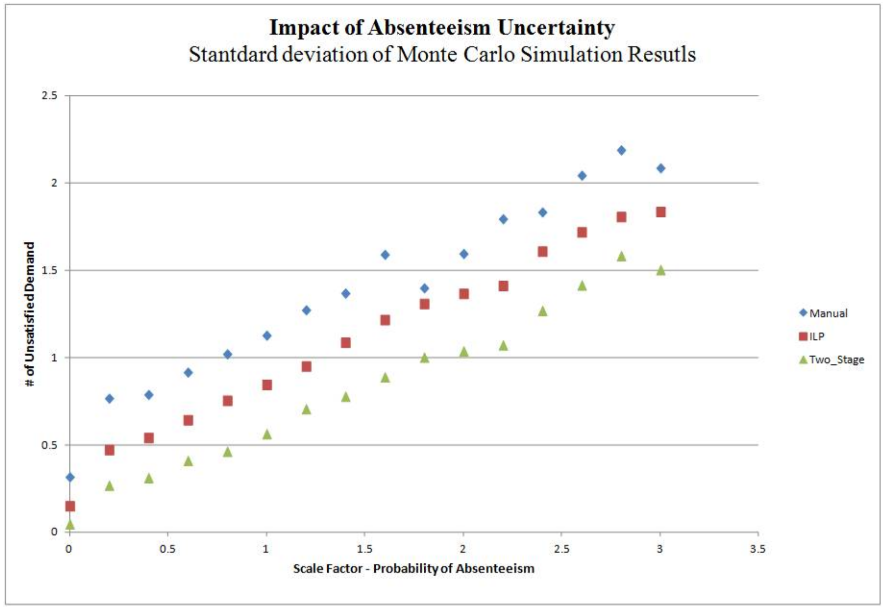

Figure 4.9: Impact of absenteeism uncertainty - Standard deviation of Monte Carlo Simulation Results

CHAPTER V: Conclusion

In this paper, a resilient two-stage dynamic programming model was proposed that considered demand uncertainty, nurse absenteeism uncertainty and nurses inclination to work overtime. For the case study the optimal solution is always giving the best performance compared with manual work and deterministic ILP model among 500 Monte Carlo simulation instances. The performance is heightened for the two-stage dynamic programming model with the increase of uncertainty. Conversely, the performance difference between the ILP model and the manual work is less obvious with the increase of uncertainty, which means the proposed model will work better under a high level of uncertainty.

The idea of the rerostering problem has received very limited attention in the literature. In this paper, it also helps the nurse manager to build a guideline to find the best overtime nurses that not only meet the overtime rules, but also minimize the cost when considering uncertainty.

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APPENDIX A: Nurses’ Preferences

In table A.1, a list of the nurses’ preference collected by the unit manager of Unit 31 at Henry Ford Macomb.

Table A.1: Nurses’ Preference for Unit 31 at Henry Ford Macomb

Payroll -10/29/17 to 11/11/17														Unit 31	Payroll- 11/12/17 to 11/25/17													
S	M	T	W	T	F	S	S	M	T	W	T	F	S		S	M	T	W	T	F	S	S	M	T	W	T	F	S
														RN-DAYS														
			R	R					R	R				N1				R							R			
														N2						R								
		R												N3						R	CTO							
				R				R	R		CTO			N4					R									
		R							R					N5										R				
		R							M					N6									M		R			
		R												N7														
	E												R	N8														R
		R	R	R						R	R	R		N9			R	R	R						R	R		
			BC											N10		R												
														N11														
														RN - NIGHTS														
		R			R									N25														R
		R												N26														
														N27														
														N28														
														N29	R				R	R							R	
														N30														
												R		N31						R							R	
			R			R				R				N32		R		R							R			
	R	R			R			R	R		R			N33		R			R	R			R		R		R	
		R		R	R	R			R		R	R	R	N34	CTO	R	R	R	CTO	CTO				R	R		R	R
		R	R						R					N35			R		R									
		R		R	R			R	R					N36				R	R	R			R	R			R	
		R				R				M	M			N37										R				
														N38														

R: Nurse’s request
E: Education
M: Meeting
CTO: Vacation

APPENDIX B: Roster created by the two-stage dynamic programming model

Payroll -10/29/17 to 11/11/17															Payrol- 11/12/17 to 12/25/17													
S	M	T	W	T	F	S	S	M	T	W	T	F	S	Unit 31	S	M	T	W	T	F	S	S	1	T	W	T	F	S
29	31	3	1	2	3	4	5	6	7	8	9	10	11	1N - DAYS	12	13	14	15	16	17	18	19	20	21	22	23	24	25
0	1	1	0	0	1	0	1	1	0	0	1	0	0	N1	0	1	1	0	0	1	0	0	0	1	0	1	1	0
0	1	1	0	0	0	1	0	0	1	0	0	1	1	N2	0	0	1	0	1	0	1	1	1	0	1	0	0	0
1	1	0	0	1	0	0	1	1	1	0	0	0	0	N3	0	0	1	1	1	0	0	1	1	0	0	0	0	1
0	0	0	1	0	1	1	1	0	0	1	0	1	0	N4	0	0	1	1	0	1	0	1	1	0	0	0	0	1
1	0	0	1	1	0	0	0	1	0	1	1	0	0	N5	1	1	0	1	0	0	0	0	1	0	1	1	0	0
0	1	0	1	1	0	0	0	1	0	1	1	0	0	N6	0	1	0	1	1	0	0	0	0	1	0	1	0	1
1	0	0	1	1	0	0	0	1	0	0	1	0	1	N7	0	0	0	0	0	0	0	1	0	0	0	0	1	1
1	0	1	1	0	0	0	0	0	1	0	1	1	0	N8	1	1	0	0	1	0	0	0	0	0	0	1	1	0
1	1	0	0	0	1	0	1	1	1	0	0	0	0	N9	0	1	0	0	0	1	1	0	0	1	0	0	1	1
1	0	0	0	1	1	0	0	0	1	1	0	0	1	N10	1	0	1	0	0	0	1	1	0	0	0	0	0	0
1	0	0	0	1	0	1	0	0	0	0	0	1	1	N11	1	0	0	0	1	0	0	0	0	1	1	0	1	0
0	1	1	1	0	0	0	0	0	0	1	0	1	1	N12	0	0	1	1	0	1	0	0	1	1	1	0	0	0
0	1	0	0	1	1	0	0	0	1	0	0	1	1	N13	1	0	0	0	0	0	1	0	0	0	1	1	0	0
0	0	1	0	1	0	1	0	1	0	1	1	0	0	N14	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Figure B.1: Roster created by the two-stage dynamic programming model, with $\omega = 1$