Incentive Contracts in Multi-agent Systems: Theory and Applications

by

Qi Luo

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Doctoral Committee:

Associate Professor Robert Cornelius Hampshire, Co-Chair
Professor Romesh Saigal, Co-Chair
Associate Professor Cong Shi
Assistant Professor Joline Uichanco
Professor Yafeng Yin

Qi Luo

luoqi@umich.edu

ORCID iD: 0000-0002-4103-7112

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ABSTRACT

This thesis studies incentive contracts in multi-agent systems with applications to transportation policy. The early adoption of emerging transportation systems such as electric vehicles (EVs), peer-to-peer ridesharing, and automated vehicles (AVs) relies on governmental incentives. Those incentives help achieve a specific market share target, prevent irregular behaviors, and enhance social benefit. Yet, two challenges may impede the implementation of such incentive policies. First, the government and subsidized organizations must confront the uncertainty in a market; Second, the government has no access to the organizations' private information, and thus their strategies are unknown to it. In the face of these challenges, a command-and-control incentive policy fails.

In §II, we revisit the primary setting in which a government agency incentivizes the OEM for accelerating the widespread adoption of AVs. This work aspires to offset the negative externalities of AVs in the "dark-age" of AV deployment. More specifically, this chapter designs AV subsidies to shorten the early AV market penetration period and maximize the total expected efficiency benefits of AVs. It seeks a generic optimal AV subsidy structure, so-called "two-threshold" subsidy policy, which is proven to be more efficient than the social-welfare maximization approach.

In §III, we develop a multi-agent incentive contracts model to address the issue of stimulating a group of non-cooperating agents to act in the principal's interest over a planning horizon. We extend the single-agent incentive contract to a multi-agent

setting with history-dependent terminal conditions. Our contributions include: (a) Finding sufficient conditions for the existence of optimal multi-agent incentive contracts and conditions under which they form a unique Nash Equilibrium; (b) Showing that the optimal multi-agent incentive contracts can be solved by a Hamilton-Jacobi-Bellman equation with equilibrium constraints; (c) Proposing a backward iterative algorithm to solve the problem.

In §IV, we obtain the optimal EV and charging infrastructure subsidies through the multi-agent incentive contracts model. Widespread adoption of Electric Vehicles (EV) mostly depends on governmental subsidies during the early stage of deployment. The governmental incentives must strike a balance between an EV manufacturer and a charging infrastructure installer. Yet, the current supply of charging infrastructure is not nearly enough to support EV growth over the next decades. We model the joint subsidy problem as a two-agent incentive contract. The government observes two correlated processes – the EV market penetration and the charging infrastructure expansion. It looks for an optimal policy that maximizes the cumulative social benefit in the face of uncertainty. In our case study, we find that the optimal dynamic subsidies can achieve 70% of the target EV market share in China by 2025, and also maintains the ratio of charging stations per EV.

§V ends the thesis with conclusions and promising future research directions. In summary, this thesis provides a new approach to appraise transportation and energy policies against exogenous and endogenous risks.

CHAPTER I

Introduction

1.1 Incentive Design in Principal-Agent Model

1.1.1 Rational for this Thesis

The principal-agent problem stems from a conflict between insurance (i.e., risk-sharing) and incentive [118]. The principal, the authority party in this relationship, proposes a *contract*, and the agent can either accept or reject it. Contracts ensure the agent works for the principal under the uncertainty in nature (i.e., risk) and provides incentives for the agent to reveal private information. The private information studied in this thesis is the actual action taken by the agent that affects the probability distribution of the output and thus the risk. For example, an employer (principal) hires employees (agents) to put certain effort into a project. The level of effort is unknown to the employer, but the stochastic output affected by the agent's effort is public information. Whenever such an action is *observable* but not *verifiable*, the *moral hazard* [63] arises. The solution to this moral hazard problem is to design so-called "incentive contracts" used throughout this thesis.

The principal-agent problem in its moral-hazard form arises in a broad range of practical issues from corporate finance, strategic behavior in politics, supply chain management to organizational design [11, 23, 28, 33, 44, 46, 54, 81, 87, 93, 136]. Incentive contracts are not merely the study of the legally binding agreement, as the name implies. Depending on the applications, it appears in forms such as governmental subsidies, revenue-sharing production plans, and managerial compensations.

Finding the optimal incentive contracts with a risk-averse agent is challenging because two self-interested parties' decisions are coupled. When the outcome of the hidden action is a random process, *dynamic incentive contracts* turn out to be considerably more complicated. Nevertheless, it is worth investigating under what conditions the optimal dynamic incentive contracts exist and how different organization forms affect the effectiveness of incentives. The following simple examples illustrate why dynamic contracts are more challenging compared with its static counterpart but worth studying.

Example I.1 (Static v.s. Dynamic Incentive Contracts)**.** Optimal dynamic contracts have memory if the payoff is history-dependent.

In the probably simplest two-period principal-agent model [103], the output $x_t \sim f(x|a_t), t \in \{1, 2\}$, i.e., x_t only depends on the action at period t. A contract defines compensations as a function of observed output $c_1(x_1)$ and $c_2(x_1, x_2)$. The risk-averse agent's utility is separable as $u(c_t) - g(a_t)$. The risk-neutral principal thus maximize the total payoff satisfying the incentive-compatible (IC) and individual-rational (IR) constraints.

(1.1)

$$\max_{c_1,c_2} \mathbb{E}_a[x_1 + x_2 - c_1(x_1) - c_2(x_1, x_2)]$$

$$s.t. \quad \{a_1, a_2\} \in \arg\max_{a_1,a_2} \mathbb{E}_{\tilde{a}_1,\tilde{a}_2}[u(c_1(x_1)) + u(c_2(x_1, x_2)) - g(\tilde{a}_1) - g(\tilde{a}_2)] \quad (IC)$$

$$\mathbb{E}_{a_1,a_2}[u(c_1(x_1)) + u(c_2(x_1, x_2)) - g(a_1) - g(a_2)] \geq 2\bar{u}, \quad (IR)$$

where $\bar{u}$ is the minimum utility received per period.

It can be proved that the optimal contract satisfies the following condition: [103]:

$$\frac{1}{u'(c_1(x_1))} = \mathbb{E}_{a_1}[\frac{1}{u'(c_2(x_1,x_2))}]. \tag{1.2}$$

We observe that, if $c_2(x_1, x_2) = c_2(x_2)$, then the LHS of (1.2) is equal to a constant, thus no incentive is given in period 1. (1.2) implies the front-load consumption – the agent has motivation to consume more in the first period to keep its continuation wealth low. This example reveals a universal characteristic of dynamic contracts – they affect the agent's decision over the planning horizon, so the agent tends to overwork at the beginning. Static analysis is unable to obtain those managerial insights.

The challenges of finding optimal contracts increase exponentially when the principal faces more complex organizations. This thesis aims to do so – the organization is extended from single-agent to multi-agent systems (in §III and §IV).

Example I.2 (Multiagent Contracts in Matrix Games)**.** The following example demonstrates the importance of investigating the existence conditions in a static matrix game setting. A principal chooses to compensate c to two agents as either low (L) or high (H) payoff, i.e., $c \in \{L, H\}$. Agents indexed by $i \in \{1, 2\}$ put effort into a project and the output of agent i's effort is denoted as $X_i \in \{A, B\}$ at levels A or B.

- The principal desires to stimulate agent 1 to exert output A and agent 2 to exert output B. The outcomes of signing contracts can be represented by the matrices in Table 1.1 where each entry is principal's and agent's utility received from the contract. If these two contracts are signed separately, the unique equilibria are $\{L, A\}$ with agent 1, and $\{L, B\}$ with agent 2.

Table 1.1: Static incentive contracts with two uncorrelated agents.

	A	B
L	4,2	2,1
H	3,3	1,2

	A	B
L	2,1	4,2
H	1,2	3,3

- We now assume that two agents outputs are aggregate in a linearly additive way. In this case, the principal's dominant policy is $[c_1, c_2] = [L, L]$. Notice that the existence and the number of equilibrium may vary with agents' utility functions $u_i(c_i, X_i, X_{-i})$. Three possible outcomes for contracts are below:

1. Unique Nash equilibrium: Assume that the utility of each agent is only dependent of its payoff, i.e., $u_i(c_i, X_i, X_{-i}) = c_i$. The agents' best responses are $[X_1, X_2] = [A, B]$. With a fixed $[c_1, c_2] = [L, L]$, their utility follows Table 1.2.

Table 1.2: Correlation between agents' outcomes with unique equilibrium

	A	B
A	2,1	2,2
B	1,1	1,2

2. Multiple Nash equilibria: Assuming that the principal rewards whoever delivers B an additional unit of compensation, there exists two Nash equilibria: $[X_1, X_2] = [A, B]$ and $[X_1, X_2] = [B, B]$ in which their utility follows Table 1.3.

Table 1.3: Correlation between agents' outcomes with multiple equilibria

	A	B
A	2,1	2,3
B	2,1	2,3

3. No Nash equilibrium: Assuming that the utility of each agent is affected by the other agent's action such that the principal would reward the agents when their output match, i.e., $u_i(c_i, X_i, X_{-i}) = c_i + 2$ if $X_i = X_{-i}$. Then there is no Nash equilibrium as seen in Table 1.4.

Table 1.4: Correlation between agents' outcomes with no equilibrium

	A	B
A	4,3	2,2
B	1,1	3,4

With two simple examples from above, we conclude that incentive contracts in multi-agent systems are a significant and general model with many applications, and solving to the optimal contracts is more complicated than it may seem on the surface. This thesis investigates a solvable framework for dynamic incentive contracts that can be applied to differently structured organizations.

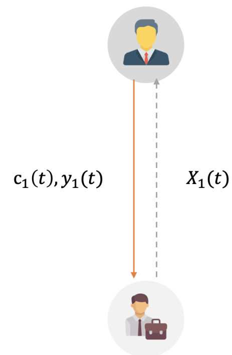
1.1.2 Statement of the Problem

This thesis covers the following three types of incentive contract(s) models in Figure 1.1. The planning horizon is either finite or infinite in continuous time. The single-agent model (Figure 1.1a)) that initiated this field of study is motivated by Holmstrom and Milgrom [61] and completed in the celebrated work of Sannikov [104, 105]. The case with infinitely many agents, i.e., mean-field approximation (Figure 1.1c) of incentive contracts, has recently been solved by Elie et al. [37]. This thesis first explores the realm in between – the number of agents on the same level (Figure 1.1b) is finite, and their correlation is not trivial as in Thakur [117]. We characterize under what conditions this type of multi-agent contracts exist, and then seek an efficient method to find these contracts satisfying the IC- and IR-constraints as in Example I.1.

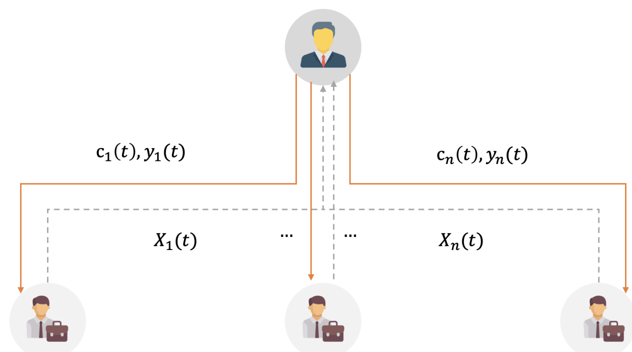
The sequence of decisions unfold as follows:

1. The principal offers contracts to agents simultaneously.

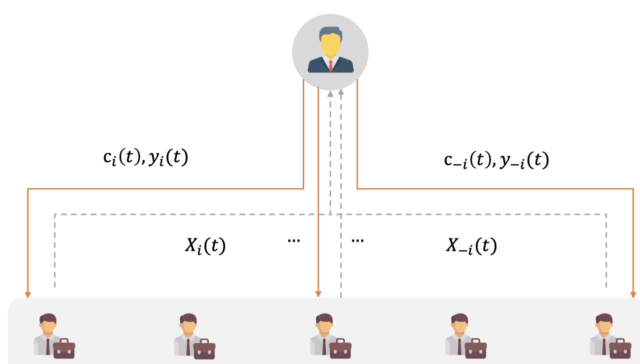
2. The agents evaluate the contracts the offered contracts and either accept or decline them. The multi-agent equilibrium (defined in §III) guarantees that no



(a) Single-Agent Incentive Contract Model (Chapter II)



(b) Multi-agent Incentive Contracts Model (Chapter III)



(c) Mean-Field Incentive Contract Model

Figure 1.1: Organizational alternative structures in incentive contracts

agent has the motivation to deviate from the group's decisions.

3. If agents accept the contracts, they exert the effort in each period.

4. External randomness occurs, which affects the agents' output processes.

5. The principal observes the output and remunerate the agents according to the terms in the signed contracts.

The information available for each decision-maker is the key research question that differentiates this thesis from the other decentralized control or dynamic games literature. Intuitively, the less information shared between different parties, the more challenging to find the optimal contracts. Two basic rules followed in this thesis are: First, the principal should have no direct access to the action taken by the agents working for it, videlicet, the moral hazard problem exists. Second, agents have less power in choosing between different optimal contracts at equilibrium. The latter assumption is natural as the principal has the absolute authority in designing contracts [73].

1.1.3 Research Objectives

1. Characterizing the sufficient conditions for the existence and uniqueness of multi-agent incentive contracts.

2. Analyzing the government incentives for automated vehicles (AVs) as incentive contracts with a market penetration process.

3. Designing the government incentives for electric vehicles (EVs) as two-agent contracts that coordinating the sales of EVs and the number of EV charging stations nationwide.

Besides, this dissertation uses teal-world data in the numerical experiments aiming to provide managerial insights for transportation policymakers.

1.1.4 Structure of the Thesis

This thesis is organized in logic order, as in Figure 1.1. We review the related literature in the remainder of this chapter to set the stage for our contribution. §II revisits the basic setting – the government agency incentivizes a AV manufacturer for the fast adoption of AVs and the mitigation of negative externalities due to low market penetration. §III proves the existence and the uniqueness of multi-agent incentive contracts. §IV demonstrates the power of this model with an application to EV subsidies. We end the thesis with conclusions and future research directions in §V.

1.2 Literature Review

1.2.1 A Brief Review of Incentive Contract Theory

Reviewing the broader contract theory literature is beyond the scope of this thesis; hence this section only summarizes the evolution of incentive problem in its principal-agent form. This problem is a special case of stochastic Stackelberg differential games played between a principal and an agent. The principal expects the agent to exert a targeted level of effort, and knows *ex ante* that, once accepted the contract, the agent will have no incentive to deviate from this level (i.e., satisfying the incentive-compatible condition) over the planning horizon. As a result, incentive contracts bypass any moral hazard. However, finding such a globally optimal contract is not trivial. Spear and Srivastava [113] studied the dynamic moral hazard problem in a discrete-time setting, where the state space explodes exponentially in the size of the planning horizon (the curse of dimensionality in dynamic programming). Holmstrom and Milgrom [61] proposed a continuous-time model. In this setting, the agent's output process is represented by a stochastic differential equation (SDE) whose *drift* term is controlled by the agent's effort. As a result, the continuous-time incentive contract problem is a limit of discrete-time dynamic games whose number of stages becomes unbounded in any finite interval. Some extensions of their work include Schättler and Sung [106], Sung [115], and Müller [91].

In recent years, following the groundbreaking work of Sannikov [104], there has been a resurgence of interest in the dynamic contract theory. As a consequence, we can decouple the principal's and the agent's problems as the agent's effort as a function of the parameter. The resulting principal's problem can then be solved by dynamic programming (more specifically, a Hamiltonian-Jacobi-Bellman equation) for the incentive contract [23–25]. The single-agent incentive contract problem has then been explored in many settings [29, 54, 57, 88, 100, 125]. A contracted agent can multitask with endogenous risk-taking [126].

Incentive contract in multi-agent systems is a natural extension considering the nature of teamwork in the enterprise. Keun Koo et al. [71] presented the first extension of multi-agent incentive contracts that initiated a stream of literature for team incentives using the Martingale approach [36, 37, 48, 117]. Similarly, the incentive-compatible condition is satisfied if the agents' actions form a *subgame perfect Nash equilibrium* over the planning horizon. However, in the multi-agent setting, new challenges arise due to varied interactions between agents. For example, an arbitrary agent may compare both its effort and payoff with others – such a phenomenon is called inequity aversion [38]. Goukasian and Wan [52] showed that the inequity aversion is present in multi-agent incentive contracts, and the agents' comparisons may lead to envy or guilt and lower their morale over the planning horizon. This thesis answers an unanswered problem in prior work: What are conditions for the existence and uniqueness of multi-agent Nash Equilibrium? It is a premise for implementing this approach to a miscellaneous collection of problems on this avenue.

1.2.2 Empirical Evidence for Incentive Contracts

Many empirical studies have tested the incentive contract theory. Bloom and Van Reenen [9] found that applying incentive contracts raised the productivity of the firm and increased the variance of individual performance. Benabou and Tirole [7] gave an opposite example in the financial industry. It turned out that contingent rewards (extrinsic motivation) may conflict with the individual's desire to perform tasks (internal motivation). Chiappori and Salanié [19] presented a comprehensive review of the econometric of contract theory in a static setting. The reverse causality differentiates the test of adverse selection and moral hazard: in an adverse selection context, the choice of contract arises from the agent's type; in a moral hazard context, the agent's action affects contracts' performance.

Incentive contracts in a dynamic setting were also tested in insurance, information technology, and corporate finance [1, 20, 34, 127]. New features such as memory, learning, and commitment are revealed in the dynamic empirical approach. Note that continuous-time models are notoriously hard for implementation, and few empirical studies verify it in literature. He [56] tested the executive compensation problem where the firm size follows a geometric Brownian motion. The firm size is affected by this executive manager's effort, which then awards his or her equity shares. Giat and Subramanian [50] developed a discrete-time approximation approach to fit principal's posterior belief in continuous-time contracts. Their study found that the interaction between asymmetric beliefs, risk-sharing, and adverse selection costs caused the time-paths of the agent's incentive intensities to be either increasing or decreasing. Similar empirical studies in firm theory include [2, 6, 18].

1.2.3 Gamification in Emerging Transportation Systems

Emerging transportation refers to advanced technologies, including electric vehicles (EVs), connected and automated vehicles (CAVs), and emerging shared mobility services springing around them. The early adoption of these technologies is usually supported by governmental incentives, financial or non-financial, so that today's cash grants endogenize the future social benefit. Gamification applies game-design elements and game principles in the non-game context of transportation. Stackelberg games are most relevant to this field – the government agency plays as a leader who designs networks, offers incentives, or allocate resources to the controlled transportation sector.

In classic transportation literature, gamification is applied on a network level. For example, Yang et al. [129] studied the user equilibrium in a Stackelberg routing game with the existence of oligopoly Cournot-Nash firms. The system optimal player is the leader and the user equilibrium, and the Cournot-Nash players are the followers. Öner et al. [97] formulated the operations of automated highway systems as a mean-field Stackelberg game in which a control center as a leader imposes fixed policies on follower vehicles to minimize the total consumed energy and travel time. In transportation regulatory literature, gamification is applied on an aggregate level. Shinde and Swarup [112] studied the demand response for EV charging problems where the leaders are the utility company, and EV aims to reduce the power generation cost and charging cost. Other demand response users are followers. Luo et al. [81] used a dynamic game model that relaxed the information-sharing assumptions in the design of governmental incentives for automated vehicles [14]. More detailed review on each application can be found in §II and §IV, respectively.

In summary, incentive contracts are a theoretical economic model abstracted from different applications. This thesis expands the setting from contracting with one agent to multi-agent, which opens doors to modeling the collaboration and competition in complex industrial organizations. Compared with static incentive theory, dynamic contracts can provide new managerial insights such as the function of memory and commitment in repeated games. This dissertation applies these results to the regulations of automated vehicles (§II) and electric vehicles (§IV) aiming to accelerate the adoption of emerging transportation.

CHAPTER II

Single-Agent Incentive Contracts for Automated Vehicles

2.1 Transportation Incentive Design: an Overview

Transportation is one of the most publicly subsidized sectors in the United States and much of the world. Conventional transport such as public transit highly relies on capital and operating subsidies from federal, state, and local sources. Recent research has been paying attention to the governmental incentives for emerging transportation technology such as electric vehicles (EVs) [60], car-sharing [90], and emerging connected and automated vehicles (CAVs) [81]. For example, despite the low market penetration rate, the total cost of EV subsidies is substantial. In the United States, the federal subsidy could end up costing as much as $15 to $20 billion in 2019 [111]. Notice that most of these conventional and emerging transportation technologies or services are operated exclusively, i.e., at the cost of validating payment, and everyone in the area pays for the subsidies whether or not using it. On the other hand, there are positive traffic network externalities associated with these applications. Using public transit or CAVs can potentially reduce traffic congestion; transportation electrification can reduce emissions; hence each taxpayer benefits them. The overuse of subsidies begs a proper justification for why subsidies in place are effective and efficient – if not so, how to improve them. This chapter, along with the following chapters aim to create a dynamic game model for evaluating, designing, and improving the planning of incentive policies.

The remainder of this chapter focuses on subsidies for emerging automated vehicle technology and is organized as follows. In §2.3, we formulate the AV subsidy problem by integrating a DOI model and DSG. The optimal subsidy policy is computed by dynamic programming with a set of implementability constraints. We then prove the structure of optimal AV subsidy policies and demonstrate their robustness by numerical experiments. In §2.4, the model is extended to a changing AV market potential with price incentives. Finally, we conclude the chapter in §2.5.

2.2 Accelerating the Adoption of Automated Vehicles

Automated Vehicles (AVs) are anticipated to tremendously enhance the efficiency, safety, and convenience of the existing transportation system, with new businesses springing up around them. Pioneering companies such as Waymo and GM Cruise have been working to develop, test, and pilot commercial services. A stream of research has been conducted to quantify the profound and far-reaching implications of AVs on the transportation system, society, and the economy [15, 17, 22, 42, 53, 89, 120]. In general, the efficiency gains from AVs are from the following three sources: (a) Vehicle platooning can improve substantially the throughput of highway facilities; (b) Advanced traffic management schemes (e.g., adaptive speed control and harmonization) that leverage vehicle connectivity can further increase the throughput and improve the stability of the traffic stream; (c) Automation can make on-demand shared mobility services more cost-effective. Low-cost shared mobility services have the potential to yield higher vehicle occupancy and reduce overall vehicular traffic demand.

All these efficiency gains hinge on the market penetration level of AVs being sufficiently high. At low market shares, AVs exert little impact on enhancing transportation system efficiency [86]. Worse yet, the early deployment of AVs will likely compromise efficiency. At early stages of the deployment, car manufacturers (original equipment manufacturers or OEMs) will likely configure their AVs with a lower operation speed and excessive safety clearance, i.e., longer time gap or headway, to ensure safety and avoid liability. Undoubtedly, the presence of these types of AVs in the traffic stream will slow other vehicles down (imposing so-called congestion externalities) and thus compromise the efficiency of transportation systems [12, 13, 49, 107]. Such an efficiency degradation could last for a very long time until the market penetration of AVs reaches a certain threshold [107]. In addition, the benefits promised by AVs can be offset by the increase in vehicle miles traveled (VMT) generated by empty trips of AVs and induced travel demand [21, 43, 79, 83], particularly at the early stage of AV deployment when necessary transportation policies are not in place and the shared mobility market is not large enough to facilitate ride-sharing.

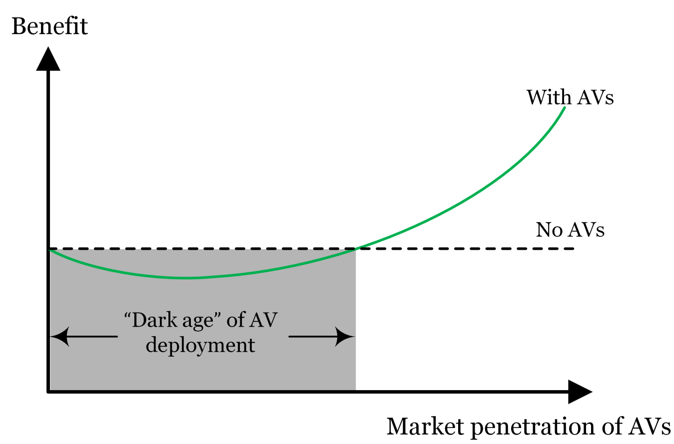


Figure 2.1: Efficiency benefit of AV deployment

With all these considerations, we envision that the overall efficiency benefit offered by AVs with various market shares likely follow a trend depicted in Figure 2.1. The benefit initially drops and then rises with the market penetration of AVs. For lack of a better word, we call the initial stage of performance drop the "dark age" of the AV deployment. *AV market failure* may occur if the market share stagnates in this "dark age" for a long time. Such an initial drop is pretty unusual and does not arise in the deployment of other new technologies such as electric or connected vehicles.

A powerful policy to address the aforementioned negative impacts of AVs is to price their utilization in a way that internalizes their external costs incurred by longer time headway or empty trips, for example. However, the timing of implementing this policy is critical, since such a congestion charge or Pigouvian tax may increase the cost of using AVs, thereby discouraging their early adoption. It appears plausible that we need to endure short-term pains for long-term benefits.

A suite of policies needs to be in place to fully correct the possible AV market failure. However, the objective of this chapter is modest. Given the social benefit curve as in Figure 2.1, we investigate a subsidy policy to accelerate the deployment of AVs adaptively from lower to higher market penetration rates. The objective of AV subsidies is to maximize the total *expected* efficiency benefits from the AV deployment over the planning horizon. The questions we are interested in examining are who should be incentivized, and how much subsidies should be enacted. While subsidizing customers can nudge them to adopt the AV technology, subsidizing manufacturers can directly motivate them to innovate the technology. The latter strategy may effectively mitigate the negative externalities of prototype AVs. It thus remains an open question, which side should be directly incentivized. With a time-varying market share, we also need to consider when and how long subsidies should be implemented. Note that previous research treating it as a static problem [10, 16] failed to answer the optimal timing for enacting AV subsidies. In contrast, we investigate how to implement AV subsidy policies that are adaptive to the AV market share. Different from previous work [69, 94, 108, 119], our model can also capture the uncertainty in forecasting the diffusion of AV technology.

The contribution of this chapter is to develop a new approach to find the optimal AV subsidy policy that shortens the "dark age" of AV deployment. Our approach is able to solve for *non-myopic* subsidy policies with the *information asymmetry* between the government agency and AV manufacturer. Subsidizing new technologies is never a one-way command-and-control process [4, 27], while full information is widely implied in the previous literature. For instance, [66] recently explored optimal subsidy paths to make a technology competitive at a given future period. [75] established a subsidy scheme to induce customers to adopt new technology over time. Results showed that, if customers are myopic, an increasing subsidy scheme is preferred; otherwise, subsidies should decrease over time to induce the adoption in early periods. The effectiveness of those suggested AV subsidy policies may be reduced because of the information asymmetry. This chapter considers two essential problems: (a) How to model the non-myopic decision makings of the government agency and the manufacturer; (b) How to avoid the information asymmetry that may lead to an unintended AV market failure. The asymmetric information is caused by government agency's unawareness of the subsidized entity's actual effort in promoting AVs. The new approach integrates two modeling techniques cohesively: a *Diffusion of Innovations* (DOI) model that describes the evolution of the AV market share and captures how the AVs' efficiency benefit varies with the market share, and *dynamic Stackelberg games* (DSG) in continuous time.

2.3 Accelerating AV Market Penetration by Subsidies

Our AV subsidy design framework integrates a DSG model with a DOI process. The main idea is as follows. In the near future, the evolution of AV's market share follows a DOI model with uncertainty. The government agency is the leader (denoted by "G") who establishes statutory subsidy policies with the intention of accelerating the adoption of AVs. AV manufacturers are the followers (denoted by "A") who respond to the subsidy policies by enhancing the AV innovations. Both are non-myopic decision makers because they consider each other's decisions with regard to the time-varying AV market dynamics. The information asymmetry occurs when the government agency expects to advance the AV technology by subsidizing the manufacturers. However, the manufacturer's exact effort in AV innovations is not observable. Such asymmetric information on AV innovations compromises the cost-effectiveness of the AV subsidy policies. On the other hand, given that more advanced AV innovations will accelerate the AV market penetration process, the government agency can use the time-varying AV market share as an imperfect indicator of the manufacturer's effort. The government agency's objective is to find an optimal AV subsidy policy that maximizes its total expected payoff (i.e., the efficiency benefit of AVs) over the planning horizon. In what follows, we first introduce how to model the AV market penetration process with uncertainty, then we integrate this process into a DSG setting.

2.3.1 AV Market Penetration with Uncertainty

The DOI model depicts the process by which the AV technology spreads in the transportation system. The AV market size, i.e., the cumulative number of AVs sold by time t, is denoted by $N(t)$. The AV market potential, i.e., the population of potential AV consumers, is denoted by $M(t)$. Both AV market dynamics variables are observable to the government agency. The DOI model presumes that AV consumers consist of two groups: innovators who are early users of AVs and imitators whose tendency to purchase AVs depends on the size of innovators. Therefore, the market penetration rate is determined by two diffusion parameters: (a) the coefficient of external influence a (the "power of innovation") that represents the manufacturer's effort in AV innovations, and (b) the coefficient of internal influence b (" the power of contagion") that represents the word-of-mouth effect within the consumers' social network. Although many extensions of the model have been proposed, we simplify the following analysis by using a (generalized) Bass diffusion model [5]. The Bass diffusion model is widely recognized as a seminal work that initiated a stream of DOI models. Nevertheless, our subsidy design paradigm can easily adopt more complex market penetration models [94, 109, 116]. At time t, the AV market penetration rate $dN(t)/dt$ follows the dynamics below,

$$\frac{dN(t)}{dt} = a\Big(M(t) - N(t)\Big) + bN(t) \cdot \frac{M(t) - N(t)}{M(t)},$$

which is a combination of the innovation effect (i.e., the rate attributes to innovators as the first term) and the imitation effect (i.e., the rate attributes to imitators as the second term).

The AV market forecast may overestimate or underestimate the AV market growth in the future. It is natural to assume that the uncertain AV market dynamics follows a Gaussian process. Each AV market penetration sample path is generated by an extraneous Brownian motion $B(t)$ (Figure 2.2), and the AV market dynamics is described by the following stochastic differential equation:

$$dN(t) = \left(a + b\frac{N(t)}{M(t)}\right)(M(t) - N(t))dt + \sigma N(t)dB(t), \tag{2.1}$$

where the drift term follows the DOI model above, and the diffusion term has a constant volatility σ. Note that the uncertainty aggravates as the time horizon expands in Figure 2.2.

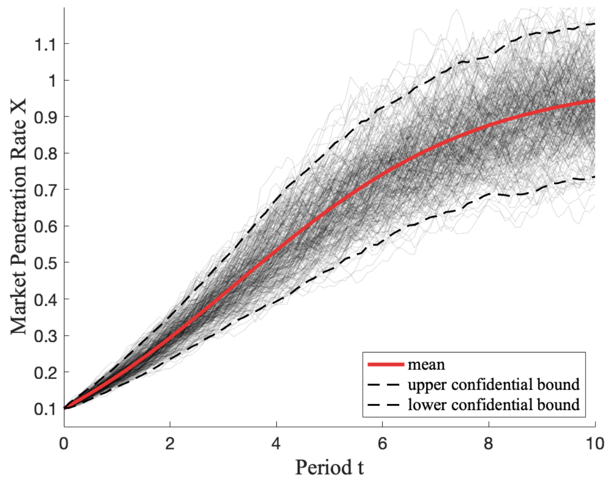


Figure 2.2: Sample paths, mean and 95% confidence interval of the DOI model for AV market penetration

The uncertainty in the AV market penetration process implies the possible inefficiency of AV subsidy policies because of information asymmetry. Over the planning horizon $t > 0$, the government agency continuously receives the efficiency benefit of AVs represented by a function of market size $g(N(t))$. As shown in Figure 2.1, $g(N(t)) < 0$ in the "dark age" of AV deployment. To accelerate the early adoption of AVs, the government agency would like to incentivize the manufacturer to realize a large $a(t)$ during the "dark age" by a sequence of AV subsidies.

2.3.2 AV Subsidy Policies for Dynamic Stackelberg Games

In what follows, we formulate the AV subsidy problem as a DSG in the simplest setting (Figure 2.3). An extension that includes varying market size and price discount mechanisms for consumers are discussed in §2.5. When the manufacturers' responses are additive, we can use a single agent to present a group of agents with linearly aggregated responses. We discuss more complicated interactions between agents at the end of this chapter.

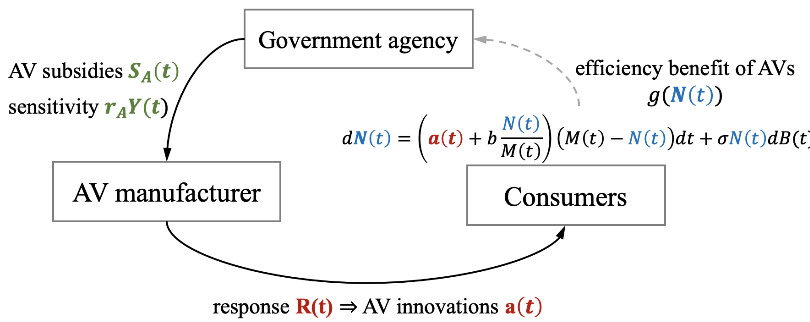


Figure 2.3: DSG model for the AV subsidy problem

After implementing the AV subsidies, the efficiency benefit $g(N(t))$ is what the government agency gains, and the total subsidies are what it pays to the manufacturer. As in [69], we suppose that the government agency's instantaneous payoff is a linear combination of the benefits and costs. The risk-neutral government agency's goal is to initiate a sequence of per-unit AV subsidies $\{S_A(t)\}_{t\geq 0}$ to maximize the total expected discounted payoff:

$$\mathbb{E}\left[r_G \int_0^\infty e^{-r_G t}\left[g(N(t)) - S_A(t)\cdot \frac{dN(t)}{dt}\right]dt\right],$$

where we use a continuously compounded discount factor $e^{-r_G t}$. Note that r_G in front of the integral normalizes the government agency's total payoff to annuity payments.

After receiving the subsides, the AV manufacturer reacts with a response $R(t)$. The response represents a target action that affects the DOI process, and the exact value of $R(t)$ is not observable by the government agency. In practice, the government usually lacks the access to AV manufacturer's private information on innovations, or monitoring the effort is too costly. Thus, the government agency who only observes the AV market size $N(t)$ (i.e., an imperfect indicator of $R(t)$) needs to specify the amount of subsidies S_A and the *sensitivity level* $r_A Y$ in the policy (Figure 2.3).

In the simple setting, we assume that $R(t)$ directly controls the power of innovation as $R(t) \equiv a(t)$. While most of the DOI literature focuses on controlling the imitation effect represented by b, promoting the AV innovation represented by a is the objective of AV subsidies. This is because the innovation effect is the main driving force in the early adoption of AVs. In the extension, we also discuss how $R(t)$ can include other factors like price mechanisms. The manufacturer's instantaneous utility function is $h(S_A(t), R(t))$. With a given AV subsidy policy, the AV manufacturer's objective is to maximize the expected total discounted utility:

$$\mathbb{E}\left[r_A \int_0^\infty e^{-r_A t} h(S_A(t), R(t)) dt\right].$$

To guarantee the existence of the best response, the AV manufacturer is assumed to be risk-averse, i.e., the instantaneous utility h is concave and satisfies $\partial h/\partial S_A > 0$ and $\partial h/\partial R < 0$. Risk aversion expressed by the concavity of the utility function h is a realistic assumption. Non-myopic AV manufacturers who are exposed to the AV market uncertainty prefer not to overexert their innovations now for future market growth. We will give a more rigorous proof of this in Proposition II.5.

The *optimal* AV subsidy policy characterizes a sequence $\{S_A(t)\}_{t\geq 0}$ that maximizes the government agency's expected total discounted payoff if only the AV manufacturer cooperates with the best responses. Following the literature of mechanism design, we call the conditions that specify the manufacturer's best responses the IC-constraint (incentive-compatible) and the IR-constraint (individual-rationality), respectively. The IC-constraint guarantees that the AV manufacturer's best response solves the utility maximization problem above. The IR-constraint guarantees that the AV manufacturer stays in the market as long as the cumulative expected utility over the horizon exceeds some pre-defined quantity W_0. In summary, the optimal

AV subsidy policy solves the following optimization problem:

$$
\begin{aligned}
(2.2) \qquad & \max_{S_A(t):t\geq 0} \mathbb{E}\left[r_G \int_0^\infty e^{-r_G t}\left[g(N(t)) - S_A(t)\frac{dN(t)}{dt}\right] dt\right] \\
s.t. \quad & a(t) \in \arg\max_a \mathbb{E}\left[r_A \int_t^\infty e^{-r_A u} h(S_A(u), a(u)) du\right], \forall t \geq 0 \ \text{ (IC-constraint)}, \\
& \mathbb{E}\left[r_A \int_0^\infty e^{-r_A t} h(S_A(t), a(t)) dt\right] \geq W_0 \ \text{ (IR-constraint)}.
\end{aligned}
$$

Plugging the AV market penetration process (2.1) into the objective function of (2.2) yields $\mathbb{E}[\int_0^\infty \sigma N(t) dB(t)] = 0$. This does not mean that the asymmetric information is eliminated because $a(t)$ is still not directly controlled by the government agency. The government agency's objective function can be rewritten as:

$$
\max_{S_A(t):t\geq 0} \mathbb{E}\left[r_G \int_0^\infty e^{-r_G t}\left[g(N(t)) - S_A(t)\left(a(t) + b\frac{N(t)}{M}\right)(M - N(t))\right] dt\right].
$$

Before investigating how to find the optimal AV subsidy policies, we first introduce a hierarchy of feasible policies that are in the government agency's interest. Suppose that the joint domain of the government agency's decision $S_A(t)$ and the manufacturer's response $R(t)$ is a known compact set $\mathcal{U} = \mathcal{U}_{S_A} \times \mathcal{U}_R \in \mathbb{R}^2$. The support $\mathcal{U}_{S_A}$ and $\mathcal{U}_R$ are known a priori due to the limited budgets of both sides at time t. In what follows, we define the *admissible*, *implementable*, and *optimal* subsidy policy, respectively.

Definition II.1. $\{S_A(t), R(t)\}_{t\geq 0}$ are admissible if $(S_A(t), R(t)) \in \mathcal{U}$ for all $t \geq 0$ along any path generated by the stochastic process $B(t)$.

The implementable policies guarantee that the AV manufacturer will realize the target responses after receiving the subsidies. With the presence of information asymmetry, the attainable implementable policies are second best.

Definition II.2. $\{S_A(t), R^*(t)\}_{t\geq 0}$ is implementable if:

1. $\{S_A(t), R^*(t)\}_{t\geq 0}$ is admissible,

2. For a given $S_A(t)$, $R^*(t)$ are optimal for the AV manufacturer's utility maximization problem (i.e., the policy is incentive-compatible).

Finally, our goal is to find the optimal subsidy policy defined below.

Definition II.3. $\{S_A^*(t), R^*(t)\}_{t\geq 0}$ is optimal if it is implementable and maximizes the government's expected total payoff.

The optimal subsidy policy is dependent on the AV market dynamics and the expected total payoff. Hence, the decisions are adaptive and non-myopic over the planning horizon. Since the bilateral decisions $S_A(t)$ and $R(t)$ are coupled in (2.2), and there are infinite number of IC-constraints in continuous time, below we shall introduce a tractable scheme to decompose the optimal AV subsidy problem.

2.3.3 Solving for Optimal AV Subsidies

The optimization for the AV subsidy problem (2.2) can be transformed into a dynamic program. This tractable scheme for solving the DSG is inspired by the celebrated results in differential games [25, 104]. The road map for the dynamic program transformation includes three steps. First, we can find an equivalent representation of the IC-constraint (Theorem II.4). Second, the representation can be parameterized and incorporated into the objective function with mild smoothness assumptions (Proposition II.5). Finally, the optimal AV subsidy policies can be computed by dynamic programming in continuous time, i.e., a Hamilton-Jacobi-Bellman (HJB) equation (Theorem II.6).

We denote the manufacturer's response as a generic variable $R(t)$. One of the central ideas in the DSG is *continuation value*, which is the expected discounted total payoff/utility at time t with the optimal subsidy policy followed to the end of the horizon. This function is equivalent to the value function in dynamic programming or reinforcement learning. More specifically, the AV manufacturer's continuation value at time $t \geq 0$ is given by

$$W(t) = \mathbb{E}\left[r_A \int_t^{\infty} e^{-r_A(s-t)} h(S_A^*(s), R^*(s)) ds | \mathcal{F}_t\right],$$

where the filtration $\mathcal{F}_t$ represents the information collected by time t (including the decisions S_A and Y, and the market dynamics X). $W(t)$ is a state variable in the dynamic program. Applying the Martingale representation theorem, we can derive the dynamics of $W(t)$ as follows:

$$dW(t) = r_A \left(W(t) - h(S_A^*(t), R^*(t))\right) dt + \sigma r_A Y(t) dB(t),$$

where $r_A Y(t)$ is the sensitivity level of the manufacturer's continuation value $W(t)$ with respect to the AV market size $N(t)$. $Y(t)$ measures the marginal utility gained by increasing the response $R(t)$. It is a well-defined variable because the dynamics of $W(t)$ and $N(t)$ are adapted to the filtration generated by the process $B(t)$, i.e., the extraneous noise in the AV market forecast.

Since $\mathcal{U}$ is compact, we denote the set $\mathcal{U}_R = [\underline{R}, \bar{R}]$. For instance, $R(t) \equiv a(t) \in [0.01, 0.04]$ in this setting [84]. Over the infinite planning horizon, we assume a transversality condition $\lim_{s\to\infty} \mathbb{E}[e^{-r_A s} W(t+s)] = 0$. Then we can derive the following theorem that converts the subsidy policies respecting $R(t)$ to $Y(t)$ by the comparison principle.

Theorem II.4 ([104])**.** *For any given subsidy policy $\{S_A(t), Y(t)\}$ at any $t \geq 0$, the IC-constraint that derives the manufacturer's optimal decision in* (2.2) *is equivalent to:*

$$h(S_A(t), R(t)) + Y(t)R(t) \geq h(S_A(t), \hat{R}(t)) + Y(t)\hat{R}(t) \text{ for all } \hat{R}(t) \in [\underline{R}, \bar{R}]$$

Proof. Theorem II.4 Given the dynamics of $N(t)$ in equation 2.1, we can compute the dynamics of the market penetration rate $X(t)$ when (a) the AV manufacturer control the power of innovation, i.e., $R(t) = a(t)$; (b) the AV manufacturer controls the pricing incentives, i.e., $R(t) = p(t)$. For brevity, we only prove the more complicated case $R(t) = p(t)$ in below.

$$\begin{aligned} dN(t) &= M(t)\left(a + b\frac{N(t)}{M(t)}\right)(1 - \frac{N(t)}{M(t)})dt + \sigma N(t)dW(t), \\ dM(t) &= M(t)p(t)dt. \end{aligned}$$

Defining $X(t) = (\frac{N(t)}{M(t)})$ and applying Ito's lemma yields

$$\begin{aligned} dX(t) &= (a + b\frac{N(t)}{M(t)})(1 - \frac{N(t)}{M(t)})dt + p(t)\frac{N(t)}{M(t)}dt + \sigma(\frac{N(t)}{M(t)})dW(t), \\ &= (a + (b - a + p(t))X(t) - bX(t)^2)dt + \sigma X(t)dW(t). \end{aligned}$$

Assume that at time $t > 0$, the AV manufacturer follows the target responses that are obtained by solving the optimization problem (2.2). We assume that $r_A = r_G = r$ and define

$$V_A(t) = \int_0^t re^{-rs}h(s)ds + e^{-rt}W(t)$$

where $W(t)$ is the manufacturer's continuation value at time t.

It is easy to see that $V_A(t)$ is a martingale, i.e.,

$$\mathbb{E}\left[V_A(t)|\mathcal{F}_s\right] = V_A(s), \forall s < t.$$

From the Martingale representation theorem, there exists an adopted process $Y(t)$ such that

$$dV_A(t) = re^{-rt}\sigma Y(t)dB(t), \tag{2.3}$$

where $Y(t)$ is adapted to the filtration of $B(t)$, and $rY(t)$ can be interpreted as the sensitivity of $W(t)$ to $X(t)$. Applying Ito's lemma to V_A :

$$dV_A(t) = re^{-rt}h(t)dt - re^{-rt}W(t)dt + e^{-rt}dW(t).$$

Plugging into (2.3) we have the dynamics of $W(t)$:

$$dW(t) = r\left[\sigma Y(t)dB(t) + (W(t) - h(t))dt\right].$$

We use the Martingale method to show the dynamics of manufacturer's continuation value. For given time t, let the manufacturer's optimal response be $R^*(t) : t \geq 0$. Define a non-optimal control, where $\hat{R}(u)$, $u \leq t$ is arbitrary:

$$\hat{R}(u) = \begin{cases} \hat{R}(u) & 0 \leq u \leq t \\ R^*(u) & u \geq t \end{cases}$$

and

$$\hat{h}(t) = \hat{h}(S_A(t), S_C(t), \hat{R}(t)).$$

Now with

$$\begin{aligned} \hat{V}_A(t) =& r\int_0^t e^{-ru}\hat{h}(u)du + e^{-rt}W(t), \\ d\hat{V}_A(t) =& e^{-rt}\{r\hat{h}(t)dt + dW(t) - rW(t)dt\} \\ =& e^{-rt}\{(r(\hat{h}(t) - h(t))dt + rY(t)\sigma dB(t)\}. \end{aligned}$$

Define the Girsanov's kernel

$$\phi(t) = \hat{R}(t) - R(t),$$

and the measure change:

$$L(t) = \exp\{-\int_0^t \phi(u)B(u) - \frac{1}{2}\int_0^t \phi^2(u)du\}$$

and $d\hat{P}(t) = L(t)dP(t)$. It follows from the Girsanov's theorem that:

$$\sigma dB(t) = \sigma d\hat{B}(t) + \phi(t)dt.$$

Thus, under measure $\hat{P}$, the $X(t)$ dynamics becomes:

$$dX(t) = \left(a + (b - a + \hat{R}(t))X(t) - bX(t)^2\right)dt + \sigma X(t)d\hat{B}(t),$$

and the dynamics of $\hat{V}_A(t)$ becomes:

$$d\hat{V}_A(t) = e^{-rt}\left\{\left[r(\hat{h}(t) - h(t)) + rY(t)(\hat{R}(t) - R(t))\right]dt + r\sigma Y(t)d\hat{B}(t)\right\}.$$

By comparison theorem, under R^*, $\hat{V}_A$ must have a negative drift, giving the optimality condition:

$$h(t) + Y(t)R(t) \geq \hat{h}(t) + Y(t)\hat{R}(t) \text{ for all } \hat{R}(t).$$

Similar proof can be applied to the case the manufacturer determines the power of innovation a. We show the result for the case in which the manufacturer chooses the optimal response R. Assuming that function h is differentiable to control R so for any given $Y(t)$, the first-order optimality condition suggests that at optimality,

$$\frac{dh(R(t), S(t), Y(t))}{dR(t)} + Y(t) = 0.$$

In the risk-neutral manufacturer case, i.e., h is linear in R, and since a linear (more generally convex) function achieves it maximum on the boundary so that the optimal pricing policy becomes a bang-bang control:

$$R^*(t) = \begin{cases} \bar{R} & \text{if } Y(t) \geq 0 \\ \underline{R} & \text{if } Y(t) < 0, \end{cases}$$

which means that the government agency should penalize those who frequently manipulate the prices to the extremes.

For a risk-averse agent, if h is continuously differentiable, and its first-order derivative is invertible (this is the case when h is strictly concave in R), then the optimal control is

$$R^*(t) = (h')^{-1}(-Y(t)),$$

where $(h')^{-1}$ is an inverse function of the first derivative of h. □

By imposing mild smoothness and concavity conditions on the AV manufacturer's utility function, we can reduce the countably infinite IC-constraints into a parametric function of S_A and $r_A Y(t)$, as seen in the following proposition.

Proposition II.5. *R^* is the best response from the AV manufacturer if*

1. *For any given subsidy policy $\{S_A(t), Y(t)\}$, if the AV manufacturer is risk neutral (i.e., h is linear in R), then $R^*(t)$ satisfies the following optimality condition*

$$R^*(t) = \begin{cases} \bar{R} & \text{if } Y(t) \geq 0 \\ \underline{R} & \text{if } Y(t) < 0. \end{cases}$$

2. *For any given subsidy policy $\{S_A(t), Y(t)\}$, if h is continuously differentiable, concave, and nonlinear in R, then $R^*(t)$ solves*

$$\frac{\partial h(S_A(t), R(t))}{\partial R(t)} + Y(t) = 0$$

for all $t \geq 0$.

The intuition of recommending a sensitivity level $r_A Y(t)$ in the AV subsidy policy is related to the dual roles of $Y(t)$ above. The first role is to ensure no violation of the IC-constraint. In Proposition II.5, $Y(t)$ is the AV manufacturer's marginal utility for enhancing AV innovations. Besides the fixed amount of subsidies $S_A(t)$ received over the planning horizon, the manufacturer is incentivized to increase the response $R(t)$ to gain higher utility with the presence of $Y(t)$. $S_A(t)$ are still necessary to guarantee that the IR-constraint is satisfied. The second role is to penalize the information asymmetry. In the dynamics of the AV manufacturer's continuation value $W(t)$ and Theorem II.4, a higher sensitivity level $r_A Y(t)$ will potentially increase the stochasticity of its expectation on the future benefit. Therefore, the manufacturer is forced to cooperate with accelerating the early adoption of AVs to reduce the risk of increasing stochasticity. Proofs of Theorem II.4 and Proposition II.5 are as follows.

In summary, the subsidy policy including $Y(t)$ and $S_A(t)$ is sufficient to break down the information asymmetry. Proposition II.5 demonstrates how to find a mapping from a subsidy policy $\{S_A(t), Y(t)\}$ to the AV manufacturer's optimal response $R^*(t)$. Such a sensitivity level could be easily included in practice, for example, a suggested internal rate of return relevant to the advancements of AV technology. Different from the previous literature [23, 104], the best response R^* is basically a function of S_A and Y because of the DOI model.

o have a more intuitive state variable representing the AV market share, we normalize (2.1) by a constant market potential M to obtain the dynamics of the *market penetration rate* $X(t) = N(t)/M$,

$$dX(t) = (a(t) + bX(t))\,(1 - X(t))dt + \sigma X(t)dB(t).$$

Let $F(X, W)$, denote a $C^{2,2}$ function with two state variables – AV market share X and the manufacturer's continuation value W. With all these considerations, we can solve the optimal subsidy policies in (2.2) by dynamic programming in the following theorem.

Theorem II.6 (Optimal Subsidies)**.** *The government agency's continuation value is* *equal to $F(X,W)$ that solves the following HJB equation:*

$$r_G F = \max_{S_A,Y}\Big\{ r_G[g(N) - S_A \cdot M(a^* + bX)(1-X)] + (a^* + bX)(1-X)F_X + \\ r_A\Big(W - h(S_A, a^*)\Big)F_W + \frac{1}{2}\sigma^2(X^2F_{XX} + 2r_AXYF_{XW} + r_A^2Y^2F_{WW})\Big\}.$$

Proof. Theorem II.6 For notational convenience, we denote the government's instantaneous payoff function f. The government's continuation value $C_g(t)$ at time $t > 0$ and the optimal $S(u)$ is adopted in $u \in [t, \infty)$:

$$C_G(t) = \mathbb{E}^{R(y)}\{r_G \int_t^\infty e^{r_G(u-t)} f(u)du|\mathcal{F}_t\}. \tag{2.4}$$

At time $t > 0$, we can rewrite the government's optimal value as:

$$V_G(t) = r_G \int_0^t e^{-r_G u} f(u)du + e^{-r_G t}C_G(t).$$

In case an optimal strategy is adapted, $V_G(t)$ is a $\mathcal{F}_t$ martingale, and thus has a zero drift. By assumption, there exists a $C^{2,2}$ function when the manufacturers choose $a(t)$, or a $C^{1,2,2}$ function F when the manufacturers choose $p(t)$. For convenience, we suppose that $C_G(t) = F(P(t), X(t), W(t))$.

Under the assumption the continuation value of the government can be written as:

$$V_G(t) = r_G \int_0^t e^{-r_G u} f(u)du + e^{-r_G t}F(P(t), X(t), W(t)).$$

Using Ito's lemma the dynamics of $V_G(t)$ can be derived as:

$$dV_G(t) = r_G e^{-r_G t} f(t)dt - r_G e^{-r_G t}F(P, X, W) + e^{-r_G t}dF(P, X, W),$$

where the drift term is $[pF_P + (a + (b - a + p)X - bX^2)F_X + r_G(W - h)F_W + \frac{1}{2}\sigma^2(X^2F_{XX} + r_G^2Y^2F_{WW} + 2r_GXYF_{XW})]dt$.

By setting the drift term to zero, we have the HJB equations in Theorem II.6. It is easy to verify the following upper bound for the government's continuation value:

$$\bar{F}(P) = F(P, C, 1, W) = \sup \mathbb{E}\{r \int_t^{\infty} e^{r(u-t)} f(P, C, 1) du | \mathcal{F}_t\} = r\frac{1}{r}\bar{g}(N) = g(e^{-P} M_0).$$

This means that W along with other state variables are in a compact set. □

The proof of Theorem II.6 shows that it is sufficient to solve the above HJB equation to solve for the optimal subsidies in (2.2). The proof also shows that W is finite if the manufacturer's utility function h is finite. Let $\bar{W}$ denote the upper bound of W. It is clear that $\{S_A^*(t)\}_{t\geq 0}$ are adaptive to the filtration generated the market dynamics for all $t \geq 0$. We can characterize the property of F in the following proposition.

Proposition II.7. *For a given X, there exists $W^* \in [0, \bar{W}]$ such that:*

1. *$F(X, W^*) = \max_W F(X, W)$.*
2. *For any $W \in [0, W^*]$, $F_W \geq 0$.*
3. *For any $W \in [W^*, \bar{W}]$, $F_W \leq 0$.*
4. *For any $W \in [0, \bar{W}]$, $F_{WW} \leq 0$.*

Proposition II.7 shows that the government agency's continuation value F is partially concave with regard to the AV manufacturer's continuation value W. A brief proof is as follows. Note that F is assumed to be a twice differentiable function in W with uniformly bounded derivatives over S_A and Y. By definition, $F(0) = 0$ and $F_{WW}(0) < 0$. With the assumption that h is a concave function (i.e., a risk-averse manufacturer), and that there is a $w \in [0, \bar{W}]$ such that $F_{WW}(w) = 0$, it follows that the entire solution is a linear function, which is a contradiction. So F_{WW} is nonpositive everywhere and F is a concave function in W. With $F_W(0) > 0$, the properties above are proved.

2.4 Main Results

The main analytical results are (a) comparing the cost-effectiveness of the AV subsidy policies using the DSG model with a standard welfare maximization approach, and (b) characterizing the special structure of optimal AV subsidies. We use numerical methods to verify those results.

2.4.1 Optimal v.s. Welfare-Maximization Subsidies

Social welfare maximization seems to be a more natural approach for designing subsidies. However, the presence of information asymmetry can cause inefficiency and friction in implementing those policies. Since Pareto efficiency is a necessary condition for welfare maximization [26], we want to compare our results with the subsidy policies that obtain the most efficient Pareto outcomes. The government agency who has the flexibility to choose a different approach will prefer the one that gives a higher payoff.

The comparison also shows how, in practice, to implement the optimal subsidy policy. The policies derived from Theorem II.6 are implementable because: (a) The mapping from the subsidy policy to the manufacturer's best response in Proposition II.5 holds; (b) The manufacturer's continuation value $W(t)$ and the AV market share $X(t)$ are observable as state variables. To convey the AV subsidy policies with the state $W(t)$, we propose a simple mechanism to evaluate the IR-constraint below.

The mechanism uses a Pareto optimality argument in Figure 2.4. We divide the state space of $W \in [0, \bar{W}]$ into three regions, $\mathcal{W}_1, \mathcal{W}_2$, and $\mathcal{W}_3$ as follows:

1. $\mathcal{W}_1 = [0, W^*]$,

2. $\mathcal{W}_3 = \{W \in [0, \bar{W}] : F(W) < 0\}$,

3. $\mathcal{W}_2 = [0, \bar{W}] \setminus \{\mathcal{W}_1, \mathcal{W}_3\}$.

With the computed subsidy policies $\{S_A^*, Y^*\}_{t \geq 0}$, the government agency can decide the continuous value $F(W)$ regarding the entry threshold W_0. The optimal subsidy policies over the three regions are:

1. If $W_0 \in \mathcal{W}_1$, the optimal subsidy policies automatically drive the AV manufacturer to obtain W^* approximately. The government agency can enforce a higher $W \approx W^*$ such that both the government agency and the manufacturer obtain higher payoff/utility.

2. If $W_0 \in \mathcal{W}_2$, the government can obtain the exact continuation value $F(W_0)$ by implementing the optimal subsidies.

3. If $W_0 \in \mathcal{W}_3$, then $F(W) < 0$ so the subsidy policy is rescinded. In other words, no feasible AV subsidy policy exists in this region.

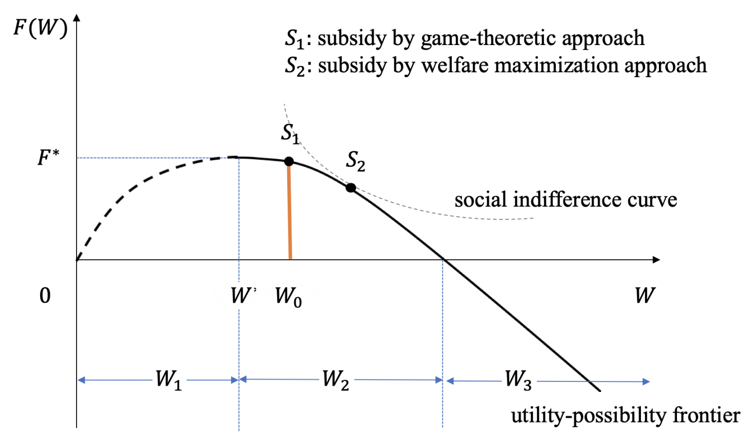


Figure 2.4: Relationship between the government agency's and the manufacturer's continuation value

Theorem II.8. *For the government agency, the DSG optimal subsidy policy dominates a welfare-maximization subsidy policy by providing a higher expected discounted* *total payoff for any given market share* $X \in [0, 1]$*.*

Proof. In Proposition II.7, we know that F is a partially concave function in W (Figure 2.4). Then we have the following observations:

1. The solid curve in Figure 2.4 is the payoff possibility frontier.
2. The welfare-maximization subsidy policy is obtained within $\mathcal{W}_2$.
3. Because the government agency has the flexibility of choosing which policy on the frontier to deploy, and both contracts are implementable, it will prefer to implement the DSG subsidy policy.

Hence, if the IR-constraint threshold W_0 is not large (which is chosen to be 0 by default), the continuation value with DSG subsidy policy dominates that of the welfare-maximization subsidy policy. □

2.4.2 Structure of Optimal AV Subsidy Policy

Previous literature found the optimal subsidy policies to be monotonic [66, 69, 75]. This chapter discovers a unique structure of the optimal AV subsidies $\{S(t)\}_{t\geq 0}$ in Proposition II.9 below. The presence of this two-threshold structure is unprecedented because of the existence of the "dark age" of AV deployment. The structure is also relevant to the weight of innovation and imitation effects in the DOI model (see Figure 2.5).

Proposition II.9. *The optimal subsidy policy has a two-threshold structure during the product life cycle – an early subsidy and a late subsidy.*

1. *The early subsidy is implemented when the AV market share is low, and decreases with the ascending AV market penetration rate* X*.*

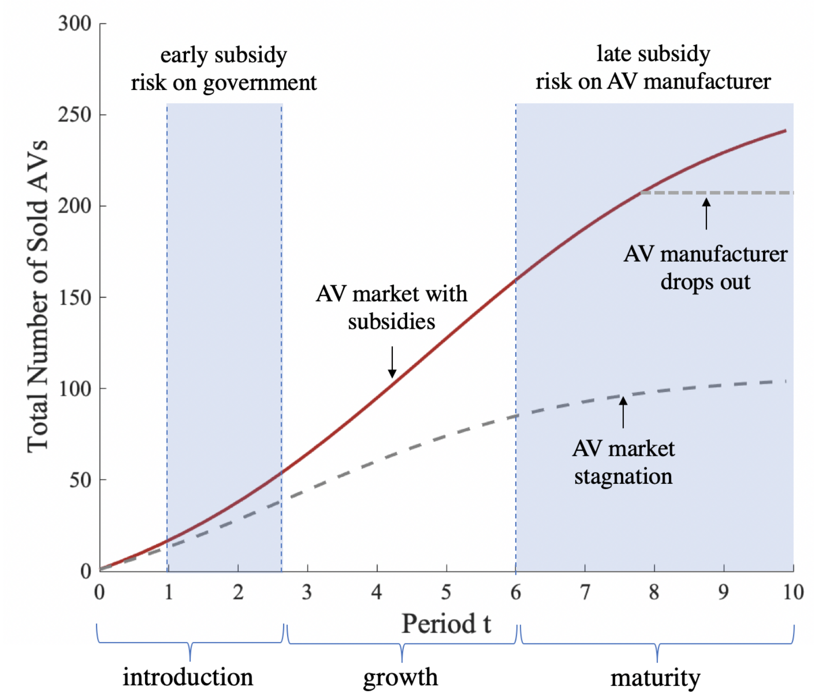


Figure 2.5: Optimal AV subsidy policy over the AV market penetration process

2. *The late subsidy is implemented when the AV market share is high, and increases with the ascending AV market penetration rate X.*

Proof. Applying the first-order optimality condition with regard to S_A on the right-hand side of the HJB equation in Theorem II.6,

$$S_A^* = (h')^{-1}\left(\frac{M(a^* + bX)(1 - X)}{\frac{1}{2}\sigma^2 r_G F_W}\right).$$

The inverse function of an increasing continuous function is also increasing, thus h^{-1} is increasing in its argument. Let the domain of S_A be $\mathcal{U}_{S_A} = [\underline{S}, \bar{S}]$. Because of the S-shape of the DOI model, S_A^* converges to the lower bound $\underline{S}$ when the numerator is large and $F_W < 0$ (W_0 is small). When $F_W = 0$, it is easy to see that $S_A^* = 0$ in Theorem II.6. The monotonicity of the h^{-1} also implies a monotonic subsidy policy in early subsidy and late subsidy respectively. Therefore, the optimal subsidies has the two-threshold structure described above. □

The transition of risk-sharing is another dimension of the subsidy policy that is not mentioned in previous literature. Such a transition over the planning horizon is because of the information asymmetry associated with the DOI process. At a low market share, the government agency shall undertake a high risk when it implements the early subsidy policies and creates the early demand for AVs. Such a relationship reverses when the AV market penetration reaches the near-saturation stage. At a high market share, the declining market growth stimulates the AV manufacturer to reduce or even terminate the production of AVs. In response, the late subsidy policy incentivizes the manufacturer to take more risk so to gain efficiency benefit in the future.

2.4.3 Iterative Algorithm for Optimal AV Subsidy Policy

Since the optimal AV subsidies in Theorem II.6 can only be computed numerically, we propose an iterative algorithm that provably converges to the optimal subsidies. We can compute a menu of subsidy policies as a look-up table for different state variables, so the government agency can easily implement them in practice. For notational convenience, let the decision variables be $u = [S_A, Y]$ and $\Lambda(u)$ be the left-hand side of the HJB equation in Theorem II.6. The gradient can be evaluated because $\Lambda(u)$ is assumed to be differentiable with regard to the control variables.

- Input data: DOI coefficients b and M, the initial AV market share $X(0)$, parameters r_G, r_A, σ, the stopping criteria $\epsilon > 0$, and functions g and h.

- Step 0: Initialization. For given $u^0 = (\underline{S_A}, \underline{Y}) \in \mathcal{U}$ at iteration $n = 0$, we solve $\Lambda(u^0) = r_G F$ by an implicit finite difference method. This gives the value of F^0 over discretized state space.

- Step 1: Search direction – compute the approximate value of gradient $\nabla_u \Lambda$ by:

$$\nabla_u \Lambda^n = \left[\frac{\Lambda(S_A + \triangle S_A, Y) - \Lambda(S_A, Y)}{\triangle S_A}, \quad \frac{\Lambda(S_A, Y + \triangle Y) - \Lambda(S_A, Y)}{\triangle Y} \right]$$

  If $\|\nabla_u \Lambda^n\| < \epsilon$, stop; else go to Step 2.

- Step 2: Line search – find the step size γ^n

$$\Lambda(u^n + \gamma^n \nabla_u \Lambda^n) = \max_{\gamma} \Lambda(u^n + \gamma \nabla_u \Lambda^n).$$

- Step 3: Update the AV subsidy policies S_A and Y by the gradient ascent $u^{n+1} = u^n + \gamma^n \nabla_u \Lambda^n$. Go to Step 1.

The concavity of F ensures that the algorithm converges to the global optimal solution. If changing the subsidies in real time is not possible, we can approximate the subsidy policies with a sequence of subsidies in discrete time. [110] showed that the gap of optimality also converges in implementation. Note that, with an infinite horizon, we do not need to resolve the menu of AV policies using the renewal theory [23].

Proof. Convergence of the iterative algorithm We have shown that function F derived from HJB equation has a numerical solution that converges to an unique weak solution. Next, we show that the optimization also converges to (S_A^*, Y^*). We show the convergence of sequences as follows. Set $\Gamma = \{u : \Lambda(u) \leq \Lambda(u^0)\}$ be a compact set and there exists a subsequence N such that $\{u^n\}_{n \in N}$ converges to $\bar{u}$ and $\triangledown \Lambda(\bar{u}) = 0$.

Step 2 $\rightarrow$ Step 4 satisfies that $\Lambda(u^{n+1}) > \Lambda(u^n)$ and $\triangledown_u \Lambda$ is the steepest ascent direction. Thus the sequence $\{u^n\} \subset \Gamma$. Since Γ is a compact set, there exists the required subsequence and limit point $\bar{u}$.

Now assume that $\Lambda(\bar{u}) \neq 0$ and $\triangledown\Lambda(\bar{u}) = \bar{d}$. There exists a step size $\bar{\gamma}$ such that

$$\Lambda(\bar{u} + \bar{\gamma}\bar{d}) = \max_{\gamma} \Lambda(\bar{u} + \gamma\bar{d}),$$

and for some $\delta > 0$, Λ is a $C^{1,1}$ function

$$\Lambda(\bar{u}) = \Lambda(\bar{u} + \bar{\gamma}\bar{d}) - \delta.$$

Now define

$$\begin{cases} z^n = u^n + \bar{\gamma}d^n \\ \bar{z} = \bar{u} + \bar{\gamma}\bar{d} \end{cases}.$$

Note that $z^n \to \bar{z}$ thus the difference between the current point and the limit point, $v^n = z^n - \bar{z} \to 0$.

Let $n \in N$, and use the first-order approximation by the Taylor's formula:

$$\Lambda(u^n + \bar{\gamma}d^n) = \Lambda(z^n) = \Lambda(\bar{z}) - \nabla\Lambda(\bar{z})^T v^n.$$

As the sequences converge $z^n \to \bar{z}$ and $u^n \to \bar{u}$, the gradient $\nabla\Lambda(u^n) \to \nabla\Lambda(\bar{z})$. As $v^n \to 0, \nabla\Lambda(\bar{z})^T v^n \to 0$. Hence, for sufficiently large $n \in N$, $\|\nabla\Lambda(\bar{z})^T v^n\| < \frac{\delta}{2}$. So

$$\Lambda(u^n + \bar{\gamma}d^n) > \Lambda(\bar{z}) - \frac{\delta}{2} = \Lambda(\bar{u}) + \frac{\delta}{2}.$$

Since γ^n is a maximizer, thus

$$\Lambda(u^{n+1}) = \Lambda(u^n + \gamma^n d^n) \geq \Lambda(u^n + \bar{\gamma}d^n) > \Lambda(\bar{u}) + \frac{\delta}{2}.$$

This contradicts the fact that $\Lambda(\bar{u}) > \Lambda(u^{n+1})$. Thus such a sequence of solutions in the iteration set exists within such a compact set. This completes the proof of convergence. $\square$

2.4.4 Numerical Results

We shall validate the analytical results by numerical experiments and gain some managerial insights using the AV market forecast data. By running sensitivity tests on the optimal subsidies regarding these input data, we can identify the critical research directions that may effect the AV subsidy policies.

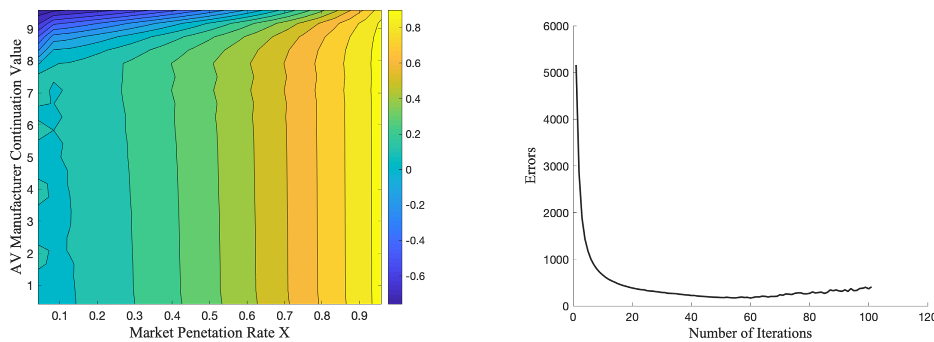
The following instances are common to all numerical experiments.

1. The AV market penetration process refers to the DOI model in [109]. They used a survey-based approach to estimate the individuals' propensities to adopt AVs. The estimated power of innovation was $a = 0.108$, and the estimated power of contagion was $b = 0.957$. The AV market saturation was expected to be 71.3% of the current automotive market in the United States, and thus the AV market potential is $M = 200$ million cars [96].

2. The aggregate efficiency benefit of AVs that is measured by the unit reduced value of driving time refers to [107]. The benefit function is quadratic: $g(X) \approx -650 + 6000 \cdot (X - 0.33)^2$. The positive efficiency benefit is not obtained until the AV market surpasses 67% market share. We term the market share that gives most negative efficiency benefit ($X = 33.5\%$) the *worst-case market share*, and the constant 6000 the *social benefit multiplier*.

3. The AV manufacturer's continuation value W is bounded if subsidies S are bounded. Hence, we use a trivial upper bound for the government's continuation value $\bar{F} = g(M)$.

– **Solving Optimal AV Subsidies** With these considerations, we computed the optimal subsidies with varying states X and W by the above iterative algorithm. The government's continuation value $F(X, W)$ is shown in Figure 2.6a. The convergence of the algorithm is validated by tracking the error of the government agency's continuation value F between two subsequent iterations in Figure 2.6b. The normalized government agency's continuation value $F/F_{\max}$.



(a) Government agency's relative continuation value

(b) Convergence of the iterative algorithm

Figure 2.6: Solving the optimal AV subsidy policy by the iterative algorithm

Our observations include: (a) When the government's continuation value $F(X, W) > 0$, there exists an optimal subsidy policy (S_A^*, Y^*) that accelerates the early adoption of AVs. In the case that $F < 0$, the AV market failure is unavoidable; (b) Figure 2.7 verifies the two-threshold structure of optimal subsidies during the market penetration process.

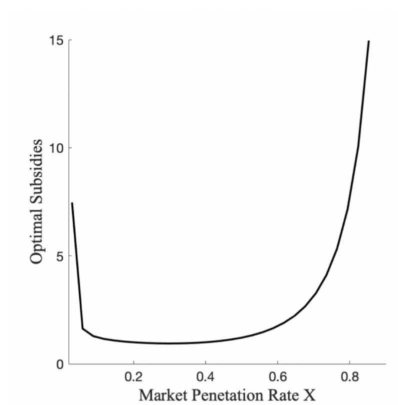
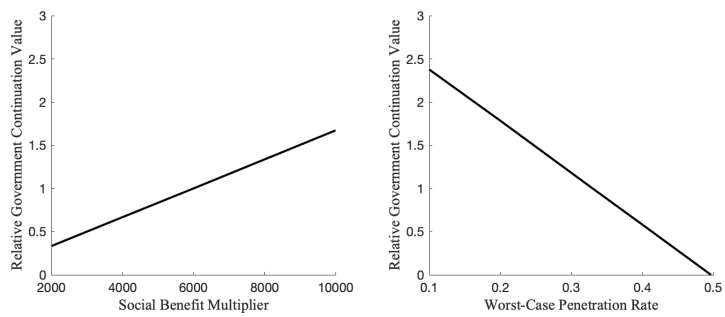


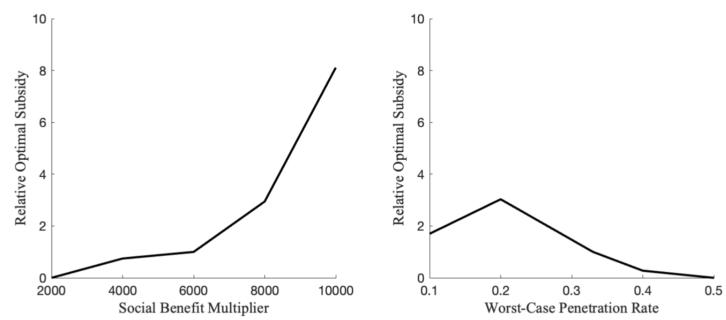
Figure 2.7: Optimal AV subsidy (in unit of $\$1,000$) regarding the market penetration rate X

2.4.5 Sensitivity Analysis of Optimal Subsidies

The government agency may be inquisitive about the robustness of optimal AV subsidy policies with regard to the input data. We considered various uncertainties in the data, among others: (a) the AV efficiency benefit function $g(N)$, (b) the parameters in the DOI model, and (c) the noise of AV market forecast. For convenience, we ran a sensitivity test on a fixed state ($W = 4.2, X = 0.8$), and used the measures of relative objective values and subsidies in respect to the original instance.



(a) Sensitivity of the government agency's continuation value F.



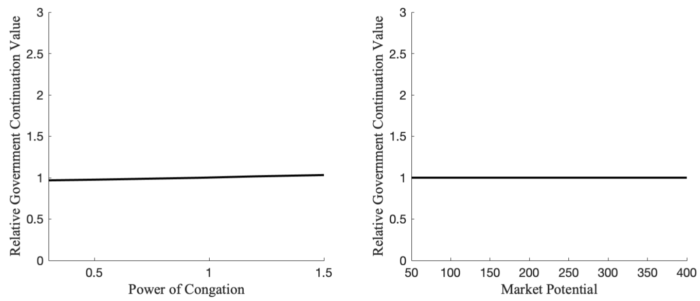
(b) Sensitivity of the optimal AV subsidy policy S_A.

Figure 2.8: Sensitivity of the optimal AV subsidy policy to the efficiency benefit function

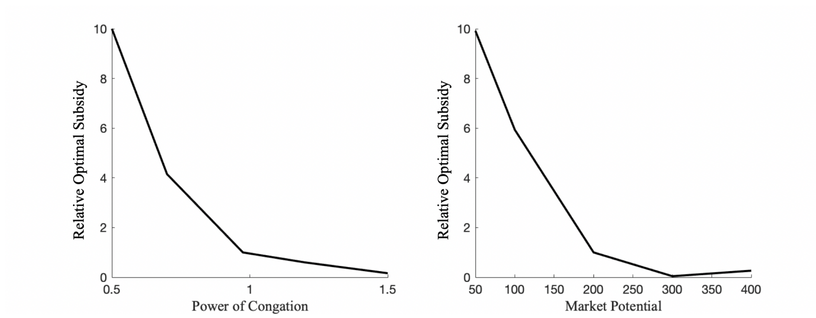
We have the following observations from the sensitivity analysis:

1. The government agency's continuation value and optimal subsidy policies are *sensitive* to the AV efficiency benefit function $g(N)$.

   (a) With an increasing social benefit multiplier (i.e., the magnitude of AV's efficiency benefit), the government's expected total payoff increases (Figure



(a) Sensitivity of the government agency's continuation value F.



(b) Sensitivity of the optimal AV subsidy policy S_A.

Figure 2.9: Sensitivity of the optimal AV subsidy policy to the market penetration DOI model

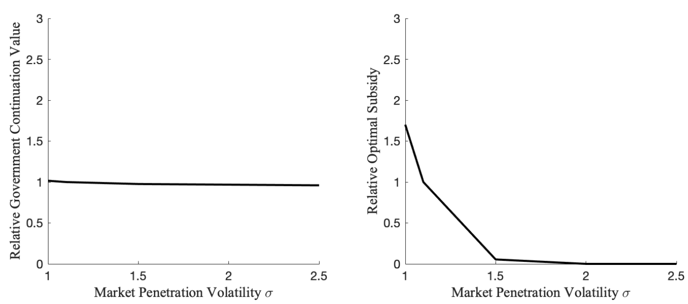


Figure 2.10: Sensitivity of the government agency's continuation value and the optimal AV subsidy policy to the market penetration volatility

2.8a).

(b) With an increasing worst-case penetration rate (i.e., the market share with most negative efficiency benefit), the government agency's total payoff decreases (Figure 2.8a).

(c) With an increasing efficiency benefit, the government agency is willing to pay higher subsidies (Figure 2.8b).

Hence, it is noticeably valuable to improve the operational rules of AVs, e.g., the time headway, in order to improve the efficiency benefit of AVs.

2. The government agency's objective value is *insensitive* to the parameters of the market penetration process (Figure 2.9a).

3. The optimal subsidy policies are *sensitive* to the parameters of the market penetration process. If the initial adoption rate of AVs during the launching period is fast, the required amount of AV subsidies can be reduced significantly (Figure 2.9b).

4. The optimal subsidy policies are *sensitive* to the AV market forecast uncertainty. Greater uncertainty (represented by the market penetration volatility σ) may increase the gap of information asymmetry (Figure 2.10). Therefore, with large noise in the AV market forecast, the government should reduce the subsidies to penalize the hidden information of $R(t)$.

2.5 Extension: Changing AV Market Potential

2.5.1 AV Subsidy Policies with Concerns of Car Ownership and Incentivizing Consumers

We aim to relax the assumption that the AV market potential M (i.e., the population of potential AV consumers) is fixed. With the rising of shared mobility, previous literature has predicted the possible demise of private car ownership because of the widespread of AVs [76, 119, 132]. This implies an extrinsic declining number of the private-car consumers. On the other hand, AVs may be highly priced at the beginning so that the initial AV market potential is restricted, which causes an intrinsic ascending or declining trend of car ownership. In response, the government agency can directly compensate consumers by offering a per-vehicle subsidy $S_C(t)$ to consumers. We model the market potential with a simple exponential function of the extrinsic car ownership trend $\beta(t)$ and the retail price multiplier $P(t)$ [5]

$$M(t) = M_0 e^{-\beta(t) - P(t) + S_C(t)}.$$

$\beta(t)$ is a known process that represents the decreasing car ownership because of the widespread use of AVs [109].

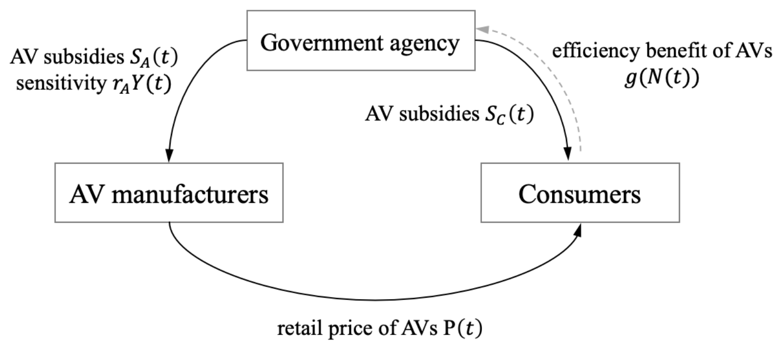


Figure 2.11: Extension to the DSG model for AV subsidy problem

Now the AV manufacturer can control the AV market potential $M(t)$ by modifying the price multiplier $p(t) = -dP(t)/dt$. $p(t)$ is termed as the "pricing incentives" for AVs. In the extension, price multiplier is the AV manufacturer's response, i.e., $R(t) \equiv p(t)$. The extension to the DSG model is depicted in Figure 2.11. It is easy to see that:

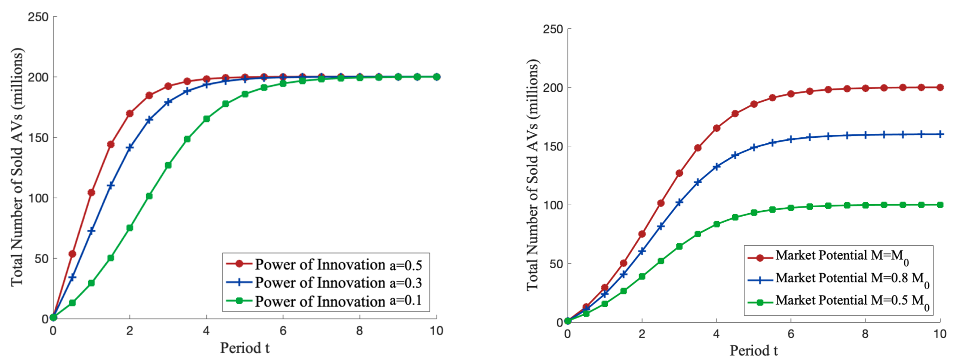
$$dM(t) = -M(t)dP(t) = M(t)p(t)dt, \qquad M(0) = M_0.$$

Applying Ito's lemma to (2.1), we can characterize $X(t)$ that is controlled by $p(t)$ as:

$$dX(t) = [a + (b - a - p(t))X(t) - bX(t)^2]dt + \sigma X(t)dB(t).$$

Compared to the simpler case above, the impacts of subsidies on the AV market penetration processes are different (Figure 2.12). When the manufacturer's response

$R(t)$ controls the AV innovations (i.e. the value of $a(t)$), the AV market growth rate is tempered at the early adoption stage but ultimately the market size will reach the saturated market share. In contrast, when the manufacturer provides pricing incentives for AVs (the value of $p(t)$), the market potential $M(t)$ is changed. It is worth mentioning that the market failure with inadequate pricing incentives (i.e., $p(t)$ is lower than expected) has more serious consequences than that with inadequate AV innovations (i.e., $a(t)$ is lower than the target). With inadequate AV innovations, the slow market penetration only generates finite negative externalities during the "dark age"; with inadequate pricing incentives, the AV market's long stagnation in low market size may continuously generate negative externalities.



(a) AV manufacturer changes the power of innovations $a(t)$.

(b) AV manufacturer changes the market potential $M(t)$ by pricing incentives.

Figure 2.12: The mean value of the AV market penetration process under controls

Similar to (2.2), the government agency solves the following an optimization problem with with an IC-constraint for the optimal pricing incentives $p(t)$. It is trivial to include other responses $R(t)$, e.g., a mixture of $a(t)$ and $p(t)$. With all these considerations, we can formulate the optimal subsidy problem as:

$$\max_{S_A(t),S_C(t)} \mathbb{E}\left[r_G \int_0^\infty e^{-r_G t}\left[g(N(t)) - (S_A(t)+S_C(t))\left(a + b\frac{N(t)}{M(t)}\right)(M(t)-N(t))\right]dt\right]$$

$$s.t.\, p(t) \in \arg\max_p \mathbb{E}\left[r_A \int_t^\infty e^{-r_A u} h(S_A(u), S_C(u), p(u))du\right], \forall t \geq 0 \text{ (IC-constraint)} \tag{2.5}$$

$$\mathbb{E}\left[r_A \int_0^\infty e^{-r_A t} h(S_A(t), S_C(t), p(t))dt\right] \geq W_0 \text{ (IR-constraint)}$$

With the extension of decision variables $S_C(t)$ and $p(t)$, the state variables of the dynamic programming should also be expanded. Let $F(P,X,W)$ be a $C^{1,2,2}$ function. Besides the AV manufacturer's continuation value $W(t)$ and AV market penetration rate $X(t)$, government agency also needs to observe the retail price of AVs $P(t)$. With Proposition II.5, the best response p^* can be found as a function of S_A, S_C and Y. The following theorem solves the optimal AV subsidy policies.

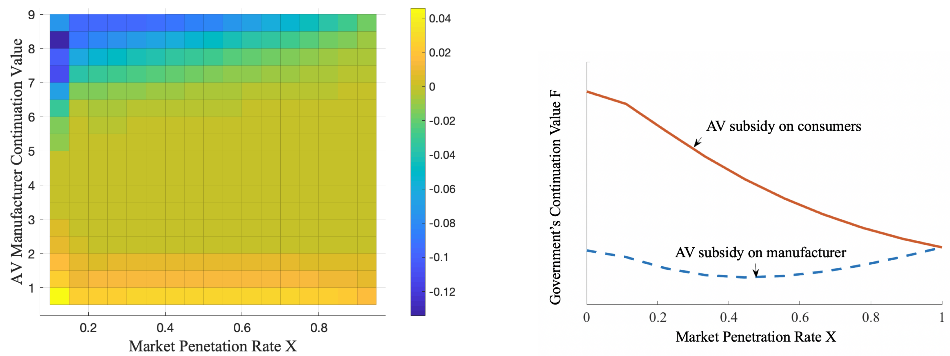
Theorem II.10. *The government agency's continuation value is equal to* $F(P,X,W)$ *that solves the following HJB equation:*

$$r_G F = \max_{S_A,S_C,Y}\Big\{r_G[g(N)-(S_A+S_C)M_0e^{-\beta-P+S_C}(a+bX)(1-X)]+p^*F_P+ (a+(b-a-p^*)X-bX^2)F_X+r_A(W-h(S_A,S_C,p^*))F_W + \frac{1}{2}\sigma^2(X^2F_{XX}+2r_GXYF_{XW}+r_A^2Y^2F_{WW})\Big\}.$$

2.5.2 Result and Discussion

We can use the same iterative method to solve for the optimal AV subsidy policies in Theorem II.10. The arguments for two-threshold structure of the AV subsides $S_A(t)$ also hold. However, $S_C(t)$ does not retain this special structure.

Now we briefly discuss the new insights about whom should be subsidized in the AV market. Because of the complexity of the state variables, we find that there is no seemingly simple answer to this question. In numerical experiments, we compared the relative continuation values of giving a fixed amount of subsidies to consumers (F_{consumer}) and to manufacturers ($F_{\text{manufacturer}}$). In Figure 2.13, we can observe that there are regions where subsidizing AV manufacturers promises a larger expected total payoff, and vice versa. In general, the government agency should give a higher weight on the consumers' subsidies when the market penetration rate is small, or the AV manufacturer's continuation value is low. Otherwise, it is more beneficial to subsidizing AV manufacturers.



(a) Relative difference $(F_{\text{consumer}} - F_{\text{manufacturer}})/F_{\text{manufacturer}}$.

(b) Government agency's continuation value over the market penetration process.

Figure 2.13: Comparison of subsidizing AV manufacturers and consumers

In sum, our main results provide the following insights for designing AV subsidy policies:

1. The DSG optimal subsidy has a two-threshold structure because of the presence of a "dark age" in AV deployment. The government agency needs to subsidize AVs in their early deployment as well as the near-saturation stage.

2. The most crucial factor for AV subsidies is the efficiency benefit related to AV deployment. To mitigate the negative externalities of AVs, the government agency needs to consider developing and implementing operational strategies that improve the efficiency of AV technology. Otherwise, the government agency has to pay high subsidies to avoid the potential AV market failure.

3. The optimal AV subsidies also depend on the parameters in the market penetration process. Hence, conducting more research on forecasting the AV market is necessary.

4. The iterative algorithm can compute a menu of optimal subsidy policies for the government agency with a different data instance.

2.6 Conclusion

The potential efficiency loss during the "dark age" of AV deployment calls for policies and strategies to accelerate the deployment of AVs. As it is extremely difficult, if not impossible, to establish a single modeling framework to prescribe these policies and strategies, this chapter investigates an optimal subsidy policy that maximizes the benefits of AV deployment throughout a planning horizon. The possible AV market failure is because of the information asymmetry between the government agency and the AV manufacturers. To prevent this failure due to the static and monopolistic subsidy design practice, we develop a new dynamic games approach. This approach can compute the adaptive subsidies based on the state of the AV market penetration process with uncertainty. Given the optimal subsidies, the AV manufacturer is incentivized to enhance the AV innovations, and provide pricing incentives to potential consumers.

This approach opens the door to many promising research directions and interesting applications in transportation policy. For example, the AV subsidies can be imposed to evolving generations of technology with different efficiency benefit functions. We can model it by replacing the DOI model with a system of diffusion processes. This model can also capture more complex interactions between multiple manufacturers with different utility functions, e.g., a competition for the future AV market. We refer the interested audience to the extension to multi-agent systems [80]. Since DSG is a general framework of prescribing optimal transportation policies, we can generalize our approach to other policy-making processes to include the regulated entity's response into the consideration.

With this new approach, we gain new insights about how to accelerate the adoption of AVs through adaptive subsidies. First, the DOI model and information asymmetry induce a two-threshold structure in the optimal subsidies, i.e., decreasing subsidies at low market share, and increasing subsidies at near-saturation market share. Second, the cost efficiency of the optimal subsidies obtained by this dynamic games approach dominates other approaches. Third, the sensitivity analysis of the optimal subsidies addresses the necessity for more AV market studies. The government agency should consider policies that enhance the AV operations and improve the accuracy of the AV market forecast models. With all considerations, the policy makers can strategically mitigate the negative externalities of AVs while embracing the advances of the technology.

CHAPTER III

Multi-agent Incentive Contracts

3.1 Statement of the Multi-agent Incentive Contracts Problem

In this chapter, we consider the problem of a single party, called the principal, creating contracts to delegate a task to a group of different agents. Incentive contracts stimulate the agents to act in the principal's interest by compensating them for achieving two goals: (i) they accept the offered contract (i.e., the contract is subject to the *individual rational* (IR) constraint); and (ii) they exert the effort at a *desired* level determined by the compensation spelled out in the contract (i.e., the contract is subject to the *incentive compatible* (IC) constraint). Such incentive contracts have been used for many practical problems ranging from corporate finance to strategic behavior in politics to institutional design [11, 23, 29, 44, 46, 54, 81, 87, 93].

In a dynamic setting, the goal as before is to incentivize agents to exert the desired effort over the planning horizon. To achieve this, each contract defines a stream of payoff amounts which depend on the effort exerted by the corresponding agent. In the framework we consider in this chapter, the agent's effort process is not perfectly observable, possibly due to the cost or the difficulty of monitoring it. Instead, the principal observes a noisy output process, which is a result of the effort exerted by the agent. This proxy results in *information asymmetry* about the agent's effort (the agent knows it, but the principal can only infer it from a proxy). The asymmetric information can create a potential *moral hazard* problem in the contract design [73]. The system efficiency is degraded as the first-best contract is not admissible. Given all these considerations, the incentive contract must solve the moral hazard problem and maximize the principal's utility.

A significant extension of the single-agent incentive contract is *multiagent incentive contracts*. For example, a company hires multiple employees to collaborate on a project. Since employees with correlated responses may have different capabilities and utility functions, designing contracts separately for each is not viable. Koo et al. [71] presented the first extension of multiagent incentive contracts that initiated a stream of literature for team incentives using the Martingale Approach [36, 37, 48, 117]. In the multiagent setting, new challenges arise due to varied interactions between agents. For example, an arbitrary agent may compare both its effort and payoff with others – such a phenomenon is called inequity aversion [38]. Goukasian and Wan showed that inequity aversion is present in multiagent incentive contracts [52] and agents' comparisons lower their exerted effort levels.

The critical condition for the existence of multiagent incentive contracts is that agents' actions at each epoch must form a *Nash Equilibrium*. This equilibrium then incentivizes each agent to choose the principal's desired actions and nullifies the moral hazard in the contract. The conditions for the existence of this equilibrium is still an open question. Prior work [36] assumed away these conditions by stating that the agents' action matrix that appeared on both sides of agents' optimality condition exists. The agents' actions constituting a Nash Equilibrium, thus led to a circular argument. Yet, characterizing the existence of Nash Equilibrium in multiagent contracts is non-trivial [30, 62, 82], more so in the dynamic setting considered in our work.

In Example I.2 in §I, we see that even if the existence problem is settled, the uniqueness of the Nash equilibrium must still be tackled. In at least one prior work the uniqueness issue has been assumed away, see Elie and Possimai [36] where the following claim is made to resolve the situation with multiple Nash equilibria: "Since the uniqueness of a Nash equilibrium is more the exception than the rule, we also need to assume that the community of agents has agreed on a common rule to choose between different equilibria."

Our goal in this chapter is to find conditions that guarantee the existence of a unique multiagent Nash Equilibrium in incentive contracts. We see that even if the existence problem is settled, the uniqueness of the multiagent Nash Equilibrium must still be tackled [36]. Characterizing unique equilibrium has practical value as coordinating agents to select the optimal Nash Equilibrium is improbable; it also has theoretical value as the optimal contracts with a set of equilibria is computationally intractable. Under the assumption that all agents are risk-averse, and interactions of all agents' actions on other's output follow a concave function, using a fixed point theorem (specifically the Kakutani fixed point theorem), we prove the existence of a subgame perfect Nash Equilibrium. With a slight strengthening of the condition on the Hessian matrix of the interaction functions, with the use of the theorem of Gale and Nikaido [47] and Kojima and Saigal [72], we prove that the equilibrium is unique. These results then enable us to develop a provably convergent iterative procedure to solve for the incentive contracts.

Unlike the infinite horizon setting of [104], we consider the problem with a finite horizon where the terminal condition may be path-dependent. Such terminal conditions are widely used in modeling options, mortgage defaults, and car leasing, thus enhancing the applicability of the methodology.

General notation used in the rest of the chapter is as follows. A set of indices $[n] = \{1, 2, \dots n\}$. Bold variables are vectors or matrices of random variables or functions. In equilibrium analysis for the i^{th} agent, we denote a vector as $\boldsymbol{x} = [x_1, \dots, x_i, \dots, x_n] = [\boldsymbol{x}_{-i}, x_i]$, where x_i indicates the variable associated with the i^{th} agent. x_P indicates that the variable is associated with the principal. $\tilde{x}$ is a variable that deviates from x in the domain of x. $D_x F$ is the Jacobean and $D_x^2 F$ is the Hessian of the C^2 function F of x.

The remainder of this chapter is organized as follows. In §3.2, we describe the setting of multi-agent incentive contracts. In §3.3, we characterize the agents' optimal responses and prove the existence of unique Nash equilibrium. We then formulate the principal's problem as a Hamilton-Jacobi-Bellman equation in §3.4. We also give an iterative procedure to implement the optimal incentive contracts. In §3.5, we draw final conclusions.

3.2 Setting

There is a single principal and n agents (indexed by $i \in [n]$) entering the contracts simultaneously at epoch $t = 0$. A contract signed between the principal and each agent i specifies the payoff $c_i(t)$ that the agent will receive by outputting $X_i(t)$, a proxy for the the agent's action $a_i(t)$ in working for the principal over the horizon $t \in [0, T]$. The vectors of n agents' actions and compensations are denoted as $\boldsymbol{a}(t)$ and $\boldsymbol{c}(t)$, respectively. Since the principal's goal is to incentivize n agents to collaborate on one project, these n contracts are correlated in many ways. The principal's decision, the payoff $c_i(t)$ for agent i, is in a domain $\mathcal{C}_i \subseteq \mathbb{R}$; the agent i's decision, the effort level $a_i(t)$, is in a domain $\mathcal{A}_i \subseteq \mathbb{R}$. The size of domains may vary for each

$i \in [n]$. The Cartesian products of compensations and efforts are denoted as $\mathcal{C}$ and $\mathcal{A}$, respectively.

3.2.1 Output Processes and Terminal Conditions

In an environment of uncertainty, the principal can only observe output processes $\boldsymbol{X}(t) = [X_1(t), \ldots, X_n(t)]^T \in \mathcal{X}$, which are imperfect observations of agents' actions. We assume that the dynamics of $X_i(t)$ follows a SDE that depends on n agents' actions $\boldsymbol{a}(t)$:

$$dX_i(t) = f_i(\boldsymbol{a}(t))dt + \sigma_i dB_i(t), \forall i \in [n], \tag{3.1}$$

which follows the following assumptions that are general extension of multiagent contract in [36, 71, 110, 117].

1. The drift term $f_i : \mathcal{A} \to \mathbb{R}_+$ in (3.1) is in L^2 space such that $\int_0^T f_i^2 ds < \infty$ for all $i \in [n]$.
2. f_i is partially differentiable almost everywhere with respect to $a_i(t)$ for all $i \in [n]$.
3. The diffusion term σ_i are known constants for all $i \in [n]$.
4. The Brownian motions $\boldsymbol{B}(t) = [B_1(t), \cdots, B_n(t)]^T$ are correlated with the correlation matrix $E(\boldsymbol{B}(t)\boldsymbol{B}(t)^T) = \boldsymbol{\Sigma}$ strongly positive definite, i.e., $\boldsymbol{x}^T\boldsymbol{\Sigma}\boldsymbol{x} \geq \alpha\|\boldsymbol{x}\|^2$ for all $\boldsymbol{x} \in R^n$ and some constant $\alpha > 0$.

For each agent $i \in [n]$, there is a path-dependent terminal payoff Φ_i at the end of planning horizon $T < \infty$. In other words, $\boldsymbol{\Phi}$ is a vector of functions of $\{\boldsymbol{X}(t), \boldsymbol{c}(t)\}_{0 \leq t \leq T}$. Path-dependent terminal conditions strengthen the commitments in contracts. Each agent could be charged a penalty if its cumulative outputs do not reach a specified target at termination. Similarly, the principal may rectify the payoff if the cumulative compensations do not reach a certain threshold. Let $\boldsymbol{Z}(t)$ denote the cumulative measures along the sample paths whose dynamics dZ_i for $i \in [n]$ follows

$$dZ_i(t) = \mu_{Z_i}(\boldsymbol{X}(t), \boldsymbol{c}(t))dt + \boldsymbol{\sigma}_{Z_i} dB_{Z_i}(t), \tag{3.2}$$

where $\mu_{Z_i}, \boldsymbol{\sigma}_{Z_i}$ are deterministic functions of appropriate dimension, and B_{Z_i} are independent Brownian motions. Two sets of processes $\boldsymbol{B}(t)$ and $\boldsymbol{B}_Z(t)$ ($\boldsymbol{B}_Z(t) = (B_{Z_1}(t), \cdots, B_{Z_n}(t))^T$) are also independent.

An example of a path-dependent terminal condition is an Asian-options type, i.e., $\boldsymbol{Z}(t) \in \mathbb{R}^n$ represents the total observed output from n-agents from 0 to t.

$$\boldsymbol{Z}(t) = \int_0^t \boldsymbol{X}(s) ds, \tag{3.3}$$

and this can be derived from (3.2) by letting $\mu_{Z_i}(\boldsymbol{X}, \boldsymbol{c}) = X_i$ and $\boldsymbol{\sigma}_{Z_i} = 0$.

The two systems of SDEs, (3.1) and (3.2) are adapted to the filtration generated by the Brownian motions B_i and B_{Z_i} for all $i \in [n]$. It is a well-known result that the vector $(\boldsymbol{X}(t), \boldsymbol{Z}(t))$ is a Markov process.

3.2.2 Solving Optimal Contracts by Optimization

$u_i : \mathcal{A} \times \mathcal{C}_i \to \mathbb{R}$ is i^{th} agent's instantaneous utility, i.e., utility in $[t, t + dt)$ and $u_P : \mathcal{X} \times \mathcal{C} \to \mathbb{R}$ is the principal's instantaneous utility. Note that u_i is possibly a function of all agents' actions.

The principal's and the agents' goals are to maximize the respective *expected total discounted utility* over the finite horizon $[0, T]$. We denote the i^{th} agent's expected total discounted utility by U_i and the principal's expected total discounted utility from contracting with n agents by U_P as follows:

$$U_i = \mathbb{E}^{\boldsymbol{a}}\left[r_i \int_0^T e^{-r_i s} u_i(\boldsymbol{a}(s), c_i(s)) ds + r_i e^{-r_i T} \Phi_i(Z_i(T)) \right], \forall i \in [n],$$
$$U_P = \mathbb{E}^{\boldsymbol{a}}\left[r_P \int_0^T e^{-r_P s} u_P(\boldsymbol{X}(s), \boldsymbol{c}(s)) ds - r_P \mathbf{1}^\intercal \cdot e^{-r_P T} \boldsymbol{\Phi}(\boldsymbol{Z}(T)) \right],$$

where $r_i \in (0,1)$ and $r_P \in (0,1)$ are the discount rate of the i^{th} agent and the principal, respectively. The discount rates in front of the integral normalize the utility to annuity costs [113]. In the case that the principal is risk-neutral, i.e., u_P is a linear function of $\boldsymbol{X}$, we can reduce the principal's problem using the following observation. After taking expectations on the integral of i^{th} agent's output process $\mathbb{E}[\int X_i(t)dt] = \mathbb{E}[\int f_i(\boldsymbol{a}(t))dt] + \mathbb{E}[\int \sigma_i dB_i(t)] = \mathbb{E}[\int f_i(\boldsymbol{a}(t))dt]$, using the fact that the expectation of an Ito's integral is zero. Thus we can write U_P in terms of $\boldsymbol{a}$ only in this special case [117].

Optimal multi-agent contracts should maximize the principal's expected total discounted utility U_P subjected to (a) n individual-rational (IR) constraints at $t = 0$, and (b) n incentive-compatible (IC) constraints at any $t \in [0, T]$. The IR-constraints guarantee that agents would agree to enter the contracts if the expected utility exceeding certain thresholds; the IC-constraints guarantee that agents would realize the target efforts at each epoch of the horizon. In the presence of interactions between agents, we have one supplementary constraint that the n agents' best responses constitute a Nash equilibrium at each $t \in [0, T]$. In summary, optimal multi-agent contracts can be solved as follows:

$$
\begin{aligned}
(3.4) \qquad & \max_{\boldsymbol{c}(t), t\in[0,T]} \mathbb{E}\left[r_P \int_0^T e^{-r_P t} \left[u_P(\boldsymbol{X}(t), \boldsymbol{c}(t)) \right] dt - r_P \mathbf{1}^\intercal \cdot e^{-r_P T} \boldsymbol{\Phi}(\boldsymbol{Z}(T)) \right] \\
& s.t. \quad U_i \geq \underline{\mathrm{W}}_i, \quad \forall i \in [n] \quad \text{(Individual-Rational constraint)}, \\
& \qquad a_i^*(t) \in \arg\max_{a_i} \mathbb{E}\Big[\int_t^T r_i e^{-r_i s} u_i(a_i, \boldsymbol{a}_{-i}^*, c_i) ds + r_i e^{-r_i T} \Phi_i(Z_i(T)) \Big], \\
& \qquad\qquad \forall i \in [n], \forall t \in [0, T] \quad \text{(Incentive-Compatible constraint)}.
\end{aligned}
$$

3.3 Incentive-Compatible Constraint

In this section, we characterize an individual agent's optimum action within given multiagent contracts.

3.3.1 Parametrization of Individual Agent's Problem

We analyze an arbitrary i^{th} agent's optimum action given the other agents' optimum actions. Without loss of generality, we reformulate the analysis of the prior work [23, 104] under the new multiagent contracts setting.

In dynamic Stackelberg games, one commonly defines the continuation value $W_i(t)$ (the value function in dynamic programming) when the optimal actions $\boldsymbol{a}$ are taken by all agents in $[t, T]$, i.e., the agent i's conditional expected optimal discounted utility received from t to T, as follows,

$$W_i(t) = \mathbb{E}^{\boldsymbol{a}}\left[\int_t^T r_i e^{-r_i(s-t)} u_i(\boldsymbol{a}(s), c_i(s))ds + r_i e^{-r_i(T-t)}\Phi_i(Z_i(T))|\mathcal{F}_t^{\boldsymbol{B},\boldsymbol{B}_z}\right]. \tag{3.5}$$

where $\mathcal{F}_t^{\boldsymbol{B},\boldsymbol{B}_z}$ is the filtration generated by the Brownian Motions $\boldsymbol{B}$ and $\boldsymbol{B}_z$.

We now describe the dynamics of $W_i(t)$ for a single agent with a path-dependent terminal condition as follows:

Proposition III.1. *There exists an $\mathcal{F}_t^{\boldsymbol{B},\boldsymbol{B}_z}$ adapted process $\boldsymbol{Y}_i(t) = (Y_{i1}(t), Y_{i2}(t))$ such that the continuation value $W_i(t)$ of the i^th agent is represented by the process*

$$dW_i(t) = r_i\Big[W_i(t) - u_i\left(\boldsymbol{a}(t), c_i(t)\right)\Big]dt + r_i Y_{i1}(t)\sigma_i dB_i(t) + r_i Y_{i2}(t)\sigma_{Z_i} dB_{Z_i}(t),$$

Conversely, a process $W_i(t)$ satisfying the SDE is the i^{th} agent's continuation value.

Proof. Given fixed and optimal n-agents efforts $\{\boldsymbol{a}(t) : t \geq 0\}$, and the filtration $\mathcal{F}_t = \mathcal{F}_t^{\boldsymbol{B},\boldsymbol{B}_z}$, we have

$$U_i(t) = \mathbb{E}^{\boldsymbol{a}}\left[\int_0^T r_i e^{-r_i s} u_i(\boldsymbol{a}(s), c_i(s))ds + r_i e^{-r_i T}\Phi_i(Z_i(T))|\mathcal{F}_t\right], \tag{3.6}$$

$U_i(t)$ is a $\mathcal{F}_t$-Martingale, i.e., for any $s < t$, using (3.6) and the iterated conditional expectation, it is readily seen that $\mathbb{E}^{\boldsymbol{a}}(U_i(t)|\mathcal{F}_s) = U_i(s)$. From the Martingale Representation theorem [23], we obtain the existence of adapted processes $Y_{i1}(t)$ and $Y_{i2}(t)$ such that:

$$dU_i(t) = r_i e^{-r_i t} Y_{i1}(t) \sigma_i dB_i(t) + r_i e^{-r_i t} Y_{i2}(t) \sigma_{Z_i} dB_{Z_i}(t).$$

From (3.5), it is easily seen that the (3.6) can be rewritten as:

$$U_i(t) = \int_0^t r_i e^{-r_i s} u_i(\boldsymbol{a}(s), c_i(s)) ds + e^{-r_i t} W_i(t),$$

and using Ito's lemma we obtain the dynamics

$$dU_i(t) = r_i e^{-r_i t} u_i(\boldsymbol{a}(t), c_i(t)) dt + e^{-r_i t} dW_i(t) - r_i e^{-r_i t} W_i(t).$$

Equating the above two dynamics of $dU_i(t)$ gives the result. □

The expansion of state space (when compared to [104]) is needed to accommodate the path dependent terminal condition, requiring the vector $(\boldsymbol{X}(t), \boldsymbol{Z}(t))^T$ to be a part of the state space. Dynamic contracts between the principal and the i^{th} agent must specify: (a) the instantaneous compensations $c_i(t)$, and (b) two processes $Y_{i1}(t)$ and $Y_{i2}(t)$ as the sensitivity of the agent's continuation value $W_i(t)$ to the output $X_i(t)$ and terminal process $Z_i(t)$, respectively.

Given a contract $\{c_i(t), \boldsymbol{Y}_i(t)\}_{t\in[0,T]}$, we use the one-shot deviation principle to derive the necessary condition for the optimality of the effort $\{a_i(t)\}_{0\le t\le T}$ with given $\{\boldsymbol{Y}_i(t)\}_{0\le t\le T}$. This optimality condition is equivalent to the IC-constraint in (3.4). Such an optimality condition holds for an arbitrary i^{th} agent's $a_i(t)$ given $\boldsymbol{a}_{-i}$.

Proposition III.2. *For any fixed $\boldsymbol{a}_{-i}(t)$, the contracted compensation $c_i(t)$ for the agent i is implementable if and only if $\{a_i(t)\}$ satisfies*

$$a_i(t) = \arg\max_{\tilde{a}_i(t)\in\mathcal{A}_i} \left[Y_{i1}(t) f_i(\boldsymbol{a}_{-i}(t), \tilde{a}_i(t)) + u_i(\boldsymbol{a}_{-i}(t), \tilde{a}_i(t), c_i(t)) \right], [] \tag{3.7}$$

for all $t \in [0, T]$.

Proof. Let $\boldsymbol{a}(t)$ be the optimal effort vector, and let the effort of the i^{th} agent, for a fixed $t > 0$, be

$$\tilde{a}_i(s) = \begin{cases} \tilde{a}_i(s) & \text{if } s < t \\ a_i(s) & \text{if } s \geq t. \end{cases}$$

We denote $\tilde{\boldsymbol{a}} = (\boldsymbol{a}_{-i}, \tilde{a}_i)$. Choosing actions $\tilde{\boldsymbol{a}}$ will change the dynamics of X_i and W_i. To obtain the new dynamics we apply the Girsanov's theorem with the kernel $\phi(t) = f_i(\tilde{\boldsymbol{a}}(t)) - f_i(\boldsymbol{a}(t))$. The new dynamics adapted to Brownian motions $\tilde{B}_i$ and $\tilde{B}_{Z_i}$ on the space $(\Omega, \mathcal{A}, \tilde{P})$ are given by

$$\begin{cases} \sigma_i dB_i(t) & = \sigma_i d\tilde{B}_i(t) + \phi(t)dt, \\ \sigma_{Z_i} dB_{Z_i}(t) & = \sigma_{Z_i} d\tilde{B}_{Z_i}(t). \end{cases}$$

Substituting in (3.1) and Proposition III.1 under $\tilde{\boldsymbol{a}}$, the dynamics of $U_i(t)$ becomes:

$$\begin{aligned} d\tilde{U}_i(t) = & r_i e^{-r_i t} \left(u_i(\tilde{\boldsymbol{a}}(t), c_i(t)) - u_i(\boldsymbol{a}(t), c_i(t)) + Y_{i1}(t)(f_i(\tilde{\boldsymbol{a}}(t)) - f_i(\boldsymbol{a}(t))\right) dt + \\ & Y_{i1}(t)\sigma_i d\tilde{B}(t) + Y_{i2}\sigma_{Z_i} d\tilde{B}_{Z_i}(t). \end{aligned}$$

Since a_i is optimal, the drift of this SDE must be non-positive. This completes the proof. □

These two propositions decouple the principal's and an arbitrary i^{th} agent's problem. To specify the target efforts that are not observable, the principal can incentivize the agent by recommending a sensitivity level $r_i \boldsymbol{Y}_i(t)$. With n agents, the Nash Equilibrium is equivalent to finding the optimal $\mathbf{Y}(t) = [\boldsymbol{Y}_1(t), \ldots \boldsymbol{Y}_n(t)]^\intercal$ jointly. The principal can create a contract with: (a) functions for $\{c_i(\boldsymbol{W}(t), \boldsymbol{X}(t), \boldsymbol{Z}(t))\}_{i\in[n]}$ for each agent i; and (b) functions of the sensitivity $\{r_i \boldsymbol{Y}_i(t)\}_{i\in[n]}$ that specify the target effort processes. Hence create multiagent contracts that provide consistent information for all agents over the planning horizon which are thus *implementable*.

Characterizing implementable multiagent contracts require that the actions of the agents $\boldsymbol{a}(t)$ form a multiagent Nash Equilibrium at each epoch $t \in [0, T]$. We note that in our formulation there are interactions among n-agents both in the instantaneous utility u_i and drift term of output processes f_i for all $i \in [n]$. The principal thus chooses a target effort level $\boldsymbol{a}(t)$ which form a a Nash Equilibrium amoung agents, so that each agent $i \in [n]$ is disincentivized to deviate from the target $a_i(t)$ when the other agents do not, i.e., implementing the targeted $a_i(t)$.

3.3.2 Nash Equilibrium in Multi-agent Contracts

We now prove the existence of a Nash Equilibrium among n-agents best responses (3.7) at a fixed epoch t. The Bellman's principle of optimality guarantees that it is sufficient to show the existence of Nash Equilibrium within the Hamiltonian of IC-constraint to prove the existence of subgame perfect Nash Equilibrium.

We need the following assumptions on the functions u_i and f_i for all $i \in [n]$:

1. $u_i : \mathcal{A} \times \mathcal{C}_i \to \mathbb{R}$ is twice continuously differentiable, increasing in c_i, and concave.

2. $f_i : \mathcal{A} \to \mathbb{R}_+$ is twice continuously differentiable, increasing and concave.

3. For each i and $\boldsymbol{a}$, $\frac{\partial f_i(\boldsymbol{a}_i, 0)}{\partial a_i} \neq 0$ and $f_i(\boldsymbol{a}) \to \infty$ while $\frac{\partial f(\boldsymbol{a})}{\partial a_i} \to 0$ as $a_i \to \infty$.

4. The set $\cap_i \{(\boldsymbol{a}, \boldsymbol{c}) : u_i(\boldsymbol{a}, c_i) \geq 0 \text{ for all } i\}$ is nonempty and compact.

5. There exists an $m > 0$ such that $m < \sup_x u_i(\boldsymbol{a}_{-i}, x, c_i)$, and $u_i \to -\infty$ as $x \to \infty$, for all i and $\boldsymbol{a}_{-i}, c_i$.

6. $u_i(\boldsymbol{a}_{-i}, 0, c_i) \geq 0$ for each $\boldsymbol{a}_{-i}, c_i$.

The single-agent contract in [41, 104] and the multi-agent contracts in [117] are special cases of the functions above with u separable in $a(t)$ and $c(t)$ and $f(a(t)) = a(t)$. Assumption 4 is satisfied because an arbitrary agent can choose effort $a_i(t) = 0$

to have zero utility. Assumption 6 is valid because $a_i(t) \notin \mathcal{A}_i$ if $u_i < 0$. With these assumptions, we can show the following lemmas.

Lemma III.3. *Let* $\boldsymbol{\alpha}_i = (\boldsymbol{a}_{-i}, c_i)$, *and we define*

$$g_i^{\boldsymbol{\alpha}_i}(x) = \frac{-u_i'(\boldsymbol{a}_{-i}(t), x, c_i(t))}{f_i'(\boldsymbol{a}_{-i}(t), x)}.$$

$g_i^{\boldsymbol{\alpha}_i}$ *is continuously differentiable and monotonically increasing as a function of* x *in the domain* $\mathcal{A}_i$. *Also, there exist* $0 \leq \beta_i < \gamma_i$ *such that for each* $\beta_i < y < \gamma_i$ *and* $\boldsymbol{\alpha}_i \in \mathbb{R}^n$, $g_i^{\boldsymbol{\alpha}_i}(x) = y$ *has a solution.*

Proof. $g_i^{\boldsymbol{\alpha}_i}$ is well defined from Assumption 3 on f_i', i.e., it is nonzero and its monotonicity follows from the concavity of u_i and f_i. We define

$$\hat{g}_i(x) = \inf_{\boldsymbol{\alpha} \in \mathbb{R}^n} g_i^{\boldsymbol{\alpha}}(x), \quad \beta_i = \max\{0, \sup_{\boldsymbol{\alpha} \in \mathbb{R}^n} g_i^{\boldsymbol{\alpha}}(0)\}.$$

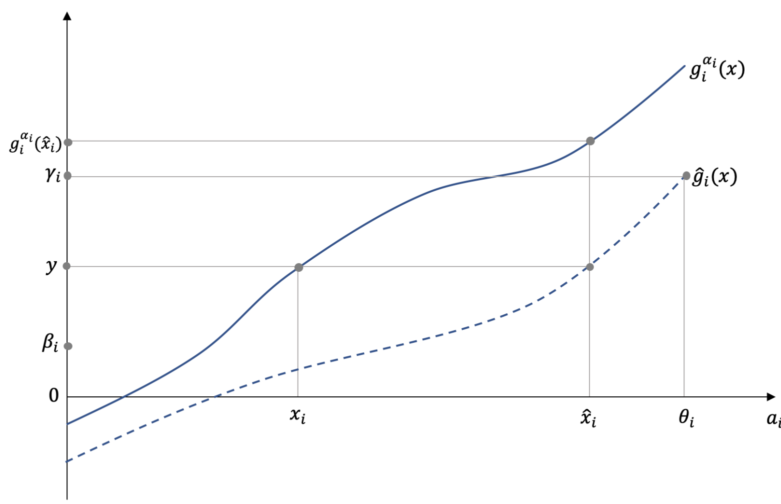


Figure 3.1: Proof for existence of $g_i^{\boldsymbol{\alpha}_i}(x) = y$ in Lemma III.3.

Let θ_i be the i^{th} agent's greatest effort, i.e., $\theta_i = \sup \mathcal{A}_i$. Define $\gamma_i = \hat{g}(\theta_i)$ and θ_i sufficiently large so that $[\beta_i, \gamma_i]$ is nonempty. This exists as $\hat{g}_i$ is an increasing function in Figure 3.1.

For arbitrary $y \in [\beta_i, \gamma_i]$, we define $\hat{g}_i(\hat{x}) = y$. Such an $\hat{x} \in \mathcal{A}_i$ exists because the function $\hat{g}_i$ is monotonically increasing. Now, for any $\boldsymbol{\alpha}_i$, the function $g_i^{\boldsymbol{\alpha}_i}(\hat{x}) \geq y$ and $g_i^{\boldsymbol{\alpha}_i}(0) \leq \beta_i$. The result follows from the continuity of $g_i^{\boldsymbol{\alpha}_i}$ and the intermediate value theorem. □

Applying Lemma III.3 to all agents $i \in [n]$, we define a set $\mathcal{Y} = \prod_i [\beta_i, \gamma_i]$. We can now rigorously define the multiagent Nash Equilibrium as follows.

Definition III.4. The multi-agents' effort $\boldsymbol{a}$ is called a Nash equilibrium if and only if an arbitrary agent's deviation from the stipulated effort level in $\boldsymbol{a}$ while the other agents follow their stipulated actions will result in a loss to the agent, i.e., for each $i \in [n]$,

$$a_i \in \Gamma_i(\boldsymbol{a}_{-i}, c_i, y_i) = \Big\{ \hat{x} : \hat{x} = \arg\max_x \left[y_i f_i(\boldsymbol{a}_{-i}, x) + u_i(\boldsymbol{a}_{-i}, x, c_i) \right] \Big\}. \tag{3.8}$$

We now prove a simple lemma to characterize the equilibrium:

Lemma III.5. *For all $t \in [0, T]$, and each $\boldsymbol{y}(t) \in \mathcal{Y}$ and $\boldsymbol{c}(t) \in \mathcal{C}$, if $\boldsymbol{a}(t)$ satisfying 3.8 exists, it lies in the set $\bigcap_i \{(\boldsymbol{a}, c_i) : u_i(\boldsymbol{a}, c_i) \geq 0\}$.*

Proof. For any given contract $\boldsymbol{y}(t), \boldsymbol{c}(t)$, let $\boldsymbol{a}(t)$ be a Nash equilibrium for each $t \in [0, T]$ and let $u_i(\boldsymbol{a}(s), c_i(s)) < 0$ for some $i \in [n]$, and $s \in (t_1, t_2)$. Thus $\int_{t_1}^{t_2} u_i(\boldsymbol{a}_{-i}(s), a_i(s), c_i(s)) ds < 0$. But, from Property 6,

$$\int_{t_1}^{t_2} u_i((\boldsymbol{a}_{-i}(s), 0, c_i(s)) ds \geq 0.$$

Thus $\boldsymbol{a}(t)$ is not a Nash equilibrium for $t \in (t_1, t_2)$, a contradiction. The result follows from the fact that as $u_i(\boldsymbol{a}_{-i}(t), a_i(t), c_i(t))$ is continuous and thus it cannot be strictly negative on a set of measure 0 in $[0, T]$. □

The following corollary shows that agents continue to abide by the conditions of the contracts until the termination epoch.

Corollary III.6. *A consequence of implementation of Nash equilibrium is that no agent has an incentive to leave the contracts before the terminal epoch T.*

Proof. As is seen in the proof of Lemma III.5, when agents' actions form a multiagent Nash equilibrium, each agent receives a positive utility in any finite interval, thus making the each agent's total utility an increasing function of its continuation value. Thus, no agent is motivated to deviate from the target action before the termination epoch T. □

Theorem below establishes the existence of such an equilibrium in each given epoch t.

Theorem III.7. *For each given $\boldsymbol{y}(t) \in \mathcal{Y}$ and $\boldsymbol{c}(t) \in \mathcal{C}$, there exists a subgame perfect Nash Equilibrium $\boldsymbol{a}(t) \in \mathcal{A}$ for every $t \in [0, T]$.*

Proof. For a fixed agent $i \in [n]$, given the concavity of the functions in Proposition III.1, a necessary and sufficient condition for $\hat{x}$ to solve the optimization problem is that $g_i^{\boldsymbol{a}(t),c_i(t)}(\hat{x}) = y_{i1}(t)$. We note that as defined in (3.8), $\Gamma_i(\boldsymbol{a}(t), c_i(t), \boldsymbol{y}_i(t)) = \{x : g_i^{\boldsymbol{a}(t),c_i(t)}(x) = y_{i1}(t)\}$.

We now define a point-to-set map,

$$\Gamma(\boldsymbol{a}(t)) := \Gamma^{\boldsymbol{c}(t),\boldsymbol{y}(t)}(\boldsymbol{a}(t)) = [\Gamma_1(\boldsymbol{a}(t), c_1(t), \boldsymbol{y}_1(t))), \cdots, \Gamma_n(\boldsymbol{a}(t), c_n(t), \boldsymbol{y}_n(t)))].$$

Note that $\Gamma : \mathcal{A} \to \mathcal{A}^*$, where $\mathcal{A}^*$ is the set of all compact and convex subsets of $\mathcal{A}$. To see that Γ is an upper hemi-continuous point to set map, let $\boldsymbol{a}^k$ be a sequence in $\mathcal{A}$ that converges to $\boldsymbol{a}$. Also let $x^k \in \Gamma(\boldsymbol{a}^k)$ for each k such that $\boldsymbol{x}^k$ converges to $\boldsymbol{x}$. To see that $\boldsymbol{x}$ is in $\Gamma(\boldsymbol{a})$, we note that x_i^k is such that $g_i^{\boldsymbol{\alpha}^k(t)}(x_i^k) = y_{i1}(t)$. From the definition of g_i in Lemma III.3, it is a continuous function of $\boldsymbol{\alpha}$ and x, thus $y_{i1}(t) = \lim_{k\to\infty} g_i^{\boldsymbol{\alpha}^{\boldsymbol{k}}(t)}(x^k) = g_i^{\boldsymbol{\alpha}(t)}(x)$ for each i. The existence of Nash Equilibrium now follows from Lemma III.5, Property 4 in the assumptions, and the Kakutani Fixed Point Theorem [98]. □

3.3.3 On Uniqueness of Nash Equilibrium in Multi-agent Contracts

The individual incentive contract assumes that, if multiple subgame perfect Nash Equilibrium exists, the principal has the power to choose her preferred one. However, if multiple equilibria exist in the multiagent contracts, first all equilibria must be found, and then to look for a plausible selection criteria to convince the agents to implement a specific chosen equilibrium. To avoid this computational problem at each epoch t, we impose reasonable and mild additional conditions to guarantee a unique Nash Equilibrium. We now state these conditions:

1. u_i is strictly concave in $\boldsymbol{a}$ and $u_i'(\boldsymbol{a}_{-i}, a_i, c_i) := \frac{\partial u_i(\boldsymbol{a}_{-i}, a_i, c_i)}{\partial a_i} < 0$ for each $i \in [n]$, and each $\boldsymbol{a}_{-i}$.

2. Let for each $i \in [n]$, $u_{ij}^{''} := \frac{\partial^2 u_i}{\partial a_i \partial a_j}$ for each i, j, and similarly $f_{ij}^{''}$. The negative definite matrix (negative semi-definite matrix) $D^2 u_i$ ($D^2 f_i$) is such that the ith row is strictly diagonally dominant (diagonally dominant) in variables $\boldsymbol{a}$, i.e.,

$$-u_{ii}^{''} > \sum_{i \neq j} |u_{ij}^{''}| \quad (-f_{ii}^{''} \geq \sum_{i \neq j} |f_{ij}^{''}|).$$

Remark III.8. Comments on the uniqueness conditions of the Nash equilibrium of agents:

1. Condition 1 stipulates that the optimal effort the agents exert are unique, also have a negative effect on their instantaneous utility, i.e., the marginal utility as a function of the agent i's effort a_i is negative.

2. Condition 2 states that agent i's particular decision mostly affects the decrease in his marginal utility. In contrast, the other agents' efforts have a minor effect

(note the strict concavity implies that $u_{ii}^{''}$ is negative).

3. The signs of u''_{ij} is related to whether a_i is *strategic complement* or *strategic substitute* [98]. Diagonal dominance thus assumes that the magnitude of the effect of any agent's actions exceeds the magnitude of the combined strategic effects of all the other agents' actions.

We now prove a result:

Lemma III.9. *Let u_i and f_i satisfy Conditions 1 and 2 above and $g_i^{\boldsymbol{\alpha}_i}$ be as defined in Lemma III.3, and $g(\boldsymbol{a}) = [g_1^{\boldsymbol{\alpha}_1}(\boldsymbol{a}), \cdots, g_n^{\boldsymbol{\alpha}_n}(\boldsymbol{a})]^T$. The Jacobean matrix of g, $D_{\boldsymbol{a}}g(\boldsymbol{a})$ is then a P-matrix, i.e., has all principal minors positive.*

Proof. We first show that $D_{\boldsymbol{a}}g(\boldsymbol{a})$ is a strictly row diagonally dominant Jacobean matrix. Note that, suppressing the argument $\boldsymbol{a}$,, $\boldsymbol{c}$, we obtain

$$\begin{aligned}
\frac{\partial g_i}{\partial a_i} &= \frac{1}{f'_i}\{-u_{ii}^{''} - \frac{-u'_i}{f'_i} f_{ii}^{''}\}, \\
\frac{\partial g_i}{\partial a_j} &= \frac{1}{f'_i}\{-u_{ij}^{''} - \frac{-u'_i}{f'_i} f_{ij}^{''}\}.
\end{aligned}$$

The row dominance now follows from Condition 2 and the observation that $f_i > 0$, $-u_{ii}^{''} > 0$, $-f_{ii}^{''} \geq 0$ and $g_i^{\alpha_i} = \frac{-u'_i}{f'_i} > 0$ in the domain $g^{-1}(\mathcal{Y}) \subset \mathcal{A}$. Also, it is easy to see that each principal submatrix of $D_{\boldsymbol{a}}g$ is also strictly row diagonally dominant. Using Gershgorin's theorem [64], it follows that all the principal submatrices of $D_{\boldsymbol{a}}g$ are nonsingular. We now let B be any such principal submatirx, and let I_B be the diagonal matrix of its diagonal elements and A_B the matrix of its off-diagonal elements. Define $B(t) = I_B + tA_B$ for each $t \in [0, 1]$. $A(t)$ is strictly row diagonally dominant for each t, and since $det(B(0)) > 0$, $det(B(1))$ is also positive. Thus $D_{\boldsymbol{a}}g$ is a P-matrix. □

Theorem III.10. *Assume conditions 1 and 2 above hold. Then, for each epoch $t \in [0,T]$, the Nash equilibrium is unique.*

Proof. From the strict concavity of u_i and (3.8), we see that for given $\boldsymbol{y}$ and $\boldsymbol{c}$, $\boldsymbol{a}$ is a Nash equilibrium if and only if

$$g(\boldsymbol{a}) = \boldsymbol{y}.$$

Let θ_i be the largest effort agent i can put, as found in Lemma III.3, and define $\hat{\mathcal{A}} = \Pi_i[0,\theta_i]$ and consider the set $g(\hat{\mathcal{A}}) = \{\boldsymbol{y} : g(\boldsymbol{a}) = \boldsymbol{y}, \boldsymbol{a} \in \hat{\mathcal{A}}\}$. Using the P-matrix property of $D_{\boldsymbol{a}}g$, the fact that $\hat{A}$ is a hypercube and the Gale-Nikaido theorem [47] (or [72]), we see that g maps $\hat{\mathcal{A}}$ homeomorphically onto $g(\hat{\mathcal{A}})$. The uniqueness follows as $\mathcal{Y} \subset g(\hat{\mathcal{A}})$. □

3.4 The Optimal Multi-agent Contracts

In this section, we solve the optimal multiagent contracts given that n-agents put effort at equilibrium. We denote the principal' controls as $\boldsymbol{v}(t) = (\boldsymbol{c}(t), \boldsymbol{y}(t))$. With the parameterized IC-constraints and a well-defined set of Nash Equilibria $\Theta(\boldsymbol{v})$ for given $\{\boldsymbol{v}(t)\}$ for all $t \in [0,T]$, the principal's problem is as follows:

$$U_P = \max_{\{\boldsymbol{v}:\boldsymbol{a}\in\Theta(\boldsymbol{v})\}_{0\le t\le T}} \mathbb{E}^{\boldsymbol{v}}\Big[\int_0^T r_P e^{-r_P s}\left(u_P(\boldsymbol{X}^{\boldsymbol{v}}(t), \boldsymbol{c}(s)\right) ds - r_P e^{-r_P T}\mathbf{1}^\intercal \cdot \boldsymbol{\Phi}(\boldsymbol{Z}(T))\Big]. \tag{3.9}$$

With the assumption that the solution $\boldsymbol{v}^* = (\boldsymbol{y}^*, \boldsymbol{c}^*)$ to the principal's problem exists, we define the present value of the principal's continuation value at some epoch $t \in [0,T]$ as

$$R_P(t) = \mathbb{E}^{\boldsymbol{v}^*}\left[\int_t^T r_P e^{-r_p u}\left(u_P(\boldsymbol{X}^{\boldsymbol{v}^*}(u), \boldsymbol{c}^*(u)\right) du - r_P e^{-r_P T}\mathbf{1}^\intercal \cdot \boldsymbol{\Phi}(\boldsymbol{Z}(T))|\mathcal{F}_t^{\boldsymbol{B},\boldsymbol{B}_z}\right].$$

We now make the following assumption about this principal's continuation value,

Assumption III.11. *We assume that the value $R_p(t)$ has the following $C^{1,2,2,2}$ functional form $F(t,\boldsymbol{W}(t),\boldsymbol{X}(t),\boldsymbol{Z}(t))$ in variables t, the n-agents' continuation vector $\boldsymbol{W}(t)$, the observed output vector $\boldsymbol{X}(t)$, and the termination value descriptor vector $\boldsymbol{Z}(t)$ (C^k represents the differentiability class).*

In what follows, for the ease of exposition, we will shorten $F(t,\boldsymbol{W}(t),\boldsymbol{X}(t),\boldsymbol{Z}(t))$ to F_t whenever there is no possibility of confusion.

Remark III.12. The state space includes $\boldsymbol{Z}(t)$ to assure that the vector process $(\boldsymbol{W}(t),\boldsymbol{X}(t),\boldsymbol{Z}(t))$ is Markov. In the special case that $T \to +\infty$ (i.e., an infinite time horizon with the transversality condition), the state space does not contain t, as in [23].

Then the optimum utility received by the principal, following the optimal control $\boldsymbol{v}^*$ can also be written as:

$$U_P(t) = \int_0^t r_P e^{-r_P s} u_P(\boldsymbol{x}^{\boldsymbol{v}^*}, \boldsymbol{c}^*(s))ds + F(t,\boldsymbol{W}(t),\boldsymbol{X}(t),\boldsymbol{Z}(t)). \tag{3.10}$$

Note that, at epoch t, $\{\boldsymbol{x}(s)\}_{s\le t}$ is realized, and $\{\boldsymbol{X}_{\boldsymbol{v}}(s)\}_{s>t}$ is dependent on controls $\boldsymbol{v}$. Following the same argument in Proposition III.1, we can see that $U_P(t)$ defined by (3.10) is a $\mathcal{F}^{\boldsymbol{B},\boldsymbol{B_Z}}$-adapted Martingale, and thus has drift zero. Applying Ito's multidimensional lemma and the dynamics of $\boldsymbol{W}(t)$, $\boldsymbol{X}(t)$ and $\boldsymbol{Z}(t)$, we obtain the dynamics of $U_P(t)$. Thus, we can solve for F by setting the drift term of the dynamics of U_P to 0.

To obtain the drift term we recall the dynamics of the state variables. For notational convenience, we let $\boldsymbol{\sigma} = \text{diag}((\sigma_1,\cdots,\sigma_n)$, $\boldsymbol{Y}_1(t) = \text{diag}(r_1\sigma_1\boldsymbol{Y}_{11}(t),\cdots,r_n\sigma_n\boldsymbol{Y}_{1n}(t))$ and $\boldsymbol{Y}_2(t) = \text{diag}(r_1\sigma_{Z_1}\boldsymbol{Y}_{12}(t),\cdots,r_1\sigma_{Z_n}\boldsymbol{Y}_{2n}(t))$ and $\boldsymbol{r} = \text{diag}(r_1,\cdots,r_n)$. Let $\boldsymbol{L}$ be the Cholesky factor of $\boldsymbol{\Sigma}$ (i.e., $\boldsymbol{\Sigma} = \boldsymbol{L}\boldsymbol{L}^T$), the covariance matrix of $\boldsymbol{B}(t)$. There exists a process $\hat{\boldsymbol{B}}$, a vector of n independent Brownian motions, with

$$\begin{cases} \boldsymbol{B}(t) = \boldsymbol{L}\hat{\boldsymbol{B}}(t), \\ \mu(t, \boldsymbol{W}(t)) = \boldsymbol{W}(t) - \boldsymbol{u}(\boldsymbol{a}(\boldsymbol{y}(t)), \boldsymbol{c}(t)), \\ \boldsymbol{\sigma}(\boldsymbol{y}(t)) = \boldsymbol{Y}_1(t)\boldsymbol{L}. \end{cases}$$

Using Proposition III.1, we get

$$d\boldsymbol{W}(t) = \boldsymbol{r}\left[\boldsymbol{W}(t) - \boldsymbol{u}(\boldsymbol{a}(\boldsymbol{y}(t)), \boldsymbol{c}(t))\right] dt + \sigma(\boldsymbol{y}(t))d\hat{\boldsymbol{B}}(t) + \boldsymbol{Y}_2(t)d\boldsymbol{B}_{\boldsymbol{z}}(t), \tag{3.11}$$

and similarly for the dynamics of $\boldsymbol{Z}(t)$ using (3.2).

We define a differential operator $\mathcal{H}^{\boldsymbol{v}}$ as a function of the control vector $\boldsymbol{v} = (\boldsymbol{y}, \boldsymbol{c})^T$ as follows,

$$\begin{aligned} \mathcal{H}^{\boldsymbol{v}} F_t \;=\; & \boldsymbol{r}\Big(D_{\boldsymbol{w}} F_t \mu(t, \boldsymbol{w}(t))\Big) + D_{\boldsymbol{x}} F_t f(\boldsymbol{a}(\boldsymbol{y}(t))) + D_{\boldsymbol{z}} F_t \mu_{\boldsymbol{z}}(\boldsymbol{x}(t), \boldsymbol{c}(t)) + \\ & \frac{1}{2}\mathrm{trace}\Big(\boldsymbol{\sigma}(\boldsymbol{y}(t))^{\intercal} D^2_{\boldsymbol{w}} F_t \boldsymbol{\sigma}(\boldsymbol{y}(t)) + \boldsymbol{Y}_2(t) D^2_{\boldsymbol{w}} F_t \boldsymbol{Y}_2(t) + \boldsymbol{L}^T \boldsymbol{\sigma}^{\intercal} D^2_{\boldsymbol{x}} F_t \boldsymbol{\sigma} \boldsymbol{L} + \\ & \boldsymbol{\sigma}(\boldsymbol{y}(t)) D^2_{\boldsymbol{w}\boldsymbol{x}} F_t \boldsymbol{\sigma} \boldsymbol{L} + \boldsymbol{\sigma}_{\boldsymbol{z}}^{\intercal} D^2_{\boldsymbol{z}} F_t \boldsymbol{\sigma}_{\boldsymbol{z}}\Big), \end{aligned} \tag{3.12}$$

where $D_{\boldsymbol{x}} F_t$ and $D^2_{\boldsymbol{x}} F_t$ are the first and second derivative matrices of F_t with respect to $\boldsymbol{x}$.

Applying the multidimensional Ito's lemma to (3.10), we get the drift of the dynamics of $U_P(t)$ as

$$\frac{\partial}{\partial t} F_t + r_P e^{-r_P t} u_P(\boldsymbol{x}^{\boldsymbol{v}^*}, \boldsymbol{c}^*(t)) + \mathcal{H}^{\boldsymbol{v}^*} F_t. \tag{3.13}$$

We now prove the theorem that verifies the Assumption III.11:

Theorem III.13. *The principal's problem can be formulated as the Hamilton-Jacobi-*

Bellman equation:

$$\frac{\partial}{\partial t}F_t + \max_{\boldsymbol{v}=(\boldsymbol{y},\boldsymbol{c})} \left\{ r_P e^{-r_P t} u_P(\boldsymbol{x}(t), \boldsymbol{c}(t)) + \mathcal{H}^{\boldsymbol{v}} F_t \right\} = 0 \tag{3.14}$$

$$s.t. \quad F(T, \boldsymbol{w}, \boldsymbol{x}, \boldsymbol{z}) = -r_P e^{-r_P T} \mathbf{1}^\intercal \cdot \boldsymbol{\Phi}(\boldsymbol{z}), \forall \boldsymbol{w}, \boldsymbol{x}, \boldsymbol{z},$$

$$\boldsymbol{a}(\boldsymbol{v}(t)) \in \Theta(\boldsymbol{v}(t)), \forall t \in [0, T].$$

Also, its solution $F_t^{\boldsymbol{v}^*} (= F_t)$ *is such that* $F_t = U_P(t)$ *and the control* $\boldsymbol{v}^* := \boldsymbol{v}^*(t, \boldsymbol{x}, \boldsymbol{w}, \boldsymbol{z})$ *solves principal's optimization problem 3.9, thus verifying the Assumption III.11.*

Proof. For ease of notation, we define $\boldsymbol{S}^{\boldsymbol{v}} = (\boldsymbol{w}^{\boldsymbol{v}}, \boldsymbol{x}^{\boldsymbol{v}}, \boldsymbol{z}^{\boldsymbol{v}})^T$ and let $F_t^{\boldsymbol{v}} = F(t, \boldsymbol{S}^{\boldsymbol{v}}(t))$ be the weak solution of the equation (3.14) under control $\boldsymbol{v}$.

Now, using an arbitrary control law $\boldsymbol{v}$, such that $\boldsymbol{a}(\boldsymbol{v}(t)) \in \Theta(\boldsymbol{v}(t)), \forall t \in [0, T]$ at the arbitrary time t, with the state dynamics of $\boldsymbol{S}^{\boldsymbol{v}}$ governed by the Brownian motions $\boldsymbol{B}, \boldsymbol{B_Z}$ and when F solves the HJB equation, we see that

$$\frac{\partial}{\partial t}F_t + r_P e^{-r_P t} u_P(\boldsymbol{x}(t), \boldsymbol{c}(t)) + \mathcal{H}^{\boldsymbol{v}} F_t \leq 0,$$

for all $\boldsymbol{v}$. Thus we have, for each time $s \in [0, T]$

$$\frac{\partial}{\partial t}F_s + r_P e^{-r_P t} u_P(\boldsymbol{x}^{\boldsymbol{v}}(s), \boldsymbol{c}^{\boldsymbol{v}}(s)) + \mathcal{H}^{\boldsymbol{v}} F_s^{\boldsymbol{v}} \leq 0,$$

From the boundary condition, we also have $F_T = -r_P e^{-r_P T} \mathbf{1}^\intercal \boldsymbol{\Phi}(\boldsymbol{z}^{\boldsymbol{v}})$. Integrating the above expression we obtain the inequality

$$F_t^{\boldsymbol{v}} \geq \mathbf{E}^{\boldsymbol{v}} \left[\int_t^T r_P e^{-r_P s} u_P(\boldsymbol{x}^{\boldsymbol{v}}(s), \boldsymbol{c}^{\boldsymbol{v}}(s)) ds - r_P e^{-r_p T} \mathbf{1}^\intercal \boldsymbol{\Phi}(\boldsymbol{z}^{\boldsymbol{v}}(T)) | \mathcal{F}^{\boldsymbol{B}, \boldsymbol{B}_z} \right] = R_P^{\boldsymbol{v}}(t)$$

Since the control $\boldsymbol{v}$ was chosen arbitrarily, we have:

$$F_t \geq \sup_{\boldsymbol{v}} R_P^{\boldsymbol{v}}(t) = R_P(t) \tag{3.15}$$

Using the Ito' formula, we will see that

$$F_t^{\boldsymbol{v}} = \mathbb{E}^{\boldsymbol{v}} \Big[\int_t^T r_P e^{-r_P u} \left(u_P(\boldsymbol{x}^{\boldsymbol{v}}(u), \boldsymbol{c}(u) \right) du - r_P e^{-r_P T} \mathbf{1}^\intercal \cdot \boldsymbol{\Phi}(\boldsymbol{z}^{\boldsymbol{v}}(T)) | \mathcal{F}^{\boldsymbol{B}, \boldsymbol{B_Z}} \Big],$$

Using Ito's lemma on F_t and then integrating, we obtain:

$$\int_t^T \left(\frac{\partial}{\partial t}F_s + \mathcal{H}^{\boldsymbol{v}^*}F_s\right) ds = e^{-r_P T}\mathbf{1}^\intercal \cdot \mathbf{\Phi}(\boldsymbol{z}^{\boldsymbol{v}^*}(\boldsymbol{T})) - F_t,$$

Combining (3.15), we get:

$$F_t \geq R_P(t) \geq R_P^{\boldsymbol{v}}(t) \geq F_t$$

The theorem now follows the verification theorem that $F_t = R_P(t)$ for arbitrary t and v^* is the optimal control law. □

Proposition III.14. *For some $\epsilon > 0$ but small, perturb the boundary condition to $-\mathbf{1}^\intercal\mathbf{\Phi}(\boldsymbol{z}) - \epsilon||\boldsymbol{s}||^2$, and let F^ϵ solve the HJB problem. If the boundary condition functions $\Phi_i(Z_i(T))$ are convex, $F^\epsilon(t,)$ is strictly concave for each $t \in [0,T]$.*

Proof. Note that $-\mathbf{1}^\intercal\mathbf{\Phi}(\boldsymbol{z}) - \epsilon||\boldsymbol{s}||^2$ is strictly concave in $\boldsymbol{s}$. By definition of concave function, $D_{\boldsymbol{s}}^2 F_T$ is negative definite and is thus negative semi-definite on an open neighborhood of T. We now show that F is concave at $t < T$ by contradiction. Assume that $D_{\boldsymbol{s}}^2 F_{\hat{t}}$ has a zero eigenvalue at given $\hat{t} < T$ and $\hat{\boldsymbol{s}} \in \mathbb{R}^{3n}$, and let the associated eigenvector be $\boldsymbol{q}(\hat{t}, \hat{\boldsymbol{s}})$. Considering the function $F_{\hat{t}}$ along the ray $\{\boldsymbol{s} : \boldsymbol{s} = \hat{\boldsymbol{s}} + \lambda\boldsymbol{q}(\hat{t}, \hat{\boldsymbol{s}})\}$. Using the Taylor expansion on $\boldsymbol{s}$, we have

$$F(\hat{t}, \boldsymbol{s}) = F(\hat{t}, \hat{\boldsymbol{s}}) + \lambda D_{\boldsymbol{s}} F(\hat{t}, \hat{\boldsymbol{s}})\boldsymbol{q}(\hat{t}, \hat{\boldsymbol{s}}) + o(\lambda^2) = \hat{a} + \hat{b}\lambda + o(\lambda^2),$$

where $\hat{a}$ and $\hat{b}$ are constant in λ and possibly functions of $\hat{t}$. Since F satisfies the HJB equation in Theorem III.13, we can represent $\frac{\partial F_t}{\partial t} - r_P F_t = \bar{c}\lambda + \bar{d}$. If both u_P and μ_z are affine functions of $\boldsymbol{s}$, the left-hand side of (3.14) is affine in λ. We denote function $\Psi^{\boldsymbol{v}} := r_P u_P + \mathcal{H}^{\boldsymbol{v}}\hat{F}_t = \bar{a}(\boldsymbol{v})\lambda + \bar{b}(\boldsymbol{v})$. Thus, there exists $\boldsymbol{v}$ such that

$$\max_{\boldsymbol{v}}\{\bar{a}(\boldsymbol{v})\lambda + \bar{b}(\boldsymbol{v})\} + \bar{c}\lambda + \bar{d} = 0. \tag{3.16}$$

Note that if $\boldsymbol{v}^*(\lambda)$ maximizes (3.16), the function for this choice of $\boldsymbol{v}$ must be affine. Thus assuming that the function at the optimum solution has the form $a(\boldsymbol{v}^*)\lambda + b(\boldsymbol{v}^*)$, we note that since the HJB holds for any $\boldsymbol{s}$, the equation is satisfied if and only if $a(\boldsymbol{v}) = -\bar{c}$, $b(\boldsymbol{v}) = -\bar{d}$ and $\lambda D\bar{a}(\boldsymbol{v}) + D\bar{b}(\boldsymbol{v}) = 0$. Such a control $\boldsymbol{v}$ almost surely does not exist, since it must satisfy the above system which has $n+1$ variables and $n+2$ nonlinear equations. Thus, a contradiction, and $D^2_{\boldsymbol{s}} F_t$ is negative definite and F_t is strictly concave in $\boldsymbol{s}$ for all $t \in [0, T]$. □

3.4.1 Iterative Algorithm for Solving Multi-agent Contracts

Since adding an equilibrium constraint causes new computational issues, we propose a simple iterative algorithm to solve the optimal multi-agent contracts in Theorem III.13. The main idea is to integrate a numerical method for HJB (i.e., Howard's algorithm [67]) with a fixed-point algorithm (i.e., Eaves-Saigal's algorithm [32]). For brevity, we denote the state variable at time t by a time-generic vector $\boldsymbol{s} = (\boldsymbol{w}, \boldsymbol{x}, \boldsymbol{z}) \in \mathbb{R}^{3n}$ (note that the mesh width for each type of state may vary) and the control at time t by $\boldsymbol{v} = (\boldsymbol{c}, \boldsymbol{y}) \in \mathbb{R}^{2n}$. We discretize the $\boldsymbol{s} - t$ plane by choosing uniform mesh widths $\Delta\boldsymbol{s} = (\Delta w^n, \Delta x^n, \Delta z^n) \in \mathbb{R}^{3n}$ and a time step Δt such that $T/\Delta t \in \mathbb{N}$. We define the discrete mesh points $\boldsymbol{s}^{\boldsymbol{\tau}}_{\boldsymbol{i,j,k}}$ by:

$$(3.17) \qquad \begin{aligned} &\boldsymbol{s}_{\boldsymbol{i,j,k}} = (\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k})^\intercal \Delta\boldsymbol{s}, && (\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}) = (i_1, ..., i_n, j_1, ...j_n, k_1, ..., k_n)^\intercal \in \mathbb{N}^{3n}, \\ &t_\tau = \tau\Delta t, && \tau \in [\frac{T}{\Delta t}]. \end{aligned}$$

Our goal is to use a finite difference method in state space to produce approximations $F^\tau_{\boldsymbol{i,j,k}}$ to the solution $F(t, \boldsymbol{w}, \boldsymbol{x}, \boldsymbol{z})$ in (3.14).

It will be useful to define the approximation for the Hamiltonian operator $\mathcal{H}^{\boldsymbol{v}} F_t$ in (3.12) as $\mathcal{H}^{\boldsymbol{v}} \hat{F}_{t_\tau}$ (we use a forward-in-time and central-in-space scheme) with the following approximations for gradient:

$$\begin{cases} \frac{\partial \hat{F}_{t_\tau}}{\partial t} = \frac{F^{\tau+1}_{\boldsymbol{i},\boldsymbol{j},\boldsymbol{k}} - F^{\tau}_{\boldsymbol{i},\boldsymbol{j},\boldsymbol{k}}}{\Delta t} \\ D_{\boldsymbol{w}} \hat{F}_{t_\tau}|_\ell = \frac{F^{\tau}_{\boldsymbol{i}+\boldsymbol{e}_\ell,\boldsymbol{j},\boldsymbol{k}} - F^{\tau}_{\boldsymbol{i}-\boldsymbol{e}_\ell,\boldsymbol{j},\boldsymbol{k}}}{2\Delta w}, \quad \forall \ell \in [n] \\ D_{\boldsymbol{x}} \hat{F}_{t_\tau}|_\ell = \frac{F^{\tau}_{\boldsymbol{i},\boldsymbol{j}+\boldsymbol{e}_\ell,\boldsymbol{k}} - F^{\tau}_{\boldsymbol{i},\boldsymbol{j}-\boldsymbol{e}_\ell,\boldsymbol{k}}}{2\Delta x}, \quad \forall \ell \in [n] \\ D_{\boldsymbol{z}} \hat{F}_{t_\tau}|_\ell = \frac{F^{\tau}_{\boldsymbol{i},\boldsymbol{j},\boldsymbol{k}+\boldsymbol{e}_\ell} - F^{\tau}_{\boldsymbol{i},\boldsymbol{j},\boldsymbol{k}-\boldsymbol{e}_\ell}}{2\Delta z}, \quad \forall \ell \in [n] \end{cases},$$

where $\boldsymbol{e}_\ell \in \mathcal{R}^n$ is a unit vector with 1 in ℓ^{th} entry and 0 elsewhere. The ℓ^{th} entry of the approximation for hessian (we only present the hessian regarding $\boldsymbol{w}$) is:

$$D^2_{\boldsymbol{w}} \hat{F}_{t_\tau}|_{\ell,\ell'} = \begin{cases} \dfrac{F^{\tau}_{\boldsymbol{i}+\boldsymbol{e}_\ell+\boldsymbol{e}_{\ell'},\boldsymbol{j},\boldsymbol{k}} - F^{\tau}_{\boldsymbol{i}+\boldsymbol{e}_\ell-\boldsymbol{e}_{\ell'},\boldsymbol{j},\boldsymbol{k}} - F^{\tau}_{\boldsymbol{i}-\boldsymbol{e}_\ell+\boldsymbol{e}_{\ell'},\boldsymbol{j},\boldsymbol{k}} + F^{\tau}_{\boldsymbol{i}-\boldsymbol{e}_\ell-\boldsymbol{e}_{\ell'},\boldsymbol{j},\boldsymbol{k}}}{4\Delta w^2} & \text{if } \ell \neq \ell', \\ \dfrac{F^{\tau}_{\boldsymbol{i}+\boldsymbol{e}_\ell,\boldsymbol{j},\boldsymbol{k}} - 2F^{\tau}_{\boldsymbol{i},\boldsymbol{j},\boldsymbol{k}} + F^{\tau}_{\boldsymbol{i}-\boldsymbol{e}_\ell,\boldsymbol{j},\boldsymbol{k}}}{\Delta w^2} & \text{otherwise.} \end{cases}$$

We define the function $\Psi^{\boldsymbol{v}} := r_P u_P + \mathcal{H}^{\boldsymbol{v}} \hat{F}_t$ and the principal's value function under optimal control at time t as $F^* := F^{\boldsymbol{v}*}(t, \boldsymbol{s})$. We initialize with the boundary condition $F^P(T, \boldsymbol{w}, \boldsymbol{x}, \boldsymbol{z}) = -\mathbf{1}^\intercal \boldsymbol{\Phi}(\boldsymbol{z})$ as the terminal conditions, and the well-posed conditions for the state space. Especially, we note that, in a n-agents contract, when n_1-agents have zero continuation values w, we need to first solve an $(n - n_1)$-agents subproblem as a boundary condition. In the m^{th} step in the policy iteration, policy evaluation under controls v^m is conducted by solving the approximation of the PDE below:

$$\left(\frac{\partial \hat{F}_{t_\tau}}{\partial t} \right)^m + \Psi^{\boldsymbol{v}^m} - r_P \hat{F}^m_{t_\tau} = 0.$$

Since the PDE under an arbitrary control is well-posed, we can find a weak solution to F_t [40]. In the next iteration, we need to solve two problem:

1. Solve a fixed-point problem to find the unique agents' optimal responses $\boldsymbol{a}^*(t) \in \Theta(\boldsymbol{v}^m)$;

2. Use a greedy algorithm to improve the policy as

$$\boldsymbol{v}^{m+1} = \arg\max_{\boldsymbol{v}' \in V} \Psi^{\boldsymbol{v}'}.$$

In summary, we can solve the optimal multi-agent contracts by adopting a backward scheme:

1. Initialize the terminal condition $F(T, \boldsymbol{s}) = -\mathbf{1}^\intercal \boldsymbol{\Phi}(\boldsymbol{z})$.

2. While $t = T - \tau\Delta t \geq 0$, with a fixed $\epsilon > 0$,

   (a) For each state $\boldsymbol{w}, \boldsymbol{x}, \boldsymbol{z}$, start with an arbitrary contracts $\boldsymbol{v}_0 = \{\boldsymbol{c}_0, \boldsymbol{y}_0\}$.

   (b) Solve a fixed point problem such that $\boldsymbol{a}^*(t) \in \Theta(\boldsymbol{v}_0)$. If the conditions in 3.3.3 are satisfied, the equilibrium is unique.

   (c) Solve for the boundary conditions as a single-agent contract in [104]. We then solve a parabolic PDE within (3.14), i.e., with fixed contracts, to obtain $\tilde{F}(t, \boldsymbol{s})$ [32].

   (d) Optimize the objective value $\tilde{F}(t, \boldsymbol{s})$ for all states $\boldsymbol{s} = (\boldsymbol{w}, \boldsymbol{x}, \boldsymbol{z})$ by the gradient ascend method. The gradient is $\nabla_{\boldsymbol{v}} \tilde{F} \in \mathbb{R}^{2n}$, and the step size γ can be determined by a line-search method. If $\|\nabla_{\boldsymbol{v}} \tilde{F}\| \geq \epsilon$, go back to (ii) with the new contracts $\boldsymbol{v} = (\boldsymbol{c}, \boldsymbol{y})$.

   (e) Go to step 3 if $\|\nabla_{\boldsymbol{v}} \tilde{F}\| < \epsilon$.

3. Update the contracts $\{\boldsymbol{c}(t), \boldsymbol{y}(t)\}$ and continuation value $F(t, \boldsymbol{s})$. Go to Step 2 with $\tau \leftarrow \tau + 1$.

The algorithm converges because:

1. At each epoch, the Nash equilibrium of agents $a^*(t)$ exists.

2. With a unique Nash equilibrium, the boundary conditions are well-posed as the convexity of F with regard to $(w_i, 0)$ for all $i \in [n]$.

3. The numerical methods for a parabolic PDE converges to a weak solution under any given contract.

4. Howard's algorithm guarantees that the sequences of F^m converges to F^* and $\boldsymbol{v}^m$ converges to $\boldsymbol{v}^*$ as $m \to \infty$ [67] for dynamic programming.

Note that our problem in (3.14) only if the Nash equilibrium for agents is unique; otherwise, the policy evaluation has a looping pattern and the convergence result does not hold. There are many alternative methods to solving the problem [45, 95].

3.5 Conclusion

Multiagent incentive contracts with broad applications are hard to solve in general. We characterize the sufficient conditions under which the Nash Equilibrium of agents exists most certainly and additional requirements for the Nash Equilibrium to be unique. We develop a backward iterative algorithm to find the optimal contracts. The implication of our result is two-fold. First, comparing to the single-agent setting, multiagent contracts can model either team collaborations or competitions depending on the context. Second, those conditions of existence and uniqueness contain new insights about the inertia of effective contracting in multiagent systems.

The limitations of the multiagent incentive contracts model include:

1. The Martingale approach is restricted to the SDE output process, where the each agent's decision only affects the drift term. Extension to controlling the diffusion of output process may cause significant technical difficulties even in the single-agent caes.

2. The coupled gradient-based and fixed-point optimization restrict the computational efficiency of solving the contracts, and thus the algorithm can only compute optimal local contracts. Developing more efficient algorithms for multiagent contracts is a meaningful future direction.

Our model has potentially opened doors to implementing dynamic contracts with a wide range of applications in quantitative finance, economics, operations research, and decentralized controls.

CHAPTER IV

Incentive Design for EV and Infrastructure

4.1 Introduction

Many countries worldwide have made massive investments and instituted significant regulations on the deployment of electric vehicles (EVs). The EV adoption provides various benefits ranging from emission reduction to dependence on imported crude oil [65]. National governments around the world, such as Germany, Norway, China, and India, have proposed banning the future sale of gasoline-powered vehicles by 2030 [8]. The approaches to achieving this goal vary from market to market, which can be categorized into non-financial and financial incentives. This work focuses on the latter approach including subsidy, tax reductions, tax credits, and other waivers on fees. Subsidies such as purchase rebates or cash grants are most widely used for promoting the electrification of transport worldwide. For example, European countries have been offering governmental subsidies for EVs since 2009 [135]. Chinese government implemented generous EV incentives to achieve its ambitious goals such as EVs will account for 40 percent of the countrys automobile sales by 2025.

However, the adoption of EV is facing significant uncertainties ahead as governments are phasing out EV subsidies worldwide [137]. Worse yet, with EVs predicted to see substantial growth in next decades, government and industry are faced with the challenge of ensuring that enough charging infrastructure is in place to meet the future demand. The surge of EV adoption in large part helped by the government incentives may fail these goals with the EV subsidy elimination [59]. In contrast, this chapter aims to develop a new EV subsidy policy with the following two elements: First, the policy must enhance the gradual transition to higher EV market penetration in the competition with the Internal Combustion Engines (ICE) vehicles. The effect of EV subsidies implemented now will have a long-lasting effect due to the long lifespan of automobiles. Second, the policy must achieve an effective coordination between charging station (CS) installers and OEMs. EV expansion relies heavily on the availability and affordability of the charging infrastructure and the reliability of the electric grid [114]. How does government design subsidy policies to maximizes EV's social return over the planning horizon? How to allocate the limited budget between OEMs and charging station installers to ensure and accelerate an enhanced market for EV acceptance? This work will answer these questions that which not been addressed in the current policy-making literature.

4.1.1 Case Study: China's EV Subsidy Policy

This work proposes a data-driven EV and CS subsidy policy and verifies its effectiveness in the context of Chinese EV market. China has become the largest automobile market in the world since 2009 [65]. Its ambition to become the world leader in EVs is impeded by regulatory delays. Figure 4.1 demonstrates the timeline of Chinese EV subsidy policies and the scope of this work. Despite the early subsidy policies implemented in 2009, only 8,159 EVs were sold in China in 2011 [131]. Owing to the generous subsidy and tax reductions issued in 2014 [121], China's EV industry has achieved rapid growth and become the world's largest EV market by volume [99]. While it remains true that the expansion of EV charging infrastructure

(Figure 4.4b) greatly lags the increase of demand.

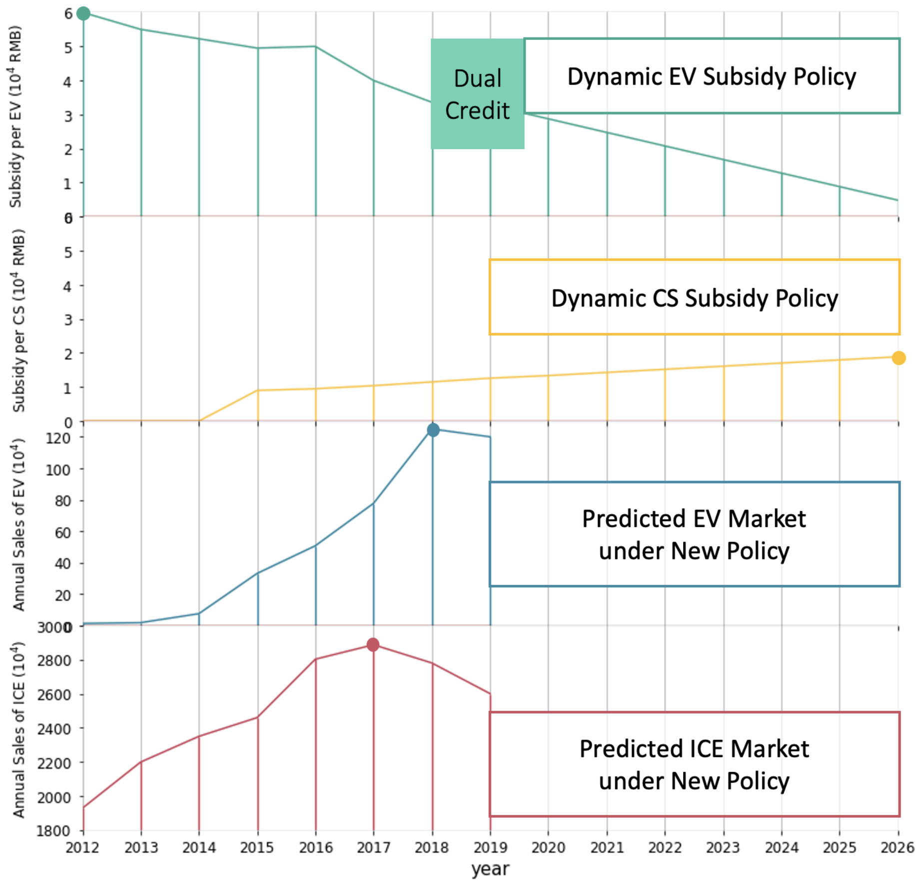


Figure 4.1: Timeline of China's EV policy and new policy proposed in this work.

On the other side, inappropriate EV subsidy policies might lead to two unintended consequences [74]: First, OEMs have less incentive to improve the fuel economy of conventional vehicles; Second, EV subsidies spawned the "cheat compensation" problem as the government's supervision costs rose considerably and the automobile industry became overheated. These issues resulted in a subsidy cut across China. Besides the policies that are still in operation, the Chinese government has reformed the EV subsidy policy. It officially promulgated the "Dual-Credit" Policy in 2018 [99]. The new policy rewards OEMS for both the EV sales and the car model's fuel consumption. The implemented Dual-Credit Policy is, despite being effective in solving those issues, is unable to tackle the decline of overall EV adoption in China. Although many extraneous factors attributed to the ineffectiveness of Dual-Credit Policy, this chapter focuses on designing a pro-active dynamic subsidy policy that assures to achieve the government's goal in 2025.

4.1.2 Main Results and Contributions

This work aspires to present an integrated game-theoretic paradigm for EV and charging infrastructure subsidy policy design. This game model captures how the government coordinates two industries to strike a balance between the EV market growth and the charging infrastructure expansion. The resulting subsidy policy aims to significantly improve the effectiveness of existing EV subsidy policies in China. Our work has three main contributions to the EV regulation policy literature:

1. The suggested policy is the first financial incentive model that considers the interactions between promoting EV market penetration process and expanding charging infrastructure.

2. This work uses China's EV market data to analyze the EV market dynamics and compute the optimal EV subsidies over the next decade.

3. This work develops an innovative and general multi-agent dynamic incentive model that can be deployed with asymmetric information between the government and industry.

The remainder of this chapter discusses how to design an alternative EV and charging infrastructure subsidy policy. We first review the related literature in §4.2. A dynamic game theory model is proposed in §4.3 that optimizes the EV subsidy policy based on the EV market predictions. We consider how to allocate part of the subsidies to EV charging station installers so that the potential demand for EVs are fully exploited. §4.3.3 - §4.3.5 conduct numerical experiments with real-world data and carry out a sensitivity analysis of the proposed dynamic EV subsidy policy. We draw general policy implications in §4.5 and final conclusions in §4.6.

4.2 Literature Review

The burgeoning research interest in government incentives for EVs has arisen along with the sales of first mass-production plug-in EVs in 2010 [135]. With tens of billions of dollars already invested in building up an EV-friendly infrastructure, China has strived to be a leader in EV technology as well as EV-related public policies. Hao et al. [55] presented the rationale of Chinas two-phase EV subsidy scheme and estimated its impacts on EV market penetration, with a focus on the ownership cost analysis of EVs. Du et al. [31] identified key problems regarding the sustainable growth of EV market penetration by market-acceptance indicators and a cluster analysis method. They concluded that the early adoption of EVs in China relied mainly on fiscal incentive policies. Admitting the limited effect of former policy incentives on EV adoption in China, recent research has started to rethink the EV policy in a broader socioeconomic context as in this work.

Beyond considering the governmental incentive problem as a command-and-control process, a stream of literature considered the inefficiency of EV subsidies because of strategic OEMs or customers. Egbue et al.[35] used a survey-based approach to test how consumer attitudes and perceptions of EV affected the adoption of EVs. This early work believed that the widespread adoption of EVs could still be decades away due to the issues of range anxiety and high upfront cost. The rapid expansion of EVs and generous subsidies provided to the advanced EV technology worldwide proved the falsity of this claim. Thenceforth modelling customers' and industry's strategic behaviors become a central task in studies. For example, Zhang [133] investigated the influence of EV subsidies and the strategic consumers' model on EV production. Liu et al.[77] presented an evolutionary game to show that subsidy plays a key role in stimulating EVs industry development. Yang et al. [130] applied a two-stage optimization model to identify that there was a positive relationship between government's subsidy scheme and consumers acceptance of EVs. These above-mentioned studies mainly focused on examining the performances of government incentives for EVs as a static problem. They paid little attention to the interactions between different stakeholders in the EV market.

Our work aligns with the prior work that studied the EV market as a two-sided market – a network consisting of government, OEMs, and charging station installers. *Network Effect* presents in the EV market as the expansion of EV infrastructure can attract a wider user base, and vice versa. Theoretical economic models on indirect network effects date back to Katz and Shapiro [70], Rochet and Tirole [102], and Armstrong [3]. These early studies focused on pricing and coordination issues in two-sided markets. Subsequent work, such as Weyl [124], extended the modeling framework to examine different market structures and types of platforms. The theory of two-sided market aspired to facilitate technology diffusion in emerging energy market such as fuel cell [58] and utility service [101].

Since the adoption of EVs is a technology diffusion process, this works formulates the dynamic EV subsidy problem as a principal-agent problem. There is a reviving interest in the principal-agent problem. Sannikov [104] proposed a solvable framework that reduced the dynamic problem as a large static variational problem in continuous time. This new method has been used for the dynamic subsidy analysis of automated vehicles. To the best of our knowledge, this chapter is the first work that fills the gap between the empirical analysis of EV regulation practice and recent theoretical progress in incentive policy design.

4.3 Dynamic Subsidy Model for EV and CS Market Coordination

Finding an alternative EV subsidy policy that promotes the synergy of EV market and CS infrastructure development follows the following road map:

1. Fitting a correlated forecast model of EV market penetration and CS infrastructure expansion processes (§4.3.1).
2. Formulating the EV subsidy problem as a multi-agent dynamic game (§4.3.2).
3. Solving the optimal EV subsidies by dynamic programming (§4.3.3).
4. Finding an implementable approximation of the optimal EV subsidy policy that improves the Dual-Credit Policy (§4.3.4).
5. Evaluating the properties of the suggested EV subsidy policies in different scenarios (§4.3.5).

Notation used in this chapter is summarized in the following table:

The structure of the game-theoretical model for EV subsidies is presented in Figure 4.2. We summarize the notation of parameters used in our model in Table 4.1.

4.3.1 Benchmark: China's Dual-Credit Policy

This work presents a new EV subsidy scheme based on the Dual-Credit Policy in China. Despite today's low EV market share (e.g., EV ownership in China was only 1.37% in 2019 [78]), the government has implemented a wide array of incentives to promote the production of "New Energy Vehicles" (NEVs), the term used to

Table 4.1: Summary of notation in EV-CS subsidy model

Symbol	Description
EV	Electric vehicle
CS	Charging station
N_{EV}	Total sale of EVs
N_{CS}	Total number of installed CSs
t	Time period during the decision horizon, $t \in [0, \tau]$
ICE	Internal Combustion Engine powered vehicle
a_1	Efforts of EV OEMs
a_2	Effprts of Chargins Station installers
b_1	Internal Influence of EV
b_1	Internal Influence of Charging Station
X_{EV}	Market Share Rate of EV
X_{CS}	Market Share Rate of Charging Station
α_{11}	EV self-coefficient
α_{12}	EV to CS coefficient
α_{21}	EV self-coefficient
α_{22}	CS to EV coefficient
ζ	EV and CS market interaction coefficient
r_G	Government's discount factor
r_A	EV OEM's discount factor
r_B	CS installer's discount factor
γ	Buffer ratio in government's payoff
ϕ_{Te}	Target fuel consumption of EV
ϕ_{Tf}	Target fuel consumption of ICE
ϕ_{Ae}	Actual fuel consumption of EV
ϕ_{Af}	Actual fuel consumption of ICE
h	Scores for each EV in the Dual-Credit policy
ω	Amplification factor in the Dual-Credit policy
β	NEV score ratio requirements
S_{EV}	NEV scores Price
S_{CS}	Charging Station Subsidy
C_{EV}	Cost of environmental impacts per EV
C_{ICE}	Cost of environmental impacts per ICE
P_{EV}	The profit OEM obtained per sale of EV
P_{ICE}	The profit OEM obtained per sale of ICE
P_{CS}	The profit CS installer can get in each charging station
r_1	Conversion ratio of efforts to revenue utility in EV manufacturing
r_2	Conversion ratio of efforts to revenue utility in CS installation

designate plug-in EVs eligible for public subsidies. The subsidies are paid directly to OEMs rather than customers. For example, a new "Dual-Credit" Policy was formally implemented in 2018, aiming to promote a more sustainable development

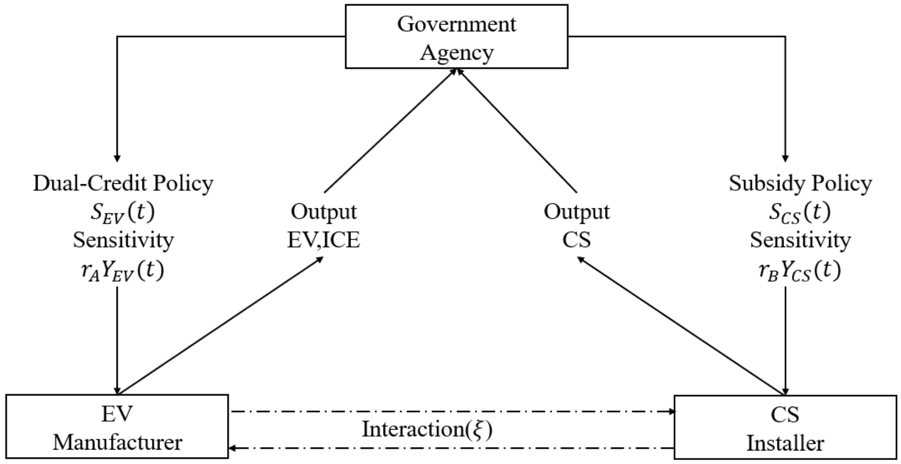


Figure 4.2: DSG model for the EV and CS subsidy problem at time t

of EV technology [99]. As a result, more EVs were sold in China than in the rest of the world in 2018 [31]. The Dual-Credit Policy is constructed in such a way that the lower vehicle fuel consumption performance can be compensated by the heavier reliance on EVs. It assesses two scores separately for each OEM: $CAFC$ and NEV as in the flow chart in Figure 4.3.

- $CAFC$ ("Annual Vehicle Fuel Consumption Scores"):

  1. If a $CAFC$ score is positive, it can be carried over to the next year or transferred to the affiliates;

  2. If the $CAFC$ score is negative, there are 3 ways to resolve it: a) using the previous year's carry-over scores, b) using the scores earned by the affiliates, and c) purchasing NEV scores.

- NEV ("New Energy Vehicle Scores"):

  1. If an NEC score is negative, it can be made up by trading with a company with a positive score;

  2. If the NEV score is positive, it can be sold to a company with a negative

score.

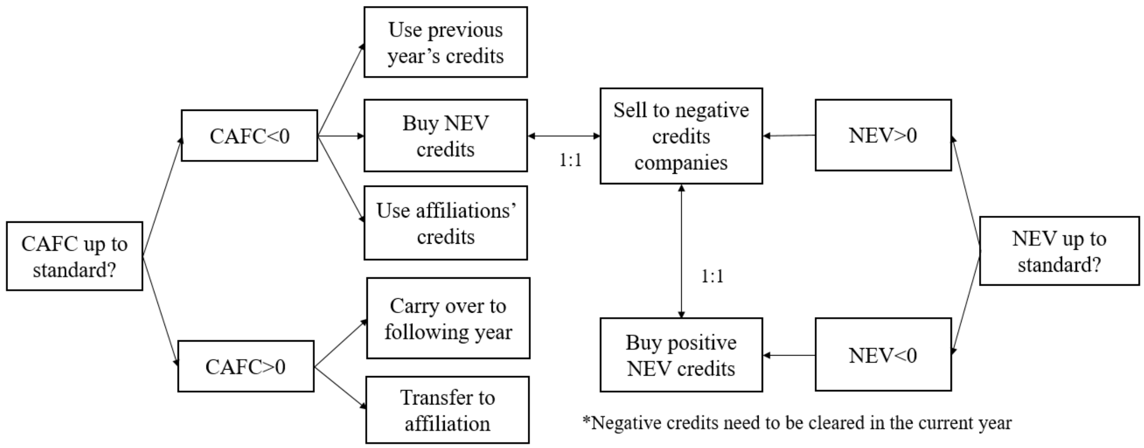


Figure 4.3: Flow-chart of Dual-Credit Policy

The Dual-Credit score can be computed by equations as follows:

$$CAFC = \frac{\sum_{i=1}^{N} FC_i \times V_i}{\sum_{n=1}^{N} V_i - W_i}, \tag{4.1}$$

$$T_{CAFC} = \frac{\sum_{i=1}^{N} T_i \times V_i}{\sum_{n=1}^{N} V_i}, \tag{4.2}$$

$$C_{CAFC} = (\alpha \times T_{CAFC} - CAFC) \times \sum_{i=1}^{N} V_i, \tag{4.3}$$

where $CAFC$ is the actual average fuel consumption of the enterprise; i is the index of the passenger vehicle model; FC_i is the fuel consumption of the i-th model; $V_i$is the annual production of the model; W_i is the multiplier of the i-th model; T_i is the fuel consumption target value for the i-th model; α is the average fuel consumption requirement (in the "Methods and Indicators for Evaluation of Passenger Vehicle Fuel Consumption"); T_{CAFC} is the target value of the average fuel consumption of the enterprise; C_{CAFC} is the average fuel consumption of the company.

$$C_{NEV-ACTUAL} = \sum_{i=1}^{N} C_i \times V_{i-NEV}, \tag{4.4}$$

$$C_{NEV-TARGET} = \beta \times \sum_{i=1}^{N} V_{i-NEV}, \tag{4.5}$$

$$NEV = C_{NEV-ACTUAL} - C_{NEV-TARGET}, \tag{4.6}$$

where C_i is the new energy passenger vehicle model scores; V_{i-NEV} is the annual production or imports of the i-th new energy passenger vehicle model; β is the EV scores ratio requirement; V_{i-CV} is the annual production (excluding exports) or imports of the i-th conventional fuel vehicle. This chapter does not consider the exchange of scores between OEMs. Transferring credit scores to the next year is not allowed. Each OEM's final CAFC score and NEV score must be equal by the end of each year. We refer readers to [99] for more details about the Dual-Credit Policy. This policy can thus assure the automobile industry to reach a proper balance between low-energy consumption and energy diversification in production plans.

4.3.2 Correlated EV and CS Market Dynamics

The main contribution of this work is to consider subsidies for EV and CS as a joint problem. We model the EV market penetration as a Diffusion of Innovation (DOI) process [85] affected by the implemented subsidy policies. The DOI model assumes that the adoption rate of EVs is determined by two diffusion parameters: (a) the coefficient of external influence a that represents the OEMs effort in innovation, and (b) the coefficient of internal influence b that represents the word-of-mouth effect within consumers. At time t, the EV market share rate $dX_{EV}(t)/dt$ is defined by the dynamics in equation (3.7), as follows:

$$\frac{dX_{EV}}{dt} = (a + bX_{EV})(1 - X_{EV}), \tag{4.7}$$

where $X_{EV}(t) = N_{EV}(t)/M_{EV}$ is the EV market share (growing from 0 to 1 if all vehicles in the market are EVs). The aggregate EV market size, i.e., the cumulative number of EVs sold by time t, is denoted by $N_{EV}(t)$; the EV market potential, i.e., the saturation population of AV consumers, is M_{EV}.

We simplify the design of a state-level EV subsidy policy by aggregating all vehicle models into one group. This macroscopic DOI model have been widely used in the EV policy literature [39, 51, 128]. However, the aggregate model in (4.7) has two obvious defects:

1. It assumes that EV adoption is solely affected by the intrinsic tendency to purchase EVs. Nevertheless, customers will not choose EVs over ICEs if there is no convenient access to charging stations [114]. Hence the government should also promote to build more charging stations as the EV market share increases.
2. It assumes that the market forecast is perfect while in fact any predictive model has uncertainty.

This chapter extends the DOI model to two-dimensional differential equations:

$$\begin{cases} \dfrac{dX_{EV}}{dt} = (\alpha_{11}a_1 + \alpha_{12}a_2 + \zeta a_1 a_2 + b_1 X_{EV})(1 - X_{EV}) \\ \dfrac{dX_{CS}}{dt} = (\alpha_{21}a_1 + \alpha_{22}a_2 + \zeta a_1 a_2 + b_2 X_{CS})(1 - X_{CS}) \end{cases}, \tag{4.8}$$

where a_1 and a_2 represent OEM's effort in promoting EVs and CS installer's effort in expanding CS infrastructure respectively. b_1 and b_2 represent the fixed coefficient of internal influence. $\alpha_{11}a_1$ indicates the direct impact of OEM's own decision on EV production, $\alpha_{12}a_2$ indicates the indirect impact of CS installer's decision on EV production (α_{12} can be set to 0 if irrelevant), and $\zeta a_1 a_2$ indicates the degree of correlation between the EV market and charging station infrastructure due to customers' choice. These parameters are designed to capture the synergy between the sales of EVs and the expansion of CS infrastructure.

The deterministic model is unrealistic when predictions are made over long periods of time. Thus we model the uncertainty of this prediction by converting (4.8) to a stochastic model which is driven by a Gaussian process (in particular, a Brownian Motion). Measuring uncertainty in the future expansion of the markets by a Gaussian process is appropriate when a large number of factors affect this expansion. The EV and CS market dynamics are thus generalized to the following system of stochastic differential equations:

$$\begin{cases} dX_{EV} = (\alpha_{11}a_1 + \alpha_{12}a_2 + \zeta a_1 a_2 + b_1 X_{EV})(1 - X_{EV})dt + \\ \qquad \sigma_1(1 - X_{EV})dB_1(t) \\ dX_{CS} = (\alpha_{21}a_1 + \alpha_{22}a_2 + \zeta a_1 a_2 + b_2 X_{CS})(1 - X_{CS})dt + \\ \qquad \sigma_2(1 - X_{CS})dB_2(t) \end{cases}, \tag{4.9}$$

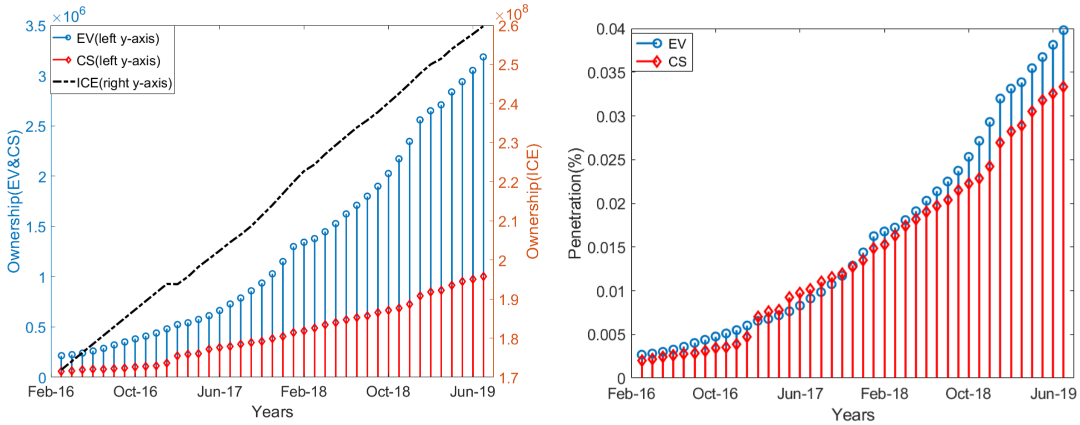
where the drift term follows the DOI model, and the diffusion terms have constant volatilities σ_1 and σ_2 respectively. EV and CS penetration paths are generated by extraneous Brownian Motions $B_1(t)$ and $B_2(t)$, respectively.

4.3.3 Determining Parameters of EV-CS Joint Diffusion Dynamics

In this section, we use the collected EVs and charging infrastructure data in China's NEV market from February 2016 to June 2019[1](Figure 4.4) to model the dynamic processes in (4.9). The government goal is to use incentives to control the rate of the market penetration with the aim of increasing the social benefit.

The Chinese government's projection of the future EV market [65] suggests that by 2035, the number of EVs in China will reach 80 million, and the number of

[1]The collected date is between 2016 and 2018 including the periods that Dual-Credit policy was in effect. This chapter assumes that the dynamic market model is unaffected in the observations considering the short time window that the new policy was deployed.



(a) Ownership of EV and CS in China

(b) Penetration of EV and CS in China

Figure 4.4: Ownership and penetration of EVs and CSs in China

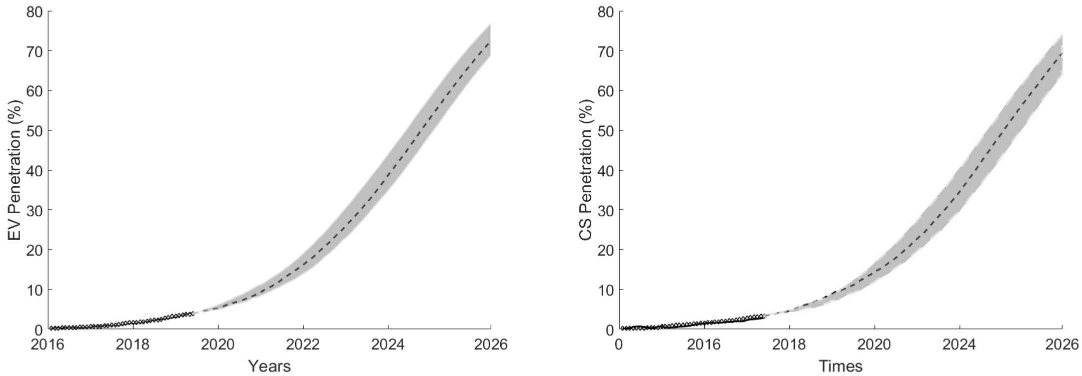
charging stations will reach 30 million. Therefore, we assume the saturation market potential to be $M_{EV} = 80$ million and $M_{CS} = 30$ million, respectively. After fitting the collected data to equation (4.9), the fitted parameters are shown in Table 4.2. The parameters such as α_{11}, α_{12} are not homogeneous in time, but we found these changes to be relatively small in the best-fit values. Thus in the remaining analysis, we assume these parameters as constant in time.

Table 4.2: EV market parameter fitting results

Fitted Parameters	α_{11}	α_{12}	α_{21}	α_{22}	ζ	σ_1	σ_2
	0.001	0.001	0.001	0.001	3	0.000982	0.001094
Fitted Decisions	a_1	a_2	b_1	b_2			
	0.009	0.009	0.049	0.049			

Figure 4.5a and Figure 4.5b show the forecast for EVs and CSs from January 2020 to December 2026. It can be seen that by December 2026, both EV's and CS's market penetration rates are close to 70% of the market potential. The confidential intervals of predicted market growth rates are shown in Figure 4.5.

With the predicted EV and CS market penetration processes, the EV-CS ratio gradually reaches the government's target in Figure 4.6. Note that, due to data avail-



(a) EV Market Penetration Forecast (b) Charging Station Market Penetration Forecast

Figure 4.5: Market forecast to 2026 based on 2016-2019 EV market data

ability, we only consider the public-owned CS infrastructure in this work. This work investigates how a proper EV-CS subsidy policy guarantees the supply of reliable EV charging infrastructure over time.

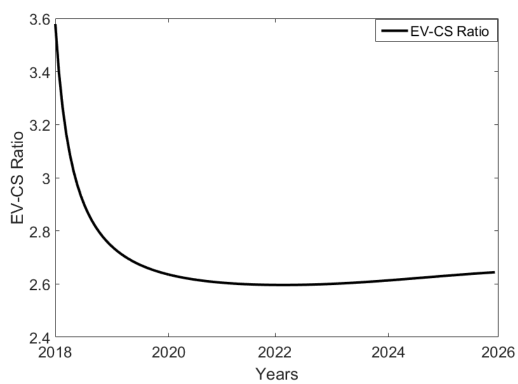


Figure 4.6: EV-CS ratio to 2026 based on 2016-2019 EV market data

4.3.4 EV-CS Subsidy Policy as Dynamic Stackelberg Game

After implementing the EV's Dual-Credit Policy and the charging station subsidy policy, the government gains an overall efficiency benefits $f(X_{EV}(t), X_{CS}(t))$. This function includes the environmental and social benefit accrued by EV adoption.

Since OEMs and CS installers are strategic, which was a main reason for the unintended consequence of EV subsidies in China [99], this work studies how subsidized parties react to the EV subsidies received. We model this process as a dynamic Stackelberg Game in which the government is the principal who offers subsidies and the OEM and CS installers are agents who independently promote their markets. The dynamic game model for EV-CS subsidies is summarized in Table 4.3.

In this game, a risk-neutral government pays a sequence of per-unit EV and charging station's subsidies $S_{EV}(t)$ and $S_{CS}(t)$ to the OEM and the CS installer to maximize the total expected discounted payoff in equation (4.10):

$$E[\int_0^\tau e^{-r_G t} f(X_{EV}(t), X_{CS}(t), S_{EV}(t), S_{CS}(t))dt], \tag{4.10}$$

where r_G is the discount factor of the government agencys total payoff.

After implementing subsidy policies, the government is unable to observe the exact decision $a_1(t)$ and $a_2(t)$ made by the OEM and CS installer. Given OEM's instantaneous utility function $h_1(S_{EV}(t), a_1(t))$ and the CS installer's instantaneous utility function $h_2(S_{CS}(t), a_2(t))$, the government observes the signals X_{EV} and X_{CS} to infer their optimal decisions. With a given EV and CS subsidy policy, the OEM's and CS installer's objective is to maximize their expected total discounted utility, respectively:

$$\begin{cases} E[\int_0^\tau e^{-r_A t} h_1(S_{EV}(t), a_1(t))]dt] \\ E[\int_0^\tau e^{-r_B t} h_2(S_{CS}(t), a_2(t))]dt] \end{cases} . \tag{4.11}$$

The optimal EV and CS subsidy policies characterize sequences $S_{EV}(t)$ and $S_{CS}(t)$ that maximizes the government's expected total discounted payoff only if the EV OEM and CS installer cooperate and adopt the optimal responses. Following the literature of mechanism design [104], we call the conditions that specify the OEM and CS installer's best responses the Incentive-Compatible (IC)-constraint and the IR-constraint respectively. The IC- constraint guarantees that the OEM and CS

Table 4.3: Multiagent dynamic Stackelberg games for EV subsidy policy

Decision-makers	Government (Principal)	EV OEM	CS Installer
Decision	Subsidies $S(t) = [S_{EV}(t), S_{CS}(t)]$	Promoting EVs $a_1(t)$	Building Infrastructure $a_2(t)$
Shared Information	$dX_{EV} = (\alpha_{11}a_1 + \alpha_{12}a_2 + \zeta a_1 a_2 + b_1 X_{EV})(1 - X_{EV})dt + \sigma_1(1 - X_{EV})dB_1(t)$ $dX_{CS} = (\alpha_{21}a_1 + \alpha_{22}a_2 + \zeta a_1 a_2 + b_2 X_{CS})(1 - X_{CS})dt + \sigma_2(1 - X_{CS})dB_2(t)$		
Utility	$f(X_{EV}, X_{CS}, S_{EV}, S_{CS}) = -f_1(X_{EV})S_{EV} - f_2(X_{EV})S_{EV} - f_3(X_{CS})S_{CS} - f_4(X_{EV})$ $\begin{cases} f_1(X_{EV}) = \gamma\phi_{T_f}(N - M_{EV}\frac{dX_{EV}}{dt}) + \gamma\phi_{T_e}(M_{EV}\frac{dX_{EV}}{dt}) - [\phi_{Af}(N - M_{EV}\frac{dX_{EV}}{dt}) + \phi_{Ae}M_{EV}\frac{dX_{EV}}{dt}] \cdot \frac{(N - M_{EV}\frac{dX_{EV}}{dt}) + M_{EV}\frac{dX_{EV}}{dt}}{(N - M_{EV}\frac{dX_{EV}}{dt}) + \omega(M_{EV}\frac{dX_{EV}}{dt})} \\ f_2(X_{EV}) = (h - \beta)M_{EV}\frac{dX_{EV}}{dt} \\ f_3(X_{CS}) = M_{CS}\frac{dX_{CS}}{dt} \\ f_4(X_{EV}) = C_{EV}M_{EV}\frac{dX_{EV}}{dt} + C_{ICE}(N - M_{EV}\frac{dX_{EV}}{dt}) \end{cases}$	$h_1(S_{EV}, a_1) = [f_1(X_EV) + f_2(X_{EV})]S_{EV} + P_{EV}M_{EV}\frac{dX_{EV}}{dt} + P_{ICE}(N - M_{EV}\frac{dX_{EV}}{dt}) - (r_1 a_1)^2$	$h_2(S_{CS}, a_2) = (P_{CS} + S_{CS})M_{CS}\frac{dX_{CS}}{dt} - (r_2 a_2)^2$
Objective	$\max_{S_{EV}(t), S_{CS}(t)}$ $\{E[\int_0^\tau e^{-r_G t} f(S_{EV}(t), S_{CS}(t), a_1(t), a_2(t)dt]\}$	$\max_{a_1}$ $E[r_A \int_0^\tau e^{-r_A t} h_1(S_{EV}(t), a_1(t))]dt]$	$\max_{a_2}$ $E[r_B \int_0^\tau e^{-r_B t} h_2(S_{CS}(t), a_2(t))]dt]$

installer's best responses solve their respective utility maximization problem. The Individual-Rational (IR)-constraint guarantees that OEM and CS installer stay in the market as long as the cumulative expected utility over the horizon exceeds some pre-defined quantity W_0 and W_1. In summary, the optimal EV and CS subsidy policies solve the following optimization problem in equation (4.12):

$$\max_{S_{EV}(t),S_{CS}(t)} \{E[\int_0^{\tau} e^{-r_G t} f(S_{EV}(t), S_{CS}(t), a_1(t), a_2(t) dt]\} \tag{4.12}$$

$$s.t. \begin{cases} a_1(t) = \underset{a_1}{argmax}\, E[\int_0^{\tau} e^{-r_A t} h_1(S_{EV}(t), a_1(t))]dt] \\ a_2(t) = \underset{a_2}{argmax}\, E[\int_0^{\tau} e^{-r_B t} h_2(S_{CS}(t), a_2(t))]dt] \end{cases} \qquad (IC)$$

$$\begin{cases} E[\int_0^{\tau} e^{-r_A t} h_1(S_{EV}(t), a_1(t))]dt] \geq W_0 \\ E[\int_0^{\tau} e^{-r_B t} h_2(S_{CS}(t), a_2(t))]dt] \geq W_1 \end{cases} \qquad (IR)$$

4.3.5 Solving the Optimal EV-CS Subsidy Policy

The optimization for the EV and CS subsidy problem is notoriously difficult because (4.12) includes three coupled subproblems. Recent results in differential games [80, 104] propose a tractable scheme for solving the problem by transforming (4.12) into a dynamic program. The scheme includes three main steps. First, we can find an equivalent representation of the IC-constraint. Second, the representation can be parameterized and incorporated into the objective function with mild smoothness assumptions. Finally, the optimal EV and charging station subsidies can be computed by dynamic programming, i.e., a Hamilton-Jacobi-Bellman(HJB) equation in Theorem IV.2.

One of the main differences between static subsidy and dynamic subsidy analysis is the including the continuation value into the state space. Continuation values are the expected discounted total payoff or utility at time $t > 0$ with the optimal subsidy policy followed through the horizon. This function is also termed as the value function in dynamic programming or reinforcement learning. More specifically, OEM's and CS installer's continuation values at time $t > 0$ are given by equation (4.13):

$$\begin{cases} W_{EV}(t) = E[\int_t^\tau e^{r_A(s-t)} h_1(S^*_{EV}(s), a_1(s))ds|\mathcal{F}_t] \\ W_{CS}(t) = E[\int_t^\tau e^{r_A(s-t} h_2(S^*_{CS}(s), a_2(s))ds|\mathcal{F}_t] \end{cases}, \tag{4.13}$$

where the filtration $\mathcal{F}_t$ represents the information collected by time t (including decisions and markets dynamics observed by time period t). W_{EV} and W_{CS} are added as state variables in dynamic program.

After augmenting the state space, two control variables, the sensitivity level Y_{EV} of the OEM's continuation value W_{EV} with respect to the EV market penetration rate $X_{EV}(t)$ and the sensitivity level Y_{CS} of the CS installer's continuation value W_{CS} with respect to the charging station market penetration rate $X_{CS}(t)$, are introduced to the optimization of subsidy policy. Y_{EV} and Y_{CS} represent the government's control over the OEM's and CS installer's payoff in the future.

Given a subsidy policy $[S_{EV}(t), Y_{EV}(t)]$ and $[S_{CS}(t), Y_{CS}(t)]$ at time t, if h_1 and h_2 are continuously differentiable, concave, and nonlinear in a_1 and a_2 respectively, we can define a Nash Equilibrium as follows:

Definition IV.1. OEM's and CS installer's responses a_1^* a_2^* form a Nash Equilibrium defined in (4.14) for all $t \in [0, \tau]$:

$$\begin{cases} a_1^* = \arg\max_{a_1} Y_{EV}(\alpha_{11}a_1 + \alpha_{12}a_2^* + \zeta a_1 a_2^* + b_1 X_{EV}) + h_1(a_1, a_2^*) \\ a_2^* = \arg\max_{a_2} Y_{CS}(\alpha_{21}a_1^* + \alpha_{22}a_2 + \zeta a_1^* a_2 + b_2 X_{CS}) + h_2(a_1^*, a_2) \end{cases} \tag{4.14}$$

Using the first-order optimality condition, we have

$$\begin{cases} \dfrac{\partial h_1(a_1, a_2^*)}{\partial a_1} + Y_{EV}(\alpha_{11} + \zeta a_2^*)(1 - X_{EV}) = 0 \\ \dfrac{\partial h_2(a_1^*, a_2)}{\partial a_2} + Y_{CS}(\alpha_{22} + \zeta a_1^*)(1 - X_{CS}) = 0 \end{cases}. \tag{4.15}$$

Plugging in the utility functions, we have:

$$\begin{cases} a_1 = \frac{Y_{EV}}{2r_1^2}(\alpha_{11} + \zeta a_2^*)(1 - X_{EV}) \\ a_2 = \frac{Y_{CS}}{2r_2^2}(\alpha_{22} + \zeta a_1^*)(1 - X_{CS}) \end{cases}$$

As long as the slopes of two curves satisfy

$$\frac{\zeta Y_{CS}}{2r_1^2} \leq \frac{2r_2^2}{Y_{EV}\zeta},$$

we have that the unique solution to (4.15) $a_1^* \geq 0$, $a_2^* \geq 0$ forms a Nash Equilibrium. It is easy to check that the data fitted in Table 4.2 guarantee this condition.

The solution to (4.15) is a significant step towards promoting EVs via the network effect, which is a new finding not addressed in prior work. The government allocates a fixed amount of subsidies between the OEM and the CS installer to ensure that EV customers can have a stable access to charging infrastructure without oversubsidizing either side. It also designates that both the OEM and CS installer can not be better off if they do not follow the optimal a_1^* and a_2^*.

Let $F(X_{EV}, W_{EV}, X_{CS}, W_{CS})$ denote a second-order continuous function with four state variables – EV market share X_{EV}, OEM's continuation value W_{EV}, charging station market share X_{CS}, CS installers continuation value W_{CS}. In Theorem IV.2, we show that F calculates the government's continuation value, i.e., the cumulative payoff gained from the new EV subsidy policy. The government's goal is thus to maximize the value of F and achieve the target EV market penetration rate over the planning horizon.

Theorem IV.2. *With all these considerations, we can obtain the optimal EV-CS subsidy policies by solving the following HJB equation:*

$$r_G \frac{\partial F}{\partial t} = \max_{\{S_{EV}, S_{CS}, Y_{EV}, Y_{CS}\}_{t \in [0,\tau]}} \{f(X_{EV}, X_{CS}, S_{EV}, S_{CS}) + L_1 \triangledown F_1 + \frac{1}{2} H_1^T \triangledown^2 F_2 H_1 + L_2 \triangledown F_2 + \frac{1}{2} H_2^T \triangledown^2 F_2 H_2\} \tag{4.16}$$

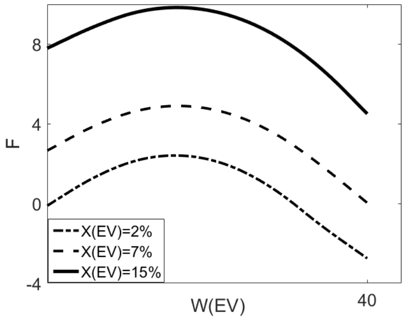
The operators in (4.16) are defined below:

$$\begin{cases} \triangledown F_1 = [\frac{\partial F}{\partial X_{EV}}, \frac{\partial F}{\partial W_{EV}}]^\intercal \\ \triangledown F_2 = [\frac{\partial F}{\partial X_{CS}}, \frac{\partial F}{\partial W_{CS}}]^\intercal \\ \triangledown^2 F_1 = \begin{bmatrix} \frac{\partial^2 F}{\partial {X_{EV}}^2} & \frac{\partial^2 F}{\partial X_{EV} \partial W_{EV}} \\ \frac{\partial^2 F}{\partial X_{EV} \partial W_{EV}} & \frac{\partial^2 F}{\partial {W_{EV}}^2} \end{bmatrix} \\ \triangledown^2 F_2 = \begin{bmatrix} \frac{\partial^2 F}{\partial {X_{CS}}^2} & \frac{\partial^2 F}{\partial X_{CS} \partial W_{CS}} \\ \frac{\partial^2 F}{\partial X_{CS} \partial W_{CS}} & \frac{\partial^2 F}{\partial {W_{CS}}^2} \end{bmatrix} \\ L_1 = [(\alpha_{11} a_1^* + \alpha_{12} a_2^* + \zeta a_1^* a_2^* + b_1 X_{EV})(1 - X_{EV}), W_{EV} - h_1(S_{EV}, a_1^*)] \\ L_2 = [(\alpha_{21} a_1^* + \alpha_{22} a_2^* + \zeta a_1^* a_2^* + b_2 X_{CS})(1 - X_{CS}), W_{CS} - h_2(S_{CS}, a_2^*)] \\ H_1 = [X_{EV} \quad Y_{EV}]^\intercal \\ H_2 = [X_{CS} \quad Y_{CS}]^\intercal \end{cases} .$$

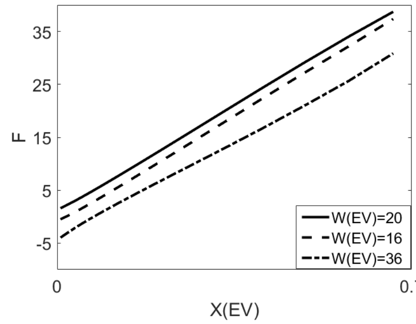
The HJB equation obtains the optimal subsidies S_{EV} for the OEM and the optimal subsidy S_{CS} for the CS installer that also maximizes their own utility in equation (4.14) along with the control variables. In conclusion, the derived EV subsidy policy resolves the "cheating compensation" problem amid the current EV subsidy policy.

We use the following boundary conditions in the government's continuation value to solve the HJB equation:

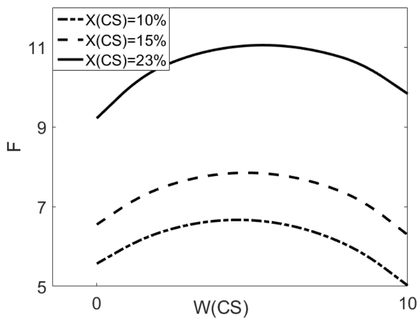
These boundary conditions assure that:



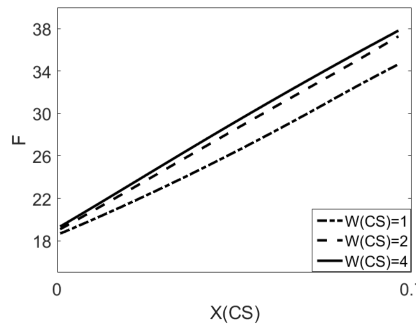
(a) Functional relationship between F and W_{EV}



(b) Functional relationship between F and X_{EV}



(c) Functional relationship between F and W_{CS}



(d) Functional relationship between F and X_{CS}

1. When the penetration rate of X_{EV} and X_{CS} hits 1, and the OEM's continuation value W_{EV} and W_{CS} hit W^*_{EV} or W^*_{CS}, the government's continuation value reaches the maximum.

2. When the penetration rate of X_{EV} and X_{CS} hits 0, the government agency cannot realize its Aim of Promoting EVs and CSs. Hence, it's continuation value reaches the minimize.

3. When the continuation values W_{EV} or W_{CS} hit $\underline{W}$, the government's continuation value at its lowest value.

4.4 Result

In this section, we develop a new EV-CS subsidy policy using the dynamic game approach developed above. The computed optimal subsidy policy has a complex form over the planning horizon, which is difficult to implement in real-time. We seek simple but efficient alternative EV subsidy policies in what follows. The alternative policy is a two-stage linear approximation to the optimal policy, which means the government either phases in or out during a certain period of time. It is almost as efficient as the optimal policy and the cumulative social benefit collected from the suggested EV subsidy policy is substantial compared with the Dual-Credit Policy. We conduct several sensitivity analyses in China's EV market to check how the policy works with market changes.

The following assumptions about data aggregation are made through out the analysis:

1. We compute the cash grant per unit credit as in the Dual-Credit Policy. Different car models with different fuel consumption and mileage range will receive different credit.

2. The total environmental pollution costs are calculated from the average EV fleet statistics in 2019 [68].

3. The parameters in Table 4.4 are from the fitted aggregated nationwide market dynamics in Table 4.2 and other relevant work [92, 122, 123].

Table 4.4: Values of parameters defined in Table 4.1 for numerical experiments

Parameters	N	M_{EV}	M_{CS}	ϕ_{Ae}	ϕ_{Te}	ϕ_{Af}
Values	2.27	80	30	2	6	8
Parameters	ϕ_{Tf}	γ	h	ω	β	C_{EV}
Values	6	1.2	4.5	3	0.1	2
Parameters	C_{ICE}	P_{EV}	P_{ICE}	P_{CS}	r_1	r_2
Values	8	-1	2	2	3	2

4.4.1 Optimal Subsidies for OEM and CS Installer

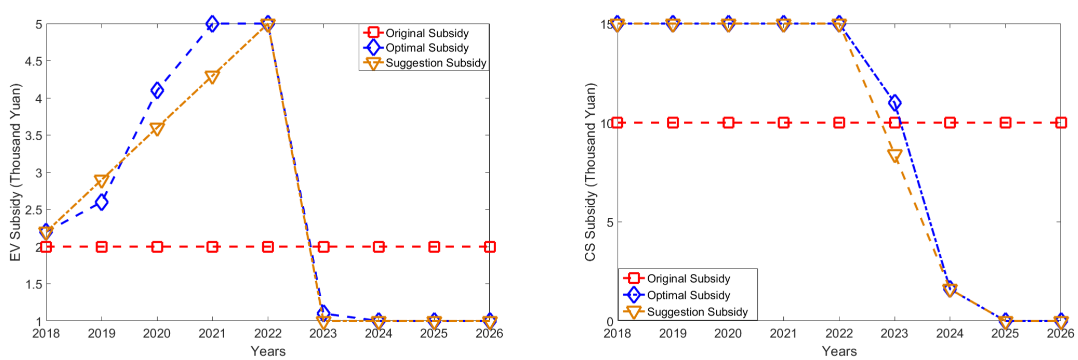
With EV and CS market data, we first compute the optimal EV-CS subsidies from 2019 to 2026 for a benchmark. The optimal subsidies per EV or per CS are shown in Figure 4.7. In response, the OEM and CS installer's optimal controls of innovation coefficient a_1 and a_2 are shown in Figure 4.10, respectively. We draw the following observations:

1. The EV credit score first increases monotonically to its peak from 2019 to 2022, and then declines to a constant low level after 2023.

2. The CS subsidy remains a high constant value until 2022 due to the shortage of charging infrastructure, and then phases out after 2024.

3. OEM reacts concurrently to the increase or decrease of EV subsidies, i.e., EV market's growth is proportional to the provided subsidies.

4. CS installer's response is inconsistent with maximum amount of subsidies offered in 2019-2023. Due to the constraint on the target EV-CS ratio, expanding the charging infrastructure will induce high costs. CS installer prefers to delay the initial investment and leverages the correlated EV and CS market to achieve the set goal in 2025.



(a) Dual-Credit Policy Score (b) Charging Station Subsidy

Figure 4.7: Comparison of the optimal EV subsidy policy and the Dual-Credit EV subsidy policy

The structure of the optimal policy has interesting implications which are not discovered in prior work. To achieve a reasonable EV-CS ratio as in Figure 4.6, the government should accelerate the sales of EVs at the early stage. On the other hand, the phase-out of existing EV subsidy policy will potentially enlarge the gap between the current EV-CS ratio (∼3.6) and the target value (∼1.5). Our model can coordinate the interest of OEMs and CS installers more competently.

4.4.2 Simple-to-Implement EV-CS Subsidies

The optimal subsidy policy computed above, although maximizing the government's total payoff, is hard to interpret and implement in practice. Observing the increasing and decreasing trends of S^*_{EV} and S^*_{CS}, we propose a simple piecewise linear approximation of the the optimal policy (termed as "suggested policy"). To show the efficiency of the proposed alternative policy, we conduct numerical experiments on the following policies:

1. Case 1 (Constant Dual-Credit Policy): The government keeps using the existing Dual-Credit Policy until 2026 (The EV subsidy per credit score is $S_{EV} = 0.2$ and the CS subsidy is $S_{CS} = 1$).

2. Case 2 (Suggested EV-CS Subsidy Policy): The piecewise linear function is a proxy to the optimal subsidy above[1].

3. Case 3 (Optimal EV-CS Subsidy Policy): Implementing the optimal policy computed in §4.1.

4. Case 4 (Price-Rebate Policy): The government extends the phase-out of the EV subsidy and continues the increasing CS incentives (shown in Figure 3.1).

[1] The approximate subsidy policy follows:

$$S_{EV} = \begin{cases} 0.7t - 1410.4 & 2018 \le t < 2023 \\ 1 & t \ge 2023 \end{cases}, \qquad S_{CS} = \begin{cases} 15 & t < 2022 \\ -6.6t + 13360.2 & 2022 \le t \le 2024 \\ 0 & t > 2024) \end{cases}.$$

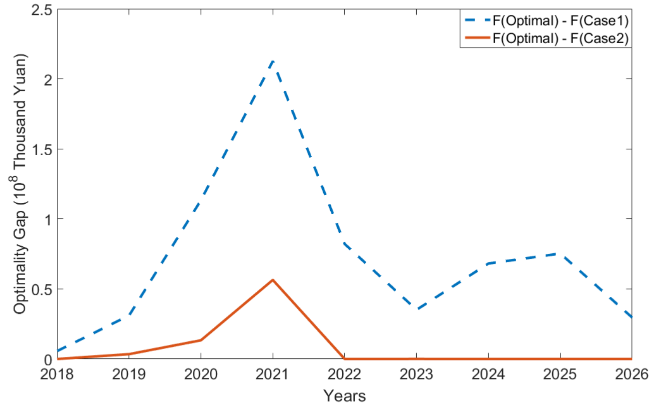
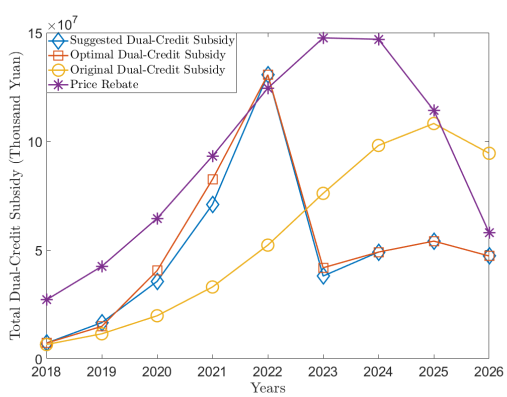
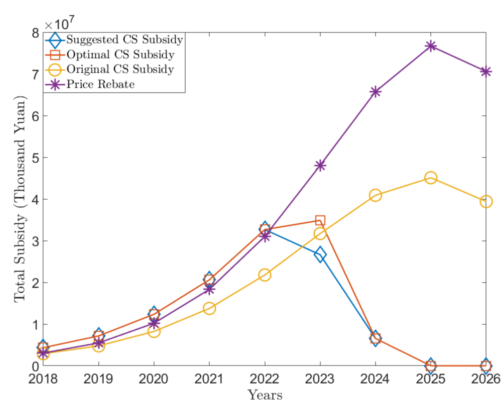


Figure 4.8: Difference between the optimal EV-CS subsidy and Case 1 / Case 2 policies

Figure 4.8 shows how the government's loss of cumulative payoff changes compared with the optimal subsidy policy (termed as the "optimality gap"). The difference of government's continuation value F using the Case 1 policy and that under the optimal policy reaches the apex in 2021. While it is much smaller under the suggested easy-to-implement policy in Case 4. As the constant credit disregards the unmet demand in the early adoption of EVs, the optimality gap in Case 1 climbs quickly. This comparison implies the misread of governmental incentives worldwide – the phase-out of EV subsidies has a far-reaching negative impact on the future EV market.



(a) Total EV Subsidy



(b) Total CS Subsidy

Figure 4.9: Comparison of cumulative EV and CS subsidies in Case 1 - Case 4

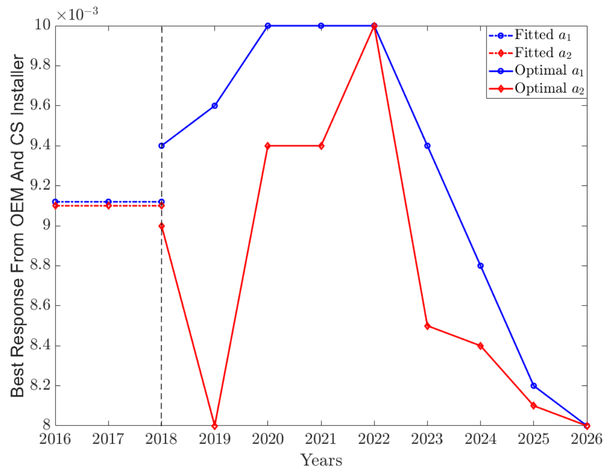


Figure 4.10: OEM and CS installers' best response

The government usually faces a budget constraint in the EV-CS incentive problem. A common trade-off is between the effectiveness of the policy, measuring by the proportion of the actualized EV and CS market share, and the annually spent budget. Figure 4.9 shows that, to reach the same predicted EV and CS market size from 2019 to 2026, our suggested policy requires the government to offer significantly less grant. In other words, the performance of EV subsidy is not a matter of how much but a matter of when – by following the derived timing of policy, the government can save budget while achieving the same goal over the planning horizon. The key issue is making a pro-active policy that subsidizes the early adoption of EVs and prepares charging infrastructure before the year of 2022. Despite the case study of China's EV market, it is a general conclusion applied to other EV markets worldwide.

4.4.3 Interactions between EV Market and CS Market

As the growth of EV market and the expansion of CS infrastructure are closely related, we now investigate how the strength of this interaction affects the optimal subsidy policy. This work use the interaction factor ζ between two market penetration processes X_{EV} and X_{CS} as an aggregate parameter for a variety of factors such as customers' choice [114] and the spatial proximity to the charging infrastructure. A greater value represents stronger synergy between these two markets. The above numerical experiments set $\zeta = 3$ from the fitted market data. We test the scenarios that the interaction becomes weaker or stronger, or even the opposite, with $\zeta = -30, -6, -3, 0, 3, 6, 30$ respectively. The impact of interaction strength on the EV subsidies is shown in the Table 4.5.

Table 4.5: Impact of EV and CS market interactions on optimal subsidy policies

	Dual-Credit Policy Score /CS Subsidy(Thousand Yuan)								
ζ	2018	2019	2020	2021	2022	2023	2024	2025	2026
30	1.6/−	1.7/−	2.6/+	+/+	+/+	1.8/+	−/4.3	−/−	−/−
−6	1.9/−	2.5/+	3.7/+	+/+	+/+	1.3/13	−/2.6	−/−	−/−
−3	1.9/+	2.6/+	3.6/+	+/+	+/+	1.2/11.8	−/2	−/−	−/−
0	2/+	2.5/+	4/+	+/+	+/+	1.2/11	−/1.8	−/−	−/−
3	2.2/+	2.6/+	4.1/+	+/+	+/+	1.1/11	−/1.6	−/−	−/−
6	2.3/+	3.3/+	4.4/+	+/+	+/+	1/10.3	−/1.3	−/−	−/−
30	2.5/+	3.3/+	4.4/+	+/+	+/+	1/7	−/−	−/−	−/−

+ : Subsidies for EV or CS reach the upper bound.
− : Subsidies for EV or CS reach the lower bound.

Since the expansion of CS lags behind the growth of EV market in China, the impact of interaction has has different implications for these two markets:

1. EV: From 2018 to 2022, the optimal Dual-Credit score for EVs will increase with the correlator ζ. During the period of 2022-2024, as the optimal amount of subsidy per EV declines, ζ has an inverse impact. It is because the overheat of EV will otherwise enlarge the EV-CS ratio.

2. CS: From 2018 to 2023, when the correlator ζ is negative and small, the optimal CS subsidies will increase from 0 to the maximum and then remain a constant thereafter. From 2023 to 2025, the optimal CS subsidy is negatively correlated with ζ. The government eliminates the subsidy for CS after 2025.

In summary, the government can intervene the interaction between EV and CS market in many ways. For example, improving the network design of charging infrastructure so that EV becomes a more attractive product. Nevertheless, enhancing the correlation does not always mean the drop of subsidies.

4.4.4 Sensitivity Analysis of Optimal Subsidy Policies

The computed EV and CS subsidy policies may be sensitive to the input EV and CS market data. Among the high dimensional input data, we choose five parameters that are most important in the Dual-Credit policy. The experiment below computes the relative impact of these parameters on the government's cumulative payoff over the planning horizon. Positive impact means that increasing the parameter will enhance the government's total payoff.

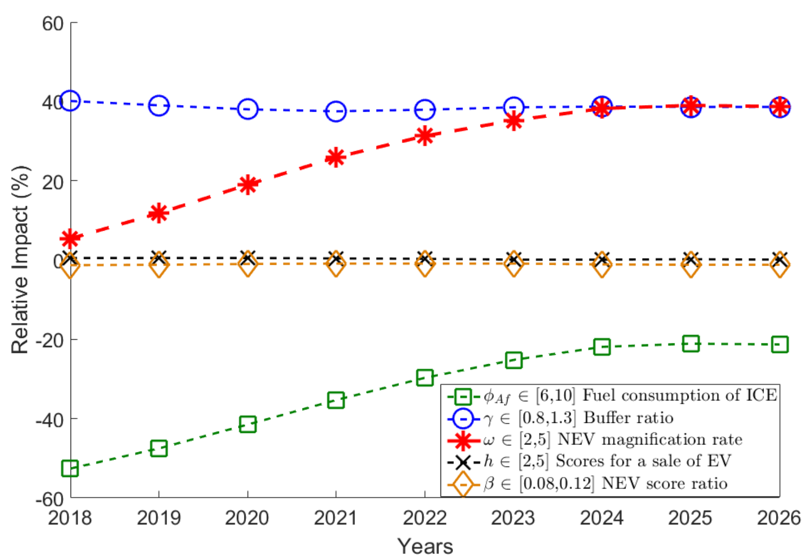


Figure 4.11: Sensitivity analysis of optimal subsidies

Figure 4.11 shows that ϕ_{Af}, γ and ω have a great impact on the government's continuation value F. The definition of these parameters can be found in Table 4.1. ϕ_{Af} and β have a negative correlation with the government's objective while γ, ω and h have a positive correlation. The impact of h and β can be ignored. We explain the impact of ϕ_{Af}, γ and ω as follows:

1. ϕ_{Af}: The relative impact of ϕ_{Af} on the government's objective is decreasing over the planning horizon. This is because the proportion of ICEs in sales of automobiles decreases. ϕ_{Af} has the greatest impact on the government's objective value in 2018-2020, which suggests the government to allocate more budget to promote the improvement of fuel consumption.

2. γ: The relative impact of buffer ratio on the government's objective value is about 40% over the planning horizon. As the most influential parameter from 2021 to 2024, the government can increase the value of γ by incentivizing OEMs to deploy more advanced technology to enhance the environmental benefit of EVs.

3. ω: The relative impact of ω increases over the years. It represents that the NEV's weight the CAFE score becomes the most influential parameter after 2025. The government can directly control ω in the Dual-Credit Policy to promote the adoption of EVs more effectively after 2025.

Although the impact of h and β on the government's objective is small, they are critical for the Dual-Credit Policy. For example, h determines the OEM's effort in improving EVs' mileage per charge, which is an important factor in customers' purchasing decision. More customers' behavior research is needed in the subsidy policy.

4.5 Policy Implication

We now summarize the key policy implications. First, the government should integrate the subsidies for EVs and charging infrastructure as a joint decision. The range anxiety is one of the significant barriers to large-scale adoption of EVs [35]. Since the expansion of the charging infrastructure lags behind the growth of the EV market, this work suggests boosting the uptake of CS in the early stage and phasing out both financial subsidies afterward. This asynchronous EV-CS subsidy policy leverages the network effect theory [102] to reduce the governmental spending on the adoption of EVs. These results outperform the existing strategies in China's EV market case study.

Second, the government should encourage the OEM and the infrastructure installer to share the subsidy information and production plans. This information-sharing is critical for accelerating the buildup of charging infrastructure for EVs. China's separate incentive policies for EVs and their charging infrastructure caused the difficulty of coordinating two parties' investments. The mistrust in the other party's decision has induced huge market frictions. This work's equilibrium-based approach will meet the EV-CS ratio standards by 2025 in a noncooperative way, i.e., their revenue-oriented strategies will automatically serve for the best of the EV adoption process.

Finally, the government should facilitate the sustainable growth of the EV market with proactive incentive policies. Many unintended consequences of previous EV subsidy policies resulted from using static analyses. Those analyses suggested using a fixed amount of subsidy that have yielded substantial market inefficiency. For example, OEMs overproduced EVs due to the bull-whip effect, i.e., producing more EVs to meet the new demand than needed. This work shows the benefits of implementing a mixture of dynamic EV-CS subsidy policies under uncertain market conditions.

4.6 Conclusion

This chapter develops a new multi-agent dynamic game approach to design EV subsidy policy. The optimal EV subsidy policy increases the synergy between the EV market growth and charging infrastructure development. It thus significantly enhances the effectiveness of governmental incentives amid the phase-out of subsidies worldwide. China's EV market case study validates the long-term value of using a proactive EV-CS subsidy policy. We recommend an alternative easy-to-implement subsidy policy, which gains more social benefit than the currently implemented Dual-Credit Policy in China. Besides, the sensitivity analysis uncovers three important input parameters that affect the government's payoff over the planning horizon, either positively or negatively.

We do not intend to conclude that the suggested EV subsidy policy is superior to other options. Our goal is to outline a uniform game-theoretic scheme to test and compare different policies. Interesting future research directions include, among others: First, extending the aggregate analysis to more specific policies considering more factors such as car type and fuel economy; Second, exploring regional EV subsidy policies because the EVs' level of impact on emissions varies across China [65]; Third, including more decision-makers into dynamic games such as customers. The last task requires to develop a new fundamental approach to solve the optimal subsidies because adding customers into games violate the multi-agent structure in this work.

CHAPTER V

Conclusion

This thesis studies incentive contracts in multi-agent systems. We proved the existence and uniqueness conditions of the model in a general setting. Incentive contracts are most suitable for designing pro-active policies for complex organizations with mutual or conflicting interests. Compared with static incentive design, dynamic contracts are in favor of the principal because of following two new features: First, characterizing conditions for termination (principal terminates the incentives) or retirement (agent withdraws from the contracts); Second, exploiting the uncertainty of future profit through the front-loading compensation as agents possessing memory tend to consume more in advance.

This thesis also demonstrated the capability of incentive contract theory with two applications in emerging transportation systems. Governments around the world have been underwriting EVs for a decade, and AV is an emerging technology that promises to benefit the society even more. The subsidy is one of the most significant, if not the only, regulatory techniques to help these markets to comply with the transportation electrification and (potential) automation mandate. With the phasing out of EV subsidies and uncertain AV market in the future, it is necessary to rethink the best practice of transportation policy under the umbrella of this thesis.

To close this doctoral thesis, I would like to mention some promising while challenging research avenues. Two significant extensions of this dissertations are information explosion and organizational structure.

First, existing incentive contracts literature solves a planning problem. The principal faced with asymmetric information (i.e., the agent's action is unobservable) is assumed to have access to both the output process and the agent's utility functions. In a single-agent setting, prior works studied the case that principal and agent share a common prior on the project's unknown profitability (i.e., the output process) and applies a Bayesian learning approach to update their belief on the profitability [29, 57]. Learning splits the optimal incentive contracts to a two-phase form with the agent's information rent. Nevertheless, Demarzo and Sannikov [29] eliminates the hidden utility function assumption by contracting with a risk-neutral agent, which limits the direct extension to this thesis's scope of multi-agent systems.

Second, the organization can be expanded vertically to a hierarchical structure. Hierarchies appear in organizations with a branching structure such that direct links exist only between the immediate superiors and subordinates. Almost every system of organization applied to the world is arranged in hierarchies as they are one of such forms that agents on the higher level have a greater power of authority than those on the lower level. In contrast, agents on the same level have the same relative amount of authority. To the best of my knowledge, Zhou [134] is the only related work that studied a dynamic signaling game in which each subordinate observes only the superior's signals before determining its action. It found that the welfare-optimal hierarchical structure is a chain structure. Hence, it is interesting to study how incentive contract theory can be applied to hierarchies.

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