What? How? Why? Resource Use by Michigan Two-Year College Instructors When Planning and Teaching the Fundamental Theorem of Calculus

by

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Dedication

To my family.

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Abstract

This dissertation examines the resources that two-year college instructors use to aide their teaching. In particular, this dissertation is an investigation into the resources used by two-year college calculus instructors in Michigan when they plan and teach the Fundamental Theorem of Calculus (FTC), how they use those resources and why they use them. Resources are broadly defined as assets that instructors access that impact their planning and instruction. While there are many resources available for teaching the FTC, they are often minimally used. The FTC connects the two major calculus concepts of differentiation and integration, yet it is difficult for students to understand the significance of the theorem. Traditionally, the FTC is presented as two theorems, in one section of one chapter of a textbook. One theorem describes the inverse relationship between differentiation and integration, and the other theorem explains how to calculate a definite integral. Theoretical underpinnings for this study come from documentational and instrumental genesis, as articulated by Gueudet and Trouche (2009). This theory articulates the dual understanding of how instructors use resources and how resources affect instructors. The study uses a mixed method design with three levels of data collection: a survey of all community college calculus instructors at all the Michigan community colleges; 14 interviews with instructors, selected to represent a variety of experience levels; and two classroom observations of instructors who identified the FTC as important and identified themselves as comfortable teaching it. Findings from this dissertation indicated that most instructors use the textbook for planning and homework, and use their personal background and student feedback when teaching a lesson on the FTC. Despite the calculus reform movement in

the 1980s that encouraged teachers to incorporate technology into their classroom, and the availability of technologies that would help explain concepts vital to understanding the FTC, this study found that the theorem is often presented without technology. An examination of instructor descriptions of the importance of the FTC revealed that instructors tended to consider the FTC as two disconnected theorems, similar to how it was presented in the textbooks. Instructors also allocated the same amount of time to the FTC as they did to other sections of the textbook. One implication from these findings is that if instructors wished to emphasize the FTC in their calculus classes by spending more time on it, the resources that they use may be inadequate. This dissertation contributes to research that focuses more broadly on higher education mathematics curriculum research.

Chapter 1: Introduction

This dissertation investigates the resources that two-year college instructors[1] use when they plan and teach the fundamental theorem of calculus (FTC). The relationship between two-year college mathematics instructors and the resources they use is complex and worthy of study. What I want to know is what resources faculty use that affect how they plan and teach the fundamental theorem of calculus. How do they use these resources? Why do instructors use (or not use) those resources? My interest in understanding resources stems from experiences I had in my own teaching, and my interest in the FTC stems from my experience as a student. This study contributes to and extends body of research around calculus instruction. The unique contribution of this study is its focus on the resources involved during the planning and teaching of one topic with one group of instructors, and has implications for researchers and instructors.

My interest in the resources used by two-year college instructors stems from two particular experiences I had while teaching where resources changed my teaching. A few years ago I was asked to teach a beginning algebra course that I had taught several times in the past. I had previously enjoyed teaching this course, and was excited to teach it again. There was a new textbook for the course, but I didn't expect that it would make much difference in my teaching. When I asked other instructors about the new textbooks, I was told, "the problem sets are harder." When I started teaching, I was surprised by my reliance on the textbook for my teaching. The new textbook treated some topics differently than the old textbook, and these

[1] Throughout this dissertation I refer to "teachers" of K-12 mathematics and "instructors" of college mathematics. I make this distinction because the requirements to teach K-12 include a teaching certification, while college instructors are not required to have any pedagogical training. Instructors may also hold a teaching certification, but it is not required. I refer to the activity of both teachers and instructors as "teaching."

changes prompted me to change how I taught that topic. For example, the old textbook considered the use and manipulation of exponents as an entire chapter. The new textbook condensed exponents as one section at the end of chapter one. When I taught this with the new textbook, I shortened my discussion of this topic to a portion of one class period. I eliminated the introduction that I used to include for exponents, gave fewer examples during class, and assigned less homework. Where I used to break up simplifying exponential expressions and solving exponential equations over two class periods, I now taught everything in one period, because it was one section of the textbook.

The second experience I had involved a student complaint. I was teaching an algebra course and the college required students to purchase a graphing calculator for the course. However, I learned mathematics via paper and pencil. Although I could use a graphing calculator, I did not see a great deal of added value for it in the classroom. One student complained that he had been required to pay for this calculator, but we were not using it. As a result of that complaint, I paid careful attention to opportunities for calculator use beyond computation. I asked other instructors what topics they taught where the graphing calculator was most helpful and I searched the internet for ideas about integrating calculator use in the classroom. I incorporated instructions on how to use the graphing calculator in my teaching, and taught them how to use their calculator to do linear regression. Without the capabilities of the graphing calculator, the computations involved in linear regression discouraged me from introducing it to students. As I investigated appropriate ways to incorporate the graphing calculator into my teaching, I came to realize that the resource that influenced my initial teaching in this instance was my personal background. Although the graphing calculator impacted my teaching later, the trigger for the change in my teaching and planning was not the graphing calculator, but the feedback I received from a student.

These two experiences motivated a desire to understand resources and not only how they can influence teaching, but how they can be used. I was surprised by how much the resource of the textbook impacted my teaching. I reduced classroom time for one topic based on the layout of the textbook. Based on student feedback, I changed how I used the graphing calculator in my teaching and planning. Based on the capabilities of the graphing calculator, I was able to teach a new topic. This experience led me to think about the resources I was given as an instructor, and how resources may be used by other instructors.

Within mathematics education research there is a growing body of scholarship on resource use and the interaction between instruction and resources, raising questions about the interactive nature of resource use. How does resource use impact instruction? How does instruction impact resource use? An underlying assumption for this work is that

> resources are essential for mathematics teachers, and teachers use different kinds of resources which shape the mathematical content presented to, and used by, pupils in their mathematics learning. Moreover, when appropriating resources, teachers adapt them to their needs and customs. (Pepin, Gueudet, & Trouche, 2013).

In other words, what teachers and instructors do with resources influences their instruction as well as student learning. Understanding how and why instructors use available resources is crucial for the development of new resources or the implementation of existing resources. Some researchers have chosen to study a particular resource (such as the graphing calculator), and how instructors use that resource (e.g., Doerr & Zanger, 2000; Remillard, 2000). Others focus on understanding and categorizing the resources instructors use (e.g, Adler, 2000; Remillard, 2005), and some researchers study a particular topic in combination with a particular resource or resources (e.g., Bowman, 2018), while others have worked on proposing and developing frameworks for understanding resource use (Gueudet & Trouche, 2009; Rabardel & Yaern, 2003). In my experiences, the resources I was exposed to impacted a very specific aspect of my teaching. The textbook impacted how I taught exponents, and the graphing calculator allowed me to introduce linear regression; the resources and the topic of instruction were tightly linked. Therefore, in studying resource use by other instructors, I focused on a specific topic that had the potential for resources to impact teaching.

My interest in the FTC came gradually. I initially took calculus in high school, and much of my memory of that course is vague. In my undergraduate work, I learned about the fundamental theorem of algebra and the fundamental theorem of arithmetic. In my graduate mathematics courses, years later, these two theorems stuck with me, but I did not remember the FTC until I took a course in complex analysis and was confronted with a use of Green's theorem, and then an advanced calculus class where I spent a great deal of time trying to understand and reproduce a proof of Stokes Theorem. These two theorems are generalizations of the FTC[2], and the proof of Stokes theorem that I focused on required the use of the FTC. As I studied where the FTC fit into this proof, I realized that even though it was called "fundamental", I didn't recall either the theorem or the proof. Why could I recite and explain (although not prove) the fundamental theorem of integers and the fundamental theorem of algebra, but not the fundamental theorem of calculus? In the course of reviewing the FTC, I discovered that the fundamental theorem of calculus occupied the unique position of being a fundamental theorem that could be introduced, applied, and proven within the same course of first semester calculus. This was not the case for the other two theorems. Proof of the fundamental theorems of algebra and integers required upper level college mathematics. So why was it that I had to relearn the

[2] There are four theorems of vector calculus that are considered fundamental: 1) Green's Theorem, 2) Gradient Theorem for Line Integrals, 3) Divergence Theorem, and 4) Stokes' Theorem. Green's theorem is sometimes called the two-dimensional version of Stokes' theorem.

FTC? It is a *fundamental* theorem, clearly used later in mathematics courses, and I had considered myself a better than adequate mathematics student. I found myself frustrated with having to relearn a theorem that I felt I should already know. As I gained a growing understanding of the importance of this theorem that I didn't have in my undergraduate studies, I wondered why I hadn't learned it well the first time around. I learned calculus via paper, pencil, and a textbook, and students now had graphing calculators and computers. Was it just me? How did resources matter? As an instructor, I understood that resources had influenced my teaching, but what about this particular topic? Was teaching it now, with many more available resources, significantly different than when I had learned it many years ago? This dissertation combines my interest in resources with my interest in the teaching of the FTC by examining what resources two-year college instructors use in teaching the FTC, how they use those resources, and why they use them.

Resources used in Teaching

Early research into resource use in mathematics classrooms focused on material resources; primarily the textbook. Much of the motivation for the study of how curriculum materials such as textbooks are used dates back more than fifty years. Through two large-scale international studies in the 1960s and 1970s, the importance of textbooks became evident. In the early 1960s, the First International Mathematics Study (FIMS) was designed to consider how various "inputs" such as money, teacher competence, materials, teaching time and method of instruction related to the "outputs" of student achievement and attitudes (Husén, 1979). FIMS was designed with the implicit assumption that better inputs should result in better outputs, and that different inputs results in a different student achievement. Unfortunately, rather than a comparison of inputs to student achievement, the study was perceived as an international mathematics contest, with winners and losers. An effort was made to point out that this was an invalid contest, because different countries had different curricula. For example, students surveyed from selective schools in Germany and England had been exposed to algebra at the time of the test, while students in general schools in Sweden and the United States did not have that exposure (Husén, 1979). Anticipating this reaction, researchers had asked teachers about the about whether their students had the "opportunity to learn" the material, which turned out to be an ambiguous question with answers that could not be reliably compared. For example, did a teacher who agreed that students had an 'opportunity to learn' some content mean that the material was a part of the intended curriculum or simply that the material was in the textbook?

These questions led researchers in the 1970s to re-design the study as the Second International Mathematics Study (SIMS) and focus on the relationship between the inputs of FIMS, how those inputs were implemented, and then student achievement. This was articulated as the intended, implemented and attained curriculum (Brown, 1996; Pepin, 1999). SIMS targeted students around 8^{th} and 12^{th} grade, and was much more focused on curriculum. A critical feature of the Second International Mathematics Study was that of obtaining reports from the teachers of the classes on whether the mathematics on the test had been taught, either during the year or in a prior year. Rather than asking a vague question about whether students had the opportunity to learn the material, teachers were asked directly whether or not they had taught the mathematics on the test. The resulting data, called "opportunity-to-learn," was found to have more impact on the attained curriculum than all other variables studied, including teaching methods (Brown, 1996). As a result of the answers to the survey provided by teachers as well as examinations of textbooks, researchers found that teachers in the US were more tied to the textbook than instruction in most other countries. In particular, items that were taught, as well as strategies used for teaching, were primarily found in the textbook. However, "exclusion from the textbook made it virtually certain that the strategy or representation would not be used." (McKnight, et al 1978 p. 75). Results from SIMS indicate that what students learn is highly correlated with what is taught (opportunity to learn), and what is taught can be found in the textbook. Based on these results, understanding how teachers and instructors use their textbooks, and what parts of the textbook they use can contribute to our understanding of how and why students learn what we intend for them to learn.

Results from FIMS, SIMS, and later studies continue to show a strong link between what is in the textbook and what is taught in the classroom (Schmidt, Houang, & Cogan 2002). Several studies after SIMS have focused on how mathematics teachers use their textbook in the classroom, but most of these have focused on elementary and middle-school mathematics (e.g., Remillard & Bryans, 2004; Sherin & Drake 2009; Sosniak &Stodolsky, 1993). Few studies, however, have examined how teachers or instructors of more advanced mathematics use their curriculum materials. There is some research that implies that the textbook and curriculum materials are important, even in undergraduate mathematics education (Nichols, 2009; Weinberg et al, 2012), but little research on how they are used (Mesa & Griffiths, 2010; Mesa, Wladis & Watkins, 2014).

The idea that the textbook is the primary resource for mathematics teaching is still prevalent, but changing. Many mathematics classrooms are incorporating technology such as graphing calculators and computer algebra systems (CAS) into the curriculum (Crowe & Zand, 2000; Buteau, Marshall, Jarvis & Lavicza, 2010). Some textbooks are written to accommodate these technologies. There is abundant research into the challenges and affordances offered by these technologies by teachers/instructors and students from elementary school through college (e.g., Ball, Ladel, & Siller, 2018; Buteau, et al 2010; Crow & Zand, 2000; Kilicman, 2010), but very little that examines how college instructors—who may not have training with technology—are using those technologies in the classroom. The definition of resources has slowly expanded from primarily tangible material resources to include human resources (such as students) as well as cultural resources (Adler, 2000). Remillard (2005) published an extensive literature review about research into the use of curriculum materials which found that many researchers included the intangible resource of instructor background and knowledge as a factor in how curriculum resources were used.

For this work, I am interested in more than curriculum resources; I am interested in all the resources instructors use when they plan and teach the FTC, what those resources are, how they use those resources, and why they use the resources in particular ways. With that in mind, it is helpful to define what constitutes a resource. The Oxford English dictionary (2018) defines resources as assets that can be drawn on by a person or organization in order to function effectively. For this study, I define resources as assets that instructors draw on that impact their planning and teaching of the FTC. I define teaching as time spent with students either in class or in office hours, and I define planning as time spent getting ready to teach, including choosing homework. Similar to other researchers (Adler, 2000; Pepin, Gueudet, & Trouche, 2014), I consider resources and their uses together. I describe the conceptualization of resources in more detail in the literature review.

Two-year college context

I chose to look at two-year college instructors for two reasons. First, two-year colleges educate a significant number of students. Of the 3,852,000 students taking mathematics in fall 2015, 14% (556,000) of them were enrolled in a first semester calculus course at a two-year college. Of all students taking *first* semester calculus, approximately 15% are enrolled at a two-year college (Blair, Kirkman, & Maxwell, 2018).

Second, I believe that the opportunity to learn about the greatest variety of instruction is at the two-year college level. Calculus is offered in high schools, two-year colleges, four-year colleges, and universities. Instructors at two-year colleges have important similarities with instructors at these other institutions. Two-year colleges are also known as teaching institutions because teaching is their priority (Cohen & Brawer, 2003), so two-year college instructors may share a focus on teaching with K12 teachers. However, they may not have had the pedagogical training that K12 teachers are required to have. A master's degree and/or doctorate degree in the field taught is sufficient to become a community college instructor: no teaching certification is required—making these instructors similar to post-secondary instructors at other institutions. As a result, these instructors often bring adequate disciplinary knowledge, and an interest in teaching, but may have little pedagogical training and experience (Grubb, 1999).

There are reasons to believe that calculus may be a desirable course to teach at two-year colleges, because it is one of the most advanced math courses available and often taught by full-time instructors. Within a two-year college mathematics department, there are fewer sections of calculus than sections most other courses, sometimes only one section per year. Because of this, instructors may have more autonomy over calculus instruction than a course that must be coordinated among multiple sections and multiple instructors.   There is some evidence that calculus instructors at two-year colleges are more interested in teaching calculus more than calculus instructors at other institutions (Bressoud, 2012). Thus, the combination of caring about teaching with the potential autonomy of teaching a calculus level course offers motive and opportunity for instructors to use a variety of resources.

Much of the growing research into two-year college mathematics education involves developmental mathematics (Waycaster, 2001; Bahr, 2008; Melguizo, Kosiewicz, Prather & Bos, 2014). Some attention has been paid to calculus at the two year college, including the resources that instructors use, particularly as part of the extensive NSF study titled Characteristics of Successful Programs in College Calculus I (CSPCC). There are also smaller studies that explore college mathematics instructors' use of resources, but these generally lack a focus on a specific mathematical topic (Gueudet & Pepin, 2018).

Calculus and the FTC

Two-year college instructors have things in common with both K12 teachers (a focus on teaching) and post-secondary instructors at other institutions (no required pedagogical training), and all of these instructors teach calculus. Calculus is the culmination of mathematics courses for some students and the beginning of more intensive math courses for others. Calculus 1 is often positioned as a gatekeeper course for students wishing to enter STEM fields (Treisman, 1992; Blair, Kirkman, & Maxwell, 2018), making the passing of calculus high stakes for these students. Yet a recent national study of calculus students found that 38% of those enrolled in two-year college calculus either failed or withdrew from the course (Bressoud, 2015).

The calculus reform movement from the 1980s and 1990s encouraged instructors and teachers to break away from traditional lecture and paper/pencil problem solving to teaching for understanding (Hughes-Hallett, n.d). Since then, resources and technologies have been developed to support these changes. For example, graphing technologies have evolved from hand-held calculators to sophisticated online programs and three-dimensional (3D) printers. These resources have been tested with students and shown to offer the *opportunity* for increased conceptual understanding and procedural fluency (Artigue, 2002; Dunham & Dick, 1994).

Even with the motivation of the calculus reform movement, students are still struggling with calculus, and calculus is still functioning as a gatekeeper course (Bressoud, 2012; Suresh, 2006). A recent study involving more than 3,000 calculus students in the U.S. found that their confidence and enjoyment of mathematics dropped sharply from the beginning to the end of their first semester calculus course (Bressoud, 2015). However, the drop in confidence level was mitigated by what was called 'good teaching.' Indicators of good teaching "largely reflect the rapport between student and instructor" (p. 184), and include such things as the instructor listening to student questions and making students feel comfortable in class. The use of educational technologies was not associated with good teaching, but was considered neutral in most cases (Sonnert, *et al.*, 2015).

Increasing student confidence and enjoyment of mathematics (or at least, not decreasing it) is a desirable goal, but is not an indicator that students understand the mathematics being taught. There is considerable evidence that even successful students in calculus do not have a good understanding of some of the concepts of calculus (e.g., Byerley, Hatfield, & Thompson, 2012; Fisher, Samuels, & Wangberg, 2016; Grundmeier, Hanson, & Sousa, 2006; Park, 2013; Sealey, 2014, Serhan, 2015).

Some of the primary concepts of calculus are limits, derivatives, and integrals (Burn & Mesa, 2015; Sofronas, *et al*., 2011). Within a first semester calculus course, the fundamental theorem of calculus (FTC) links the primary concepts of integration and differentiation. Concepts involved in understanding the FTC are used in physics, engineering, and other related fields. The FTC is the historical grand breakthrough of calculus and links the branches of differential and integral calculus (Bressoud, 2011). This theorem prepares students to learn later mathematical theorems such as Green's Theorem and Stokes' theorem. If students struggle with the initial FTC, they may struggle with later mathematical theorems, and with mathematical concepts in other fields.

Understanding the FTC is particularly challenging. For example, a student may understand that a derivative can represent the velocity from a graph that displays a relationship between distance and time. A student may also understand that taking the integral of a function means doing the opposite of taking the derivative. Yet most students consider an integral to represent an area "under" a curve. If the curve that is being integrated is a velocity function, then the integral would represent a distance. However, connecting the idea of area with a distance can be difficult.

There is some research into ways to teach the FTC for understanding (Byerley, et al., 2012) that recommends introducing the ideas of rate of change (differentiation) and accumulation (integration) toward the beginning of a calculus class. This same study recommends graphing technologies as vital to teaching the understanding of the FTC. Yet most instructors are teaching differentiation and integration separately and there is inconsistent use of graphing technologies in calculus classes.

This dissertation uniquely connects scholarship on calculus, two-year colleges, instructors' use of resources, and the FTC by investigating the resources used by two-year college instructors when they plan and teach this important theorem. I investigated what resources are used, how they are used, and why they are used in that way. Examining the resources used by two-year college instructors to plan and teach the FTC affords an in-depth look at an important topic in a mathematics course that many students do not successfully complete.

Overview of the dissertation

In the chapters that follow, I explore how Michigan two-year college instructors described their use of resources when planning and teaching the FTC. In Chapter Two I begin by reviewing and synthesizing the literature available on mathematics teachers and instructors use of resources, including the use of technology as a resource in calculus classrooms. This is followed by a historical description of the FTC as well as an explanation of the mathematics needed to fully understand the conceptual underpinnings of the FTC. In this chapter I also outline my theoretical stance and the frameworks that I drew upon to analyze the data. Chapter 3 details the methods used to collect the data for this dissertation, including the rationale for why the mixed methods approach was appropriate for this study. The methods for analysis of each type of data are explained as well. This chapter concludes with a description of my own subjectivity and the limitations of this study. Chapter 4 details the findings about what resources instructors are using, how they are using them, and why they are using them. I begin by describing the ways that instructors talk about the FTC in their interviews and then outline the major resources, as well as how and why they are used by instructors. Chapter 5 is a discussion of the findings and Chapter 6 includes the potential implications of these findings.

Chapter 2: Literature Review

What resources do two-year college instructors use when planning and teaching the fundamental theorem of calculus (FTC)? How do they use those resources? Why do they use them in that way? In this chapter I reviewed literature relevant to resources, technology and calculus teaching, and the FTC. For this dissertation, I examine the resources that instructors are using when planning and teaching the FTC. For the purposes of this work, I define teaching as direct interactions with students, including class time and office hours. I define planning as getting ready to teach, including such things as planning a semester, planning lessons, and choosing and grading homework.

In order to define what constitutes a resource for teaching, I adjusted Adler's (2000) framework for conceptualizing resources. For this study, I define resources as assets that instructors draw on that impact their planning and teaching of the FTC. In her work, Adler categorizes resources into two categories, which she calls basic resources and "other" resources. Basic resources are those resources that are needed for schooling. These include material resources such as electricity and basic building infrastructure, and human resources such as class size and teacher qualifications. Other resources can be human, material, and social-cultural. Other human resources include the teacher knowledge base, parents, and colleagues. Other material resources include such material items as technologies and textbooks. A new type of resource in the "other" category is the cultural resource. Cultural resources are primarily language and time. Language includes as the language spoken, the mathematical language of instruction, and communication between students. Time as a resource includes the organization of the time periods and the length of periods. In terms of how instructors use resources, Adler argues, "the functioning of a resource in and for school mathematics lies in its use in context, and not in the mere presence of the resource" (Adler, 2000, p. 221). In other words, for a resource to be used for learning, the focus will need to be removed from the resource itself, and onto the resource in combination with the mathematics. In this way the resource becomes the means through which mathematics is learned.

For the purposes of this chapter, I use Adler's conceptualizations of "other" resources in the context of two-year college calculus (see Table 1). For each of these categories, I review some of the literature available, in the context of two-year college calculus when possible.

Table 1: Resource Categories (Adapted from Adler, 2000)

Resource	**Examples**
Material: Technologies	Computers, internet
Material: School mathematics materials	Graphing calculators, textbooks, associated computer software
Human: Persons	Teacher background and experience
Social Cultural: Language	Mathematical and everyday language, communication
Social Cultural: Time	Total class-time, length of class period

The first category is material resources. Material resources include technologies such as graphing calculators, computers, websites, and software. Material resources also include curriculum materials such as the textbook and associated software. The second category of resources to consider is human resources. This includes the background and experience of instructors and how that affects their teaching and planning. The third category of research are social-cultural resources. These resources are largely intangible and include both every-day language and mathematical language. Social-cultural resources also include time; both time spent in class in terms of the length of each lesson, and the total time spent in class over a semester. Finally, all of these resources have a relevant context. In this dissertation there is an institutional context as well as a mathematical context. The institutional context is two-year colleges in Michigan and the mathematical context is the FTC in a first semester calculus class. This chapter examines the literature on resources within each of these categories, while always keeping in mind, “resources are not self-explanatory objects with mathematics shining clearly through them. Mathematical meaning comes in their mediated use” (Adler 2000, p. 209).

This chapter has two parts: In the first part, I present literature on the research into the context of two-year colleges and calculus instruction, followed by literature on calculus and the FTC. In the second part I present literature on material, human, and social-cultural resources, in terms of calculus and two-year colleges when relevant.

Context: two year colleges in Michigan

This dissertation examines calculus instructors at two-year colleges in Michigan. Like many two-year colleges throughout the country, two-year colleges in Michigan serve multiple needs. In general, two-year colleges are open access, offering anyone who wants it the opportunity to enroll in the college, regardless of former grades and educational background. They serve a dual purpose of technical or vocational training and general college education. This dual-purpose two-year college institution is uniquely North American (Cohen & Brauer, 2003), and place two-year colleges in the position of being not quite like high schools and not quite like other colleges and universities. Although I often refer to these institutions as “two-year colleges” in this document, many other sources call them “community colleges.” These terms are used interchangeably, particularly in this section, because so much of the literature uses the term “community colleges.” Community colleges in Michigan are defined in the Public Act 306 of 2003 as follows:

> "Community college" means an educational institution providing collegiate and noncollegiate level education primarily to individuals above

> the twelfth grade age level within commuting distance. The term includes an area vocational-technical education program that may result in the granting of an associate degree or other diploma or certificate. (MCL 389.105(c))

Within two-year college mathematics departments in Michigan, calculus is offered as part of the collegiate level education and be taken by students intending to transfer to four-year colleges and universities.

The two-year colleges (formerly known as junior colleges and now community colleges) began in the early twentieth century as primarily local institutions, set to serve local populations. The local control for community colleges that is so much a part of Michigan's current system is not the case in most other states[3], and can be seen as part of the history of two-year colleges in Michigan.

Michigan initially adopted community colleges as outgrowths of local school districts, and that history can be seen in the decentralized nature of current two-year colleges in Michigan. As early as 1852, the president of the University of Michigan called for early college courses to be offered in secondary schools (Tappan, 1852). The first junior college in Michigan was established as part of the Grand Rapids school district in 1914. In 1917, Michigan passed its first junior college law (Public Act 146), authorizing large school districts to offer advanced courses to high school graduates that did not exceed two years of collegiate work. In 1955, a Legislative Study Committee on Higher Education was formed to survey higher education in Michigan, with "a major section on the community college" (Martorana, 1957, p. ii). This report found that by 1956 there were over 16,000 student enrolled in Michigan's 14 community colleges (Martorana, 1957), and noted the extraordinary growth of enrollment in two-year colleges (154%) compared

[3] In a review by the Education Commission of the United States that examined state-level coordinating agencies for higher education, they write, "Michigan does not really have a state-level coordinating or governing agency for postsecondary education." The only other state that has no coordinating state level agency is Vermont. See https://www.ecs.org/postsecondary-governance-structures/

with growth in all higher education (53%). One of the recommendations of this report was that the law be adjusted to provide a mechanism to found community colleges that span two or more school districts. Public acts in 1954 and 1955 had established the possibility to operate community colleges independently from the local school districts (Kolins, 1999), but did not require community colleges to operate separately.

The operation and founding of community colleges was fully removed from local school districts with the adoption of the state constitution in 1963. The 1963 constitution of the State of Michigan, removed the requirement that community colleges be tied to school districts, stating, “The legislature shall provide by law for the establishment and financial support of public community and junior colleges which shall be supervised and controlled by locally elected boards.” (State Constitution of Michigan, Article VIII, Section 7, 1963). In other words, the founding of two-year colleges was to remain local and public, but should not be tied to a particular district of a particular size. Community colleges continued to expand in Michigan, and by the early 1970s, Michigan was one of seven states to have a “mature” system of community colleges, with 90-95% of the state’s residence living within “reasonable” commuting distance (Cohen & Brawer, 2003, p. 17). Currently, Michigan has 28 two-year colleges spread throughout the state, each with their own locally elected boards. The reliance on local governance means that two-year college math departments in Michigan can generally set their own requirements in terms of resources that instructors’ are required or recommended to use and courses that are offered.

Context: Mathematics

This section of the literature review focuses on the Fundamental Theorem of Calculus (FTC). I begin with an explanation of why calculus, and the FTC specifically, is a good topic for understanding instructors' use of curriculum resources. I then offer a historical overview of the FTC, and I conclude with the mathematics involved.

In general, calculus is a well-researched area of mathematics at both the secondary and post-secondary levels (e.g., Bressoud, Carlson, Mesa, & Rasmussen 2013; Sofronas et. al, 2011; Tall, 1996; White & Mitchelmore, 1996). The calculus reform that started in the late 1980s gave a major push to technology (Cole 1996; Hughes Hallet, n.d; Murphy, 2006). Calculus teachers were encouraged to incorporate technology into their teaching on a regular basis, and curriculum materials were designed that included technology (Ferrara, Pratt, & Robutti, 2006; Ferrini-Mundy & Graham, 1991). Many calculus textbooks have included sections for using a graphing calculator or programming in Maple for years (e.g., Finney, Thomas, Demana, & Waits, 1994; Larson, Hostetler, & Edwards, 2007). Thus, calculus offers an opportunity to study how instructors use curriculum resources in a solidly researched course with an extended history of encouraging the use of curriculum resources beyond the textbook.

History of the Fundamental Theorem of Calculus. In an article published in 2011, Bressoud described the history of the Fundamental Theorem of Calculus (FTC) as the historically "grand breakthrough" of calculus that relates differentiation and integration. Tradition attributes the discovery of the FTC to Newton and Leibniz around the same time, yet two earlier mathematicians, Isaac Barrow and James Gregory, applied the FTC to calculating a curve length without recognizing the importance of their discoveries. According to Bressoud (2011), Barrow and Gregory are not credited with the discovery of the FTC because they "never married [algorithmic] technique to the general application of this theorem […] it is precisely in the combination of algorithmic technique with a grasp of the full meaning of the [Fundamental

Theorem of Integral Calculus [4]] FTIC that calculus becomes a useful tool" (p. 103). This "full meaning" of the FTIC includes the idea of relating a rate of change and the accumulation of area under a curve. It also includes "broad applications to variable phenomena" (Bressoud, 2011, p. 106) and the use of calculus to analyze curves.

Both Newton and Leibniz recognized the importance of relating a rate of change and accumulation. Newton was analyzing curves while Leibniz was focused on the integral as a sum of infinitesimals under a curve. Bressoud (2011) and Thompson (1994a) have suggested that students should have exposure to these early dynamic explanations of the FTC—relating rate of change and accumulation of area. However, textbooks today do not generally make the connection between the rate of change of a function and the accumulation of the area under the function, and thus do not reflect how the FTC was discovered (Bressoud, 2011; Tall, 1996).

The FTC is the "glue" of calculus. But *how* it is the "glue" is lost without considering the context of how it came about by studying the problems that Newton/Leibniz (and others) were trying to resolve (Bressoud, 2011; Thompson 1994a). The history of the FTC is important in this study because common college textbooks devote only one section of one chapter to the FTC. If instructors are primarily using their textbooks for this topic, the importance and relevance to the rest of calculus may be underrepresented. However, if instructors recognize the textbook as providing only limited information about the FTC, they might consider a wide variety of resources in addition to the textbook and course materials when teaching the FTC. Thus, the FTC offers a bounded area to research that is rich with mathematical and historical context and has the potential for a variety of resource use.

[4] Bressoud (2011) refers to the FTC by its historical designation of FTIC, Fundamental Theorem of *Integral* Calculus, which was in use until approximately 1970 (p. 109)

Mathematics of the FTC. This section begins with a brief description of where the fundamental theorem of calculus fits within the calculus curriculum, then a brief description of the FTC followed by a description of some applications in other fields.

The fundamental theorem of calculus is central to the study of calculus. Calculus has many applications to other fields, such as physics and engineering. In these fields, the fundamental theorem of calculus is used to describe the relationships between the rate of change of a quantity and the accumulation of that quantity.

Although I refer to *the* fundamental theorem of calculus throughout this paper, the theorem is sometimes referred to as two theorems, or one theorem in two parts. These two parts provide the conceptual link between differentiation and integration. One part details the link between a function describing a rate of change and the accumulation of the area under the graph of that function, whereas the second part states that *any* continuous function is the derivative of some other function, independently of whether or not we can define that function in simple terms. The first part is useful for determining exact values in application problems. For example, if I have a container that can be described by an integrable function, I can determine the exact amount of a material needed to fill that container. The second part states that for a function f (e.g., e^{-x^2} which describes the standard normal distribution), there is another function g that describes the area under the graph of f, and that we can approximate g (although we may not be able to describe it in simple terms). In the case of the standard normal distribution, the area represents the percentage of the sampled population between two points.

Two small-scale research studies into applications of the fundamental theorem of calculus have found that students have difficulty applying the concepts that underlie the theorem. Bajracharya and J. Thompson (2014) asked students in physics and calculus to solve physics problems that were best solved by applying the FTC. They found that these students “failed to use the FTC to determine physical quantities, e.g., the change in internal energy, when the question did not include an algebraic function explicitly” (p. 5). This was concerning because students in advanced physics are often expected to be able to find connections between the rate of change and the accumulation of a quantity based on a graphical representation. This research suggests that students may be learning how to use FTC only superficially, and without sufficient understanding that would allow them to transfer its use to new contexts.

A second study (Jones, 2015) investigated eight physics and engineering students’ understanding of the integral. The student responses to integration questions were interpreted in terms of three possible conceptual understanding of the integral. One of these understandings, ‘function matching’, interprets the integral as coming from the derivative of some original function, as in the FTC. Jones found that these students were able to make productive use of function matching for decontextualized problems, but did not use this notion productively when given problems in context. For example, students were asked to explain the meaning of the integral $\int_0^{600} R dt$ where R represented the varying revolutions-per-minute of a motor. Students who used function matching were able to correctly explain that the integral resulted in a measure of revolutions, but were not able to explain why the integral of a velocity function in meters/second, results in a length (just meters). Students could not explain why the units changed using the function matching conception. In contrast, students who visualized the problem as adding up pieces (taking infinitely many rectangles) were able to algebraically justify the change in units.

These two studies suggest that students may superficially understand the connection between the derivative and the integral as illustrated in the FTC and have difficulties applying this knowledge. One thing that could be relevant to interpret the findings from these studies is information about students' experiences learning the FTC. I would like to know, what curriculum resources their instructors used when teaching the FTC? Did the instructors make appropriate use of technology? The next portion of this literature review explains the mathematics needed to understand and apply the FTC, illustrates a problem a standard textbooks' explanation of the FTC, and describes some research being done with alternative ways to understand the FTC.

Mathematical concepts needed to understand the FTC. In this section I describe the fundamental theorem of calculus in more detail, including some of the relevant mathematical concepts and applications. I detail other research that has been done on the fundamental theorem of calculus, and I conclude with how my research questions fit into this research.

The fundamental theorem of calculus may be considered as the link between differentiation and integration; evidence that integration and differentiation are inverse operations. It may be stated formally in two parts as follows:

> **Fundamental Theorem of Calculus I**: If a function f is continuous on the closed interval $[a, b]$ and F is an antiderivative of f on the interval $[a, b]$ then $\int_a^b f(x)dx = F(b) - F(a)$. (Larson, Hostetler, & Edwards 2007 p. 282)
>
> **Fundamental Theorem of Calculus II**: If f is continuous on an open interval I containing a, then, for every x in the interval, $\frac{d}{dx}\left[\int_a^x f(t)dt\right] = f(x)$. (Larson, Hostetler, & Edwards 2007 p. 289)

There are several mathematical concepts needed to understand the FTC; specifically functions, anti-differentiation, differentiation, rate of change, continuity, and limits. The two primary concepts that research attributes as challenging for understanding the FTC are the concept of function and the concept of rate of change (Bajracharya & J. Thompson, 2014; Carlson, Smith, & Persson, 2003; P. Thompson, 1994).

In Larson et al (2007), the proof of the first part of the theorem is paraphrased as follows:

Consider a partition of $[a, b]$ as follows: $a = x_0 < x_1 < \cdots < x_{n-1} < x_n = b$. Then rewrite $F(b) - F(a)$ as $F(x_n) - F(x_{n-1}) + F(x_{n-1}) - \cdots - F(x_1) + F(x_1) - F(x_0)$

$$= \sum_{i=1}^{n} [F(x_i) - F(x_{i-1})].$$

By the Mean Value Theorem, there exists a number c_i in each ith sub-interval such that

$$F'(c_i) = \frac{F(x_i) - F(x_{i-1})}{x_i - x_{i-1}}.$$

Choose $\Delta x_i = x_i - x_{i-1}$, and because $F'(c_i) = f(c_i)$,

$$F(b) - F(a) = \sum_{i=1}^{n} f(c_i)\Delta x_i.$$

This says that by applying the Mean Value Theorem, the **constant** $F(b) - F(a)$ is a Riemann sum of f on [a,b] and taking the limit as $\Delta x_i \to 0$ yields $\int_a^b f(x)dx = F(b) - F(a)$.

(adapted from Larson, Hostetler, & Edwards, 2007, p. 283)

Although this proof is accurate and understandable, it presents a conceptual challenge. Using the mean value theorem in this way involves considering the rate of change at a single point. This interpretation assumes a static situation, as if nothing is changing. Yet calculus is the study of how things change. Relying on a relatively simple proof that uses a static model in this way may inhibit the conceptual understanding of the FTC, its importance to calculus, as well as the link between differentiation and integration (P. Thompson, 1994a). A report by the Committee on the Undergraduate Program in Mathematics (CUPM, a part of the Mathematical Association of America, or MAA) recommends against defining the definite integral strictly as a limit of Riemann sums, stating that the traditional approach is to define the definite integral as a limit of Riemann sums and then explain FTC as stating that integration and differentiation are inverse processes. Most students, however, never grasp the formal definition and understand integration as antidifferentiation, thus removing any meaning from FTC. A better approach is to explain FTC as stating the equivalence of two ways of understanding the definite integral: as the change in the value of an antiderivative or as the limit of a summation. (CUPM, 2004)

Table 2: Mental Actions of the Covariational Framework. Adapted from Carlson, Jacobs, Coe, Larsen, and Hsu (2001, p. 357).

Mental Action (MA)	Description of mental action	Behaviors
MA1: Coordination	Coordinating the value of one variable with changes in the other	Labeling the axes with verbal indications of coordinating the two variables (e.g., *y* changes with changes in *x*)
MA2: Direction	Coordinating the direction of change of one variable with changes in the other variable	• Constructing an increasing straight line • Verbalizing an awareness of the direction of change of the output while considering changes in the input
MA3: Quantitative Coordination)	Coordinating the amount of change of one variable with changes in the other variable	• Plotting points and constructing secant lines • Verbalizing an awareness of the amount of change of the output while considering changes in the input.
MA4: Average Rate	Coordinating the average rate-of-change of the function with uniform increments of change in the input variable	• Constructing contiguous secant lines for the domain • Verbalizing an awareness of the rate of change of the output (with respect to the input) while considering uniform increments of the input
MA5: Instantaneous Rate	Coordinating the instantaneous rate of change of the function with continuous changes in the independent variable for the entire domain of the function	• Constructing a smooth curve with clear indications of concavity changes • Verbalizing an awareness of the instantaneous changes in the rate of change for the entire domain of the function (direction of concavities and inflection points are correct)

An alternative way of understanding the FTC uses the concept of covariation, specifically stating that accumulation and rate of change are related. The concept of covariation refers to coordinating “an image of two varying quantities, while attending to how they change in relation to each other” (Carlson, Persson, & Smith 2003). Covariation is described by Carlson, Jacobs, Coe, Larson, and Hsu (2002) as essential for understanding the concepts of calculus. The mental actions and behaviors that indicate covariational reasoning are described in **Error! Reference source not found.**.

Students in this study were given a level classification if their behavior indicated a mental understanding at that level and all previous levels. For example, a student classified as having Level 3 (covariational reasoning) needs to display behavior at Mental Actions 1, 2, and 3. P. Thompson (1994b) implied that covariational reasoning is vital for students’ understanding of the Fundamental Theorem of Calculus: “The Fundamental Theorem of Calculus—the realization that the accumulation of a quantity and the rate of change of its accumulation are tightly related is one of the intellectual hallmarks in the development of the calculus” (p. 130). However, Carlson et al. (2002) found that students who had recently completed a second semester of college calculus with a grade of A had difficulty understanding that an instantaneous rate of change resulted from smaller and smaller refinements of the average rate of change. The idea that instantaneous rate of change can be described by smaller refinements of the average rate of change is what is used in the mean value theorem proof of the FTC. Yet even good students have demonstrated difficulty with this idea.

P. Thompson and his colleagues developed an experimental course using technology to explore covarying relationships between accumulation and rate of change (P. Thompson, Byerly, & Hatfield, 2013). This course was shown to offer students a more robust understanding of FTC. Stated goals of this course included that students “formalize the relationship between accumulation and rate of change—that has been employed throughout—by stating it as the

Fundamental Theorem of Integral Calculus" and "understand the fact that every rate of change function has an accumulation function. Some accumulation functions can be expressed in closed form; most cannot" (p. 127). The course used technology that allowed students to explore visually and transform functions that model relationships between accumulation and rate of change. The problems model real-world situations and highlight the interaction between accumulation and rate of change. This course did not use a commercially available textbook; instead technology (Graphing Calculator 4.0, Avitzur, 2011) was extensively used. These authors suggested that a traditional textbook should not be used, because the fundamental theorem of calculus is present from day one, and it is not limited to a small section of the course (Byerly, personal communication, March 6, 2015). The authors concluded that this course was successful in getting students to understand the fundamental theorem of calculus and covariation.

I choose to consider the FTC to research how instructors use their resources for three main reasons. First, the FTC is "fundamental" to calculus, with a great deal of historical context. Second, the mathematical context for the FTC is extensive, including concepts of limits, derivatives, integration, functions, and covariation. Third, the treatment of the FTC in traditional textbooks is short, generally one section of one chapter. This means that instruction on the FTC may span only one or two class sessions, yet the mathematical and historical context offer a potential for a wide variety of resource use.

In summary, the FTC is important and can be taught with a wide variety of resources. There is some evidence that college instructors may use resources differently than K-12 teachers, but research into what resources college instructors use and how they use them is scant. There is evidence that the resources instructors use can affect their teaching. However, in teaching and planning, the same resource may be used very differently by different instructors, and the same instructor may use the same resource differently depending on the context. Thus it is important to focus on how instructors use their resources within one mathematical context.

Material resources

Scholarship on resources used in mathematics classrooms primarily focuses on tangible resources, such as curriculum materials and textbooks. Studies have shown that curriculum materials and their uses are correlated with student outcomes (McKnight, 1987; Schmidt, Huoang, & Cogan, 2002; Stein, Remillard, & Smith, 2007), and that mathematics instruction in the United States is driven by the textbook (Begle, 1973; McKnight, 1987; Schmidt, Huoang, & Cogan, 2002). The most common material resource for instructors and students in the mathematics classroom is a textbook (Nichols, 2009; Stark, 2000). Most of the scholarship that considers how teachers and instructors use material resources is at the elementary and middle school mathematics level, with some investigations into high schools and college.

Within this scholarship, I focus on two types of literature: 1) studies that focus on curriculum materials and their intended use compared with their actual use; and 2) studies that focus on teacher/instructor interaction with curriculum materials and resources, and how teachers/instructors and curriculum resources changed with that interaction. I expand on these areas in the following sections.

Intended and actual use of curriculum materials. Initial studies of curriculum were concerned with how curriculum materials (which were often new or "reformed") were implemented by teachers. One of the first studies that used this approach was the International Association for the Evaluation of Education Achievement's (IEA's) Second International Mathematics Study (SIMS) (McKnight et al., 1987; Brown, 1996; Pepin 1999). SIMS was implemented between 1977 and 1981, and targeted students in 20 countries at two levels; students at age 13 and students studying mathematics who were in their final grade of secondary education. SIMS was developed in part as an attempt to understand some of the results of the First International Mathematics Study (FIMS). For example, FIMS included a broad set of potential questions for students, which none were expected to have completely covered. In order to mitigate that, teachers were asked to indicate which questions students had the opportunity to learn. However, the phrase "opportunity to learn" was open to so much interpretation that this measure for understanding curriculum was unreliable (Freudenthal, 1975, Brown 1996). SIMS addressed these criticisms by narrowing their focus to curriculum, specifically, "international variations in the mathematics curriculum, intended and implemented as well as attained" (Brown, 1996, p.205).

A major contribution of the SIMS study to current mathematical education research is the distinction between the intended, the enacted or implemented, and the attained curriculum. This distinction is described by McKnight et. al, 1987 in defining the curriculum:

> The curriculum is not only what is intended to be taught, as reflected in syllabi and textbooks, it is also what is actually taught (implemented) in classrooms and what is attained by students as a result of that instruction. (p. 85)

The separation of the curriculum into intended, enacted, and attained is a framework that has persisted to the present day.

A consideration of the intent of the curriculum materials has been an important area of research. For example, the intent of curriculum materials may encompass student mastery of knowledge, student skills, and/or increased interest in the subject. This type of research includes textbook analysis (such as Mesa, 2010; Mesa, Suh, Blake, & Whittemore, 2012) and research on the explicit or implicit purpose and potential effect of curriculum materials for both teachers/instructors and students (Cohen & Ball, 1996; Remillard, 2014; Herbel-Eisenmann, 2007).

I include here studies that speculate on the unanticipated effects of curriculum materials, because they are not focused directly on how students learn from the materials, but rather on the potential effect of the materials themselves. For example, some curriculum studies have found that many mathematics textbooks tend to be “closed,” meaning that problems can be solved using a rule or formula (Boaler, 1998; Brown, 1996; Cooper, 1992). The closed nature of problems in these textbooks may encourage students to solve problems procedurally without necessarily understanding the concepts underlying the mathematics that they are using. Although procedural knowledge may not be the stated intent of the materials, this may be the effect.

In addition to considering the intent of curriculum materials in terms of students, several studies have considered the intent and possible effect of curriculum materials on teachers. Research into reform curriculum materials have suggested that features of some curriculum materials are intended to influence teacher understanding and in turn support teachers' innovative instruction (e.g., Ball & Cohen, 1996; Collopy, 2003; Davis & Krajcik, 2005; Remillard & Bryans, 2004). In particular, Remillard and Bryans (2004) examined eight elementary teachers who were using the same reform mathematics curriculum. They examined the teachers' enactment of curriculum materials with a goal of “developing a detailed and nuanced understanding about the relationship among curriculum material, the enacted curriculum, and the possibilities for teacher learning” (p. 356). They interviewed and observed instructors during the first two years of their adoption of a new curriculum. They analyzed observations and interviews and found that the orientation of teachers toward the curriculum implementation included their ideas about mathematics and how it is learned, ideas about the teachers' role, ideas about the general role of curriculum materials in teaching, and views of that particular curriculum material. They found that views of mathematics, teaching, and learning were not closely connected to how a teacher enacted the curriculum. However, views of the general role of curriculum materials in teaching and views of the particular curriculum being used were connected to the use of the materials. This was also connected to whether teachers were open to learning from the curriculum.

What these studies have in common is that they assume curriculum materials can help both teachers and students learn, and that curriculum materials are primarily written for teachers. However, at the college level, the primary audience for curriculum materials is students. In spite of evidence showing that college mathematics instructors use the textbook for planning a course (Stark, 2000), and that they commonly require students to purchase curriculum materials (such as a textbook, Nichols, 2009), these instructors generally perceive the curriculum materials to be written for the students and not for them (Leckrone 2014; Lockwood, Johnson, & Larson, 2013; Mesa & Griffiths, 2012; Weinberg, et al, 2012).

There is promising research that suggests that curriculum materials, if designed properly, can influence college instructors, but this research is very limited. Lockwood, Johnson, and Larson, (2013) developed a software tool for abstract algebra instructors that described anticipated student responses to abstract algebra questions, with varying levels of detail. Although the four pilot instructors in their study commented that they found the tool very helpful, they did not explain *how* the tool was helpful, nor *how* the instructors used the tool when planning and teaching. Lockwood, Johnson and Larson's (2013) study is promising because it suggests that college instructors, at least in this study, are not averse to using well-designed resources to help them teach. By investigating the research question, "How do instructors use their resources in teaching and planning the FTC?," I seek to make explicit how these materials contribute (or do not contribute) to community college instructors' planning and teaching.

Interaction with curriculum resources. More recent studies have considered teachers as participants with the curriculum. These studies offer explanations and frameworks for understanding how teachers use curriculum resources as a tool for teaching (Brown 2009; Remillard 2005; Owens 2014). Studies that focus on the interaction between teachers and their curriculum resources tend to fall into two somewhat overlapping categories, which I call 1) teacher-focused and 2) tool-focused. The teacher-focused studies consider teachers and how their teaching may change as a result of curriculum materials. They also consider how teachers may change and learn due to curriculum materials. In this group of studies, the teacher is foregrounded and the curriculum materials are backgrounded. Curriculum materials in these studies are often seen as static and unchanging. The tool-focused group of studies attend primarily to the purpose of curriculum materials (or tools) and how the purpose (and thus, the nature of the tool) changes based on the tool-users. The tool-focused group of studies sees curriculum materials as dynamic and changing. Although these foci for research are not mutually exclusive, there are differences, primarily in the frameworks used for analysis. I expand on this distinction next.

Teacher-focused. Studies in this sub-group have investigated the implementation of curriculum materials and how the implementation of reform materials may affect teachers and organizations, as well as how curriculum materials can support innovative teaching (Davis & Krajcik, 2005; Davis, Palincsar, Arias, Bismack, Marulis, & Iwashyna, 2014; Fullan & Pomfret, 1977; Schneider & Krajcik, 2002; Schneider, Krajcik, & Blumenfeld, 2005). The findings from this scholarship include two main ideas, that teachers can learn both mathematics and teaching strategies from their textbook, and that textbook use by teachers may change over time (Drake & Sherrin, 2009; Silver, Ghousseini, Charalambous, & Mills, 2009).

Although teachers may learn both mathematics and teaching strategies from their textbooks in grade school, post-secondary mathematics instructors usually have advanced degrees in their subject (Blair, Kirkman, & Maxwell, 2018). This makes it less likely that these instructors will admit to using their textbooks to understand the mathematics, so the finding that instructors may learn both mathematics and teaching strategies from their textbook may not apply at this level. It may also be that instructors do not recognize if and whether they are learning from the materials.

Teacher-focused studies also suggest that new curriculum materials may more directly influence teaching early in their implementation. Teachers may first focus on understanding the materials and deciding what to add or omit from the curriculum resources available. However, as teachers become familiar with the curriculum materials, they seem to focus their attention on the mathematical content in a lesson that is relevant for upcoming lessons. These studies suggested that textbooks may matter more in the first year of teaching with the textbook, but less in subsequent years.

There is some research on the use of textbook by college *students*, with implications for faculty textbook use (e.g., Berry, Cook, Hill, & Stevens, 2010; Durwin & Sherman, 2008; Weinberg, et al, 2012). These studies pointed to a link between the curriculum resource (often a textbook), how instructors chose that resource, and how students were asked (by instructors) to use that resource. A study with college finance students (Berry, Cook, Hill, & Stevens, 2010) indicated that over 50% of the students spent less than one hour per week reading their textbook. However, students felt they would increase their reading if instructors told them exactly what was important. In general, the researchers found that students felt that the professor should be guiding their learning, not the textbook. “Our survey reveals that many students… feel that [the textbook] is a ‘substitute’ for the lecture material rather than an enhancement of the learning process.” Weinberg et al. (2012) surveyed over 1,000 first and second year college mathematics students on how they use their textbook and they found that students primarily looked at examples and answers, not expository text. Similar to the study of finance students,

> when [math] students thought their instructor asked them to read the chapter text frequently…, they were generally more likely to report using the text for various purposes than if they thought the instructor asked them to look at the chapter text infrequently. (p. 164)

In other words, if a math instructor requests that students read their textbook, they may or may not read it as intended, but they will likely do more with the textbook than just look at examples and answers. These few studies of research on textbook use by students have implications for instructors. These studies indirectly link the curriculum resource (often a textbook) with advice for instructors to consider how students are likely to use (or not use) the resource when teaching. In other words, these studies imply that use of resources during class time by instructors should include a discussion with students on how they are expected to use that resource.

There are very few studies that have directly examined textbook use by college mathematics instructors. I found only one study that considered post-secondary mathematics instructors and their use of textbooks. Mesa and Griffiths (2012) interviewed 15 full-time instructors at nine different post-secondary institutions, to find out how their textbook influenced their teaching. They found that although instructors used the textbook for designing the syllabus, preparing classes or assigning homework, these instructors perceived the textbook as written for students, not for them. The instructors also felt the need to work outside the textbook during class time, in order to provide students reasons for coming to class (p. 97). Mesa and Griffiths also noted that instructors seemed to differentiate their instruction based on whether they were teaching “undergrad students” or “math students,” which included honors students. The “math students” were generally expected to read the expository text, whereas the “undergrad students” were expected to work the exercises (p. 96). In the two-year college setting, calculus is one of the highest mathematics courses offered; thus students are not easily classified as “undergraduate” or “math” students.

Other studies linking mathematics instructors with curriculum resources have been less detailed, looking at whether or not the resource is used by instructors, without considering how the resource may be used. For example, an internal survey by the University of Michigan found that 98% of natural sciences faculty survey respondents reported that they “always/often/sometimes” use textbooks in teaching their courses (Nicholls, 2009). An investigation into the general use of textbooks in higher education (Stark, 2000) found, perhaps unsurprisingly, that the textbook is a strong influence on course planning in terms of the structure of the course. However, these studies did not provide details about how instructors actually used curriculum materials within a particular discipline, and therefore they cannot answer the question of how instructors use curriculum resources when planning and teaching students a particular topic, such as the fundamental theorem of calculus.

Tool-focused. Studies in this sub-area of teacher-curriculum interaction are theory-based, and have been influenced by Cultural Historical Activity Theory (CHAT), as well as by scholarship on human-computer (or artifact) interaction. CHAT draws on work from Vygotsky and Leont’ev, and considers both an activity and the cultural historical background of both humans and objects (Engeström, Miettinen, & Punamäki, 1999; Engeström, 2001). While examining teacher interaction with curriculum materials from this perspective, neither the teacher nor the tool is given prominence; what matters is the activity, in this case, teaching.

Rabardel (2003) described the idea that an artifact can be changed by humans into what he calls an "instrument." He described an artifact as "an intermediary mediating position between the subject and the object" (p. 665) and an instrument as "a mixed functional unit made up of components born of the artifact and of others born of the subject" (p. 670). In other words, what may start out as a simple curriculum artifact, such as a textbook, is changed by its use (by an instructor) into something more functional that depends on both the original textbook and the instructor/user.

The idea of an artifact being changed by its use was built upon by Gueudet and Trouche (2009) who described the "instrument" succinctly as "Instrument = Artifact + Scheme of Utilization" (p. 204), and then used this idea to develop a theoretical model for what happens with curriculum resources when they are used by the teacher. They defined the end product as a "document," which is not a static object, but incorporates the resource, how it is used and how it is intended to be used (called "operational invariants," p. 209). They described the change from resource to document as a process called documentational genesis. However, they cautioned that "a documentational genesis must not be considered as a transformation with a set of resources as input, and a document as output. It is an ongoing process" (Gueudet & Trouche, 2009, p. 206).

This way of considering teachers' use of curriculum resources does not ignore the teacher, but focuses more on the activity of teaching and the use of curriculum materials as the tool which mediates teaching. This theoretical approach considers both the curriculum materials *and* their purpose and it sees this purpose to be dynamic and changing. This approach allows for the study of mathematics instructors interaction with resources beyond commercial curriculum materials.

The studies reviewed in this section fall into broad categories that consider how teachers/instructors use curriculum materials, what the intent of the curriculum materials is, and how teachers/instructors interact with resources. Most of the studies reviewed in this section focus either broadly on teachers' use of resources or narrowly on one resource such as a new textbook, and they all have the underlying assumption that the resources used can affect teaching. What is missing is research that focuses on how instructors use a variety of resources within one mathematical topic.

Technology as Resource

In this section I examine literature on technology in the undergraduate mathematics classroom, with an emphasis on the technology used in the calculus classroom. A common technology used that is particular to mathematics is graphing technology. This technology has advanced from the handheld graphing calculators available 30 years ago to sophisticated software the can model 3D equations and manipulate algebraic symbols.

During the calculus reform movement in the 1990s, graphing technologies were widely assumed to help students with the conceptual understanding of mathematics (Dunham & Dick, 1994). The premise was that if students were not bogged down in calculations, they could spend more time understanding the concepts and applications of mathematics. There was some backlash to this view, with some educators and researchers expressing concern that graphing technologies were inhibiting procedural (computational) ability in mathematics (Doerr & Zanger, 2000). Both claims—for an increase in conceptual understanding of mathematics and a decrease in procedural ability were widely researched. A number of studies concluded that students did not show a decrease in procedural ability when graphing technologies were used and that the influence of graphing technologies on students' conceptual understanding of mathematics might be positive (e.g. Palmiteer, 1991; O'Callaghan, 1998; Artigue, 2002). In part because of mixed reviews about whether or not graphing technologies can help students' conceptual understanding of mathematics, more research was done. Much of this research has focused on *how* teachers can integrate graphing technologies into their lessons in order to improve conceptual knowledge (e.g. Conners & Snook, 2001; Meagher, 2010).

Forster (2004) examined how graphing calculators were used in a high school calculus class. Foster chose a particular class to study where students had traditionally high scores on the Western Australian University entrance exams. She wanted to understand how calculuators were used in that class, and looked for efficient use of calculators in 21 lessons. For this paper, efficiency was defined as quick and easy calculation. She found that the teacher led class discussion about problems that can arise with calculators and often provoked discussion of technology by asking for methods to solve. There was some discussion about when the calculators may fail, and how students still needed to make judgments even with a calculator (in case a possible answer doesn't fit the problem). This article showcased a way to use graphing technology in a way that does not detract from traditional mathematics. However, this study focused on a high school class, and interpreted part of the success of that class as teacher-led discussions about the technology. In post-secondary mathematics courses, students often have less face-time with the instructor, meaning that time for meaningful discussions about technology may be limited.

Buteau, Marshall, Jarvis, & Lavicza, (2010) examined 204 published papers that explicitly discuss computer algebra systems at the tertiary level. Although the sources for this literature review were limited, this paper provided an excellent overview of the issues around using computer algebra systems in post-secondary mathematics. Because 90% of the papers reviewed were practitioner based (and not educational research based), the issues and themes listed in this review are primarily from the instructors' perspective. Calculus teachers have been encouraged to incorporate graphing technologies into their classrooms for years, yet we have very little information about how (or if) these technologies are being incorporated by post-secondary calculus instructors. This study gives insight into potential reasons that instructors may have for implementing or not implementing graphing technologies.

This review found that the top three stated instructional purposes of CAS-based technology in the articles reviewed were (1) Experimentation and Exploration, (2) Visualization, and (3) Real and Complex Problems. The top three challenges for CAS were (1) Assessment, (2) Syntax, and (3) Unexpected Behavior of CAS. These purposes and challenges are answered, in part, by the following studies.

Conners & Snook (2001) describe an experimental study that took place at the United States Military Academy at West Point. Notably, this is the only study that clearly distinguishes between a graphing calculator that will handle algebraic manipulation (TI-89) and calculators that will not. One hundred students were randomly assigned to use a TI-89 calculator during the last three (of four) mathematics classes. Another 100 students were randomly chosen as the control group, and had calculators without a qwerty keyboard. The research question examined was whether a calculator with algebraic manipulation capabilities would make a difference in student learning. For all students in all classes, technology had been a part of the curriculum for over a decade; students regularly used MathCAD outside of class. The authors demonstrate that the experimental and control group are both similar to each other and to the whole population of students by comparing first semester grades as well as SAT and ACT scores. All students took a common final exam, and the answers from the experimental and control group were evaluated against each other and previous exams. Conners and Snook found statistically significant improvement for the experimental group on 8 out of 13 questions. The most significant improvement was on items classified as application problems.

Lavicza (2010) conducted a mixed method study that began with a qualitative study of 22 mathematicians in Hungary, the United Kingdom, and the United States in order to examine the use of graphing technologies. A survey was developed and sent to 4,500 mathematicians in participating countries, and 1,103 responded, an unexpectedly high response rate. Lavicza found that over half the mathematicians who responded used CAS in teaching, and over two thirds used CAS in their own research. During teaching, the CAS was primarily used to visualize conceptions during lecture. Another purpose given for CAS was to engage students in experimental activities and solving real-world problems, usually in computer lab settings.

Lavicza interprets the high level of response to the survey as an indication that mathematicians are interested in teaching. Tertiary math instructors often have a reputation not caring about teaching (or at least not being good at it). This interpretation of the response rate opens up a space for more collaboration between mathematicians and math educators. Lavicza also suggests that the fact that more instructors use CAS in their research than their teaching could mean that "mathematicians accept that CAS is part of the literacy, but at the same time they are reluctant to accept that CAS shapes mathematical knowledge" (p. 111).

Manouchehri (2004) observed the discourse in a math class for secondary teachers with 16 pre-service math teachers. Beginning the second week, the instructor introduced NuCalc, a computer algebra system which has no special syntax. The class discussions were analyzed before and after the introduction of NuCalc. Manouchehri found that discourse before introducing NuCalc was dominated by two students and the instructor. She then found that using

NuCalc was associated with an increase in student reflection and challenging of unsupported statements.

Based on her discourse analysis, there is a transfer of authority from the instructor to the software. Instead of looking to the instructor for answers and confirmation, the students turned to the software. In addition, the software became a tool for extending mathematical thinking and constructing more sophisticated mathematical explanations than were offered before NuCalc was used. The discourse analysis showed that after NuCalc was introduced, all students participated equally and the instructor spoke less. Because of this, Manouchehri suggests that the technology may be considered not just for presentation, but as a discourse participant in the analysis.

Using technology changes things. Instructors in Lavicza's study were reluctant to acknowledge that using technology may shape mathematical knowledge. The students in Manouchehri's study used technology to augment their knowledge. Perhaps technology changes the discourse as well as shapes knowledge.

Berger (2010) examined the use of a CAS with a particular mathematical task. The author gave 203 pairs of South African students four related tasks to solve using Mathematica. She examined the fourth task, which was to determine the interval needed on a second order polynomial in order for the accuracy of the Maclaurin polynomial to be within 0.1 of the actual function.[5] Her research questions were,

> With regard to Task 4, to what extent and how are students in the Mathematics I Major class able to use CAS (together with pencil and paper, if required) as a tool with which to construct representations of pertinent mathematical objects, to experiment upon these representations and to observe and interpret the mathematical relationships or results required by the activities? In particular, what sort of a difference (if any) is there in the performance of pairs of CL students, pairs of NCL students

[5] The Maclaurin polynomial is a carefully designed polynomial that looks like the original function, in this case cos(x), near zero. The higher the degree, the more accurate the polynomial is to the original function. See http://mathworld.wolfram.com/TaylorSeries.html. This type of task may be done toward the end of calculus 1 or 2.

> and pairs consisting of one CL student and one NCL student. (p. 324) (CL = Computer Literate. NCL = Not Computer Literate).

Berger based her analysis of the mathematical activity on construction of a representation, transformation of that representation, and interpretation of the observed representations. Berger reports that in South Africa, a lack of computer literacy can be understood as a proxy for a disadvantaged educational background. She found that prior computer experience did not give an advantage to students during the construction phase, which "bodes well for equity concerns: a lack of prior exposure to technology did not diminish the ability of students to construct CAS-based signs in the given task" (p. 331). In other words, the syntax of the software did not cause extra problems for less advantaged students, which was a concern raised in the literature review by Buteau, et al. (2010).

These studies of computer algebra systems in calculus classrooms indicate that, similar to other curriculum resources, technologies can help facilitate conceptual understanding and can have unexpected side effects.

Human Resources

Understanding what is necessary for future teachers to know has been an active area of research in mathematics education (Ball, Thames, & Phelps, 2008; Hill, Rowan, & Ball, 2005; Shulman, 1986). However, two-year college mathematics instructors are not required to have any pedagogical training. There is limited evidence about how the background of mathematics instructors influences their teaching but there is evidence about the overall characteristics of two-year college mathematics faculty, and calculus instructors in particular. In this portion of the chapter I summarize the information available about two-year college mathematics faculty and review two studies that examine how the background and training of post-secondary faculty influenced their teaching.

Two-year college mathematics faculty have strong educational and mathematics background, with 95% of full-time faculty having a master's degree or higher, and 86% holding their highest degree in mathematics(73%) or mathematics education(13%). Full-time instructors teach an average of 18 contact hours per week, and 85% of calculus courses are taught by full-time instructors. Although the majority of calculus courses are taught by full-time instructors, part-time instructors teach 15% of them, and 36% of all two-year college math courses. For part-time faculty, 83% have a master's degree or higher, and 77% have their highest degree in mathematics(58%) or mathematics education(19%), and 64% of them teach 6 or more contact hours per week (Blair, Kirkman, & Maxwell, 2018). A recent national study of calculus instruction found that 80% of two-year college instructors indicated a high interest in teaching calculus, compared with only 39% of research university calculus instructors. Over 60% of two-year calculus instructors are male (Bressoud, 2012), yet women make up 55% of full-time math instructors at two-year colleges (Blair, Kirkman, & Maxwell, 2018).

Instructors at two-year colleges may not have formal pedagogical training, yet 82% of institutions require some kind of ongoing professional development or continuing education for full-time faculty. Most of this requirement (94%) is met by activities either provided by the institution or other professional organizations (Blair, Kirkman, & Maxwell, 2018). However, faculty development may not be effective in changing instruction and often does not have long-term effects (Murray, 2002). One interesting study by Hébert (2001) examined the learning outcomes of 1,833 Florida college students who had been dual-enrolled in a community college mathematics course during high school. She examined whether their grade in their first college math course was different based on the status of their teacher in high school. Of the students examined, 920 of them had been taught by a high school mathematics course and 913 of them had been taught by a community college instructor. She found that students who had been taught by high school teachers had better grades in college than those who had been taught by college faculty:

> Students in Group A (dual enrollment taught by high school teachers) earned significantly better grades in subsequent coursework in mathematics after high school graduation than students in Group B [dual enrollment taught by community college faculty] and received more high grades (A's and B's) in the subsequent coursework than expected. The trend held true regardless of the university attended or the gender or ethnicity of the students (p. 33).

One way that this becomes even more significant is because these high school teachers must qualify to teach at community colleges in order to instruct students who are dual-enrolled. However, the reverse is not true. Community college instructors need not be qualified to teach at the high school level. Hébert states:

> High school teachers who teach dual enrollment classes may have an educational advantage over college faculty. While college faculty are considered experts in their field, possessing a minimum of a master's degree in the discipline, often high school teachers have an additional credential. Most high school teachers, in addition to the master's degree in the discipline, have a degree in education. Unlike many college faculty, most high school teachers have a background in such things as learning styles, teaching techniques, developmental stages, and assessment and evaluation (p. 34).

This study suggests that pedagogical training may help prepare students, and that instructors may rely on their educational background when teaching. Yet two-year college instructors are not required to have teaching certifications.

A study by Oleson and Hora (2013) interviewed 53science, technology, engineering, and mathematics (STEM) faculty at three research institutions in order to understand how their background affected their teaching. After examining interview data, they found that the influences on how faculty taught was more complex than simply an "apprenticeship of observation" (Lortie, 1975). In particular, "faculty reported four distinct types of influences:

experiences as a student, as a teacher, as a researcher, and from their personal lives" (p. 30-31). Experiences as a student influenced faculty teaching both in terms of how they were taught and how they learned. Experiences as a teacher influenced instruction in terms of learning what worked or did not work in the classroom, both for students and for the instructors themselves. However, Oleson and Hora point out:

> the data do not uniformly reveal a willingness of faculty to continuously learn and revise their teaching behaviors based upon evidence of ineffectiveness. For some, years of experience in the classroom has resulted in a recipe for instruction that is satisfactory and does not require any adjustment (p. 42).

Some faculty may use their prior teaching experiences only up to a certain point. Experiences in as a researcher and in their personal lives also influenced instructors teaching and decision making. Instructors rely on multiple resources to inform their teaching, including their own background.

Social-cultural resources

Social-cultural resources that instructors draw on include two main areas, language and time. The language resource includes the language of mathematics as well as communication in the classroom. Time includes how often an instructor is in the classroom as well as the duration of time in front of students. The language of mathematics and everyday language often overlap. For example, the word "integral" has a specific meaning in a calculus class, but it can also mean essential or necessary. The word "leg" can refer to one-third of a triangle, or it can refer to a body part. The word "log" can be short for logarithm, or part of a tree that has been cut off, or refer to an official record of events. Multiple meanings of vocabulary words are present throughout mathematics, and vocabulary difficulties are documented to cause confusion for students, regardless of their native language (Barwell, 2005; Schleppegrell, 2007). The standard recommendation for teachers is to be precise in their language when teaching mathematics (Leung, 2005). Within a mathematics classroom, students are expected to learn and use mathematical language, and instructors are expected to help them learn the language. Research on the language of mathematics includes more than vocabulary. Symbolism in mathematics is another form of language that must be understood and interpreted. For example, $(-a)^2$ is not the same as $-a^2$, but they may both be described as "the opposite of a squared." The complexity of language in the mathematical classroom and how instructors should teach mathematical language has long been a study of research (Pimm, 1989; Simpson & Cole, 2015). Some of the recommendations for helping students learn mathematical language include having students communicate mathematically (NCTM, 2000; Leung, 2005), and studies have examined peer-instruction in calculus classrooms as well as i-clickers (Bode et al, 2009; Miller, Vega, & Terrell, 2006).

Communicating mathematically is a form of student engagement that instructors may want to see. However, learning mathematical communication takes time. In Michigan, two-year college calculus courses are either four or five credit hours, are taught between two and five days per week, and range from 14 to 16 weeks per semester. The means a total of 56 to 80 hours per week in the classroom. Because many calculus instructors express concern about having enough time to cover all the topics needed, Johnson, Ellis, and Rasmussen (2014) investigated the relationship between coverage expectations, coverage concerns, and instructional practices in calculus classrooms. They considered 47 instructors at five selected institutions (see Bressoud, Carlson, Mesa, & Rasmussen, 2013) where calculus term lengths ranged from 7 to 15 weeks, with similar content coverage requirements. The pacing of the institutions was also examined, with instructors in the 7 week course expected to cover 4 sections per class, and instructors in the

15 week course expected to cover less than 2 sections per class. They found no statistical correlation between concerns about coverage and intended pacing, suggesting that the amount of material that instructors were expected to cover and the amount of time they had to cover that material did not affect how concerned instructors were about covering all the material. They found small correlations between instructional practices and the level of concern that instructors felt about pacing. For example, instructors who were more concerned about coverage were more likely to lecture, but they were also likely to continue student centered practices such as group-work and having students explain their thinking. They explain:

> Even when instructors include more lectures, they did not do so by eliminating other instructional practices. Instead, these findings suggest that instructors at selected institutions pair student-centered practices with lecture when they are pressured for time. This is an important distinction to make because, when paired with other activities that engage students, lecture can be a highly productive instructional practice (p. 500).

They also examined similar data from non-selected institutions, and found that pressure to cover the material did not change instructional practices. They conclude that feeling variable levels of pressure to cover material may not explain differences in instructional practices, and that other factors (such as class size) should be considered when examining reasons for instructional practices.

There is ongoing debate in secondary mathematics programs about block scheduling vs. traditional scheduling and the effect of scheduling on students and teachers (Zapeda & Mayers, 2006; Zelkowski, 2010). However, little has been examined in terms of scheduling in post-secondary institutions. A study by Diette and Raghav (2016) compared the GPA of students in classes who met more frequently for shorter periods with students in classes who met less frequently for longer periods. They found no difference in learning outcomes for classes that met twice per week for longer periods of time compared with classes that met more frequently for shorter periods of time.

Theoretical Stance

The theoretical framework for examining resources in study is grounded in the literature and my own experiences while teaching. Similar to Rabardel (2003), Gueudet et al. (2014), Brown (2009), and Remillard (2004), I believe that there is a relationship between instructors and their resources that cannot be defined by looking just at instructors or just at their resources. I see the instructor as a co-constructor of the curriculum, with both the instructors and the resources having impact in the classroom. The critical elements of this framework include three main ideas: first, that one instructor may use the same resource in multiple ways (e.g., Brown, 2009); second, that multiple instructors may teach the same topic with the same resources in multiple ways; and third, that a resource and how it is used should be understood together (Gueudet et al, 2014). I am primarily interested in the instructors' interaction with the content of calculus and resources and how those resources impact instruction, in the environment when they are teaching and planning to teach, as well as their justifications for resource use.

In light of this literature review, I propose the following questions:

1) What resources do two-year college calculus instructors use to assist in their planning and teaching lessons in FTC?
2) How do two-year college calculus instructors use their resources to assist in their planning and teaching lessons in FTC?
3) Why do two-year college calculus instructors use their resources in the ways that they do when planning and teaching lessons in the FTC?

Chapter 3: Methods

In this chapter I detail the methods used for answering each research question, in five sections. The first section is a brief explanation of the context used for this dissertation and explains the overall data collection methods. The second section details the quantitative data collection and analysis done, including a rationale for each section of the survey. Likewise, the third section details the qualitative data collection and analysis done on that data. The fourth section describes the analysis done on both types of data and summarizes the methods used in this dissertation. The final section details my subjectivity as a researcher and the limitations of these methods.

I used a two-phase mixed methods explanatory design for the study. In this type of study, the researcher collects quantitative data first and qualitative data second, “to help explain or elaborate on the quantitative results.” The quantitative data provided a general picture of the research problem, and the qualitative data were used “to refine, extend or explain the general picture” (Creswell, 2012, p. 542). I chose this design because mixed methods explanatory designs can provide a fuller answer to a research question; each type of data has strengths that build on the other. For example, the quantitative section provided some insight into what resources the instructors used while the qualitative section asked instructors how and why they used those resources. The mixed methods design allowed me to provide a much richer picture of how instructors describe their teaching of the FTC.

Because I expected instructors to rely fairly heavily on their textbooks treatment of the FTC (they didn’t), I examined a variety of calculus textbooks in order to prepare for data collection. I made copies of the Fundamental Theorem of Calculus section from eight different textbooks. These textbooks were among a list of textbooks indicated by the national calculus study to be used by college instructors. I considered how these textbooks treated the FTC, if there was a proof for the evaluation portion of the FTC, and how it was proved (see Table 3).

Table 3: Calculus Textbooks and their treatment of the FTC

Textbook Author (edition)	Section for FTC	Evaluation Portion	Proof Type
Spivak (3rd)	Chapter 14	FTC II	Uses 1st FTC
Edwards/Penny (3rd)	Section 5.5	FTC II	Uses 1st FTC
Adams (5th)	Section 5.5	FTC II	Uses 1st FTC
Thomas (10th)	Section 4.5	FTC II	Uses 1st FTC
Stewart (5th)	Section 5.3	FTC II	Uses 1st FTC
Ostebee-Zorn (2nd)	Section 5.3	FTC II	Uses 1st FTC
Hughes-Hallet (4th)	Sections 5.3 and 6.4	FTC I	Proof in supplement uses MVT
Larson/Edwards (10th)	Section 4.4	FTC I	Uses MVT
Briggs/Cochran/Gillett	Section 5.3	FTC II	No proof, but discussion is close to MVT proof

As discussed in the literature review, the FTC is often presented as two theorems. One of the theorems involves the relationship between integration and differentiation, and the other theorem is used to evaluate a definite integral (often called the "evaluation portion" of the FTC). I examined the textbooks to see in which order they presented the two theorems (if they were separated). From this examination, I noted that two common textbooks had very different ways of proving the evaluation portion of the FTC (see Appendix B). The proof from the Larson textbook partitioned an interval and invoked the mean value theorem (MVT). The proof from the Stewart textbook used what they called the first fundamental theorem of calculus (the idea that the derivative of an integral is the integrand) without referring to partitions or the MVT. Once I understood similarities and differences between textbooks treatment of the FTC, I began the process of data collection, which proceeded in three phases.

In the first phase, I administered a survey to all 136 Calculus I instructors in Michigan to obtain the trends in these instructors' use of curriculum materials (See Appendix A). In the second phase, I used the survey responses to select 14 instructors for an interview study in order to better explain the trends identified from the survey. In the third phase, I selected two participants from the interview study, and observed those two instructors while they taught the Fundamental Theorem of Calculus. In the next sections I describe each phase in terms of the instrument, the participants, and the analysis done.

Phase 1: Quantitative Phase

The first phase of the data collection involved a 28-question online survey of two-year college calculus instructors. This survey was used to describe trends in the population and was used to answer the research question:

> What resources do two-year college calculus instructors use to assist in their planning and teaching lessons in FTC?

The Survey Instrument.

The survey (see Appendix A) was used to identify the most common textbooks and other curriculum resources used by community college calculus faculty. Findings from a previous study (Leckrone, 2014) suggest that community college calculus instructors use their textbook more than they recognize. Instructors in that study used their textbooks as a resource and reference for themselves and their students. They deferred to the textbook for things like notation and homework problems. One instructor, who stated that he did not use the textbook, was able to refer students to various sections of the textbook for help. Given this information about textbook use, presenting specific questions about textbook use, as identified in this earlier study, helped instructors identify the various ways in which textbooks can be used. To complement the information about textbooks, the survey asked about technology use and other resources in general and when teaching the FTC. The survey was used to identify which curriculum resources were being used by instructor in various ways. This information adds to a growing body of research on undergraduate mathematics instructors' use of curriculum resources and contributes to information about how those resources are used when teaching the FTC.

The survey (see Appendix A) began by asking instructors to confirm that they were currently teaching Calculus I or had taught Calculus I in the previous semester. If the answer to both of those questions was no, the survey sent the participant forward to the end. The body of the survey had 26 questions and covered three areas: course information, FTC, and demographic information. The six demographic questions in the last section of the survey matched the Conference Board of Mathematical Sciences (CBMS) reports of this population (Blair, Kirkman, & Maxwell, 2013). These questions were used to determine the degree to which the sample of respondents is representative of the US community college population in terms of gender, educational background, and experience. I describe the questions in each section of the survey next.

Course information section. The 10 questions in this section (questions 3-12) are a combination of semi-structured and open response questions that ask instructors about the materials they use in the classroom. Question 3 asks the participant how the textbook the instructor uses was chosen. This question was used to compare responses to later questions about how often they use their textbook and how much they like the way the textbook handles treatment of the FTC. Questions 4 and 5 asked the instructor to choose which textbook they use from a list taken from the Characteristics of Successful Programs in College Calculus (CSPCC) Instructor Start of Term Survey, and how many semesters they have taught with that book. These initial questions served to orient the participant to the purpose of the survey.

The next four questions (questions 6-9) asked participants to rate how they used the textbook on a scale from 1 (never) to 7 (always), in terms of assigning homework problems, following the symbols and formulas, using examples from the textbook during class, and if the participant is comfortable changing the order of topics in the textbook. The answers to these questions are on a 1-7 Likert scale, from Never (1) to Always (7), with a N/A option if the question does not apply. Structuring the answers in this way allowed for the variables in the analysis phase to be considered as continuous for statistical purposes (Groves, et. al, 2011). Question 9 asked participants if they used examples from the textbook during class time, and was intended to compare with existing research (Mesa & Griffiths, 2012; Leckrone, 2014).

Question 10 was an open-ended question that asked about other resources (such as graphing calculators, other books, websites, etc.) that participants used when planning to teach first semester calculus. Questions 11 and 12 in this section asked about students, and what materials students were required to purchase for a first semester calculus class.

Fundamental theorem of calculus. This section consisted of eleven questions that probed the participant’s perception of the treatment of the FTC in their textbook, the technologies used when teaching the topic, which of two possible proofs of the FTC they preferred and why, and their opinion of the textbook and technologies used during teaching (See Appendix B).

The first five questions in this section (Q13-17) asked participants how often they use their textbook and about their perceptions of their textbooks treatment of the FTC. Similar to the questions about their use of textbook in the course materials question, the four questions about how they perceived their textbooks treatment of the FTC asked participants to answer on a 1-7 Likert scale, from Positive (1) to Negative (7), in order to facilitate analysis of the survey later

(Groves, et. al, 2011). Following the textbook questions, participants were asked about technology, including what technology they permit and require students to use, and whether or not they feel that the FTC is easier for students to understand without added technologies. Because the idea of technology can range from mechanical pencils to computers and robots, instructors were given a list to choose from, including graphing calculators that do (or do not) perform symbolic algebra, a computer algebra system such as Maple, Mathematica, MATLAB, and "other." If an instructor chose "other", they were asked to describe the technology.

Following questions on technology, instructors were shown two possible proofs of one portion of the FTC, each of which was adapted from common textbook proofs, but were very different from each other (see Appendix B). The first proof assumed that the evaluation portion of the FTC was the first fundamental theorem and used the Mean Value Theorem and Riemann sums to prove it. The second proof assumed that the evaluation portion of the FTC was the second fundamental theorem and used the first fundamental theorem to prove it. Question 22 asked instructors which proof they preferred and question 23 asked them why they preferred that proof.

The last question in this section (Q24) asked participants to explain anything else that they felt was relevant to teaching the FTC. Less than half of the instructors responded to this question, and although this question added to the picture of how instructors think of the FTC, it was not compared with responses to other questions.

Demographic information. This section consisted of eight questions about instructors' position, demographics, and professional background (see Figure 4). Research has shown that demographic information is more effectively gathered at the end of a survey than at the beginning, which may allow for a greater response rate (Groves, *et al.,* 2011).

The first five questions of this section (Q25-29) were compared with demographic information reported by Blair, Kirkman, and Maxwell (2013) about two-year college instructors in the United States. Question 30 asked for a self-identification of expertise. This question allowed for comparison with questions asking about the number of sections of calculus taught using a particular textbook and the overall number of years of calculus taught.

Participants. In order to find participants for the survey, I compiled a list of all two-year college calculus instructors by going through each website for all 27 Michigan two-year colleges and looking at their course schedules for Fall, 2015 and Winter, 2016. For each named calculus instructor, I looked on the website for their e-mail. One community college listed five calculus courses as being taught by "staff," and I was unable to find out who taught those courses. If an e-mail for a specific calculus instructor could not be found on the college website, I called the college to request it. In all cases, the e-mail was provided to me.

Prior to administering the survey, I piloted the questions with five people, two former calculus tutors, one former calculus instructor, and two other math instructors in order to determine potential misunderstanding of questions and to estimate the completion time. The survey took between 10 and 30 minutes to complete; I refined language as well as the look and feel of the survey before administering it to participants.

I choose to limit the sample to Michigan instructors for two reasons, although this survey could be easily expanded to other instructors in other states. First, community colleges in Michigan offer a diverse set of potential respondents. Unlike some other states (such as Ohio and California), community colleges in Michigan are decentralized, with informal communication between the colleges. Innovative uses of resources are neither dictated nor discouraged by a central authority. The decentralization of community colleges means a potentially wider variety of resource use than might be found in a centralized system.

Second, I expected that community college instructors in Michigan would be more likely to respond to a research request coming from a “local” university than a general survey request. A major problem for survey research is response rate. According to Shih and Fan (2009), surveys embedded in e-mail may have a response rate of from 10-50%, with an average of 30%. Other studies suggest that web-based surveys (such as the one given here) may have even lower response rates than surveys embedded in e-mail (Cook, Heath, & R. Thompson, 2000; Manzo & Burke, 2012). Of the 136 instructors who were e-mailed the link to the survey, 86 (63%) responded, and of those 86 responses, 50 (37% of the total) were useable.

In order to secure the best participation rate possible, I e-mailed instructors an invitation to complete the survey, with a two-week deadline, and a reminder near the end of the two weeks. At that point, there were some instructors who had started, but not completed the survey. I contacted each those instructors with a direct link to their survey and a request to finish.

After collecting instructor responses, an error in my IRB approval jeopardized a portion of data collection. After rectifying the error and meeting with the IRB board, it was determined that I would be able to use responses collected from instructors if they were able to provide me with an e-mail confirmation that they agreed that I use their data. I contacted every instructor again, and all but 5 gave me permission to use their data. This left me with approximately[6] 50 responses to analyze. Responses came from all around the state, as shown in Figure 1. Out of 27 two-year colleges contacted, instructors from 22 colleges responded.

[6] The number is approximate because not every instructor answered every question. For example, while most questions had 52 responses and some had 47.

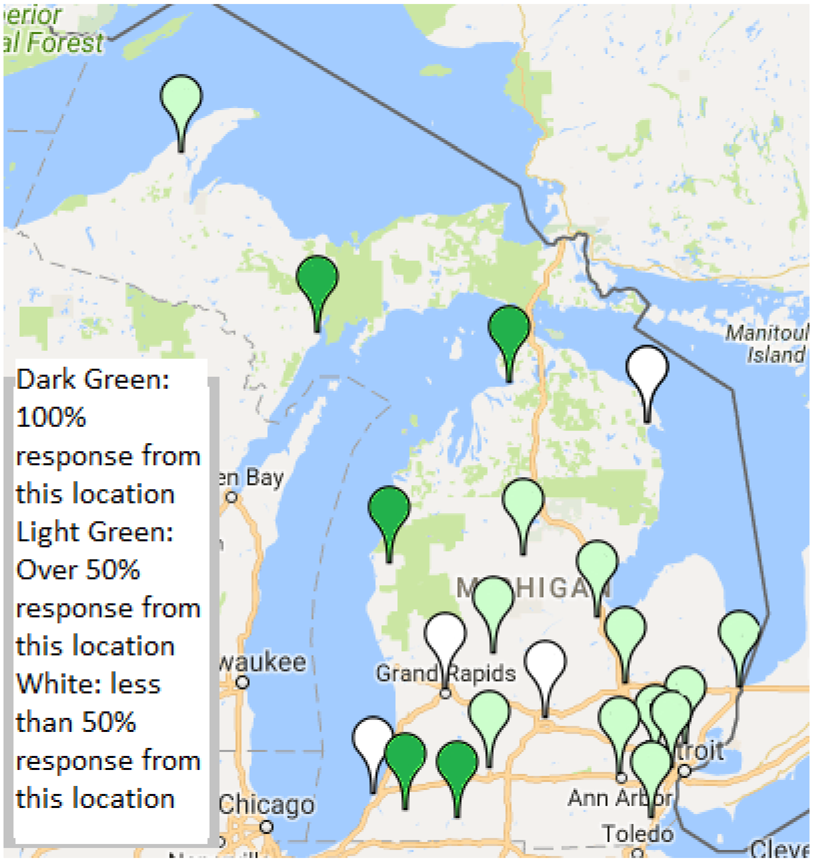

Figure 1: Geographic distribution and proportions of survey responses. Developed at zeemaps.com and adapted.

As shown in Figure 1, participants in the survey came from all over the state. Five two-year colleges had a 100% response rate, and 12 colleges had a response rate of 50% or better. An additional four colleges had some instructors respond, and only five colleges had no instructors respond.

Analysis.

Before I could analyze responses, I needed to clean the data and validate the answers in the survey. This was done by removing all responses with no data in questions 3-25, because these participants indicated that they did not teach calculus I during the 2015-2016 school year. I also hid all identifying information and assigned each participant a number rather than a name.

Some of the survey responses were edited (Groves, *et al.*, 2009) as follows: Four records had a text answer to Question 30 ("Including this year, approximately how many semesters have you taught college calculus?"), so I changed that to a number (i.e., "3 semesters" was changed to "3"). For the same question, one instructor said "400", which I changed to 40, because 400 semesters implied more years than the average life-span. After editing the data for this question, there were 12 records with no answer to that question. However, those 12 records had a response to Question 5 ("Including this semester, approximately how many semesters of first semester calculus have you taught using that book?"). I input that number into question 30 as a proxy for how many semesters that a instructor had taught calculus.

Once the data was cleaned and edited, I considered the short answers. I used open coding for responses to the short answer questions 23 and 24. For each question, I printed the answers and sorted them into piles with similar answers. Responses to Q23 ("Why do you prefer that proof?" were put into five categories: math, students, both math and students, self and other (see **Error! Reference source not found.**). Responses coded as math generally included some mathematical reason why the proof was preferred. Responses coded as student focused more student understanding as a reason for preferring one proof over the other. Responses coded as both math and students included both of these types of reasons. Responses coded as "self" did not include specific references to math or students but implied that it was a personal preference. There were five responses that were coded as "other" that did not fit into any of the previous categories. These categories were used to compare with other responses in the survey to look for patterns.

Table 4: Categorization of participant responses to their FTC proof preference

Category	Examples
Math	1) MVT is an important part of Calculus and is used elsewhere 2) I appreciate that this gives us a chance to touch on families of functions as

Category	Examples
	antiderivatives (listed as [6] in the proof). Also, it is common in proofs later in the undergraduate degree to begin by defining a function, and I mention this while proving the statement.
Students	1) Students seem to understand the connection between the antiderivative and the definite integral when I draw graphs of simple functions and shade and compute areas between the graphs of simple functions and the x-axis than when I attempt to formally use the Mean Value Theorem method 2) I like that it builds upon the idea of sums of areas which students are familiar with from the introduction to integration.
Math and Students	1) It (1) reviews the mean value theorem from earlier in the course, (2) it shows the power of adding zero in a clever way, (3) it leads directly to the common method of evaluating definite integrals that the students will use far more than derivatives of functions defined by integrals 2) I think it is more clear as to the connection between derivative and integrals. It also more clearly shows to students how we evaluate definite integrals.
Self	1) I learned it that way (50 years ago) 2) elegance, ease of understanding
Other	1) We do not teach a proof oriented course, so I really would answer "Not applicable." 2) I don't have a preference either way. The majority of my students do not want to see a proof. They just want to know the theorem, know what it means "in layman's terms", and know how to use it. When I try to show them proofs in class, they just get more confused than they already were.

Responses to Question 24 were also put into five categories (see **Error! Reference source not found.**).

Table 5: Categorization of participant responses about other FTC importance

Category	Examples
Importance	1) I still remember my calculus teacher … immediately after proving the FTC, asking "Did we just waste some time? Or did this change the world?" Then he declared "It changed the world!" 2) History.
Inverse Operations	1) I focus on the relationship between the derivatives and anti-derivatives for definite integrals since they are "inverse" of eachother. 2) Emphasizing the inverse relationship between integrals and derivatives.
Other Math	1) The connection between the riemann sum of f and what it represents in terms of F. 2) An intuitive understanding that we treat f(x) as a 'derivative' function and that the sum of many little changes in f(x) equals a big change in F(x).
Students	1) The more that you can appeal to student knowledge of common geometric area formulas and other usual student knowledge, the better.

	2) technology helps with understanding
Other	1) Think before you do anything, like integrate and differentiate. 2) I prefer the titles fundamental theorem of calculus and integral evaluation theorem.

Responses coded as importance included some information about the FTC being important in and of itself, without reference to the mathematics involved. Reponses coded as inverse operations mentioned the relationship between integration and differentiation as opposites or inverses. Responses coded as “other math” indicated some portion of mathematics other than the idea of inverse operations. Responses coded as students indicated something about student learning. There were three responses coded as “other” that did not fit any of these categories. These categories of responses were compared with interview findings about the importance of the FTC.

Trustworthiness. Certain survey questions were designed to enhance survey reliability and trustworthiness. In particular, Questions 14-17 were expected to have a high internal consistency, and they did. A Cronbach’s alpha of .89 indicated that these four questions were answered similarly, although not completely in the same way. In order to reduce response error, some questions were worded positively (Q6-9). Question 21 asked about teaching the FTC *without* (considered negatively worded)) added technologies. This question was designed so its answers could be compared to those of questions 18-20, which asked about what technologies were used (positively worded). This type of design allows the researcher to compare answers and validate responses. If a participant answered that they did not use technology when teaching the FTC, but that they strongly disagreed that explaining the FTC was easier without added technologies, it could be an indication of a potential problem. Finally, based on the pilot, the survey was expected to take at least 10 minutes, and the time instructors spent on the survey was recorded in order to examine how long it took instructors to respond.

Phase 2: Interview Study

I conducted 14 semi-structured interviews with instructors using a pre-written protocol. I designed the protocol to enrich the survey information by examining how and why instructors use resources when teaching and planning the FTC. In this section, I explain the design of the interview protocol, how participants were chosen for interviews, and how the analysis of the interviews was done.

Interview Protocol. I began the interview process with questions about the instructor's educational background and teaching experience. These questions were designed to establish rapport and give instructors a chance to elaborate on similar questions that were asked in the survey. After gathering background information, the interview protocol consisted of four sections: general teaching and planning of calculus I, teaching FTC, discussing a proof of the FTC, and hypothetical questions (see Appendix C for full list of interview questions). I will briefly outline each interview section and then describe the analysis done.

General Teaching and Planning. This section included three questions with several potential sub-questions. Before beginning this section of the interview, I read the paragraph at the front of the interview protocol to instructors in order to define teaching and planning. Questions 1-3 were designed to obtain information from instructors about how they plan for this course in general, what a typical class looked like, and what resources they used. There questions were asked in order to get a feel for how teaching the FTC was similar or different to what an instructor described as typical.

General FTC. This section consisted of five questions that probed instructors' understanding about the importance of the FTC and their resources when planning and teaching the FTC.

Question 4 asked instructors how important the FTC is to calculus, and question 7 asked them how they introduce the FTC to students.

Teaching FTC. This section consisted of four questions designed obtain information about what instructors perceived as the importance of the FTC in terms of student understanding. As described in the literature review, the FTC requires understanding important mathematical ideas, and instructors may privilege one idea over another. Questions 9-12 were designed to give instructors the opportunity to share not only what they felt was important about the FTC, but what they felt students should take away from learning about the FTC.

Also in this section, I gave instructors the two potential proofs of the evaluation portion of the FTC that they had seen on the survey, and asked them to select the one that they preferred. I asked them why they preferred that proof. This information was triangulated with their responses in the survey. I then asked instructors to explain one of the proofs to me. The explanation of the proof as well as questions 4, 7, and 11 were designed to give instructors the opportunity to display covariational reasoning (Carlson, Persson, & Smith 2003).

Hypothetical questions. This section consisted of two questions about hypothetical situations. Question 14 asked instructors what resource, real or imagined, they would like to have when teaching FTC. This question was designed to obtain participants' thinking about possible resources that they may not have mentioned yet. Question 15 asked instructors to give advice to a hypothetical new instructor (me) at their school. By asking instructors to respond to a hypothetical instructor, I am encouraging them to "teach" me how they teach. Responses to this question offered an opportunity for instructors to confirm the resources that they said they used as well as any they may have neglected to mention during the course of the interview.

Participants. Out of 50 survey responses, 24 instructors supplied their e-mail on the survey and indicated they were open to an interview. From these 24 possibilities, I chose 14 instructors from around the state to interview. One of the selection criterion was experience with textbook used. This criterion was chosen because there is literature that notes a difference in perceived reliance on the textbook from the first to the second and third year of teaching (Behm & Lloyd, 2009; Drake & Sherin, 2009; Silver, Ghousseini, Charalambous, & Mills, 2009). These studies suggest that as instructors become more familiar with curriculum materials, their attitudes towards them and use of them changes. I chose instructors to interview (see **Error! Reference source not found.**) in order to maximize variation in terms of resource use (for example, one instructor did not use a textbook), semesters of experience, and educational background. I also selected instructors so that the sample had both full-time and part-time instructors and included instructors from around the state.

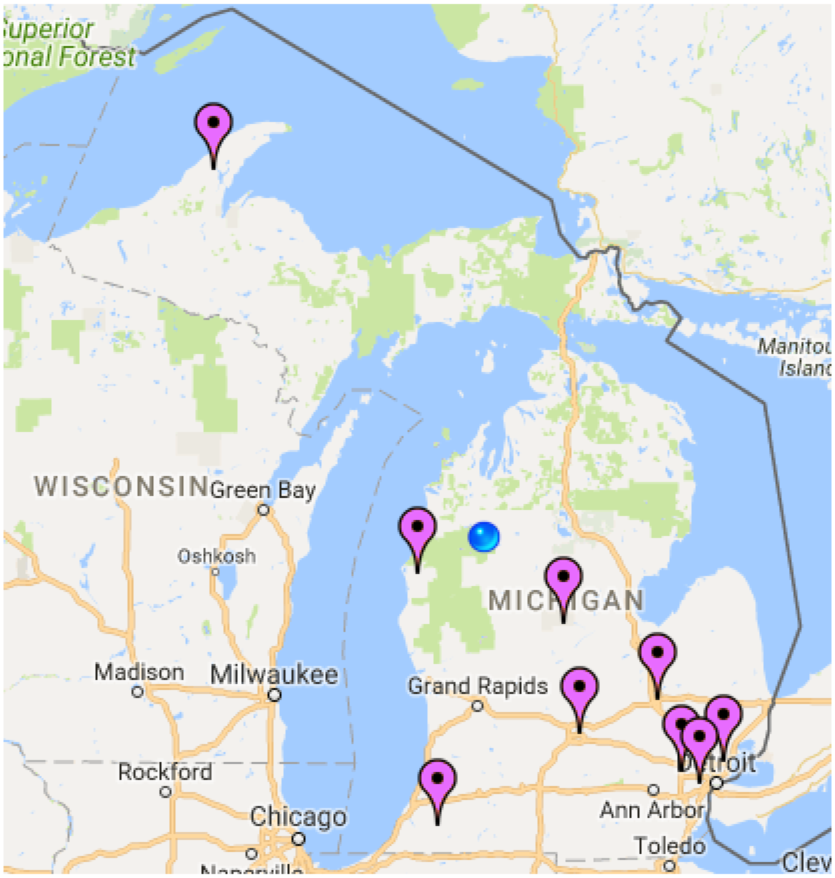

Figure 2: Map of interview locations. Developed at zeemaps.com and adapted.

All interviews were conducted in person; the instructors taught in locations across the state (see Figure 2).

Interviews were audio-recorded and the bulk of the interview was transcribed. After interviewees signed a consent form, the interviews began with questions about instructor education and background. These questions repeated and expanded on some survey information and were designed to set instructors at ease. Because it was not directly related to my research questions, I made notes and charts (see **Error! Reference source not found.**) rather than transcribe the information.

Table 6: Interviewees

Pseudonym	Educational Background	Semesters teaching	Full-time/ Part-time	Other

		Calculus I		
Frederick	BA in Math	8 semesters	Full-time	Teaches at a High School
George	Master's in Math	<1 semester	Part-time	
Harold	JD and Master's in Math	>12 semesters	Full-time	
Ian	Master's in Math	5 semesters	Full-time	
James	PhD in Math	65 semesters	Full-time	Dept Chair
Karl	Master's in Math	15 semesters	Full-time	Dept Chair
Leopold	Master's in Math and Stats	15-25 semesters	Part-time	Has FT non-teaching job
Marcus	Master's in Math	10 semesters	Full-time	Applied Calc
Nathan	Master's in Math	63 semesters	Full-time	Dept Chair
Oliver	Master's in Math	5 semesters	Part-time	Business Calc
Philip	Master's in Math	8 semesters	Full-time	
Richard	Master's in Math	11 semesters	Full-time	
Suppiluliumas	PhD in Math	20 semesters	Part-time	Retired from non-teaching job
Theresa	Master's in Engineering	50 semesters	Part-time	

Analysis. These four portions (not background) of the interview were transcribed and coded in Hyperresearch. I coded the transcripts in stages, using a combination of open coding and constant comparisons (Corbin & Strauss, 2008). First, I identified any mention of resources to discover just what these instructors were using. At the same time, when the resource was identified, an annotation was attached to each code with a note about how the resource was used. For example, a typical annotation is: "Resource: Textbook. In planning for a class, he reviews the textbook section and considers what problems to do." (annotation for Harold, 94-98). This level of coding allowed me to quickly access what resources instructors were using and how they were using it.

In the second phase of coding, I reviewed all the codes to determine why the instructor used the resources in the way that they do. In order to do this, I began by identifying areas of one instructor's transcript where he gave reasons for using a resource in a certain way. For each section of the interview text that I coded as a reason, I included a four part annotation with more detail, including the original resource (what) and how it was used, then a code for why, and a comment that explains why I chose that code. For example, Nathan explained that he sometimes took students to the computer lab when he didn't want to do a lot of calculation by hand.

> R: When do you spend a class in the computer lab?
> Nathan: Generally, when it's a—like numerical methods for integration. Where I want them to use the software to actually do the problem. Or I'm not gonna have them do a whole lot by hand.
> R: How do you decide which sections to do that?
> Nathan: Usually the ones that involve a lotta number-crunching that they're never gonna do by hand, **that I don't wanna do by hand**. (Nathan, 908-914)

My annotation for that section of text was:

> What: computer lab
> How: spends a class in the computer lab for a number crunching heavy section
> Why: self
> Comment: if the section involves a lot of number crunching that he doesn't want to do by hand, that would be a time he might take students to the lab

Initially, I based the "why" codes on Herbst and Chazan's (2011) practical rationality and obligations of teaching. This framework describes consists of "four professional obligations can organize the justifications (or refutations) that participants might give to actions that depart from a situational (or contractual) norm." (p. 450). However, I ran into two issues that led me to depart from this coding scheme. First, the *individual* obligation (attending to the needs of the individual student) and the *interpersonal* obligation (attending to the needs of the class and classroom as a whole) did not appear separately. This may be because the obligations framework based on "interpreting teacher (and students) actions *in the classroom*," (p. 428). I was analyzing interviews instead of observations, and I was focusing on resource use rather than general actions. Second, because I was talking to instructors rather than observing them in the classroom, many instructors explained reasons that had to do with their background, training, or personal preference. After my first attempt to classify the reasons instructors gave me for why they used resources in a particular way, I combined the classifications of individual and interpersonal obligations as “student,” and I considered explanations regarding background and personal preference as “self.” The final coding scheme is shown in **Error! Reference source not found.**.

Table 7: Coding categories for "why" from interview transcripts

The primary reason given for resource use was…	Example
Institution	[the graphing calculator] does the integration exactly. That would be perfect so they could actually check their answers and make sure everything is good. And then they can get the numerical value that you’re looking for…But our [department] says, “No, no, no.” (Marcus 584-587)
Math	“I like [graphing calculators] because it forces kids understand domain and range of functions” (Frederick, 715-716)
Self	“I always try to pick one new question [from the textbook] a semester to talk about in class so it’s not completely boring for me.” (Philip, 87-89)
Students	“I start [planning a lesson] from the textbook as my base because this is obviously what the students are going to be using and referencing. So if there is notation, I am using consistent notation with the text book because I don’t want to confuse the students on that note” (Ian, 39-41)

The code of “math” matches the idea of disciplinary obligation from Herbst and Chazan (2011) and “institution” matches the institutional obligation.

My third phase of coding the interview data was done to capture nuances of how the instructors referred to the FTC and teaching it for students (see **Error! Reference source not found.**). For example, some instructors mentioned FTC and finding the area under a curve. This was coded as “area.”

Table 8: Instructors' explanations of the FTC

Code	Meaning	Example
Area	Mentioning area under a curve when talking about teaching FTC.	I like area problems, 'cause I wanted to be able to draw the pictures quickly on the board without having, 'okay, so now let's graph this or that.' And the other thing is that for finding areas there are very few shapes we can find areas of without calculus. So I was sticking to simple stuff. (Karl, 513-515)
Accumulation	Mention of an accumulating function (that calculates area)	So I start by defining accumulation function. So we start by having *f* be a continuous function on an [interval] and we define *g* of *x* as the definite integral … from *a* up to *x*. So we start by defining this thing which we call an accumulation function. And we look at GeoGebra and you can do some visualizations where this thing's fixed and you can dynamically move this back and forth and [the accumulation function] calculates this area. (Ian, 412-416)
Anti-differentiation	Mention of an anti-derivative or using anti-differentiation when teaching FTC	Yeah. I start with the idea of an anti-derivative. They tend to pick that up, but they don't like it 'cause they know it's gonna be tough. 'Cause they're like, "Oh, that's gonna be hard." (Harold, 872-874)
Inverses	The FTC shows the inverse relationship of derivatives and integrals (or anti-derivatives)	[I tell students] remember at the very beginning where the book said there was two main problems in calculus, like the slope of the tangent line and finding the area under the curve. Well, those two things are inverses of one another and this is what this theorem will show (George, 798-801)
Other Context	Putting the FTC in context, talking about the history, etc.	I try to make it historically interesting. History of calculus sometimes. Not in a boring way, too much. Maybe ten minutes. I'll say, "Greeks knew about this stuff. They knew this rate of accumulating rectangles and things. They didn't quite put it together until much later."(Harold, 656-658)

Although the ideas of anti-differentiation and inverses are very similar, the way that instructors used these terms merit distinction. Anti-differentiation, when mentioned, was often more about process than concepts, and did not explicitly include the idea that the fundamental theorem of calculus linked derivatives and integrals. Inverse, on the other hand, was used when instructors specifically mentioned that processes of integration and differentiation undid each other.

After coding all the transcripts, I considered the primary resources that were mentioned in the survey and looked for patterns in the interview data from those resources. In particular, I examined reasons for the use of graphing calculators, textbooks, and intangible resources. I examined patterns of what, how, and why instructors said they used each resource.

Phase 3: Observations. Of those 14 instructors interviewed, I chose three to ask for observations based on availability, experience, and textbook use. I could not observe the first instructor (he was teaching Calculus I for the first time), because the IRB approval was not in place until after he taught the fundamental theorem of calculus. The second instructor (first observation) was the head of the department with over 15 semesters of experience teaching Calc I with a variety of textbooks. The third instructor (second observation) had experience teaching Calc I, but was teaching from a new (to him) textbook at the time of observation.

Neither of the two observations were audio or video recorded, so I took field notes, composed classroom maps, and collected any handouts given to students. I observed instructors on the days that they were teaching the fundamental theorem of calculus. Observation data were used to triangulate with interview and survey data. The resources observed during the observation were compared to the codes that captured *what* instructors said they used during their interviews. Their treatments of the FTC and its proof in the lesson were compared to their interviews.

Overall, the survey provided information about what resources instructors used when planning and teaching the FTC, and provided a hint of how they thought about the FTC. The interview data provided more detail about how and why those resources were used, and the observations were used to compare with survey and interview data.

Subjectivity

My background includes working in business for eight years, first as technical support and later as a software trainer and trouble-shooter. I have a secondary teaching certification in the State of Michigan(2004-2009) for math and German, with an elementary rider for German. I have three years of experience teaching high-school German, and 10 years of experience teaching two-year college mathematics. When I returned to graduate school, I worked simultaneously on my doctorate in mathematics education and master's in mathematics. I took calculus in high-school, and never in college. I view calculus as a subject that is difficult, but understandable. I have never taught first semester calculus. I view the understanding of mathematics as a combination of conceptual and procedural understanding. I believe that to have one without the other is incomplete. My experience with students in the two-year college setting has generally been positive. I view students in this setting as often having other commitments outside of school, and I do not expect college and homework to always be their first priority. I have been a participant in a research study, and enjoyed it. My colleagues at the two-year college where I taught have been congenial and helpful, in part because of our shared office location. All part-time faculty shared an office space and we often met each other and discussed our classes. As part-time faculty at only one location (many of my colleagues taught at multiple campuses), I felt that I had an advantage in the amount of time and energy that I could spend on my students. The primary disadvantage I experienced as a part-time instructor was compensation (pay) and a limited choice of what I could teach, as well as an uncertainty about scheduling. We did not get a teaching contract until we had been in the classroom for two weeks. My experiences as a part-time mathematics instructor, as a mathematics graduate student, a research participant, and as a two-year college instructor may color what I am seeing and not seeing in the data. I am comfortable with mathematics departments and with part-time faculty. I used my status as a former instructor to approach the interviewees as a colleague rather than a researcher.

Chapter 4: Findings

This dissertation examines what, how, and why two-year college calculus instructors in Michigan use the resources that they do when planning and teaching the fundamental theorem of calculus (FTC), as well as how these instructors describe the FTC. The most common resource used by instructors was the textbook, followed by graphing technologies, and intangible resources such as the instructor's background and feedback from students. The ways instructors used these resources and their reasons for doing so varied greatly. In this chapter, I present the findings from each phase of data collection, followed by a summary of themes found from the overall analysis of these data.

Survey Results

The survey was used to investigate primarily what resources instructors used when planning and teaching the FTC. Answers to questions on this survey gave insight into not only what instructors were using, but also began to explain how these resources were used and gave insight into how instructors considered the FTC. The survey was organized into three sections. The first section collected information about resources used for general teaching and planning. The second section asked about resources used specifically when planning and teaching the FTC, and the third section asked about participant demographics. Within the second section, instructors were asked to examine two different proofs of one part of the FTC, and report their preference and why. This particular part of the survey was not directly tied to resources (although it was expected to confirm that instructors preferences matched the resource of the textbook that they used), and yielded some unanticipated findings. I begin this section of findings with a description of participants, which shows a wide variety of instructors. I then report findings from the survey first by resource. For each resource I describe the findings that apply to planning and teaching Calculus I, then planning and teaching the FTC. I then report findings directly related to the FTC that are not tied to any specific resource, before moving on to the next section on interview results.

Fifty instructors from 22 community colleges responded to the survey. Thirty-five indicated that they were teaching calculus in Winter 2016 while 15 were teaching the course in Fall 2015[7]. The majority of the instructors who responded to this survey were full-time (37/50). These proportions are comparable those reported by the 2015 survey of two year college mainstream calculus programs, (79% of TYC calculus instructors were full time; see Blair, Kirkman, Maxwell, 2018, p. 17&19 [8]). The majority of the respondents were male (28/48) and had a master's degree (38/49) in mathematics (33/49); the other respondents had master's in mathematics education, statistics, engineering, or law. Seven respondents had a PhD in a STEM field (math, physics or engineering); one respondent's PhD was in higher education, and another in law, and another had all the requirements for a PhD in mathematics education except for the dissertation. One respondent had an undergraduate degree in math. The majority of the participants (28) reported being in their 30s or 40s; 16 participants indicated being in their 50s or 60s; a small number of participants were under 30 years old (2) or over 69 years old. The participants in the sample reported having taught between 1 and 65 semesters of college calculus (average = 6 terms, SD = 17 terms), with a median of four and a mode of three terms; it is then

[7] Respondents who indicated that they were teaching calculus in the winter term were not asked if they had taught in the fall term. I was looking for instructors who taught calculus at least once during the 2015/2016 school year (not summer), so there was no need to ask about a prior semester once they said yes to teaching in the winter term. A question by question summary of responses to questions can be found in Appendix G

[8] This percentage was not directly included in this report, but calculated from the reports of percentage of main-stream and non-mainstream Calculus I faculty as well as total enrollment and average section size.

accurate to say that the majority of the respondents had about two years of experience teaching college calculus. Thus the sample included respondents with a wide range of ages and of college calculus teaching experience, which gives confidence that their responses may be useful in characterizing their practices. For a complete list of participants' demographic information, see Appendix F. I now present the overall synthesis of responses in the survey regarding its three major areas, their textbook use, their use of technology and other resources, and their use of the FTC proofs.

Textbook use.

Survey results about textbooks indicated which calculus textbook instructors were using and how they were chosen and gave some insight into how instructors used them. The majority of instructors used textbooks mandated by the department (41), although eight instructors chose their own textbook, and one wrote his own textbook. The majority of the calculus textbooks mentioned by 35 instructors were authored or or co-authored by three people: Edwards (2006, 2007, 2013[9]), was a co-author on textbooks used by 14 instructors; Stewart (2009, 2010), was the author on textbooks used by 11 instructors; and Briggs (2012, 2014) was the co-author on textbooks used by 10 instructors. The instructors reported that they had been teaching with their current textbook for 6 semesters on average (SD=5).

Four questions in the survey (Q6-9) asked for the respondents' general use of their textbook, specifically, whether they used the same formulas and symbols as in the textbook, if they were comfortable changing the order of topics, their assignment of homework from textbook or software; and their use of examples from the textbook during class time. The respondents used a 7-point Likert scale, from never to always, in responding to these statements.

[9] Instructors on the survey did not include the edition they were using. These dates represent the most recent editions available prior to Fall, 2015.

In Table 9, the responses are presented as frequencies in 3 categories: 1-2 (Never or Infrequently); 3-4-5 (Occasionally); and 6-7 (Almost Always or Always). All but one instructor assigned homework from the textbook, with 37 instructors indicating that they always assigned homework from the textbook.

Table 9: Instructors use of their textbook; (N=49).

Question	Never or Infrequently	Occasionally	Almost Always or Always
6) It is important to me to use the same formulas and symbols as my textbook	4	16	29
7) I am comfortable changing the order of topics in the textbook.	5	18	26
8) I assign homework from the textbook or software associated with the textbook	1	1	47
9) I use examples from the textbook during class time	16	18	15

Instructors used their textbook for assigning homework and were generally comfortable changing the order of topics. Instructors generally followed the notation and symbols that their textbooks used. The responses to Question 7 suggest that most instructors feel some agency over the order of topics that they teach. These responses suggested to me that some instructors might introduce the FTC in the first few weeks of the semester, rather than follow the order in the textbook and present it in the last few weeks. The responses to Question 8 suggest clear tendencies of instructors to use their textbooks or related software to assign homework problems for students. However, these responses do not imply that the textbook is the only source of homework problems, even if it is always used for homework, some instructors may use additional materials.

The last item is shows a more even distribution than the first three. As discussed previously, there were problems in the wording for Question 9. Instructors may have interpreted “example problems” to include the problem section of the textbook or they may have interpreted “example problems” as only the sample problems in the exposition text. Thus, the 16 instructors who indicated that they Never or Infrequently use example problems from their textbook during class time may actually use the problems from the problem set during class time.

Textbooks were also used by some instructors when planning to teach the FTC. As part of the questions about planning and teaching the FTC, instructors were asked how they used their textbooks, specifically how often they referred to their textbook when planning to teach the FTC. Six instructors indicated that they never refer to their textbook when planning to teach the FTC, 29 indicated that they sometimes use their textbook, and 13 instructors said they always refer to their textbook when planning to teach the FTC (Survey Question 13, n = 48). On this survey, I did not ask how or why instructors use or do not use their textbook when planning to teach the FTC, and these results should be treated with caution. That 42 out of 48 instructors sometimes or always refer to their textbook when planning to teach the FTC does not mean that most instructors are familiar with how their textbook treats the FTC. An instructor who always uses the textbook to plan to teach the FTC may refer to the FTC section of their textbook for notation only, and not be familiar with the entire section. This finding is another indication that the textbook is a resource that is frequently used by instructors, not just in overall planning and teaching Calculus I, but also in planning to teach the FTC.

Each instructor was also asked to evaluate their textbook’s presentation of the FTC in terms of the explanation, proof, problem sets and overall treatment. These four items were assessed on a 7-point Likert scale from positive (1) to negative (7) in terms of their impression of the textbook in those categories. In Table 10, the responses are presented as frequencies in 3 categories: 1-3 (Positive); 4 (Neutral); and 5-7 (Negative).

As seen in Table 10, most instructors had a positive to neutral view of all aspects of their textbooks treatment of the FTC. All four questions have a similar distribution of results, with 31 instructors having a generally positive impression of their textbooks overall treatment of the FTC, and 31 also having a positive view of their textbook's problem sets and explanation of the FTC.

Table 10: Impression of the textbook's treatment of the FTC (n=48)

Question	Positive	Neutral	Negative
(14) What is your general impression of your textbook's overall treatment of the Fundamental Theorem of Calculus?	31	12	5
(15) What is your general impression of this textbook's explanation of the Fundamental Theorem of Calculus?	31	11	6
(16) What is your general impression of this textbook's proof(s) of the Fundamental Theorem of Calculus?	27	15	6
(17) What is your general impression of this textbook's problem sets relating to the Fundamental Theorem of Calculus?	31	7	10

To understand whether the same instructor felt positively (or negatively, or neutral) toward each aspect of the textbook's treatment of the FTC, the results of these questions were compared with each other. A Cronbach's alpha of 0.89 indicated that these questions had a high internal consistency. In other words, the attitudes of an instructor toward their textbook's section of the FTC were fairly consistent across the explanation, proof, problem sets and overall treatment.

Graphing technologies and other resources.

Participants were asked about the resources they used when planning the semester (Question 10). Twenty-two reported using only curriculum materials (such as textbooks) and calculators. One instructor indicated that he spoke with other instructors. Six respondents said they used materials that they created themselves and 18 said they used Google and/or mathematics software such as Maple, Desmos, and Wolfram Alpha.

The survey asked instructors what students were required to purchase for a first semester calculus course. Forty-one respondents indicated that students were required to purchase a textbook; 23 indicated that the department required a graphing calculator, with an additional three instructors indicating that a graphing calculator was highly recommended, but not required. The primary resources that instructors mentioned for planning and teaching a first semester calculus course were textbooks, the software associated with the textbook, and some kind of graphing technology (either calculators or software). Participants did not mention blackboards, whiteboards, chalk, markers, or a document projector, perhaps because these resources are common enough to be overlooked.

When teaching and planning to teach the FTC, instructors were asked specifically about resources other than the textbook. Questions 18-21 asked instructors about the technologies that they and students used when the instructor taught the FTC. Eighteen instructors indicated that they used no technologies when teaching the FTC, and four of them did not permit students to use technology when learning the FTC. One instructor who used Desmos to teach the FTC did not permit students to use technology when learning the FTC. Although only five instructors did not permit students to use technology when learning the FTC, 29 did not require students to use any technology when learning.

For instructors who did use technology when teaching the FTC, a graphing calculator was the only extra technology used by 21 instructors in the survey. An additional six instructors used a computer algebra system and two instructors used other technologies such as Geogebra Applets. Thirty-six instructors permitted students to use a graphing calculator when learning the

FTC, and 16 of these instructors required students to use a graphing calculator. Of two instructors who indicated they used a computer algebra system to teach the FTC, one required students to also use a computer algebra system when learning the FTC. Overall, results to questions about technology use when teaching the FTC indicates that instructors teach the FTC in a variety of ways, and may not be consistent between their use of technology when teaching and what they permit students to use.

Question 22 asked whether they agreed or disagreed with the statement “In my experience, explaining the Fundamental Theorem of Calculus to students is easier without added technologies.” Based on a 7 point Likert scale (See Appendix F), 18 instructors agreed (5-7) and 19 disagreed (1-3). Ten instructors were neutral or had no opinion (4) on whether or not it was easier to teach the FTC without added technologies. Of the 18 instructors who agreed that explaining the FTC was easier without added technologies, 11 did not use technology when teaching the FTC, five used a graphing calculator and 1 used Desmos. Of the instructors who disagreed that it was easier to teach the FTC without added technologies, three of them did not use technology when teaching the FTC. I did not ask why they used or did not use technology when teaching the FTC, and these inconsistent results about how they used technology compared with their stated preferences warrants further investigation. Instructors may have institutional requirements requiring or forbidding a calculator, or instructors may have simply read some of the questions wrong. Question 22 was posed in a negative fashion (“without added technologies”), and the first option was to disagree with this statement. For questions 14-17, the first option was to agree with the statement. In addition, one instructor (who chose “other” when asked about technology and the FTC) indicated that the technology use depended on the problem. For some problems technology use was required, and for some it was not permitted.

Instructors may have interpreted questions about students learning the FTC to apply only to classroom time or to include working on homework problems. Classroom time and homework may have different expectations and requirements regarding technology use. Therefore inconsistencies between instructor and student use of technologies when teaching and learning the FTC may not be as large as they seem based on responses to this survey.

FTC Proof Preference.

I presented survey participants with two different proofs of the evaluation portion of the FTC (Appendix B). The first proof, called here the MVT proof, was adapted from Larson (2007) and used the mean value theorem. The second proof, called here the I-to-II proof was adapted from Stewart (2003) and used the first FTC in proving the second FTC. Because I anticipated that instructors' preferences would match the proof in the textbook that they used, I examined the textbooks that instructors said they used, and considered the version of the proof of the evaluation portion of the FTC that was presented in that textbook. With limited exceptions, those version of proof either matched the Larson version (proof 1, uses MVT, see Appendix B) or the Stewart Version (proof 2, uses FTC I, Appendix B). The exceptions were the Briggs textbooks and Tan's applied calculus, which did not have a proof of the FTC. However, the Briggs textbooks had a rationalization for the FTC that mirrored the Stewart version (I-to-II).

After presenting both proofs to instructors, they were asked which proof they preferred, and why. Overall, 21 instructors preferred the MVT proof and 20 instructors preferred the I-to-II proof. Four instructors chose "other" proof preferred, based on no proofs given in class or proof by other means. I compared those responses with the proof in their textbook. I had anticipated that their preference would match what was in their book. However, this was not the case (see Table 11). The textbook seemed to have little influence in determining which proof of the FTC instructors preferred. Table 11 lists how many instructors used each textbook, which proof type was in the text (if known), and which proof type the instructors who used that text preferred. Not every instructor who chose a textbook indicated a preference of proof, so the numbers are not consistent. For example, although nine instructors indicated that they used a Larson/Edwards text, only eight of those instructors chose which proof they preferred.

Table 11: Textbooks, proof type, and instructor proof preference

Textbook Author(s)	**Proof type in text**	**Number of Instructors using text**	**Number prefer proof 1 (MVT)**	**Number prefer proof 2 (FTC)**
Larson/Edwards	MVT	9	4	4
Larson/Edwards ET	MVT	4	3	0
Edwards Penny ET	MVT	1	1	0
Anton/Bivens/Davis ET	MVT	5	1	3
Stewart ET	I-to-II	7	4	3
Stewart Concepts and Contexts	I-to-II	4	1	1
Briggs/Cochran ET	I-to-II	4	1	3
Briggs/Cochran/Gillett	I-to-II	6	1	3
Hass/Weir/Thomas ET	I-to-II	3	1	1
Thomas/Weir/Hass/Giordano ET	I-to-II	1	1	0
Thomas/Weir/Hass/Giordano	I-to-II	1	1	1
Hughes/Hallet	I-to-II	1	1	0
Munem	I-to-II	1	0	1
Own text (written by teacher)	unknown	1	1	0
Applied Calculus by Tan	n/a	1	Prefer not to prove	
Total MVT	*MVT*	*19*	*9*	*7*
Total FTC	*I-to-II*	*28*	*11*	*13*
Total	*n/a*	*49*	*21*	*20*

It is apparent from participants' responses to survey questions 14-17, that they generally approved of their textbook's treatment of the FTC in terms of the explanation, proof and problems, however the book that they were using did not seem to influence which proof they said they preferred. Because this result was unexpected, I compared proof preference with number of semesters teaching calculus, employment status, gender, age range, and degree field. All categories had some instructors that preferred the MVT proof and some that preferred the I-to-II proof (see Table 12). Based on the information gathered on the survey, there is no way to describe either a typical instructor who preferred one of the proofs or to predict which proof would be preferred by an instructor with particular characteristics.

Table 12: Proof preference based on experience, status, gender, age, and degree field

	Semesters Experience		Employment Status		Gender		Age Range		Degree Field		Total
	0-7	>7	Full-Time	Part-Time	Female	Male	<50	50+	Math	Other	
MVT Proof	12	8	17	3	5	15	14	6	15	5	20
I-to-II Proof	15	6	14	7	4	16	12	9	15	6	21

The category with the most differences between proof preference was part-time employment status. More part-time instructors preferred the I-to-II proof than preferred the MVT proof, by a ratio of 7:3. However, because there were only 10 participants in this category, these preferences are not statistically significant.

Instructors on the survey were asked why they preferred the proof that they chose, and I analyzed these responses (see **Error! Reference source not found.**, methods chapter). The two most common reasons instructors gave for preferring a proof was because of the math in the proof (13 instructors) or because the proof was better for students (11 instructors). This is shown in Table 13, below.

Table 13: Proof preference by reason and type of proof

Reason for preference	Number of instructors who gave that reason	Number of instructors with that reason who preferred MVT proof	Number of instructors with that reason who preferred I-to-II proof
Math	13	8	5
Students	11	2	9
Self	6	4	2
Math and Students	5	3	2
Other	5	2	0
No preference	5	1	3

Of the 13 instructors whose reason was math, eight preferred the MVT proof and five preferred the I-to-II proof. Of the 11 instructors whose reason was students, two preferred the MVT proof and nine preferred the I-to-II proof.

Instructors were asked about their personal preference of proof, and why they preferred that proof. They were not asked if they proved that portion of the FTC during class time, and if so, which proof they used. These results indicate that personal proof preference is not shaped by the textbook used, but the results do not indicate whether or not instructors are teaching their preferred proof.

Overall, survey results indicated that the main resource instructors used was their textbooks, and that textbooks were used for assigning homework and to examine notation. Most instructors had positive impressions of their textbooks treatment of the FTC, but that did not translate to their personal preference of an FTC proof. The second most common resource that survey instructors used was graphing technologies, but the survey did not give information about how instructors used these resources. The survey provided a starting point for interviewing instructors about how and why they used the resources that they did. The next section describes the results from the interview data.

Interview Results

I chose 14 instructors to interview from the 21 survey respondents who indicated they would like to be interviewed. The interviews ranged from 77 minutes to 160 minutes, with an average of 110 minutes. The purpose of the interview was to understand how and why instructors used their resources for planning and teaching the FTC, as well as confirm what resources instructors were using, and find out if they used more resources than they mentioned on the survey. The analysis of interview data revealed nuances about how instructors used their textbooks and graphing technologies that was not present in the survey data, and revealed the intangible resources that instructors used. Intangible resources such as and instructor's personal background, feedback from students, and the affordances and limitations of their home institution were mentioned very rarely on the survey, but all interviewed instructors mentioned some of these intangible resources and how they affected their teaching. The analysis of the interview data also revealed patterns about how instructors talked about the FTC. I present an overview of the findings here and expand on each of those in the subsequent sections.

All instructors mentioned using the textbook for planning the full semester or a single lesson and for designing homework by using the problem sets available in the textbook. When teaching, instructors said they primarily used the board and graphing technology to display functions and to perform calculations quickly and accurately. Some instructors also reported using a graphing tool while teaching the FTC. The analysis suggests that the way instructors described their use of the graphing calculator was related to how they thought about the FTC. However, other resources were used regardless of how the instructors viewed the FTC. The reason that instructors gave for using these resources was primarily out of concern for students. Instructors also indicated that their teaching and use of recourses was sometimes driven by a concern for accurate mathematics as well as the influence of their own background and their home institution. I expand on this throughout the interview section of this chapter.

The findings in this section, similar to the findings in the survey section, are arranged by resource. I describe the findings by three major resource categories: textbooks, intangible resources, and graphing technologies. For each resource, I separate out ways that instructors said they used those resource. For each resource and way of using that resource, I discuss why instructors use that resource. When applicable, I link the resource, how it is used, and why it is used to teaching the FTC. Then I describe the findings related to how instructors discussed the FTC. Finally, I give a summary of the reasons instructors gave for using all resources and an overall summary of the findings.

Textbooks

Textbook use was ubiquitous among instructors interviewed and surveyed. Even Richard, who wrote his own course pack, said he brought the textbook to class with him. Most instructors used their textbook in very similar ways. Based on findings from survey data that indicated the textbook was used by all instructors, I examined interview text for mentions of how and why the textbook was used. From this, I found four themes from the interviews about how and why instructors used their textbooks. Instructors use their textbooks for: 1) planning a semester, 2) preparing to teach a lesson, 3) to assign homework, and 4) for notation.

Planning a semester. Of the 14 instructors interviewed, all instructors said they used the textbook in some way for planning. Overall, every instructor except Richard indicated that they used their textbook in some way when planning or outlining a semester. For example, George, Leopold, and Frederick received an outline of first semester calculus topics that they must cover by the college. These outlines included book sections. In order to plan what to teach on a certain day, they took that outline and used the textbook to build a schedule "I try and go on a pace where I'm breakin', maybe, a section down—every two days, I'm goin' through a section." (Frederick, 10-11). George's students get a syllabus from the college, with a list of sections in the book to be covered and a note saying that the instructor may adjust the schedule, but not the content (see Appendix D). Oliver based his semester schedule on the six chapters in his textbook, "I would do two chapters, a review class, a test class, two chapters, review. I just broke [the semester] into three bits" (Oliver, 1192-1193). Leopold, Harold, Philip, George, and Karl shared their schedules with me and each of these schedules included sections of the textbook that needed to be covered on various days (see Appendices D and J for examples). Nathan indicated that he based his lessons on sections of the textbook, implying that his semester planning was closely linked to the book: “I’ll do one [section] a day, generally. Sometimes I’ll do two sections a day. Sometimes, maybe one in two days. Or two in three days, but generally it’s one a day.” (Nathan, 862-863).

Not every teacher indicated a reason for using the textbook to plan a semester. However, two teachers were explicit that they used the textbook to plan a semester based on institutional requirements. For example, Leopold is given a list of textbook sections that he needs to cover throughout the semester, and then is able to organize his sections.

> With only 32 days—16 weeks, two days a week. I’ve got 32 days to teach all these sections… Again, [I’m] trying to get creative in what I can possibly combine with what are the topics that are natural pairings based on how I can see where they would progress upward. It is not necessarily the order of the textbook. (Leopold, 232-233, 303-305)

Thus, he used his textbook to plan his semester in order to follow the sections that the institution said he has to cover, but he has some flexibility in how he arranges the sections to be covered. This passage also illustrates an important nuance in how instructors used their textbooks to plan the semester. Using the textbook to plan a semester did not mean that instructors followed the order of topics that was presented in the textbook. Leopold organized the topics that he was required to teach by what makes mathematical sense, using the textbook the institution requires.

Several instructors modified the order of textbook topics and explained to me why they did so. Leopold (see Appendix K) described his reasons for adjusting the order of sections in the topics as “I needed to look at it from, if I’m going to have to combine sections, what topics are realistic that I can combine together. Then at the same time, what order of the topics makes the most sense?" (Leopold, 234-236). Theresa also shifted topics based on what she feels makes sense:

> when I look at a textbook, I go through what the chapters are, and I'm one of these that likes continuity when I teach. So if I find that, in chapter 1, there's a piece of chapter 1 that I don't feel is continuous with the other stuff, I may hold off on that until I find something that connects with that piece. (Theresa, 134-138)

George also got a syllabus from the department (see Appendix D), and modified it slightly. " [at this institution] there's five pages of stuff that I have to have in [the syllabus] and I modify it a little bit …the schedule's mostly set in stone, although I had to adjust it for twelve weeks instead of fifteen weeks" (George, 216-218).

One of the consequences of using the textbook to plan the semester is that the FTC is in only one section of one chapter of all textbooks [10]that these instructors were using[11]. Instructors who used their textbook to plan their semester were unlikely to devote more than an average section's worth of time to the FTC. One of the interview questions asked instructors to advise me, as a potentially new instructor at their institution, on teaching the FTC. The advice I received regarding how many class periods or hours I should devote to teaching it ranged from two to four hours. For a four credit-hour 14-16 week course, this allocation corresponds to between 1/32nd and 1/14th of the time spent in front of students.

Planning a lesson. In addition to planning a semester, nine out of 14 instructors mentioned reading or reviewing the textbook for lesson planning. I asked instructors toward the beginning of the interview how they typically planned a lesson. Nine of the 14 instructors explained how they would read or review the textbook. For example, Philip said, "I'll read back through the

[10] Richard did not use a textbook; he used his own course pack. However, the time he devoted to teaching the FTC did not differ from instructors who used a textbook.
[11] The only textbook that devoted two sections to the FTC was the Hughes-Hallet, which was not used by any of the instructors I interviewed.

section myself before I go teach it" (Philip, 72). Harold said, "I just look through [the textbook]. I circle some examples that I think are good. Some that I wanna avoid" (Harold, 83-84). Frederick, George, Ian, James, Nathan, Oliver, and Suppiluliumas also reviewed their textbook when planning a lesson.

Of the five instructors who did not mention their textbook during lesson planning, Richard said that he used his course pack, Karl reviewed his notes, Leopold reviewed his course pack and notes that he created from the textbook before the semester started, Theresa did not say if she used her textbook in planning a lesson or not, and Marcus, who taught applied calculus, used activity sheets to plan a lesson.

Although Marcus was very familiar with the textbook and often taught from it, he felt that applied calculus should be taught differently than regular calculus. When planning lessons, he was often frustrated with the textbook:

> Here's the problem I have with applied calculus. They give the stupid equation from nowhere. The students have no idea where this equation is coming from. So why don't they make it, 'here is the data we have. Why don't you do a regression analysis and [find] what would fit.' That's the way I think the applied calculus should go… But there's no book that'll do that (Marcus, 491-497). I don't think [students] get a very intuitive knowledge about what's going on when they use the text. (Marcus 924-925)

When teaching the FTC, he used his own worksheets and avoided the book (see appendix E). In particular, although Marcus used the textbook to plan most lessons and assign most homework, he did not use the textbook for homework when teaching the FTC.

The reasons that instructors gave for using the textbook during lesson planning primarily had to do with students. Ian was clear that he started with the textbooks because that was what students used, saying "I start from the textbook as my base because this is obviously what the students are going to be using and referencing" (Ian, 39-40). Harold chose his problems from the textbook based on what he felt that students could understand. Marcus avoided using the textbook for the FTC because he didn't feel it was appropriate for what students needed to know in applied calculus.

However, although the textbook was a common resource that instructors used to plan a lesson, it was not the only resource used. Instructors also relied on their background knowledge, experience teaching, worksheets and notes from past years, etc. Some instructors indicated that they avoided their textbook for certain topics. Thus, the influence of the textbook on an individual lesson may vary by content.

Assigning homework. One of the ways that the interview was able to build on the survey results was by examining how instructors used their textbooks to assign homework. In the survey, all but one instructor indicated they used the textbook to assign homework. During the interviews, I confirmed that the most common way of using the textbook was to assign homework. All of the interviewed instructors used the textbook for this purpose. Even Richard, who stated on the survey that he never assigned homework from the textbook, used textbooks in this way. While Richard stated that he did not assign homework directly from one textbook, his course pack included homework with problem sets from multiple textbooks.

In the survey (Question 8) I did not differentiate between on-line homework that is linked to the textbook such as MyMathLab or WebAssign and the one taken from a bounded copy of the textbook because each on-line homework question can be found in the textbook. However, on-line homework often excludes problems in the textbook that ask for a proof or an explanation.[12] During the interviews, I asked whether the instructors used their textbook or the online software (or both) to assign homework.

[12] This is slowly changing, depending on the software and course. The last time I set up online homework (in 2014) there was an option to include these types of questions as well as an option to write my own questions. The software

Of the 14 instructors interviewed, all of them used the problem sets in the textbook for some or all of the homework. When I asked about homework software, the participants were clear that there was a difference between problem sets in the physical textbook and the online homework both in terms of content, and in terms of advantages and disadvantages for students. Three instructors, Ian, Suppiluliumas, and Leopold, offered students a choice between online or textbook problems, because they were basically the same[13], but noted that the feedback to students was different. "On the online homework you can hit 'show me' and then watch an example, try another. It tells you immediately if you did it right or not. Versus the written homework they have to come to me." (Ian, 137-138). George avoided online homework because he had seen students get frustrated when an answer did not match exactly what the online system expected, and he felt that students should not pay extra for an online system when they were already paying a great deal for their textbook:

> I've seen students do WebAssign and they get really frustrated with that. It takes a really long time. "What? What's wrong with this answer? It's right?" And I would say, "Well, if a human looked at that they would grade that right. But there's something that the computer doesn't like about it, so it's really frustrating."… "I don't want you to pay more money for the homework separately." (George, 303-306, 315-316)

Karl, Philip, and Ian appreciated how online homework gave immediate feedback to students and saved them time in grading (although Ian still did some grading for students who chose to do their homework from the book). In speaking about an anticipated change from online to book homework that was coming in the next year, Philip said, "Well, the bigger difference is going to be in the grading. Because here it's graded and it's done. Here if I had to collect homework for all hundred and twenty students every semester and grade it, they'd never get anything back"

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did not grade these questions automatically and the instructor had to log in to look at the work of each student. Thus students did not receive immediate feedback and the set-up demanded more time on the part of the instructor.

[13] The online homework systems and the problem sets in the book are tightly linked. See Appendix I

(Philip, 334-336). James mentioned that some of the more conceptual problems he wished to assign were not available online, so he used both the physical textbook and the online homework system. “Not every problem in the textbook is available on the online homework systems and sometimes the better ones aren’t available so I assign those” (James, 72-74). Two other instructors, Nathan and Philip, also noted that the online system was not exactly like the book, but did not say that they assigned additional problems from the text.

The two most common reasons that instructors gave for assigning homework from the textbook or online system were because of themselves (the online homework saved them grading time) and for students. Some instructors saw the online homework system as an advantage to students in that it gave them immediate feedback, and some instructors saw the online homework system as a detriment to students because it cost more and because not every problem was available to students. Thus, the same concern for students resulted in different uses of the textbook and online homework system.

Although the textbook was often used as a bank of problems, and all instructors used it for homework in some way, one instructor avoided the textbook for FTC homework. As noted above, Marcus felt the textbook was inappropriate for applied calculus and used his own worksheets for homework in that particular section (see Appendix E). Another instructor, Richard, pulled his homework from a variety of sources, including textbooks.

Using the textbook as a bank of problems for homework has implications for teaching the FTC because mathematics textbooks are commonly set up so that one topic depends on the previous topics. In calculus books, the FTC section appears in the first chapter on integration, after differentiation is completed. If an instructor wanted to teach the FTC before differentiation, he or she may have difficulty finding homework problems that would not rely on prior textbook sections of derivatives.

Notation. On the survey, most instructors indicated that it was important to match the notation in their textbook. I followed up on this during the interview and asked instructors about if and why they wanted to follow the notation in their textbook. All instructors said they knew their textbook well enough to discuss its notation and tell students why they adopted it or strayed from it. Instructors who did not match notation were clear that they let students know which notation (such as f' or ${}^{dy}/_{dx}$) was in their book, and why they were deviating from that notation. Most instructors indicated a concern for students when they talked about how concerned they were with matching the notation from their textbook.

Ian, Karl, and Suppiluliumas were clear that they tried to match notation in order to avoid confusion for students. Karl said, “I definitely don’t want to give them anything that’s too different from the book, [because] I think at this level, I think of my own difficulties in understanding stuff at first.” He was concerned that students would not be able to follow along if his notation strayed too far from what the textbook used. Richard matched notation because there was a department final and he wanted students to be familiar with what would be on the final exam.

Harold, James, Marcus, and Theresa did not match notation from the textbook, and this was also out of concern for students. Harold, James, and Marcus wanted students to understand multiple notations. Harold said he used the notation he prefers, and expected students to be able to understand what he was teaching. “I just let them know that’s another thing. I tell them, there are different ways to write the same thing. Don’t get confused” (Harold, 440-441). James wanted students to be exposed to a variety of notations so that they would be less confused in the future:

> I think it's important for the students to see a variety of notations if a variety of notations is what they're gonna see in the future. For example, in derivatives, the Stewart book used prime notation almost exclusively until they couldn't anymore. The Hass book uses Leibnitz notation quite a bit. Hardly any of them use dot notation (James, 413-416)

He goes on to say that he exposes students to dot notation because it is used in physics and he uses it in physics application problems.

Theresa felt that her textbook used confusing notation, particularly in the section about the FTC. She didn't care for any prime notation (f'), and wanted students to understand problems with letters other than x, which she felt the book used too often. I asked her what she felt was hard about teaching the FTC, and she answered, "Well, sometimes the way the book puts it. See, I find that a lot of students have trouble with formulas… and with notation" (Theresa, 842-845). Because of this, Theresa created her own formula sheets for students, where she could use the notation she felt was easier for students to understand.

The other instructors interviewed did not discuss notation in their textbooks in relation to the FTC, but only whether or not they followed the notation in the textbook. In general, the reasons instructors gave for paying attention to the notation from the textbook had to do with concern for students.

In summary, as noted by Stark (2000), most instructors in this study used their textbook for planning the overall semester as well as individual lessons. In addition, the textbook and online homework were used as a bank of problems, both for homework and for as a bank of problems to solve during class. Instructors spoke about matching the notation in the textbook, and pointing out differences to the class when their notation differed from that of the textbook.

Human and Social-Cultural resources

In this section, I describe the intangible resources that instructors described during their interview that influenced their teaching and planning. Intangible resources are defined as both human and socio-cultural resources (Adler, 2000). They include such things as noticing students during class, the length of a lesson, and the frequency of the lessons. The three primary intangible resources that instructors described were 1) background in learning and teaching, 2) cues from students during class time, and 3) affordances and limitations of the institution. In this section I discuss each of these briefly, with a mention of how and why instructors used these resources. When possible, I include how these intangible resources affected how instructors said that they taught the FTC.

Background. Instructors indicated that their past experiences in teaching and learning influenced how and what they present to the class and what resources they use. Six instructors said they used their experiences as students or their business background to explain why they taught or planned in a certain way. For example, James said he reordered the book sections based on how he would have learned best as a student. Karl said he used the notation from the book because changing it would have confused him as a student. Philip and Richard avoided using examples from the book during class time because they had prior instructors who taught that way and found it to be a poor way for them to learn. Philip also avoided extra technology (such as CAS) because he had to relearn math after relying on the calculator in college.

Oliver and Suppiluliumas had strong business backgrounds that influenced how they taught calculus. For example, Suppiluliumas said that he has students explain how they solved various problems to each other by asking them how they got an answer:

> How did you get that answer? "Oh, I typed some numbers into my calculator." No, you don't tell your boss that ever. How do you talk to an executive, or a senior engineer or something like that? They have to learn how to do that, and that gets into part of the classroom labors. … How did you do it? 'I didn't do it' Okay, that's a good reason. No problem then. The next time I say, 'how'd you do it', they'll show me how they did it. Now that's another technique you get in the business world. You do not

get to say I didn't do it because if you say that, you don't get to be in the next meeting (Suppiluliumas, 184-187, 1185-1188).

His rationale for having students practice explanations was that students needed to learn how to explain what they were doing to potential bosses and coworkers. Oliver taught business calculus, and had a background in business. He viewed business calculus differently because of his background: "I've worked commercially around the world designing, so [I'm] very practical orientated, and so I view the business calc as not a pure calculus class" (1516-1518). He was clear in his interview that he explained theorems, but did not require students to derive them.

Two instructors explained how their teaching experience affected how they prepared for class. Harold said, "If I know they're gonna ask about this problem, 'cause every class does, I'll have that prepared [on the projector]" (Harold, 297-299). Leopold chose which homework problems to emphasize to students based on his past experience teaching:

> Online it will say… all these are gonna be your homework problems for every section. The first time I taught it… I was like, you had to do all of these and you're going to do them all for paper homework.…Then what I did... I'm going to pick five problems, maybe six, from the section that I would say are going to be, what I would call the most—not necessarily the most critical, but these are the ones where they had the fundamental concept in mind. That if you can do these, if you know how to do these five or six problems, you're gonna be okay for the test and you're going to have a basic understanding of what's going on here. (Leopold, 114-126)

Leopold was given a set of homework problems by his institution, and told to have students do those problems. In his first semester teaching, he had students do all of those problems, but later he chose which of the problems he wanted to emphasize, based on his prior experiences teaching.

Thus instructors' learning and teaching experiences influenced how they planned and taught. Experiences of struggle as students led them to avoid duplicating that experience for their students; past professional experiences also led them to emphasize some problems over others.

Interviewed teachers said that their personal background impacted how they approached the FTC. Oliver's background in business affected what he felt was the purpose of a business calculus class, which in turn affected how he taught it.

> [The FTC is] really important because it proves the sum of a rectangle can be done [to find an area]. It's actually the formal proof from the sum of those infinite number rectangles. I try to explain it, and we do a bit of it, but I can't go into the full depth (Oliver, 1786-1788).

Oliver was clear that he finds the theorem important, but that he explains it rather than proves it.

Karl's explained how his background meant that he did not test on the theory of FTC.

> So I'm not actually going to give them a quiz or test problem where I say 'state this' or where I say, 'differentiation and integration are connected in some way. Comment.' ... I won't do that at this level… Because I don't think that for this level it's appropriate. .. I know that I didn't understand [the FTC] when I first took calculus… even though I did fine on the AP test, I know I didn't have this theory down. I just *know* I didn't. … And I'd made it through analysis and everything. So I'd even done proofs with this stuff. And I still didn't get it. So I don't think it's fair to expect a student to understand it. (Karl, 404-416)

Karl stated that he feels that it wasn't fair to ask his students to do things he didn't understand until he was teaching. George (who was teaching calculus I for the first time) remembered learning FTC where his instructor said, "we just changed the world" and he intended to present the FTC that way to his class.

Overall, the background and experiences that instructors had in teaching and learning influenced how they taught calculus in general, and the FTC in particular by influencing how they organized topics for the semester, how they prepared problems to present in class, what type of problems they gave on quizzes, and what types of homework problems they chose to emphasize.

Student feedback. One of the most common intangible resources that instructors said they used was feedback from students during class time. This feedback included facial expressions of confusion and nods of understanding, verbal feedback, and questions from students. Instructors used this feedback during class time, and it impacted their teaching.

Four instructors described a typical classroom day as dependent upon whether or not students ask questions. George talked about noticing when a student stops asking questions as a way for him to know when it was time for him to move on to a new topic:

> There's one girl that asks a lot of questions, which is good for me because …if there's a girl in the class that's constantly asking questions, like, good. I can constantly answer questions. And then once she stops asking questions, okay, now I'm ready to move on. I kind of use that as a guideline for the whole class. (George, 545-550)

He paid particular attention to feedback from one particular student and let that guide his instruction. During my observations of Karl and Philip, I noticed that they paid attention to non-verbal cues such as looks of confusion from students.

> A lotta what I do also… depends on what the students' questions are. 'Cause I don't wanna have an hour lecture prepared, where I just go through that stuff. Usually what I've done, is maybe ten minutes' worth of stuff I wanna say. The rest of it is reacting to what they ask, what they need (Nathan, 94-97).

Theresa did not post her homework until after teaching because it depended on how much she was able to teach during class. She was also specific that she did not stop students from asking homework questions, because if there were a lot of homework questions, it meant that she needed to explain things better.

> And I will never tell students, "Oop, no more homework questions," because I figure, as soon as you do that, you have made it so that no students are gonna ask questions in class. And I figure, if there's enough questions in class about the homework, then there's something go, wrong. They didn't understand it, or they didn't catch on. And so then I have to go back and, and figure out another way to explain it. … if it turns out that

> thirty minutes of class is used to go over homework questions, there was a reason for it. (Theresa, 401-410).

Theresa was concerned for student understanding, and student questions were one way that she interpreted their understanding. In general, two-year college instructors are understood to care about students, and this was evidenced by the way that the instructors used student feedback in their teaching.

Institutional Requirements. The instructors mentioned several institutional requirements or policies that impacted their teaching; of these the most prominently discussed was the course schedule; which was mentioned by nine instructors. I define the course schedule as the number of times a week the course met and the length of each session. Instructors also mentioned policies regarding calculator use and homework, and the physical classroom space. Instructors with the same course schedule evaluated it both positively and negatively, and instructors were more critical of the other institutional requirements.

The instructors said that the course schedule influenced their teaching in both positive and negative ways. Nine instructors mentioned the course schedule as impacting how they taught and planned. Six instructors went further and evaluated that impact as positive or negative. Among the six instructors who evaluated the impact of the course schedule, four of them mentioned the impact positively, even though they did not have the same schedule: Two instructors taught five days per week for approximately 50 minutes each day (Harold and Karl) and two taught two days per week for approximately 2 hours (Ian and James). These four instructors each said that their course schedule gave them more time to spend with students or on a certain topic. Harold said that the "extra" time means that he could give a quiz every week as well as review before an exam. " … 'cause again the luxury of having five classes a week… On Fridays, I can do a 20-minute quiz. Then still have time to go over things. I can have time to review before an exam (Harold, 361-364). Karl was able to teach concepts on one day and practice on the next day because of the institutional schedule: “Most lessons in this book take two days; first day theoretical, second day computational” (Karl, 49). Ian and James, who taught two days per week, saw an advantage to that schedule. Ian appreciated teaching for a longer period at a time, because he had at least an hour at a time to do an activity:

> We do an activity every week. We always have time to do a group activity every week, pretty much…. [Classes are] two hours. So usually we do the activity for at least an hour. Sometimes it’s more, depending on how big the activity is (Ian, 180-184).

James appreciated the longer period of time that he was able to spend on one concept. In particular, he appreciates how the course schedule lets him spend time talking about the FTC: “I try to do it in one [class]. We have a two-hour class period, and so there's time to linger” (James, 760).

Philip and Suppiluliumas said that the course schedule was a constraint on their teaching. Both taught two days per week for approximately two hours, but would prefer to teach one hour, four days per week:

> I’d rather have four one-hour periods than two two-hour periods by a long shot… People need to breathe. If you cover too many topics, somebody’s gonna miss something. They don’t have time to digest it. The recovery is not so good (Suppiluliumas, 1125-1131).

Philip taught two days per week at the time of our interview, but had taught Calculus I four days per week in the past. He preferred a four-day schedule because,

> The students don’t have to synthesize so much at once. If I cover a lesson and a half in a day–which I have no choice but to do some days–they have no time to go home and practice before I say, “Okay, remember what we talked about an hour ago? Now you gotta use it.” And that wouldn’t happen if we were on a four day a week. Even though it’s only going home and having that evening, I don’t have to cover more than one lesson in a day. I can split some lessons over to two days and let them practice, maybe, fundamental theorem part one before we look at part two. It would be nice to have that opportunity (Philip, 1117-1127).

Both Philip and Suppiluliumas were clear that they felt a four-day schedule would be better for students because teaching fewer topics per day would help students understand the concepts better.

Leopold, George and Oliver discussed how the schedule affected their planning, but did not express positive or negative thoughts about the schedule itself. For example, Leopold said,

> I've got 32 days to teach all these sections. I need to be able to cover it all and be able to have enough time towards the end to review the material for the final exam. … I needed to look at it from, if I'm going to have to combine sections, what topics are realistic that I can combine together. Then at the same time, what order of the topics makes the most sense? (Leopold, 232-236).

In addition to Leopold, George, and Oliver, Frederick mentioned how the institutional setting (although not the schedule) impacted his teaching. He taught in a high school setting, which meant that students were not allowed to leave class if the lesson finished early.

> Kids aren't allowed to leave early. I think that's a little bit different than the college setting cuz, once a professor's done lecturing, kids typically get up and leave, but not here. … it's not free time. It's problem solving time. Get the homework out and get to work (Frederick, 108-111).

Frederick needed to prepare enough material to cover the time he had with students or have work for them to do when they finished early. Other instructors could let students who were finished leave early, and he could not. He did not mention this requirement as a constraint of his teaching, but the institutional setting impacted his teaching and planning.

Overall, the course schedule impacted instructors planning and teaching in ways that were sometimes perceived as positive and sometimes negative. Teaching fewer days per week for longer times and teaching more days per week for shorter times could both have advantages and disadvantages, depending on the perspective of the instructor. It was clear that course schedules made a difference in teaching, but not what an ideal schedule would be for instructors.

Graphing Technologies

In this section, I describe how instructors use (or do not use) their graphing calculators and other graphing technologies. The use of graphing technologies in the classroom by instructors fell into three broad categories. First, graphing calculators (or a related computer program) can be used to show visual images of functions. Second, graphing technologies can be used for calculation, either to avoid doing complex calculations by hand, or to verify that answers you have calculated by hand are correct. Third, the use of graphing technologies in the classroom may be avoided by instructors.

Six of the instructors interviewed mentioned that they used the graphing calculator in order to show visual images of functions (Harold, Karl, Theresa, Leopold, Philip, and Marcus). Harold and three more instructors (Ian, Nathan, and Richard) felt that the graphing calculator did not consistently show the visual function well enough and they used Maple or Desmos, but the purpose was the same, to display the visual image of a function:

> I prefer [Desmos]... It's very dynamic. … So what I did was I made a function and then I gave it an initial input which was variable, which was dynamic. You could change the initial input like you do for Newton's method. You give it a guess, you know. And then I just reiterated Newton's method process symbolic and so that if you changed your initial guess, I mean, you can change the function and then you can also change the initial guess. And you'd see the tangent lines moving around. And I think I reiterated it four times so you could see the four different tangent lines and the intercepts sort of bouncing around and then homing in on zero. And then you could copy, paste, and reiterate as many tangent lines as you want. And then moving the initial guess around you see it dynamically in front of you just moving around. (Ian, 82-98)

In this passage, Ian is describing an example of how he used Desmos to dynamically explore Newton's method of finding the zeros of a function. In total, nine instructors indicated that an important use for the graphing calculator or graphing software was to offer a visual display of functions.

The second main use of the graphing calculator by instructors was to perform numerical calculations. Seven of the interviewed instructors mentioned managing problems with intensive calculations, such as left-hand or right-hand sums as a reason to use calculators in the classroom. Four of those instructors explained that the calculator always was generally faster than manual calculations, whether or not the calculations were intensive. Three more instructors mentioned using the calculator to check or verify calculations that were preformed manually. In total, 10 out of 14 instructors used graphing calculators as a calculating device.

Three instructors actively discouraged the use of graphing calculators by students and themselves. They wanted for students to think about the magnitude of the numbers and be able to estimate answers with some degree of accuracy. In addition to wanting students to developing number sense, Harold and Suppiluliumas felt that *not* using a graphing calculator helped students organize their work and thinking, and they wanted to see students' work (not just answers). Suppiluliumas was particularly adamant that students be able to explain how they got their answers without saying "I just typed some numbers in a calculator." Discouraging calculator use during class time meant that instructors had to be careful that they did not model problems with complex computations or give students those problems on exams. However, these instructors did not say that they chose homework problems based on their ease of computation, and all of these instructors accepted and expected that students would use a calculator on their homework.

Ways of talking about the FTC

Although my initial research questions had to do with what, how, and why instructors used resources when talking about the FTC, in the course of analyzing those results, themes emerged about how instructors talked about the FTC. In this section I describe how instructors discussed the FTC with me, and then examine the resources they said they used to see if there is any difference in how resources are used based on how instructors talked about the FTC.

Instructors talked about the FTC in different ways, which I grouped into three broad categories: evaluation, inverses, and combination. In order to understand these categories, it is important to recall that the FTC is often given as two theorems. One of those theorems describes how to evaluate a definite integral function by using the endpoints of the antiderivative. This theorem basically says: if $F(x)$ is any antiderivative of $f(x)$, then $\int_a^b f(x)dx = F(b) - F(a)$. [14] The description of this theorem often involves finding the area under a curve of a function by evaluating the endpoints of its antiderivative. The other theorem says that (under certain circumstances) the derivative of an integral function is the integrand of that function (inverses). In other words, given certain conditions, $\frac{d}{dx}[\int_a^x f(t)dt]$ is $f(x)$.[15] The third category, which I call combination, links the first and second fundamental theorems together by using the idea of an accumulation function to describe the area under the curve of a derivative function.

During their interview, instructors mentioned both theorems of the FTC, with all describing area (evaluation) as part of the FTC and all, except Oliver, describing the relationship between integration and differentiation as included in the FTC. In order to understand what instructors emphasize in their teaching of the FTC, I asked instructors what was the main point of the FTC that they wanted students to understand. Overall, thirteen out of 14 interviewed

[14] If a function f is continuous on the closed interval $[a, b]$ and F is an antiderivative of f on the interval $[a, b]$ then $\int_a^b f(x)dx = F(b) - F(a)$. (Larson, Hostetler, & Edwards, 2007, p.282)

[15] If f is continuous on an open interval I containing a, then, for every x in the interval, $\frac{d}{dx}\left[\int_a^x f(t)dt\right] = f(x)$. (Larson, Hostetler, & Edwards 2007 p. 289)

instructors described the main purpose of the FTC for students in terms of inverses or evaluations, but did not combine the ideas:

> I would want them to say that it is a description of the relationship between the derivative and the integral. I think that's imperative. I would want them to say something to the – if we're talking the whole fundamental theorem – I would want them to say something about evaluation of definite integrals. I think that's kind of the biggest takeaway in terms of practical use. I think those two things are really kind of the key components that I would hope that they would get out of it (Philip, 753-759)

Philip was clear that there were two parts to the FTC, and both were important, but he described them as separate ideas and theorems.

Two instructors linked the inverse and evaluation properties of the FTC by discussing functions, but only one (Ian) described this link as the main purpose for students to understand.

> I think the main thing I want them to walk away with is just that there is a fundamental relationship between the instantaneous rate of change of a quantity and the area under a curve. Not of that quantity, but of its derivative. They might forget that. They might forget that little detail two years from now or whatever, that it's not the area under the curve and the rate of change of the same curve. They'll probably forget that. But I want them to remember that there is a fundamental relationship between instantaneous rate of change and accumulation. (Ian, 682-687)

Ian was very clear that the FTC was more than the area under a curve, more than evaluating a definite integral, and more than the inverse relationship between integration and differentiation. He links the instantaneous rate of change (differentiation) with the area under a curve, thus linking the two fundamental theorems of calculus in a way that other instructors did not articulate.

Six of the 14 instructors interviewed focused on the inverse relationship between the integral and derivative as the main purpose of the FTC for students. Frederick, George, James, and Nathan described integration and differentiation as "inverse operations" that were linked by the FTC. When asked, "What do you think is important for students to know about the fundamental theorem?" Nathan responded, "How to differentiate an integral" (Nathan, 799-801). Philip and Suppiluliumas had a related main idea for students: they described the FTC as explaining the relationship between the derivative and the integral. They did not use the words "inverse operations," but were clear that FTC connected the two operations of differentiation and integration. Philip said he wanted students to walk away with the idea that "[the FTC] is a description of the relationship between the derivative and the integral" (Philip, 753-754). In addition to these six instructors, another five (Harold, Ian, Karl, Leopold, and Marcus) mentioned the idea of inverse operations when talking about FTC. The main idea of the FTC that they wanted students to understand was either evaluation or combination.

Seven participants said that the most important purpose of the FTC for students was to evaluate the definite integral, especially in terms of area under the curve of a function. When examining all FTC related codes in their transcript, Harold, Karl, Leopold, Marcus and Oliver had this as the most common way of considering FTC. Richard and Theresa also considered area as one of the main purposes of FTC, but they were very careful to point out that using FTC to evaluate a definite integral can get you a negative value, and area cannot be negative.

> I'm very, very careful in explaining to them that value of the definite integral is not always the area, because they always have an issue with that. Where we'll do an integral, and the value of the integral is not the value of the area. Because they talk about this, oh, negative area. I'm like, "No, negative area is not negative. You're getting the value of the integral. The area is not negative." (Richard 1133-1137)

In other words, area is a useful concept for understanding the evaluation portion of the FTC, but not enough to fully understand it.

In addition to these seven instructors, four more (George, Ian, James, and Philip) also mentioned area when talking about FTC. "My intention is to get to that evaluation part, and run with it. Start doing integrals, applications of integrals, and then we can go to areas between [curves]" (James, 834-835). On the survey, two instructors mentioned area as an additional idea of importance for the FTC.

One instructor, Ian, did not describe the main purpose of the FTC as either evaluation or inverse operations. His explanation of how he taught the FTC had to do the combination of both theorems. He talked about an accumulation function and the process of differentiation, and how they relate to each other:

> So we start by defining this thing which we call an accumulation function. And we look at GeoGebra and you can do some visualizations where this thing's fixed and you can dynamically move this back and forth and it calculates this area… So then we say it's a function like any other function so can we differentiate it? … We're talking about differentiation. What is the rate of change in the value of this function? So what is the rate of change in this total area? And so we go back to the definition of the derivative." (Ian, 414-432)

Ian described teaching the FTC by first defining an accumulating function, and displaying it to students. Students were then to notice that the accumulating function was continuous and well-defined. He then challenged students to come up with a way to differentiate the accumulating function. In this way, he emphasized both area and the relationship between the accumulating function (integral) and the derivative. His explanation of the main purpose of FTC was different from other instructors in that it combined both the inverse and evaluation portions of the FTC. Harold also mentioned accumulating functions, and Philip mentioned the importance for students of understanding that the integral is a function–and that it is one that we can differentiate, but the main purpose of the FTC for both Harold and Philip was not about functions. Ian's explanation (above) of accumulating functions and the FTC matches closest to what Carlson *et al*. (2002) find important in understanding the FTC.

Overall, Instructors' descriptions of how they taught the FTC, and what they wanted students to remember about the FTC, generally aligned with the two theorems that were presented in their textbooks. These two FTC theorems can be broadly described as an evaluation (area) portion and an inverse portion. Ian was the only instructor who attempted to combine those two theorems by considering the rate of change of an accumulation function. Next I examine the way instructors talked about the FTC and the way that they used their resources.

For each resource that instructors said they used, I examined the way that instructors described the importance of FTC for students to see if it related to how they said they used their resource. The only resource that had any relationship to the way instructors talked about the FTC was graphing technologies. The relationship between how instructors describe the importance of FTC for students and how they use the graphing calculator is summarized in Table 14. In this table, the columns represent the how instructors interpreted the main purpose of the FTC, either as the inverse relationship between differentiation and integration, as the idea of area or evaluation of a definite integral, or as a combination of the two (functions). The rows represent the main ways that graphing technology were used or not used. Instructors are listed within each cell by initial.

Table 14: Graphing and Calculator Use by FTC interpretation

How instructors used graphing or computer software	Instructors ways of talking about the FTC		
	Area or Evaluation: H, K, L, M, O, R, T	Relationship or Inverses: F, G, J, N, P, S	Combination: I (H, P, but this was not primary for them)
Visualization	H, K, L, M, O, R, T	P, N	I
Calculation	M, L, R, O	F, G, J, N, P	
Calculator use discouraged	H	N, S	

Each instructor who interpreted the main purpose of the FTC for students as understanding the area under a curve or evaluating a definite integral found it important to visualize functions during class with a graphing calculator or graphing software. Of the six instructors who found the most important part of the FTC to be the relationship between integration and differentiation, only two used the graphing calculator to display functions when teaching FTC. Of the three instructors who did not allow students to use their calculator on quizzes or tests, two of them expected students to use a graphing calculator either to calculate or visualize graphs. Only one, Suppiluliumas, did not express this expectation of his students. In general, what instructors found to be the purpose of the FTC for students seemed to relate only to whether or not they used graphing calculators to visualize functions when teaching the FTC.

Why do instructors use those resources

Instructors in this dissertation study used a variety of resources when planning and teaching the FTC. My analysis of the interview transcripts sought to identify statements that reflected a reason why they use those resources. I coded 428 passages[16] across the 14 interviews and identified five different reasons, Students, Self, Math, Institution, and Background. In talking about students as a reason, instructors demonstrated a concern for student well-being and understanding. For example, Nathan gave students an extra day to do their homework, "so if they don't know the first day, they can just ask me in class, and then do it the next day" (Nathan, 167-168). In talking about mathematics as a reason, instructors indicated a concern for upcoming mathematics or the profession of mathematics. For example, James wanted students to learn from wrong answers because,

> If you don't get it immediately, fine. Giving up is not okay… Going back to it, looking for similar things, filling holes in the background. I mean, this is how mathematics is done in the research level. It's how it should be done in the classroom. (James, 231-234)

In talking about institution, instructors indicated that the reason they used or did not use a certain resource in a particular way was due to the requirements or expectations of the institution. For example, Harold appreciates that the institution sets up Calculus 1 to be taught five days per week, so he can give a quiz "Every Friday, 'cause again the luxury of having five classes a

[16] Some passages were double coded. The total number of codes was 479.

week" (Harold, 361). When the reason for a passage was coded as self, instructors mentioned that they taught a certain way because it was easier for them or because they liked it better. For example, when I asked Ian why he taught the FTC differently than how his book does, he said, "I think that it makes more sense to me to approach it in a different order." (Ian, 296) Passages coded as self occasionally overlapped with background codes, which I defined as an instructor's experience teaching and learning. In Table 15, I present the frequency with which these reasons were discussed organized alphabetically by instructor.

Table 15: Frequency of reason code by instructor

	Reasons				
Instructor	Students	Math	Institution	Self	Background
Frederick	30	10	10	12	0
George	9	5	7	10	2
Harold	18	13	6	7	1
Ian	6	7	2	10	0
James	10	5	3	7	0
Karl	7	2	4	9	3
Leopold	11	8	8	2	2
Marcus	4	3	7	2	0
Nathan	10	8	1	4	1
Oliver	13	5	8	5	2
Philip	16	6	5	7	3
Richard	26	9	8	12	0
Suppilulumas	17	0	7	12	5
Theresa	17	4	0	6	0
Total	194	85	76	105	19

The most common reason instructors gave for using resources in the way they did had to do with concern for students (41%), followed by concerns for self (22%), followed by concerns for math (18%) and institution (16%). About 4% had to do with "background." With two exceptions (Ian and Marcus), instructors mentioned students as reasons more often than any other reason for using a resource. Overall, this confirms other sources (Grubb, 2002) that indicate that two-year instructors care about their students.

Observation Results

I observed two teachers, Philip and Karl. Philip taught calculus two days a week, in lessons of 110 minutes long. I observed him during week 12. Our interview was in August, prior to the term I observed. Karl's section was offered five days per week, in lessons that were 52 minutes long. I observed him on Thursday and Friday of week 13, and our interview was between the two observations. Philip and Karl were both observed two weeks before the end of their term. I had different purposes for observing these instructors. For Philip, my purposes were first to observe an instructor who was teaching from a new (to him) textbook, and second to triangulate what he did in the classroom with what he said that he did in the survey and interview. For Karl, my purpose was first to observe an instructor with extensive experience with the textbook. Because I was able to interview him immediately following my first observation, my secondary purpose was to use some of the observation data to guide portions of the interview. In what follows I describe (1) what transpired in the observations, (2) how they used resources; and (3) how I used the observation data for corroborating findings from the surveys and interviews.

A primary finding, which can be surprising given the literature on the mismatch between reported and observed practice documented in K-12 studies of teaching, is that that what instructors did during class aligned very well with what they described in their survey responses and in their interviews. Newfield (2003) and Koziol and Burns (1986) have documented that the accuracy of teachers self-reporting is high under certain conditions.

Philip. Philip's calculus class was a four credit class, taught two days per week in the morning, for 110 minutes. My observation was more than three months after our August interview, and by that time, he had changed textbooks from Hass, Weir, & Thomas (2012) to Stewart (2015).

Observation. I arrived in Philip's office a few minutes before the beginning of class, and we walked to the classroom. He brought donuts to class that day in part (he said) because my coming to visit was an excuse for him to bring snacks, and in part because students had just finished an exam in the testing center. His course was in a larger lecture hall that looked like an auditorium. Students sat in the first three rows, I sat one row behind them with at least three empty rows behind me. Before the official beginning of class Philip offered students donuts and mentioned that it was his mom's birthday. Students asked about Philip's wife, who was pregnant. One student mentioned that his parent was making a quilt for the new baby.

Class officially started at 8am, and after reminding students that their test should have been taken by the day before, Philip asked students to leave their books closed because he was going to prove things that day a bit differently than their text. He indicated that a different version of the proof would be extra credit, and that students would have to do one of those proofs on the final exam. His motivation for the FTC was the connection between derivative and integral, and that it was more convenient than taking the limit of a Riemann sum. He then asked students to remember the intermediate value theorem (IVT), and suggested they open their books for that to pages 122-123. I noticed he said the page number without looking it up, even though he was teaching from that text for the first time; this suggested that he had memorized the page for the theorem. He reminded students that they needed to make sure that the function was continuous in order to apply the IVT. He then reviewed the MVT, told students to look at page 291 for the MVT, and told them to consider what the MVT might say about anti-derivatives. From 8:10 to 8:24 he continued reviewing the MVT, the idea of continuous functions, and leading students toward the proof of what he called FTC I. Within those 14 minutes he included some meta-language about proving in general. He said that in order to prove things it was sometimes easier to prove a slightly related result and use that to prove the theorem, and that a popular technique for proving is called bounding. He also asked how many students were planning to take second semester calculus (about half raised their hands).

At 8:24 he wrote $F(x) = \int_a^x f(t)dt$ and drew a picture on the board (see Figure 3) to represent F(x). He asked students to think about why he used “t” and if “capital F” was a function. His verbal information was extensive, but he did not write down all of this on the board.

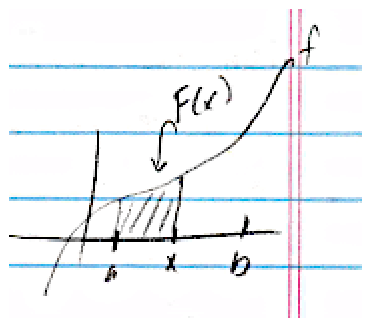


Figure 3: Philip's first picture to motivate FTC I

At 8:29 he wrote out the complete FTC I on the board (see Figure 4).

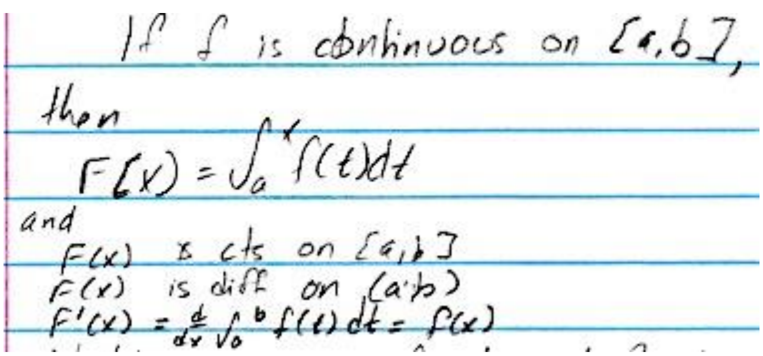


Figure 4: FTC I from Philip's observation

He then asked students to think about inverse functions and inverse processes. He discussed this for two minutes. At that point (8:31), he mentioned that the final exam was in 2.5 weeks and that they should make notes about what was coming on the final, such as the proof of the FTC. He discussed study techniques and recommended that students draw a picture of what is going on.

Philip then asked students how much of the homework on section 5.2 they have done (which dealt with evaluating definite integrals), and drew Figure 5 on the board.

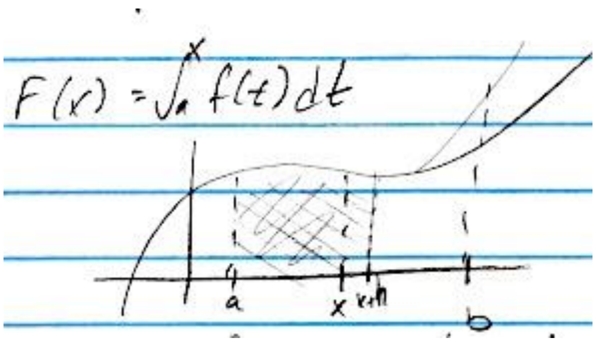


Figure 5: Philip's picture of FTC I

He asked students to combine their knowledge of "section 5.2" with the idea of taking a limit, and wrote the intermediate and mean value theorems on the left side the board. While proving the FTC, he said that if he was appealing to a concept in the left column (the IVT and MVT), he would use a red marker. The rest of the explanation and proof was done in black marker. He then worked through the proof of the FTC I by using the limit definition of a derivative (and Figure 5) to calculate the derivative of $F(x)$. He said, "What does it mean [for a function] to be differentiable? It has a derivative. And differentiable implies continuous." His explanation and proof of FTC I took 19 minutes, until 8:50am, when he began showing sample problems. At that point he wrote, "Suppose $g(x) = \int_0^x \sqrt{t + t^3}dt$" on the board. Then he wrote: $g'(x) = \sqrt{x + x^3}$ and called this the "punchline" of the theorem. He introduced a second problem, $h(x) \ = \int_3^x sin^2 tcostdt$. He asked students if the 3 matters (it doesn't, but he did not write down why not), and wrote: $h'(x) = sin^2 xcosx$. He said, "Isn't that nice? Seems simple enough. Let's throw a little bit of a monkey wrench."

For the next example he wrote $g(x) = \int_x^1 \ln(t + t^2)dt$. He referred students to section 5.2 in the book, and asked, "what's the problem?" He then wrote, $= -\int_1^x \ln(t + t^2)dt$. Immediately following that, he wrote $g'(x) = -\ln(x + x^2)$. He then introduced "one more kink" and said he would choose a problem from the book. After looking at his book, he put Figure 6 on the board:

$$h(x) = \int_1^{\sqrt{x}} \frac{z^2}{z^4+1}\,dz$$

Figure 6: Philip's first FTC problem from the textbook.

At this point students had been in class for 60 minutes, and he told students to take a quick break and think about that problem, "don't just randomly square things." The break lasted five minutes, during which some students got up and some stayed at their desks. One student asked him if the problem involved *u* substitution. At 9:05 he began solving the problem on the board using *u* substitution (see Figure 7).

9:05 Let $u = \sqrt{x} = x^{1/2}$

$$h'(x) = \frac{d}{dx}\int_1^{\sqrt{x}} \frac{z^2}{z^4+1}\,dz$$

$$\underbrace{\left(\frac{d}{du}\int_1^{u} \frac{z^2}{z^4+1}\right)}\frac{du}{dx}$$

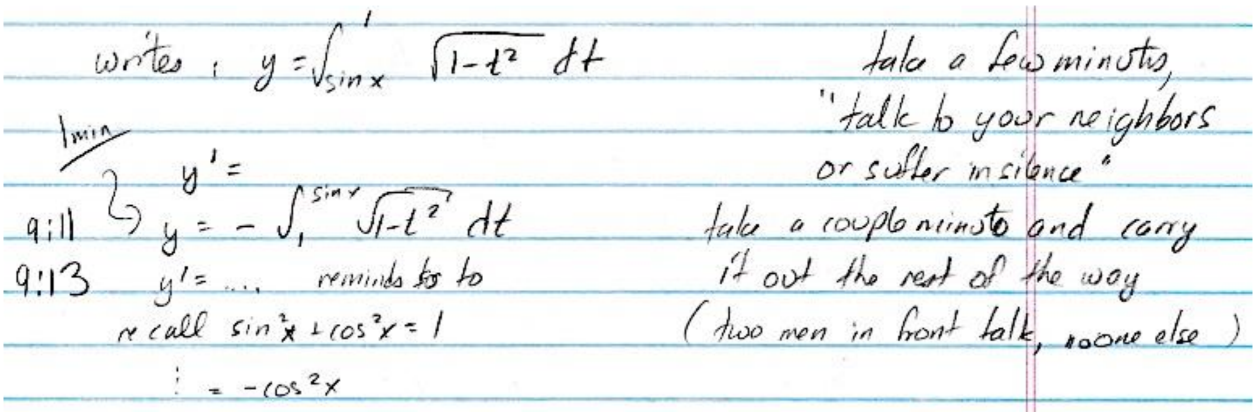
$$\frac{u^2}{u^4+1}\cdot\frac{1}{2\sqrt{x}}$$

$$= \frac{(\sqrt{x})^2}{(\sqrt{x})^4+1}\cdot\frac{1}{2\sqrt{x}} = \frac{x}{x^2+1}\cdot\frac{1}{2\sqrt{x}}$$ "if you want to leave it that way, that's fine."

$$= \frac{\sqrt{x}}{2(x^2+1)}$$ key is chain rule...

Figure 7: Philip's solution to first textbook problem, with comments.

After working through the fourth example, he looked in his textbook for an example to give students to try on their own. He put a problem on the board and gave students one minute to think about it before putting additional information on the board (see Figure 8).

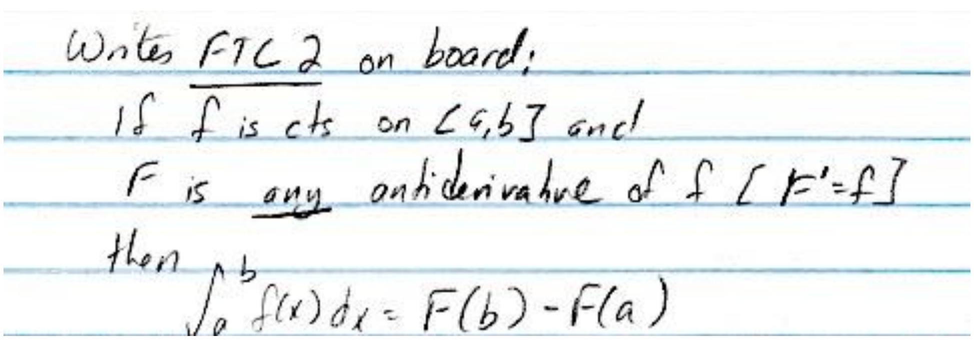


writes: $y=\int_{\sin x}^{1}\sqrt{1-t^2}\,dt$ take a few minutes,
1min "talk to your neighbors
$y'=$ or suffer in silence"
9:11 $y=-\int_{1}^{\sin x}\sqrt{1-t^2}\,dt$ take a couple minutes and carry
9:13 $y'=\ldots$ reminds to to it out the rest of the way
recall $\sin^2 x+\cos^2 x=1$ (two men in front talk, noone else)
$\vdots \; = -\cos^2 x$

Figure 8: Philip's first problem for students to try on their own, with comments and times

After one minute, Philip rewrote the problem to reverse the limits of integration, and then waited another two minutes before writing more on the board.

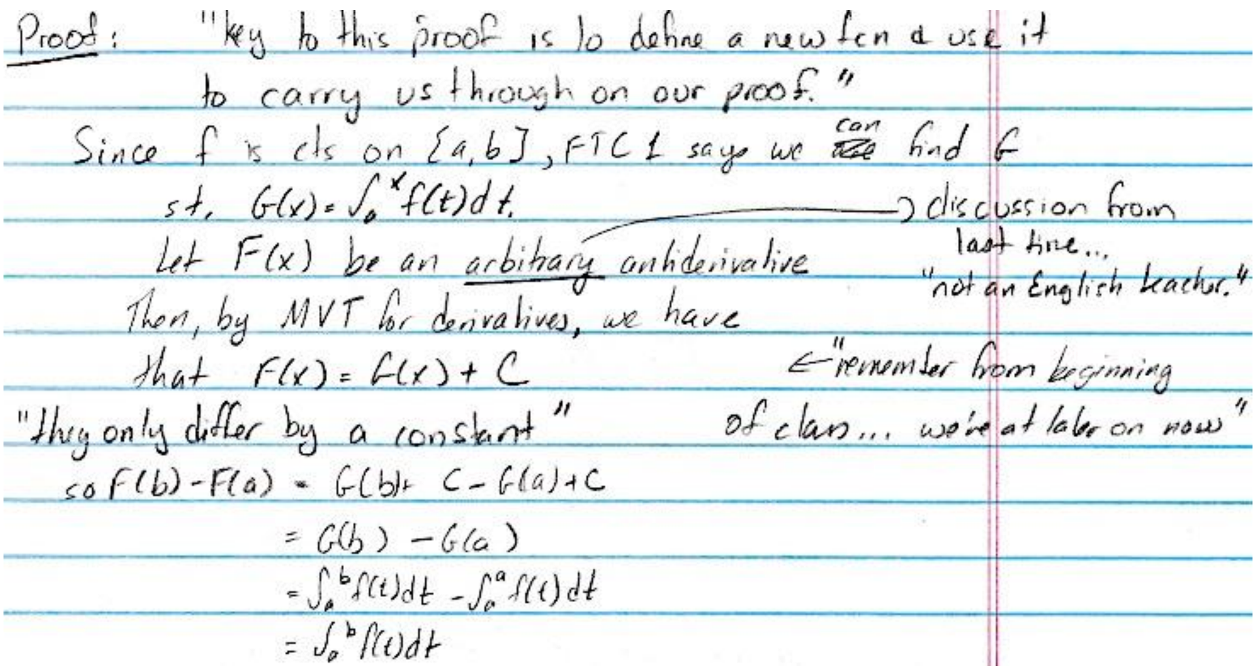
At 9:15, while students were finishing that problem, he wrote the second FTC on the board (see Figure 9).



Writes FTC 2 on board:
If f is cts on $[a,b]$ and
F is any antiderivative of f $[F'=f]$
then $\int_a^b f(x)\,dx = F(b)-F(a)$

Figure 9: FTC II from Philip's observation

He asked students to cover up their books, and told them that the proof for this one is "pretty easy" and that he would walk them through it. He then walked through the I-to-II version of the proof, with comments along the way to point out things that students should notice.



Proof: "key to this proof is to define a new fcn & use it to carry us through on our proof."

Since f is cts on [a,b], FTC I says we can find G s.t. $G(x) = \int_a^x f(t)\,dt$.

discussion from last time... "not an English teacher."

Let F(x) be an arbitary antiderivative

Then, by MVT for derivatives, we have that $F(x) = G(x) + C$

"remember from beginning of class... we're at later on now"

"they only differ by a constant"

so $F(b) - F(a) = G(b) + C - G(a) + C$

$= G(b) - G(a)$

$= \int_a^b f(t)\,dt - \int_a^a f(t)\,dt$

$= \int_a^b f(t)\,dt$

Figure 10: Phillip's FTC II proof, with comments.

Similar to his proof of FTC I, he discussed a general way of proving before beginning the proof. For the FTC I proof he mentioned proving a simpler theorem first and using that. For the FTC II proof he said that the key was to define a new function and use it. The proof of the FTC II took eight minutes, from 9:15 to 9:23. After finishing the proof, he asked students to go back to their notes and give him an example of a definite integral that they solved previously. A student gave him an example, and he reminded them that the problem had originally taken up the whole board. He solved the definite integral using FTC II, and pointed out how much faster it was than Riemann sums. He then returned to the textbook and asked students to pick a question. They picked #42, which he then put on the board. He solved the problem on the board, while making references to the linear properties of the definite integral (by pulling out a constant) and the unit circle. At that point, a student asked him about when it would be appropriate to use the calculator, which could compute an answer for definite integrals. He stated that a calculator answer only on the next exam would not be worth any points, and gave general calculator advice.

At 9:40, a student asked about problem #30 in the textbook, and Philip spent six minutes solving that problem. While solving that problem, he used his calculator to check his arithmetic. After solving that problem, Philip discussed the upcoming test with students. He told them the types of problems that would be on the test and put a sample True/False test problem on the board (see Figure 11)

Writes: "If this were a test question:

5a) $\int (f(x) \cdot g(x))\, dx = \int f(x)\, dx \int g(x)\, dx$

Figure 11: Philip's sample test problem

He said that in order to to prove that the statement was false they should use a counterexample; he had a mistake in notation that a student pointed out. Philip thanked her and fixed the mistake, and finished class by telling students that they could get extra credit by coming to see him during office hours between then and the next test, and describing another proof of the FTC. He told students that they would have to know the other proof well enough to discuss it with him. At 9:55 the class ended and a student reminded Philip to call his mom to wish her a happy birthday.

Resources. There were three main resources that I observed Philip using during class. First, he used the textbook as both a bank of problems and as a resource for students. Not only did four of his example problems during class came from the FTC section of the textbook (bank of problems), he also mentioned sections and page numbers for previous theorems and previous work that students had done. Second, he used the graphing calculator to verify computations that he had done by hand, but not to display graphs of functions. He also discussed appropriate uses of the graphing calculator. Third, Philip used student feedback during class time by paying attention to students. When students asked him questions, he responded. When a student caught one of his mistakes, he fixed it and thanked her for noticing. Other resources that Philip used include colored markers, a whiteboard, and a document projector.

Corroboration. On the survey, Philip had indicated that he always used the textbook for example problems during class and that he used a graphing calculator during teaching and planning. Thus, Philip's survey responses matched what I observed in class, including the colored pens, which Philip described during our interview:

> I do use different colored pens. Particularly since I don't go heavy on the proofs a lot of times I'll do proof by picture. And so a lot of my proof by picture I'll use multiple colors to kind of illustrate the point. (Philip, 560-562).

Also during our interview, Philip discussed the importance of the proof of the FTC in showing that something is differentiable by proving it has a derivative. He was clear that he wanted students to understand that idea:

> The fundamental theorem is one of the explicit times where you are explicitly showing that something is differentiable by proving it has a derivative. And I don't think that students see that a whole lot, so that's kind of a neat portion of how this book handles it. Anyways, I like that they are proving that something is differentiable, and I point that out to them (Philip, 799-804).

Because the idea that proving differentiability can be shown by proving that a function has a derivative was so important to Philip, he said he was explicit about that during his instruction. This is what happened. He introduced the idea of the theorem by appealing to differentiability, exactly as he had described in the interview. Overall, Philip's enactment of his FTC lesson was consistent with what was anticipated by the responses he gave in the survey and the interview.

Karl.

Karl's calculus class was a five credit class, taught five days per week in the morning, for 52 minutes. Karl's observation and interview were much closer together than Philip's. I observed Karl over the course of two days, and I interviewed him between these observations.

Observation day 1. I arrived in Karl's office a few minutes before the beginning of class, and we walked into the classroom arriving four minutes before the official start of class. At the official start time, Karl opened the door and students came in. He closed the door one minute later. The classroom was set up with long desks, and there was a chalkboard on three sides. Windows looking onto the courtyard were on the fourth side. Students spaced themselves out, with the four female students sitting in the front right corner. I sat in the back left corner.

At the start of class, the textbook was on the overhead, displaying the problem set. He shut the projector off and referred to a piece of paper with his notes (see Appendix I). Karl began by writing the "Section 5.3 Fundamental Theorem of Calculus (p. 354)" on the board (see Figure 12). He drew a linear function on the board, and labeled it $f(x) = x + 3$.

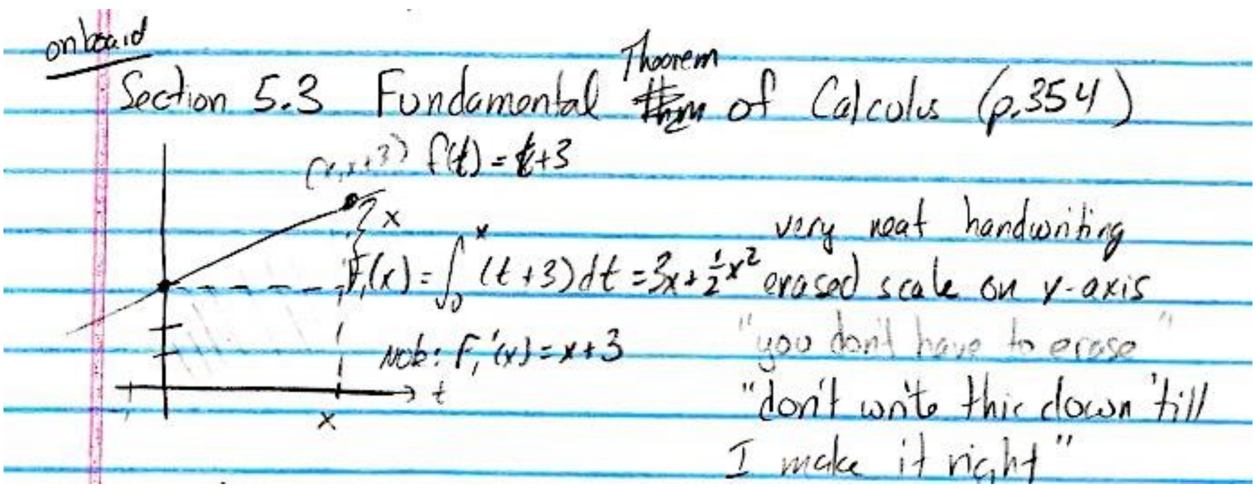


Figure 12: Karl's first picture to motivate the FTC

He then drew the graph and asked students to find the area under the graph from zero to *x*. He then deliberately replaced the *x* in the initial function and rewrote it as $f(t) = t + 3$, pointing out to students that they shouldn't write things down until he made it right.

Next, he wrote the $F(x) = \int_0^x (t+3)dt$ on the board and defined that function as the area between the line defined by $f(t) = t + 3$ and the *t*-axis. He calculated that area using the area of a square and a rectangle, both formulas which were known to students. He then pointed out to students the derivative of that area function.

After calculating the area manually, Karl changed the limits of integration, called the initial function $F_1(x)$ and wrote $F_2(x) = \int_{-1}^{x}(t+3)dt$. He had students calculate the area indicated by that function manually (sum of two triangles), and then find the derivative of that area function. He then asked, “If I change the limits of integration, does the derivative change?” And pointed out to students that the area function was, “in some sense,” an anti-derivative of $x + 3$. At that point, he had students to consider the relationship between the derivative of an integral function and the integrand. He was careful not to state the relationship was only assumed, and not necessarily true (see Figure 13).

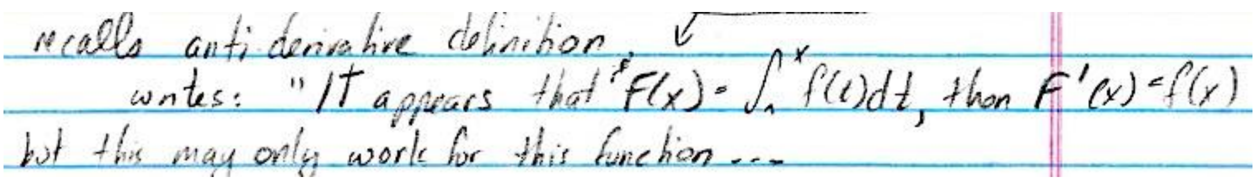


Figure 13:Karl's connection between derivative and integral

Following that introduction, he draws a generic continuous function on the board, and considers the area between that function and the *x*-axis (see Figure 14).

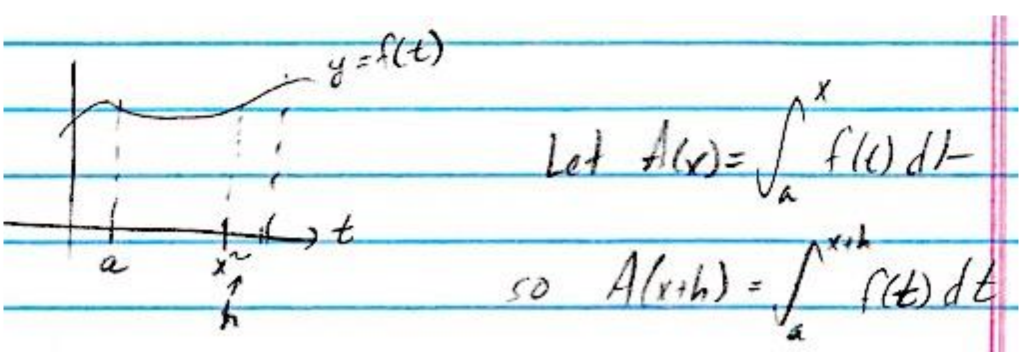


Figure 14: Karl's introduction to FTC I proof

He says, “could we agree that, if h is small enough, then $A(x+h) - A(x) = hf(x)$? Because I would like to divide by h.” He then paused and asked if there were any questions. A female student asked why he wanted to divide by h, and Karl explained that if he could divide by h, he would have $\frac{A(x+h)-A(x)}{h} \approx f(x)$. He wrote this on the board, and had students consider the limit of both sides as h goes to zero, to “get rid of the wavy equals.” After writing that, he asked students, “What does it take to be a derivative?” and helped them recall the limit definition of a derivative. Karl then put Figure 15 on the board, with a box around it, saying that he did not want to use prime notation.

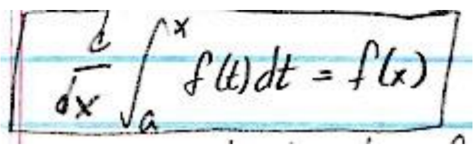


Figure 15: Not using “prime” notation

He pointed at his original picture of $f(t) = t + 3$ (Figure 12) and then his second picture (Figure 14) and explained out that both functions were continuous, connecting that idea to what students had done earlier with the mean value theorem and needing functions to be continuous. This process of introducing the area function, calculating areas, and leading up to the FTC took 26 minutes. At 9:34 he wrote FTC (part I) on the board (see Figure 16).

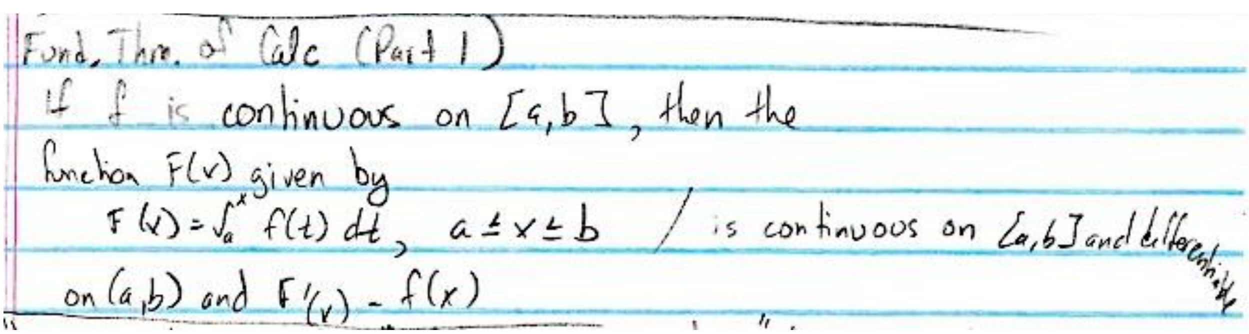


Figure 16: Karl's board for FTC I

Then he told students, “it’s the thing in the little box over there [Figure 13] that’s used the most. He then gave students three problems to try (see Figure 17).

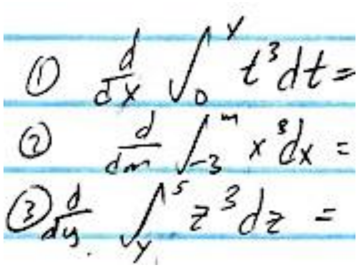


Figure 17: Karl's first three FTC I problems

He wrote the answers to the first two problems on the board, and tied the third one back to a prior student question about opposites. This took five minutes. At 9:39, he wrote another function on the board, $y(x) = \int_0^{x^3} t\,dt$ on the board and drew a picture of the triangle formed by that function (see Figure 18).

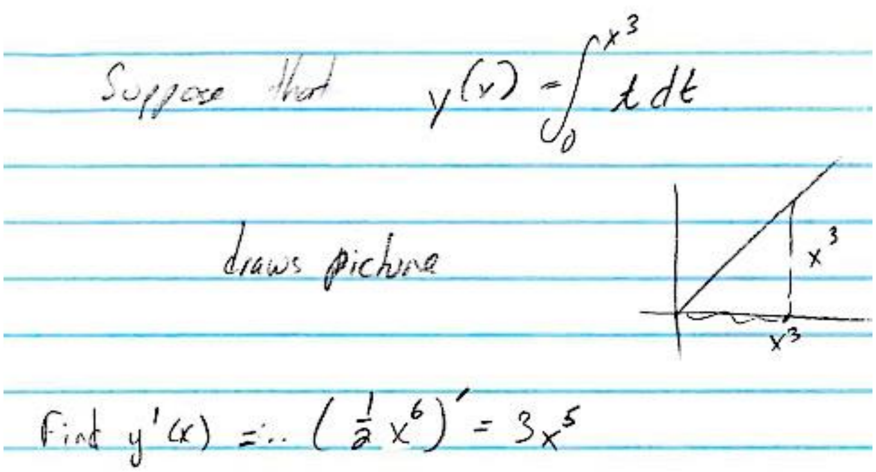


Figure 18: Karl's introduction to chain rule for FTC I

He asked students to find the derivative of $y(x)$ by finding the area of the triangle in terms of x and then taking the derivative of the area function. He cautioned students not to just “plop” in an integrand for the derivative. He had students consider $\frac{d}{dx}\int_0^{x^3} t\,dt$, and solved this using the chain rule, pointing out that they were looking for the derivative with regard to x, not x^3. One student commented "that's easier," to which Karl responded, "yes, if I use the chain rule."

At 9:45, he began instruction on the second FTC. He wrote "Find $\int_a^b x\,dx$ on the board, and drew a picture (see Figure 19) to motivate the second FTC.

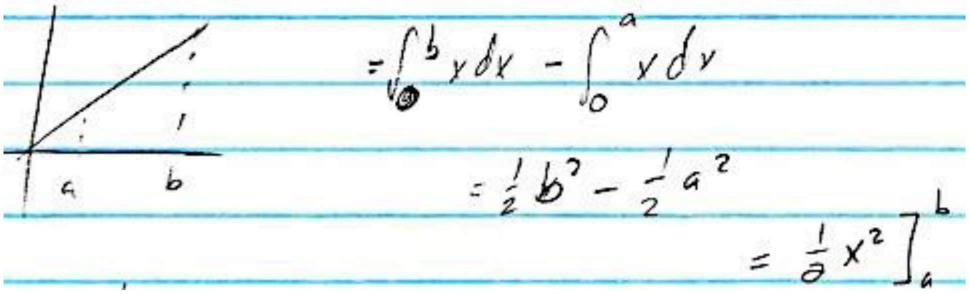


Figure 19: Karl's beginning of FTC II

He then solved the problem using the area of a trapezoid, as shows in Figure 19) and then asked students to give him the "easiest" anti-derivative for *x*, while reminding students that there were a lot of anti-derivatives for *x*. Students offered $\frac{1}{2}x^2$. He then said "you might wonder if I cooked this up so that it works" and drew a more generic function, $f(t)$ on the board.

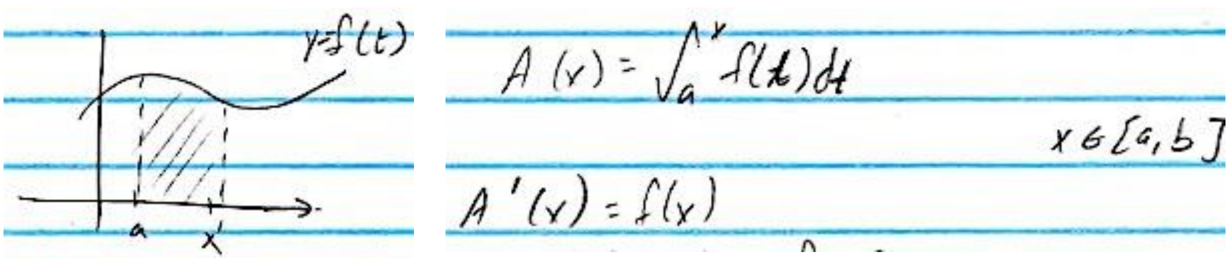


Figure 20: Karl's beginning of FTC II explanation

He then defined an area function for $f(t)$ using the definite integral, pointing out that the previous theorem meant that the derivative of the area function was easy to identify as $f(x)$. He reminded students that the area function was an anti-derivative of f on the interval $[a, b]$ and that they could get another anti-derivative by adding a constant, $F(x) = A(x) + C$. He then wrote a very basic argument for the second FTC on the board, as Figure 21.

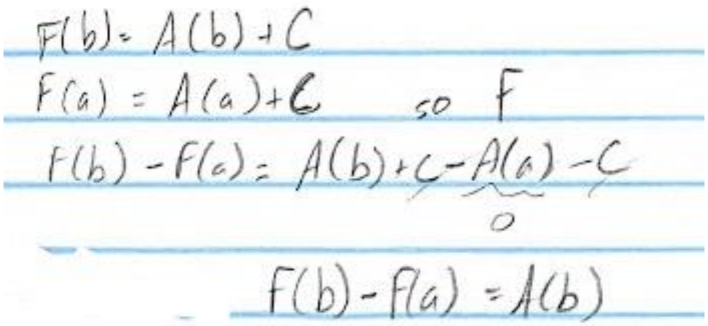


Figure 21: Karl's justification for FTC II

While writing on the board, Karl verbally pointed out that $A(a)$ equaled zero. The process of finding the area of the trapezoid and leading up to FTC II took 13 minutes. At 9:58, he acknowledged that he was running out of time, wrote the second fundamental theorem of calculus on the board (see Figure 22), and released students from the class.

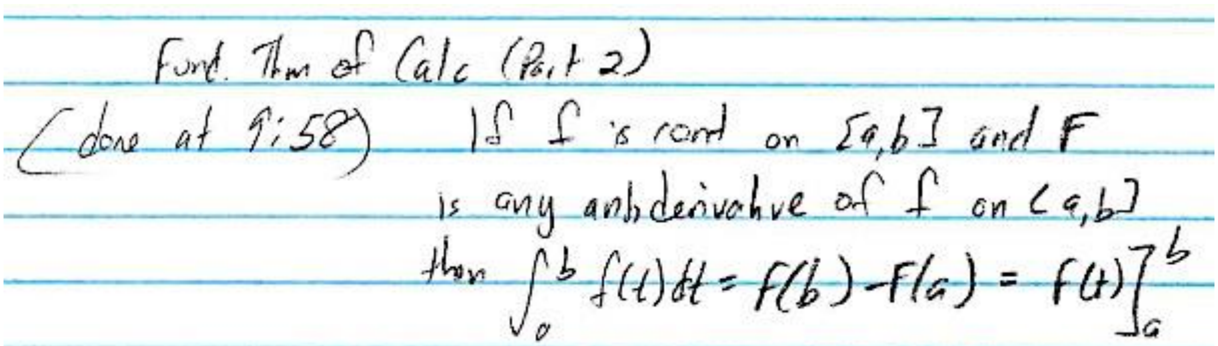


Figure 22: Karl's FTC II

After students left the classroom, he took pictures of the board and left.

Observation day 2. Similar to the first day, Karl displayed the textbook book or problems on the overhead as students were walking in. To begin this class, he wrote “5.3, continued” on the board. He solved two area problems on the board. A student asked a question about the second one (see Figure 23), and he drew a picture to demonstrate the area.

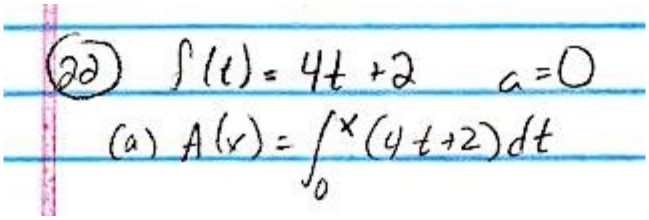


Figure 23: Second area problem from Karl, day 2

After calculating the area from the picture, he pointed out that he hadn't used the FTC at all in that problem, and solved it a second way using the FTC. Students were more vocal on this day than the previous day, and asked questions about computations as Karl worked through these problems. In this lesson, Karl demonstrated a total of 12 problems, all of which were from the textbook that was being projected on the overhead. As he went through the problems, Karl pointed out prior mathematical topics that students should recall (e.g., properties of logs, integrals, and trig functions). For most of definite integral problems 3 to 12, Karl set up the anti-derivative but did not compute a final answer. For the 7th problem, a student asked a question about inverse sine that prompted Karl to get out his graphing calculator and use the overhead to show the graph of the function to the class.

On the next problem, Karl solved the problem on the board, and his solution had a common mistake (that he seemed to notice). He asked students "Am I OK here?" and waited for them to notice that a minus sign should be a plus sign. For problem 10, Karl drew a graph of $y = 6\cos x$ on the board and spoke about net area compared to actual area (see Figure 24).

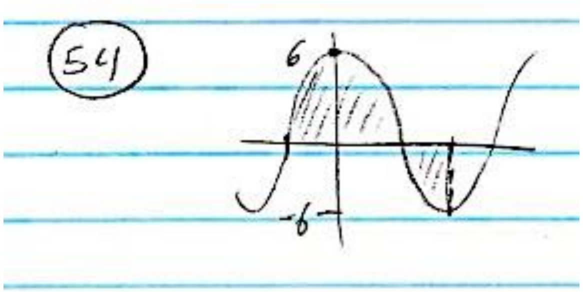


Figure 24: Karl's example to demonstrate net area

Karl spent time with problem 10, discussing why the definite integral and the area were not quite the same thing, and defining net area compared with actual area. He also used this problem to give an example of different ways to set up the definite integral. He set up four more problems, leaving the last problem set up for students as a "suggested problem."

Resources.

There were four main resources that I observed Karl using during class: his notes, the textbook, a graphing calculator, and student feedback. During the first day, he did not use his textbook or a graphing calculator, but relied on his notes and student feedback. Karl watched his students while he was teaching, often pausing to look at them before moving on, and sometimes waiting for a verbal response. On the second day, Karl used textbook as a bank of problems, and the graphing calculator for displaying a function and for computing. He also used student feedback, similar to the first day. Other resources that Karl used were chalk, the chalkboard, and a document projector.

Corroboration.

On the survey, I asked participants if there was anything else that they felt was relevant to teaching the FTC. Karl wrote, "The more that you can appeal to student knowledge of common geometric area formulas and other usual student knowledge, the better." I saw this in how Karl approached both parts of the FTC. He drew linear functions that allowed for quick calculations of the area between the line and the x-axis, based on triangles, squares, and trapezoids. I also asked participants on the survey to write anything else about their background that they felt was relevant to their teaching of the FTC (Question 33). Karl wrote about how he tries to support students in his class:

> I taught high school for 6 years and watched students struggle with algebra. I see some of those same struggles among my calculus students. While some of them should never have had a respectable grade in an algebra class recorded on any transcript for them, the fact remains that they are now in my class, and I try to support them as much as is practical. When I _know_ that something is going to cause trouble for students weak in algebra, I point that out explicitly and try to address common errors. (Karl, survey)

I saw this attention to students during my observations of him when he answered every student question, and waited for students to find a common error that he made in a calculation. He addressed an area that he knew was going to cause trouble in the beginning of the first day, although I didn't realize it until I interviewed him.

During each interview, I asked instructors how they chose the examples they used during the lesson. For Karl, I was able to ask about specific examples. I asked him about his first example, when he changed $f(x) = x + 3$ to $f(t) = t + 3$. He said the change from *x* to *t* was deliberate: "I know I wanted to hit the idea of why we use the *t* versus the *x*. For the limit of integration it has to be *x*. It's going to be *x*, so we need some other variable" (Karl, 190-191). In other words, he deliberately introduced the function with *x*, and then erased it and changed it to *t* as an indication to students that the limit of integration is not the same as the variable in the integrand.

I asked Karl about the first three examples that he put on the board (Figure 17) and he explained why he chose them in that order: "That's actually why I started with zero [in the first example] and then I backed up to negative one. Because I want them to see that the initial point doesn't matter" (Karl, 123-124). He was able to tell me that those three examples were chosen in order to help students become aware that the initial point of integration doesn't matter for finding a derivative.

I also asked Karl about an interaction with a student that said something about "opposites." He explained:

> Yeah. And she didn't mean opposites. And I was trying to correct her. She meant inverses there. But it's a theoretical idea. She does get it, if you are careful with the language. 'Okay, no. Three is the opposite of negative three. There's where we use opposites. But differentiation and integration are not… we don't want to say they're opposites as much as they're inverses'. So I was trying to correct her there and she'll be okay with that. (Karl, 261-265)

During my observation of this interaction, Karl did not directly correct the student, and never told her that she was wrong or that her language was inaccurate. However, during class he referred to this interaction when talking about his third example, and pointed out that switching the limits of integration yielded the *opposite* (he emphasized this) value. This exchange indicated that Karl paid attention to students during class and that he knew his students. He cared for what they were learning and understood common errors. He used non-verbal cues from students during his teaching as well, which I observed and he expanded on, saying: "They're at a point now where nobody who's left is sitting there smiling and nodding and not understanding" (Karl, 525-526). Karl watched his students during his teaching, often pausing to look at them before moving on, and sometimes waiting for a verbal response.

Given that the interview and one of the observations happened on the same day, the fact that what Karl said he did and what he actually did were closely aligned may not be surprising. The advantage to interviewing Karl immediately after an observation was not corroboration of what he said and what he did, but rather that the timing of the interview allowed me to ask him details about what I saw during class. Karl's observation offered insight into why he used particular resources and why he chose particular examples to highlight. Overall, the two observations confirmed the resources that instructors used and confirmed their care for their students.

Summary

All instructors used their textbooks to some degree and were familiar with its notation. Other resources that instructors used included graphing technologies such as calculators, Maple, Wolfram Alpha, and Desmos. They used these technologies to display functions and to perform complicated calculations. The ways that instructors described the FTC related only to whether or not they used their calculator to display functions when teaching the FTC. The intangible resources that instructors used included their background, requirements of their institution, and cues from their students. Instructors described the way in which those resources impacted their teaching. The reasons that instructors gave for using the resources in the way that they did was primarily out of concern for students. I offer some interpretations of these findings in the next chapter.

Chapter 5: Discussion

This mixed methods study examined first semester calculus instructors at Michigan Two-Year colleges. I investigated what resources instructors use, how they use them, and why they use them. In particular, I focused on the fundamental theorem of calculus (FTC) and the resources that are used or not used when teaching and planning this important theorem. In order to answer these questions, I sent a survey to all 136 Michigan two-year college instructors who taught calculus in the 2015-2016 school year. Of the 50 instructors who participated in the survey, I interviewed 14 of them in depth. Of those 14, I observed two instructors while they were teaching the FTC. I begin this chapter with a review of my research questions and a summary of the findings from all three data sources that answer those questions. I then explore interpretations for those findings.

Research Questions

1) What resources do two-year college calculus instructors use to assist in their planning and teaching lessons in FTC?

When teaching FTC, instructors primarily used the blackboard or whiteboard and graphing technology. A slight majority of instructors surveyed preferred to teach FTC without added technologies. Instructors said they primarily used their textbook to assign homework (planning), although two interviewed instructors created their own homework specifically for the FTC. Instructors said they also relied on their background and experience when planning how to present the FTC to their students. Interactions with students guided some of the teaching of the FTC.

2) How do two-year college calculus instructors use their resources to assist in their planning and teaching lessons in FTC?

The textbook was used primarily during planning the overall course and in order to assign homework. Instructors paid attention to the notation in their textbook, and noted when their notation differed. Instructors' background and experience did not seem to influence the types of resources they used, but did influence how they used them. The FTC was primarily presented with no more sophisticated technology than a graphing calculator and PowerPoint slides.

3) Why do two-year college calculus instructors use their resources in the ways that they do when planning and teaching lessons in the FTC?

The most common reason (41%) instructors gave for using (or not using) their resources in a variety of ways had to do with concern for students. The second most common reason (24%) instructors gave for using or not using resources was because of personal preferences. The third and fourth most common reasons for resource use were a concern for accurate mathematics (18%) followed by requirements of the institution (16%).

This information about what, how, and why, while important, fell short in describing and explaining how instructors used of resources when teaching the FTC. While investigating the first three research questions, a pattern emerged about how instructors discussed their conceptions of the FTC which led to a fourth research question:

4) How do these teachers discuss the FTC? What are their conceptions of this theorem?

Most instructors in this study discussed the FTC in ways that matched the two parts of the theorem, each being considered a theorem on its own. To these instructors, the FTC is either a theorem about the inverse relationship between integration and differentiation, or it is a theorem that allows the calculation of a definite integral by taking an anti-derivative, or it is both of those things, but not at the same time. Rarely did instructors connect these two theorems in any conceptual way.

The findings from this dissertation connect and expand on scholarship on calculus, two-year colleges, instructors' use of resources, and the FTC by investigating the resources used by two-year college instructors when they plan and teach this important theorem. I investigated what resources are used, how they are used, and why they are used in that way. Examining the resources used by two-year college instructors to plan and teach the FTC gave me an in-depth look at an important topic in a mathematics course that many students do not successfully complete. In this chapter I examine the findings through the lens of instrumental genesis.

Instrumental Genesis

In this study, I did not look at resources in isolation. I considered how resources were used and why instructors used their resources in the way that they did, in particular when they taught the FTC. In the course of answering questions about how and why they used various resources, instructors indicated various ways that resources shaped their teaching. This dual understanding of how instructors use resources and how resources affect instructors is articulated by the conceptual framework of instrumental genesis (Gueudet & Trouche 2009; Rabardel & Bourmaud, 2003). In the instrumental genesis framework, what I call a resource is called an artifact, and the combination of the resource (artifact) and its scheme of utilization is considered a document:

> Instrumental geneses have a dual nature. On one hand, the subject guides the way the artifact is used and, in a sense, shapes the artifact: this process is called instrumentalization. On the other hand, the affordances and constraints of the artifact influence the subject's activity: this process is called instrumentation. (Gueudet & Trouche, 2009, p. 204)

In other words, the process of instructors using their resources is instrumentalization, and the process by which the resources influence the instructor is called instrumentation. One important component in this framework is time. The use of resources and their effect on planning and instruction is not static. Resource use may moderate instruction, instruction in turn may influence resource use, which may change the instruction yet again. For example, the background and experience of an instructor may influence how a lesson is taught, which in turn contributes to that instructor's background, which then may influence the instruction of lessons differently. Instructors in this study used their resources in a variety of ways, and the use of those resources impacted their teaching.

Instrumental genesis has its roots in cultural historical activity theory (CHAT). CHAT draws on work from Vygotsky and Leont'ev, and considers both an activity and the cultural historical background of both humans and objects (Engeström, Miettinen, & Punamäki,1999; Engeström, 2001) over time. In the case of this dissertation, the activities being investigated are the planning and teaching of the FTC.

Using the lens of instrumental genesis requires that resources and their use be considered together, over time. In the next section, I discuss seven combinations of resources and their uses that came from the findings of this dissertation; 1) textbook and planning the semester, 2)textbook and planning a lesson, 3) textbook and notation, 4) textbook and assigning homework, 5) background and planning, 6) student feedback and teaching, and 7) graphing technologies and teaching. Each of these combinations reflects the instrumentalization of these resources; how they were used by instructors. For each combination I point out the possible instrumentation process that may occur from using the resource in that particular way, particularly with regard to the planning and teaching the FTC.

Textbook and planning the semester

Instructors indicated that the textbook and the semester schedule were closely entwined. All interviewed instructors said that they spend no more than two class hours (one or two days) on teaching the FTC, and all but one indicated that they use their textbook in some way to plan the semester. I examined the table of contents for every textbook that participants said they used to teach the FTC and noticed that almost all of them have one section of one chapter devoted to the FTC.[17] In addition, all the textbooks used by the participants presented the FTC after differentiation was completed and within the first chapter of integration.

Prior research indicates that students have difficulty understanding the FTC (Bajracharya, & J. Thompson, 2014; Schnepp & Nemirovsky, 2001; P. Thompson, 1994), and that one way to improve student understanding of the FTC is to introduce the topic early in the course (P. Thompson, Byerley, & Hatfield, 2013), with the ideas of rate of change (differentiation) and accumulation (integration) introduced together. All instructors in this study indicated some level of comfort with changing the order of topics in the textbook, yet none of them indicated that they introduced integration and differentiation together. If all the textbooks available to instructors devote only a small portion of their text to the FTC, and none of them introduce the concepts of accumulation and rate of change together, this may influence how instructors introduce these topics. Thus, the textbook may be a resource that inhibits a richer teaching of the FTC.

Textbook and planning a lesson

Nine of the 14 instructors interviewed explained that they read the textbook before planning a lesson. The textbooks that these instructors used generally presented the FTC in two distinct parts, which matches how instructors talked about the FTC. When instructors discussed

[17] One instructor used a textbook by Hughes and Hallett (2012), which was the only textbook to have the FTC in two sections.

the FTC with me, they considered the two theorems to be about the inverse roles of integration and differentiation, or a way to evaluate a definite integral, or both, but only Ian linked the ideas present in both theorems when discussing what he wanted students to understand

> I think the main thing I want them to walk away with is just that there is a fundamental relationship between the instantaneous rate of change of a quantity and the area under a curve. Not of that quantity, but of its derivative. They might forget that. They might forget that little detail two years from now or whatever, that it's not the area under the curve and the rate of change of the same curve. They'll probably forget that. But I want them to remember that there is a fundamental relationship between instantaneous rate of change and accumulation. (Ian, 682-687)

In other words, Ian was very clear that the FTC was more than the area under a curve, more than evaluating a definite integral, and more than the inverse relationship between integration and differentiation.

Instructors did not say that they linked their conceptions of the FTC with the way that the theorems were presented in the textbook; it is more likely that the textbook authors present the theorems that way because the FTC is commonly understood to be two theorems. However, textbooks and instructors are both presenting the FTC as two separate theorems that may not be conceptually linked. In my two observations, the link between the first and second FTC was in one small portion of the proof of the second FTC. The conceptual link between the theorems was not developed, and it was not in the textbooks that the instructors used, either.

The textbook was a common resource that instructors used to plan a lesson, and this is part of the instrumentalization of the textbook. However, the instrumentation process, in this case, is unclear. The textbook was not the only resource used for planning a lesson, and not even the primary resource in some cases. Instructors also relied on their background knowledge, experience teaching, worksheets and notes from past years, etc. Marcus in particular avoided his textbook when teaching the FTC. Thus, the influence of the textbook for planning an individual lesson is unclear and may vary by content.

Textbook and assigning homework

All instructors used their textbooks in some way as a bank of homework problems. Some instructors used the on-line homework that is linked to their textbook, some required paper and pencil homework from the textbook, and some did both. Although instructors indicated that they chose homework problems carefully, using the textbook as a bank of problems may impact the timing of when the FTC is taught, due to the structure of mathematics textbooks.

There is a perception that mathematics builds on prior knowledge and that it is organized hierarchically. Thus, materials in later chapters may draw on material from earlier chapters. All the textbooks that instructors used introduced rates of change and derivatives before introducing accumulation and integration. There may not be a good way to choose homework from textbooks that reflect both rate of change and accumulation, at a level that students can understand without significant prior calculus background. In other words, if derivatives are covered in chapters two and three, and the FTC is introduced in chapter four, instructors may have difficulty finding appropriate homework problems within the textbook for a concept from chapter four that don't rely on the concepts of chapters two and three.

However, finding appropriate homework did not discourage instructors from rearranging other textbook sections when they chose. Most of the survey participants and all of the interviewed instructors felt comfortable changing the order of topics in the textbook. Yet despite this comfort, changes to the textbook order generally involved putting one section later or (less often) pulling one section forward (see Appendix J for an example of how Leopold rearranged some of his textbook sections). However, none of the instructors who rearranged their textbook sections introduced the idea of integration and differentiation together, and none of them introduced the FTC toward the beginning of class, as Bressoud (2011) and P. Thompson (1994a) suggest.

An instructor choosing to select homework from textbooks is another instrumentalization process embedded in using the textbook for planning a semester and using the textbook to plan a lesson. However, the instrumentation process involved in using the textbook to select homework is not as clear. It is possible that using the textbook as a bank of problems affects the timing of when the FTC is taught, but there was no evidence for or against such possibility in the findings. None of the instructors in this study indicated that they taught the FTC in the first few weeks of the semester, rather than in the last few weeks when it is usually taught. It is possible that a view of the textbook as a source for homework discouraged instructors from such a change because problems in the homework sections typically build on previous sections; but it also possible that instructors simply did not consider the possibility of teaching the FTC towards the beginning of the semester.

Textbook and notation

Instructors in this study indicated that following the symbolic notation in the textbooks was important to them. Mathematical notation can pose particular challenges for students (MacGregor & Stacey, 1997, Rubenstein & D. Thompson, 2001), and the level of complexity of notation in calculus is high. For example, one of the FTC theorems refers to the derivative of an integral function, which is expressed with two different notations:

$$\frac{d}{dx}\left(\int_a^x f(t)dt\right) \tag{1}$$

$$F'(x) \text{ when } F(x) = \int_a^x f(t)dt \tag{2}$$

These two notations offer different advantages, although their meaning is the same. Expression (1) can be used to emphasize that the derivative is to be taken with respect to *x* (rather than *t*, for example). Expression (2) can be used to emphasize that a definite integral is a function, which can be conceptually difficult for students. The textbooks that instructors use may use either (or both) of these notations. Instructors in this study found it important to know what the textbook notation was, independently from what they used during instruction. When instructors diverged from the textbooks notation, they signaled the change to students, alerting them of the difference and asking them to be cautious (e.g., Harold). Instructors wanted to be familiar enough with the notation in order to explain to students when and if their notation was different. This is one way that the instrumentation of the textbook could be seen. Instructors indicated that they paid attention to what the textbook was doing for the sake of their students, and because of this attention, the notation within the textbook had an impact on their planning and teaching.

Background and planning

In this dissertation I examined the resource of instructor background in terms of their experiences teaching and learning. The intangible resource of instructor background is very different from a tangible resource such as the textbook. The textbook mediates instruction, and instruction mediates the use of the textbook, but the textbook itself remains unchanged. While the use of a textbook may change from day to day, the textbook itself does not change. However, the background of instructors is dynamic. Any attempt at understanding an instructors' background as a resource must necessarily come with the understanding that background is not a static resource and only represents a snapshot in the background of instructors.

Instructors' past experiences teaching and learning mathematics contributed to how they planned a lesson. On one hand, an instructor's past negative experiences could be offered as a reason for not doing something, (e.g., Karl does not require students to learn the FTC proofs), whereas on the other hand a past experience might have suggested a negative impact of a practice (e.g., Philip's past experience led him to avoid technologies when possible). Experiences as a student are, for these instructors, in the past; it is the memories of these experiences that shape planning.

In this study instructors' teaching experiences were more dynamic resources than their experiences as students. Based on prior semesters teaching, instructors planned things like how and which examples to present during class, from the problems that are selected to be solved to the notation that is used. In this way, the instrumentation is visible in the way that experience shapes teaching. The instrumentalization is clear here as well. Without prior experience, instructors can't learn that a certain sequence or a specific notation (e.g., $f(t)$ instead of $f(x)$ will work . The act of teaching contributes to the resource of experience. The resource of instructor background shows the cyclical nature of the instrumentation and instrumentalization process. Experience contributes to planning and teaching, which in turn contributes to experience.

Student feedback and teaching a lesson

Students impacted teaching and were, presumably, impacted by teaching. While student feedback as a resource may be incorporated into an instructor's background experiences, I chose to classify it as a separate resource, because it is not solely dependent on the instructor. Instructors respond to questions from students as well as looks of confusion, and adapt their teaching accordingly. The instrumentation of student feedback is immediately apparent when observing classes. Instructors paid attention to students, noticed if they looked confused, and called them by name. Instructors answered all questions from students, and gave them opportunities to ask questions. Instructors changed the course of their plans based on a student's confusion in the first half of the lesson.

Student feedback clearly impacts instruction, yet the instrumentalization of student feedback is more subtle. Although student feedback is a resource that depends on students, the activity of teaching influences student feedback. Students are more likely to ask questions when an instructor gives them time and space to ask questions. Instructors can elicit student feedback by calling on them by name as the instructors in this study did. They can also suppress student feedback by ignoring student questions or not giving appropriate "wait time" for responses to questions directed toward students (Larson & Lovelace, 2013; Wagner & Herbel-Eisenmann, 2008). Similar to instructor background, student feedback is a cyclical resource. Student feedback can immediately impact instruction (instrumentation), and the instructors response to feedback can, in turn, influence student feedback (instrumentalization). Student feedback is an example of a dynamic resource that illustrates the dual nature of resource use.

Graphing technologies and teaching the FTC

Unlike textbooks, not all instructors used graphing technologies in order to teach first semester calculus. Within this data set, about half of the instructors used graphing technologies, and the way that they used them varied. As a material resource, graphing technologies are significantly newer than textbooks, and there can be a learning curve where the focus is more on the resource than the mathematics available with the resource. Instructors who finished their education before 1985 were not likely to have access to these technologies in their own schooling, which can make the learning curve for teaching even steeper. Yet teaching the FTC may seem like an ideal place to introduce graphing technologies. Technologies can quickly and accurately display a great variety of functions, as well as visually compare an accumulation function and a rate of change. In addition to visually displaying mathematics functions, technologies can remove the burden of onerous computations for many types of calculus problems. This allows instructors and students to focus on conceptual understanding of important theorems such as the FTC. However, over-reliance on graphing technologies can hinder an instinctive understanding of some of the most common parent functions as well as procedural fluency. The tension between conceptual understanding and procedural fluency was mentioned by instructors in this study, and this may be one explanation why the technologies are inconsistently used.

These seven combinations of resources and their uses illustrate the instrumentation and instrumentalization of resources for planning and teaching. In the next section I discuss the reasons that instructors gave for why they used resources and how the context of two-year colleges may contribute to those reasons.

Two-Year College Context

All instructors in this study taught at Michigan two-year colleges. There are two aspects of this context that may be relevant to the findings from this study. First, the status of calculus at two-year colleges allowed instructors some autonomy over the structure of their calculus courses, and second, the primary mission of two-year colleges is teaching, often leading to high teaching loads by instructors.

Status of calculus. Calculus at two-year colleges is often the pinnacle of the sequence of math courses offered at the college (other courses include differential equations or linear algebra), and, therefore, desirable to teach. Instructors may prefer to teach calculus over other courses, and there may be a certain status to being a calculus instructor. There is evidence that two-year college instructors are much more interested in teaching calculus than instructors at other post-secondary institutions (Bressoud, 2012). At two-year colleges, calculus is an upper level course, and there are usually fewer sections of calculus offered than sections of lower level mathematics. The prestige of teaching an upper level course with fewer sections may offer instructors some autonomy over how they use their resources to teach the FTC. In this study, instructors felt comfortable changing the order of topics in their textbooks (such as in Appendix J). The most common reasons instructors gave for using (or not using) their resources in a variety of ways had to do with concern for students (41%). Similar to other research on community college instructors (Grubb, 2002), the instructors in this study demonstrated a concern for student well-being and understanding. Because instructors have some autonomy over the resources that they use, it makes sense that the reasons they gave for choosing resources are primarily out of regard for students. Teaching calculus at the two-year college context means that instructors who are concerned for their students are able to make resource choices based on those concerns. Class sizes are limited, and instructors may know many students personally.

Their choice of resources was often based on how they felt it would help their students. This didn't always lead to the same use of resources, particularly in the case of technologies. Some instructors felt that graphing technologies were critical to enhance student understanding of calculus concepts, and incorporated this into their classroom. Other instructors felt that technologies inhibited students' ability to think mathematically, and restricted their use. Regardless of how they used the resource, instructors indicated much of their reasoning as a concern for students.

The status of calculus at two-year colleges may also explain why the third most common reason instructors gave for using resources was math (18%). Calculus is an upper-level mathematics course at a two-year college, and may be the last time that instructors have a chance to impart mathematical wisdom to students. This may explain why some of instructors' choices of resource use were based on their concern for mathematics.

The second most common reason instructors gave for using resources was personal preference (24%). In order to indulge personal preference, some autonomy in resource use is required, but the figure of 24% may not be explained simply by autonomy.

Focus on Teaching. One of the primary missions of two-year colleges is teaching (Cohen & Brawer, 2003). This is reflected in the teaching loads of full-time instructors, who typically teach fifteen credit hours each semester. Part-time two-year college instructors have limited teaching loads at an institution, but they may be teaching at multiple institutions. Some instructors in this study indicated that they chose ways of assigning homework (e.g., online or paper and pencil) based on their available time for grading. Instructors with high teaching loads may have less time to plan instruction and less inclination to experiment with a variety of resources. High teaching loads may make it more enticing for instructors to rely on outside resources to ease the work of planning instruction. Because course (re)design is time consuming and complex, it is understandable that instructors may prefer to accept the order provided in their textbooks: differentiation first followed by integration with one section of one chapter focused on the FTC. It makes sense that in a context of a heavy teaching load, personal preference may drive some of the instructors' choice of resources.

Limitations

There are four main limitations in the study. First, the survey was e-mailed to all instructors who had taught first semester calculus in the 2015-2016 school year. However, many two-year college instructors are part time, and may only teach one semester, so some instructors may not have had access to their college e-mail at the time the survey was mailed. Question 9 on the survey had ambiguous wording, and “examples from the textbook” could be interpreted as either the examples that the textbook used before the exercise problems, or examples that were used in class that were pulled from the exercise problems.

Second, my background as a two-year college mathematics instructor may have influenced the way that instructors talked to me. They may have assumed that I understood much of the mathematics we discussed without careful explanation, so they may have displayed less knowledge about the FTC than they possess. In order to mitigate this, I asked for explanations often. Instructors may have assumed that our experiences in teaching mathematics at a two-year college were similar in terms of resources available. I asked instructors detailed questions about all resources, including ones such as the overhead projector and chalk. Some instructors were surprised that I had to mention those types of resources at all, but I did not list any resources that they did not mention. In this sense, my experience teaching may have had the advantage of allowing me to question instructors on resources that are common in mathematics classrooms. I also e-mailed each instructor in the interview with a summary of resources and how I understood them to explain the FTC (member checking). This e-mail allowed them an opportunity to correct and remove or add resources that we missed discussing in the interview and explain their thinking about the FTC further if needed.

A third limitation is that I have never taught first semester calculus. This meant that writing the survey and interview questions were not based on experience. In order help ensure that I asked the right questions, I had my survey and interview questions reviewed by calculus instructors that I know in person. However, I still had a problem with the wording on Question 9 in the survey. My lack of experience teaching first semester calculus worked to my advantage in having instructors explain how they taught the FTC. I had no frame of reference from having taught it before for how it could be taught, or common student errors to watch for. Because of this, instructors may have explained their teaching techniques in more detail than they might have to someone with more experience.

A final limitation is in terms of the data. For my definition of resources, I used Adler's (2000) framework for instructor resources. I considered resources to be assets that impacted how an instructor planned and taught the FTC. Tangible resources were fairly easy to identify, but intangible presented more of a challenge. Because much of my data included instructors' discussion of resources, some obvious resources were likely overlooked. The resources mentioned in the findings section of this document include only those resources that were obvious enough to be observed, or resources that instructors were aware enough of to mention. Thus, some self-awareness on the part of instructors was expected, yet ubiquitous resources (such as electricity) tend to be assumed and not mentioned. In addition , as Adler (2000) states, "Resources in school mathematics practice need to be seen to be used (visible) and seen through to illuminate mathematics (invisible)" (p. 214). Resources that are invisible enough may not have been mentioned by instructors. One common example was the use of instructor background. For example, instructors would mention, in passing, that they taught a certain way because it was how they learned it. But when asked how they go about planning and teaching the FTC, they would not mention thinking about their own background. In order to limit this type of oversight in the data, I structured interview questions to include questions to trigger ideas of intangible resources. At the beginning of the interview I deliberately mentioned "intangible" resources, and gave examples such as "conversations with colleagues" in order to trigger a thought of non-material resources on the part of the instructor. When I asked, "How do you go about planning a lesson for this course?" I listened closely for any mention of intangible resources. As interviews progressed, I periodically summed up what the instructor was telling me, including as “resource” those items that instructors may not have mentioned explicitly. This often included their background and student feedback. Instructors would confirm that yes, these were resources.

Chapter 6: Conclusion and Implications

This mixed methods study investigated the resources that Michigan two-year college instructors use when they plan and teach the fundamental theorem of calculus. The resources were considered in combination with their uses by instructors and the effect that they had on instructor teaching and planning. In this dissertation I define resources as assets that instructors were able to draw on to help them plan and teach the FTC. Resources were both tangible and intangible, and were identified as resources if they impacted planning or instruction. This impact was described by instructors on their surveys and during their interviews, and seen by me during the observations. An examination of all three data sets (surveys, interviews, and observations) yielded the identification of five common resources: textbooks, graphing technologies, instructor background, institutional requirements, and student feedback. Textbooks were used by instructors to plan a semester and as a bank of problems. Graphing technologies were used to illustrate functions and do calculations. The instructors' background impacted their planning, as did the requirements by the institution, and feedback from students impacted teaching. This chapter explores each of these findings in terms of affordances and limitations for instruction, and implications for instructors, administrators, and researchers. I conclude with directions for future research.

Affordances and Limitations of Resources

Each resource offered some affordances and limitations with its instrumentation and instrumentalization. In the instrumentalization process, textbooks were often used as a source for problems to do during class and to give students for homework. The trust placed in textbooks to include problems that are appropriate for a first semester calculus class was not questioned by any of the instructors in this study. Textbooks with appropriate problems made the work of teaching easier; instructors did not have to create problems appropriate to what was being taught. However, using the textbook as a bank of problems can also have limitations. Mathematics textbook sections and chapters tend to build on each other. With all of the textbooks used by instructors in this study, the FTC was taught in one or two sections of one chapter, and only after a study of differentiation was concluded. This meant that instructors who used the textbooks as a bank of problems might have difficulty choosing problems if they wished to introduce the FTC significantly earlier than it appears in the textbook. As part of the instrumentation process, the textbook was often used to plan out the semester. None of the instructors from this study indicated that they introduced the FTC towards the beginning of a semester, and none of the instructors indicated that they spent more time on the FTC than they would normally spend on one section of the textbook. While the textbook definitely offers advantages in terms of the potential organization of a course and a bank of problems to be drawn on, the instrumentation of the textbook may also limit how calculus is taught. It may not occur to instructors to introduce the FTC before finishing differentiation; yet some research suggests doing just that in order to facilitate a better understanding of the FTC and calculus over all (P. Thompson, Byerly, and Hatfield, 2013). Because almost all calculus textbooks[18] follow a similar formatting of derivatives before integration, it may not occur to instructors that they can change the order. The implication is that in the absence of knowledge of the research cited above or a significant change in the way textbooks are written, instructors are unlikely to offer any significant change

[18] I examined the table of contents for all the calculus textbooks listed by instructors on the survey, as well as 10 different online calculus textbooks and 3 more calculus textbooks that were not used by any instructors in this data set. Of those, only one textbook, Saxon Calculus (not used by any instructors in this study and marketed to homeschoolers), did not follow the typical calculus formatting.

in the way the FTC is handled. In addition, choosing to use the textbook to plan a semester may mean having only one day to teach the FTC.

Future research may consider the impact of the instrumentalization of the textbook on other aspects of teaching beyond scheduling a semester. What is the level of impact of the textbook on the instruction of college level classes. Instructors generally used problems from the textbook, but what about the exposition text? Does a significant difference in the textbooks presentation of a topic (such as different proofs of the FTC) affect how instructors present that topic to classes?

The second resource common to all data sets was graphing technologies. Instructors approached graphing technologies in one of three ways: 1) avoiding them, 2) using them primarily for computation, or 3) using them for computation and the visual display of functions. A significant portion of the instructors who were surveyed or interviewed avoided graphing technologies and calculators. The consequence of avoiding these technologies meant that instructors had to choose problems to demonstrate that could be displayed by hand and that were relatively quick to compute. When students are primarily exposed to problems that have easy computations and simple fraction or integer answers, they may be reluctant to work on problems or accept answers that don't conform to what they have seen in class. One of the advantages to using graphing technologies is that it offers the students the opportunity for increased efficiency and conceptual exploration (Artigue, 2002; Dunham & Dick, 1994). However, graphing technologies can also become a crutch to students, enabling them to get answers without understanding the underlying mathematical process (E. Brown, *et al*., 2007).

Instructors who used graphing technologies to teach the FTC either used them primarily for computation (e.g., Philip who used the calculator to check his arithmetic) or for both computation and to display functions. According to P. Thompson, Byerly, and Hatfield (2013), graphing technology (specifically Grapher 4.0, Avitzur, 2011) was necessary in order to have students understand the relationship between the accumulation function and the rate of change, and understanding that relationship was foundational to an understanding of the FTC. Without this connection and with little technology used for teaching the FTC, students' conceptual understanding of the FTC may be lacking.

Some of the implications from these findings are that administrators, researchers, and textbook authors cannot assume that instructors are using the graphing technologies recommended or required by the institution or curriculum materials. Even if a college requires the use of graphing technologies, instructors may struggle to implement them coherently into the classroom. Future research could focus in on how technologies are used when teaching the FTC and the impact of those technologies on student learning, both conceptual and procedural.

The third resource common to all data sets was instructors' background. The professional and educational background of instructors impacted how they planned and taught the FTC. An instructor's personal and professional background, used in order to facilitate planning and teaching, can help instructors reflect on their teaching; what works and what doesn't. An instructor who has a strong background in business may have valuable insights to offer students. However, by relying on their own background, instructors may misinterpret where students struggle, especially in their early teaching careers. For example, an instructor who had no trouble understanding the FTC may not expect students to struggle with this theorem. The implications for instructors relying solely on their personal and professional background are that they may limit their effectiveness in teaching topics such as the FTC. The instructors in this study did not always consider their personal backgrounds as a resource, and only with careful questioning and reading of the data did this resource become visible. Future research could consider numerous instructors' backgrounds and examine whether similarities in backgrounds lead to similarities in teaching.

The fourth resource found across all data sets that impacted instruction was the requirements of the institution where the instructor taught. There were two primary requirements of institutions that impacted instruction. The first was the course schedule. The instructors in this sample, whether or not they were the head of the department, were required to abide by the contact hours set for the class as well as the times for the class to be held. Most instructors did not get to choose whether a class was four or five credits, and how many days per week it was offered. Some instructors who had two-hour blocks felt that this was ideal because it gave them time for an activity every day. Some instructors with two-hour blocks would have preferred to see students more often for less time, because they felt that students needed time to digest the information between classes. In other words, there are advantages and disadvantages to various types of schedules.

The second institutional requirement that impacted instruction was the requirement of curriculum materials for calculus classes. Many institutions had a required textbook, and some had requirements for graphing calculators, online software, and common exams. Instructors in this study used the material requirements of the institution, but often adapted them in ways that the institution may not have anticipated. Although institutional requirements may mean that instructors have fewer decisions to make, those same requirements may also mean that instructors may miss some opportunities to have students explore new ways to view the FTC.

Future research could include a comparison of institutional requirements among two-year colleges, and whether (and how) those requirements impact instruction. Research into how adopt or subvert the requirements of an institution should include the impact of those requirements (and how they are implemented) on students. For scheduling, future research should compare advantages and disadvantages to various types of schedules for students, instructors, and institutions. The implication for institutions is that instructors may have strong opinions about what is best for students, and it could be helpful to be transparent about the advantages and reasons for various types of schedules. Michigan two-year colleges have a great deal of autonomy, and can choose what type of calculus schedule is offered as well as how many credit hours a calculus course requires. Smaller institutions with one or two full-time instructors could involve them in the scheduling process. Larger institutions may offer calculus with different types of schedules. Researchers should consider that the schedule may impact how a topic is taught. A topic that is taught for two hours on one day may not be taught the same way if it is taught for two days, one hour each. Future research could explore the differences in these types of schedules, particularly around important topics such as the FTC.

The last resource mentioned by instructors in all data sets was student feedback. This resource is primarily used during instruction. For example, a confused look from students may lead an instructor to re-explain a concept. However, student feedback can also affect planning. A poor showing on an exam may lead an instructor to re-teach a topic. In this study the FTC was taught near the end of a semester, which implies that instructors may not see much written feedback (in the form of quizzes or exams) from students about this topic. If students are struggling with a topic in differentiation, instructors have multiple opportunities to address it, whether or not a student looks confused when the topic is introduced. Yet instructors do not have as much opportunity to re-teach topics that come toward the end of a course. This means that student feedback during class is especially important at this time. Instructors may offer more in-class activities as a way to explore student understanding. Student feedback regarding the FTC can be invaluable in guiding instruction, particularly because it is taught at the end of a semester. Future research could examine the impact of both verbal and non-verbal student feedback on instruction, and consider whether (or how) the timing of that feedback within a course may change how instructors take-up student feedback. For example, do instructors pay more attention to student expressions of confusion at the beginning or end of a course? How does the timing of when a topic is taught impact how instructors respond to student feedback?

Overall, the findings from this study expand a body of research around calculus instruction. The unique contribution of this study is its focus on the resources involved during the planning and teaching of one topic with one group of instructors. Beyond the implications for instructors and researchers suggested by the resource use above, I offer some other implications for researchers, instructors, and the institution.

Other implications

It was not the intention of this study to focus on instructor understanding of the FTC, but rather the resources that were used by instructors to plan and teach the FTC. Within the context of this study, I offered some findings about the ways instructors discussed the FTC, but this was not the focus of the study. Instructors in this study generally discussed the FTC as either the inverse relationship between integration and differentiation, or the evaluation of a definite integral. Only rarely did they connect these two ideas when talking with me. This does not mean they do not make these connections, but does imply questions about their understanding and what they convey to students. Future research should probe instructors' understanding and how that understanding impacts instruction.

Findings from this study included the impacts that instructors' use of resources had on their planning and teaching of the FTC. However, I did not examine the impact of the use of these resources on student learning. Future research should consider if and how the combinations of resources and their use affect student learning.

Findings from this also study indicated a possible relationship between the way that instructors used graphing technologies and what they felt was the main purpose of the FTC for students. Instructors who focused on the inverse relationship between differentiation and integration tended primarily use graphing technologies for computation. Instructors who focused on the evaluation portion of the FTC tended to use the graphing capabilities as well as computational abilities of the technologies. The conceptual idea of an inverse relationship is not easily demonstrated visually, yet P. Thompson, Byerly, and Hatfield (2013) imply that a visual understanding of this relationship is key to understanding the FTC, and demonstrate how it can be done. Instructors may benefit from professional development around the idea of using graphing technologies to demonstrate this inverse relationship. As more textbooks are developed on-line with interactive tools, this type of exploration could be built in to the textbook software.

Some of the instructors in this study were given a course outline that included sections from the textbook that needed to be taught. Institutions should consider that presenting a course outline with textbook sections may imply a recommendation for how long each topic should be taught. If this is not a desired recommendation, institutions may consider presenting course outlines based on topics rather than textbook sections. Institutions should also consider the resources that they require instructors to use may have unexpected impacts on their planning and instruction.

Finally, this study illustrates certain advantages to researching instruction at two-year colleges. The focus on teaching at two-year colleges also meant that instructors in this study were able to articulate to me what they were doing and why they were doing it. They were also interested in talking to me about their teaching. Over 1/3 of all two-year college instructors in Michigan who were contacted in the initial survey e-mail responded, and almost half of the survey respondents expressed an interest in being interviewed. Mathematics instruction at two-year colleges is under-researched (Mesa, Wladis & Watkins, 2014), which may be one of the reasons instructors were so willing to talk to me. If researchers have a choice of institutional settings for their research, two-year colleges should be considered.

Concluding implications

By the time students reach calculus, they have probably been exposed to two other fundamental theorems. The fundamental theorem of arithmetic can be explained in elementary school, but not proved until number theory. The fundamental theorem of algebra can be understood in pre-calculus, but not proved without an understanding of complex analysis. The fundamental theorem of calculus is first fundamental theorem that can be proved during the semester when it is taught. Yet this theorem is more conceptually difficult to understand than the previous two "fundamental" theorems that students may have been exposed to. Unfortunately, instructors in this study did not spend very much time ensuring conceptual understanding. Many students already struggle with math, and if mathematicians call something fundamental but don't treat it as fundamental, students may miss the key concepts embodied there. It may be wise to consider dramatically changing the way that the FTC is taught, and considering how the resources that instructors use can contribute to a possible perception that a "fundamental" theorem is not important. Language matters, and the message being sent to students when they are told something is fundamental, but it isn’t treated that way, does not cast a flattering light on the mathematics profession.

This dissertation examined the resources that instructors used when planning and teaching the fundamental theorem of calculus. I explained how instructors navigate (and sometimes subvert) an occasionally flawed set of resources, and why they use those resources in the way that they do. Instructors are doing the best that they can, under often challenging circumstances. They used their resources in different ways, and the resources they chose to use (and the way that they used them) impacted instruction. Future research must consider both the instrumentation and the instrumentalization of resources. This dissertation offers an example of how that can be done, around one important mathematical topic.

Appendices

Appendix A: Survey

Q1) Are you teaching Calculus I in the Winter, 2016?
Yes
No

Q2) Did you teach Calculus I in Fall, 2015?
Yes
No

COURSE INFORMATION

Q3) The Calculus textbook I use is:

a. A common textbook selected by the department
b. A textbook I chose from an appointed list
c. A textbook of my own choosing
d. Other (please specify) ____

Q4) What textbook is used for your Calculus I course? Select from the list below or specify a different text if your book is not on the list.

Note the distinction between "Early Transcendentals" and standard editions. No distinction is made between single-variable and combined single- and multivariable volumes.

1. Anton/Bivens/Davis – Calculus
2. Anton/Bivens/Davis - Calculus: Early Transcendentals
3. Blank/Krantz – Calculus
4. Edwards/Penney - Calculus: Early Transcendentals
5. Hass/Weir/Thomas - University Calculus
6. Hass/Weir/Thomas - University Calculus: Alternate Edition
7. Hass/Weir/Thomas - University Calculus: Elements with Early Transcendentals
8. Hughes Hallett et al. - Calculus
9. Larson/Edwards - Calculus
10. Larson/Hostetler/Edwards - Calculus: Early Transcendentals
11. Larson/Hostetler/Edwards - Essential Calculus
12. Rogawski - Calculus
13. Rogawski - Calculus: Early Transcendentals
14. Salas/Hille/Etgen - Calculus
15. Smith/Minton - Calculus

16. Smith/Minton - Calculus: Concepts and Connections
17. Smith/Minton - Calculus: Early Transcendentals
18. Stewart - Calculus
19. Stewart - Calculus: Concepts and Contexts
20. Stewart - Calculus: Early Transcendentals
21. Stewart - Essential Calculus
22. Stewart - Essential Calculus: Early Transcendentals
23. Swokowski - Calculus
24. Thomas/Weir/Hass/Giordano - Thomas' Calculus
25. Thomas/Weir/Hass/Giordano - Thomas' Calculus: Early Transcendentals
26. Varberg/Purcell/Rigdon - Calculus
27. Varberg/Purcell/Rigdon - Calculus: Early Transcendentals
28. Other (Please specify Title and Author(s))________________________________

Q5) Including this semester, approximately how many semesters of first semester calculus have you taught using that book?

Q6-9) The next four questions ask about your general use of your textbook

	1 (Never)	2	3	4	5	6	7 (Always)	N/A
(6) It is important to me to use the same formulas and symbols as my textbook (N/A means you do not use a textbook)								
(7) I am comfortable changing the order of topics in the textbook. (N/A means you do not use a textbook)								
(8) I assign homework from the textbook or software associated with the textbook (N/A means you do not assign homework)								
(9) I use examples from the textbook during class time (N/A means you do not use a textbook)								

Q10) In addition to the textbook, what other resources (such as graphing calculators, computer software, other books, websites, etc) do **you** use when planning to teach a first semester calculus class? Please be specific.

Q11) Does your department require **students** to purchase a common textbook for Calculus I?

Yes
No
Unknown

Q12) What are **students** required to purchase (textbook, software, graphing calculators, etc.) for your first semester calculus course? Please be specific.

FUNDAMENTAL THEOREM OF CALCULUS

Q13) When planning to teach the Fundamental Theorem of Calculus, how often do you refer to your textbook?

Never
Sometimes
Always

Q14-17) Please describe your general impressions of your textbook's treatment of the fundamental theorem of calculus.

	Positive (1)	2	3	4	5	6	Negative (7)
(14) What is your general impression of your textbook's overall treatment of the Fundamental Theorem of Calculus?							
(15) What is your general impression of this textbook's explanation of the Fundamental Theorem of Calculus?							
(16) What is your general impression of this textbook's proof(s) of the Fundamental Theorem of Calculus?							
(17) What is your general							

impression of this textbook's problem sets relating to the Fundamental Theorem of Calculus?	

Q18) What technology do **you** use when teaching the Fundamental Theorem of Calculus?

a. I do not use technology
b. Graphing calculators that do not perform symbolic algebra
c. Graphing calculators that perform symbolic algebra
d. Computer algebra system (Maple, Mathematica, MATLAB, etc)
e. Other (please describe) ____

Q19) What technology are your *students* **permitted** to use when learning the Fundamental Theorem of Calculus?

a. Technology not permitted.
b. Graphing calculators that do not perform symbolic algebra
c. Graphing calculators that perform symbolic algebra
d. Computer algebra system (Maple, Mathematica, MATLAB, etc)
e. Other (please describe) ____

Q20) What technology are your students **required** to use when learning the Fundamental Theorem of Calculus?

a. Technology not required.
b. Graphing calculators that do not perform symbolic algebra
c. Graphing calculators that perform symbolic algebra
d. Computer algebra system (Maple, Mathematica, MATLAB, etc)
e. Other (please describe) ____

Q21) In my experience, explaining the Fundamental Theorem of Calculus to students is easier without added technologies.

(Strongly Disagree) 1 2 3 4 5 6 7 (Strongly Agree)

The fundamental theorem of calculus is often given in two parts, but the order of those parts may vary. Two proofs of a portion of the fundamental theorem of calculus are given below. Please consider both of these and then answer the following questions. Note that option 1 uses the mean value theorem in the proof and option 2 uses the other ("first") fundamental theorem of calculus in the proof. (See Appendix B)

Q22) Which proof do you prefer?

Option 1 (uses the mean value theorem)
Option 2 (uses the "first" fundamental theorem of calculus)
Other (please describe) ____

Q23) Why do you prefer that proof? Specifically, what elements of that proof do you like?

Q24) Please explain anything else you feel is relevant to teaching the Fundamental Theorem of Calculus

DEMOGRAPHICS

Q25) Your current position is best described as

Full-time
Part-time
Other (describe) ____

Q26) Gender

Male
Female

Q27) Age

20-29
30-39
40-49
50-59
60-69
over 69

Q28) What is your highest degree attained?

PhD
EdD
Master's Degree
Bachelor's Degree
other (please describe) ____

Q29) In what field is your highest degree? (check all that apply)

Mathematics
Mathematics Education
Statistics
Physics
Engineering
Other Field (please describe)____

Q30) Including this year, approximately how many semesters have you taught college calculus?

Q31) I consider myself a(n) __________________ teacher of calculus I:

Beginner
Novice
Advanced
Expert

Q32) Why did you choose that category?

Q33) Is there anything else about your background that you feel is relevant to your teaching of calculus?

Q34) As part of this research, I will be interviewing community college teachers about their experiences teaching calculus and teaching the Fundamental Theorem of Calculus. Would you like to be considered for an interview?

Yes (provide e-mail address) ____
No

Appendix B: Proofs given to instructors for comparison

Fundamental Theorem of Calculus Proof Option 1:

Fundamental Theorem of Calculus I: If a function f is continuous on the closed interval $[a, b]$ and F is an antiderivative of f on the interval $[a, b]$ then $\int_a^b f(x)dx = F(b) - F(a)$. (Larson, Hostetler, & Edwards 2007 p. 282)

Fundamental Theorem of Calculus II: If f is continuous on an open interval I containing a, then, for every x in the interval, $\frac{d}{dx}\left[\int_a^x f(t)dt\right] = f(x)$.

(Larson, Hostetler, & Edwards 2007 p. 289)

Proof of FTC I

Consider a partition of [a,b] as follows: $a = x_0 < x_1 < \cdots < x_{n-1} < x_n = b$. Then rewrite $F(b) - F(a)$ as $F(x_n) - F(x_{n-1}) + F(x_{n-1}) - \cdots - F(x_1) + F(x_1) - F(x_0)$

$$= \sum_{i=1}^{n} [F(x_i) - F(x_{i-1})].$$

By the Mean Value Theorem, there exists a number c_i in each ith sub-interval such that

$$F'(c_i) = \frac{F(x_i) - F(x_{i-1})}{x_i - x_{i-1}}.$$

Choose $\Delta x_i = x_i - x_{i-1}$, and because $F'(c_i) = f(c_i)$,

$$F(b) - F(a) = \sum_{i=1}^{n} f(c_i)\Delta x_i.$$

This says that by applying the Mean Value Theorem, the **constant** $F(b) - F(a)$ is a Riemann sum of f on [a,b] and taking the limit as $\Delta x_i \to 0$ yields $\int_a^b f(x)dx = F(b) - F(a)$.

(adapted from Larson, Hostetler, & Edwards, 2007, p. 283)

Figure 25: MVT Proof

Fundamental Theorem of Calculus Proofs Option 2:

Fundamental Theorem of Calculus I: If f is continuous on $[a, b]$, then the function g, defined by

$$g(x) = \int_a^x f(t)dt \qquad\qquad a \leq x \leq b$$

is continuous on $[a, b]$ and differentiable on (a, b) and $g'(x) = f(x)$

Fundamental Theorem of Calculus II: If f is continuous on $[a, b]$, then

$$\int_a^b f(x)dx = F(b) - F(a)$$

where F is any antiderivative of f, that is, a function such that $F' = f$.

(Stewart, 2003 p. 342 – 344)

Proof of FTC II:

Let $g(x) = \int_a^x f(t)\,dt$. We know from Part I that $g'(x) = f(x)$; that is, g is an antiderivative of f. If F is any other antiderivative of f on $[a, b]$, then we know (from a previous theorem) that F and g differ by a constant:

$$F(x) = g(x) + C \qquad [6]$$

for $a \leq x \leq b$. But both F and g are continuous on [a,b] and so, by taking limits of both sides of equation 6 (as $x \to a^+$ and $x \to b^-$), we see that it also holds when x=a and x=b. If we put $x = a$ in the fomula for $g(x)$, we get $g(a) = \int_a^x f(t)dt = 0$. So using equation 6 with $x = b$ and $x = a$, we have

$$F(b) - F(a) = [g(b) + C] - [g(a) + C]$$

$$= (g(b) - g(a) = g(b) = g(x) = \int_a^b f(t)dt$$

(paraphrased from Stewart, 2003, p. 345)

Figure 26: I to II Proof

Appendix C: Interview Protocol

Give consent form and describe the study in brief: "I am researching how you use various resources like textbooks and software when you plan and teach the Fundamental Theorem of Calculus. For the purposes of this interview, I define teaching as being in front of students, either during class time or during office hours. Planning refers to preparation for teaching, to get ready to be in front of students and lead class sessions as well as choosing homework. If you have any questions about the words I am using, please ask. Also, I would like to remind you that all questions are optional and that you can stop the interview at any time for any reason or for no reason at all. Do you have any questions?"

Background questions

1) What is your educational background?
    a. Degree? Institution? Year?
2) How many years have you been teaching?
    a. At this location?
    b. Elsewhere?
    c. Any other subjects?
3) Age, Gender, Race (optional)
4) What sorts of training in teaching did you have prior to teaching?
    a. Teaching certification?
5) Is there anything else about your background that you feel is relevant to your teaching?

Interview questions (general teaching/planning):

1) How do you go about planning a lesson for this course?
    a. What materials or resources (textbook, software, internet, other) do you use when planning lessons?
    b. What do you want a typical lesson of yours to look like?
        i. Describe the structure of a lesson
    c. How do you decide what examples to use during a lesson?
    d. How and when do you decide what to give as homework?

2) <Textbook> is the textbook assigned to the class. On the survey you said that you <do/do not> use this textbook during your teaching? Please explain why.
    a. When do you bring the book to class (if at all)?
    b. When do you refer to the book for notation (depending on survey response)?
    c. Are there some times or topics when you use the book more or less?
        i. If so, Why? When? What topics? If not, why not?

3) Per the survey, you said you use (do not use) <technology>. Please explain more about this.
    a. Why do you use/not use <technology>?
    b. How do you use <technology>?
    c. Give an example?

Interview questions (general FTC)

4) How important is the FTC to calculus?
   a. Why?

5) What are the advantages and disadvantages with <textbook from survey> for teaching FTC?

6) What are the advantages and disadvantages with <technology from above> for teaching FTC?

7) How do you introduce the FTC to your students?
   a. Approximately how many class periods do you spend on the FTC?
   b. Do you use <technology from above> when introducing the FTC?
      ii. How?

8) How do you choose homework problems for FTC?

Interview questions (Teaching FTC)

9) After you first lesson on FTC, what is the main point that you want students to walk away with?
   a. What if anything do you do to make sure that students understand ______ (response to previous question)

10) What do you think is important for students to know about the FTC?
    a. Why?

11) What else is important about the FTC?
    a. If you asked a student about the FTC, what would you (ideally) want them to say?

12) In which ways does <textbook> help students gain this understanding of the FTC?
    a. In what ways does <technology from above> help students gain this understanding of the FTC?
    b. What kinds of things do you do in class that the <textbook and technology> lacks to help students understand the FTC?

13) How do you know that a student understands the FTC?

Interview questions (Summary and hypothetical)

14) If you money was no object, and you could have any resource, real or imagined, you wanted for teaching the FTC, what would you like?

15) I have never taught first semester calculus, but I would like to. Imagine that I am a new calculus teacher in your institution and come to you for advice on teaching the FTC. What would you tell me?

a. What do I need to make sure students understand?
b. How much time should I spend?
c. How should I choose examples to present?
d. How should I assign homework?

Appendix D: Institutional Schedule from George

Schedule

NOTE TO STUDENTS:

The basic topics in this course must be covered but may be accomplished using a modified version of the schedule listed below.

Session	Topics	Target Competencies	Assignments/Assessments	Due Date
Week 1	Chapter 0: Functions Review		0.1 – 0.3, Appendix B, 0.4, 0.5	
Week 2	Chapter 1: Limits and Continuity		**Exam 1 (Chapter 0)** 1.1 – 1.3	
Week 3	Chapter 1: Limits and Continuity Review Chapter 2: The Derivative		1.5, 1.6, 2.1, 2.2	
Week 4	Chapter 2: The Derivative Review		**Exam 2 (Chapter 1)** 2.3 – 2.6	
Week 5	Chapter 3: Topics in Differentiation		**Exam 3 (Chapter 2)** 3.1 – 3.3	
Week 6	Chapter 3: Topics in Differentiation Review Chapter 4: The Derivative in Graphing and Applications		3.4 – 3.6, 4.1, 4.2	
Week 7	Chapter 4: The Derivative in Graphing and Applications		**Exam 4 (Chapter 3)** 4.3 – 4.5	
Week 8	Chapter 4: The Derivative in Graphing and Applications Review Chapter 5: Integration		4.5, 4.8, 5.1. 5.2	
Week 9	Chapter 5: Integration		**Exam 5 (Chapter 4)** 5.3, 5.4	
Week 10	Chapter 5: Integration Review		5.5, 5.6, 5.8, 5.9	
Week 11	Review for Final Exam		**Exam 6 (Chapter 5)** Catchup/Review	
Week 12	Review for Final Exam		Catchup/Review **Comprehensive Final Exam**	

Updated by Curriculum Committee

January 2015

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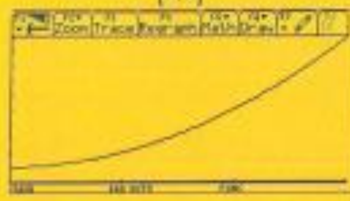
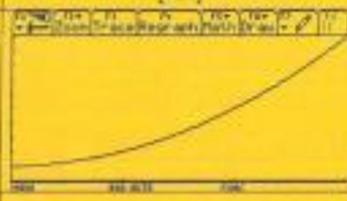
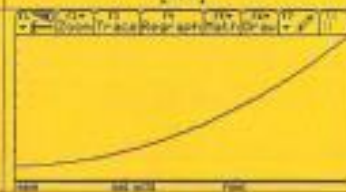
Figure 27: Sample Institutional Schedule

Appendix E: Sample worksheets from Marcus

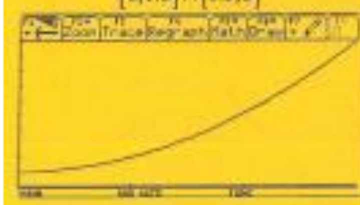
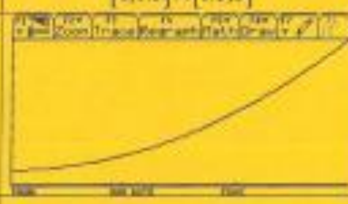
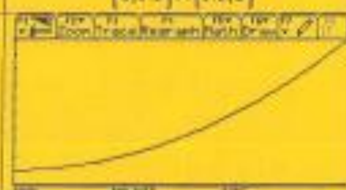
Marcus gives these worksheets to students when teaching the FTC

MATH 165 Applied Calculus **Activity 11**

Given the graph $y = x^2 + 1$ over $[0,3]$, a guided tour will be given for a process of finding the area between the curve and the x-axis over this interval. The first estimation will be using left endpoint rectangles (first column) and right endpoint rectangles (second column) on subintervals and averaging the results. The second estimation will be using center point rectangles (third column).

Left Endpoint Rectangle(s) $[0,3]$	Right Endpoint Rectangle(s) $[0,3]$	Center point Endpoint Rectangle(s) $[0,3]$
		
Sum of the area of left endpoint rectangle(s) (SALER) is ______.	Sum of the area of right endpoint rectangle(s) (SARER) is ______.	Sum of the area of center point rectangle(s) (SACPR) is ______.

The average of the above SALER & SARER is ________.

Left Endpoint Rectangle(s) $[0,1.5]\wedge[1.5,3]$	Right Endpoint Rectangle(s) $[0,1.5]\wedge[1.5,3]$	Center point Endpoint Rectangle(s) $[0,1.5]\wedge[1.5,3]$
		
Sum of the area of left endpoint rectangle(s) (SALER) is ______.	Sum of the area of right endpoint rectangle(s) (SARER) is ______.	Sum of the area of center point rectangle(s) (SACPR) is ______.

The average of the above SALER & SARER is ________.

Left Endpoint Rectangle(s) $[0,1]\wedge[1,2]\wedge[2,3]$	Right Endpoint Rectangle(s) $[0,1]\wedge[1,2]\wedge[2,3]$	Center point Endpoint Rectangle(s) $[0,1]\wedge[1,2]\wedge[2,3]$
		
Sum of the area of left endpoint rectangle(s) (SALER) is ______.	Sum of the area of right endpoint rectangle(s) (SARER) is ______.	Sum of the area of center point rectangle(s) (SACPR) is ______.

The average of the above SALER & SARER is ________.

Using your answers from the front of this sheet over the given intervals, fill in the table below and then complete the statement below the table.

Interval(s)	Average of SALER & SARER	SACPR
$[0,3]$		
$[0,1.5]\wedge[1.5,3]$		
$[0,1]\wedge[1,2]\wedge[2,3]$		

Based on the above information in the table, an estimation for the exact area between the x-axis and the graph of the curve $y = x^2 + 1$ is ________________.

Figure 28: In class activity for FTC

HW

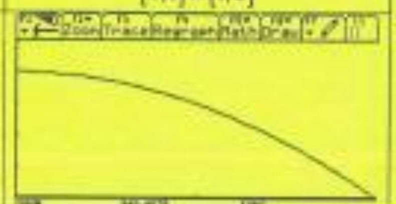
MATH 165 Applied Calculus **Activity 12**

Given the graph $y = 4 - x^2$ over $[0,2]$, a guided tour will be given for a process of finding the area between the curve and the x-axis over this interval. The first estimation will be using left endpoint rectangles (first column) and right endpoint rectangles (second column) on subintervals and averaging the results. The second estimation will be using center point rectangles (third column).

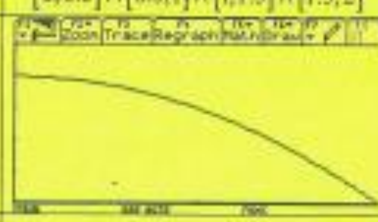
Left Endpoint Rectangle(s) $[0,2]$	Right Endpoint Rectangle(s) $[0,2]$	Center point Endpoint Rectangle(s) $[0,2]$
		
Sum of the area of left endpoint rectangle(s) (SALER) is ______.	Sum of the area of right endpoint rectangle(s) (SARER) is ______.	Sum of the area of center point rectangle(s) (SACPR) is ______.

The average of the above SALER & SARER is ________.

Sum area center point rectangle

Left Endpoint Rectangle(s) $[0,1] \wedge [1,2]$	Right Endpoint Rectangle(s) $[0,1] \wedge [1,2]$	Center point Endpoint Rectangle(s) $[0,1] \wedge [1,2]$
		
Sum of the area of left endpoint rectangle(s) (SALER) is ______.	Sum of the area of right endpoint rectangle(s) (SARER) is ______.	Sum of the area of center point rectangle(s) (SACPR) is ______.

The average of the above SALER & SARER is ________.

Left Endpoint Rectangle(s) $[0,0.5] \wedge [0.5,1] \wedge [1,1.5] \wedge [1.5,2]$	Right Endpoint Rectangle(s) $[0,0.5] \wedge [0.5,1] \wedge [1,1.5] \wedge [1.5,2]$	Center point Endpoint Rectangle(s) $[0,0.5] \wedge [0.5,1] \wedge [1,1.5] \wedge [1.5,2]$
		
Sum of the area of left endpoint rectangle(s) (SALER) is ______.	Sum of the area of right endpoint rectangle(s) (SARER) is ______.	Sum of the area of center point rectangle(s) (SACPR) is ______.

The average of the above SALER & SARER is ________.

Using your answers from the front of this sheet over the given intervals, fill in the table below and then complete the statement below the table.

Interval(s)	Average of SALER & SARER	SACPR
$[0,2]$		
$[0,1] \wedge [1,2]$		
$[0,0.5] \wedge [0.5,1] \wedge [1,1.5] \wedge [1.5,2]$		

Based on the above information in the table, an estimation for the exact area between the x-axis and the graph of the curve $y = 4 - x^3$ is ________________.

Figure 29: Homework for FTC

Appendix F: Survey Participants

Table 16: List of Survey Participants

Participant	Gender	Status	Age	Highest Degree	Subject	Semester Calculus Teaching Experience	Proof Preference	Textbook
1	F	FT	60-69	Master's	Mathematics	18	MVT	2
2*	F	PT	60-69	Master's	Engineering	50	FTC I	2
3		FT	60-69	Master's	Mathematics	40	FTC I	2
4*	M	PT	30-39	Master's	Mathematics	1	FTC I	2
5						3		2
6	M	FT	40-49	Master's	Mathematics	24	MVT	4
7*	M	FT	50-59	PhD	Mathematics	65	MVT	7
8*	M	FT	50-59	Master's	Mathematics	63	FTC I	7
9	M	FT	40-49	Master's	Mathematics	18	MVT	8
10	M	FT	40-49	Master's	Mathematics	12	MVT	9
11	M	PT	over 69	Master's	Mathematics	3	MVT	9
12*	M	FT	30-39	JD	Law	12	MVT	9
13	M	FT	40-49	Master's	Mathematics	32	MVT	9
14	F	PT	30-39	Master's	Mathematics	1	FTC I	9
15	M	PT	30-39	Master's	Mathematics	3	FTC I	9
16*	M	FT	40-49	Bachelor's	Mathematics	14	FTC I	9
17	M	FT	50-59	PhD	Physics	36	FTC I	9
18	F	FT	30-39	Master's	Mathematics	5	other	9
19	M	FT	40-49	PhD	Higher Education	30	MVT	10
20	M	PT	over 69	Master's	Law	10	MVT	10
21	M	FT	40-49	Master's	Math Education	12	MVT	10
22						25		10
23*	M	PT	40-49	Master's	Math and Statistics	3	MVT	19
24*	M	FT	29-30	Master's	Mathematics	5	FTC I	19
25	M	FT	60-69	ABD	Math Education	60	other	19
26						4	other	19
27	F	FT	30-39	Master's	Mathematics	14	MVT	20
28	F	FT	40-49	Master's	Mathematics	18	MVT	20
29	M	FT	60-69	Master's	Mathematics	3	MVT	20
30	F	FT	30-39	Master's	Mathematics	7	MVT	20
31	M	FT	40-49	Master's	Mathematics	12	FTC I	20
32	M	PT	60-69	PhD	Mathematics	2	FTC I	20
33*	M	FT	50-59	Master's	Mathematics	55	FTC I	20
34	F	FT	40-49	Master's	Mathematics	10	FTC I	24

35	M	FT	40-49	Master's	Mathematics	40	MVT	25
36**	M	FT	29-30	Master's	Mathematics	8	FTC I	25
37	M	FT	40-49	Master's	Mathematics	3	MVT	28
38	F	FT	60-69	Master's	Mathematics	20	MVT	28
39	M	FT	30-39	Master's	Mathematics	2	MVT	28
40	M	PT	over 69	PhD	Engineering	10	FTC I	28
41*	M	FT	30-39	Master's	Statistics	11	FTC I	28
42	F	FT	40-49	PhD	Mathematics	12	FTC I	28
43	M	FT	40-49	Master's	Math Education	24	FTC I	28
44	M	FT	50-59	Master's	Mathematics	12	FTC I	28
45**	M	FT	40-49	Master's	Mathematics	25	FTC I	28
46	M	PT	50-59	Master's	Engineering	1	FTC I	28
47*	M	FT	60-69	Master's	Mathematics	10	other	28
48						4		28
49	M	PT	60-69	PhD	Mathematics	3		28
50						8		28
51	M	FT	30-39	Master's	Mathematics	0		
52*	M	FT	50-59	Master's	Mathematics	5		
53	M	FT	30-39	PhD	Mathematics			
54	M	PT	40-49	Master's	Mathematics			

* indicates that the participant was interviewed
** indicates that the participant was interviewed and observed

Appendix G: Survey Results

Survey of Michigan Two-Year College Calculus I Teachers

Q1) Are you teaching Calculus I in the Winter, 2016?
Yes **35**
No

Q2) Did you teach Calculus I in Fall, 2015?
Yes **15**
No

COURSE INFORMATION

Q3) The Calculus textbook I use is:

a. A common textbook selected by the department **41**
b. A textbook I chose from an appointed list **0**
c. A textbook of my own choosing **8**
d. Other (please specify) __**1 (self-written text)**______________________________

Q4) What textbook is used for your Calculus I course? Select from the list below or specify a different text if your book is not on the list.

Note the distinction between "Early Transcendentals" and standard editions. No distinction is made between single-variable and combined single- and multivariable volumes.

2. Anton/Bivens/Davis - Calculus: Early Transcendentals **5**
4. Edwards/Penney - Calculus: Early Transcendentals **1**
7. Hass/Weir/Thomas - University Calculus: Elements with Early Transcendentals **3**
8. Hughes Hallett et al. - Calculus **1**
9. Larson/Edwards - Calculus **9**
10. Larson/Hostetler/Edwards - Calculus: Early Transcendentals **4**
19. Stewart - Calculus: Concepts and Contexts **4**
20. Stewart - Calculus: Early Transcendentals **7**
24. Thomas/Weir/Hass/Giordano - Thomas' Calculus **1**
25. Thomas/Weir/Hass/Giordano - Thomas' Calculus: Early Transcendentals **2**
28. Other (Please specify Title and Author(s))___**13**______________________________

Other: Briggs – Calculus Early Transcendentals **1**
Briggs/Cochran – Calculus Early Transcendentals **3**
Briggs/Cochran/Gillett – Calculus for Scientists and Engineers **6**
Munem – Calculus with Analytic Geometry **1**
Self written - **1**
Tan – Applied Calculus **1**

Q5) Including this semester, approximately how many semesters of first semester calculus have you taught using that book?

Average: 6.12, standard deviation (sample) 5.4, Standard deviation (population) 5.34. Range: 1-25, median is 4, mode is 3.

Q6-9) The next four questions ask about your general use of your textbook

	1 (Never)	2	3	4	5	6	7 (Always)	N/A
(6) It is important to me to use the same formulas and symbols as my textbook (N/A means you do not use a textbook)	**0**	**4**	**4**	**2**	**10**	**21**	**8**	
(7) I am comfortable changing the order of topics in the textbook. (N/A means you do not use a textbook)	**2**	**3**	**2**	**2**	**14**	**14**	**12**	
(8) I assign homework from the textbook or software associated with the textbook (N/A means you do not assign homework)	**1**	**0**	**0**	**0**	**1**	**10**	**37**	
(9) I use examples from the textbook during class time (N/A means you do not use a textbook)	**4**	**12**	**6**	**6**	**6**	**5**	**10**	

Q10) In addition to the textbook, what other resources (such as graphing calculators, computer software, other books, websites, etc) do **you** use when planning to teach a first semester calculus class? Please be specific.

Q11) Does your department require **students** to purchase a common textbook for Calculus I?

Yes **41**
No **5**
Unknown **3**

Q12) What are **students** required to purchase (textbook, software, graphing calculators, etc.) for your first semester calculus course? Please be specific.

Also graphing calculator: 23
Graphing Calculator highly recommended but not required: 3

Q13) When planning to teach the Fundamental Theorem of Calculus, how often do you refer to your textbook?

Never **6**
Sometimes **29**
Always **13**

Q14-17) Please describe your general impressions of your textbook's treatment of the fundamental theorem of calculus.

	Positive (1)	2	3	4	5	6	Negative (7)
(14) What is your general impression of your textbook's overall treatment of the Fundamental Theorem of Calculus?	**7**	**13**	**11**	**12**	**5**	**0**	**0**
(15) What is your general impression of this textbook's explanation of the Fundamental Theorem of Calculus?	**6**	**16**	**9**	**11**	**6**	**0**	**0**
(16) What is your general impression of this textbook's proof(s) of the Fundamental Theorem of Calculus?	**4**	**14**	**9**	**15**	**3**	**3**	**0**
(17) What is your general impression of this textbook's problem sets relating to the Fundamental Theorem of Calculus?	**7**	**13**	**11**	**7**	**6**	**4**	**0**

Q18) What technology do **you** use when teaching the Fundamental Theorem of Calculus?

a. I do not use technology **18**
b. Graphing calculators that do not perform symbolic algebra **18**
c. Graphing calculators that perform symbolic algebra **3**
d. Computer algebra system (Maple, Mathematica, MATLAB, etc) **2**
e. Other (please describe) **4: MyMathLab, Handouts, GeoGebra, don't like the question**

Q19) What technology are your *students* **permitted** to use when learning the Fundamental Theorem of Calculus?

a. Technology not permitted. **5**
b. Graphing calculators that do not perform symbolic algebra **27**
c. Graphing calculators that perform symbolic algebra **9**
d. Computer algebra system (Maple, Mathematica, MATLAB, etc) **3**

e. Other (please describe) **3: WolframAlpha and YouTube, both a and b, question is too vague**

Q20) What technology are your students **required** to use when learning the Fundamental Theorem of Calculus?

a. Technology not required. **29**
b. Graphing calculators that do not perform symbolic algebra **13**
c. Graphing calculators that perform symbolic algebra **3**
d. Computer algebra system (Maple, Mathematica, MATLAB, etc) **1**
e. Other (please describe) **1: both a and b, depending on the question**

Q21) In my experience, explaining the Fundamental Theorem of Calculus to students is easier without added technologies.

1 Strongly Disagree	2	3	4	5	6	7 Strongly Agree
4	**6**	**9**	**10**	**4**	**10**	**4**

The fundamental theorem of calculus is often given in two parts, but the order of those parts may vary. Two proofs of a portion of the fundamental theorem of calculus are given below. Please consider both of these and then answer the following questions. Note that option 1 uses the mean value theorem in the proof and option 2 uses the other ("first") fundamental theorem of calculus in the proof. (See Appendix B)

Q22) Which proof do you prefer?

Option 1 (uses the mean value theorem) **20**
Option 2 (uses the "first" fundamental theorem of calculus) **21**
Other (please describe) **4 (3: no proof given, 1: proof by other means)**

Q23) Why do you prefer that proof? Specifically, what elements of that proof do you like?

Reasons for proof: 5 “other”, 11 “students”, 6 “self”, 5 “both”, 13 “math” and 9 no answer. (n=49, but 45 answered the which proof do you prefer, which means 5 gave a preference but no reason)

Q24) Please explain anything else you feel is relevant to teaching the Fundamental Theorem of Calculus

Q25) Your current position is best described as

Full-time **37**
Part-time **12**
Other (describe) **0**

Q26) Gender

Male **38**
Female **10**

Q27) Age
20-29 **2**
30-39 **11**
40-49 **17**
50-59 **7**
60-69 **9**
over 69 **3**

Q28) What is your highest degree attained?
PhD **8**
EdD **0**
Master's Degree **38**
Bachelor's Degree **1**
other (please describe) **2 (1: ABD, 1: JD)**

Q29) In what field is your highest degree? (check all that apply)
Mathematics **39**
Mathematics Education **3**
Statistics **2**
Physics **1**
Engineering **2**
Other Field (please describe) **3 (1: Higher Ed, 2: JD Law)**
(Note: one person had a double degree in Math and Stats)

Q30) Including this year, approximately how many semesters have you taught college calculus?

Average: 16.6, standard deviation (sample) 17, Standard deviation (population) 16.9.
Range: 1-65, median is 11.5, mode is 3.

Q31) I consider myself a(n) __________________ teacher of calculus I:
Beginner **1**
Novice **7**
Advanced **29**
Expert **12**

Q32) Why did you choose that category?

Content Knowledge	**6**
Deficit	**7**
Experience	**24**

Teaching	
Other	**1**
Students	**8**

Q34) As part of this research, I will be interviewing community college teachers about their experiences teaching calculus and teaching the Fundamental Theorem of Calculus. Would you like to be considered for an interview?

Yes **21**
No **27**

Appendix H: Philip's Schedule

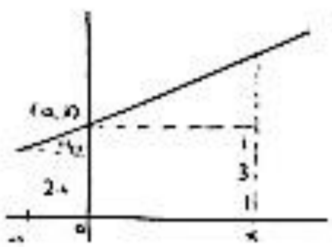
COURSE OUTLINE

Date	Reading/Assignments	Exercises
Note: some sections may not be covered in lecture, but you should read each section **before** it is covered in class and do all assigned problems after the lecture. Note: Projects will be periodically assigned throughout the semester, as time permits.		
9/8	REVIEW	Chapter 1 Exercises
9/10	2.1, 2.2	
9/15	2.2, 2.3	
9/17	2.4, 2.5	Quiz 1 – 2.1 – 2.3 (In Class)
9/22	2.5, 2.6	
9/24	Review Chapter 2, 3.1	Quiz 2 – 2.4 – 2.6 (In Class)
9/25 – 10/1	EXAM 1	Take Exam 1 in the Testing Center
9/29	3.2	
10/1	3.3, 3.4	
10/6	3.4, 3.5	Quiz 3 – Project – 3.3 – 3.4 (DUE 10/13)
10/8	3.6	
10/13	3.7, 3.8	Quiz 4 – 3.1 – 3.6 (Take Home DUE 10/15)
10/15	3.8, 3.9	
10/20	3.10, 3.11	
10/22	3.11, Review Ch. 3	Quiz 5 – 3.7 – 3.11 (In Class)
10/23 – 10/29	EXAM 2	Take Exam 2 in the Testing Center
10/27	4.1	
10/29	4.2	
11/3	4.3, 4.4	
11/5	4.4, 4.5	Quiz 6 – 4.1 – 4.4 (Take Home)
11/10	4.6, 4.7	
11/12	4.7, 4.8	Quiz 7 – Project – Chapter 4 Review (Take Home)
11/17	Review Ch. 4, 5.1	
11/17 – 11/23	EXAM 3	Take Exam 3 in the Testing Center
11/19	5.2, 5.3	
11/24	5.3, 5.4	Quiz 8 – 5.1 – 5.2 (In Class)
12/1	5.5, 5.6	
12/3	5.6, Review chapter 5	Quiz 9 – 5.3 – 5.4 (In Class)
12/4 – 12/9	EXAM 4	Take Exam 4 in the Testing Center. Note the shortened window so that exams can be returned to students by 12/10!
12/8	Review	Quiz 10 – 5.5 – 5.6 (In Class)
12/10	Review	Let's remember what we covered all semester!
12/17	FINAL EXAM	8:00 am – 9:55 am

Figure 30: Schedule from Philip

Appendix I: Karl's notes for teaching FTC

5.3 Fundamental Theorem of Calculus (p.354)



Try $f(x)=x+3$. But x is on horiz. axis. So $f(t)=t+3$

Let $F_1(x)=\int_0^x f(t)dt = \int_0^x (t+3)dt$

Then $F_1(x) = \frac{1}{2}x^2+3x$.

$F_1'(x) = x+3 = f(x)$.

Let $F_2(x)=\int_{-1}^x f(t)dt = \int_{-1}^x (t+3)dt$.

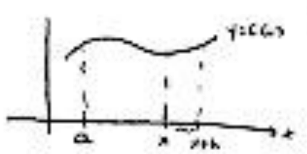
Then $F_2(x) = \frac{1}{2}x^2+3x+\frac{5}{2}$

$F_2'(x) = x+3 = f(x)$

It appears that if

$F(x) = \int_a^x f(t)dt$, then $F'(x) = f(x)$

Consider



$A(x) = \int_a^x f(t)dt$. $A(x+h) = \int_a^{x+h} f(t)dt$

$A(x+h) - A(x) \approx hf(x)$

$\frac{A(x+h)-A(x)}{h} \approx f(x)$

$\underbrace{\lim_{h\to 0} \frac{A(x+h)-A(x)}{h}}_{A'(x)} = \underbrace{\lim_{h\to0} f(x)}_{f(x)}$

Can argue for $h<0$ also

$$\boxed{\frac{d}{dx}\int_a^x f(t)dt = f(x)}$$

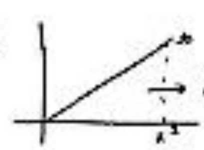
Pt 1 of FTC:

If f is cont. on $[a,b]$, then the function $F(x)$ given by $F(x) = \int_a^x f(t)dt$, $a\le x\le b$, is continuous on $[a,b]$ and differentiable on (a,b), and $F'(x) = f(x)$.

[The deriv. of the integral of a function is the function. Inverse processes.]

OO $\frac{d}{dx}\int_0^x t^3 dt$; $\frac{d}{dm}\int_{-3}^m x^3 dx$; $\frac{d}{dy}\int_y^5 z^3 dz$

Suppose $y(x) = \int_0^{x^3} t\,dt$. Find $y'(x)$.



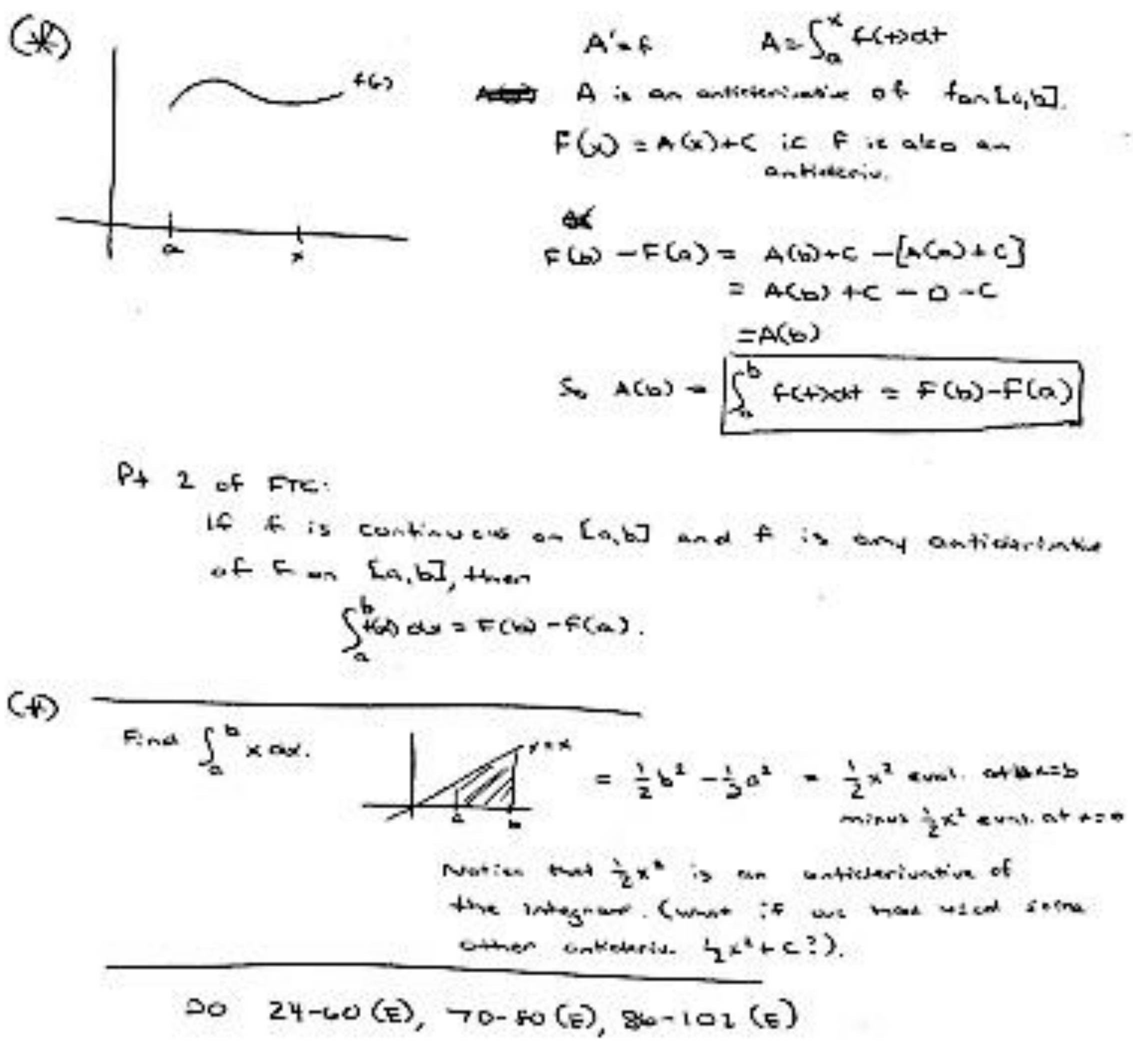
$A = \frac{1}{2}x^3 \cdot x^3 = \frac{1}{2}x^6$

$\frac{dA}{dx} = 3x^5$

$y = \int_0^u t\,dt$ where $u = x^3$

$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx} = u\cdot 3x^2 = x^3\cdot 3x^2 = 3x^5$

Do #62 – 68



(*)

$A' = f$ $\quad A = \int_a^x f(t)\,dt$

~~Also~~ A is an antiderivative of f on [a,b].

$F(x) = A(x) + C$ if F is also an antideriv.

$F(b) - F(a) = A(b) + C - [A(a) + C]$

$= A(b) + C - 0 - C$

$= A(b)$

So $A(b) = \boxed{\int_a^b f(t)\,dt = F(b) - F(a)}$

Pt 2 of FTC:

If f is continuous on [a,b] and F is any antiderivative of f on [a,b], then

$\int_a^b f(x)\,dx = F(b) - F(a)$.

(#)

Find $\int_a^b x\,dx$. $\quad = \frac{1}{2}b^2 - \frac{1}{2}a^2 = \frac{1}{2}x^2$ eval. at x=b minus $\frac{1}{2}x^2$ eval. at x=a

Notice that $\frac{1}{2}x^2$ is an antiderivative of the integrand. (What if we had used some other antideriv. $\frac{1}{2}x^2 + C$?).

Do 24-60 (E), 70-80 (E), 86-102 (E)

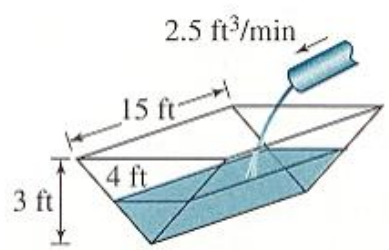
Figure 31: Teaching notes from Karl for FTC

Appendix J: Sample WebAssign Problem

The only differences between this problem and the problem in the textbook are the numbers in red.

1.FinCalc2 4.6.012.-

A trough is 15 ft long and 4 ft across the top as shown in the figure.



Its ends are isosceles triangles with height 3 ft. Water runs into the trough at the rate of 2.5 ft^3/min. How fast is the water level rising when it is 1.39 ft deep? Give your answer correct to 3 decimal places.

ft/min

Grade This | Show Answer | Try Again

*Click **Grade This** after you answer a question, and then click **Show Answer**. After answering all question parts, you can click **Try Again**.*

Figure 32: Sample problem from WebAssign

Note: Clicking on "Try Again" changes the 1.39 ft measurement.

Appendix K: Leopold's schedule

Week	Monday	Tuesday	Wednesday	Thursday	Friday
1	**Jan. 11** Chapter 1 Review	**Jan. 12**	**Jan. 13** 2.2 and 2.3	**Jan. 14**	**Jan. 15** **Last day to drop with 100% refund (no course shown)**
2	**Jan. 18** MLK DAY (no class)	**Jan. 19**	**Jan. 20** 2.5	**Jan. 21**	**Jan. 22** **Last day to drop with 50% refund (no course shown) or to change audit/credit status**
3	**Jan. 25** 2.4	**Jan. 26**	**Jan. 27** 2.1, 2.6	**Jan. 28**	**Jan. 29**
4	**Feb. 1** 2.7	**Feb. 2**	**Feb. 3** 3.1, Review	**Feb. 4**	**Feb. 5**
5	**Feb. 8** Test 1 (Chapter 2)	**Feb. 9**	**Feb. 10** 3.2 and 3.3	**Feb. 11**	**Feb. 12**
6	**Feb. 15** 3.4	**Feb. 16**	**Feb. 17** 3.5 & 3.7	**Feb. 18**	**Feb. 19**
7	**Feb. 22** 3.6	**Feb. 23**	**Feb. 24** 4.5, Review	**Feb. 25**	**Feb. 26**
8	**Feb. 29** Test 2 (3.1 – 3.7)	**Mar. 1**	**Mar. 2** 4.1, Proj Part I Due	**Mar. 3**	**Mar. 4** **Last day to drop with W regardless of current grade**
9	**Mar. 7** **Spring Break**	**Mar. 8**	**Mar. 9** **Spring Break**	**Mar. 10**	**Mar. 11**
10	**Mar. 14** 4.2	**Mar. 15**	**Mar. 16** 4.3	**Mar. 17**	**Mar. 18**
10	**Mar. 21** 2.8 and 4.4, CS 1 due	**Mar. 22**	**Mar. 23** 3.8 and 4.6, CS 2 due	**Mar. 24**	**Mar. 25**
11	**Mar. 28** 3.9, CS 3 due	**Mar. 29**	**Mar. 30** 4.7, CS 4 due	**Mar. 31**	**Apr. 1**
12	**Apr. 4** Test 3 (3.8 – 4.6)	**Apr. 5**	**Apr. 6** 4.8, Proj Part II Due	**Apr. 7**	**Apr. 8**
13	**Apr. 11** 5.5 (part 1)	**Apr. 12**	**Apr. 13** 5.1 and 5.2	**Apr. 14**	**Apr. 15**
14	**Apr. 18** 5.3	**Apr. 19**	**Apr. 20** 5.4, 5.5 (part 2)	**Apr. 21**	**Apr. 22**
15	**Apr. 25** Review, **Last day to drop with W if current grade is at least 1.0**	**Apr. 26**	**Apr. 27** Test 4 (4.7 – 5.5)	**Apr. 28**	**Apr. 29**
16	**May 2** FERP, Review, PP2 Due	**May 3**	**May 4** FERP, Review Day	**May 5**	**May 6**
17	**May 9** Final Exam	**May 10**	**May 11**	**May 12**	**May 13**

Figure 33: Schedule from Leopold

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