Temporal Scaling of Carbon Emission and Accumulation Rates: Modern Anthropogenic Emissions Compared to Estimates of PETM-Onset Accumulation

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Key Points:

- Rates are often time-scale or denominator dependent and must be compared on the same scale of time
- Modern carbon emission rates on short time scales are 9–10 times higher than estimates for carbon accumulation during onset of the PETM
- If carbon emissions continue at increasing rates we can expect to reach PETM accumulations in as few as 140 to 259 years

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Abstract

The Paleocene-Eocene thermal maximum (PETM) was caused by a massive release of carbon to the atmosphere. This is a benchmark global greenhouse warming event that raised temperatures to their warmest since extinction of the dinosaurs. Rates of carbon emission today can be compared to those during onset of the PETM in two ways: (1) projection of long-term PETM rates for comparison on an annual time scale; and (2) projection of short-term modern rates for comparison on a PETM time scale. Both require temporal scaling and extrapolation for comparison on the same time scale. PETM rates are few and projection to a short time scale is poorly constrained. Modern rates are many and projection to a longer PETM time scale is tightly constrained — modern rates are some 9–10 times higher than those during onset of the PETM. If the present trend of anthropogenic emissions continues, we can expect to reach a PETM-scale accumulation of atmospheric carbon in as few as 140 to 259 years (about 5 to 10 human generations).

Plain Language Summary

The Paleocene-Eocene thermal maximum (PETM) is a global greenhouse warming event that happened 56 million years ago, causing extinction in the world's oceans and accelerated evolution on the continents. It was caused by release of carbon dioxide and other greenhouse gases to the atmosphere. When we compare the rate of release of greenhouse gases today to the rate of accumulation during the PETM we must compare the rates on a common time scale. Projection of modern rates to a PETM time scale is tightly constrained and shows that we are now emitting carbon some 9–10 times faster than during the PETM. If the present trend of increasing carbon emissions continues, we may see PETM-magnitude extinction and accelerated evolution in as few as 140 years or about five human generations.

1 Introduction

Modern carbon release rates are known for every year from 1959 through 2015, a time series spanning 57 successive years (Fig. 1). Masses of carbon are measured in petagrams, where one petagram is equivalent to 10^{15} grams, 10^{12} kilograms, and 10^{9} metric tons. Release rates have risen steadily from 2.454 petagrams of carbon per year (PgC/yr) in 1959 to 9.897 PgC/yr in 2015, a four-fold increase in 56 years (Boden et al., 2016). The 57 modern release rates on one-year intervals can be combined to yield many additional rates on time scales ranging from two to 57 years. Modern rates considered together (57 rates on 1-year intervals + 56 rates on 2-year intervals + … + 1 rate on a 57-year interval = 1653 total rates) enable quantification of their dependency on interval length and categorization of the process involved. However, the rates have little meaning in isolation.

One way to appreciate the rates, process, and risk of present-day carbon release to the earth's atmosphere and oceans is to compare current emissions to those in earth history. The Paleocene-Eocene Thermal Maximum (PETM) 55.8 million years ago is an appropriate benchmark (Kennett & Stott, 1991; Koch et al., 1992; Zachos et al., 2001). During onset of the PETM some 2,300–12,000 petagrams of carbon (PgC) were released to the atmosphere from methane hydrates (Dickens et al., 1995), circumpolar permafrost (DeConto et al., 2012), and/or North Atlantic volcanism (Gutjahr et al., 2017). This happened within a span of 3 to 20 thousand years (Table 1).

The PETM raised global temperatures by 5–8° C, to the warmest temperatures since extinction of the dinosaurs 66 million years ago. The PETM altered the earth's carbon cycle, climate, ocean chemistry, and marine and continental ecosystems (McInerney & Wing, 2011). Benthic foraminifera suffered a major extinction (Thomas, 1989). Salient effects on land included dwarfing, floral change, and the first appearance of mammalian groups such as artiodactyls, perissodactyls, and primates that rapidly dominated later faunas (Gingerich, 1989; Clyde & Gingerich, 1998; Wing et al., 2005; Smith et al., 2009; Secord et al., 2012).

2 Modern Carbon Release Rates from the Perspective of the PETM

Eight recent modeling studies quantify rates of carbon release during the onset of the PETM. Carbon accumulations are generally estimated from the masses of carbon required to explain differences in carbon isotopic ratios before and during the event. Rates of accumulation can be calculated by dividing an estimate of total accumulation with an estimate for the corresponding interval, but extracting information from the literature is complicated when authors fail to match accumulations, intervals, and rates explicitly.

1. Zeebe et al. (2009, p. 579) estimated that some 3,000 petagrams of carbon (PgC) accumulated during onset of the PETM spanning some 5,000 years. The corresponding rate is 0.600 PgC/yr on a time scale of 5,000 years.
2. Cui et al. (2011, p. 483, fig. 4c, and table S3) estimated that 2,503 to 12,974 petagrams of carbon (PgC) accumulated during an onset interval of about19,000 years. The median accumulation appears to be 7,126 PgC and the median rate is thus about 0.375 PgC/yr on a time scale of 19,000 years.
3. Bowen et al. (2015, p. 44–45) estimated that some 3,000 PgC accumulated in two pulses during an onset interval of about 3,000 years, for a rate of about 1.000 PgC/yr on a time scale of 3,000 years.
4. Kirtland Turner and Ridgwell (2016, p. 12, table S1) made 78 estimates of carbon release rates for different masses of carbon and different release intervals. Their estimates cover all reasonable possibilities and as a result provide little constraint on these.
5. Frieling et al. (2016, p. 12,062) estimated carbon emissions during onset of the PETM to have reached 3,000 PgC over an interval of 5,000 years, for a rate of 0.600 PgC/yr on a time scale of 5,000 years.
6. Zeebe et al. (2016, p. 328) estimated that 2,500 to 4,500 PgC accumulated during a PETM onset interval of 4,000 years. Median accumulation for the interval is 3,500 PgC and the median rate is 0.875 PgC/yr on a time scale of 4,000 years.
7. Gutjahr et al. (2017: extended data table 1b) estimated an accumulation of 6,141 PgC for the 20,000-year onset duration of their assumed age model, yielding an average rate of 0.307 PgC/yr on a time scale of 20,000 years.
8. Finally, in a recent review, Kirtland Turner (2018, fig. 4a, table 1) estimated a PETM accumulation of some 4,500 PgC in 3,000 years for a rate of 1.500 PgC/yr on a time scale of 3,000 years.

The numbers here and in Table 1 for studies 1–3 and 5–8 are the masses of carbon, onset intervals, and accumulation rates extracted from each report. All should be stated and matched explicitly, however when two of the quantities are given (e.g., mass of carbon and accumulation rate), the third (e.g., onset interval) can be calculated. Each PETM mass is an average mass for a given interval and rate, each PETM interval is an average interval for a given mass and rate, and each PETM rate is an average rate for a given mass and interval.

PETM rates range from about 0.3 to 1.5 PgC/yr. For comparison, the current rate of carbon release to the atmosphere is nearly 10 PgC/yr on a time scale of one year (Boden et al., 2016), which, on the face of it, exceeds all of the PETM rates by a factor of more than six. The time scale associated with each rate is emphasized in the list above because rates are often dependent on their time scale, and this can be expected for carbon emission and accumulation rates.

Whether a median PETM rate of 0.600/yr on a median PETM time scale of 5,000 years is more or less than a modern rate of 9.897 PgC/yr on a time scale of one year is an empirical question more subtle than some people realize. A definitive answer requires that we know how the rates scale with their corresponding time intervals (denominators of the rates), and the rates must be compared on the same time scale (comparison of rates on different scales of time is a common statistical deception).

3 Spatial and Temporal Scaling

Most people understand that measured geographic features, e.g., river lengths, coastline lengths, topographic relief, etc., are scale dependent — in the sense that calculated values depend on the scale of measurement (Steinhaus, 1954; Richardson, 1961; Mandelbrot, 1967, 1983). Comparisons, to be valid, must be made on, or projected to, the same scale of measurement. It is seemingly less widely known that natural features in the temporal domain, e.g., river flow, flooding, sediment accumulation, and evolutionary change, are also scale dependent — in the sense that calculated values depend on the scale of time involved (Hurst, 1951; Sadler, 1981; Gingerich, 1983). Here again comparisons, to be valid, must be made on, or projected to, the same scale of time.

Romans (2007) , writing on the temporal scaling of Sadler (1981), introduced what he called a ‘Sadler effect’ of “measurement interval bias” implying that Sadler’s empirical relationship of rates and intervals is somehow an artifact. Gould (1984), writing on the temporal scaling of Gingerich (1983), labeled the scaling a psychological and mathematical artifact. The ‘Sadler effect’ is not a tendency to underestimate rates when averaging over long time scales, as some believe, but rather an empirical demonstration that a rate on any time scale is determined by, and remains dependent on, the time scale represented in its denominator. Rates must be brought to the same time scale for comparison. Inverse relationships of measured differences and calculated rates to their associated intervals are not artifacts, but they are widely observed and now expected features in the natural world (Mandelbrot, 1983). Further, the relationships of such differences and rates to their spatial and temporal scales in nature are proportional — linear when plotted on log-log axes — whether the values are accumulated differences or calculated rates.

The easiest way to visualize and quantify the dependence or independence of a set of accumulated differences with regard to time is to plot the logs of the accumulations against the logs of the corresponding time intervals. Accumulations and differences are used interchangeably here because a carbon accumulation is the difference between the amount of carbon present at the start of an interval and the amount present at the end of an interval. Temporal scaling requires that at least two accumulations be measured over different intervals, or at least two rates be calculated for different intervals. There is a common understanding that change expressed as difference or accumulation depends on the length of the interval involved, but a misperception that calculating rates removes this dependence. Rates are only independent of interval length in the special case when the underlying differences are wholly dependent on interval length (as when driving an automobile at a constant speed). The temporal scaling of differences and the temporal scaling of rates derived from the differences are complementary in the sense that the slopes always differ by one and the intercept is always the same (Gingerich, 2019).

Stationary time series have accumulated differences that scale with a slope at or near 0 on a log-difference versus log-interval (LDI) plot, and have rates that scale with a slope at or near −1.0 on a log-rate versus log-interval (LRI) plot. Random-walk time series have differences that scale with a slope at or near 0.5 on an LDI plot, and have rates that scale with a slope at or near −0.5 on an LRI plot. Directional time series have differences that scale with a slope at or near 1.0 on an LDI plot, and have rates that scale with a slope at or near 0.0 on an LRI plot. Temporal scaling on LDI and LRI plots shows whether and how differences and rates are influenced by corresponding intervals of time. This is true for longitudinal time series such as modern carbon emissions and emission rates, and for cross-sectional equivalents such as PETM accumulations and accumulation rates when estimates are available for independent intervals.

4 PETM and Modern Carbon Emissions and Rates

Figure 2a is a combined LDI-LRI plot for temporal scaling of the seven PETM-onset carbon accumulations and rates listed in Table 1. The seven PETM rates range from 0.307 to 1.500 PgC/yr, on time scales or intervals (rate denominators) of 3,000 to 20,000 years. A common pattern is evident: the higher rates are those calculated on shorter time scales, and the lower rates are those calculated on longer time scales. A line fit to the PETM rates (blue diamonds in Figure 2a) has a slope of −0.611, which is close to the slope expected for a random time series. A line fit to the PETM accumulations (light blue circles) has a slope of 0.389 (the complement of −0.611). Lines fit to the PETM rates and to the PETM accumulations, both on time scales ranging from 3,000 to 20,000 years, have an intercept of 2.130. This common intercept corresponds to a predicted PETM rate of $10^{2.130}$ = 135 PgC/yr on a time scale of one year.

The problem with this prediction is that it is based on relatively few (seven) PETM accumulation estimates, or rate estimates derived from these, all characterizing a single PETM-onset event. Multiple estimates represent the event itself and the associated accumulation, interval, and rate values, but they do not constrain extrapolations of accumulations or rates to different scales of time. Thus the extrapolated PETM rate of $10^{2.130}$ = 135 PgC/yr on a time scale of one year is poorly constrained and may or may not be significantly greater than modern carbon emissions on a time scale of one year.

Figure 2b is a combined LDI-LRI plot for temporal scaling of the 1,653 modern carbon emissions and emission rates based on the 57 annual values published by Boden et al. (2016). The 1,653 modern rates range from 2.454 to 9.897 PgC/yr (0.390 to 0.996 on a $\log_{10}$ scale), on time scales or intervals (rate denominators) of 1 to 57 years. A line fit to the modern rates (red diamonds in Figure 2b) has a slope of 6e-04 or 0.001, which is almost exactly the slope (0.000) expected for rates in a directional time series. A line fit to the modern accumulations (light red circles) has a slope of 1.001 (the complement of 0.001 and again the slope expected for differences in a directional time series). Both have an intercept of 0.755. This common intercept yields a predicted modern rate of $10^{0.755}$ = 5.689 PgC/yr, on a time scale of one year. The intercept for modern emissions has a narrow bootstrapped confidence interval, with limits (dashed red lines in Fig. 2) ranging from $10^{0.727}$ = 5.338 to $10^{0.781}$ = 6.038 PgC/yr.

When the modern rate of $10^{0.755}$ = 5.689 PgC/yr on a time scale of one year is projected to a median PETM time scale of 5,000 years the modern rate on this time scale is still approximately 5.689 PgC/yr. Thus, compared on the same scale of time, the modern rate of carbon emissions is significantly different and 9–10 times the median PETM rate of 0.600 PgC/yr on a PETM time scale (Table 1). Figure 2b shows this graphically by the vertical distance between the dashed double-red-line confidence interval and the open gray diamonds. Extrapolation of modern emissions (light red circles) to a PETM time scale in Figure 2b yields a similar result, where modern emissions are again projected to be some 9–10 times greater than PETM emissions (open gray circles).

5 Modern Carbon Emissions Projected Forward in Time

The temporal scaling slope of modern carbon emission rates is 0.001, which is almost exactly the slope (0.000) expected for a directional process. There is nothing stationary or random about modern carbon emissions. Emissions may change in the future, but the 57-year record of anthropogenic carbon emissions shows that we have been adding carbon to the atmosphere at annual rates increasing steadily through time (Fig. 1). The process being directional means we are, in effect, manufacturing carbon and adding it to the atmosphere as efficiently as any factory makes widgets.

Where will this lead? The increase in rates of modern emissions shown in Figure 1 is linear with slight deviations, and R . 0.1248 $t - 0.242$ is a reasonable model for the time series as a whole. If the present trend of increasing emissions continues, how long will it take to reach a PETM-magnitude carbon accumulation? A simple extrapolation is illustrated in Figure 3. Emission rates for the years from 1959 through 2015 are shown in red. The sum of carbon emissions for the years from 1959 through 2015 is 335 PgC (2.53 on the $\log_{10}$ ordinate of Fig. 3). Projecting emissions forward in time, we can expect to reach the estimated minimum PETM accumulation value of 3,000 PgC (3.477 on a $\log_{10}$ scale) in the year 2159, and we can expect to reach the maximum PETM accumulation value of 7,126 (3.853 on a $\log_{10}$ scale) in the year 2278. The year 2159 is only 140 years or about five human generations in the future, while 2278 is 259 years or about 10 generations in the future. To put these intervals in perspective: my grandfather was born 140 years ago, and Benjamin Franklin was inventing the three-wheel clock showing hours, minutes, and seconds some 259 years ago.

6 Conclusions

Temporal scaling of emission and accumulation rates can be used to compare present-day carbon emissions to carbon accumulations in the geological past. The Paleocene-Eocene thermal maximum (PETM) raised global temperatures by 5–8° C, the warmest temperatures of the past 66 million years, and the PETM altered the earth's carbon cycle, climate, ocean chemistry, and marine and continental ecosystems. Temporal scaling of PETM-onset carbon accumulation rates on long time scales might lead one to expect higher carbon emission rates than we see today on short time scales. However, the PETM rates are relatively few and temporal-scaling projection of these is poorly constrained.

The statistical advantage of projecting forward from the present to anticipated PETM-onset values in the future is that the modern samples are many, and the temporal scaling of this projection is tightly constrained. Modern carbon emission rates are increasing steadily. If this continues we can expect PETM-onset values of carbon accumulation within 140 to 259 years. A second PETM-scale global greenhouse warming event is on the horizon if we cannot lower anthropogenic carbon emission rates.

Acknowledgments, Samples, and Data

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Table 1. Published estimates for the time interval, carbon accumulation, and carbon accumulation rate during onset of the Paleocene-Eocene Thermal Maximum (PETM). Accumulation estimates from Cui et al. (2011) and Zeebe et al. (2016) are medians (see text). Maximum, median, and minimum values for the estimates are tabulated in the bottom rows of each column.

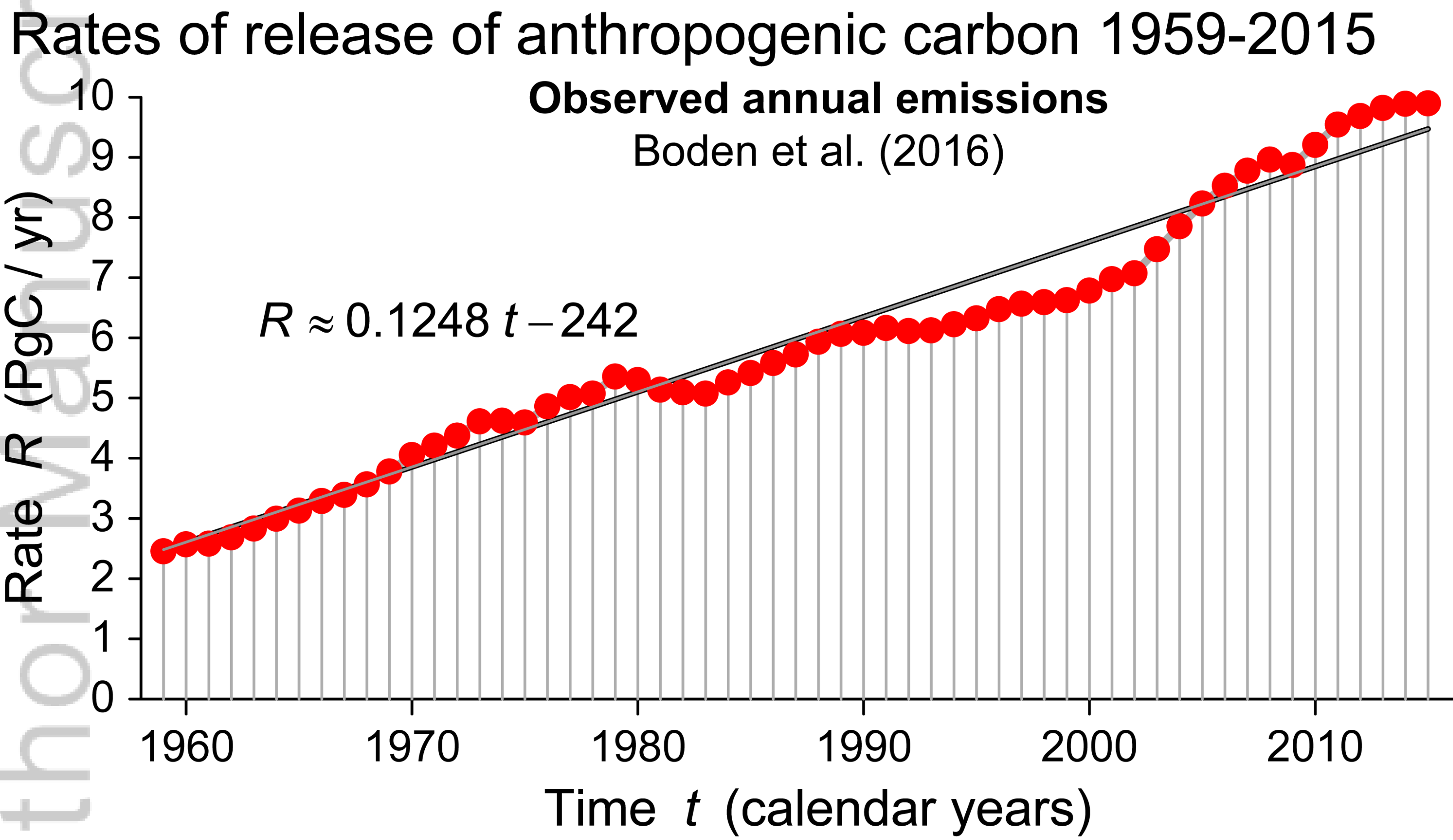
Source	Interval (yr)	Accumulation (PgC)	Rate (PgC/yr)	Log_{10} interval	Log_{10} accumulation	Log_{10} rate
Zeebe et al. (2009)	5,000	3,000	0.600	3.699	3.477	−0.222
Cui et al. (2011)	19,000	7,126	0.375	4.279	3.853	−0.426
Bowen et al. (2015)	3,000	3,000	1.000	3.477	3.477	0.000
Frieling et al. (2016)	5,000	3,000	0.600	3.699	3.477	−0.222
Zeebe et al. (2016)	4,000	3,500	0.875	3.602	3.544	−0.058
Gutjahr et al. (2017)	20,000	6,141	0.307	4.301	3.788	−0.513
Kirtland Turner (2018)	3,000	4,500	1.500	3.477	3.653	0.176
Maximum estimate	20,000	7,126	1.500	4.301	3.853	0.176
Median estimate	**5,000**	**3,500**	**0.600**	**3.699**	**3.544**	**−0.222**
Minimum estimate	3,000	3,000	0.307	3.477	3.477	−0.513

Figure 1. Global annual anthropogenic carbon emissions and carbon emission rates for the years 1959 through 2015 (Boden et al., 2016, in the Global Climate Budget of Le Quéré et al., 2016). Emission rates are now nearly 10 PgC/yr on a time scale of one year. Line fit to the points shows the long term trend: R . 0.1248 · *t* – 242.

Figure 2. Carbon accumulations and accumulation rates estimated for the onset of the PETM compared to modern carbon emissions and emission rates. (**a**) Light blue circles are PETM-onset accumulations that range from 3,000 to 7,126 petagrams of carbon (PgC). Dark blue diamonds are corresponding PETM onset rates that range from 0.307 to 1.500 PgC/yr (−0.513 to 0.176 on a $\log_{10}$ scale). These are rates on time scales (intervals or rate denominators) of 3,000 to 20,000 years (3.477 to 4.301 on a $\log_{10}$ scale; values from Table 1). Dotted lines are fit to PETM accumulations and rates for corresponding intervals, pointing to a common short-term PETM rate (intercept) of $10^{2.130}$ = 135 PgC/yr on a time scale of one year. The PETM slopes of 0.389 for accumulations and −0.611 are closest to those expected for random processes (0.500 and −0.500). (**b**) Modern carbon emissions (light red circles) range from 2.454 to 335.5 PgC, and modern emission rates (red diamonds) range from 2.454 to 9.897 PgC/yr (0.390 to 0.995 on a $\log_{10}$ scale) — on time scales of 1 to 57 years (0 to 1.756 on a $\log_{10}$ scale; annual rates are from Boden et al., 2016). Note that the slope for accumulations or emissions and the corresponding slope for rates in each panel are complementary (differing by one), while intercepts for the two are the same. PETM-onset accumulations and rates extrapolated to an annual time scale (intercepts in panel a) are about 24 times higher than those for modern emissions (intercepts in panel b), with an uncertain confidence interval. Modern emission rates extrapolated to a PETM time scale are about 9–10 times higher than PETM rates on this time scale (open gray diamonds representing the blue diamonds in panel a), and the extrapolation has a very narrow 95% confidence interval (parallel dashed red lines).

Figure 3. Model for carbon accumulation as the sum of carbon emissions, based on the steady increase in emissions and emission rates shown in Figure 1. Red circles are annual accumulations through 2015. If the recent trend in emissions continues, we can expect to reach the minimum estimate for PETM-scale carbon accumulation in the year 2159 and the maximum estimate for PETM-scale carbon accumulation in the year 2278. The light red band illustrates the range of PETM values for carbon accumulation (Table 1). The range of PETM values brackets the mass of carbon thought to remain in fossil fuel reserves (Archer et al., 2009). Finally, the PgC trajectory shown here, logged, resembles the upper bound for carbon emissions in the Representative Concentration Pathway or RCP 8.5 model of the Intergovernmental Panel on Climate Change (Ciais et al., 2013).

Figure 1.



Rates of release of anthropogenic carbon 1959-2015
Observed annual emissions
Boden et al. (2016)
R ≈ 0.1248 t − 242
Rate R (PgC / yr)
0
1
2
3
4
5
6
7
8
9
10
1960
1970
1980
1990
2000
2010
Time t (calendar years)

Figure 2.

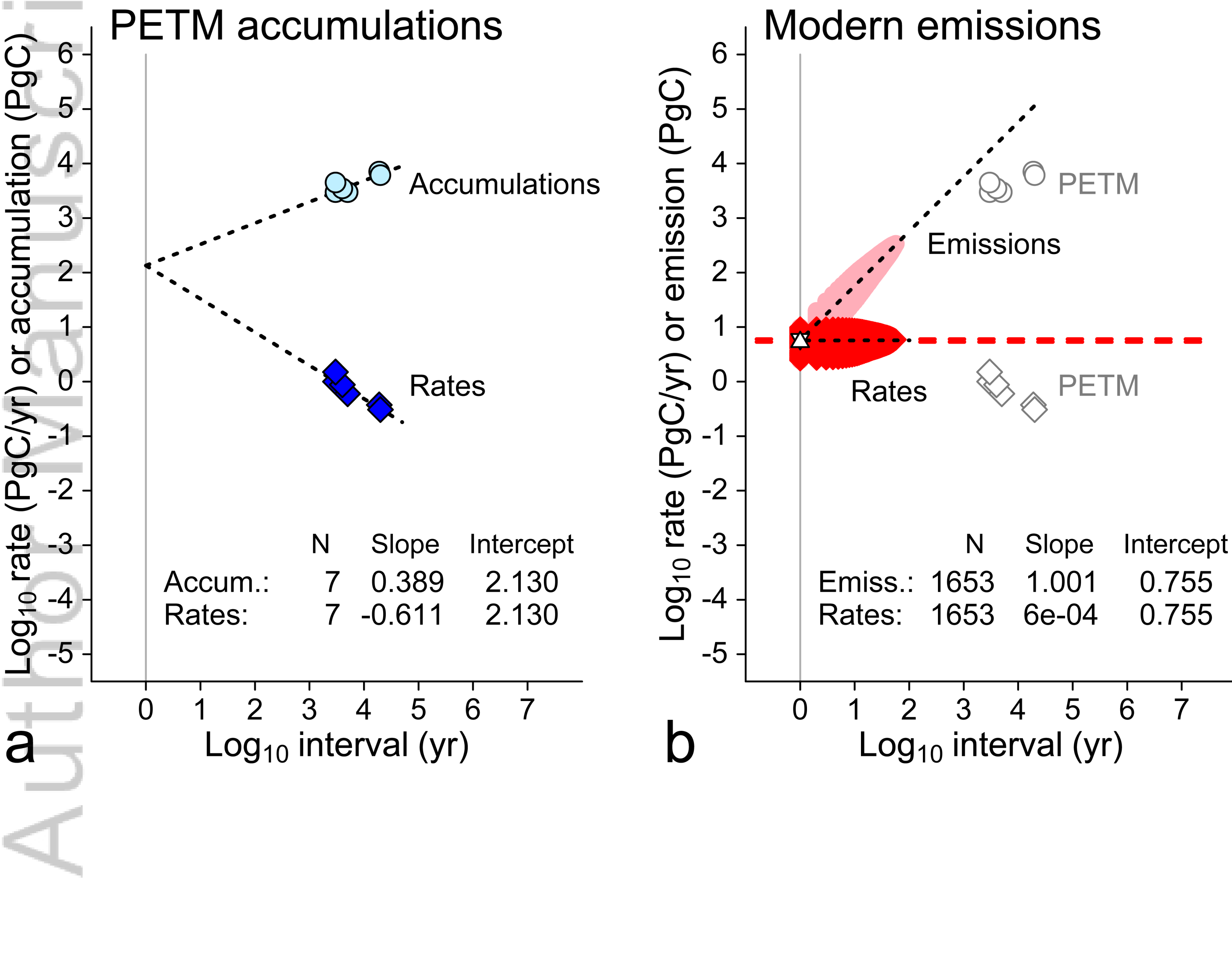
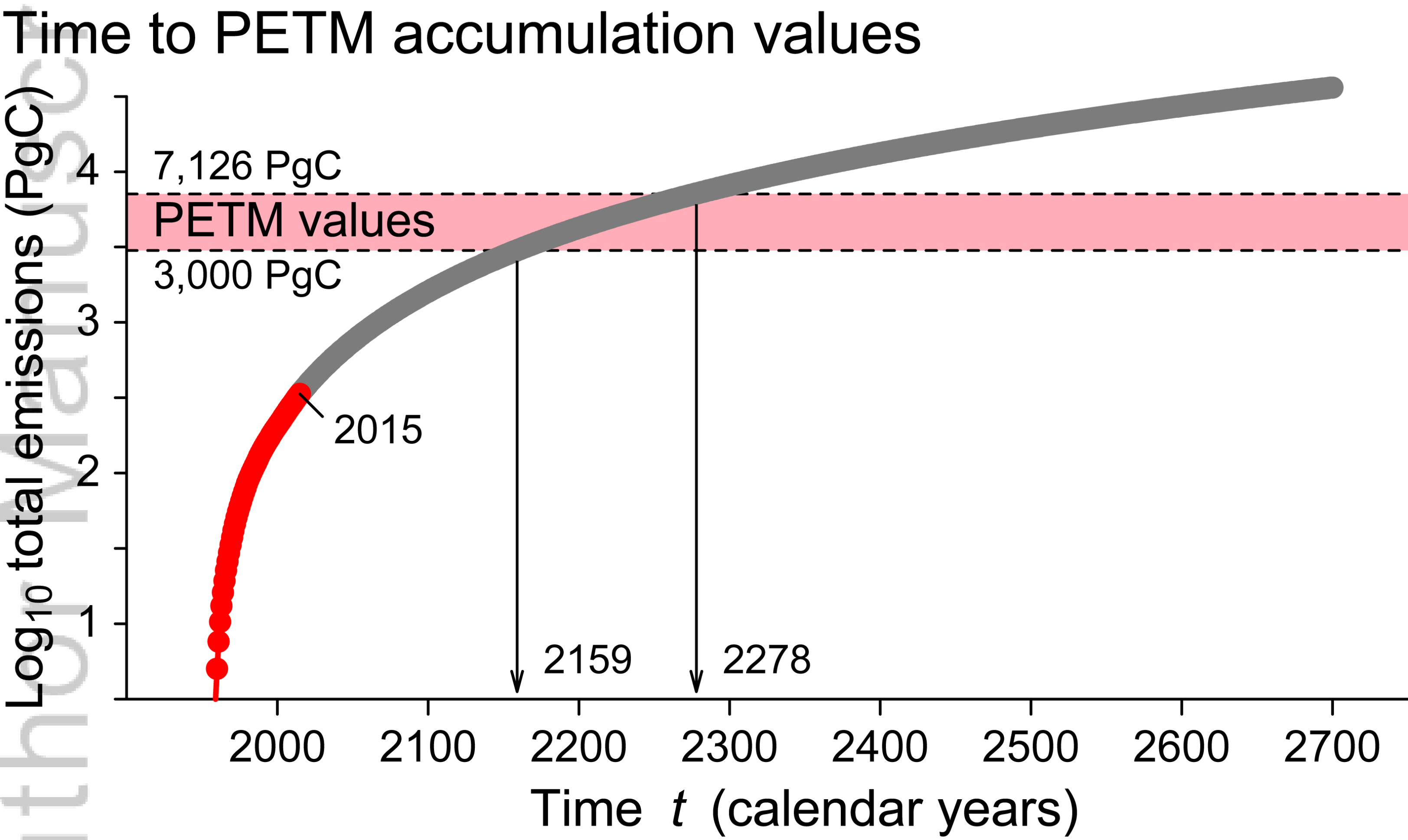

PETM accumulations
Log10 rate (PgC/yr) or accumulation (PgC)
Accumulations
Rates
N Slope Intercept
Accum.: 7 0.389 2.130
Rates: 7 -0.611 2.130
Log10 interval (yr)
a
Modern emissions
Log10 rate (PgC/yr) or emission (PgC)
PETM
Emissions
Rates
PETM
N Slope Intercept
Emiss.: 1653 1.001 0.755
Rates: 1653 6e-04 0.755
Log10 interval (yr)
b

Figure 3.


Time to PETM accumulation values
7,126 PgC
PETM values
3,000 PgC
2015
2159
2278
Log10 total emissions (PgC)
Time t (calendar years)
1
2
3
4
2000
2100
2200
2300
2400
2500
2600
2700