

Optimal Fulfillment Strategies in an Omnichannel Retail Supply Chain

by

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Abstract

With the development of digital technologies, more and more brick-and-mortar stores are starting to offer the online channel to sell their products. For example, Walmart and Whole Foods are selling fresh groceries from both their websites and store locations. As a result, such omni-channel retailers need to serve both online and in-store demand. To do that, the retailer may choose to fulfill online demand from a centralized distribution center (DC), or by utilizing inventory of stores. In this thesis, I explore the optimal fulfillment strategies of an omni-channel retailer. Firstly, consider customers' behavior when they face online and in-store purchase options. Using utility theory, model customers' behavior in preferring either channel. Secondly, I explore the impacts of retailers' fulfillment choices on its inventory cost, shipping and delivery cost, as well as overall profitability. This thesis identifies conditions under which either fulfillment strategy (i.e., from DC or stores) is optimal. And find that the optimal fulfillment strategy is dependent on the total number of stores, unit inventory cost at the stores and DC, unit delivery cost, product prices and number of stores. Case studies based on Manhattan and Los Angeles are provided to further investigate the retailer's fulfillment decision as well as the impacts of its pricing decision, and geographic and cost characteristics. For Manhattan, for both exogenous and endogenous price cases, the regions where store fulfillment are optimal first decrease and then increases as the total number of stores increases. For Los Angeles, the region where store fulfillment is optimal always increases with the total number of stores.

Chapter 1: Introduction

For a retailer, how to make the suitable products available to targeting customer is a very important issue (Tsay and Agrawal, 2004). Traditionally, it offers products via stores located in proximity to its customers, providing them a chance to experience and touch the product. On the other hand, with the advancement in digital technologies in recent years, offering products via the online channel has become an increasingly popular route that many retailers take these days. The online channel has the ability to provide product information, and therefore less search cost for customers. This advantage of online channel allows the online retailer provides a large range of products. In the same way, the Internet can improve the retailers' decision efficiency through the quick information feedback and make the collaboration between different partners (Giménez and Ramalhinho, 2004). Customers are increasingly choosing to place orders online and getting products directly delivered to their doorsteps, saving time associated with travel, by just paying a small fee or sometimes for free (Brynjolfsson et al., 2003).

As a result, a growing number of retailers, such as Walmart and Whole Foods are adding or switching completely to online channel (Nunes and Cespedes, 2003, Lang and Bressolles, 2013, Bayram and Cesaret, 2020, Singh et al., 2005). While omni-channel retail provides a significant opportunity for providing better customer service, it also introduces another dimension of complexity in the retailer's fulfillment strategy (Aspray et al., 2013). A successful fulfillment strategy in an omni-channel retail supply chain can not only reduce the operation cost and increase the profit, but also provide competitive advantages in the market (Nicholls and Watson, 2005). Within omni-channel operations, fulfillment is commonly thought as one of the most expensive and crucial (De Koster, 2002, Lummus and Vokurka, 2002). To fulfill online demand, retailers have two options, using inventory from either stores or distribution centers (DC). The benefit of fulfilling from stores is saving last-mile delivery cost due to their closer proximity to customers'

homes, especially when there are large number of stores. However, doing so incurs extra shipping cost related to the transportation of the online orders from DC to stores. More importantly, storing inventory at stores versus the DC results in a loss of pooling effect between different locations. Conversely, fulfilling online orders directly from DC leads to lower shipping cost, while allowing the retailer to better leverage the inventory pooling effect across different locations. Due to these tradeoffs, it is not an easy decision for retailer on fulfillment sources. In addition to these cost tradeoffs, fulfillment strategy will also affect consumers' purchasing behavior, which in turn can influence the demand from either channel as well as retail prices. In this research, following important research questions are examined. First, what is the optimal fulfillment strategy for an omnichannel retailer facing online (and in-store) demand? Second, how do important drivers, such as the total number of stores, inventory costs, transportation cost and retail price, impact the optimal fulfillment strategy and the retailer's profit? Finally, how do fulfillment and pricing decisions influence one another?

This thesis considers a retailer with one DC and several stores who faces online and in-store demands. I examine its optimal fulfillment strategies to maximize the total profit. I first apply utility theory to model customers' channel selection decision when shopping with the retailer. Then explore the retailer's optimal fulfillment strategy for maximizing profitability while expecting the customers' behavior. This thesis considers inventory holding cost, transportation cost from DC to stores, and online delivery cost from stores to customers for the retailer. In addition, this thesis performs analysis on the impacts of various factors, such as the number of stores, product price, and costs related to transportation and customer inconvenience, on the performance of the omnichannel supply chain. I find that the retailers optimal fulfillment strategy depends on the store's inventory cost h_s , DC's inventory cost h_c , delivery cost ϕ_d and the total number of stores N . The retailer prefers store fulfillment when the store's inventory cost is low, the delivery cost is high, the number of stores is neither too small nor too large. On the contrary, if the delivery cost is high and number of stores is either high or low, store fulfillment is only preferred if DC has very

high inventory cost. Store fulfillment is also preferred when DC's inventory cost is sufficiently high and the delivery cost is sufficiently low. Interestingly, I find that store fulfillment may be optimal even when the stores' inventory cost is high. In particular, this happens when DC's inventory cost is very high, delivery cost is high, and the number of stores is either small or large.

Finally, this thesis compares the results using case studies of Manhattan and Los Angeles with distinctive characteristics, with which study the impacts of pricing decision, and geographic and cost characteristics. For Manhattan, for both exogenous and endogenous price cases, the region where store fulfillment is optimal first decrease and then increase as the total number of stores increases. For Los Angeles, the region where store fulfillment is optimal always increases with the total number of stores.

The contributions are as follows:

- 1) This thesis provided a model framework for identifying the optimal fulfillment strategy of an omnichannel retail supply chain.
- 2) This thesis identifies conditions under which either fulfillment strategy is optimal.
- 3) This thesis explains the main tradeoffs between the fulfillment strategies in terms of transportation cost, inventory cost, demand and revenue.
- 4) This thesis illustrates the impacts of pricing decision, and geographic and cost characteristics using case studies of two US cities with differing characteristics.

The remainder of this thesis is organized as follows. In Chapter 2, I review the related researches. In Section 3, I describe the model setup, and provide analytical solutions and insights for the retailer's optimal fulfillment strategy under exogenous demand. Section 4 offers case studies of Manhattan and Los Angeles to the model and further illustrates the fulfillment strategy decision when the pricing decision is also endogenous. Sections 5 concludes and summarizes future research directions.

Chapter 2: Literature Review

I have witnessed the growth of omni-channel supply chains in various industries, especially with the development of e-commerce. With online sales increasing globally, many bricks-and-mortar retailers like Walmart are constructing online systems to serve more customers (Biggs and Suhren, 2013). Meanwhile, online retailers like Amazon are also increasingly opening or collaborating with physical stores to expand their distributions (Verhoef et al., 2007). An omni-channel retailer operates both bricks-and-mortar stores and online channel, with grocery retailers seeing the fastest expansion into omni-channel supply chains (Hübner et al., 2016), while non-food products still comprise the majority of omni-channel supply chains (Forrester 2014). Carroll (2018) (Carroll, 2018) reports that about 91% of retailers have or plan to choose omnichannel. What this thesis is concerned about is non-perishable products that can be food or non-food items.

The literature explores omnichannel from various aspects, such as pricing strategy, distribution, technology adoption, etc. Managing an omnichannel supply chain leads to increasing complexity in the retailer's operations and therefore making the fulfillment decision tradeoff more complicated. With the coexistence of online and in-store channels, the retailer now also faces the decision of how to best satisfy online customer demand. This decision can have important implications for the retailer's distribution cost, inventory, pricing decision, and customer behavior. The fulfillment strategy is the focus of this thesis.

2.1 Omnichannel Retail Supply Chain Management

This thesis reviews the literature on omnichannel retail supply chain management. Several papers focus on online retail supply chains. Småros et al. (2000) (Småros et al., 2000) study two factors that they believe are most important in online grocery business, with the first being the improvement of purchase opportunities and the second being the optimization of physical distribution. They conclude that providing various products and flexible services to customers

matter for online grocery, and that online grocery retailer needs to factor in the customers' acceptance when provide service. Swaminathan and Tayur (2003) (Swaminathan and Tayur, 2003) present a model for online retail supply chain, and point out that due to the prevalence of online channel, bricks-and-mortar retailers needs to consider not only the impact of each single parameter, but also the interaction between multiple parameters, such as supplier relationship, customization, real-time decision, distribution and pricing. They also suggest that integrating online and offline operations is becoming more important for groceries sellers. Xing et al. (2010) (Xing et al., 2010) carries out an empirical study to test a conceptual framework for physical distribution service quality on non-food supply chain. They compare the customer service of an omni-channel retailer and a purely online retailer, and argue that omni-channel retailing is a good strategy to improve retailer's service and revenue.

The most relevant to this thesis is the literature on omni-channel supply chain management. In particular, inventory fulfillment strategy is the emphasis. As a result of the physical retailing's growing overlap with online retailing, the fulfillment process is becoming increasingly nonlinear (Beck and Rygl, 2015). A number of papers study the challenges associated with fulfillment strategies in omni-channel supply chains (Melacini et al., 2018). For example, Khouja (2001) (Khouja, 2001) assumes that when the store has a stock out only part of customers accept drop-shipping, and develops a newsvendor model for the optimal fulfillment strategies. He shows that the optimal strategy is to use a mix of the fulfillment options. Beamon (2001) (Beamon, 2001) has a similar conclusion, while describing the major issues and challenges in hybrid distribution system that can serve retailers, in-store customers and online customers, with multiple order-entry system, demand pattern, transportation types, inventory and information systems, and performance measurement methods being parts of the challenge. Agrawal and Smith (2015) (Agrawal and Smith, 2015) study empirically two major furniture retailers to analyze the supply chain planning process and materials flow, and conclude that omnichannel is good method to improve the supply chain performance. Gao and Su (2017) (Gao and Su, 2017) examine the channel choice for a retailer

who offer the option to buy online and pick up in store. They find that store fulfillment may hurt online fulfillment's profit when store fulfillment costs more. Thus, allocating revenue into multiple channels can benefit the retailer. Agatz et al. (2008) offer a review of papers on the fulfillment process in omni-channel environment. They provide conceptual and quantitative reviews in purchasing, warehousing, delivery and sales functions in omnichannel supply chains, and point out that few dedicated models on omnichannel retailing exist. Alptekinoğlu and Tang (2005) (Alptekinoğlu and Tang, 2005) model a distribution system with stochastic demand. They study how orders should be placed, and how demand should be allocated between different sales locations. They also apply the model to find a cost-effective way to distribute products after two retailers merge. Compared to their paper, this thesis work considers the addition of an online sales channel in which products are directly delivered to customers, and the thesis studies how the online demand should be allocated among different facilities. Li et al. (2015) (Li et al., 2015) compare the influences of assortment, selling price and delivery time for online retailers and bricks-and-mortar retailers. They conclude that it is optimal for the retailer choose the online channel when delivery cost is low and delivery speed is fast. On the contrary, if the customer is impatient and delivery cost is high, traditional channel is preferred. This thesis also studies the channel fulfillment decision for retailer in omni-channel, but extends their work by investigating the joint influence of inventory, shipping and delivery. Ishfaq et al. (2016) (Ishfaq et al., 2016) consider the case when a traditional retailer adds an online channel, and show that the capability and configuration of retailer's distribution network influent the choice of fulfillment policy, and that integrating the online channel can generate scaling effect with a large store network. This thesis also considers the influence of retailer's distribution network to fulfillment decision and distribution system. In contrast, this thesis focuses on the fulfillment strategy of online orders. In the next section, I provide more detailed reviews on papers that focus on retailers' fulfillment decisions.

2.2 Online Demand Fulfillment

The fulfillment decision in terms of online order preparation locations generally fall into one of two methods. The first method is to fulfill online orders from distribution center or warehouse, with all the orders are picked, packed and delivered by the DC or warehouse (*DC Fulfillment*). DC fulfillment can be beneficial for omni-channel retailers because of the DC's ability to aggregate inventory, and thus provide consistently high service level (Boyer et al., 2003). Moreover, it is also much easier for the DC to set up stock levels, including cycle and safety stock, as a result of the its large warehouse space (Agatz et al., 2008), and tailored inventory levels based on the type of the product (Chiang and Monahan, 2005). In contrast, stores may need to decrease their inventory space in order to increase the selling space (De Koster, 2002). DeValve et al. (2018) (DeValve et al., 2018) aim at providing a method for online retailers to decide when and what should they do when they choose DC fulfillment. They develop a data-driven method with stochastic demand that consider local fulfillment constraints and customer abandonment to find the optimal strategy. Hübner et al. (2016) (Hübner et al., 2016) focus on solving the last-mile delivery problem as the fulfillment strategy for a single channel, either online channel or bricks-and-mortar channel.

The second method of fulfilling online demand is for all orders and products to be prepared and delivered from existing retail stores (*Store Fulfillment*). A key benefit of this is that it utilizes the close proximity of retail store and end consumers, thus reducing last-mile delivery distance and delivery time. The convenience of backward distribution system, such as exchange and return, makes it can provide higher backward customer service (Lang and Bressolles, 2013). Smith and Sparks (2009) (Smith and Sparks, 2009) use Tesco as an example to illustrate the store fulfillment strategy. They discuss the success story of Tesco's supply chain transformation into omnichannel operations and provide lessons for other omni-channel retailers. Bayram and Cesaret (2020) (Bayram and Cesaret, 2020) assume that the online orders are fulfilled from inventory of nearby stores, and investigate the dynamic decision of where to fulfill an incoming order from through a heuristic method. Uncertain demand, handling cost and shipping cost are involved in their dynamic model. Different from their work, this thesis focuses on the higher-level strategic decision of

whether online orders are to be fulfilled from stores or a centralized warehouse. In addition, this thesis also considers the role of transportation and inventory costs in the retailer's decision.

Bailey and Rabinovich (2005) (Bailey and Rabinovich, 2005) study online book retailer that fulfill orders from store inventory or by drop-shipping, where they assume the costs include fixed and linear cycle costs. Their results suggest that using both fulfillment options at the same time is beneficial for the omni-channel retailer. The objective of the retailer is to minimize the inventory cost while maximizing the customer's order fill rate. This thesis extends their setting considering the combination of in-store and online channels, while adding other operational cost including shipping and delivery. Bendoly et al. (2007) (Bendoly et al., 2007) study the operational strategy for an omnichannel retailer based on the assumption that the stores hold either all or none of the inventory. This thesis extends their discussion by allowing stores to hold only part of the inventory, and model customer channel choice as well as last-mile delivery. Jalilipour Alishah et al. (2015) (Jalilipour Alishah et al., 2015) explore the allocation of inventory between one store and one fulfillment distribution center while assuming that the offline channel holds extra inventory for fulfilling demand from the online channel. Then they increase the store number based on previous setting in 2018 (Alishah et al., 2018). They show that the online channel can benefit more from inventory pooling effect than the physical channel because it can more easily change the inventory quantity and location according to customers' distribution, and that the inventory from multiple locations can be shared to satisfy online demand. In contrast from their work, this thesis assumes that the online order inventory can be stored in distribution centers or stores, DC does not hold store inventory, and also models the last-mile delivery of online demand to end consumers. In addition to the pooling of inventory from different locations, this thesis also points out another type of inventory pooling effect associated with the pooling of online and in-store demand.

2.3 The Traveling Salesman Problem

Order fulfillment strategy for online channel is different from physical channels, as a result of the small order size from individual customers and the large total volume of orders (Tarn et al., 2003). Thus, the logistics of delivering customer orders to them is an important aspect of the

fulfillment strategy. This thesis takes into consideration of shipment from the distribution center to stores and last-mile delivery to customers' homes, and finds that this transportation cost plays an important role in determining fulfillment strategy.

The transportation from DC to multiple retail stores in this work is modeled as a traveling salesman problem (TSP), which is concerned with finding the optimal route that minimizes transportation cost or distance to visit a given set of locations and return to the starting point. TSP has been studied extensively in the literature. Dantzig and Ramser (1959) (Dantzig and Ramser, 1959) consider the problem when there are a number of service stations with given demand between two points. They study how to design routing to let a fleet of vehicles satisfy the demand while minimizing total travel distance. Beardwood et al. (1959) (Beardwood et al., 1959) prove that the shortest distance through N locations in a unit bounded region is asymptotically proportional to \sqrt{N} . Daganzo (1984a, 1984b) (Daganzo, 1984, 1984) develop formulas for estimating the vehicle's travel distance as a function, the function considers the influences of the depot's area and shape. Lawler et al. (1985) (Lawler, 1985) discuss the theory, solution methods and application on this problem. Haimovich and Rinnooy Kan (1985) (Haimovich and Rinnooy Kan, 1985) study the property and solution methods of a capacitated vehicle routing problem where a fleet of vehicles with limited capacities is serving some customers who are located in Euclidean plane. Some researchers (Burns et al., 1985, Federgruen and Zipkin, 1984, Gallego and Simchi-Levi, 1990) extend the problem by considering inventory management.

Matai et al. (2010) (Matai et al., 2010) divides research on TSP into seven directions based on the applications. The first direction is drilling of printed circuit boards problem that minimizes the movement time of drill heads (Grötschel and Holland, 1991). Lim et al. (2014) (Lim et al., 2014) propose a combinatorial cuckoo search algorithm and apply it to three cases to optimize the path of drill holes. The second direction is overhauling gas turbine engines while guaranteeing gas is uniformly distributed through the turbine. Plante et al. (1987) (Plante et al., 1987) develop a heuristic algorithm for placing the vanes in the turbine. Third, TSP can be used in X-Ray

crystallography to determine the sequence of detectors. Bland and Shallcross (1989) problem (Bland and Shallcross, 1989) compare different TSP algorithms in solving the. Fourth, TSP problem is used to minimize the length of wires in computer wiring. Lenstra and Kan (1975) (Lenstra and Kan, 1975) provide the TSP formulations for not only computer wiring and vehicle routing, but also clustering and job-shop scheduling problems. Grötschel and Holland (1991) (Grötschel and Holland, 1991) point out another application of TSP which is mask plotting in printed circuit boards. Another application and the most relevant to this thesis is vehicle routing, for example to determine the route and capacity assignment for delivering products to a number customers with a fleet of trucks with the objective of minimizing the length of the delivery distance. In addition, TSP has been applied to solve order warehouse order picking problems, where the sequence and routes for the pickers to pick up multiple products for one or more orders are determined. For example, Theys et al. (2010) (Theys et al., 2010) compare the solution generation using two algorithms and eight hybrid heuristics for a warehouse order picking problem with multiple parallel aisles. They recommend the hybrid algorithm in order to improve effectiveness.

The traveling behavior among end customers is another important strategic issue for TSP. In general, the customers desire the convenience of nearby stores. Pancras et al. (2012) (Pancras et al., 2012) show that customers consider their travel cost when shopping, and estimate that the fast food customers' inconvenient travel cost per mile is \$0.6 on average. As a result, many large grocers like Walmart, have been increasing the total number of stores (FORM 10-K, 2016). Cachon (2014) (Cachon, 2014) studies a retailer's facility location problem while modeling the shipping from a warehouse to stores as a TSP. He also studies the impact of an emission tax on this decision and its impact of overall emissions.

Due to the complexity of TSP, few papers in the literature study the combination of TSP with other problems. Cachon (2014) combines TSP with a retailer's facility location decisions. However, he does not consider the fulfillment of omnichannel demand or the last-mile delivery decisions. This thesis contributes to the literature on TSP by studying an integrated model that synthesizes

shipping, omnichannel order fulfillment, last-mile delivery and consumer channel purchasing behavior.

2.4 Last-mile Delivery

The cost efficiency of delivering online demand to end consumers is a major challenge for the omnichannel retailer. A high-efficiency delivery system is an determining factor of the retailer's business viability (Agatz, Fleischmann and Van Nunen, 2008). Meanwhile, customer service, especially for last-mile delivery service, is an important factor of customer satisfaction (Boyer et al., 2005). There is a growing number of researches focusing on the quality of the last-mile delivery service (Rabinovich and Bailey, 2004). In the highly competitive retail market, finding the balance between cost efficiency and customer service is key for the retailer (Boyer, Hult and Frohlich, 2003). As a result, there has been an increasing number of research papers on this topic. They (Belavina et al., 2017, Hsu and Li, 2006, Punakivi and Saranen, 2001, Punakivi and Tanskanen, 2002, Punakivi et al., 2001, Yrjo, 2001) compare the distribution costs among several fulfillment strategies, and find that the retailer should gradually expand the traditional stores' capabilities to satisfy the customers. Hsu and Li (2006) (Hsu and Li, 2006) study lead-time dependent demand, and develop a non-linear profit model to find the optimal delivery shipment cycle to balance the delivery cost and customer service. Their numerical analysis suggests that compared to imposing a static policy, adjusting shipping frequencies to temporary and regional demand is a better strategy. Belavina et al. (2017) (Belavina et al., 2017) study the tradeoff between two revenue models for an omnichannel retailer, including the per-order model and the subscription model. They also study the retailer's last-mile delivery decision and the resulting environmental performance. Lin and Mahmassani (2002) (Lin and Mahmassani, 2002) perform a simulation to illustrate the influence of potential cost on customer service, where cost is estimated based on the delivery time window for different delivery policies. Robusté et al. (2003) (Robusté et al., 2003) use continuous approximation method to analyze the delivery time window and efficiency. They show that the influence of time window increases with the increase of delivery vehicle capacity.

In this thesis, according to the review of last-mile research in Olsson et al. (2019) (Olsson et al., 2019), there are very little literatures on the last-mile fulfillment problem. Leung et al. (2018) (Leung et al., 2018) use genetic algorithm to re-engineer the last-mile order fulfillment process in a distribution center. Other papers (Letnik et al., 2018, Daniela, 2017, Nathanail et al., 2016, Handoko et al., 2016) related to last-mile fulfillment direction mainly focus on urban freight terminals problems. This thesis enriches this research direction by combining the retailer's fulfillment strategy decision with last-mile delivery problem. Specifically, this thesis considers retailer's shipping cost, last-mile delivery cost, customers' channel choice and demand. The impacts of various factors such as the geographic characteristics of the service region, unit inventory holding cost, and customer valuation are also studied.

Chapter 3: The Model

Consider a retailer selling a generalized product for price p to customers in a given service region of size A . Note that the price can either be an exogenous value set by market competition or an endogenously determined by the retailer. This thesis discusses the exogenous price case in this section, and then considers endogenous price cases in Section 4.2. According to Proposition B.1 (see in Appendix B), the optimal profit can be got in either store fulfillment or DC fulfillment. As a result, this thesis limits the attention to discrete fulfillment strategy only in all following discussion. The notations of the paper are shown in Table 3.1.

Customers are uniformly distributed throughout the service region. Without loss of generality, this thesis normalizes the population density to be 1. The retailer has a single distribution center (DC) at the geometric center of the region, which fulfills orders from N identical and uniformly distributed retail stores. The retailer offers both in-store and online channels. In-store customers travel to the stores in their personal vehicles, while online orders are delivered to customer's homes by the retailer's trucks. Figure 3.1 illustrates the service region.

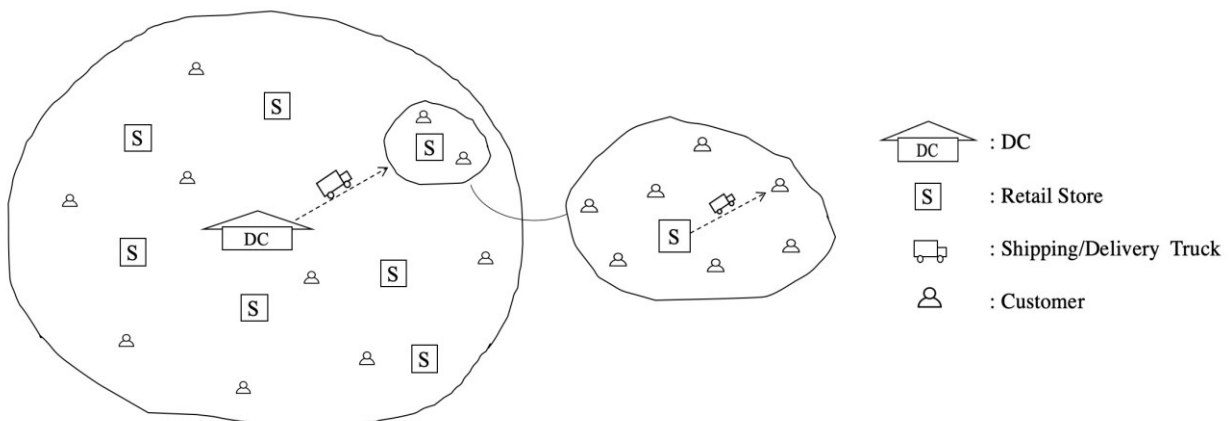


Figure 3.1: Service region

Table 3.1: Notations

Symbol	Meaning
v	Customer's valuation of a product, v is uniform distributed on $[0, V]$
p	A product's exogenous selling price
N	Number of retail stores
c_o	Online shopping cost for online customers
c_i	Inconvenience cost for in-store customers, c_i is uniform distributed on $[0, C/N]$
U^j	In-store or online customer's utility, where $j = i, o$
D^j	In-store, online and total demand rate, $j = i, o$ and t
D^s	Demand fulfilled from store
Γ_j	Average delivery distance of one order from retailers to end customers, $j = S, C$
W	Delivery distance determined by service region
λ	Delivery distance determined by truck's capacity
ϕ_d	Delivery cost from store to customer per unit distance per unit product
ϕ_t	Shipping cost from DC to customer per unit distance per unit product
Φ_t	Coefficient dependent on the area and region's tessellation type
C_t	Shipping cost per unit product
ϕ_I	A retailer's service level
h_j	Inventory cost per unit product per unit time in store or DC, $j = S, C$
H_j	Total inventory cost per unit time when online orders fulfilled from store or DC, $j = S, C$
C_S	Retailer's total cost if online orders are fulfilled from stores
C_C	Retailer's total cost if online orders are fulfilled from DC
ΔC_h	Inventory cost difference under two fulfillment strategies
ΔC_d	Delivery cost difference under two fulfillment strategies
ΔC_t	Shipping cost difference under two fulfillment strategies
ΔC_T	Total transportation cost difference under two fulfillment strategies
π_S	Retailer's profit if online orders are fulfilled from stores
π_C	Retailer's profit if online orders are fulfilled from DC

3.1 A Customer's Channel Choice

Each customer first makes channel choice between buying online versus in-store when she purchases a product. The customer has a valuation v for the product, which is assumed to be heterogeneous and follows a uniform distribution on $[0, V]$, where V is the highest possible valuation. If the customer chooses buying online, then she incurs an associated cost c_o , such as order delivery fee. Similarly, when the customer purchases in-store, then an inconvenient cost c_i

is incurred for the time and cost associated with traveling to the store and hand picking up the product. This thesis assumes that c_i is uniformly distributed on $\left[0, \frac{C}{N}\right]$, where C is the maximum inconvenient cost if there is only one store in the region. Therefore, the more stores there are in the region, the less inconvenience cost is incurred by customers to shop in-store. This is because of the distance that customers need to travel is shorter when the number of stores increases. c_i is independent from v . This thesis also assumes that $V > c_o + p$ and $\frac{C}{N} > c_o$ to eliminate the non-trivial case where online demand is non-zero.

If a customer chooses the online channel, her pay-off is $U^o = v - c_o - p$. If a customer chooses the in-store channel, her pay-off is $U^i = v - c_i - p$. Therefore, she would choose to buy online if and only if $U^o \geq U^i$ and $U^o \geq 0$. This is equivalent to $c_o \geq c_i$ and $v \geq c_o + p$. Similarly, she would choose to buy from the in-store channel if and only if $U^i \geq U^o$ and $U^i \geq 0$, or equivalently, $c_o \leq c_i$ and $v \geq c_i + p$. Otherwise, $U^i \leq 0$ and she would buy nothing. The regions for these choices are illustrated in Figure 3.2.

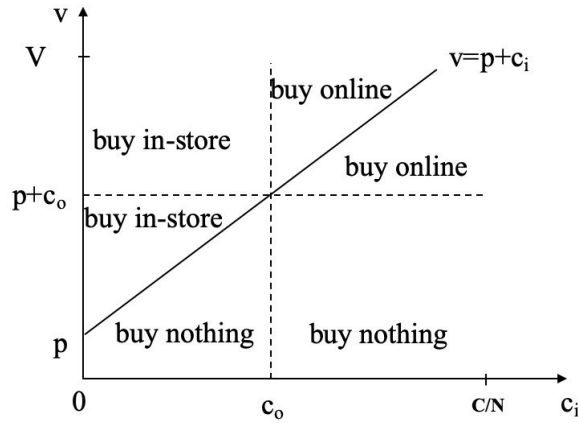


Figure 3.2: Customers' channel choice

Therefore, the expected demand for the in-store channel is given by

$$D^i(p) = \int_0^{c_o} \frac{N}{C} \int_{c_i+p}^V \frac{1}{V} dv dc_i = \left(V - p - \frac{c_o}{2}\right) \frac{c_o N}{CV}, \quad (1)$$

and the expected demand for the online channel is given by

$$D^o(p) = \int_0^{\frac{C}{N}} \frac{N}{C} \int_{c_o+p}^V \frac{1}{V} dv dc_i = (V - p - c_o) \frac{C - c_o N}{CV}. \quad (2)$$

The total expected demand rate for both channels is hence

$$D^t(p) = D^i(p) + D^o(p) = \frac{Nc_o^2}{2CV} - \frac{c_o + p}{V} + 1. \quad (3)$$

In the following Lemma 3.1, I examine the impacts of price p and the total number of stores N on the demands of each channel. All proofs are provided in the appendix for ease of exposition.

Lemma 3.1: $\frac{\partial D^i}{\partial p} < 0$, $\frac{\partial D^i}{\partial N} > 0$, $\frac{\partial D^i}{\partial v} > 0$, $\frac{\partial D^i}{\partial C} < 0$, $\frac{\partial D^i}{\partial c_o} > 0$; $\frac{\partial D^o}{\partial p} < 0$, $\frac{\partial D^o}{\partial N} < 0$, $\frac{\partial D^o}{\partial v} > 0$, $\frac{\partial D^o}{\partial c_o} < 0$, $\frac{\partial D^o}{\partial C} > 0$.

From Lemma 3.1, both online and in-store demands are decreasing in the price p and increasing in customers valuation of the product v , with the increase of max value of customer's inconvenient cost C , in-store demand decrease and online demand increase, as expected. With increase of c_o , online demand decreases and in-store demand increases. Result from increase cost in online channel, part of the customers who can't adopt online channel transfer to in-store channel. In addition, as the total number of stores increase or as C decreases, in-store demand increases, while online demand decreases. This is because the distances between customers and the stores decrease as the number of stores increases, and thus their inconvenience cost associated with in-store shopping decreases, making shopping in-store a more attractive option.

3.2 The Retailer's Problem

The retailer anticipates customers' channel choice behavior as described in the last section. This thesis assumes that customer orders for the in-store and online channels arrive following the Poisson distribution, whose averages are given by D^i and D^o respectively (I omit the term p in $D^i(p)$ and $D^o(p)$ wherever applicable from this point on for ease of exposition). The retailer then chooses the optimal fulfillment options for online orders, that is, whether to fulfill them from retail store inventory or directly from DC inventory. This thesis refers to the former as *store fulfillment* and the latter as *DC fulfillment*. Note that this thesis examines the base setting in this section, where the retail price is assumed to be exogenous (i.e., set by market competition). Figure

3.3(a) and Figure 3.3(b) illustrate the retailer's behavior if online orders are fulfilled from stores or DC, respectively.

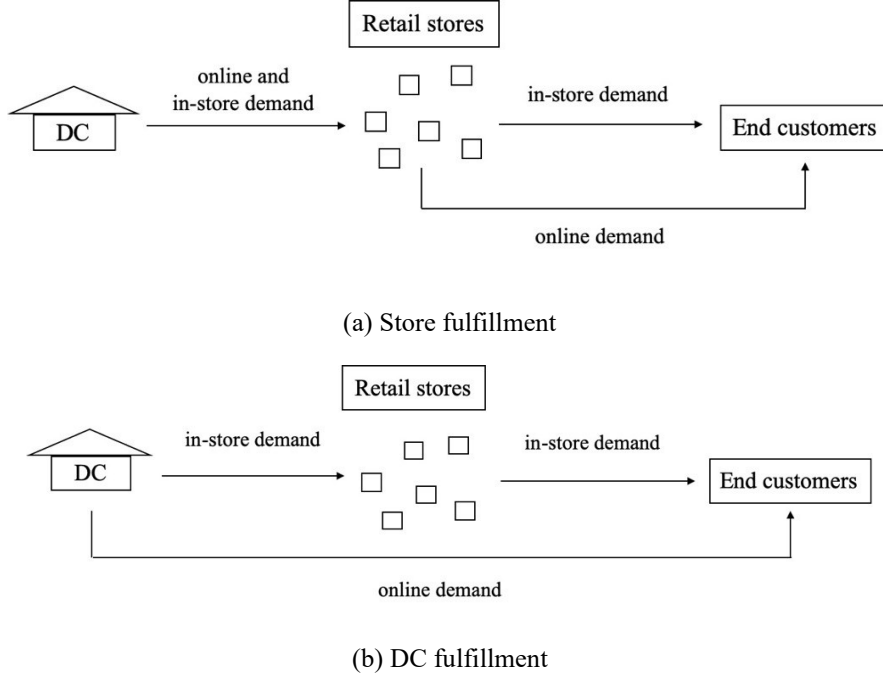


Figure 3.3: Retailer's decision behaviors under two fulfillment strategies

3.2.1 Inventory Cost

Independent of the fulfillment strategy, the retailer's inventory cost per unit of product sold per unit time is concave increasing in the number of stores N and the store's order quantity Q_s , and decreasing in total demand rate D_s fulfilled by all of the stores (Cachon, 2014). Note that D_s includes the demand from in-store, as well as the portion of online demand that is fulfilled from store inventory. Specifically, the stores' inventory cost per unit of product per unit time is given by

$$h_s \phi_I Q_s^{\frac{1}{2}} \sigma_s D_s^{-\frac{3}{2}} \sqrt{N} = h_s \phi_I Q_s^{\frac{1}{2}} D_s^{-1} \sqrt{N}, \quad (4)$$

where ϕ_I is the desired service level (assumed to be the same for DC and stores), σ_s is the standard deviation of the demand fulfilled by the stores, and h_s is the space cost of inventory per unit per unit time for the store. The derivation of this equation can be found in (Cachon, 2014).

Based on the Poisson demand assumption, $\sigma_s = D_s^{\frac{1}{2}}$, and hence the equation in (4) holds.

Therefore, the stores' total inventory cost per unit time H_S is given by

$$H_S = h_s \phi_I Q_S^2 D_S^{-1} \sqrt{N} \times D_S = h_s \phi_I Q_S^2 \sqrt{N}. \quad (5)$$

This thesis assumes lot-for-lot ordering policy at the DC, that is, the DC matches the total order size (and frequency) from the stores and only holds inventory if it fulfills online orders directly. The base stock order policy is used for online orders. Let Q_c be DC's order quantity, which consists of the quantity for fulfilling both store orders, and that for fulfilling online orders if it uses the DC fulfillment strategy. Then, the DC's inventory cost H_C per unit time is given by

$$H_C = h_c \phi_I (Q_c - Q_S)^2, \quad (6)$$

where h_c is the inventory cost per unit per unit time for the DC. This thesis assumes $h_s > h_c$ to reflect that the unit inventory cost at DC is less than that in store due to economy of scale.

Therefore, the retailer's total inventory cost per unit time is

$$H_S + H_C = h_s \phi_I Q_S^2 \sqrt{N} + h_c \phi_I (Q_c - Q_S)^2. \quad (7)$$

If the retailer chooses to fulfill online orders from stores, then the stores need to order both in-store and online demands and the DC orders the same amount. That is,

$$Q_S = Q_c = D^o + D^i. \quad (8)$$

Substituting equation (8) into (5) gives the retailer's inventory cost per unit time under the store fulfillment strategy:

$$C_h^S = h_s \phi_I (D^o + D^i)^2 \sqrt{N}. \quad (9)$$

If the retailer chooses to fulfill online orders from DC inventory, then the stores only need to order to fulfill in-store demand while the DC orders for both channels. That is,

$$Q_S = D^i, Q_c = D^o + D^i. \quad (10)$$

Substituting Equation (10) into Equation (7) gives the retailer's inventory cost per unit time under the DC fulfillment strategy:

$$C_h^C = h_c \phi_I D^{o2} + h_s \phi_I D^{i2} \sqrt{N}. \quad (11)$$

The inventory cost difference between two fulfillment strategies ΔC_h is given by

$$\Delta C_h = C_h^S - C_h^C = h_s \phi_I \sqrt{N} \left[(D^o + D^i)^{\frac{1}{2}} - D^{i\frac{1}{2}} \right] - h_c \phi_I D^{o\frac{1}{2}}. \quad (12)$$

The following Lemma 3.2 suggests that the store fulfillment strategy can lead to higher or lower inventory cost than the DC fulfillment strategy.

Lemma 3.2: $\Delta C_h \geq 0$ if $h_c \leq h_{c0}$, where $h_{c0} = \frac{h_s \sqrt{N} \left\{ \left[\frac{Nc_o^2 + 2C(V-p-c_o)}{2} \right]^{\frac{1}{2}} - \left[c_o N \left(V - p - \frac{c_o}{2} \right) \right]^{\frac{1}{2}} \right\}}{[(V-p-c_o)(C-c_oN)]^{\frac{1}{2}}}$.

$\Delta C_h < 0$, otherwise.

Lemma 3.2 suggests that fulfilling from the DC leads to lower inventory cost than store fulfillment if the unit inventory cost in DC is smaller than a threshold h_{c0} , and to higher inventory cost otherwise.

Corollary 3.1: h_{c0} defined in Lemma 3.2 has the following properties:

(i) $\frac{\partial h_{c0}}{\partial p} < 0$,

(ii) $\frac{\partial h_{c0}}{\partial C} > 0$,

(iii) $\frac{\partial h_{c0}}{\partial V} > 0$,

(iv) There exists $N_0 > 1$ such that $\frac{\partial h_{c0}}{\partial N} \leq 0$ if $N \geq N_0$, and $\frac{\partial h_{c0}}{\partial N} > 0$ otherwise

Corollary 3.1 suggests that h_{c0} increases with the total number of stores N for small N , while it decreases with N for large N . Then based on Lemma 3.2, when N is small, an increase in N leads to higher likelihood of that store fulfillment has higher inventory cost. In contrast, when N is large, an increase in N leads to lower likelihood of that store fulfillment has higher inventory cost.

This effect can be explained as follows. Two types of inventory pooling effects may come into play in this system. If online demand is fulfilled from the DC, then there is benefit due to the pooling of online demand inventory from different store locations into the DC's warehouse, to which this thesis refers as *location pooling*. If the store fulfills online demand, then a second effect as a result of the pooling of online and store inventory at each store also occurs, to which this thesis

refers as *channel pooling*. The balance of these two pooling effects determines the holding cost difference.

The total number of stores N in turn impacts the above two pooling effects in two important ways. First, an increase in N has a direct increasing effect on location pooling. Second, it reduces the portion of online demand, while increases the portion of in-store demand. When N is small, the first way dominates and hence location pooling effect is increasing in N . When N is large, the second way dominates and hence location pooling effect is decreasing in N . The channel pooling effect, that depends on the volume of online and in-store demand. Specifically, if either demand is small, then the channel pooling effect would be small. Since in-store demand increases with N and online demand decreases with N , the channel pooling effect firstly increases with N when N is small, and then decreases with N when N is sufficiently large.

When the above two pooling effects are combined, the impacts of N on the inventory cost difference between the two strategies can be observed. If N is relatively small, then both effects are increasing in N , while the rate of increase is higher for location pooling than channel pooling. Therefore, in this case, the advantage of DC fulfillment in inventory cost increases with N . If N is relatively large, then both effects are decreasing in N , while the rate of decrease is higher for location pooling than channel pooling. Therefore, in this case, the advantage of DC fulfillment in inventory cost decreases with N .

3.2.2 Logistic Cost

To fulfill the in-store demand, the DC solves a traveling salesman problem (Lin and Kernighan, 1973) and ships products to stores at a cost of ϕ_t per unit distance per unit product. this thesis follows Cachon (2014) (Cachon, 2014) and assume that $N \gg 1$, under which condition the transportation cost is solely dependent on the area of the service region denoted by A and the shape of the tessellations. Specifically, the shipping cost C_t per unit product from DC to stores is given by

$$C_t = \phi_t \Phi_t \sqrt{N}, \quad (13)$$

where Φ_t is a constant dependent only on the area of the region and the type of tessellation. For example, for square tessellation (see Figure 3.4 for an illustration), $\Phi_t = \sqrt{A}$. In what follows, this thesis considers square tessellation for ease of exposition. However, the results can be extended to other types of tessellations.

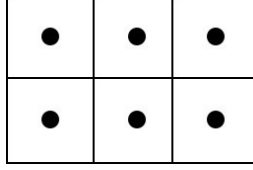


Figure 3.4: Square tessellation of stores

Obviously, the delivery distance for fulfilling online orders if they were fulfilled by stores would be different from if they were fulfilled directly from DC inventory. Following Belavina et al. (2017) (Belavina et al., 2017), the average delivery distance per order can be calculated as

$$\begin{cases} \Gamma_S = \frac{W}{\sqrt{N}} + \lambda, & \text{if orders are fulfilled from stores,} \\ \Gamma_C = W + \lambda, & \text{if orders are fulfilled from DC,} \end{cases} \quad (14)$$

where W is a constant determined by size and shape of the service region as well as the number of orders that each truck can deliver, and λ is also a constant determined by the number of orders that each truck can deliver, days in operation, the density of customers and the total annual order frequency. The detailed expressions of W and λ are provided in the appendix.

Let ϕ_d be the transportation cost per unit distance per unit product for delivery. The last-mile delivery cost is usually much higher than the transportation cost from DC to stores, due to a lack of economy of scale (Lee and Whang, 2001). To reflect this relationship, this thesis assumes that $\phi_d > \phi_t$. Therefore, the delivery cost under store fulfillment is

$$C_d^S = \phi_d D^o \Gamma_S,$$

and under DC fulfillment is

$$C_d^C = \phi_d D^o \Gamma_C.$$

I can now derive the shipping cost difference ΔC_t and delivery cost difference ΔC_d , which are respectively

$$\Delta C_t = \phi_t \Phi_t \sqrt{N} D^o, \quad (15)$$

$$\Delta C_d = \left[\left(\frac{1}{\sqrt{N}} - 1 \right) \phi_d W \right] D^o. \quad (16)$$

Therefore, the total transportation cost difference ΔC_T is given by

$$\Delta C_T = \Delta C_t + \Delta C_d = \phi_t \Phi_t \sqrt{N} D^o + \left[\left(\frac{1}{\sqrt{N}} - 1 \right) \phi_d W \right] D^o. \quad (17)$$

Lemma 3.3: The cost differences between store fulfillment and DC fulfillment strategies are as follows

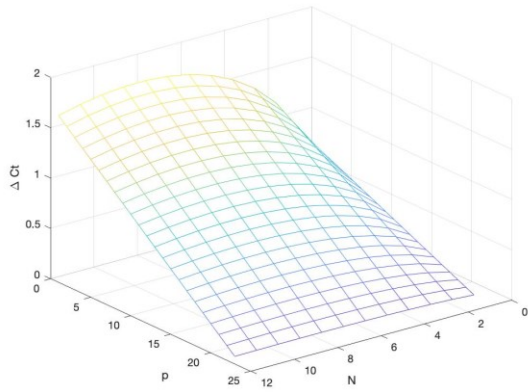
- (i) $\Delta C_t \geq 0$, $\frac{\partial \Delta C_t}{\partial p} \leq 0$; $\frac{\partial \Delta C_t}{\partial N} \geq 0$ if $N \leq \frac{c}{3c_0}$, and $\frac{\partial \Delta C_t}{\partial N} < 0$ otherwise.
- (ii) $\Delta C_d \leq 0$, $\frac{\partial \Delta C_d}{\partial p} \geq 0$; There exists a unique $N_d > 1$ such that $\frac{\partial \Delta C_d}{\partial N} \leq 0$ when $N \leq N_d$ and $\frac{\partial \Delta C_d}{\partial N} > 0$ otherwise.
- (iii) $\Delta C_T < 0$ if $\phi_d > \frac{4\phi_t \Phi_t}{W}$ and $N_1 < N < N_2$; and $\Delta C_T \geq 0$ otherwise,

where $N_1 = \frac{[\phi_d W - \sqrt{\phi_d W(\phi_d W - 4\phi_t \Phi_t)}]^2}{4\phi_t^2 \Phi_t^2}$ and $N_2 = \frac{[\phi_d W + \sqrt{\phi_d W(\phi_d W - 4\phi_t \Phi_t)}]^2}{4\phi_t^2 \Phi_t^2}$.

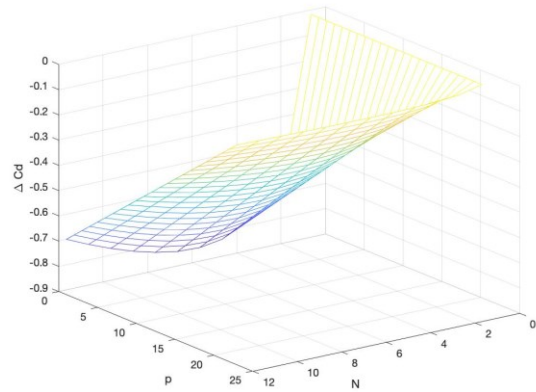
From Lemma 3.3, fulfilling from the DC always leads to lower transportation cost. This is because the DC needs to transport both online and in-store orders to stores if online orders are fulfilled from stores, whereas it only needs to transport in-store orders to stores if online orders are fulfilled from the DC. Lemma 3.3 also suggests that fulfilling from the DC always leads to higher delivery cost. This is because stores have a closer proximity to customers than the DC. When the two effects are synthesized, then store fulfillment leads to less combined transportation and delivery cost if unit delivery cost ϕ_d is sufficiently high and the number of stores N is neither too high nor too low.

Lemma 3.3 also suggests that the transportation cost difference is decreasing in the product price p . In other words, the added transportation cost from fulfilling from the DC instead of stores decreases as price increases, due to the decrease in the total demand. The added transportation cost is also increasing in N if N is small, and is decreasing in N otherwise. This is due to the

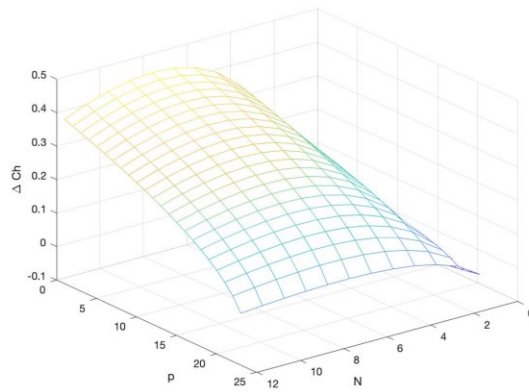
balancing of two effects as N increases. On the one hand, it increases the transportation distance from DC to stores, while on the other hand, it reduces the amount of products transported for fulfilling online orders. If N is large, the second effect dominates leading to a reduction in transportation cost difference, and vice versa. Similarly, the delivery cost increases from fulfilling from the DC instead of stores is decreasing in price, and is decreasing in N if and only if N is sufficiently small. Figure 3.5 (a) and (b) show the results of Lemma 3.3.



(a)



(b)



(c)

Figure 3.5: Cost differences between two fulfillment strategies (a) ΔC_t v.s. N, p (b) ΔC_d v.s. N, p (c) ΔC_h v.s. N, p ($\phi_d = 2$; $\Phi_t = 1$; $\phi_l = 0.5$; $\lambda = 2$; $h_s = 2$; $h_c = 1.5$; $V = 25$; $C = 25$; $c_o = 1$; $W = 1$.)

3.2.3 Total Operational Cost and Optimal Fulfillment Strategy

The retailer's total operation cost is the sum of the shipping cost from DC to stores (this thesis refers to this as shipping cost from here on for ease of exposition), the delivery cost for online demand, and the inventory holding cost at the DC and stores. These costs are dependent on the order fulfillment strategy and given by

$$\begin{cases} C_S = C_t(D^o + D^i) + \phi_d D^o \Gamma_S + h_s \phi_I (D^o + D^i)^{\frac{1}{2}} \sqrt{N}, \\ \quad \text{if orders are fulfilled from stores,} \\ C_C = C_t D^i + \phi_d D^o \Gamma_C + \left(h_c \phi_I D^{o\frac{1}{2}} + h_s \phi_I D^{i\frac{1}{2}} \sqrt{N} \right), \\ \quad \text{if orders are fulfilled from DC.} \end{cases} \quad (18)$$

The first term in Equation (18) represents the shipping cost from DC to stores, which includes the portion of online demand if they are fulfilled from store inventory and excludes it otherwise. The second term in Equation (18) represents the retailer's delivery cost, which is determined by the delivery distance associated with the given fulfillment strategy. The third term represents the holding cost incurred by the retailer's facilities (i.e., the DC and stores).

Combining Equation (13)-(18), I can simplify the retailer's total operational cost under either fulfillment strategy as

$$\begin{cases} C_S = \phi_t \Phi_t \sqrt{N} (D^o + D^i) + \phi_d D^o \frac{W}{\sqrt{N}} + \phi_d \lambda D^o + h_s \phi_I (D^o + D^i)^{\frac{1}{2}} \sqrt{N}, \\ \quad \text{if orders are fulfilled from stores,} \\ C_C = \phi_t \Phi_t \sqrt{N} D^i + \phi_d W D^o + \phi_d \lambda D^o + \left(h_c \phi_I D^{o\frac{1}{2}} + h_s \phi_I D^{i\frac{1}{2}} \sqrt{N} \right), \\ \quad \text{if orders are fulfilled from DC.} \end{cases} \quad (19)$$

In what follows, this thesis examines the retailer's optimal strategy to maximize its expected profit under either fulfillment strategy. Specifically,

$$\begin{cases} \pi_S = p[D^i(p) + D^o(p)] + c_o D^o(p) - C_S, & \text{if orders are fulfilled from stores,} \\ \pi_C = p[D^i(p) + D^o(p)] + c_o D^o(p) - C_C, & \text{if orders are fulfilled from DC.} \end{cases}$$

After plug in the demand and cost functions (1)-(19), the profit functions dependent on the online order fulfillment strategy become as follows,

$$\left\{ \begin{array}{l} \pi_S = (p - \phi_t \Phi_t \sqrt{N}) D^i(p) + \left(p + c_o - \phi_t \Phi_t \sqrt{N} - \phi_d \frac{W}{\sqrt{N}} - \phi_d \lambda \right) D^o(p) \\ \quad - h_s \phi_I \left(D^o(p) + D^i(p) \right)^{\frac{1}{2}} \sqrt{N}, \quad \text{if orders are fulfilled from stores,} \\ \pi_C = (p - \phi_t \Phi_t \sqrt{N}) D^i(p) + (p + c_o - \phi_d W - \phi_d \lambda) D^o(p) \\ \quad - h_c \phi_I D^o(p)^{\frac{1}{2}} - h_s \phi_I D^i(p)^{\frac{1}{2}} \sqrt{N}, \quad \text{if orders are fulfilled from DC.} \end{array} \right. \quad (20)$$

The following proposition describes the retailer's optimal fulfillment strategy.

Proposition 3.1: It is optimal for the retailer to fulfill online orders from stores if and only if one of the following conditions holds

- (i) $h_s \leq h_{s1}$, $\phi_d > \frac{4\phi_t \Phi_t}{W}$, $N_1 < N < N_2$,
- (ii) $h_c > h_{c1}$, $\phi_d > \frac{4\phi_t \Phi_t}{W}$, $N \leq N_1$ or $N \geq N_2$,
- (iii) $h_c > h_{c1}$, $\phi_d \leq \frac{4\phi_t \Phi_t}{W}$,
- (iv) $h_c > h_{c1}$, $h_s > h_{s1}$, $\phi_d > \frac{4\phi_t \Phi_t}{W}$, $N_1 < N < N_2$

where $h_{c1} = \frac{(C - c_o N)(V - c_o - p) \left[\left(\frac{1}{\sqrt{N}} - 1 \right) \phi_d W + \phi_t \Phi_t \sqrt{N} \right] + h_s \phi_I C V \sqrt{N} \left\{ \left[\frac{N c_o^2 + 2C(V - c_o - p)}{2CV} \right]^{\frac{1}{2}} - \left[\frac{(V - p - \frac{c_o}{2}) c_o N}{CV} \right]^{\frac{1}{2}} \right\}}{C V \phi_I \left[\frac{(V - p - c_o)(C - c_o N)}{CV} \right]^{\frac{1}{2}}}$ and

$$h_{s1} = - \frac{(C - c_o N)(V - c_o - p) \left[\left(\frac{1}{\sqrt{N}} - 1 \right) \phi_d W + \phi_t \Phi_t \sqrt{N} \right]}{C V \phi_I \sqrt{N} \left\{ \left[\frac{N c_o^2 + 2C(V - c_o - p)}{2CV} \right]^{\frac{1}{2}} - \left[\frac{(V - p - \frac{c_o}{2}) c_o N}{CV} \right]^{\frac{1}{2}} \right\}}$$

From Proposition 3.1, the retailer's optimal strategy depends on the store's inventory cost h_s , DC's inventory cost h_c , delivery cost ϕ_d and the total number of stores N .

Corollary 3.2: ΔC_h has the following properties:

- (i) Under the conditions of Proposition 3.1 (i), $h_{c0} > h_{c1}$, then $\Delta C_h \geq 0$ if $h_c \leq h_{c0}$, and $\Delta C_h < 0$ otherwise.
- (ii) Under the conditions of Proposition 3.1 (ii) and (iii), $h_{c0} \leq h_{c1}$, then $\Delta C_h < 0$.
- (iii) Under the conditions of Proposition 3.1 (iv), $h_{c0} > h_{c1}$, then $\Delta C_h \geq 0$ if $h_{c1} < h_c \leq h_{c0}$, and $\Delta C_h < 0$ if $h_c > h_{c0}$.

First, the retailer prefers store fulfillment when the store's inventory cost is low, the delivery cost is high, the number of stores is neither too small nor too large. This is because when delivery cost is high and the number of stores is neither too small nor too large, store fulfillment leads to lower total transportation (shipping and delivery) cost as suggested by Lemma 3.3. In this case, if the store's inventory cost is sufficiently low, then the savings in transportation cost from store fulfillment is larger than the associated cost increase, and therefore leads to overall cost saving associated with store fulfillment. On the contrary, if the delivery cost is high and the number of stores is either small or large, then store fulfillment leads to higher combined transportation and delivery cost based on Lemma 3.3. In this case, store fulfillment is only preferred if DC has very high inventory cost. It is straightforward to see that store fulfillment is also preferred when DC's inventory cost is sufficiently high and the delivery cost is sufficiently low.

Interestingly, case (iv) in Proposition 3.1 tells that store fulfillment may be better even when the stores' inventory cost is sufficiently high. In particular, this happens when DC's inventory cost is very high, delivery cost is high, and the number of stores is either small or large. According to Lemma 3.3, the combination of high unit delivery cost and moderate number of stores leads to lower combined shipping and delivery cost with store fulfillment. Meanwhile, if the DC's unit inventory cost is very high, the inventory cost under store fulfillment is lower than that of DC fulfillment for reasons explained after Corollary 3.1. Even if the DC's unit inventory cost is not high enough to lead to lower inventory cost by store fulfillment, the total transportation cost saving under store fulfillment still exceeds the inventory cost increase, rendering store fulfillment to be the better strategy.

It is also worthwhile to note that the threshold on unit inventory cost h_{c1} is dependent on various parameters, as detailed in the following corollary.

Corollary 3.3: Analyze h_{c1} defined in Proposition 3.1, this thesis has following corollaries,

$$(i) \quad \frac{\partial h_{c1}}{\partial p} \geq 0 \text{ if } h_s \leq h_{s2}, \phi_d \geq \frac{4\phi_t\phi_t}{w}, N_1 < N < N_2; \frac{\partial h_{c1}}{\partial p} < 0, \text{ otherwise.}$$

$$(ii) \quad \frac{\partial h_{c1}}{\partial c} \leq 0 \text{ if } h_s \leq h_{s3}, \phi_d \geq \frac{4\phi_t\phi_t}{w}, N_1 < N < N_2; \frac{\partial h_{c1}}{\partial c} > 0, \text{ otherwise.}$$

(iii) $\frac{\partial h_{c1}}{\partial V} \leq 0$ if $h_s \leq h_{s4}$, $\phi_d \geq \frac{4\phi_t\Phi_t}{W}$, $N_1 < N < N_2$; $\frac{\partial h_{c1}}{\partial V} > 0$, otherwise.

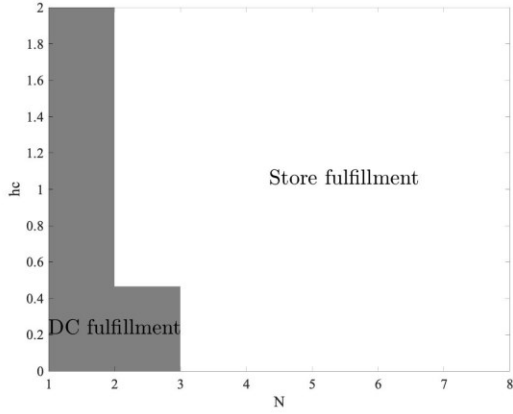
$$\text{where } h_{s2} = -\frac{2(C-c_oN)^{\frac{3}{2}}(V-p-c_o)^{\frac{3}{2}}\left[\left(\frac{1}{\sqrt{N}}-1\right)\phi_dW+\phi_t\Phi_t\sqrt{N}\right]}{\phi_I C^{\frac{1}{2}}V^{\frac{1}{2}}c_o^2N^{\frac{3}{2}}\left[y^{-\frac{1}{2}}-(1+y)^{-\frac{1}{2}}\right]},$$

$$h_{s3} = -\frac{(C-c_oN)(V-p-c_o)^{\frac{3}{2}}\left[\left(\frac{1}{\sqrt{N}}-1\right)\phi_dW+\phi_t\Phi_t\sqrt{N}\right]}{\phi_I C^{\frac{3}{2}}V^{\frac{1}{2}}N^{\frac{1}{2}}\left[y^{-\frac{1}{2}}-(1+y)^{-\frac{1}{2}}\right]}\left(V-p-\frac{c_o}{2}\right),$$

$$h_{s4} = -\frac{2(C-c_oN)^{\frac{3}{2}}(V-p-c_o)^{\frac{3}{2}}(p+c_o)\left[\left(\frac{1}{\sqrt{N}}-1\right)\phi_dW+\phi_t\Phi_t\sqrt{N}\right]}{\phi_I C^{\frac{1}{2}}V^{\frac{3}{2}}c_o^2N^{\frac{3}{2}}\left[y^{-\frac{1}{2}}-(1+y)^{-\frac{1}{2}}\right]}.$$

According to Corollary 3.3, h_{c1} is increasing with the increase of product price p , and decreasing with the increases of C and V when unit store inventory cost is low, unit delivery cost is high and number of stores is intermediate. Using Proposition 3.1, this suggests that in this case, an increase in product price or a decrease in in-store inconvenience cost and customer valuation lead the store fulfillment strategy to have worse performance. This is because the high unit delivery and intermediate number of stores lead to lower total transportation cost under store fulfillment, while low unit inventory cost at the store creates inventory cost advantage for store fulfillment. As p increases, and C and V decrease, online demand decreases, which in turn reduces the savings from store fulfillment.

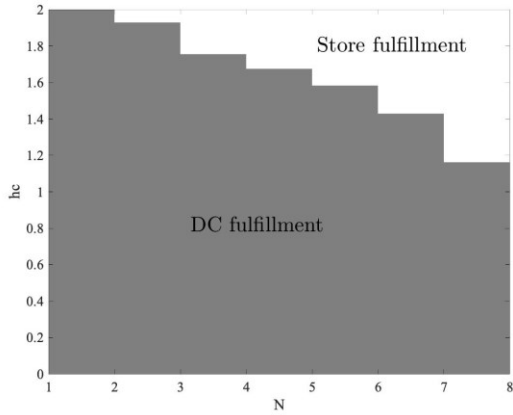
Figure 3.6 illustrates the retailer's optimal strategy under several example scenarios. Figure 3.6 (a) corresponds to case (i) in Proposition 3.1 where the unit delivery cost is high. When the store's inventory cost is low, store fulfillment strategy is optimal for intermediate number of stores N , while DC fulfillment is optimal for smaller and larger N . Case (ii) of Proposition 3.1 is illustrated in Figure 3.6 (b) when $N \in [1,2,3,6,7,8]$. When the unit delivery cost is high, the retailer prefers store fulfillment if the number of store locations is large and DC's inventory cost is high. When the unit delivery cost is low, then store fulfillment is preferred if DC's inventory cost is sufficiently high, while DC fulfillment is preferred otherwise (this corresponds to case (iii) in Proposition 3.1). Figure 3.6(d) and Figure 3.6(b) when $N \in [4,5,6]$ illustrate case (iv) in Proposition 3.1 where the unit delivery cost is high.



(a) $\phi_d = 3.5; \Phi_t = 0.9; p = 8; W = 2.1$



(b) $\phi_d = 2.9; \Phi_t = 1; p = 10; W = 1.4$



(c) $\phi_d = 2.8; \Phi_t = 1; p = 10; W = 1.4$



(d) $\phi_d = 2.8; \Phi_t = 0.9; p = 8; W = 2.1$

Figure 3.6: The retailer's optimal fulfillment strategy with respect to p and N

($\phi_t = 1; \phi_l = 0.5; \lambda = 2; h_s = 2; V = 25; C = 25; c_o = 3.$)

3.3 Special Case With $c_o = 0$

In this section, this thesis considers the special case where $c_o = 0$, i.e., the customer's online order cost is zero (the retailer delivers orders for free). In this case, all customers would choose to purchase online. Therefore, in-store demand is $D^{i0} = 0$, and online demand is $D^{o0} = \frac{V-p}{V}$. Essentially, the fulfillment problem now becomes facility location problem, i.e., whether to fulfill from a centralized warehouse or a number of smaller warehouses located closer to customers.

The shipping and delivery cost differences are now

$$\Delta C_{t0} = \phi_t \Phi_t \sqrt{N} D^{o0}, \quad (21)$$

$$\Delta C_{d0} = \left[\left(\frac{1}{\sqrt{N}} - 1 \right) \phi_d W \right] D^{o0}. \quad (22)$$

Therefore, total transportation cost difference ΔC_{T0} is given by

$$\Delta C_{T0} = \Delta C_{t0} + \Delta C_{d0} = \phi_t \Phi_t \sqrt{N} D^{o0} + \left[\left(\frac{1}{\sqrt{N}} - 1 \right) \phi_d W \right] D^{o0}. \quad (23)$$

Lemma 3.4: The logistics cost differences between store fulfillment and DC fulfillment strategies when $c_o = 0$ satisfy the following properties

$$(i) \quad \Delta C_{t0} \geq 0, \quad \frac{\partial \Delta C_{t0}}{\partial p} \leq 0, \quad \frac{\partial \Delta C_{t0}}{\partial N} \geq 0.$$

$$(ii) \quad \Delta C_{d0} \geq 0, \quad \frac{\partial \Delta C_{d0}}{\partial p} \leq 0, \quad \frac{\partial \Delta C_{d0}}{\partial N} \geq 0.$$

$$(iii) \quad \Delta C_{T0} < 0 \text{ if } \phi_d > \frac{4\phi_t\Phi_t}{W} \text{ and } N_1 < N < N_2; \quad \Delta C_T \geq 0, \text{ otherwise.}$$

where $N_1 = \frac{[\phi_d W - \sqrt{\phi_d W(\phi_d W - 4\phi_t\Phi_t)}]^2}{4\phi_t^2\Phi_t^2}$ and $N_2 = \frac{[\phi_d W + \sqrt{\phi_d W(\phi_d W - 4\phi_t\Phi_t)}]^2}{4\phi_t^2\Phi_t^2}$.

The inventory cost difference is now given by

$$\Delta C_{h0} = \phi_I D^{o\frac{1}{2}} (h_s \sqrt{N} - h_c). \quad (24)$$

$$\mathbf{Lemma 3.5:} \quad \Delta C_{h0} \geq 0, \quad \frac{\partial \Delta C_{h0}}{\partial N} \geq 0 \text{ and } \frac{\partial \Delta C_{h0}}{\partial p} < 0.$$

Lemma 3.5 suggests that when $c_o = 0$ fulfilling from stores always leads to higher inventory cost. This is because in this case, the only remaining pooling effect is location pooling.

The retailer's profits under the two fulfillment schemes can now be simplified as

$$\begin{aligned} \pi_S(p)|_{c_o=0} &= \left(p - \frac{\phi_d W}{\sqrt{N}} - \phi_d \lambda - \phi_t \Phi_t \sqrt{N} \right) D^o - h_s \phi_I (D^o + D^i)^{\frac{1}{2}} \sqrt{N} \\ &= \frac{(c - c_o N)(V - p) \left(p - \frac{\phi_d W}{\sqrt{N}} - \phi_d \lambda - \phi_t \Phi_t \sqrt{N} \right)}{CV} - h_s \phi_I D^{o\frac{1}{2}} \sqrt{N}, \end{aligned}$$

$$\begin{aligned} \pi_C(p)|_{c_o=0} &= (p - \phi_d W - \phi_d \lambda) D^o - h_s \phi_I D^{i\frac{1}{2}} \sqrt{N} - h_c \phi_I D^{o\frac{1}{2}} \\ &= \frac{(c - c_o N)(V - p)(p - \phi_d W - \phi_d \lambda)}{CV} - h_c \phi_I D^{o\frac{1}{2}} \end{aligned}$$

The profit difference when $c_o = 0$ is thus

$$\begin{aligned}\pi_S(p) - \pi_C(p)|_{c_o=0} &= -D^o \left[\left(\frac{1}{\sqrt{N}} - 1 \right) \phi_d W + \phi_t \Phi_t \sqrt{N} \right] + \phi_I D^{o\frac{1}{2}} (h_c - h_s \sqrt{N}) \\ &= - \frac{(V-p) \left[\left(\frac{1}{\sqrt{N}} - 1 \right) \phi_d W + \phi_t \Phi_t \sqrt{N} \right]}{V} + \phi_I \left(\frac{V-p}{V} \right)^{\frac{1}{2}} (h_c - h_s \sqrt{N})\end{aligned}$$

Proposition 3.2: It is optimal for the retailer to fulfill online orders from stores with $c_o = 0$ if and only if one of the following conditions holds

- (i) $h_s \leq h_{s5}$, $\phi_d > \frac{4\phi_t\Phi_t}{W}$ and $N_1 < N < N_2$,
- (ii) $h_c > h_{c2}$, $h_s > h_{s6}$, $\phi_d > \frac{4\phi_t\Phi_t}{W}$ and $N_1 < N < N_2$,

where $h_{c2} = \frac{\left[\left(\frac{1}{\sqrt{N}} - 1 \right) \phi_d W + \phi_t \Phi_t \sqrt{N} \right] (V-p)^{\frac{1}{2}}}{\phi_I V^{\frac{1}{2}}} + h_s N^{\frac{1}{2}}$, $h_{s5} = - \frac{\left[\left(\frac{1}{\sqrt{N}} - 1 \right) \phi_d W + \phi_t \Phi_t \sqrt{N} \right] (V-p)^{\frac{1}{2}}}{\phi_I N^{\frac{1}{2}} V^{\frac{1}{2}}}$ and $h_{s6} = - \frac{\left[\left(\frac{1}{\sqrt{N}} - 1 \right) \phi_d W + \phi_t \Phi_t \sqrt{N} \right] (V-p)^{\frac{1}{2}}}{\phi_I \left(N^{\frac{1}{2}} - 1 \right) V^{\frac{1}{2}}}$.

According to Lemma 3.5, store fulfillment is better than DC fulfillment when $c_o = 0$ if and only if the unit delivery cost is sufficiently high, the number of stores is neither too large or too small, and either the stores' inventory cost is sufficiently low or DC's inventory cost is sufficiently high. This result can be explained as follows.

Inventory cost under store fulfillment is always higher when the customers only shop online, but transportation cost can be higher or lower. The combination of these costs determines which strategy is better. When the unit delivery cost is high and the number of stores is intermediate, the transportation cost is lower under the store fulfillment strategy, as suggested by Lemma 3.3. When the stores' inventory cost is sufficiently low or DC's inventory cost is sufficiently high, the savings from transportation cost exceeds the cost increase due to inventory, and thus result in higher profit under store fulfillment.

The following corollary summarizes additional properties of h_{c2} .

Corollary 3.4: $\frac{\partial h_{c2}}{\partial p} \geq 0$, $\frac{\partial h_{c2}}{\partial V} < 0$.

Corollary 3.4 suggests that as the retail price increases or as customer valuation decreases, it becomes less likely for store fulfillment strategy to be the dominant strategy. This is because a

retailer price increase (or valuation of the product decrease) reduces the online demand. It will reduce the saving of total transportation cost. Although the reduction in online demand also reduces the inventory cost difference, the rate of its decrease is lower than that of the total transportation cost saving.

Chapter 4: Case Studies

In this section, this thesis examines several case studies using real world data to find the best strategy for an omni-channel retailer. Two US cities/area are selected for the analysis: Manhattan and Los Angeles. They are chosen as representations of varying geographic and demographic characteristics

The geography and population data of these two cities are listed in Table 4.1. The shape parameter ε measures the city's irregularity, indicating level of symmetry, with higher ε indicating a more irregular region (a circle has the smallest ε). To calculate ε , I approximate Manhattan with a rectangle whose height-to-length ration is 1:5, Los Angeles with a rectangle whose height-to-length is 2:3. Then use the formulas for calculating ε of rectangles and circles provided by Belavina et al. (2017), and calculate ε of Manhattan to be 1.16, that of Los Angeles to be 0.8. Between two cities, Manhattan has the smaller area A , the higher population density, and the more irregular shape.

Table 4.1: Geography and population

City	Area, A (km ²)	Population density, ρ (resident/km ²)	Shape, ε
Manhattan	59	28220	1.16
Los Angeles	1300	3077	0.8

Table 4.2 summarizes the parameters associated with shipping in the two cities. Φ_t is a constant dependent only on the area of the region and tessellation type. This thesis assumes square tessellation in the discussion, for which the value of Φ_t is \sqrt{A} . The results can be similarly derived for other types of tessellations. The estimations of ϕ_t is shown below.

The shipping cost per truck on average consists of fuel cost (39%), driver's salary cost (26%),

truck cost (17%), repair maintenance cost (10%), and other cost (8%)¹. Given that fuel and driver salary are different between two cities, I calculate the shipping cost by $\phi_d = \text{fuel cost} + \text{deiver salary} + \text{other cost}$. The 2017 American Public Transit Association's Public Transportation Fact Book reports that the average gas mileage of a shipping truck is 6.5 mpg². The 2016 gasoline price in Manhattan is \$3.04 per gallon, in LA is \$3 per gallon. Therefore, the estimation of fuel cost in Manhattan as \$0.4677/mile/truck, or equivalently \$0.2923/km/truck. Since fuel cost on average accounts for 39% of the shipping cost, the estimation the total the shipping cost to be \$0.7495/km/truck in Manhattan. This thesis assumes a delivery truck load of 20000kg (Cachon, 2014). Each parcel's weight is typically within the range [1.88kg, 8.31kg]³, and assumes each parcel is 2 kg. Therefore, I assume that each truck load contains 10000 orders. Therefore, the shipping cost per order per km is \$0.00007495 in Manhattan. Similarly, this thesis estimates that the shipping costs in Los Angeles is \$0.00007396/km/order.

Table 4.2: Shipping parameters in cities

City	Φ_t	Shipping rate, ϕ_t (\$/km/order)
Manhattan	59	28220
Los Angeles	1300	3077

Table 4.3 summarizes the delivery parameters. W is a constant determined by the size and shape of the service region and the number of orders delivered by each delivery truck (which estimated as 15). λ is another constant determined by the number of orders delivered by each delivery truck, days in operation, customer density and their order frequency. e is the retailer's market penetration rate. The thesis assumes 0.1% of the population shop at the retailer. Therefore, customer density for retailer is $\bar{\rho} = e$. Then the Delivery distances W and λ can be calculated

¹ Source: <https://www.thetruckersreport.com/infographics/cost-of-trucking/>.

² Source: <https://www.apta.com/wp-content/uploads/2017-APTA-Fact-Book.pdf>.

³ Source: <https://www.statista.com/statistics/771219/e-trade-weight-way-parcel-sent-at-l-export-france/>.

according to the formulas of W and λ , which are provided in the appendix.⁴ ϕ_d is the last-mile delivery cost from store to customers. The Bureau of Transportation statistics tells that the delivery truck's mpg is 17.4 in 2016⁵, and each delivery truck can carry 15 packages per day. Similar to the estimation of shipping cost, I can calculate that the delivery cost per truck in two cities are \$ 0.01866 /km/order and \$ 0.01842 /km/order, respectively.

Table 4.3: Delivery parameters in cities

City	W (km/order)	λ (km/order)	ϕ_d (\$/km/order)
Manhattan	1.1880	0.1077	0.01866
Los Angeles	3.8459	0.3262	0.01842

Estimates of inventory-related parameters are shown in Table 4.4. ϕ_l is retailer's service level, with $\phi_l = 2$ suggesting an in-stock probability is 97.7%. h_s is the stores' inventory cost per order per day, calculated by $h_s = \frac{v_h + f_h p_h}{q_h}$, where v_h is space cost rate, f_h is utility consumption, p_h is utility price, and q_h is weight that can be stored per square kilometer in the store space. According to the method of Cachon, electricity and natural gas consist the utility cost for mercantile retailer. $f_h p_h = f_e p_e + f_g p_g$, where f_e is electricity usage, p_e is the price of electricity, f_g is the natural gas usage and p_g is price of natural gas. I estimate v_h according to obtaining warehouse rent fee online (<https://www.loopnet.com/>). The retail grocery rent fee range in Manhattan is [1.6222, 5.1589] \$/km²/day, in Los Angeles is [0.9016, 2.8300] \$/km²/day. To simplify the calculation, I choose average values to estimate. For q_h and $f_h p_h$, I use the estimations from (Cachon, 2014). $q_h = 71$ order/km². The average electricity usage for retailers in United States is 0.5570 kWh/km²/day and the average natural gas usage is 0.6819 kWh/km²/day (Cachon, 2014). According to U.S. Energy Information Administration (EIA) report in 2016,

⁴ In W and λ , the estimation of market penetration rate $e=0.1\%$, $K=15$ \$ orders/truck, customer usually place one order per day (Belavina et al. 2017).

⁵ Source: <https://www.bts.gov/content/average-fuel-efficiency-us-light-duty-vehicles>.

commercial price of electricity is \$0.1557/kWh in New York and \$0.1701/kWh in California⁶. According to the US Energy Information Administration, the natural gas price for commercial customers is \$6.19/thousand cubic feet in New York and \$8.42/thousand cubic feet in California. Therefore $f_h p_h$ in Manhattan is $0.5570 \cdot 0.1557 + 0.6819 \cdot 0.00619 = 0.0909$ \$/km²/day. Similarly, $f_h p_h$ in Los Angeles is 0.1005 \$/km²/day. These utility-related estimates are summarized in Table 4.4.

Table 4.4: Inventory parameters in cities

City	v_h (\$/km ² /day)	$f_h p_h$ (\$/km ² /day)	h_s (\$/order/km ² /day)	ϕ_I
Manhattan	3.3905	0.0909	0.04903	2
Los Angeles	1.8658	0.1005	0.02769	2

Lastly, the thesis estimates consumer-related parameters. For customer's online ordering cost c_o , Hann and Terwiesch (2003) (Hann and Terwiesch, 2003) estimate that, on average, it costs the customer \$5 for each online order placed. Meanwhile, some retailers provide free shipping. In the numerical study, this thesis considers a range of online order cost between \$0 to \$10 per order.

Table 4.5: Utility consumption and utility price in cities

City	f_e (kWh/order)	p_e (km/order)	f_g (\$/km/order)	p_g (\$/ft ³)
Manhattan	0.5570	0.1557	0.6819	0.00619
Los Angeles	0.5570	0.1701	0.6819	0.00842

Table 4.6 shows the base case estimates of ϕ_I , C , c_o , p , V and h_c . Later, I vary their base values to study the results in alternative scenarios. Both cities turn out to have relatively high delivery cost and store holding cost, that is, they both fall into case (iv) of Proposition 3.1.

⁶ Source: <https://www.eia.gov/electricity/monthly/archive/october2016.pdf>.

Table 4.6: Baseline parameters

Parameters	Values
ϕ_I	$\phi_I = 2$
C	$C = \$50/\text{order}$
c_o	$c_o = \$5/\text{order}$
p	$p = \$4/\text{order}$
V	$V = \$25/\text{order}$
h_c	In Manhattan, $h_c = 0.036/\text{order}/\text{km}^2/\text{day}$
	In Los Angeles, $h_c = 0.020/\text{order}/\text{km}^2/\text{day}$

4.1 Exogenous Price Cases

In this section, the case studies based on exogenous pricing strategy.

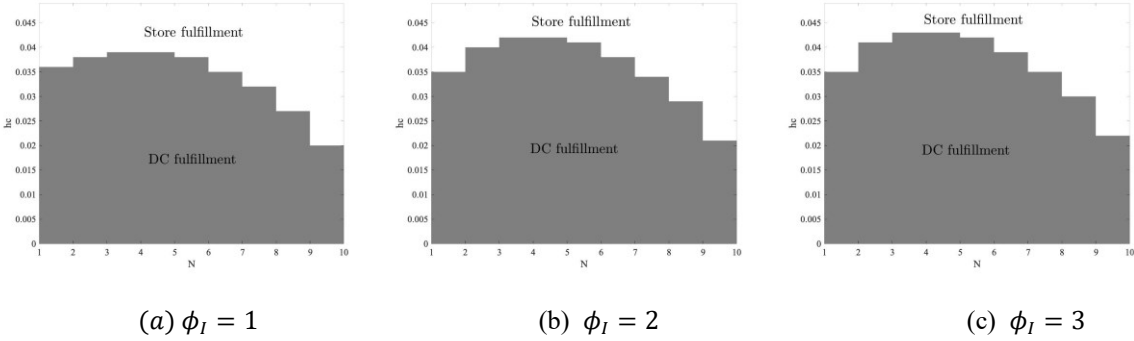


Figure 4.1: Impacts of ϕ_I and N on the retailer's optimal fulfillment strategy (Manhattan)

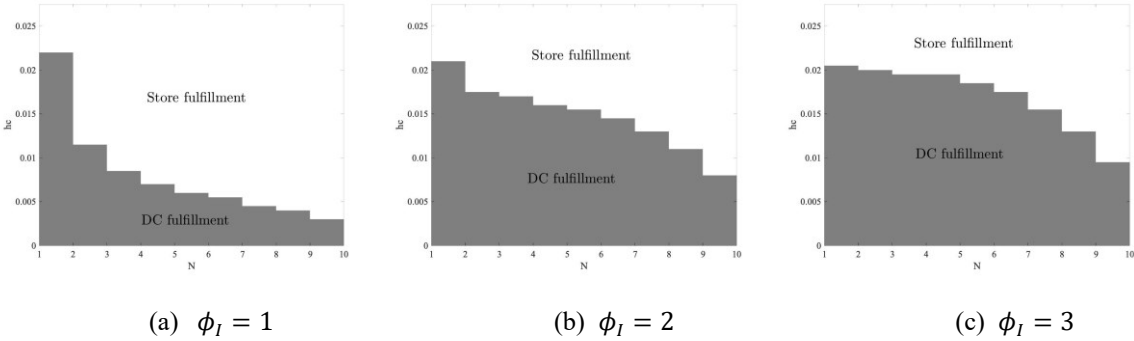


Figure 4.2: Impacts of ϕ_I and N on the retailer's optimal fulfillment strategy (Los Angeles)

Figures 4.1 and 4.2 illustrate the optimal strategies for Manhattan and Los Angeles, respectively. Note that between the two cities, their unit delivery costs ϕ_d are very close, but store inventory cost in Manhattan is much higher than in Los Angeles. Thus, for both cities, DC fulfillment is the preferred strategy with low DC inventory cost and moderate number of stores,

while store fulfillment strategy is optimal for high DC inventory cost and a large number of stores. However, the region for store fulfillment to be optimal increases with total number of stores for Los Angeles, whereas it first decreases and then increases for Manhattan. This effect can be explained as follows. When N is small, the benefit of DC fulfillment on inventory cost increases with N , while that on transportation cost decreases with N . Because inventory holding cost in Manhattan is sufficiently high, when N is small, the first effect dominates, leading to DC fulfillment becoming better as N increases. In contrast, the inventory cost in Los Angeles is too low for the inventory effect to ever dominate. Therefore, the region where DC fulfillment is optimal always decreases.

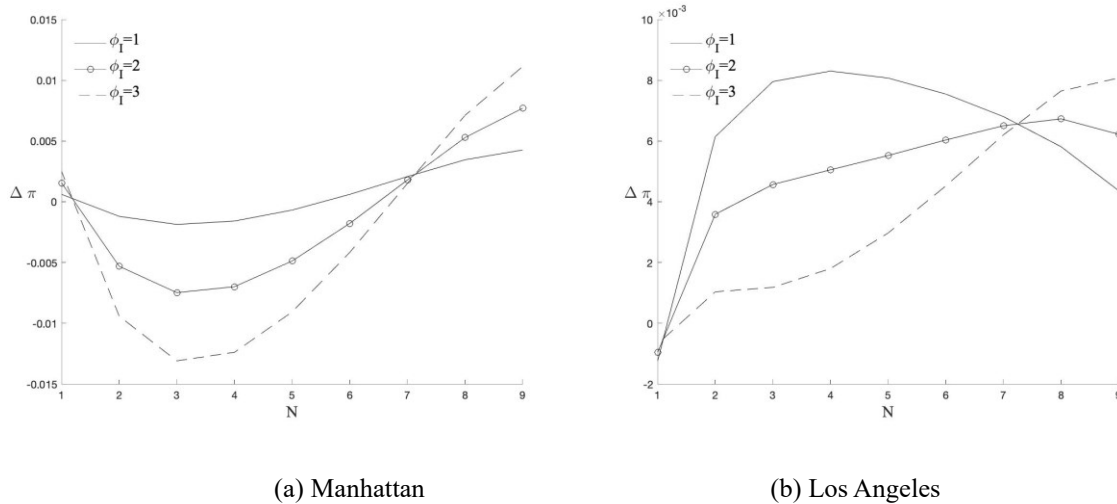


Figure 4.3: Impacts of ϕ_I and N on profit difference

Figures 4.1-4.2 also illustrate impacts of service levels ϕ_I on the retailer's optimal fulfillment strategy for the two cities. Specifically, the thesis considers ϕ_I values of 1, 2 and 3, which correspond to probabilities of no stockouts of 84.1%, 97.7% and 99.9%, respectively. With the increase of service level, the area for DC fulfillment to be optimal increase for both cities. Similar results are shown in Figure 4.3, in Figure 4.3(a), the values of $\Delta\pi$ below zero, which means DC fulfillment is optimal in Manhattan, increase with increase of ϕ_I . In Figure 4.3(b), the values of $\Delta\pi$ above zero, which means store fulfillment is optimal in Los Angeles, decrease with the increase of ϕ_I .

This is because that the retailer needs to hold more inventory to serve customers when ϕ_I increases. Due to the cheaper inventory cost at the DC, DC fulfillment becomes increasingly desirable in this situation. In the meantime, the increase for Los Angeles is more significant than that for Manhattan. This is because the shipping cost is significantly lower in Manhattan compared to Los Angeles as a result of its geographic characteristics. Therefore, the shipping cost saving from DC fulfillment is much smaller for Manhattan, and thus its benefit overall is reduced.

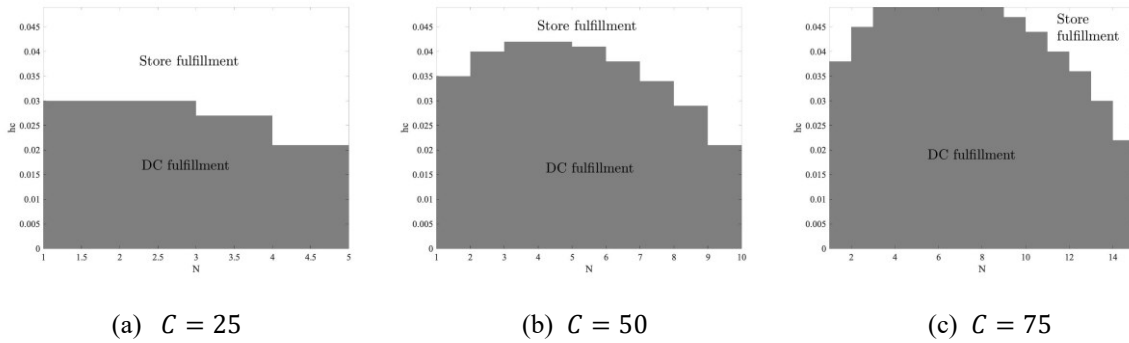


Figure 4.4: Impacts of C and N on the retailer's optimal fulfillment strategy (Manhattan)

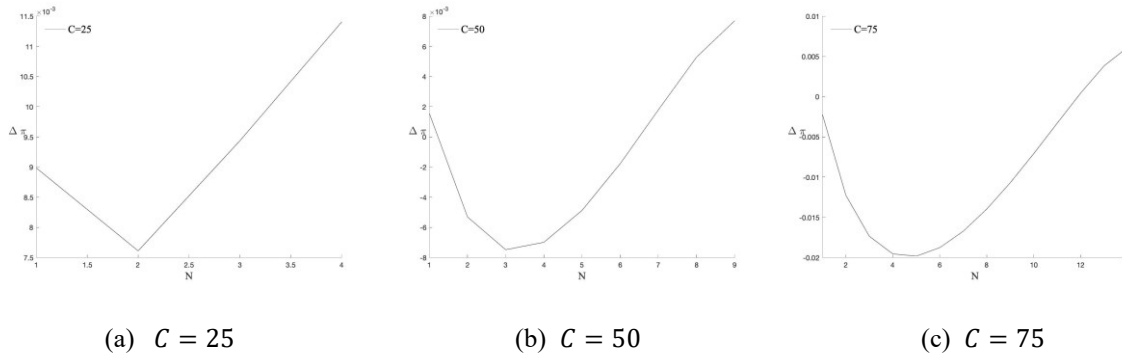


Figure 4.5: Impacts of C and N on profit difference (Manhattan)

Figure 4.4 illustrates the customer's in-store inconvenience cost C using Manhattan as an example. Higher inconvenient cost renders DC fulfillment to be increasingly beneficial. Similarly, in Figure 4.5, the positive values of profit differences are decreasing with increase of C . In addition, the total number of stores where DC fulfillment remains beneficial for the widest range of h_c also increases with C . This is because when C increases, online demand increases and

inventory holding at stores become more efficient. Therefore, for DC fulfillment to remain competitive, DC's inventory cost needs to be even more efficient.

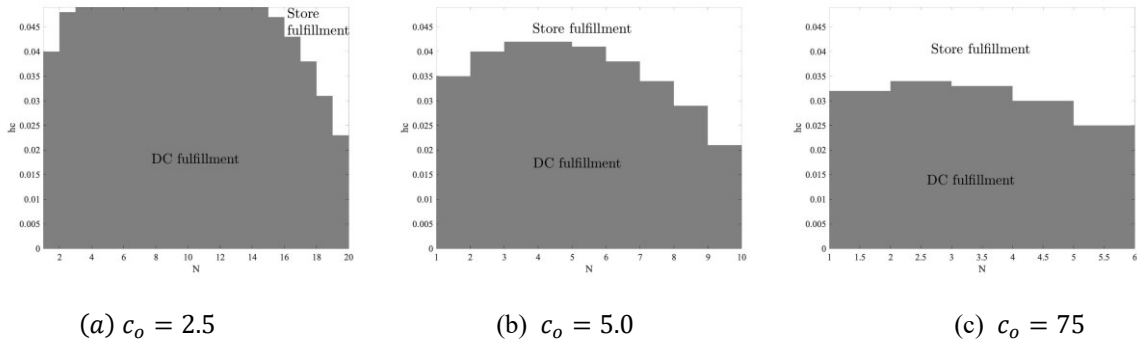


Figure 4.6: Impacts of c_o and N on the retailer's optimal fulfillment strategy (Manhattan)

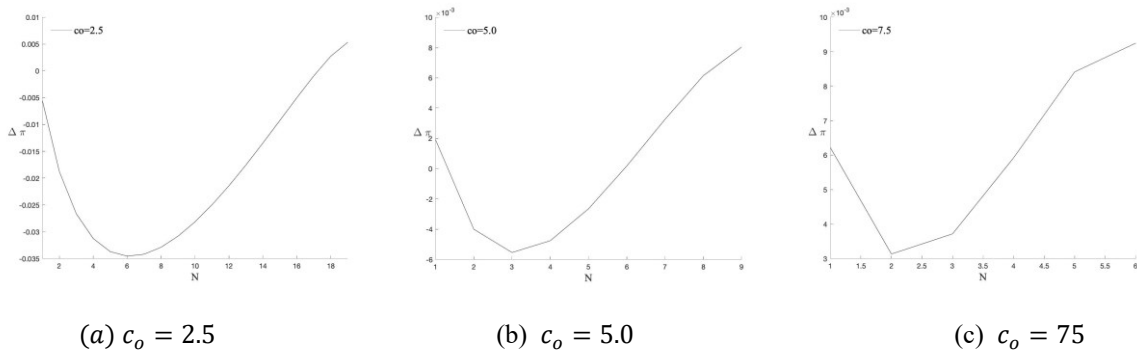


Figure 4.7: Impacts of c_o and N on profit difference (Manhattan)

Figure 4.6 and Figure 4.7 illustrate the impact of the customer's online shopping cost c_o on the retailer's optimal fulfillment strategy, again using Manhattan as an example. Specifically, we vary customer's online cost c_o between 2.5 and 7.5. The effect of c_o works in opposite direction of that of C and can be similarly explained.

Figure 4.8 illustrates the impacts of the retail price p on the retailer's optimal fulfillment strategy. With the increase of p , the region where store fulfillment strategy is optimal expands. This effect is further studied in Figure 4.9 which illustrates the impacts the profit difference between the two strategies $\Delta\pi$ behave under different p values. It can be seen that $\Delta\pi$ flattens as p increases. This is because higher price leads to lower demand both in-store and online. For Manhattan, this means that the portion of the curve below zero decreases with p , suggesting the region where DC fulfillment is better is shrinking.

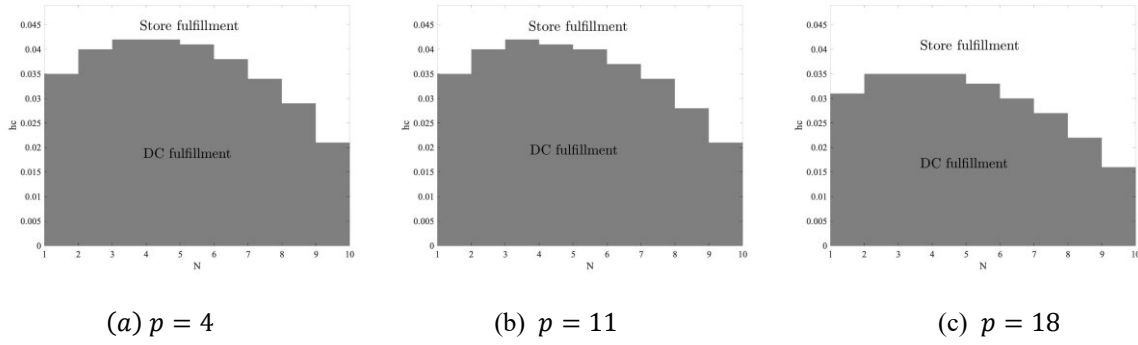


Figure 4.8: Impacts of p and N on the retailer's optimal fulfillment strategy (Manhattan)

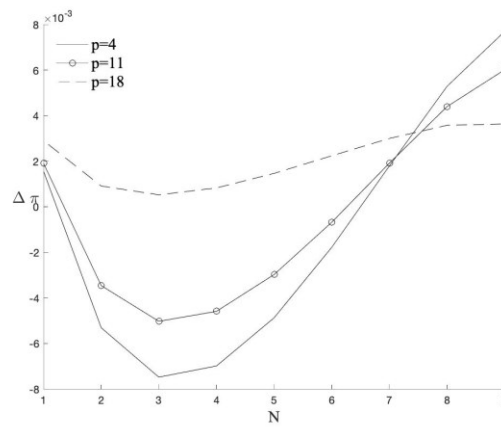


Figure 4.9: Impacts of p and N on profit difference (Manhattan)

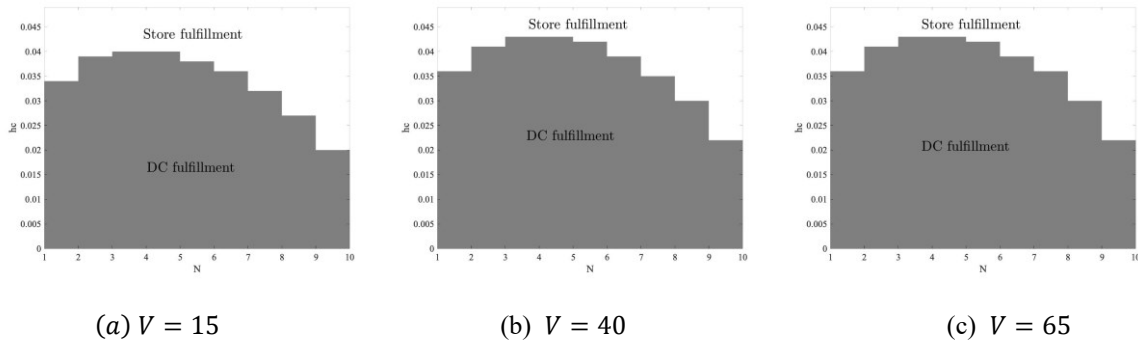


Figure 4.10: Impacts of V and N on the retailer's optimal fulfillment strategy (Manhattan)

Similarly, Figure 4.10 illustrates, for Manhattan, the impacts of customer's product valuation V on the retailer's optimal fulfillment strategy, while Figure 4.11 illustrates its effect on the profit difference between the two fulfillment strategies. The effect of V is the opposite of that of p , and can be similarly explained.

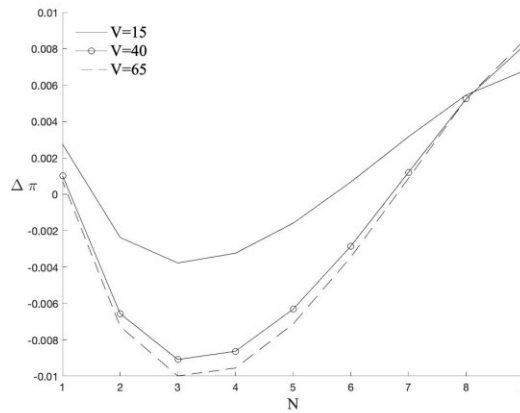


Figure 4.11: Impacts of V and N on profit difference (Manhattan)

4.2 Endogenous Price Cases

In this section, the thesis considers an extension where the price of the product is endogenous. That is, in addition to the fulfillment strategy and order quantities, the retailer also chooses the selling price under each strategy. This thesis continues to illustrate the results using the previous case study setting, and study the influences of different parameters on the optimal prices and the optimal profit differences between the two fulfillment strategies.

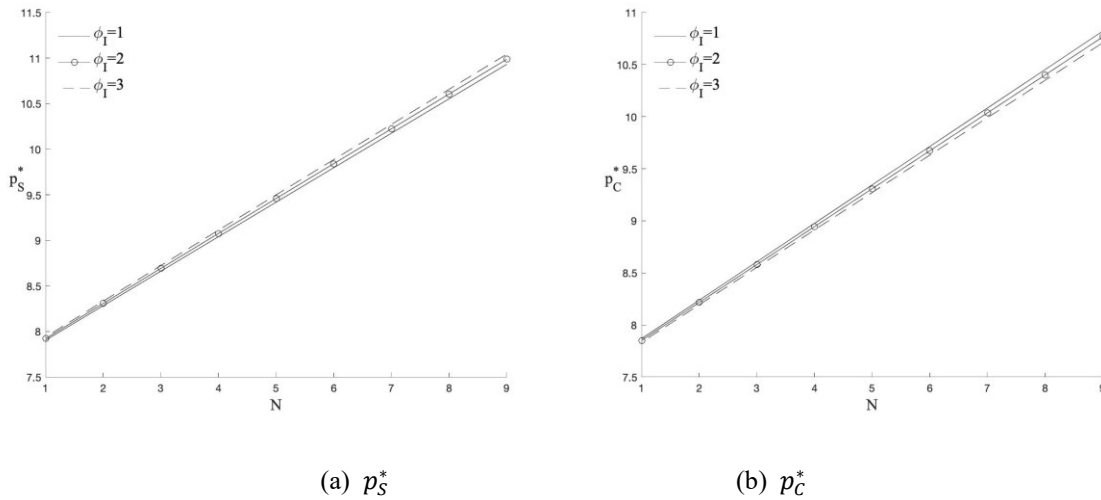


Figure 4.12: Impacts of ϕ_I and N on optimal prices under endogenous price (Manhattan)

Figure 4.12 and Figure 4.13 illustrate behavior of the optimal price. Specifically, they illustrate the impacts of ϕ_I and N on optimal prices under store fulfillment strategy and DC fulfillment strategy in Manhattan and Los Angeles, respectively. Note that p_S^* denotes the optimal price

under the store fulfillment strategy while p_C^* denotes that for the DC fulfillment strategy. Overall, the figures show that $p_S^* > p_C^*$ for Manhattan, and that both p_S^* and p_C^* are increasing with the total number of stores N under both fulfillment strategies (this is because the retailer's costs increase with the total number of stores).

It is easy to see that the impact of ϕ_I is the opposite between the two strategies for Manhattan, while they work in the same direction for Los Angeles. The optimal price increases with ϕ_I in all scenarios except for Manhattan under DC fulfillment. This is because under DC fulfillment in Manhattan, decreasing the retail price and increasing demand allows the retailer to make better use of the inventory pooling effect. In all other scenarios, in order to meeting increasing service level requirement, the retailer charges higher price to reduce the demand.

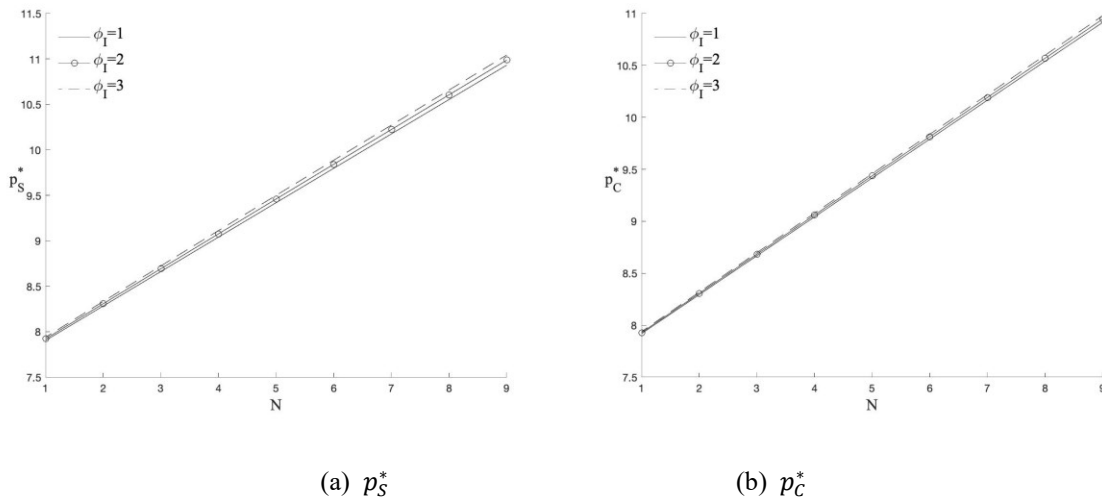
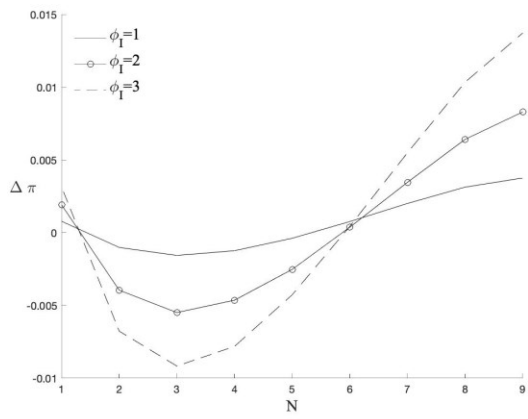
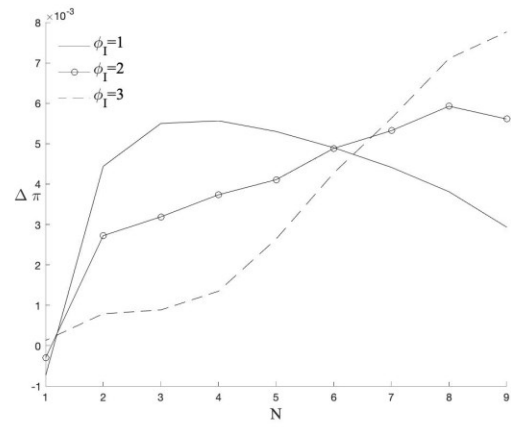


Figure 4.13: Impacts of ϕ_I and N on optimal prices under endogenous price (Los Angeles)

Figure 4.14(a) and 4.14(b) illustrate how the profit difference between the two fulfillment strategies are affected by ϕ_I and N in Manhattan and Los Angeles, respectively, when the retail optimizes its retail price. For Manhattan, the profit difference is positive for small and large N (Store fulfillment is optimal), and negative for intermediate N (DC fulfillment is optimal). For Los Angeles, the profit difference is negative for small N , and positive for large N . In fact, the behavior of the profit difference is very similar to that under the exogenous demand setting, as illustrated in Figure 4.3(a) and 4.3(b).



(a) Manhattan



(b) Los Angeles

Figure 4.14: Impacts of ϕ_I and N on profit difference under endogenous price

Chapter 5: Conclusions

With the development of digital technology, more and more retailers are operating omni-channel supply chain to meeting the changing demand. This thesis examines the optimal online order fulfillment strategy for an omni-channel retailer who is facing both in-store and online demands in order to maximize its overall profit. Two strategies including fulfilling from store inventory and fulfilling from the distribution center are considered.

An integrated model is developed, which accounts for consumers' channel choice decision, the retailer's inventory holding cost, its shipping cost from the distribution center to the stores, and the last-mile delivery cost to customers' homes. The influences of the two fulfillment strategies on various cost elements was studied. In addition, two interesting inventory pooling effects are identified, namely, channel pooling effect and location pooling effect. It is shown that both fulfillment strategy can yield lower inventory cost, determined by the tradeoff between the two pooling effects, which is in turn affected by the total number of stores.

The analytical solution to retailer's optimal fulfillment problem was provided. It was found that the retailers' optimal fulfillment strategy depends on the store's unit inventory cost, DC's inventory cost, delivery cost and the total number of stores. Specifically, if the delivery cost is high, then the retailer prefers store fulfillment when the store's inventory cost is low, the number of stores is neither too small nor too large. On the contrary, if the delivery cost is high and number of stores is either high or low, store fulfillment is only preferred if DC has very high inventory cost. Interestingly, store fulfillment may be better even when the stores' inventory cost is high. In particular, this happens when DC's inventory cost is very high, delivery cost is high, and the number of stores is either small or large.

Case studies based on Manhattan and Los Angeles are provided to further investigate the retailer's fulfillment decision as well as the impacts of its pricing decision, and geographic and cost

characteristics. For Manhattan, for both exogenous and endogenous price cases, the regions where store fulfillment are optimal first decrease and then increase as the total number of stores increases. For Los Angeles, the region where store fulfillment is optimal always increases with the total number of stores.

It is worth pointing out a few limitations of this work. First, the interaction between pricing and fulfillment decisions are complex and more analytical results should be developed to better understand it. Second, potential demand transfer between channels was not considered in the current model. This happens, for example, when customers switch from in-store to the online channel if they do not find the products in store. Third, retailer's shipping price can also be considered as a decision to further improve the results. Finally, it will be interesting to conduct empirical studies on the effect of the retailer's fulfillment decisions on various performance factors such as profitability, customer service, and customers' channel choices.

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Appendices
Appendix A: Theorem Proofs

Proof of Lemma 3.1:

This thesis gets the results from taking the first order derivation of demands.

$$(i) \quad \frac{\partial D^i}{\partial p} = -\frac{c_o N}{CV} < 0, \quad \frac{\partial D^i}{\partial N} = \frac{c_o}{CV} \left(V - p - \frac{c_o}{2} \right) > 0, \quad \frac{\partial D^i}{\partial V} = \frac{2c_o N p + c_o^2 N}{2CV^2} > 0, \quad \frac{\partial D^i}{\partial c_o} = \frac{N(V-p-c_o)}{C^2V} > 0, \quad \frac{\partial D^i}{\partial C} = -\frac{c_o N \left(V - p - \frac{c_o}{2} \right)}{C^2V} < 0.$$

$$(ii) \quad \frac{\partial D^o}{\partial p} = \frac{c_o N - C}{CV} < 0, \quad \frac{\partial D^o}{\partial N} = -\frac{c_o}{CV} (V - p - c_o) < 0, \quad \frac{\partial D^o}{\partial V} = \frac{(C - c_o)(c_o + p)}{CV^2} > 0, \quad \frac{\partial D^o}{\partial c_o} = -\frac{(C - c_o N) + N(V - p - c_o)}{C^2V} < 0, \quad \frac{\partial D^o}{\partial C} = \frac{c_o N (V - p - c_o)}{C^2V} > 0.$$

Proof of Lemma 3.2:

The expression of inventory cost difference is given as,

$$\begin{aligned} \Delta C_h &= h_s \phi_I \sqrt{N} \left[(D^o + D^i)^{\frac{1}{2}} - D^{i\frac{1}{2}} \right] - h_c \phi_I D^{o\frac{1}{2}} \\ &= h_s \phi_I \sqrt{N} \left[\left(\frac{Nc_o^2}{2CV} - \frac{c_o + p}{V} + 1 \right)^{\frac{1}{2}} - \left(\frac{c_o N (V - p - \frac{c_o}{2})}{CV} \right)^{\frac{1}{2}} \right] - h_c \phi_I \left(\frac{(C - c_o N)(V - p - c_o)}{CV} \right)^{\frac{1}{2}}. \end{aligned}$$

It is easy to say that $\Delta C_h \geq 0$ if and only if $h_c \leq h_{c0}$ where $h_{c0} =$

$$\frac{h_s \sqrt{N} \left\{ \left[\frac{Nc_o^2 + 2C(V - p - c_o)}{2} \right]^{\frac{1}{2}} - \left[c_o N \left(V - p - \frac{c_o}{2} \right) \right]^{\frac{1}{2}} \right\}}{[(V - p - c_o)(C - c_o N)]^{\frac{1}{2}}}.$$

Proof of Lemma 3.3:

The expression of shipping, delivery and total transportation cost differences are given as,

$$\Delta C_t = \phi_t \Phi_t \sqrt{N} (V - p - c_o) \frac{C - c_o N}{CV},$$

$$\Delta C_d = \left(\frac{1}{\sqrt{N}} - 1 \right) \phi_d W (V - p - c_o) \frac{C - c_o N}{CV},$$

$$\Delta C_T = \Delta C_t + \Delta C_d = \left[\phi_t \Phi_t \sqrt{N} + \left(\frac{1}{\sqrt{N}} - 1 \right) \phi_d W \right] (V - p - c_o) \frac{C - c_o N}{CV}.$$

It is easy to say that $\Delta C_t \geq 0$ and $\Delta C_d \leq 0$.

$$(i) \quad \frac{\partial \Delta C_t}{\partial p} = \frac{-(C - c_o N) \phi_t \Phi_t \sqrt{N}}{CV} \leq 0;$$

$$\frac{\partial \Delta C_t}{\partial N} = \frac{(V - p - c_o) \phi_t \Phi_t}{2CV} \left(\frac{C}{\sqrt{N}} - 3c_o \sqrt{N} \right), \quad \frac{\partial \Delta C_t}{\partial N} \geq 0 \text{ if } N \leq \frac{C}{3c_o} \text{ and } \frac{\partial \Delta C_t}{\partial N} < 0; \quad \frac{\partial \Delta C_t}{\partial N} < 0,$$

otherwise.

$$(ii) \quad \frac{\partial \Delta C_d}{\partial p} = - \left(\frac{1}{\sqrt{N}} - 1 \right) \phi_d W \frac{C - c_o N}{CV} \geq 0;$$

$$\frac{\partial \Delta C_d}{\partial N} = - \frac{(V - p - c_o) \phi_d W}{CV \sqrt{N}} \left(\frac{C}{2N} + \frac{c_o}{2} - c_o \sqrt{N} \right), \quad \frac{\partial \Delta C_d}{\partial N} \geq 0 \text{ if } \frac{C}{2N} + \frac{c_o}{2} - c_o \sqrt{N} \geq 0.$$

$$\frac{\partial \left(\frac{C}{2N} + \frac{c_o}{2} - c_o \sqrt{N} \right)}{\partial N} = - \frac{C}{2N^2} - \frac{c_o}{2\sqrt{N}} < 0 \text{ and } \frac{C}{2N} + \frac{c_o}{2} - c_o \sqrt{N} \Big|_{N=1} = \frac{C - c_o}{2} > 0. \text{ Thus, there}$$

exists a unique $N_d > 1$ such that $\frac{\partial \Delta C_d}{\partial N} \leq 0$ when $N \leq N_d$ and $\frac{\partial \Delta C_d}{\partial N} > 0$, otherwise.

$$(iii) \quad \Delta C_T < 0 \text{ if } \phi_t \Phi_t \sqrt{N} + \left(\frac{1}{\sqrt{N}} - 1 \right) \phi_d W < 0, \text{ then } \phi_d > \frac{4\phi_t \Phi_t}{W}, \quad N_1 < N < N_2;$$

$$\Delta C_T \geq 0, \text{ otherwise, where } N_1 = \frac{[\phi_d W - \sqrt{\phi_d W(\phi_d W - 4\phi_t \Phi_t)}]^2}{4\phi_t^2 \Phi_t^2} \text{ and } N_2 =$$

$$\frac{[\phi_d W + \sqrt{\phi_d W(\phi_d W - 4\phi_t \Phi_t)}]^2}{4\phi_t^2 \Phi_t^2}.$$

Proof of Corollary 3.1:

The expression of h_{c_0} is given as,

$$h_{c_0} = \frac{h_s \sqrt{N} \left[(D^o + D^i)^{\frac{1}{2}} - D^{i^2} \right]}{D^{o^{\frac{1}{2}}}} = \frac{h_s \sqrt{N} \left\{ \left[\frac{Nc_o^2 + 2C(V - p - c_o)}{2} \right]^{\frac{1}{2}} - \left[c_o N (V - p - \frac{c_o}{2}) \right]^{\frac{1}{2}} \right\}}{[(V - p - c_o)(C - c_o N)]^{\frac{1}{2}}}.$$

Let $y = \frac{D^i}{D^o} = \frac{(V - p - \frac{c_o}{2})c_o N}{(V - p - c_o)(C - c_o N)}$. After taking the first order derivation of y ,

$$\frac{\partial y}{\partial p} = \frac{c_0^2 N}{2(C-c_0 N)(V-p-c_0)^2} > 0,$$

$$\frac{\partial y}{\partial N} = \frac{(V-p-\frac{c_0}{2})C c_0}{(V-p-c_0)(C-c_0 N)^2} = \frac{C y}{(C-c_0 N)N} > 0,$$

$$\frac{\partial^2 y}{\partial N^2} = \frac{2C c_0 y}{(C-c_0 N)^2 N} > 0,$$

$$\frac{\partial y}{\partial C} = -\frac{c_0 N(V-p-\frac{c_0}{2})}{(V-p-c_0)(C-c_0 N)^2} < 0,$$

$$\frac{\partial y}{\partial V} = -\frac{c_0^2 N}{2(C-c_0 N)(V-p-c_0)^2} < 0.$$

(i) First, I calculate the square power of h_{c_0} ,

$$h_{c_0}^2 = h_s^2 N \left[(1+y)^{\frac{1}{2}} - y^{\frac{1}{2}} \right]^2 = h_s^2 N \left[1 + 2y - 2y^{\frac{1}{2}}(1+y)^{\frac{1}{2}} \right].$$

Consider the first order derivation of h_{c_0} to p ,

$$\frac{\partial h_{c_0}^2}{\partial p} = h_s^2 N \frac{\partial y}{\partial p} \left[2 - y^{-\frac{1}{2}}(1+y)^{\frac{1}{2}} - y^{\frac{1}{2}}(1+y)^{-\frac{1}{2}} \right] = h_s^2 N \frac{\partial y}{\partial p} \frac{-\left[(1+y)^{\frac{1}{2}} - y^{\frac{1}{2}} \right]^2}{y^{\frac{1}{2}}(1+y)^{\frac{1}{2}}} < 0.$$

Then $\frac{\partial h_{c_0}}{\partial p} < 0$.

(ii) For the first order derivation of $h_{c_0}^2$ to C ,

$$\begin{aligned} \frac{\partial h_{c_0}^2}{\partial C} &= h_s^2 N \frac{\partial y}{\partial C} \left[2 - y^{-\frac{1}{2}}(1+y)^{\frac{1}{2}} - y^{\frac{1}{2}}(1+y)^{-\frac{1}{2}} \right] \\ &= h_s^2 N \frac{\left[(1+y)^{\frac{1}{2}} - y^{\frac{1}{2}} \right]^2}{[y(1+y)]^{\frac{1}{2}}} \frac{c_0 N(V-p-\frac{c_0}{2})}{(V-p-c_0)(C-c_0 N)^2} > 0. \end{aligned}$$

(iii) For the first order derivation of $h_{c_0}^2$ to V ,

$$\begin{aligned} \frac{\partial h_{c_0}^2}{\partial V} &= h_s^2 N \frac{\partial y}{\partial V} \left[2 - y^{-\frac{1}{2}}(1+y)^{\frac{1}{2}} - y^{\frac{1}{2}}(1+y)^{-\frac{1}{2}} \right] \\ &= h_s^2 N \frac{\left[(1+y)^{\frac{1}{2}} - y^{\frac{1}{2}} \right]^2}{[y(1+y)]^{\frac{1}{2}}} \frac{c_0^2 N}{2(C-c_0 N)(V-p-c_0)^2}. \end{aligned}$$

(iv) Next, this thesis considers the first order derivation of $h_{c_0}^2$ to N ,

$$\frac{\partial h_{c_0}^2}{\partial N} = h_s^2 \left[1 + 2y - 2y^{\frac{1}{2}}(1+y)^{\frac{1}{2}} \right] + h_s^2 N \frac{\partial y}{\partial N} \left[2 - y^{-\frac{1}{2}}(1+y)^{\frac{1}{2}} - y^{\frac{1}{2}}(1+y)^{-\frac{1}{2}} \right],$$

$$1 + 2y - 2y^{\frac{1}{2}}(1+y)^{\frac{1}{2}} = 1 + \frac{2y^{\frac{1}{2}} \left[y^{\frac{1}{2}} - (1+y)^{\frac{1}{2}} \right] \left[y^{\frac{1}{2}} + (1+y)^{\frac{1}{2}} \right]}{y^{\frac{1}{2}} + (1+y)^{\frac{1}{2}}} = \frac{(1+y)^{\frac{1}{2}} - y^{\frac{1}{2}}}{y^{\frac{1}{2}} + (1+y)^{\frac{1}{2}}} > 0,$$

$$\frac{\partial \left[1 + 2y - 2y^{\frac{1}{2}}(1+y)^{\frac{1}{2}} \right]}{\partial N} = \left[2 - y^{-\frac{1}{2}}(1+y)^{\frac{1}{2}} - y^{\frac{1}{2}}(1+y)^{-\frac{1}{2}} \right] \frac{\partial y}{\partial N} = - \frac{\left[(1+y)^{\frac{1}{2}} - y^{\frac{1}{2}} \right]^2}{y^{\frac{1}{2}}(1+y)^{\frac{1}{2}}} \frac{\partial y}{\partial N} < 0,$$

$$2 - y^{-\frac{1}{2}}(1+y)^{\frac{1}{2}} - y^{\frac{1}{2}}(1+y)^{-\frac{1}{2}} = - \frac{\left[(1+y)^{\frac{1}{2}} - y^{\frac{1}{2}} \right]^2}{[y(1+y)]^{\frac{1}{2}}} < 0,$$

$$\frac{\partial \left[2 - y^{-\frac{1}{2}}(1+y)^{\frac{1}{2}} - y^{\frac{1}{2}}(1+y)^{-\frac{1}{2}} \right]}{\partial N} = \left[\frac{y^{\frac{1}{2}}}{(1+y)^{\frac{3}{2}}} + \frac{(1+y)^{\frac{1}{2}}}{y^{\frac{3}{2}}} - \frac{2}{y^{\frac{1}{2}}(1+y)^{\frac{1}{2}}} \right] \frac{\partial y}{\partial N} \frac{1}{2} = \frac{1}{2[y(1+y)]^{\frac{3}{2}}} \frac{\partial y}{\partial N} > 0,$$

$$\frac{\partial^2 h_{c_0}^2}{\partial N^2} = h_s^2 \frac{\partial \left[1 + 2y - 2y^{\frac{1}{2}}(1+y)^{\frac{1}{2}} \right]}{\partial N} + h_s^2 \left[2 - y^{-\frac{1}{2}}(1+y)^{\frac{1}{2}} - y^{\frac{1}{2}}(1+y)^{-\frac{1}{2}} \right] \left(\frac{\partial y}{\partial N} + N \frac{\partial^2 y}{\partial N^2} \right) +$$

$$h_s^2 N \frac{\partial y}{\partial N} \frac{\partial \left[2 - y^{-\frac{1}{2}}(1+y)^{\frac{1}{2}} - y^{\frac{1}{2}}(1+y)^{-\frac{1}{2}} \right]}{\partial N} = h_s^2 \left\{ - \frac{\left[(1+y)^{\frac{1}{2}} - y^{\frac{1}{2}} \right]^2}{[y(1+y)]^{\frac{1}{2}}} \left[2y \frac{c}{(C-c_0N)N} + Ny \frac{2Cc_0}{(C-c_0N)^2N} \right] \right\} +$$

$$h_s^2 N \frac{1}{2[y(1+y)]^{\frac{3}{2}}} \frac{c^2 y^2}{(C-c_0N)^2 N^2} = \frac{-h_s^2 c^2 y \left[(1+y)^{\frac{1}{2}} - y^{\frac{1}{2}} \right]^2}{[y(1+y)]^{\frac{1}{2}} (C-c_0N)^2 N} \left[2 - \frac{1}{2(1+y) \left[(1+y)^{\frac{1}{2}} - y^{\frac{1}{2}} \right]^2} \right] < 0$$

$$\text{Since } (1+y)^{\frac{1}{2}} - y^{\frac{1}{2}} = \frac{1}{(1+y)^{\frac{1}{2}} + y^{\frac{1}{2}}} > \frac{1}{2(1+y)^{\frac{1}{2}}} > 0,$$

$$\text{then } \frac{\partial^2 h_{c_0}^2}{\partial N^2} < \frac{-h_s^2 c^2 y \left[(1+y)^{\frac{1}{2}} - y^{\frac{1}{2}} \right]^2}{[y(1+y)]^{\frac{1}{2}} (C-c_0N)^2 N} \left[2 - \frac{4(1+y)}{2(1+y)} \right] = 0.$$

Then there exist an optimal number of stores N_0 , $\frac{\partial h_{c_0}}{\partial N} \leq 0$ if $N \geq N_0$; $\frac{\partial h_{c_0}}{\partial N} > 0$, otherwise.

As a result of $N \in \left[1, \frac{C}{c_0} \right]$, so I can calculate the limit of h_{c_0} when $N = 1$ is, $h_{c_0}|_{N=1} =$

$$h_s \frac{\left[c(v-p-c_0) + \frac{c_0^2}{2} \right]^{\frac{1}{2}} - \left[c_0(v-p-\frac{c_0}{2}) \right]^{\frac{1}{2}}}{[(v-p-c_0)(C-c_0)]^{\frac{1}{2}}} > 0. \text{ The limit of } h_{c_0} \text{ when } N = \frac{C}{c_0} \text{ is, } h_{c_0}|_{N=\frac{C}{c_0}} = 0.$$

In a word, there exists $N_0 > 1$ such that $\Delta C_h < 0$ for $N \geq N_0$, and $\Delta C_h \geq 0$, otherwise.

Proof of Proposition 3.1:

The retailer's profits under the two fulfillment schemes are respectively given by,

$$\pi_S(p) = \frac{(C-c_oN)(V-c_oN-p)(p-\frac{\phi_d}{\sqrt{N}}W-\phi_d\lambda-\phi_t\Phi_t\sqrt{N})}{CV} - h_s\phi_I(D^o + D^i)^{\frac{1}{2}}\sqrt{N} + \frac{c_oN(c_o-2V+2p)(\phi_t\Phi_t\sqrt{N}-p)}{2CV},$$

$$\pi_C(p) = \frac{(C-c_oN)(V-c_oN-p)(p-\phi_dW-\phi_d\lambda)}{CV} - h_c\phi_I D^{o\frac{1}{2}} - h_s\phi_I D^{i\frac{1}{2}}\sqrt{N} + \frac{c_oN(c_o-2V+2p)(\phi_t\Phi_t\sqrt{N}-p)}{2CV}.$$

The profit difference is thus,

$$\begin{aligned} \pi_S(p) - \pi_C(p) &= -D^o \left[\left(\frac{1}{\sqrt{N}} - 1 \right) \phi_d W + \phi_t \Phi_t \sqrt{N} \right] + h_c \phi_I D^{o\frac{1}{2}} - h_s \phi_I \sqrt{N} \left[(D^o + D^i)^{\frac{1}{2}} - D^{i\frac{1}{2}} \right] \\ &= -\frac{(C-c_oN)(V-c_o-p) \left[\left(\frac{1}{\sqrt{N}} - 1 \right) \phi_d W + \phi_t \Phi_t \sqrt{N} \right]}{CV} + h_c \phi_I \left[\frac{(V-p-c_o)(C-c_oN)}{CV} \right]^{\frac{1}{2}} - \\ &\quad h_s \phi_I \sqrt{N} \left\{ \left[\frac{Nc_o^2 + 2C(V-c_o-p)}{2CV} \right]^{\frac{1}{2}} - \left[\frac{(V-p-\frac{c_o}{2})c_oN}{CV} \right]^{\frac{1}{2}} \right\}. \end{aligned}$$

The retailer should fulfill from stores if and only if $\pi_S - \pi_C \geq 0$ or equivalently $h_c \geq h_{c1}$,

$$\begin{aligned} \text{where } h_{c1} &= \frac{D^o \left[\left(\frac{1}{\sqrt{N}} - 1 \right) \phi_d W + \phi_t \Phi_t \sqrt{N} \right] + h_s \phi_I \sqrt{N} \left[(D^o + D^i)^{\frac{1}{2}} - D^{i\frac{1}{2}} \right]}{\phi_I D^{o\frac{1}{2}}} \\ &= \frac{(C-c_oN)(V-c_o-p) \left[\left(\frac{1}{\sqrt{N}} - 1 \right) \phi_d W + \phi_t \Phi_t \sqrt{N} \right] + h_s \phi_I CV \sqrt{N} \left\{ \left[\frac{Nc_o^2 + 2C(V-c_o-p)}{2CV} \right]^{\frac{1}{2}} - \left[\frac{(V-p-\frac{c_o}{2})c_oN}{CV} \right]^{\frac{1}{2}} \right\}}{CV \phi_I \left[\frac{(V-p-c_o)(C-c_oN)}{CV} \right]^{\frac{1}{2}}}. \end{aligned}$$

There are 2 cases under above conditions,

- (i) $h_{c1} \leq 0$,
- (ii) $h_{c1} > 0$, then $h_c > h_{c1}$.

For case(i), it's easy to see that case (i) holds if and only is $h_s \leq h_{s1}$, $h_s \leq h_{s1}$ exists if and only if $h_{s1} \geq 0$, which is equivalent to $\left(\frac{1}{\sqrt{N}} - 1 \right) \phi_d W + \phi_t \Phi_t \sqrt{N} \leq 0$,

$$\text{where } h_{s1} = -\frac{D^0\left[\left(\frac{1}{\sqrt{N}}-1\right)\phi_d W + \phi_t \Phi_t \sqrt{N}\right]}{\phi_I \sqrt{N}\left[\left(D^i + D^0\right)^{\frac{1}{2}} - D^{\frac{1}{2}}\right]} = -\frac{(C-c_0 N)(V-p-c_0)\left[\left(\frac{1}{\sqrt{N}}-1\right)\phi_d W + \phi_t \Phi_t \sqrt{N}\right]}{CV \phi_I \sqrt{N}\left\{\left[\frac{Nc_0^2 + 2C(V-c_0-p)}{2CV}\right]^{\frac{1}{2}} - \left[\frac{(V-p-\frac{c_0}{2})c_0 N}{CV}\right]^{\frac{1}{2}}\right\}}$$

Similarly, for case(ii), $h_{c1} > 0$ holds when $h_s > h_{s1}$,

$$(i) \quad h_{s1} < 0,$$

$$(ii) \quad h_{s1} > 0, \quad h_s > h_{s1}.$$

It's easy to see that $h_{s1} < 0$ if and only $\left(\frac{1}{\sqrt{N}}-1\right)\phi_d W + \phi_t \Phi_t \sqrt{N} > 0$, $\left(\frac{1}{\sqrt{N}}-1\right)\phi_d W + \phi_t \Phi_t \sqrt{N} > 0$ happens when (i) $\phi_d > \frac{4\phi_t \Phi_t}{W}$, $N \leq N_1$ or $N \geq N_2$, and (ii) $\phi_d \leq \frac{4\phi_t \Phi_t}{W}$.

$h_s > h_{s1}$ exists if and only if $h_{s1} > 0$, which is equivalent to $\left(\frac{1}{\sqrt{N}}-1\right)\phi_d W + \phi_t \Phi_t \sqrt{N} < 0$.

0. $\left(\frac{1}{\sqrt{N}}-1\right)\phi_d W + \phi_t \Phi_t \sqrt{N} < 0$ happens when $\phi_d > \frac{4\phi_t \Phi_t}{W}$, $N_1 < N < N_2$.

The proposition can be summarized as,

$$(a) \quad h_s \leq h_{s1}, \quad \left(\frac{1}{\sqrt{N}}-1\right)\phi_d W + \phi_t \Phi_t \sqrt{N} \leq 0,$$

$$(b) \quad h_c > h_{c1}, \quad \left(\frac{1}{\sqrt{N}}-1\right)\phi_d W + \phi_t \Phi_t \sqrt{N} > 0,$$

$$(c) \quad h_c > h_{c1}, \quad h_s > h_{s1}, \quad \left(\frac{1}{\sqrt{N}}-1\right)\phi_d W + \phi_t \Phi_t \sqrt{N} < 0,$$

$$\text{where } h_{c1} = \frac{(C-c_0 N)(V-c_0-p)\left[\left(\frac{1}{\sqrt{N}}-1\right)\phi_d W + \phi_t \Phi_t \sqrt{N}\right] + h_s \phi_I CV \sqrt{N}\left\{\left[\frac{Nc_0^2 + 2C(V-c_0-p)}{2CV}\right]^{\frac{1}{2}} - \left[\frac{(V-p-\frac{c_0}{2})c_0 N}{CV}\right]^{\frac{1}{2}}\right\}}{CV \phi_I \left[\frac{(V-p-c_0)(C-c_0 N)}{CV}\right]^{\frac{1}{2}}},$$

$$h_{s1} = -\frac{(C-c_0 N)(V-p-c_0)\left[\left(\frac{1}{\sqrt{N}}-1\right)\phi_d W + \phi_t \Phi_t \sqrt{N}\right]}{CV \phi_I \sqrt{N}\left\{\left[\frac{Nc_0^2 + 2C(V-c_0-p)}{2CV}\right]^{\frac{1}{2}} - \left[\frac{(V-p-\frac{c_0}{2})c_0 N}{CV}\right]^{\frac{1}{2}}\right\}}$$

Combine these equations, can have the Proposition 3.1.

$$(i) \quad h_s \leq h_{s1}, \quad \phi_d > \frac{4\phi_t \Phi_t}{W}, \quad N_1 < N < N_2,$$

$$(ii) \quad h_c > h_{c1}, \quad \phi_d > \frac{4\phi_t \Phi_t}{W}, \quad N \leq N_1 \text{ or } N \geq N_2,$$

$$(iii) \quad h_c > h_{c1}, \quad \phi_d \leq \frac{4\phi_t \Phi_t}{W},$$

$$(iv) \quad h_c > h_{c1}, \quad h_s > h_{s1}, \quad \phi_d > \frac{4\phi_t \Phi_t}{W}, \quad N_1 < N < N_2.$$

Proof of Corollary 3.2:

h_{c0} defined in Lemma 3.2 and h_{c1} defined in Proposition 3.1 are

$$h_{c0} = \frac{h_s \sqrt{N} \left\{ \left[\frac{Nc_0^2 + 2C(V-p-c_0)}{2} \right]^{\frac{1}{2}} - \left[c_0 N \left(V - p - \frac{c_0}{2} \right) \right]^{\frac{1}{2}} \right\}}{[(V-p-c_0)(C-c_0N)]^{\frac{1}{2}}},$$

$$h_{c1} = \frac{(C-c_0N)(V-c_0-p) \left[\left(\frac{1}{\sqrt{N}} - 1 \right) \phi_d W + \phi_t \Phi_t \sqrt{N} \right] + h_s \phi_I C V \sqrt{N} \left\{ \left[\frac{Nc_0^2 + 2C(V-c_0-p)}{2CV} \right]^{\frac{1}{2}} - \left[\frac{(V-p-\frac{c_0}{2})c_0N}{CV} \right]^{\frac{1}{2}} \right\}}{CV \phi_I \left[\frac{(V-p-c_0)(C-c_0N)}{CV} \right]^{\frac{1}{2}}},$$

$$h_{c0} - h_{c1} = -\frac{1}{\phi_I} D^{o\frac{1}{2}} \left[\left(\frac{1}{\sqrt{N}} - 1 \right) \phi_d W + \phi_t \Phi_t \sqrt{N} \right].$$

$h_{c0} - h_{c1} \leq 0$ when $\left(\frac{1}{\sqrt{N}} - 1 \right) \phi_d W + \phi_t \Phi_t \sqrt{N} \geq 0$; otherwise, $h_{c0} - h_{c1} > 0$.

Combine the proof of Lemma 3.2 can have the Corollary 3.2.

Proof of Corollary 3.3:

According to Corollary 3.2,

$$h_{c1} = h_{c0} + \phi_I^{-1} D^{o\frac{1}{2}} \left[\left(\frac{1}{\sqrt{N}} - 1 \right) \phi_d W + \phi_t \Phi_t \sqrt{N} \right],$$

After taking the first order derivation of h_{c1} ,

$$\frac{\partial h_{c1}}{\partial p} = -\frac{h_s c_0^2 N^{\frac{3}{2}} \left[y^{-\frac{1}{2}} - (1+y)^{-\frac{1}{2}} \right]}{4(V-p-c_0)^2 (C-c_0N)} - \frac{(C-c_0N)^{\frac{1}{2}} \left[\left(\frac{1}{\sqrt{N}} - 1 \right) \phi_d W + \phi_t \Phi_t \sqrt{N} \right]}{2\phi_I [CV(V-p-c_0)]^{\frac{1}{2}}},$$

$$\frac{\partial h_{c1}}{\partial C} = \frac{h_s c_0 N^{\frac{3}{2}} \left[y^{-\frac{1}{2}} - (1+y)^{-\frac{1}{2}} \right] (V-p-\frac{c_0}{2})}{2(V-p-c_0)(C-c_0N)^2} + \frac{c_0 N (V-p-c_0) \left[\left(\frac{1}{\sqrt{N}} - 1 \right) \phi_d W + \phi_t \Phi_t \sqrt{N} \right]}{2\phi_I C^{\frac{3}{2}} V^{\frac{1}{2}} [(C-c_0N)(V-p-c_0)]^{\frac{1}{2}}},$$

$$\frac{\partial h_{c1}}{\partial V} = \frac{h_s c_0^2 N^{\frac{3}{2}} \left[y^{-\frac{1}{2}} - (1+y)^{-\frac{1}{2}} \right]}{4(V-p-c_0)^2 (C-c_0N)} + \frac{(C-c_0N)^{\frac{1}{2}} (p+c_0) \left[\left(\frac{1}{\sqrt{N}} - 1 \right) \phi_d W + \phi_t \Phi_t \sqrt{N} \right]}{2\phi_I C^{\frac{1}{2}} V^{\frac{3}{2}} (V-p-c_0)^{\frac{1}{2}}}.$$

(i) As a result of $\frac{\partial h_{c1}}{\partial p} \leq 0$ if $h_s \geq h_{s2}$, there are two situations when $h_s \geq h_{s2}$ holds,

(a) $h_{s2} \leq 0$ if $\left(\frac{1}{\sqrt{N}} - 1 \right) \phi_d W + \phi_t \Phi_t \sqrt{N} \geq 0$,

(b) $h_{s2} > 0$, $\left(\frac{1}{\sqrt{N}} - 1 \right) \phi_d W + \phi_t \Phi_t \sqrt{N} < 0$, $h_s \geq h_{s2}$.

(ii) Similarly, as a result of $\frac{\partial h_{c1}}{\partial p} > 0$, if $\left(\frac{1}{\sqrt{N}} - 1\right)\phi_d W + \phi_t \Phi_t \sqrt{N} < 0$, $h_s < h_{s2}$,

$$\text{where } h_{s2} = -\frac{2(C-c_oN)^{\frac{3}{2}}(V-p-c_o)^{\frac{3}{2}}\left[\left(\frac{1}{\sqrt{N}}-1\right)\phi_d W+\phi_t \Phi_t \sqrt{N}\right]}{\phi_I C^{\frac{1}{2}} V^{\frac{1}{2}} c_o^2 N^{\frac{3}{2}}\left[y^{-\frac{1}{2}}-(1+y)^{-\frac{1}{2}}\right]}.$$

(iii) $\frac{\partial h_{c1}}{\partial C} \leq 0$ if $h_s \leq h_{s3}$, $h_s \leq h_{s3}$ holds when $\left(\frac{1}{\sqrt{N}} - 1\right)\phi_d W + \phi_t \Phi_t \sqrt{N} \leq 0$.

(iv) Similarly, $\frac{\partial h_{c1}}{\partial C} > 0$ if $\left(\frac{1}{\sqrt{N}} - 1\right)\phi_d W + \phi_t \Phi_t \sqrt{N} > 0$, or if $\left(\frac{1}{\sqrt{N}} - 1\right)\phi_d W + \phi_t \Phi_t \sqrt{N} < 0$, $h_s > h_{s3}$,

$$\text{where } h_{s3} = -\frac{(C-c_oN)(V-p-c_o)^{\frac{3}{2}}\left[\left(\frac{1}{\sqrt{N}}-1\right)\phi_d W+\phi_t \Phi_t \sqrt{N}\right]}{\phi_I C^{\frac{3}{2}} V^{\frac{1}{2}} N^{\frac{1}{2}}\left[y^{-\frac{1}{2}}-(1+y)^{-\frac{1}{2}}\right]\left(V-p-\frac{c_o}{2}\right)}.$$

(v) $\frac{\partial h_{c1}}{\partial V} \leq 0$ if $h_s \leq h_{s4}$, $h_s \leq h_{s4}$ holds if and only if $\left(\frac{1}{\sqrt{N}} - 1\right)\phi_d W + \phi_t \Phi_t \sqrt{N} \leq 0$.

(vi) Similarly, $\frac{\partial h_{c1}}{\partial V} > 0$ if $h_s > h_{s4}$, $h_{s4} \leq 0$ if and only if $\left(\frac{1}{\sqrt{N}} - 1\right)\phi_d W + \phi_t \Phi_t \sqrt{N} > 0$;

$h_{s4} > 0$ if $\left(\frac{1}{\sqrt{N}} - 1\right)\phi_d W + \phi_t \Phi_t \sqrt{N} > 0$,

$$\text{where } h_{s4} = -\frac{2(C-c_oN)^{\frac{3}{2}}(V-p-c_o)^{\frac{3}{2}}(p+c_o)\left[\left(\frac{1}{\sqrt{N}}-1\right)\phi_d W+\phi_t \Phi_t \sqrt{N}\right]}{\phi_I C^{\frac{1}{2}} V^{\frac{3}{2}} c_o^2 N^{\frac{3}{2}}\left[y^{-\frac{1}{2}}-(1+y)^{-\frac{1}{2}}\right]}.$$

Proof of Corollary 3.4:

As a result of $h_{c2} = \frac{\left[\left(\frac{1}{\sqrt{N}}-1\right)\phi_d W+\phi_t \Phi_t \sqrt{N}\right](V-p)^{\frac{1}{2}}}{\phi_I V^{\frac{1}{2}}} + h_s N^{\frac{1}{2}}$, $\left(\frac{1}{\sqrt{N}} - 1\right)\phi_d W + \phi_t \Phi_t \sqrt{N} \leq 0$.

After taking the first order of h_{c2} to p and V ,

$$(i) \quad \frac{\partial h_{c2}}{\partial p} = -\frac{\left[\left(\frac{1}{\sqrt{N}}-1\right)\phi_d W+\phi_t \Phi_t \sqrt{N}\right](V-p)^{\frac{1}{2}}}{2\phi_I V^{\frac{1}{2}}} \geq 0,$$

$$(ii) \quad \frac{\partial h_{c2}}{\partial V} = \frac{p\left[\left(\frac{1}{\sqrt{N}}-1\right)\phi_d W+\phi_t \Phi_t \sqrt{N}\right]}{2\phi_I V^{\frac{3}{2}}(V-p)^{\frac{1}{2}}} \leq 0.$$

Proof of Proposition 3.2:

The retailer should fulfill from stores with $c_o = 0$ if and only if $\pi_s(p) - \pi_c(p)|_{c_o=0} \geq 0$, or equivalently $h_c \geq h_{c2}$, where $h_{c2} = \phi_I^{-1} \left[\left(\frac{1}{\sqrt{N}} - 1 \right) \phi_d W + \phi_t \Phi_t \sqrt{N} \right] D^{o\frac{1}{2}} + h_s N^{\frac{1}{2}} = \frac{\left[\left(\frac{1}{\sqrt{N}} - 1 \right) \phi_d W + \phi_t \Phi_t \sqrt{N} \right] (V-p)^{\frac{1}{2}}}{\phi_I V^{\frac{1}{2}}} + h_s N^{\frac{1}{2}}$.

According to $h_s \geq h_c$, then $h_s \geq h_c \geq h_{c2}$, or equivalently $h_s \leq h_{s6}$,

where $h_{s6} = - \frac{\left[\left(\frac{1}{\sqrt{N}} - 1 \right) \phi_d W + \phi_t \Phi_t \sqrt{N} \right] (V-p)^{\frac{1}{2}}}{\phi_I \left(N^{\frac{1}{2}} - 1 \right) V^{\frac{1}{2}}}$.

As a result of $\Delta C_{h0} \geq 0$, so $\pi_s(p) - \pi_c(p)|_{c_o=0} \geq 0$ happens if and only if $\Delta C_{T0} \leq 0$,

which is equivalent to $\left(\frac{1}{\sqrt{N}} - 1 \right) \phi_d W + \phi_t \Phi_t \sqrt{N} \leq 0$.

There are two cases if $h_c \geq h_{c2}$,

- (i) $h_{c2} \leq 0$,
- (ii) $h_{c2} > 0$, $h_c \geq h_{c2}$.

For case (i), $h_{c2} \leq 0$ holds if $h_s \leq h_{s5}$, $h_s \leq h_{s5}$ exists if and only if $\left(\frac{1}{\sqrt{N}} - 1 \right) \phi_d W + \phi_t \Phi_t \sqrt{N} \leq 0$, where $h_{s5} = - \frac{\left[\left(\frac{1}{\sqrt{N}} - 1 \right) \phi_d W + \phi_t \Phi_t \sqrt{N} \right] (V-p)^{\frac{1}{2}}}{\phi_I N^{\frac{1}{2}} V^{\frac{1}{2}}}$.

For case (ii), $h_{c2} > 0$ holds if $h_s > h_{s5}$. There are two situations when $h_s > h_{s5}$ holds,

- (a) $h_{s5} \leq 0$ if $\left(\frac{1}{\sqrt{N}} - 1 \right) \phi_d W + \phi_t \Phi_t \sqrt{N} \leq 0$,
- (b) $h_{s5} > 0$, $h_s > h_{s5}$ if $\left(\frac{1}{\sqrt{N}} - 1 \right) \phi_d W + \phi_t \Phi_t \sqrt{N} \leq 0$.

Then the conditions are $\left(\frac{1}{\sqrt{N}} - 1 \right) \phi_d W + \phi_t \Phi_t \sqrt{N} \leq 0$, $h_{s5} < h_{s6}$. In a word, the retailer prefers store fulfillment strategy if on only if in following conditions:

- (i) $h_s \leq h_{s5}$, $\left(\frac{1}{\sqrt{N}} - 1 \right) \phi_d W + \phi_t \Phi_t \sqrt{N} \leq 0$,
- (ii) $h_c > h_{c2}$, $h_s \leq h_{s6}$, $\left(\frac{1}{\sqrt{N}} - 1 \right) \phi_d W + \phi_t \Phi_t \sqrt{N} \leq 0$.

Proof of Lemma 3.4:

The expression of shipping, delivery and total transportation cost difference without online cost are given as,

$$\begin{aligned}\Delta C_{t0} &= \phi_t \Phi_t \sqrt{N} \frac{V-p}{V}, \\ \Delta C_{d0} &= \left[\left(\frac{1}{\sqrt{N}} - 1 \right) \phi_d W \right] \frac{V-p}{V}, \\ \Delta C_{T0} &= \Delta C_{t0} + \Delta C_{d0} = \left[\phi_t \Phi_t \sqrt{N} + \left(\frac{1}{\sqrt{N}} - 1 \right) \phi_d W \right] \frac{V-p}{V}.\end{aligned}$$

It is easy to say that $\Delta C_{t0} \geq 0$ and $\Delta C_{d0} \leq 0$.

- (i) $\frac{\partial \Delta C_{t0}}{\partial p} = \frac{-\phi_t \Phi_t \sqrt{N}}{V} \leq 0$; $\frac{\partial \Delta C_{t0}}{\partial N} = \frac{\phi_t \Phi_t (V-p)}{2\sqrt{N}V} \geq 0$.
- (ii) $\frac{\partial \Delta C_{d0}}{\partial p} = \frac{-\left[\left(\frac{1}{\sqrt{N}} - 1 \right) \phi_d W \right]}{V} \geq 0$; $\frac{\partial \Delta C_{d0}}{\partial N} = \frac{-\phi_d W (V-p)}{2VN^{\frac{3}{2}}} \leq 0$.
- (iii) $\Delta C_{T0} < 0$ if $\phi_t \Phi_t \sqrt{N} + \left(\frac{1}{\sqrt{N}} - 1 \right) \phi_d W < 0$, which is equivalent to $\phi_d > \frac{4\phi_t \Phi_t}{W}$,

$N_1 < N < N_2$. $\Delta C_{T0} \geq 0$, otherwise,

where $N_1 = \frac{[\phi_d W - \sqrt{\phi_d W (\phi_d W - 4\phi_t \Phi_t)}]^2}{4\phi_t^2 \Phi_t^2}$ and $N_2 = \frac{[\phi_d W + \sqrt{\phi_d W (\phi_d W - 4\phi_t \Phi_t)}]^2}{4\phi_t^2 \Phi_t^2}$.

Proof of Lemma 3.5:

The expression of inventory cost difference without online cost is given as,

$$\Delta C_{h0} = \phi_I \left(\frac{V-p}{V} \right)^{\frac{1}{2}} (h_s \sqrt{N} - h_c).$$

It is easy to say that $\Delta C_{h0} > 0$.

- (i) $\frac{d\Delta C_{h0}}{dN} = \phi_I \left(\frac{V-p}{V} \right)^{\frac{1}{2}} \frac{h_s}{2\sqrt{N}} > 0$.
- (ii) $\frac{d\Delta C_{h0}}{dp} > 0$ if $N < \left(\frac{h_c}{h_s} \right)^2$; $\frac{\partial \Delta C_{h0}}{\partial p} \leq 0$, otherwise. Because of $h_s > h_c$, and $N \geq 1$, then condition is $\frac{\partial \Delta C_{h0}}{\partial p} \leq 0$ if $N \geq 1$.

Proof of W and λ :

The expressions of W and λ are given as Belavina et al., (2017),

$$\Gamma(\bar{\rho}, \tau, A, K) \cong \frac{2\varepsilon\sqrt{A}}{K} + \Lambda(K) \sqrt{\frac{\delta}{\bar{\rho}\tau}},$$

where

$$\Lambda(K) = \begin{cases} \frac{(K-2)^+}{k+1} / \sqrt{\beta^* K} + \frac{K-1}{K} \phi(\beta^* K), & \text{if } K \leq 4 \\ \phi(\beta^* K) - \frac{1}{\sqrt{\beta^* K}}, & \text{if } K > 4 \end{cases},$$

$$\varphi(x) = \begin{cases} \frac{\sqrt{x}}{6} + \frac{2}{\sqrt{x}} \frac{2}{(x/4)^2} \left[\left(1 + \frac{x}{4}\right) \ln \left(1 + \frac{x}{4}\right) - \frac{x}{4} \right], & \text{if } x < 12 \\ 0.9, & \text{if } x \geq 12 \end{cases},$$

$$\beta^* = 1 \text{ for } K \in [1,7] \text{ and } \beta^* = \frac{6.7}{K} \text{ for } K \geq 7.$$

δ indicates the days that retailer needs to delivery per day. τ is the daily order number for each customer. K is the order number that each truck can delivery per day. The retailer can only serve limited customers because of the internet penetration and preference. e indicate the market penetration of the retailer. Then the targeting population density $\bar{\rho} = e\rho$. When the online orders are fulfilling from stores, it is noticeable that the region size would be $\frac{A}{N}$.

Then the expression can be simplified as

$$\begin{cases} \Gamma_S = \frac{W}{\sqrt{N}} + \lambda, & \text{if online orders are fulfilled from stores,} \\ \Gamma_C = W + \lambda, & \text{if online orders are fulfilled from DC.} \end{cases}$$

Appendix B

When consider the case where the retailer operates under a continuous fulfillment strategy, that is, it determines the proportion of online demand that is fulfilled from in-store, which in this thesis is denoted as β . The retailer chooses β and p to maximize its total expected profit,

$$\begin{aligned}
 \max_{\beta, p} \Pi &= p[D^o(p) + D^i(p)] + c_o D^o(p) - \phi_t \Phi_t \sqrt{N} [\beta D^o(p) + D^i(p)] - \phi_d \left(\frac{W}{\sqrt{N}} + \lambda \right) \beta D^o(p) \\
 &\quad - h_s \phi_I [\beta D^o(p) + D^i(p)]^{\frac{1}{2}} \sqrt{N} - \phi_d (W + \lambda) (1 - \beta) D^o(p) \\
 &\quad - [(1 - \beta) D^o(p)]^{\frac{1}{2}} h_c \phi_I \\
 &= \max_{\beta, p} \Pi = p[D^o(p) + D^i(p)] - \phi_t \Phi_t \sqrt{N} [\beta D^o(p) + D^i(p)] - \phi_d D^o(p) \left[\lambda + \frac{\beta W}{\sqrt{N}} + (1 \right. \\
 &\quad \left. - \beta) W \right] - h_s \phi_I [\beta D^o(p) + D^i(p)]^{\frac{1}{2}} \sqrt{N} - h_c \phi_I [(1 - \beta) D^o(p)]^{\frac{1}{2}} \\
 &\quad \text{s. t. } 0 \leq \beta \leq 1 \\
 &\quad D^i(p) \geq 0 \\
 &\quad D^o(p) \geq 0 \\
 &\quad p \geq 0
 \end{aligned}$$

Solving the models, I can have the Proposition B.1.

Proposition B.1: The optimal proportion of online demand that is fulfilled from in-store with continuous fulfillment strategy is 0 or 1.

Proposition B.1 tells that the optimal choice of the retailer is to fulfill the online demand entirely from the DC or from the store. As a result, I can limit the attention to discrete fulfillment strategy only, as discuss below.

The retailer's profits under the two fulfillment schemes are given by

$$\begin{aligned}
 \pi_S(p) &= \left(p + c_o - \frac{\phi_d}{\sqrt{N}} W - \phi_d \lambda - \phi_t \Phi_t \sqrt{N} \right) D^o(p) + (p - \phi_t \Phi_t \sqrt{N}) D^i(p) - \\
 &h_s \phi_I \left(D^o(p) + D^i(p) \right)^{\frac{1}{2}} \sqrt{N}, \text{ when } \beta = 1,
 \end{aligned}$$

$$\begin{aligned}
 \pi_C(p) &= (p + c_o - \phi_d W - \phi_d \lambda) D^o(p) - h_c \phi_I D^o(p)^{\frac{1}{2}} + (p + \phi_t \Phi_t \sqrt{N}) D^i(p) - \\
 &h_s \phi_I D^i(p)^{\frac{1}{2}} \sqrt{N}, \text{ when } \beta = 0.
 \end{aligned}$$

Under fulfillment strategy X ($X \in \{S, C\}$) the retailer's pricing decision can be modeled as:

$$\begin{aligned} \max_{p \geq 0} \quad & \pi_X(p) \\ \text{s. t.} \quad & D_X^o(p) \geq 0 \\ & D_X^i(p) \geq 0 \end{aligned}$$

Proof of Proposition B.1

Taking the first and second order derivations of Π ,

$$\begin{aligned} \frac{\partial \Pi}{\partial \beta} = & -\phi_t \Phi_t \sqrt{N} D^o(p) - \phi_d \left(\frac{W}{\sqrt{N}} + \lambda \right) D^o(p) - \frac{1}{2} h_s \phi_l [\beta D^o(p) + D^i(p)]^{-\frac{1}{2}} \sqrt{N} D^o(p) + \\ & \phi_d (W + \lambda) D^o(p) + \frac{1}{2} D^o(p) [(1 - \beta) D^o(p)]^{-\frac{1}{2}} h_c \phi_l, \end{aligned}$$

and

$$\frac{\partial^2 \Pi}{\partial^2 \beta} = \frac{1}{4} h_s \phi_l [\beta D^o(p) + D^i(p)]^{-\frac{3}{2}} \sqrt{N} D^o(p)^2 + \frac{1}{4} D^o(p)^2 [(1 - \beta) D^o(p)]^{-\frac{1}{2}} h_c \phi_l.$$

As a result of $\beta^* \in 0, 1$, the maximum profit happens when $\beta = 0$ or $\beta = 1$.