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# Working Paper

## Modeling and Measuring Scale Attraction Effects: An Application to Charitable Donations

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Ross School of Business Working Paper
Working Paper No. 1380
July 2017



UNIVERSITY OF MICHIGAN

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# An Application to Charitable Donations

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## Abstract

Charities seeking donations typically employ an "appeals scale," a roster of suggested amounts presented to potential donors, along with an "Other" category. Yet little is known about how the amounts comprising appeals scales affect whether a donation is made and, if so, jointly exert "pull" on its magnitude. Availing of multi-year panel data and a field experiment, we develop a model accounting for individual level donation incidence, amount, and appeals scale attraction effects. The model incorporates heterogeneity across donors in both upward and downward scale point attraction, as well as in donation patterns (e.g., seasonality), and accommodates multiple operationalizations of internal and external referents to summarize the effects of prior donation history and scale points, respectively.

Overall results suggest that scale points do exert substantial attraction effects; that these vary markedly across donors; that they are in fact referent-based effects; that donors are more easily persuaded to give less than more; and that, while all scale points exert pull, influence wanes with distance. The modeling framework applies not only in donation contexts, but whenever an ordered categorical scale is used to collect data regarding an underlying latent response.

## Introduction

Charities are, collectively, among the largest global financial entities. The National Center for Charitable Statistics lists over a million public charitable organizations in the United States alone, with $1.65 trillion in collective revenue, more than Wal-Mart, ExxonMobil, Berkshire Hathaway, and Apple combined, fully 5.3% of GDP.[1] Solicitations for donations have become a part of everyday life, with requests being made at stores, workplaces, through the mail, various traditional media, and increasingly online (e.g., e-mail, websites, social networks). Private citizens have been generous to charities, with over 95% of US households donating per annum in one form or another. To help guide potential donors to both decide to give, and to give generously, charities commonly present them with an "appeals scale"; Figure 1 presents three such scales, used for recent funding drives by the United Way (the largest US charity, with over $3B in annual donations), Wikipedia, and the UN Foundation. Each features the most common sort of appeals scale: a series of specific donation amounts, along with "Other" (i.e., an option to donate whatever amount one wishes). Donors can thereby choose to give some amount not listed on the scale, including amounts outside the range of listed values, or not at all.

Because donors can – and do, as detailed later empirically – avail of an Other amount of their own choosing, one might question why "rational" donors would comply, choosing one of the pre-established scale points instead of some other amount. Regardless, the mere presence of a scale might "pull" donors upwards or downwards (hopefully the former) from what they might have donated otherwise. Such questions are of practical concern for charities, who wish to enhance donation drive effectiveness, and so need to assess appeals scale effects accurately.

Despite their ubiquity in charitable requests and fundraising, there is a lack of model-

[1] As per 2013, the most recent year for which comprehensive statistics are available, and adopted throughout for consistency (http://nccs.urban.org/statistics/quickfacts.cfm).

based guidance as to how appeals scales affect *individual* donor behavior. Part of the problem in providing such guidance is the need for household-level, longitudinal data on both charitable requests and outcomes – "whether" and "how much" – which charities typically possess, along with a (suitably heterogeneous) statistical model for scale attraction effects, which they typically do not. Here, we formulate and estimate such a model, one that incorporates heterogeneity in individual-level "scale attraction" effects, seasonal variation in giving, and an interrelated account for whether and how much to give, calibrated on the results of a field experiment and donation history panel data from a French charity.

The remainder of the paper is organized as follows. We first provide a concise overview of prior literature on scale attraction, donation behavior, reference effects, and related areas. We then describe our empirical application, develop the model, and present both empirical results and model comparisons, followed by general conclusions and potential for additional research.

**Literature Review**

The contextual effects of scale presentation on responses have been intensively examined in social psychology over the past two decades. Schwarz's (1999) comprehensive review suggests that features of research instruments – question wording, format, and scaling, among others – can substantially affect respondents' self-reported behaviors and attitudes, echoing earlier findings summarized by Podsakoff and Organ (1986). In particular, response scales often act as far more than a simple "measurement device," serving as reference frames that influence responses (Schwarz et al. 1991).

It has long been observed that manipulating information on prior donations from others can strongly affect donation behavior (Reingen 1982), as Shang et al. (2008) and Shang and Croson (2009, 2013) found in a variety of fundraising field tests. Several studies have addressed the role of request size on donation behavior (amount and compliance) in laboratory and field data (Doob and McLaughlin 1989, Fraser et al. 1988, Schibrowsky and Peltier 1995, Weyant and Smith 1987). Although contexts and methods vary across them, these studies largely confirm scale manipulation effects, yet differ as to whether they affect donation likelihood, donation amount, or both. De Bruyn and Prokopec (2013), in reviewing this literature, emphasize both the lack of convergence in empirical studies of donation incidence and frequency (e.g., p. 500), and also the importance of individual-level summaries of prior donation behavior, noting that a "...few studies have acknowledged differences in internal reference points... but they have only done so on the segment level." In marketing specifically, such reference effects are a cornerstone and have been supported empirically in dozens of studies (Kalyanaram and Winer 1995 and Mazumdar, Raj, and Sinha 2005 provide extensive reviews for reference pricing, specifically).

We make especial use of one of the key findings from this literature: that two distinct kinds of referents – internal and external – play a role in choice decisions. In donation contexts, as discussed extensively by De Bruyn and Prokopec (2013), the former can be characterized by what the donor "intends" to give, the latter by what the donor is asked to. Specifically, the internal referent is an unobservable that must be inferred from other information (e.g., past donation behavior), while external referents are presented at the time of the request via the appeals scale. Both types of referent were extensively tested and verified by Mayhew and Winer (1992) in the context of frequently-purchased consumer goods, and modeled, using an asymmetric response function concordant with Prospect Theory, by Hardie, Johnson, and Fader (1993), whose formulation we discuss later.

By contrast, perhaps owing to the lack of individual-specific histories, prior accounts of appeals scale manipulation have led to a range of non-consistent results. For example, Weyant and Smith (1987) found no significant difference in the average donation amount between the "smaller request" and "larger request" conditions, only in donation rate. Yet Doob and McLaughlin (1989) suggested, that when the "larger request" is beyond what donors can accept (e.g., outside a latitude of acceptance; Kalyanaram and Little 1994), it exerts negligible effect: when lower amounts were substituted in the "larger request" condition, there was a significant difference in the average donation amount, but none in rate. Two points are relevant here: first, this one change in referent reversed the pattern of substantive results; and, second, researchers should consider, or model, the picture painted jointly by donation incidence and amount.

Another potential source of inconsistencies involves parametric heterogeneity. Most previous studies could avail only of aggregate data (e.g., control / experimental group, or segment level; e.g., Desmet and Feinberg 2003) to assess the mean scale manipulation effect across conditions, potentially diluting the estimated effect of scale manipulation. In this regard, De Bruyn and Prokopec (2013) were exceptional in having obtained each donor's prior donation before the field experiment, using it a proxy for the donor's internal referent. Despite this advance, the one-shot, before/after nature of their data precludes incorporating both dynamics and "unobserved" parametric heterogeneity, which likewise plagues all prior studies relying on cross-sectional data. By contrast, a panel of individual donors provides a superior and dynamic platform to detect and measure scale effects. Panel data further enables us to build up an account of individual donors' internal referents over time, as well as provide a fully heterogeneous account of scale attraction effects.

Lastly, no published study employing scale manipulation has provided a unified account of both donation incidence and donation amount. Presuming *whether* to donate and *how much* to donate are unrelated can introduce well-known measurement errors (e.g., Wachtel and Otter

2013). An especially appealing framework is a Type II Tobit model, which comprises accounts of both incidence and a conditional output of interest (e.g., amount donated). Type II Tobit models have been deployed to analyze disparate contingent consumer decisions (e.g., Ying et al. 2006 for recommendation provision and positivity, Ascarza and Hardie 2013 for usage and retention, Shi and Zhang 2014 for store visit and spending, etc.), with the connection between incidence and amount measured by a correlation parameter. Although not involving scale manipulation specifically, Donkers et al. (2006) and Van Diepen et al. (2009) used such a model in donation contexts, but with somewhat conflicting results regarding correlation; we return to this point later when discussing our own results. In the Conclusion, we discuss a number of behavioral theories that could in principle be assessed using the proposed model coupled with appropriate experimental data; given the nature of our field experiment, we do not engage in such testing here, but do indicate when our findings are consistent with prior frameworks.

**Data description**

Our data were provided by a French charity that conducted a large-scale field experiment as part of a national fundraising campaign. The charity holds three fund-raising drives a year, at Easter, June, and Christmas. Data were collected for 10 periods in total, and consist of household-level records for the appeals scale presented to donors, whether a donation was made and, if so, the donation amount. Donation appeals were made by door-to-door canvassing (and so results pertain to this relatively high involvement method) to “regular” donors, who had always been approached that way in the past; subjects were partitioned into two groups (“levels” 1 and 2) according to their average donation amounts over the two years prior to the start of the experiment. Household-averaged donations in the level 1 and 2 groups fall within 100 FF–199

FF and 200 FF–399 FF, respectively.[2]

The charity sought to better understand the role of appeals scales in donation behavior, so manipulated it in an experiment using random assignment. Throughout, scales (see Table 1) all consisted of five suggested amounts, as well as an "Other" category, which allowed donors to give what they wished. The same scale (90, 150, 250, 500, 1000 FF) was used for all subjects for the first 8 periods of the data, and thereby helps establish a baseline. The scale was then altered for all subjects in period 9, then again for half in period 10, where different "test scales" were used in groups 1 (lower level) and 2 (higher).

[TABLE 1 ABOUT HERE]

The charity thereby implemented a 2 × 2 design: (prior donation) "level 1" or "level 2" × random assignment to either a "standard" or "test" appeals scale in period 10.[3] It is important to note that the charity was collecting real donations, and therefore did not have the luxury of 'optimally' designing scales for experimental purposes, such as orthogonalizing (e.g., some donors asked for *less* than they were accustomed to), including extreme values, and the like. Thus, the points comprising the "test" scale for the level 2 (higher prior) donation group were, quite sensibly for a field test, higher than those for the level 1 group, and potentially constitute a source of endogeneity. We will explore this possibility in the sequel, by estimating the model separately on each group and comparing individual inferences via a multivariate Cramer test.

Four hundred households in each of the four "cells" were randomly selected for analysis. Table 2 presents descriptive statistics for each, average donation amount (per household and per occasion), and yield rate. Level 1 and 2 differ substantially in per-household and in per-occasion

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[2] The charity judges regularity based on donor frequency (number of donations in past two years) and recency (periods since last donation). The distinction was applied both prior to and throughout the data window. Currency is French Francs (FF), trading during the collection window at approximately 7 to the US dollar.

[3] The scale was changed twice (periods 9 and 10) for those in each of the test groups (Level 1 and Level 2), which helps identify both parametric heterogeneity and the effects of referents on the individual level.

average donation amounts ($p < .0001$); this is unsurprising, as the baseline donation amount was used by the charity to partition donors into different levels. However, yield rates are remarkably similar across the four groups, with all between 63% and 69% (differences all *ns*). Moreover, average observed donation fails to differ across the standard and test scales, within a donation level (1: 144.7 standard vs. 148.8 test, $p > .2$; or 2: 268.6 standard vs. 275.4 test, $p > .5$). One might conclude that there were *no* effects attributable to the use of the test scale. As the forthcoming analysis will show, such a conclusion based on aggregate metrics is not only premature, but highly misleading.

[TABLES 2 AND 3 ABOUT HERE]

Table 3 suggests a clear (aggregate) seasonal pattern in both yield rate and average donation amount: people give more, and more often, at Easter than during June or Christmas. The difference in yield rates is striking – nearly ¾ of respondents donate at Easter (an important holiday in France), while just under ¼ do at the other times of year – and these proportions are nearly identical in the level 1 and 2 donation groups (the latter, by construction, has higher donation *amounts* across the board). Holding aside any aggregate patterns, there is nonetheless sizable variation in *household*-level donation profiles, with many households showing a strong preference for giving at particular times of year; this will manifest in the forthcoming model as substantial heterogeneity in seasonality.

**"Model-Free" Evidence of Appeals Scale Effects**

Before building a model, one should ascertain whether there is a phenomenon worth modeling. Table 4 presents "model-free" evidence that quantities manifest unusually strongly when they appear on the scale, vs. when they do not; specifically, all points appearing on either the Standard or Test Scales, for Levels 1 and 2, separately, for the experimental period (10).

[TABLE 4 ABOUT HERE]

For Level 1, the clearest evidence for scale attraction effects can be seen when the 'unusual' amount of 120FF is substituted for 100FF: whereas only 0.2% of respondents donated 120FF when it did not appear on the (Standard) scale, 18.4% did when it was among the five suggested amounts ($p < 0.001$ by Fisher's exact test); a similar difference (0% vs. 6.9%; $p < 0.001$) is apparent for the 180FF quantity. Both of these are sensible test values, given the ~140FF average donation for the Level 1 group (see Table 2). For Level 2 donors, where the average is ~270FF, we might expect similar effects for larger values slotted into the Test scale. And this is precisely what we find: at 200FF (2.6% Standard vs. 18.8% Test; $p < 0.001$) and 350FF (1.2% Standard vs. 10.6% Test; $p < 0.001$). Something analogous happens when a value is *removed* from the Standard scale, for example 150FF in either Level 1 (17.8% Standard vs. 5.4% Test; $p < 0.001$) or Level 2 (7.2% Standard vs. 1.8% Test; $p < 0.001$).[4] By contrast for all values included on *both* scales, as well as choosing not to give – that is, "None", 250FF, 500FF in Level 1, and "None", 500FF in Level 2 – pairwise differences are all *ns* ($p > .5$ in all five cases).

Thus, it seems fair to conclude that the appeals scale points succeed in "relocating" mass in the PDF for donation amounts. But this fails to answer several critical questions: Are all donors equally susceptible to scale effects?; Do all points 'pull' equally well?; Is the pull stronger upwards or downwards?; What is the role of prior donation history?; Are these truly *reference* effects?; among others. To answer these basic questions requires that one go beyond summary "model free" metrics and fashion a model calibrated on the individual histories of many households. Although we are not the first to examine appeals scales in individual donations (e.g., De Bruyn and Prokopec 2013), the model is indeed the first, to our knowledge,

[4] This holds whenever the amount given was over 0.5%. That is, the proportions of 350 & 1000 (Level 1) and 750 & 1000 (Level 2) are *not* significantly different, but this is due to the very small numbers of "large" donations.

that attempts to quantify appeals scale attraction effects. The presentation will therefore be suitably general, although the discussion will largely be tailored to our specific context of charitable donations.

## Model Development

### *Internal and External Referents*

The model hinges on two constructs, as discussed previously and at length in the review of the literature by Mazumdar, Raj, and Sinha (2005): that, for a particular donor, each request can be associated with both an *internal* referent ($r^I$), which the analyst can relate to prior donation history, and *external* scale-point-based referents ($r^E$); if an appeals scale contains multiple points, we denote the $k^{\text{th}}$ as $r^{E,k}$.

A key modeling task is appropriately *summarizing* the effects of both the internal (IR) and external (ER) referents. Both admit different operationalizations, which can be empirically tested for a given model via standard fit metrics. Prior literature offers several options for IR, including most recent prior value (e.g., Krishnamurthi et al. 1992) and variously weighted amalgams of past realizations (e.g., the summary in Table 1 of Briesch et al. (1997). We test five such specifications, two specifically tailored to account for seasonal donation variations: the average of all prior observed donation amounts (IR-1); the last observed donation amount (IR-2); the average observed donation amount at the same time of year (IR-3); and the last observed donation amount at the same time of year (IR-4), and a geometrically-smoothed version (IR-5) that estimates the relative weight (α) on the last observed value (i.e., equation 3 of Mazumdar et al. 2005). Note that IR-2 is a special case of IR-5, with α = 1. These should be viewed not as mental constructs, which is in any case unverifiable, but as univariate autoregressive summary measures of past donation history, among which we will select the empirically best-fitting.

That the external reference (ER) points are observable might make them appear simple, or simpler, to account for. This might be so were there only a single requested amount. But, in practice, there are several, and so it is unclear how they exert their "joint pull": perhaps the extremes are differentially noticed, or discounted; or only those nearest the internal referent have any influence; or some summary measure of all points (like the average or median); or something else entirely.[5] De Bruyn and Prokopec (2013) speak directly to such weighting schemes, finding "leftmost anchor" exerted the strongest pull; this echoed a prediction of Schibrowsky and Peltier (1995), but is contrary to, for example, extremeness aversion (Simonson and Tversky 1992). We therefore consider a wide range of possibilities in the absence of prior theory to suggest how a *group* of referents exert collective influence, an intriguing open issue that our data and model may help address. Specifically, we test whether influence is exerted by: all scale points (ER-1); the two points closest the internal referent (ER-2); the largest and the smallest points (ER-3); the median (i.e., middle) point (ER-4); the mean of all points (ER-5); and all scale points with various weighting schemes, equally (e.g., with relative weights 1-1-1-1-1; ER-6); in a V-shape (3-2-1-2-3; ER-7); inverse-V (1-2-3-2-1; ER-8); increasing (1-2-3-4-5; ER-9); and decreasing (5-4-3-2-1; ER-10).

***Modeling Scale Attraction Effects***

If the appeals scale "pulls" donors' internal referents towards the presented external ones, these separate pulls can cumulate in their effects. A simple metric for scale point influence is its "compliance degree," which we describe next.

[5] Ideally, one would be able to estimate "weights" on each of the scale points, and do so heterogeneously across donors. Given donation histories and incidence rates in real charitable data, however, one must choose battles carefully in terms of where to place heterogeneity. Detailed simulations suggest that recovering more than four heterogeneous parameters for our data is precarious (in line with the findings of Andrews, Ainslie, and Currim 2008), and we reserve these for the crucial constructs of scale attraction and seasonality.

**1. Compliance Degree**

We define $CD^k$, "compliance degree" of the $k^{\text{th}}$ external reference point as the proportional increase (or decrease) from a donor's *internal* reference point ($r^I$) to an *external* one ($r^{E,k}$). Specifically (with *DA* = Donation Amount received):

$$CD^k = (DA - r^I)/(r^{E,k} - r^I) \tag{1}$$

For example, if a donor has (latent) internal referent \$100, but is asked for \$300 and partly complies by giving \$150, $CD^{k=1}$ = (\$150 - \$100) / (\$300 - \$100) = 25%. That is, the donor "came up 25%" from a \$100 baseline. It is convenient to define the *distance*, $d^k$, between the $k^{\text{th}}$ external and the internal referent as a (positive) ratio:

$$d^k = \|r^{E,k} - r^I\|/r^I \tag{2}$$

This allows both compliance degree and pulling amount (described later) to be expressed as dimensionless quantities, which in turn helps to unify the model; for example, comparing a donor planning to give \$10, but gave \$20, to one planning to donate \$100, but asked for \$200.

We model both upward and downward "compliance degree curves", which satisfy three properties: (1) $CD^k \approx 1$ for $d^k \approx 0$: "Maximal compliance occurs near donors' internal referents"; (2) $CD^k$ decreases monotonically in $d^k$: "Compliance is worse for requests further from the internal referent"; and (3) $CD^k \geq 0$: "Compliance can't be worse than zero." Properties 1 and 2 suggest donation is highly responsive to asking for amounts close to what was 'planned' (the internal referent), but increasingly less so for distant amounts. Property 3 simply suggests that requests can be ignored, but do not literally repel donors from a scale point.

Among the many ways to specify compliance degree curves satisfying these three properties, we select a translated gamma kernel function, for two reasons. First, it provides a parsimonious, yet flexible, functional form; this is important for a *heterogeneous* account to be identified, given the small number of responses per donor during the data window. Second, the gamma kernel enables the pulling *amount* curves (described later) to follow a non-multimodal, yet flexibly-shaped, distribution. Specifically:

$$CD^k = exp(-d^k/\theta) \; ; \; \theta = \begin{cases} \exp(\beta^U), r^{E,k} \geq r^I \\ \exp(\beta^D), r^{E,k} < r^I \end{cases} \tag{3}$$

where $\theta > 0$ is the gamma kernel scale parameter; shape parameter is set at 1.[6] When $r^{E,k} \geq r^I$, we have an "upward" compliance degree curve, and otherwise a "downward" one. Since the scale parameter ($\theta$) must be positive, we specify $\beta^U$ or $\beta^D = ln(\theta)$, where $\beta^U$and $\beta^D$are the "upward" and "downward" parameters in (3). Note that $\beta^U = \beta^D$ does not imply identical upward and downward curves, because the domain of the downward curve is bounded by 100%, since one cannot give less than zero (i.e., a 100% decrement).

Although our model is novel in its account of scale attraction effects, specifically, it is hardly the first to accommodate asymmetric (i.e., upward and downward) *reference* effects in an empirical context. Hardie, Johnson, and Fader (1993) built a model that explicitly encoded the possibility of different weighting of both price and quality deviations (from one's last purchase) in utility, estimating the model on packaged goods. Our formulation, while similar in some respects, further accounts for the role of multiple external referents (the appeals scale), a variety of internal referent specifications, nonlinearity in utility, latent correlation in incidence and amount, and a more flexible (hierarchical Bayesian) account of "unobserved" heterogeneity.

[6] Fixing the shape parameter at 1 yields a non-negative, monotonically decreasing, convex curve (with regard to the origin), satisfying properties 1-3. Numerous simulations showed recovery of *two* parameters (both scale and shape) was very poor, suggesting weak identification in data generated to resemble ours. Note that there are three sources of heterogeneity identification: (1) standard (MVN) distributional assumptions about the heterogeneity distribution; (2) between- group and within-donor scale variations; and (3) the nature of the internal and external referents, which each take multiple forms.

## 2. Pulling Amount

The pulling amount ($PA^k$) is the *size* of effect exerted by a scale point, the product of compliance degree and distance between the internal ($r^I$) and the $k^{\text{th}}$ external referent ($r^{E,k}$):

$$PA^k \;=\; CD^k \times ||r^{E,k} - r^I|| \qquad (4)$$

Pulling amount captures a trade-off between asking for too little and too much: If a charity asks for just a bit more than the internal referent, compliance ($CD^k$) may be high, but the potential surplus ($||r^{E,k} - r^I||$) is small. Conversely, asking for too much leads to low compliance and large surplus. This trade-off (where the extremes are literally zero) guards against 'highly influential' scale points being placed too close or too far from internal referents.

Equation (4) implies that both "upward" and "downward" pulling curves also follow a gamma kernel, with shape parameter 2 and scale parameters $\exp(\beta^U)$ and $\exp(\beta^D)$. As depicted in Figure 2, these curves can take a variety of shapes: the upward pulling curve has domain $[0, \infty)$, is unimodal (and thus has a unique maximum), with zero at the origin and asymptoting to zero for large $d$ (for any $\beta^U$). The domain of the downward pulling amount curve is [0,1]; it is unimodal (with unique maximum) if $\beta^D < 0$, and is monotonically increasing otherwise (with maximum at 1). These internal maxima map bijectively to $\{\beta^U, \beta^D\}$, and so provide an equivalent projection of the parameters onto a meaningful metric: which upward and downward scale amounts (proportions above the internal referent) provide the strongest expected deviations. Appendix C derives closed-form expressions for these, which we will use for graphical purposes.

[FIGURES 2 AND 3 ABOUT HERE]

## 3. Accumulating Scale Attraction Effects

Because real appeals scales invariably comprise multiple points, their effects need to be somehow combined. Figure 3 illustrates the "accumulated pulling amount" accruing from multiple scale points; to match our empirical application, five external referents – three greater, two lesser – are depicted, with upward and downward curves on either side of the graph.

Because the charity did not change scales many times across the 10 periods (nor within each of the four donation groups), identifying *interactions* among scale points is not possible. Thus, the effect of each scale point is modeled separately. This is partly mitigated by the weighted-averaging schemes explored for the "accumulated pulling amount", or *APA*. In general:

$$APA \ = \sum_{k=1}^{K} w^k \times I^k \ \times PA^k; \ I^k = \begin{cases} 1, if\ r^{E,k} \geq r^I \\ -1, if\ r^{E,k} < r^I \end{cases} \qquad (5)$$

Summing the scale pulls (i.e., $w^k = 1$) is simple and intuitive, but has a shortcoming in the effect of including *additional* scale points (not testable here, as the charity fixed this at 5). For example, given internal referent 50, the *APA* of the four-point scale {9, 11, 99, 101} would be about twice as strong for the two-point scale {10, 100}, which seems unrealistic. Averaging ($w^k = 1/$k) addresses this, but raises other problems. For example, if a donor is asked for $2000 when the planned amount is $100, the real effect of such a "distant ask" might be negligible. However, equal weighting suggests a sizable effect, which again seems unrealistic. A simple rescaling, i.e., $w^k = \frac{PA^k}{\sum_{k=1}^{K} PA^k}$ , addresses both issues, while retaining proportionality. As mentioned previously, data limitations (indeed, for any data likely to be available in a charity-based study) precluded measuring $w^k$, leading to empirically testing the 10 weighting schemes of ER-1 through ER-10.

### ***General Model (Type II Tobit)***

We outline the general model structure, which affords a "dimensionless" account of pulling effects, so that heterogeneity can be specified across the log-scale for donation amount. As discussed, a Type II Tobit jointly accounts for donation incidence ("selection") and amount:

$$y^{s*} = X^s\beta^s + \epsilon^s \quad (6)$$
$$y^{a*} = ln(r^I + APA) + X^a\beta^a + \epsilon^a, \text{ where:}$$
$$y^s = 1, \text{if } y^{s*} \geq 0;\ 0 \text{ otherwise}$$
$$y^a = y^{a*}, \text{if } y^s = 1;\ \text{unobserved otherwise}$$
$$(\epsilon^s, \epsilon^a) \sim BVN(0, \Sigma_\epsilon)\ ;\ \Sigma_\epsilon = \begin{bmatrix} 1 & \rho\sigma \\ \rho\sigma & \sigma^2 \end{bmatrix}$$

The subscripts *i* and *t* (for donor and time) are suppressed, and $X^s$ and $X^a$ are covariates in the selection (*s*) and amount (*a*) equations, respectively, which we detail below.

In the amount equation, $y^{a*}$ denotes the log of the latent donation amount, which is observed only when a donation is made, that is, when $y^s = 1$, which occurs when the latent variable $y^{s*} \geq 0$. The errors ($\epsilon^s$, $\epsilon^a$) are bivariate normal, with variance of $\epsilon^s$ fixed to 1 for identification. It is important to note that we model the logarithm of donation amount, for several reasons: first, it allows $\epsilon^a$ to be plausibly homoscedastic; second, it allows all effects in the amount equation to enter multiplicatively; and third, it allows for coefficient heterogeneity to act on a dimensionless quantity, which we address in detail shortly.

The amount equation (for $y^{a*}$) contains two deterministic components. The first is the sum of a donor's internal referent ($r^I$) and the accumulated pulling amount (*APA*), which can be positive or negative. The second is all factors ($X^a\beta^a$) that affect the donation, other than those stemming from the appeals scale. Scale-based effects do not appear directly in the selection equation, because in our data all scales used were set in "reasonable" ranges for every donor (recall that these were real donors, and the charity was reluctant to alienate them with unrealistically high requests, or lose funds with low ones). The appeals scale exercises influence on donation incidence via the correlation, $\rho$. [A model was estimated allowing for scale effects in selection; the *APA* coefficient in selection was *ns*.]

***Explanatory variables and Heterogeneity***

**1. Explanatory variables**

*Selection equation*

The selection equation contains three types of explanatory variable ($X^s$), which we detail subsequently: seasonal indicators, (log of) prior donation, and "level" fixed effects. Table 3 reveals strong aggregate seasonal variation in donation likelihood, by far highest at Easter. Three dummies – Easter ($X_{it}^E$), June ($X_{it}^J$), Christmas ($X_{it}^C$) – represent when the request occurred. The log of (1+ amount the donor gave on the last request), donated $X_{it}^{lag}$, is included to examine carryover effects, and is 0 when no donation takes place.[7]

Although Table 3 suggests only modest differences in yield rate between the "larger" (level 2) and "smaller" (level 1) donation groups, we include a Level dummy ($X_i^{level}$) among the selection covariates, to allow for potential differences in baseline donation likelihood *after* accounting for seasonal patterns. Coefficients for the three seasonal dummies, the log-donation lag, and the level dummy, are denoted $\beta^E$, $\beta_i^J$, $\beta_i^C$, $\beta^{lag}$, and $\beta^{level,s}$, respectively. In the experiment, donors were randomly assigned to receive either a Standard or a Test appeals scale (during period 10), so no dummies were entered for this difference (in either selection, or amount). Doing so failed to improve fit, in any case, so we do not discuss these again.

***Amount Equation***

Based on examination of the data and unimproved fit of models including them, seasonal dummies are not included in the amount equation; the somewhat higher amounts indicated at Easter in Table 3, for example, will be well-explained by other covariates, like lags in setting "internal" referents (such as in IR-3 and IR-4). The data suggested great household variation in

[7] Replacing the log-donation lag with an indicator for *whether* one donated in the previous period led to poorer in-sample fit, possibly reflecting that the continuous variable (log-donation) carries additional information.

when to give, not how much; and that household-level seasonal variation in amount (not incidence) is small for most donors. Lastly, although donation amount is mainly predicted by a donor's internal referent and scale effects, a level dummy ($X_i^{level}$) is included to account for the difference in baseline donation amount between the two groups, denoted $\beta^{level,a}$ .

***Heterogeneity***

It is critical to incorporate "unobserved" heterogeneity, which we do in several ways. First, we model heterogeneity in the seasonal dummies for the June and Christmas coefficients ($\beta_i^J$ and $\beta_i^C$).[8] Importantly, since the model intends to capture scale attraction effects, the two "pulling" parameters ($\beta_i^U$ and $\beta_i^D$) in the amount equation are heterogeneous. If for example $\beta_i^U$ were homogeneous, each donor is presumed equally 'elastic' in being cajoled upwards. Our results will in fact strongly weigh against this presumption. To test implications across models at the individual level, we will use a multivariate generalization of the Kolmogorov-Smirnov test, the Cramér-von Mises statistic, on the individual-level joint posteriors for $\{\beta_i^U, \beta_i^D\}$.

Our formulation therefore specifies four heterogeneous parameters, to be recovered from the relatively short data window of 7 occasions, roughly 3 of which resulted in donations, on average. Although this may appear ambitious, simulations showed good recovery for all four heterogeneous parameters, and excellent recovery of the others.

***Estimation***

The full model (see appendix A) is estimated using Markov chain Monte Carlo methods. Data augmentation (Tanner and Wong 1987) essentially converts the model to a Bayesian Hierarchical Seemingly Unrelated Regression. We obtain posterior draws via Metropolis-within-

[8] Extensive simulations for data generated using the proposed model failed to recover the true parameters (mean and covariance matrix) for $\{\beta_i^E, \beta_i^J, \beta_i^C\}$ heterogeneous. Restricting the most common donation period (Easter, with a >70% yield rate) to be homogeneous led to nearly perfect parameter recovery. In short, almost all households give at Easter at least occasionally, but there is large variation in giving patterns for Easter and June.

Gibbs algorithms: Gibbs sampling if the full conditional of a parameter block is of known form, and Metropolis-Hastings, with a random walk proposal (Chib and Greenberg 1995), otherwise. We set diffuse priors for all parameters of interest; detailed procedures appear in Appendix B. All estimates are based on 100,000 draws. We discard the first 50,000 draws for burn-in, and use the last 50,000 (thinned to every tenth) to calculate posterior densities. Trace plots and standard diagnostics indicated convergence of all key parameters.

**Results**

Model selection was based on LPML (log pseudo-marginal likelihood) which, as noted by Chen et al. (2008) works particularly well for GLM-type models, and more generally by Chen and Kim (2008). For brevity, we only present full estimation results for the model with IR-1 (average of all observed donation amounts) and ER-1 (all scale points), as these provided the best fit compared with all possible combinations of the other internal and external references point formulations (i.e., IR 2-5 and ER 2-10). Table 5 summarizes posterior means and standard errors for all parameters. Detailed model comparison statistics appear in Table 6.

[TABLES 5 AND 6 ABOUT HERE]

***Error Correlation in Selection and Amount equations***

The mean of the marginal posterior for the error correlation, $\rho = -0.454$, between selection and amount is negative, and the 95% highest density region (-0.516, -0.385) is far from zero. This suggests that unmeasured factors influencing selection are correlated with those influencing amount, and operate in opposite directions. For example, a donor might, for some “latent” reason, be saving up to give a larger donation, lowering frequency and raising amount; or, conversely, may compensate for not having given for a while with a larger donation. The size of the correlation is moderate: neither close to 0 nor to 1. This differs from findings in previous research using related model formulations; for example, Donkers et al. (2006) found the correlation to be very slightly negative (-0.033; $p < .001$), while Van Diepen et al. (2009) found it to be large and positive, with a 99% credible interval of (0.946, 0.970). Very small correlations may fail to correct for potential selection biases, or could reflect substantial, independent sources of error in each equation. Conversely, a large correlation might suggest nontrivial variables omitted in both equations. It is difficult to generalize such results, since our model accounts for scale attraction effects, while prior ones do not. We did, however, find significant, moderate, negative values of $\rho$ across a very wide range of candidate models, indicating that error correlation needs to be accounted for in our data.

We note in closing that $\rho$ is a *residual* correlation, and is distinct from any "model-free", observable correlation that might exist in the data, like between number and amount of donations made. This latter sort of correlation is computed *across* donors, but $\rho$ could be assessed even for a single donor, if his/her donation history were long enough. Lastly, $\rho$ is theoretically and empirically distinct from scale attraction effects *per se*: for example, although not having given in (say) the first two periods may make it more likely one will donate in the third (or donate *more* in the third), it should not make it more likely *that one will move closer to a scale point*.

***Selection: Seasonality***

Comparing the Easter coefficient (0.708) to the (heterogeneous) ones for June and Christmas (-0.505 and -0.993, respectively) accords with the observation that giving was much more likely for Easter, on average. There is a substantial seasonal heterogeneity: the SDs of individual-level parameters for June and Christmas are 0.390 and 0.777, respectively. The large (0.714) correlation between these individual-level parameters largely reflects the fact that June and Christmas yield rates are both low (Table 3) and a high proportion of donors gave at neither time; nevertheless, the concordance of the model's parameters with aggregate benchmarks is reassuring.

***Level Dummies and Lagged Log-Amount***

The level dummy is moderately significant (mean 0.108, SE 0.032) in selection, but strongly positive in amount (mean 0.260, SE 0.013). So, as aggregate statistics suggest, level 2 donors give far more than those in level 1, but with modest difference in yield rates. The coefficient of the log-donation lag in selection is significantly negative (mean -0.123, SE 0.006), indicating that a larger donation *amount* last time leads to being less likely to give *at all* this time.

***"Pulling Effects": Gamma Kernel Parameters in Donation Amount***

The values of $\beta_i^U$ and $\beta_i^D$ determine each donor's degree of compliance ("pull") to the scale points above and below the internal referent. Because the domains of the two compliance curves differ, we should not compare $\beta_i^U$ directly to $\beta_i^D$. Figure 4A in some sense encapsulates our main results: the upward and downward pulling parameters (posterior means of $\beta_i^U$ and $\beta_i^D$) for each donor. There is clearly a good deal of heterogeneity, indicating differing degrees of susceptibility to the appeals scale, *despite only modest differences in prior donation behavior*. That is, although these donors may seem similar in terms of observed donation *behavior*, they apparently are not in terms of how swayed they are by the appeals scale.

[FIGURE 4 ABOUT HERE]

By allowing a bivariate density for $(\beta_i^U, \beta_i^D)$, the model helps assess *overall* scale compliance. Specifically, we find a substantial correlation (0.446) in these values, suggesting that donors who are "upward compliant" tend to be "downward compliant" as well. There is no *a priori* reason to expect these should be correlated at all, let alone positively, and we believe this finding to be the first of its kind. As mentioned previously, the bivariate density for $(\beta_i^U, \beta_i^D)$

maps to the joint distribution of *maximal pulling amounts* (see Appendix C), those scale values associated with the strongest overall effects; we do not call these "optimal", since a large *downward* pull is to be avoided. Heterogeneity in $(\beta_i^U, \beta_i^D)$ leads to substantial variation in maximally effective scale values. The model suggests that the scale point with maximal upward pull, which varies across donors, ranges from 57.2% to 245.5%, with a mean of 94.9%, above one's internal referent, which seems reasonable.[9] This substantial variation has an important implication: that it may be possible to substantially increase donations by *personalizing* an appeals request, based on each donor's history, although such dynamic optimization is nontrivial, and has similarly stringent data history requirements.

Figure 4B translates the model's key substantive findings into the context of the original data, specifically: How much does the maximally-effective "ask" value (either up or down) pull from the internal referent? It depicts, across donors, this maximal percentage increase and decrease (see appendix C for derivation), allowing a direct comparison of upward vs. downward scale attraction "strengths"; this was not sensible using the information on $(\beta_i^U, \beta_i^D)$ in Figure. 4A, given their different domains of operation. Maximum percentage *increases* range from 21.1% to 90.3% (mean = 34.9%; SD = 5.4%); *decreases* from 31.2% to 94.5% (mean = 82.0%; SD = 3.1%). These means suggest, unsurprisingly, that donations are more readily deflected downward than upward. Figure 10 suggests that the maximum percentage decrease is greater than the analogous increase for most donors: 81.6% of the donors lie above the diagonal (dotted) line.[10] This is nonetheless reminiscent of the asymmetric effects in Desmet and Feinberg (2006), whose lack of individual-level data precluded any distributions *across* donors, and De Bruyn and

[9] Discussions with a large university's fundraising team suggested that the success of such "upping" dropped nearly to zero when appeals hit 200% above a donor's typical or last donation amount.

[10] This hypothesis about "up" vs. "down" differences can be tested. For our 1600 participants, "down > up" for 1494/1600 = 93.4% based on 90% HDRs; and for 648/1600 = 40.5% based on 95% HDRs.

Prokopec (2013), who only had one-shot (i.e., "before" and "after") data unsuited to modeling heterogeneity or carryover effects.

**Model comparisons**

The model and data together provide clear evidence of scale-based effects on the distribution of donations. But one might reasonably question whether these were strongly dependent on the particular form of the model, five of its elements in particular: 1) internal reference point specification; 2) external reference point specification; 3) including correlation (Type II Tobit), seasonality, and scale effects; 4) incorporating response heterogeneity; and, perhaps most important, (5) whether the scale should operate as *reference* effects at all, as opposed to merely summary covariates. We examine each of these in some detail, to assess relative "contribution" to overall model fit, comparing the five internal referent specifications (IR-1-5) and ten external reference formulations (ER-1-10), as described in the model development section.[11] We refer to the model with all the aforementioned components – internal and external referents; error correlation; seasonality; heterogeneity – as the "full model". Alternative models include those lacking: error correlation ("no correlation"), scale effects ("no scale effect"), both ("simple regression"), various forms of heterogeneity (i.e., homogenous seasonality, homogenous scale effects, and both), and reference effects altogether ("no reference effects"), as explained below.

Owing to short donation histories (which preclude 'squandering' an entire year for prediction purposes), we compare fit in-sample, assessed via LPML, mean absolute deviation (MAD) and root mean square error (RMSE) for donation amount predictions; these appear in Table 6. The proposed model ("full" with IR-1, ER-1) provides a better fit than all alternatives via the LPML measure, and very nearly so using MAD and RMSE, surpassed only by geometric

[11] For IR-3, IR-4, if we don't observe donation at a certain time of year in the initialization period (first full year, or three data points), we initialize using the mean of the all observed amounts in each group.

smoothing (with an optimal smoothing carryover near 0.4); moreover, including error correlation and scale effects improves fit regardless of internal reference formulation (IR1-5) and the inclusion of heterogeneity.[12]

Table 6 also allows us to judge relative contribution to overall model fit: scale effects easily best both correlation and seasonality. For example, failing to account for scaling effects ("no scale effect") inflates RMSE approximately 20% (i.e., to 0.335 from 0.279); the corresponding figure for removing correlation alone is ~2.5%. Dropping heterogeneity entirely entailed a ~6% RMSE decrease (to 0.297), but only ~0.7% of this was attributable to seasonality (RMSE = 0.281). These comparisons suggest that *scale attraction effects may explain more variation in giving than those typically modeled in prior donation research combined*, although only additional applications can verify whether this holds generally.

In terms of internal reference point specification, IR-1, the average of all prior donation amounts appeared to dominate across the board, based on LPML. The degree of dominance was nontrivial, as high as 7.1% in RMSE; to our knowledge, such a test of 'internal' referents is unprecedented in donation contexts. Given this pattern of results, we restrict our attention to the "full" model with IR-1, and the lower portion of Table 7 summarizes fits of the ten external reference specifications (ER 1-10) for this model. ER-1, with all five scale points included, clearly dominates, by RMSE degrees ranging from 8% (vs. ER-4, for the median scale point) to 66% (vs. ER-9), for linearly increasing emphasis on higher scale points. We hesitate to term this a general finding in the absence of data capable of assessing these weights (perhaps even

[12] Estimates and fit statistics for all alternative models are available from the authors. Although RMSE and MAD are computed here across the entire posterior, model selection is based on the appropriate Bayesian criterion, LPML. If the former two are used instead, the geometric smoothing model, IR-5, is slightly superior for some values, including the "best" one of $\alpha \approx 0.4$. To check whether the IR-5 specification had different substantive implications, we recalculated the individual-level joint posteriors $\{\beta_i^U, \beta_i^D\}$ and compared them to those from IR-1 via Cramér-von Mises statistics, using the `cramer` package in R. At the .05 level of significance, none of the 1600 participants showed significant differences; we therefore conclude, based on that and visual inspection of all model results, that IR-1 and IR-5 are substantively highly similar for these data.

heterogeneously) via estimation, but the degree of advantage for ER-1 over the other nine alternatives is at the very least suggestive, and differs from De Bruyn and Prokopec's (2013) finding that the lowest scale point exerts the strongest influence (e.g., ER-10: RMSE = 0.377, or a 35% increase), although their empirical setting was somewhat different.

Regardless, the "full" model with IR-1 and ER-1 was verified to provide the best fit to the data among the 2 × 2× 2 × 2 × 5 × 10 (scale effects?; scale effect heterogeneity?; seasonality heterogeneity?; error correlation?; IR1-5; ER1-10) design. However, this precludes the possibility that the scale effects were not *reference* effects, which we take up next.

**Division into Levels 1 and 2**

As reported earlier, the charity divided prior donors for the experiment in its customary manner, based on prior donation amounts. This is entirely sensible, given the potential for loss, and even the carefully randomized study of De Bruyn and Prokopec (2013) "...constructed a customized appeal scale for each donor, tailored to both his/her last donation and the assigned experimental condition", introducing a potential for endogeneity. However, as Dorotic et al. (2014) report in the context of Loyalty Programs (LP), "From discussions with the LP manager, we know that only the frequency of the mailings is endogenous; its timing is not set based on individual behavior," and so they can "easily correct for the endogeneity". In our study, timing of requests is fixed and identical across groups (Levels 1 and 2; Standard and Test), and only *groupwise* manipulations (i.e., for Levels 1 and 2 separately) are involved. It is possible to simply re-estimate the model for each Level alone: while this reduces statistical power, it ensures that the results for each individual are informed only by the scale used in that individual's group (which cannot be ensured in the hierarchical Bayes set-up used to analyze the Level groups together). We focus on the main quantities of interest, individual-level estimates of the pulling effects, $(\beta_i^U, \beta_i^D)$, and test each of the 800 Level 1 and 800 Level 2 individual's bivariate posterior from the "Levels estimated separately" model to those previously obtained: none of the 1600 showed significant differences at the .05 level.

**Testing against *Non-Referential* Scale Effects**

The model-free evidence presented at the outset strongly suggests that respondents do react to the presence of the appeals scale, even though they have the option of ignoring it and donating whatever amount they want, or nothing at all. We assessed the importance of scaling effects in our model by estimating a nested version with the scaling removed entirely (Table 6, "no scaling effects"). But this raises the question of whether the hallmark of *reference* effects – (potentially asymmetric) pulling up and down, relative to a referent – is actually present in our data. The proposed model (6) is for the deviation between the donation amount and $ln(r_{it}^I + APA_{it})$, the internal referent adjusted for (asymmetric) pulling effects. Another possibility is to retain summary measures of both IR and ER, but without *reference* effects, specifically. This entails removing $APA_{it}$ entirely, and instead including among the regressors (i.e., along with $\beta^{level,a}X_i^{level}$) summary measures of the appeals scale points (i.e., ER-1-10), heterogeneously.[13]

It is possible to compare these results (for all possible configurations of IR and ER) via LPML; in every case, the results are inferior to the proposed model (i.e., the "full" model, with IR-1 and ER-1). For the basis of explicit comparison, we replicated all the results of Table 6 for this revised model, and LPML ranges from -23419 (for IR-1, ER-2) to a best value of -22842 (for IR-1, ER-3), compared to the proposed model's best value -21968 (for IR-1, ER-1). {MAD, RMSE} were fairly stable for the "no reference effects" models, hovering near {0.298, 0.226} vs. {0.279, 0.210} for the proposed model. That this revised "scale effects, but no reference effects"

[13] We are grateful to an anonymous reviewer for pointing out this distinction, and suggesting this explicit model as a basis of comparison.

model makes use of the same data and identical operationalizations of both IR and ER, yet fits less well across the board – approximately 10% in log-donation, based on MAD – lends credence to the documents pulling effects being *reference* effects, specifically.

**Conclusion**

Charities have long relied on appeals scales as cornerstones of their donation requests, setting them based on experience and enlightened guesswork. By contrast, the model developed here offers a heterogeneous, joint account of donation incidence and amount, while accounting for the asymmetric effect of the appeals scale. Moreover, different specifications for internal and external reference point theories can be assessed via model comparison.

Results suggest that variation across donors in scale attraction effects can be substantial. Such a finding depends critically on the availability of donation *histories*, explaining its absence from prior studies. A moderate, significantly negative, correlation between donation incidence and amount indicates the potential pitfalls of providing separately accounts, echoing similar results long-accepted in brand choice (e.g., Lattin and Bucklin 1991). In terms of internal and external referents, we found that the mean of the previous donation amounts (internal referents) and including all points in an appeals scale (external referents) offered the best fit with our data, compared with a wide variety of alternatives, as suggested by prior literature (e.g., Briesch et al. 1997). The developed model can apply well beyond the domain of charitable requests, to any situation where different interval or ordered categorical scales are used. Indeed, it may be possible to leverage the model to not only detect, but correct for, many of the sorts of scaling artifacts widely documents by consumer researchers (e.g., Schwartz 1991, 1999).

Field studies of this nature entail inevitable limitations. Charities are less concerned with optimal experimental practice than in gaining some degree of insight that doesn't risk substantial losses. In our study, for example, appeals scale amounts roughly tracked prior donation level in each segment, instead of being orthogonalized or randomized. Second, lack of substantial within-donor appeals scale variation allowed us only to test various weighting schemes, as opposed to estimating the scale points' relative influence, let along heterogeneously. Third, the optimal *number* of points can only be ascertained if these were systematically varied. And finally, because the appeals scales used by the study both historically and in the experiment contained only 'reasonable' amounts, effects of extreme scale points, such as ignoring them or even of alienating donors, await verification. Despite these data limitations, the model showed clear and strong evidence for scale attraction effects, in both upward and downward directions, and that the degree of attraction varied nontrivially across donors.

We have knowingly avoided trying to engage in tight tests of specific behavior theories that, suitably interpreted, might make specific predictions about which scale points would be relatively influential. A prime example is Configural Weight Theory (e.g., Birnbaum et al. 1992), which suggests that a scale's point's influence depends on how it compares, typically ordinally, with other points and external anchors. Tests of such theories, including range theory (e.g., Janiszewski and Lichtenstein 1999), extremeness aversion (e.g., Simonson and Tversky 1992), etc., await tighter controls than are typically available in field data, as well as specific mathematical formulations amenable to statistical estimation on individual-level data, although some progress has been made on that front, e.g., for the compromise effect (Kivetz, Netzer, and Srinivasan 2004).

Some of the data limitations suggest clear directions for future experimental and field research. First and foremost would be some scheme for orthogonalizing appeals scale amounts across various donor groups, as well as the number of points on the scale; this would allow aspects of the scales themselves, like median and range, to be not merely measured in terms of influence, but optimized. Future research might also identify subtleties of weighting: do some respondents ignore endpoints, while others anchor on them? Experiments could similarly include extreme scale points, to see whether they are ignored entirely, lead respondents not to donate at all, or something more subtle. Lastly, the present data set could not address the persistence of scale attraction effects, specifically, the degree to which they may be self-correcting, which informs whether external anchors can effectively increase total contribution over a planning horizon, what fundraisers refer to as "laddering"; assessing such issues rigorously would likely require multiple independent scale manipulations in a field setting. Regardless, any such data could be analyzed through variants of the basic framework employed here, and would help validate cross-study norms about scale point attraction effects, as well as tentatively suggest individual-level directional or variational changes in appeals scale design.

## References


Andrews, R. L., A. Ainslie, I. S. Currim. 2008. On the recoverability of choice behaviors with random coefficients choice models in the context of limited data and unobserved effects. *Management Science* B54 (1) 83-99.

Ascarza, E., B. G. Hardie. 2013. A joint model of usage and churn in contractual settings. *Marketing Science* **32** (4) 570-590.

Birnbaum, M. H., G. Coffey, B. A. Mellers, R. Weiss. 1992. Utility measurement: Configural-weight theory and the judge's point of view. *Journal of Experimental Psychology: Human Perception and Performance* **18** (2) 331-346.

Briesch, R. A., L. Krishnamurthi, T. Mazumdar, S. P. Raj. 1997. A comparative analysis of reference price models. *Journal of Consumer Research* **24** (2) 202-214.

Chen, M. H., S. Kim. 2008. *The Bayes factor versus other model selection criteria for the selection of constrained models*, in Bayesian Evaluation of Informative Hypotheses (pp. 155-180). Springer, New York.

Chen, M. H., L. Huang., J. G. Ibrahim., S. Kim. 2008. Bayesian variable selection and computation for generalized linear models with conjugate priors. *Bayesian analysis*, **3** (3) 585.

Chib, S., E. Greenberg. 1995. Understanding the Metropolis-Hastings Algorithm. *American Statistician* **49** (4) 327-335.

Croson, R., J. Shang. 2013. Limits of the Effect of Social Information on the Voluntary Provision of Public Goods: Evidence from Field Experiments. *Economic Inquiry*, 51(1), 473-477.

De Bruyn, A., & Prokopec, S. (2013). Opening a donor’s wallet: The influence of appeal scales on likelihood and magnitude of donation. *Journal of Consumer Psychology*, *23*(4), 496-502.

Desmet, P., F. Feinberg. 2003. Ask and ye shall receive: The effect of the appeals scale on consumers’ donation behavior. *Journal of Economic Psychology* **24** 349-376.

Donkers, B., R. Paap, J. Jonker, P. Franses. 2006. Deriving Target Selection Rules from Endogeneously Selected Samples. *Journal of Applied Econometrics* **21** 549–562.

Doob, A., D. McLaughlin. 1989. Request Size and Donations to a Good Cause. *Journal of Applied Social Psychology* **19** (12) 1049-1056.

Dorotic, M., P. C. Verhoef, D. Fok, T. H. Bijmolt. 2014. Reward redemption effects in a loyalty program when customers choose how much and when to redeem. *International Journal of Research in Marketing* **31** (4) 339-355.

Edwards, Y., G. Allenby. 2003. Multivariate Analysis of Multiple Response Data. *Journal of Marketing Research* **40** (3) 321-334.

Fraser, C., R. Hite, P. Sauer. 1988. Increasing Contributions in Solicitation Campaigns: The Use of Large and Small Anchor Points. *Journal of Consumer Research* **15** (3) 284-277.

Hardie, B. G., E. J. Johnson, P. S. Fader. 1993. Modeling loss aversion and reference dependence effects on brand choice. *Marketing Science* **12** (4) 378-394.

Janiszewski, C., D. R. Lichtenstein. 1999. A range theory account of price perception. *Journal of Consumer Research* **25** (4) 353-368.

Kalyanaram, G., J. Little. 1994. An Empirical Analysis of Latitude of Price Acceptance in Consumer Package Goods. *Journal of Consumer Research* **21** (3) 408-418.

Kalyanaram, G., R. Winer. 1995. Empirical Generalization from Reference Price Research. *Marketing Science* **14** (3) 161-169.

Krishnamurthi, L., T. Mazumdar, S. Raj. 1992. Asymmetric Response to Price in Consumer Brand Choice and Purchase Quantity Decisions. *Journal of Consumer Research* **19** (3) 387-400.

Kivetz, R., O. Netzer, V. Srinivasan. 2004. Alternative models for capturing the compromise effect. *Journal of marketing research*. **41** (3) 237-257.

Lattin, J., R. Bucklin. 1989. Reference Effects of Price and Promotion on Brand Choice Behavior. *Journal of Marketing Research* **26** (3) 299-310.

Mayhew, G., R. Winer. 1992. An Empirical Analysis of Internal and External Reference Prices Using Scanner Data. *Journal of Consumer Research* **19** (1) 62-70.

Mazumdar, T., S. P. Raj, I. Sinha. 2005. "Reference price research: Review and propositions." *Journal of marketing* **69** (4) 84-102.

Podsakoff, P. M., D. W. Organ. 1986. Self-reports in organizational research: Problems and prospects. *Journal of management*, **12** (4) 531-544.

Reingen, P. 1982. Test of a List Procedure for Inducing Compliance with a Request to Donate Money. *Journal of Applied Psychology* **67** (1) 110-118.

Schibrowsky, J., J. Peltier. 1995. Decision Frames and Direct Marketing Offers: A Field Study in a

Fundraising Context. *Journal of Direct Marketing* **9** (1) 8-16.

Schwarz, N., H. Bless, G. Bohner, U. Harlacher, M. Kellenbenz. 1991. Response Scales as Frames of Reference: The Impact of Frequency Range on Diagnostics Judgements. *Applied Cognitive Psycology* **5** 37-49.

Schwarz, N. 1999. Self-reports – How the Questions Shape the Answers. *American Psychologist* **54** (2) 93-105.

Shang, J., A. Reed II, R. Croson. 2008. Identity Congruence Effects on Donations. *Journal of Marketing Research* **45** (3) 351-361.

Shang, J., R. Croson. 2009. A Field Experiments in Charitable Contribution: The Impact of Social Influence on the Voluntary Provision of Public Goods. *Economic Journal* **119** (540) 1243–1587.

Sherif, M., D. Taub, C. Hovland. 1958. Assimilation and Contrast Effects of Anchoring Stimuli on Judgments. *Journal of Experimental Psychology* **55** (2) 150-156.

Shi, S. W., J. Zhang. 2014. Usage Experience with Decision Aids and Evolution of Online Purchase Behavior. *Marketing Science* **33** (6) 871-882.

Simonson, I., A. Tversky. 1992. Choice in context: Tradeoff contrast and extremeness aversion. *Journal of marketing research* **29** (3) 281-295.

Tanner, M., W. H. Wong. 1987. The Calculation of Posterior Distributions by Data Augmentation. *Journal of the American Statistical Association* **82** (398) 528-549.

Van Diepen, M. (2009). *Dynamics and competition in charitable giving*. Doctoral Disseration No. EPS-2009-159-MKT. Erasmus Research Institute of Management (ERIM).

Van Diepen, M., B. Donkers, P. Franses. 2009. Dynamic and Competitive Effects of Direct Mailings: A Charitable Giving Application. *Journal of Marketing Research* **46** (1) 120-133.

Wachtel, S., T. Otter. 2013. Successive sample selection and its relevance for management decisions. *Marketing Science* B32 (1) 170-185.

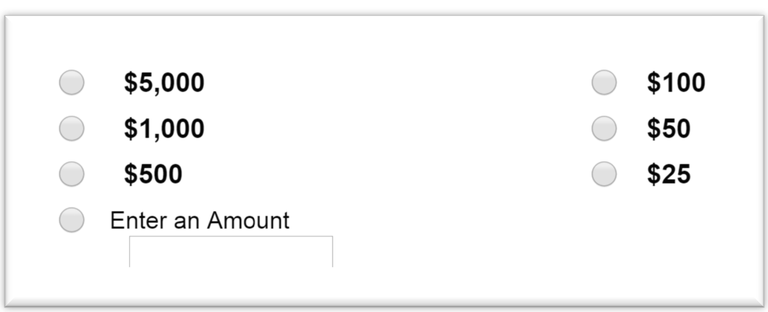
Weyant, J., S. Smith. 1987. Getting More by Asking for Less: The Effect of Request Size on Donations of Charity. *Journal of Applied Social Psychology* **17** (4) 392-400.

Ying, Y., F. Feinberg, M. Wedel. 2006. Leveraging Missing Ratings to Improve Online Recommendation Systems. *Journal of Marketing Research* **43** (3) 355-365.

**Figure 1: Appeals Scales used by Three Major Charities**

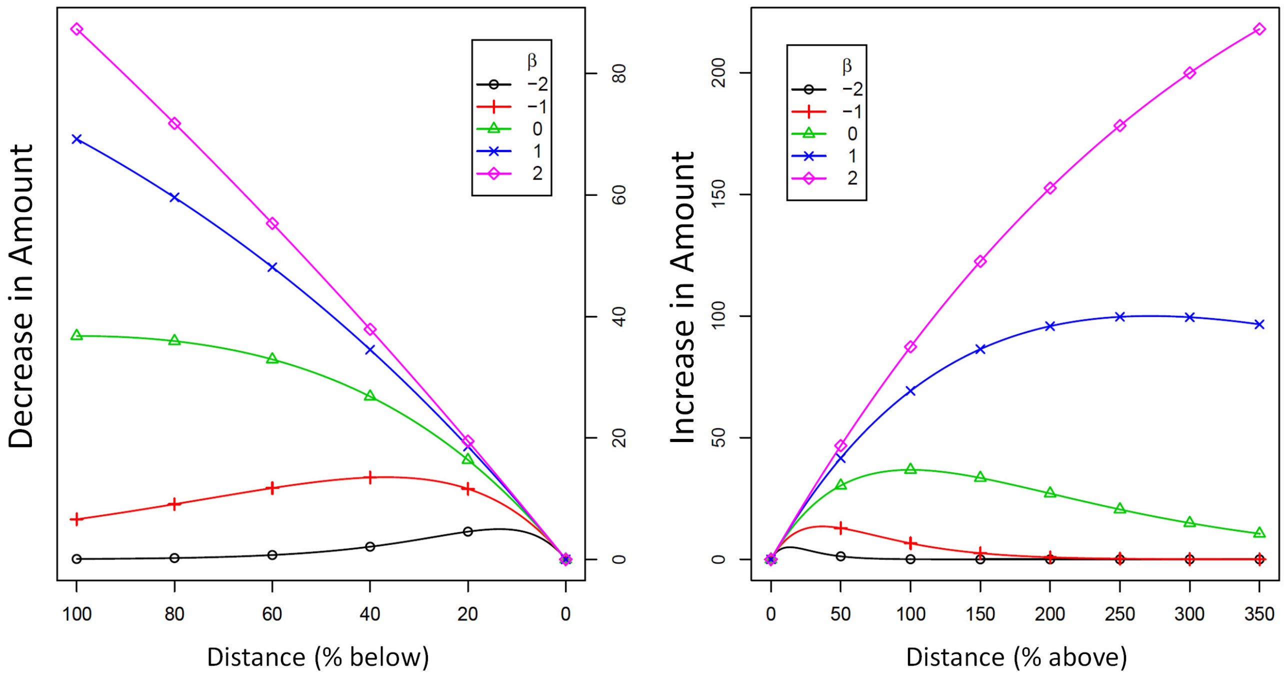

[United Way: unitedway.org]


[Wikipedia: wikipedia.org]




[United Nations Foundation: unfoundation.org]

**Figure 2: Pulling Amount Curves**




A. Downward Pulling Amount (IR = 100)

B. Upward Pulling Amount (IR = 100)

**Figure 3: Pulling amounts owing to multiple scale (external reference) points**

Note: IR normalized to 100 on Y-axis

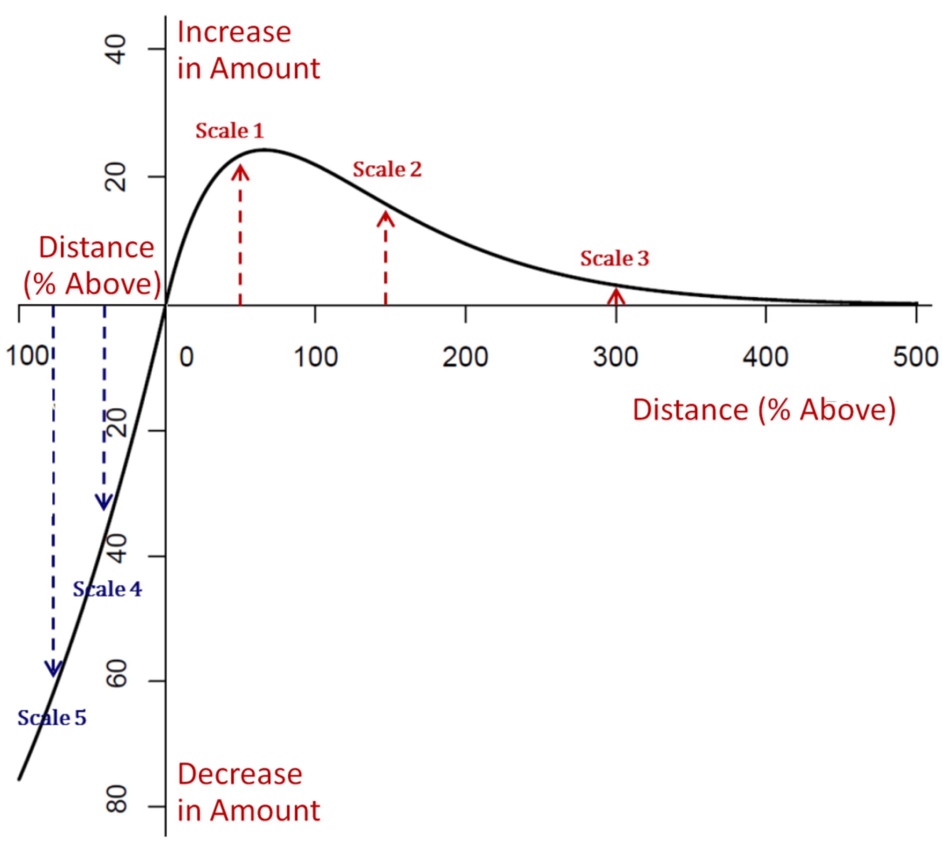


**Figure 4A: Gamma "pulling" parameters (up and down) for each donor**

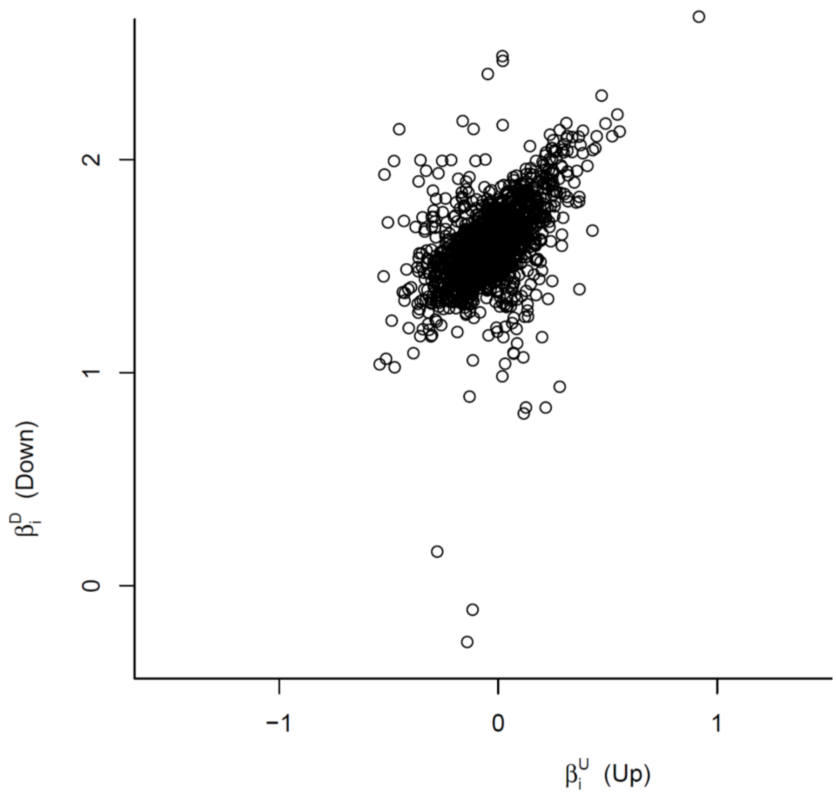



**Figure 4B: Maximum achievable pulling up and down proportions for each donor**

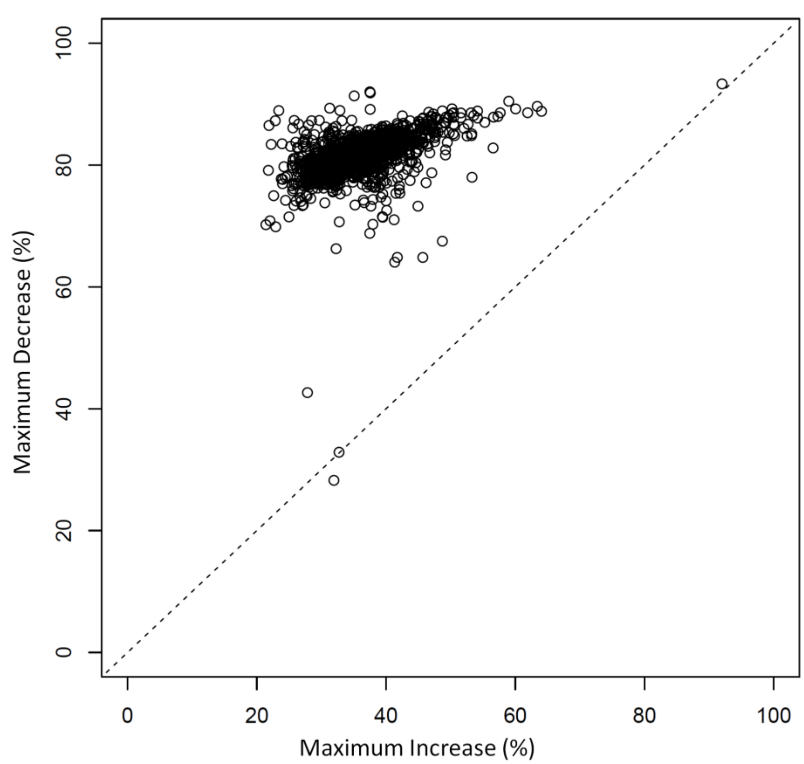


**Table 1: Appeals Scales used in the Field Experiment**

Appeals scale in periods 1-8 (all subjects)

| | | | | | | |
|---|---|---|---|---|---|---|
| *All Donors* | 90 FF | 150 FF | 250 FF | 500 FF | 1000 FF | Other |

Appeals scale in period 9 (all subjects)

| | | | | | | |
|---|---|---|---|---|---|---|
| *All Donors* | 100 FF | 150 FF | 250 FF | 500 FF | 1000 FF | Other |

Appeals scales in period 10, Standard and Test Scales

| | | | | | | |
|---|---|---|---|---|---|---|
| *Standard: Levels 1 and 2* | 100 FF | 150 FF | 250 FF | 500 FF | 1000 FF | Other |
| *Test: Level 1* | 120 FF | 180 FF | 250 FF | 350 FF | 500 FF | Other |
| *Test: Level 2* | 120 FF | 200 FF | 350 FF | 500 FF | 750 FF | Other |

**Table 2: Period 10 (Experiment) Average Donation Amounts and Frequencies**

| *Prior donation level* | *Scales* | *Yield rate* | *Average observed donations (FF)* |
|---|---|---|---|
| 1 | *Standard* | 63.3% | 143.7 |
| | *Test* | 62.5% | 148.7 |
| 2 | *Standard* | 65.3% | 268.6 |
| | *Test* | 68.6% | 275.5 |

**Table 3: Yield Rate and Average Amount of Observed Donations across Seasons**

| | *Level 1* | | | *Level 2* | | |
|---|---|---|---|---|---|---|
| | *Easter* | *June* | *Christmas* | *Easter* | *June* | *Christmas* |
| *Yield rate* | 71.9% | 19.5% | 21.7% | 73.3% | 20.1% | 24.3% |
| *Average donation per occasion* | 139.1 | 125.3 | 126.9 | 258.8 | 217.2 | 206.6 |

**Table 4: Proportion of Donations at Amounts* on Standard or Test Scales, Period 10**

| Level 1 Standard | | Level 1 Test | | |
|---|---|---|---|---|
| Scale Point | % | Scale Point | % | p-value |
| 100* | 18.6% | 100 | 7.3% | 0.000 |
| 120 | 0.2% | 120* | 18.4% | 0.000 |
| 150* | 17.8% | 150 | 5.4% | 0.000 |
| 180 | 0.0% | 180* | 6.9% | 0.000 |
| 250* | 5.1% | 250* | 4.4% | 0.743 |
| 350 | 0.0% | 350* | 0.3% | 1.000 |
| 500* | 0.3% | 500* | 0.3% | 1.000 |
| 1000* | 00.0% | 1000 | 0.0% | 1.000 |
| None | 49.4% | None | 50.2% | 0.832 |
| Other | 8.6% | Other | 6.8% | 0.342 |

| Level 2 Standard | | Level 2 Test | | |
|---|---|---|---|---|
| Scale Point | % | Scale Point | % | p-value |
| 100* | 2.6% | 100 | 1.6% | 0.450 |
| 120 | 0.1% | 120* | 2.9% | 0.000 |
| 150* | 7.2% | 150 | 1.8% | 0.000 |
| 200 | 2.6% | 200* | 18.8% | 0.000 |
| 250* | 26.7% | 250 | 9.1% | 0.000 |
| 350 | 1.2% | 350* | 10.6% | 0.000 |
| 500* | 5.3% | 500* | 4.4% | 0.627 |
| 750 | 0.0% | 750* | 0.3% | 1.000 |
| 1000* | 0.3% | 1000 | 0.2% | 1.000 |
| None | 44.1% | None | 42.2% | 0.618 |
| Other | 9.9% | Other | 8.1% | 0.373 |

* Starred amounts are those appearing on either scale (Standard or Test) within-Level.
p-values all stem from Fisher's Exact test for equality of proportions.

**Table 5: Parameter Estimates for Full Model**

| Coefficient | | *mean* | *SE* | *95% HDR* |
|---|---|---|---|---|
| ***Homogeneous*** | *correlation* ($\rho$) | -0.454 | 0.033 | (-0.516, -0.385) |
| | *sd of log amount* ($\sigma$) | 0.299 | 0.004 | (0.291, 0.307) |
| | *Easter dummy* ($\beta^{E}$) | 0.708 | 0.026 | (0.657, 0.757) |
| | *level dummy in selection* ($\beta^{level,s}$) | 0.108 | 0.032 | (0.047, 0.166) |
| | *log amount lag in selection* ($\beta^{lag}$) | -0.123 | 0.006 | (-0.136, -0.110) |
| | *level dummy in amount* ($\beta^{level,a}$) | 0.260 | 0.013 | (0.235, 0.287) |
| ***Heterogeneous*** | *pulling up* ($\beta_i^{U}$) | -0.063 | 0.039 | (-0.146, 0.007) |
| | *pulling down* ($\beta_i^{D}$) | 1.630 | 0.225 | (1.245, 2.127) |
| | *June dummy* ($\beta_i^{J}$) | -0.505 | 0.041 | (-0.583, -0.425) |
| | *Christmas dummy* ($\beta_i^{C}$) | -0.993 | 0.049 | (-1.090, -0.901) |
| | *sd of pulling up* | 0.330 | 0.022 | (0.291, 0.378) |
| | *sd of pulling down* | 0.943 | 0.127 | (0.726, 1.213) |
| | *sd of June* | 0.390 | 0.048 | (0.308, 0.497) |
| | *sd of Christmas* | 0.777 | 0.060 | (0.655, 0.895) |
| | *corr(June & Christmas)* | 0.714 | 0.083 | (0.540, 0.871) |
| | *corr(pulling up & pulling down)* | 0.446 | 0.095 | (0.243, 0.612) |
| | *corr(June & pulling up)* | -0.002 | 0.035 | (-0.070, 0.065) |
| | *corr(June & pulling down)* | 0.002 | 0.035 | (-0.066, 0.070) |
| | *corr(Christmas & pulling up)* | 0.003 | 0.034 | (-0.064, 0.070) |
| | *corr(Christmas & pulling down)* | 0.003 | 0.035 | (-0.067, 0.070) |

**Table 6: Model Fit Statistics (LPML, RMSE, MAD) for Various {IR, ER} Specifications and Restrictions, Computed over Model Posteriors**

| Internal Referent Specification (with ER-1) | LPML | RMSE | MAD |
|---|---|---|---|
| IR-1 (average of all prior donations) | -21968 | 0.279 | 0.210 |
| IR-2 (last donation) | -22482 | 0.299 | 0.221 |
| IR-3 (average of same-season donations) | -22372 | 0.288 | 0.212 |
| IR-4 (last same-season donation) | -22558 | 0.294 | 0.222 |
| IR-5: (geometrically smoothed, α = 0.4) | -22125 | 0.277 | 0.207 |
| **IR-1 Model with Restrictions:** | | | |
| No correlation (ρ = 0) | -22063 | 0.286 | 0.215 |
| No scale effect | -23095 | 0.335 | 0.226 |
| Simple regression | -23110 | 0.335 | 0.224 |
| Heterogeneous scale effect only | -22307 | 0.281 | 0.211 |
| Heterogeneous seasonality only | -22972 | 0.295 | 0.216 |
| Homogeneous scale effect and seasonality | -22716 | 0.297 | 0.218 |

| External Referent Specification (with IR-1) | LPML | RMSE | MAD |
|---|---|---|---|
| ER-1 (all scale points) | -21968 | 0.279 | 0.210 |
| ER-2 (two points closest to IR) | -22498 | 0.304 | 0.221 |
| ER-3 (largest and smallest points) | -22567 | 0.326 | 0.222 |
| ER-4 (median scale point) | -22339 | 0.301 | 0.223 |
| ER-5 (mean of all scale points) | -22445 | 0.316 | 0.218 |
| ER-6 (equal weight: {1,1,1,1,1}) | -22365 | 0.325 | 0.255 |
| ER-7 (V-shaped: {3,2,1,2,3}) | -22704 | 0.321 | 0.252 |
| ER-8 (Inverse V: {1,2,3,2,1}) | -22247 | 0.336 | 0.264 |
| ER-9 (Increasing: {1,2,3,4,5}) | -22615 | 0.463 | 0.375 |
| ER-10 (Decreasing: {5,4,3,2,1}) | -22313 | 0.377 | 0.294 |

## Appendices A - C

### *A. Full Model Specification*

As discussed above, we can write the entire model as follows (*i* = donor; *t* = time):

$$y_{it}^{s*} = \beta^E X_{it}^E + \beta_i^J X_{it}^J + \beta_i^C X_{it}^C + \beta^{lag} X_{it}^{lag} + \beta^{level,s} X_i^{level} + \epsilon_{it}^s$$

$y_{it}^{a*} = ln(r_{it}^I + APA_{it}) + \beta^{level,a} X_i^{level} + \epsilon_{it}^a$, where:

$y_{it}^s = 1, \text{if } y_{it}^{s*} \geq 0;\ 0 \text{ otherwise}$
$y_{it}^a = y_{it}^{a*} = \ln(donation\ amount)\ , \text{if } y_{it}^s = 1;\ \text{unobserved otherwise}$

$$APA_{it} = \sum_{k=1}^{K} w_{it}^k \times I_{it}^k \times PA_{it}^k\ ;\ w_{it}^k = \frac{PA_{it}^k}{\sum_{k=1}^K PA_{it}^k}, I_{it}^k = \begin{cases} 1, if\ r_{it}^{E,k} \geq r_{it}^I \\ -1, if\ r_{it}^{E,k} \leq r_{it}^I \end{cases}$$

$$PA_{it}^k\ =\ CD_{it}^k \times ||r_{it}^{E,k} - r_{it}^I||$$

$$CD_{it}^k\ = exp\left(-\frac{d_{it}^k}{\theta_i}\right)\ ;\ \theta_i = \begin{cases} \exp(\beta_i^U), r_{it}^{E,k} \geq r_{it}^I \\ \exp(\beta_i^D), r_{it}^{E,k} \leq r_{it}^I \end{cases}, d_{it}^k = \frac{||r_{it}^{E,k} - r_{it}^I||}{r_{it}^I}$$

$$(\epsilon_{it}^s\ , \epsilon_{it}^a) \sim BVN(0, \Sigma_\epsilon)\ ;\ \Sigma_\epsilon = \begin{bmatrix} 1 & \rho\sigma \\ \rho\sigma & \sigma^2 \end{bmatrix}$$

$\beta_i \sim MVN(\Delta, \Sigma_\beta)$, where $\beta_i = (\beta_i^J, \beta_i^C, \beta_i^U, \beta_i^D)$

Note that the internal reference point, $r_{it}^I$, for donor *i* can change over the course of the experiment, and is subscripted accordingly, as is the $k^{th}$ external reference point for a donor *i* at time *t*, $r_{it}^{E,k}$. Again, the variance of $\epsilon^s$ is fixed to 1 for identification. Finally, the vector of heterogeneous parameters ($\beta_i$) follows a multivariate normal distribution with mean $\mu_\beta$ and full-rank covariance matrix $\Sigma_\beta$.

### *B. MCMC Algorithm and Priors*

Here we present the prior distributions and sampling algorithm used in estimation. Because the requirement that setting error variance of the binary probit model (for donation incidence) be set to one ruins useful conjugacy properties, we instead make random draws from the unidentified space, as suggested by Edwards and Allenby (2003), and report post-processed estimates. Below, we specify $\Sigma_\epsilon$ in the unidentified space as $\Sigma_\epsilon = \begin{bmatrix} \sigma_s^2 & \rho\sigma_s\sigma_a \\ \rho\sigma_s\sigma_a & \sigma_a^2 \end{bmatrix}$.

**1. Data Augmented Likelihood**

$$\prod_{\mathrm{i}=1}^{\mathrm{n}}\prod_{\mathrm{t}=1}^{\mathrm{T}}[(y_{it}^{s*}, y_{it}^{a*})|\beta_h, \beta_i, \Sigma_\epsilon] \times \prod_{i=1}^{n}[\,\beta_i|\,\mu_\beta, \Sigma_\beta]$$

where $\beta_h = (\beta^E, \beta^{lag}, \beta^{level,s}, \beta^{level,a})$ is a vector of homogeneous parameters and $\beta_i = (\beta_i^J, \beta_i^C, \beta_i^U, \beta_i^D)$ is a vector of heterogeneous parameters.

**2. Prior Distribution**

We use proper but diffuse priors.

(1) $\beta_h \sim MVN(M, V)$, where $M = \mathbf{0}, V = 10^4 I$

(2) $\Sigma_\epsilon \sim IW(\nu_{\Sigma_\epsilon}, V_{\Sigma_\epsilon})$, where $\nu_{\Sigma_\epsilon} = 5, V_{\Sigma_\epsilon} = 5I$

(3) $\Delta \sim MVN(\bar{\Delta}, A)$, where $\bar{\Delta} = \mathbf{0}, A = 10^4 I$

(4) $\Sigma_\beta \sim IW(\nu_{\Sigma_\beta}, V_{\Sigma_\beta})$, where $\nu_{\Sigma_\epsilon} = 7, V_{\Sigma_\epsilon} = 7I$

**3. Posterior Distribution**

$$\prod_{\mathrm{i}=1}^{\mathrm{n}}\prod_{\mathrm{t}=1}^{\mathrm{T}}[(y_{it}^{s*}, y_{it}^{a*})|\beta_h, \beta_i, \Sigma_\epsilon] \times \prod_{i=1}^{n}[\,\beta_i|\,\mu_\beta, \Sigma_\beta] \times [\beta_h|\mathrm{M}, \mathrm{V}] \times [\Sigma_\epsilon|\nu_{\Sigma_\epsilon}, \mathrm{V}_{\Sigma_\epsilon}] \times [\Delta|\bar{\Delta}, \mathrm{A}] \times [\Sigma_\beta|\nu_{\Sigma_\beta}, \mathrm{V}_{\Sigma_\beta}]$$

**4. Sampling Algorithm**

*Step 1. Draw $y_{it}^{s*}$ and $y_{it}^{a*}$ (Data augmentation step)*

$[(y_{it}^{s*}, y_{it}^{a*})|y_{it}^{s}, y_{it}^{a}, \beta_h, \beta_i, \Sigma_\epsilon]$

1. If $y_{it}^{s} = 1$ then $y_{it}^{a}$ is observed. We set $y_{it}^{a*} = y_{it}^{a}$ and draw $y_{it}^{s*}$ from the truncated normal distribution below:

$$TN\left(\beta^E X_{it}^E + \beta_i^J X_{it}^J + \beta_i^C X_{it}^C + \beta^{lag} X_{it}^{lag} + \beta^{level,s} X_i^{level} + \frac{\rho\sigma_s}{\sigma_a}\left[y_{it}^{a} - (ln(r_{it}^{I} + APA_{it}) + \beta^{level,a} X_i^{level})\right], (1-\rho^2)\sigma_s^2\right), y_{it}^{s*} \geq 0$$

2. If $y_{it}^{s} = 0$ then $y_{it}^{a}$ is not observed. We draw $(y_{it}^{s*}, y_{it}^{a*})$ by following steps

a. Draw $y_{it}^{s*}$ from $TN\left(\beta^E X_{it}^E + \beta_i^J X_{it}^J + \beta_i^C X_{it}^C + \beta^{lag} X_{it}^{lag} + \beta^{level,s} X_i^{level}, \sigma_s^2\right), y_{it}^{s*} < 0$

b. Draw $y_{it}^{a*}$ conditional on $y_{it}^{s*}$ from normal distribution below:

$$N\left(ln(r_{it}^{I} + APA_{it}) + \beta^{level,a} X_i^{level} + \frac{\rho\sigma_a}{\sigma_s}\left[y_{it}^{s*} - (\beta^E X_{it}^E + \beta_i^J X_{it}^J + \beta_i^C X_{it}^C + \beta^{lag} X_{it}^{lag} + \beta^{level,s} X_i^{level})\right], (1-\rho^2)\sigma_a^2\right)$$

*Step 2. Draw $\beta_i$*

$$[\beta_i|\beta_h, \Sigma_\epsilon, \Delta, \Sigma_\beta] \propto \prod_{t=1}^{T}[(y_{it}^{s*}, y_{it}^{a*})|\beta_h, \beta_i, \Sigma_\epsilon] \times [\beta_i|\Delta, \Sigma_\beta]$$

The full conditional distribution is also of unknown form. Therefore, we use a Metropolis-Hastings algorithm with a normal random walk proposal to make draws.

*Step 3. Draw $\beta_h$*

$$[\beta_h|\{\beta_i\}, \Sigma_\epsilon] \propto \prod_{i=1}^{n}\prod_{t=1}^{T}[(y_{it}^{s*}, y_{it}^{a*})|\beta_h, \beta_i, \Sigma_\epsilon] \times [\beta_h|M, V]$$

Again, we use a Metropolis-Hastings algorithm with a normal random walk proposal to make draws.

*Step 4. Draw $\Sigma_\epsilon$*

$[\Sigma_\epsilon|\beta_h,\{\beta_i\}] \propto \prod_{i=1}^{n}\prod_{t=1}^{T}[(y_{it}^{s*},y_{it}^{a*})|\beta_h,\beta_i,\Sigma_\epsilon] \times [\Sigma_\epsilon|\nu_{\Sigma_\epsilon},V_{\Sigma_\epsilon}] \propto$

$$\prod_{i=1}^{n}\prod_{t=1}^{T} BVN(\begin{pmatrix} y_{it}^{s*} \\ y_{it}^{a*} \end{pmatrix} | \begin{pmatrix} \beta^E X_{it}^E + \beta_i^J X_{it}^J + \beta_i^C X_{it}^C + \beta^{lag} X_{it}^{lag} + \beta^{level,s} X_i^{level} \\ ln(r_{it}^I + APA_{it}) + \beta^{level,a} X_i^{level} \end{pmatrix}, \Sigma_\epsilon) \times IW(\nu_{\Sigma_\epsilon}, V_{\Sigma_\epsilon})$$

$[\Sigma_\epsilon|\beta_h,\{\beta_i\}] \sim IW(\tilde{v}_{\Sigma_\epsilon},\tilde{V}_{\Sigma_\epsilon})$

where $\tilde{v}_{\Sigma_\epsilon} = \nu_{\Sigma_\epsilon} + nT$,

$$\tilde{V}_{\Sigma_\epsilon} = V_{\Sigma_\epsilon} + \sum_{i=1}^{n}\sum_{t=1}^{T} \begin{pmatrix} y_{it}^{s*} - \left(\beta^E X_{it}^E + \beta_i^J X_{it}^J + \beta_i^C X_{it}^C + \beta^{lag} X_{it}^{lag} + \beta^{level,s} X_i^{level}\right) \\ y_{it}^{a*} - \left(ln(r_{it}^I + APA_{it}) + \beta^{level,a} X_i^{level}\right) \end{pmatrix}$$
$$\times \begin{pmatrix} y_{it}^{s*} - (\beta^E X_{it}^E + \beta_i^J X_{it}^J + \beta_i^C X_{it}^C + \beta^{lag} X_{it}^{lag} + \beta^{level,s} X_i^{level} \\ y_{it}^{a*} - (ln(r_{it}^I + APA_{it}) + \beta^{level,a} X_i^{level}) \end{pmatrix}^T$$

*Step 5. Draw* $\Delta$

$$[\Delta|\{\beta_i\},\Sigma_\beta] \propto \prod_{i=1}^{n}[\beta_i|\,\Delta,\Sigma_\beta] \times [\Delta|\bar{\Delta},A] \propto MNV_{nk}(B^*|[Z \otimes I_k]\Delta^*,\ I_n \otimes \Sigma_\beta) \times MNV_{nk}(\Delta^*|\bar{\Delta},A)$$

where $\beta_i$ is a vector of length k,

$$B = \begin{bmatrix} \beta_1^T \\ \beta_2^T \\ \vdots \\ \beta_n^T \end{bmatrix}, B_{nk\times 1}^* = vec(B^T), Z = \begin{bmatrix} \iota_1^T \\ \iota_2^T \\ \vdots \\ \iota_n^T \end{bmatrix}, \Delta^* = vec(\Delta^T)$$

$[\Delta^*|\{\beta_i\},\Sigma_\beta] \sim MNV_{nk}(\Delta^*|\tilde{\Delta},\tilde{A})$

where $\tilde{\Delta} = \tilde{A}\left([Z \otimes \Sigma_\beta{}^{-1}]B_{nk\times 1}^* + A^{-1}\bar{\Delta}\right), \tilde{A} = [(Z^T Z) \otimes \Sigma_\beta{}^{-1} + A^{-1}]^{-1}$

*Step 6. Draw* $\Sigma_\beta$

$$[\Sigma_\beta|\{\beta_i\},\Delta] \propto \prod_{i=1}^{n}[\beta_i|\,\Delta,\Sigma_\beta] \times [\Sigma_\beta|\nu_{\Sigma_\beta},V_{\Sigma_\beta}] \propto MNV_{nk}(B|Z\Delta, I_{n,}\Sigma_\beta) \times IW(\Sigma_\beta|\nu_{\Sigma_\beta},V_{\Sigma_\beta})$$

$[\Sigma_\beta|\{\beta_i\},\Delta] \sim IW(\tilde{v}_{\Sigma_\beta},\tilde{V}_{\Sigma_\beta})$

where $\tilde{v}_{\Sigma_\beta} = \nu_{\Sigma_\beta} + n, \tilde{V}_{\Sigma_\beta} = V_{\Sigma_\beta} + (B - Z\Delta)^T(B - Z\Delta)$

*C. Maximum Achievable Proportions: Upward (% Increase) and Downward (% Decrease)*

**1. Upward pulling amount.** When $r^E \geq r^I$, the upward pulling amount $PA_U$ is given by:

$$PA_U = \exp\left(-\frac{d_U}{\theta}\right)(r^E - r^I), d_U = \frac{r^E}{r^I} - 1, \theta_U = \exp(\beta_U)$$

The scale point with maximum upward pulling amount ($r^{E*}$) can be calculated by solving the first order condition for $PA_U$ with respect to $r^E$.

$$r^{E*} = (\theta_U + 1)r^I = (\exp(\beta_U) + 1)r^I$$

At the scale point $r^{E*}$, the incremental ratio in distance ($d_U^*$) is determined to be $\exp(\beta_U)$; and the maximum incremental ratio in amount as follows:

$$\exp\left(-\frac{d_U^*}{\exp(\beta_U)}\right)(r^{E*} - r^I)/r^I = \exp(\beta_U - 1)$$

**2. Downward pulling amount.** When $0 \leq r^E \leq r^I$, the downward pulling amount $PA_D$ follows:

$$PA_D = \exp\left(-\frac{d_D}{\theta}\right)(r^I - r^E), \qquad d_D = 1 - \frac{r^E}{r^I}, \qquad \theta_D = \exp(\beta_D)$$

The scale point with maximum downward pull ($r^{E*}$) can be obtained by solving the first order condition for $PA_D$ with respect to $r^E$. [Note that there is a corner solution if $\beta_D > 0$]

$$r^{E*} = \begin{cases} (1 - \theta_D)r^I = (1 - \exp(\beta_D))r^I, & \beta_D \leq 0 \\ 0, & \beta_D > 0 \end{cases}$$

At the scale point $r^{E*}$, the decremental ratio in distance ($d_D^*$) is determined to be

$$\begin{cases} \exp(\beta_D), & \beta_D \leq 0 \\ 1, & \beta_D > 0 \end{cases},$$

And the maximum decremental ratio in amount is as follows:

$$\begin{cases} \exp\left(-\frac{d_D^*}{\exp(\beta_D)}\right)(r^I - r^{E*})/r^I = \exp(-1) - \exp(\beta_D), & \beta_D \leq 0 \\ \exp\left(-\frac{d_D^*}{\exp(\beta_D)}\right)(r^I - r^{E*})/r^I = \exp\left(-\frac{1}{\exp(\beta_D)}\right), & \beta_D > 0 \end{cases}$$