Aggregate Implications of Labor-Market Composition

by

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ABSTRACT

This dissertation comprises three essays on the movements of workers into and out of employment and unemployment—in other words, the composition of the labor market. The first provides an overview. It describes the US economy's ability to create new hires from unemployment and vacancies and some implications for labor–macro models. The second considers fishery management plans in a two-sector, random search environment, where one sector harvests fish. The optimal composition of jobs is described. The third investigates how labor-market composition affects the cyclical behavior of wages informed by random search models and data from the National Longitudinal Survey of Youth 1979, the Current Population Survey, and the Current Employment Statistics program.

CHAPTER 1

Estimates of Matching Technology under Constant Returns to Scale with Implications for Fundamental Surplus

I estimate two constant-returns-to-scale matching technologies that summarize how an economy creates new matches from vacancies and unemployment. Using monthly data on matches and vacancies from the Job Openings and Labor Turnover Survey and data on unemployment from the Current Population Survey that is corrected for time aggregation, both the log-linear Cobb–Douglas and nonlinear paramterizations accurately account for the variation over time in new matches. The nonlinear paramterization has the added benefit of constraining job-finding and job-filling probabilities to be between 0 and 1. I show how the nonlinear parameterization affects the elasticity of matches with respect to unemployment and show how this elasticity affects interpretations of fundamental surplus in labor–macro models that feature random search. The constrained, nonlinear parameterization increases the cyclicality of job finding and therefore unemployment, a desirable feature for labor–macro models.

1.1 Introduction

Each month workers actively search for jobs and firms actively recruit workers. Workers go on job interviews, contact potential employers, submit resumes to employers and job websites, use services like those at university employment centers, contact job recruiters, and seek assistance from friends, relatives, and other members of the social networks. And to recruit workers, advertise in newspapers, on television, and on radio; post Internet notices; post "help wanted" signs; network with colleagues or make word-of-mouth announcements, including on platforms like Twitter; accept applications; interview candidates; contact employment agencies; and solicit employees at job fairs. Nevertheless, despite these efforts and despite workers' availability for jobs that are ready to start, workers cannot find jobs and firms cannot recruit workers. At almost any time some people are unemployed and some vacancies are unfilled.

"Workhorse" macroeconomic models capture this essential dynamic with a search friction that prevents firms from immediately hiring available works.[1] The search friction is modeled as a matching technology, which characterizes the cyclical behavior of the labor market in terms of unemployment and vacancies. But despite the centrality of matching technology to labor–macro models, nearly all empirical evidence on matching uses a Cobb–Douglas parameterization. And it is unclear how alternative parameterizes affect labor–macro modeling.

Here I report the results of estimating an alternative, nonlinear parameterization calibrated in a model by den Haan, Ramey, and Watson (2000), using data from the Current Population Survey and Job Openings and Labor Turnover Survey. The alternative parameterization fits the data as well as the Cobb–Douglas parameterization. Additionally, the nonlinear parameterization can be specified to have job-finding and job-filling probabilities fall within 0 and 1.

To assess how the alternative parameterization affects labor–macro models, I report the impact on a key statistic: the elasticity of labor-market tightness with respect to productivity, where labor-market tightness refers to the ratio of vacancies to unemployed persons. Tightness captures labor demand (vacancies) relative to labor supply (unemployed persons). I show that the nonlinear parameterization that specifies job finding to fall within the unit interval increases the cyclicality of job-finding and therefore unemployment.

To estimate the two parameterizations I correct the data for time aggregation. Matches or new hires as measured by the Job Openings and Labor Turnover Survey are the total number of new hires made throughout the month. Attributing all new hires to a point in time biases job finding upward. The correction I make suggests the bias is large.

The usefulness of modeling unemployment with a matching friction is well established. Part of that usefulness is based on the robust empirical support for constant returns to scale. Doubling the number of unemployed persons and number of vacancies appears to double the number of matches. Empirical support is documented by Blanchard and Diamond (1989), Layard, Nickell, and Jackman (1991), Blanchard et al. (1990), Bleakley and Fuhrer (1997), Bleakley, Ferris, and Fuhrer (1999), and Petrongolo and Pissarides (2001).[2] Recent estimates that support the Cobb–Douglas parameterization in-

[1] The term "workhorse" is used, for example, in Shimer (2005) and Ljungqvist and Sargent (2017).

[2] See Elsby, Michaels, and Ratner (2015) for a recent review.

clude Borowczyk-Martins, Jolivet, and Postel-Vinay (2013), who account for the idea that shifts in the matching technology may induce workers to strategically vary search, and Lange and Papageorgiou (2020), who account for firms' search intensity over the business cycle in addition to workers'. Both Borowczyk-Martins, Jolivet, and Postel-Vinay (2013) and Lange and Papageorgiou (2020), however, do not correct for time aggregation.

This empirical evidence posits that the log of matches is linearly related to a linear combination of the log of vacancies and the log of unemployment. The Cobb–Douglas parameterization can lead to trouble for labor–macro models. A simple example illustrates why.

I let $M(u, v)$ denote the number of matches as a function of the level of unemployment, u, and vacancies, v. The constant-returns-to-scalle Cobb–Douglas parameterization specifies that

$$M(u_t, v_t) = Au_t^{1-\alpha} v_t^{\alpha} \epsilon_t,$$

where A models matching efficiency (and is often associated with long-run, exogenous shifts in the Beveridge curve) and ε_t is an iid random disturbance that shifts matching efficiency. The implied job-finding probability, $M(u_t, v_t)/u_t$, is $A(v_t/u_t)^{\alpha}$. Taking $A = 1$, the ratio of vacancies to unemployment to be .25, and $\alpha = .5$ implies that the the probability a worker finds a job is $(1/4)^{.5} = .5$. These are realistic numbers: $\alpha = .5$ is close to the estimate I report using data adjusted for time aggregation, the ratio of vacancies to unemployment hovered around .25 between 2009 and 2011, and it is not unreasonable to for a worker to face a fifty percent chance of finding a job within a month.[3]

Unfortunately these parameters imply an unreasonable job-filling probability. The probability a vacancy is filled, $M(u_t, v_t)/v_t = A(u_t/v_t)^{1-\alpha}$, is $(4)^{.5} = 2$. This simple example motivates estimation of the nonlinear technology.[4]

In this paper I provide nonlinear least-squares estimates of parameterizations of M and compare them to Cobb–Douglas estimates. One parameterization specifies that the job-finding and job-filling fall between 0 and 1. To my knowledge, this is the first empirical estimate of this parameterization. These estimates are important because they have implications for understanding "fundamental surplus," which Ljungqvist and Sargent (2017)

[3]Figures 1.5 and 1.1 support these empirical claims.

[4]Labor–macro models could use A to match a steady-state job-finding probability and therefore a steady-state unemployment rate. For example, the steady state of the model in Gertler and Trigari (2009) implies a job-filling rate of above 1. Gertler and Trigari (2009) show how staggered wage setting, instead of period-by-period wage renegotiation under Nash bargaining, affects unemployment dynamics, which seems really important given what is known about how often wages are reset in the data (Barattieri, Basu, and Gottschalk, 2014). Whether a steady-state job-filling rate above 1 matters for dynamics is an open question. Ljungqvist and Sargent (2017) suggest getting around this by calibrating a model to a *daily* frequency.

use to interpret many variants of labor–macro models' ability to match the elasticity of tightness with respect to productivity.

The elasticity of tightness with respect to productivity, $\eta_{\theta,y}$, is a key statistic. It is a fundamentals-rooted statistic for how well labor–macro models match the cyclical behavior of labor markets. Ljungqvist and Sargent (2017) show that, without the influence of stochastic fluctuations in productivity in a model with exogenous separations, this elasticity can be decomposed into two factors

$$\eta_{\theta,y} = \Upsilon \frac{y}{y-c} < \frac{1}{\eta_{M,u}} \frac{y}{y-c}. \tag{1.1}$$

The second factor, $y/(y-c)$, or the inverse of the fundamental surplus fraction, is the focus of Ljungqvist and Sargent (2017). I establish that the first factor, Υ, is bounded above by $1/\eta_{M,u}$, the inverse of elasticity of matching with respect to unemployment. The inequality in (1.1) is based on a minor extension of Ljungqvist and Sargent's (2017) framework.

In the Cobb–Douglas framework, $\eta_{M,u}$, is constant, whereas the elasticity depends on tightness in the nonlinear framework. Evaluated at the average level of tightness in the US economy from December 2000 through December 2019, however, the bound is nearly twice as large using the nonlinear framework compared to the Cobb–Douglas framework.

While it is possible to evaluate $\eta_{\theta,y}$ directly, providing a bound avoids the murky issue of choosing parameters. This is more of a conceptual exercise and a first step of evaluating the consequences of using different parameterizations of matching technologies in labor–macro models. The bound in (1.1) is nearly twice as large using the nonlinear parameterization. As I show in section 1.4.1, empirically $\eta_{\theta,y}$ is very large, so the larger bound should be welcome because it puts a little less pressure on the ability of fundamental surplus to explain everything about the cyclical behavior of labor markets, an interpretation pushed in Ljungqvist and Sargent (2017).

The remainder of the paper is organized as follows. Section 1.2 describes the data used to estimate matching technology under constant returns to scale, including the bias correction associated with time aggregation. Section 1.3 reports estimates of of different parameterizations of the matching technology. These estimates are discussed in the context of canonical search model in section 1.4.2 and how fundamental parameters relate to data described in section 1.4.1.[5] Section 1.5 concludes.

[5]The canonical model is based on Pissarides (1985) using Ljungqvist and Sargent's (2017) framework.

1.2 Data

This section uses data from the Job Openings and Labor Turnover Survey and the Current Population Survey to estimate two parameterizations of constant-returns-to-scale matching functions. Before turning to the estimation, I describe the matching technologies and the data requirements.

1.2.1 Cobb–Douglas

Time is discrete and indexed by t. Data are available at a monthly frequency. Let M_t denote the number of matching occurring in month t. Let u_t denote the number of unemployed workers in month t. Let v_t denote the number of vacancies posted in month t. The Cobb–Douglas matching technology is parameterized as

$$M_t = Au_t^{1-\alpha}v_t^{\alpha}\epsilon_t, \tag{1.2}$$

where the time-invariant parameter A describes the efficiency with which the economy produces matches and ϵ_t stochastically shifts matching efficiency. This matching technology implies

$$F_t = \frac{M_t}{u_t} = \frac{Au_t^{1-\alpha}v^{\alpha}}{u_t} = Au_t^{-\alpha}v_t^{\alpha} = A\theta_t^{\alpha}\epsilon_t, \tag{1.3}$$

where $\theta = v_t/u_t$ defines tightness. This relationship can be written

$$\log F_t = a + \alpha \log \theta_t + \varepsilon_t, \tag{1.4}$$

which says that the log of the job-finding probability is linearly related to the log of the ratio of vacancies to unemployed. Equation (1.4) shows how powerful the Cobb–Douglas framework is, but it is not the only framework.

1.2.2 A Nonlinear Matching Technology

A nonlinear matching technology was suggested in an impressive, early calibration by den Haan, Ramey, and Watson (2000). This parameterization has subsequently been used in calibrated models (Ravenna and Walsh, 2012, 2008; Walsh, 2003). This matching technology is parameterized as

$$M_t = \mathcal{A}\frac{u_t v_t}{\left(u_t^{\gamma} + v_t^{\gamma}\right)^{1/\gamma}}\epsilon_t.$$

This matching technology exhibits constant returns to scale in u_t and v_t and it is increasing in both its arguments.

Under this parameterization the job-finding rate is

$$\begin{aligned} F_t &= \mathrm{A}\frac{\theta_t}{(1+\theta^\gamma)^{1/\gamma}}\epsilon_t \\ \therefore \log F_t &= \mathrm{a} + \log\theta_t - \frac{1}{\gamma}\log\left(1+\theta_t^\gamma\right) + \varepsilon_t. \end{aligned} \tag{1.5}$$

The series for F_t and θ_t can be constructed using data from the Job Openings and Labor Turnover Survey program and the Current Population Survey.

The JOLTS program collects data from approximately 16,000 business establishments to provide measures of labor demand as measured by the number of job openings or vacancies a firm is actively looking to fill. Data on the number of unemployed workers come from the Current Population Survey. The CPS asks about 60,000 households every month about information on the labor-force status of people age 16 and older, including whether a person was employed, whether they were available to work, and whether they actively searched for a job to determine whether a person was unemployed. The ratio of total non-farm vacancies, v_t, to the number of unemployed persons, u_t, defines tightness in period t.[6]

While the Bureau of Labor Statistics began tracking unemployment much earlier, the JOLTS program only began publishing data on vacancies in December 2000. The only widely available data on vacancies before this is the Conference Board's help-wanted advertising index. Even though it is possible that this index is related to the true number of vacancies, the mode of making vacancies has surely changed—for example, the shift from advertising vacancies in newspapers to advertising online—and I only consider the JOLTS data.[7]

Each month JOLTS also provides data on the number of matches, M_t. The number of matches refers to the number of new hires and includes all additions to the payroll during the month. In other words, matches refers to the *cumulative* number of new hires.[8] Attributing the cumulative number of new hires to created jobs at the beginning of the month will therefore bias the rate of job finding upward. The next section corrects for this type of time aggregation using a method similar to the time-aggregation adjustment Shimer (2012) uses to adjust short-term unemployment.

As an alternative to short-term unemployment and CPS transitions, the JOLTS pro-

[6] All series are seasonally adjusted by either the CPS or the JOLTS program.

[7] See Abraham (1987) for more information on the help-wanted index.

[8] The BLS provides a definition at https://www.bls.gov/jlt/jltdef.htm#3.

gram publishes data on new hires that can be used to measure job finding. While this new, time-aggregation-adjusted measure of job finding should be a welcome alternative, the measure nevertheless relies on the assumptions that the labor force is constant within a month and workers transition between employment and unemployment. I leave a comparison to measures constructed using CPS data for future research.

1.2.3 A New Measure of Job Finding Using JOLTS data

In this section I derive a measure of job finding that corrects for the fact that time is continuous but data are available only at discrete dates.

Workers are either employed or unemployed, transitioning between labor-market states. Time is continuous, while data are available only at discrete dates. Following Shimer (2012), refer to the interval $[t, t+1)$, for $t \in \{0, 1, 2, \ldots\}$, as "period t." Within the interval $[t, t+1)$ workers neither exit or enter the labor force.

I define the following quantities:

- $F_t \in [0, 1]$ is the job-finding probability in period t,
- $S_t \in [0, 1]$ is the separation probability in period t,
- $f_t \equiv -\log(1 - F_t) \geq 0$ is the arrival rate of the Poisson process that changes a worker's state from unemployment to employment, and
- $s_t \equiv -\log(1 - S_t) \geq 0$ is the arrival rate of the Poisson process that changes a worker's state from employment to unemployment.

To be clear, F_t and S_t are probabilities, while f_t and s_t are rates. F_t is the probability that a worker who begins period t unemployed finds a job during period t; S_t is the probability that a worker who begins period t employed loses a job during period t. I am interested in uncovering S_t and F_t.

For any $t \in \{0, 1, 2, \ldots\}$ I let $\tau \in [0, 1]$ be the elapsed time since the start of the period. I define the following stocks:

- $e_{t+\tau}$ is the number of employed workers at time $t + \tau$,
- $u_{t+\tau}$ is the number of unemployed workers at time $t + \tau$, and
- $e_t^h(\tau)$ is the number of workers who are employed at time $t+\tau$ but were unemployed at some time $t' \in [t, t + \tau)$; that is, the number of new hires or matches.

The number of new hires is zero at the start of the period:

$$e_t^h(0) = 0 \text{ for all } t.$$

The number of new hires at the end of the period is defined as

$$e_{t+1}^h \equiv e_t^h(1)$$

as the number of new hires at the end of the period, as measured by the JOLTS program.[9]

Employment within the period evolves according to

$$\dot{e}_{t+\tau} = f_t u_{t+\tau} - s_t e_{t+\tau} \tag{1.6}$$

$$\dot{e}_{t+\tau}^h = f_t u_{t+\tau} - s_t e_{t+\tau}^h. \tag{1.7}$$

Equation (1.6) states that the change in employment equals the workers who find jobs out of unemployment less the workers who separate from employment to unemployment. Equation (1.7) states that this same dynamic holds for new hires. Random search implies all workers separate at the same rate.[10]

Solving equation (1.7) for $f_t u_{t+h}$ and substituting this into equation (1.6) yields

$$\dot{e}_{t+\tau} = \dot{e}_{t+\tau}^h + s_t e_{t+\tau}^h - s_t e_{t+\tau} \text{ for } \tau \in [0, 1).$$

This is a linear first-order equation for $\dot{e}_{t+\tau}$ with $\tau \in [0, 1)$.

The generation solution is

$$e_{t+\tau} = \left[c + \int_0^\tau \left(\dot{e}_{t+z}^h + s_t e_{t+z}^h\right) e^{s_t z} dz\right] e^{-s_t \tau}. \tag{1.8}$$

The first integral on the right-hand size of the latter equation can be integrated by parts. Integrating by parts and substituting the result into (1.8) yields

$$e_{t+\tau} = c e^{-s_t \tau} + e_{t+\tau}^h.$$

[9] The series for new hires published by the JOLTS program measures all additions to the payroll during the month. Adding a worker to payroll requires a duration of employment beyond being hired and immediately fired. I therefore model new hires as workers employed at the end of the month who were not employed at the beginning of the month. This is a behavioral model of respondents. Alternatively, a behavioral model could specify that new hires refers to all new hires, including those workers to are hired and let go within the month.

[10] See footnote 9 for a description of an alternative model.

The determination of c comes from evaluating the latter at $\tau = 0$. Doing so yields $c = e_t$, as there are no new hires at the beginning of the period and $e_t^h(0) = 0$. Therefore

$$e_{t+\tau} = e_t e^{-s_t \tau} + e_{t+\tau}^h.$$

Evaluating the latter at $\tau = 1$ yields

$$e_{t+1} = e_t (1 - S_t) + e_{t+1}^h, \tag{1.9}$$

which indicates that the level of employment in the following survey period equals the employed who do not separate from their jobs plus new hires. Solving this expression for S_t yields

$$S_t = 1 - \frac{e_{t+1} - e_{t+1}^h}{e_t}. \tag{1.10}$$

To solve for the finding rate, which is the point of interest, solve equation (1.6) forward. Define $l_t = u_t + e_t$ as the labor force. The labor force is assumed to be constant during that period; that is, $u_{t+\tau} + e_{t+\tau} = l_t$ for all $\tau \in [0, 1)$. Using this assumption implies

$$\dot{e}_{t+\tau} = f_t l_t - (s_t + f_t) e_{t+\tau}.$$

This is a linear differential equation for $\dot{e}_{t+\tau}$ with constant coefficients. I solving this differential equation and evaluate the solution at $\tau = 0$ to solve for the constant of integration. The result evaluated at $\tau = 1$ is

$$e_{t+1} = \frac{\left(1 - e^{-s_t - f_t}\right) f_t}{(s_t + f_t)} l_t + e^{-(s_t + f_t)} e_t. \tag{1.11}$$

I am interested in uncovering F_t and S_t. The separation probability, S_t, is calculated using (1.10). The finding probability F_t is defined implicitly in equation (1.11). I solve equation 1.11 using a bisection method to uncover f_t and then compute $F_t = 1 - e^{-f_t}$. Details are provided in an appendix.

Figure 1.1 compares the job-finding probability corrected for time aggregation with the finding rate constructed as $\tilde{F}_t = M_t / u_t$. The correction is meaningful. The uncorrected series, for example, implies a job-finding probability above 1 in several months. While the features of data collection make this possible—for example, workers can hold multiple jobs—the uncorrected levels seem unreasonably high. The job-finding series corrected for time aggregation provides more reasonable levels.

Subsequent sections compare the results based on both the corrected and uncorrected

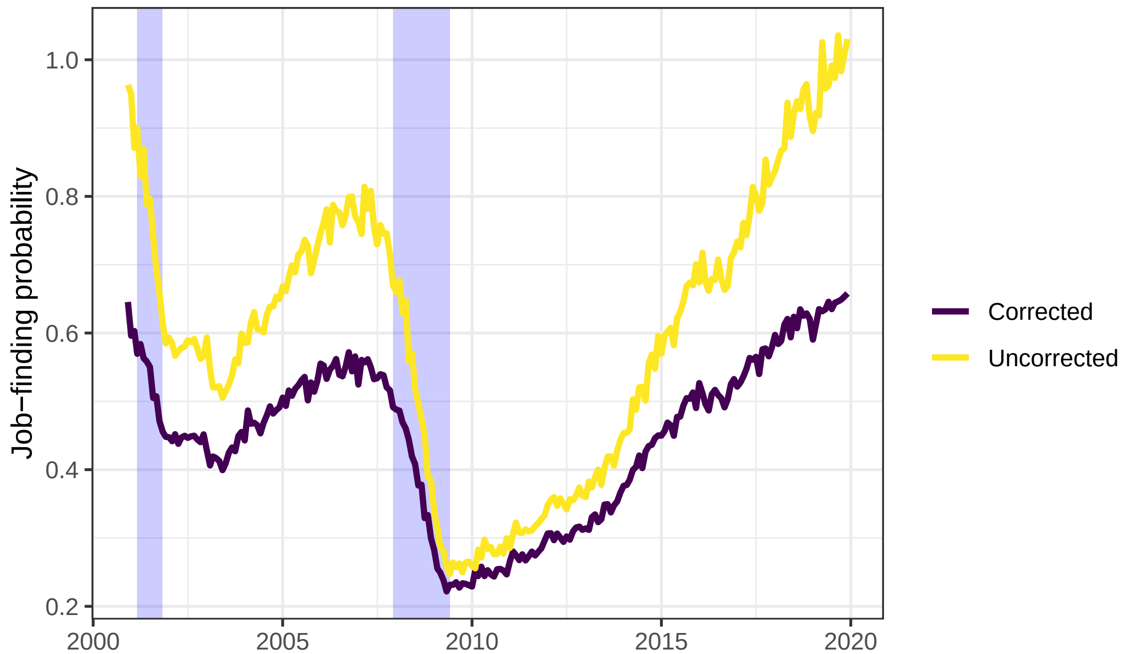


Figure 1.1: Comparison of uncorrected and time-aggregation corrected job-finding probabilities, 2000m12–2019m12.

series for job finding.

1.2.4 The Constructed Series

The relationship between finding and tightness is depicted in figure 1.2, which depicts a scatter plot of θ_t on the horizontal axis and $\log F_t$ on the vertical axis. The color of the points represents time, which provides an assessment of how the relationship shifted. The early part of the sample comprises the upper collection of points in dark blue. From those early observations, the economy transitions to the lower collection of points in light blue that compose the later sample. The shift through time is consistent with earlier shifts in the Beveridge curve identified by Bleakley and Fuhrer (1997) and Elsby, Michaels, and Ratner (2015).

Figure 1.3 classifies the points in figure 1.2 by recessions and data collected during the COVID-19 pandemic. The COVID-19 period, when the unemployment rate shot up to 14.7 percent, may have affected the March and April 2020 relationships, denoted by the yellow and green circles. I take these to be an uninformative outliers and drop all data beginning in 2020 from the subsequent statistical analysis. The other data can be understood in terms of before, during, and after the Great Recession.

Prior to the Great Recession, data are grouped into the pre–Great Recession and 2001 Recession periods. These periods are represented by dark or light purple and are nearly indistinguishable from on another. From the two early periods, however, the economy transitioned to the dates colored in blue and green, which are labeled Great Recession and Post Great Recession. The transition occurs during the Great Recession and depicted in blue. The Great Recession period comprises the collection of points that connects the two major collections of points. The Great Recession is associated with a shift in the efficiency with which workers are firms matched—a given level of vacancies per unemployed workers led to fewer matches after the Great Recession. I model this shift in matching efficiency as an exogenous shift in a in equation (1.4) and consider shifts in α during the Great Recession.

In summary, as the theory of matching technology suggests, there is a strong relationship between tightness and job finding. This relationship is analyzed statistically in the next section.

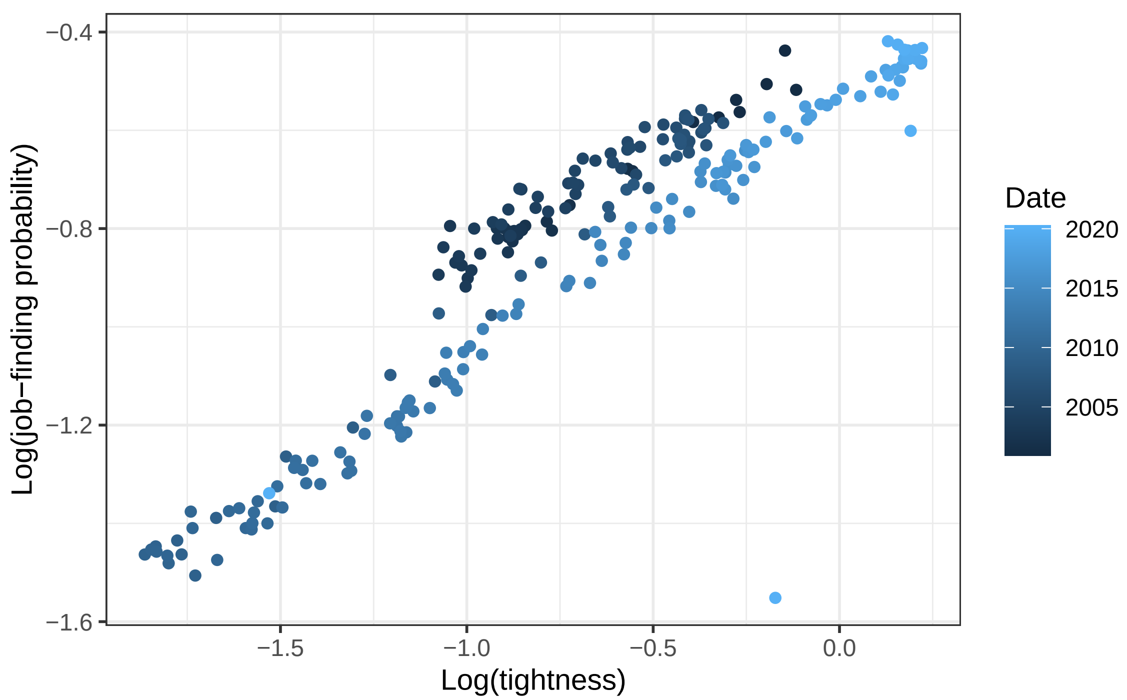


Figure 1.2: Scatter plot of job finding versus tightness, 2000m12–2020m4. The color of points denotes time.

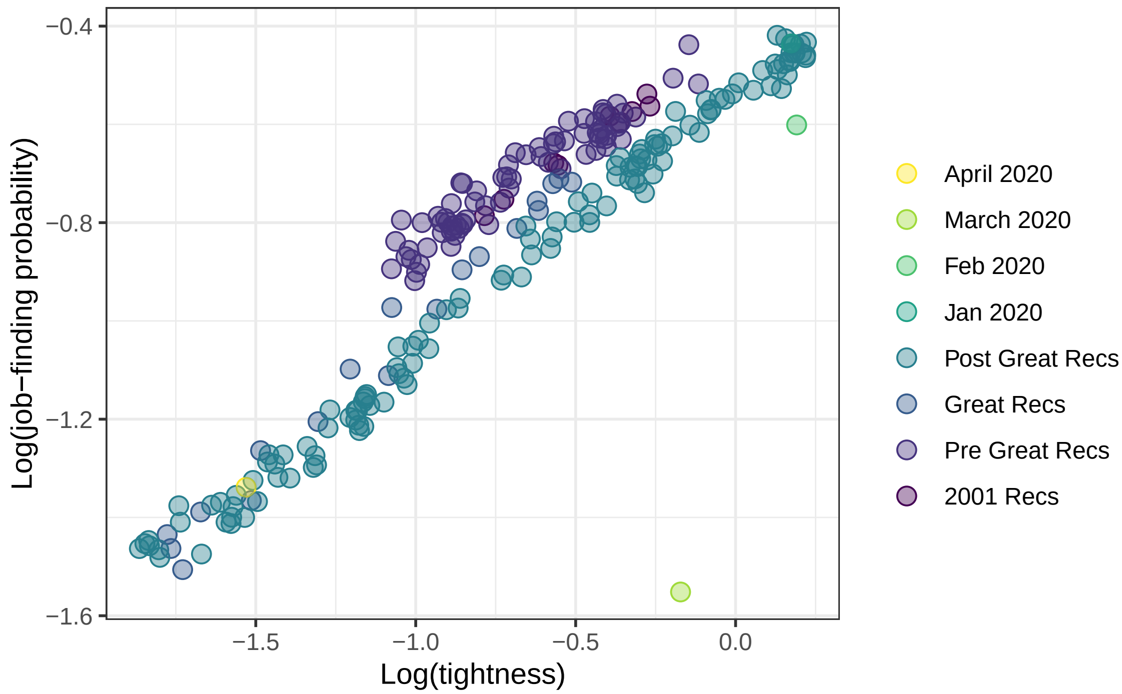


Figure 1.3: Scatter plot of job finding versus tightness, 2000m12–2020m4.

1.3 Estimation of Matching Technologies

This section reports the results of estimating the statistical relationships in (1.4) and (1.5) for both the time-aggregation-corrected and uncorrected series. I first report estimates based on the uncorrected series to compare results found in the literature.

Turning to the Cobb–Douglas model in (1.4), to make the model operational, I allow for the possibility that both the intercept and the slope shift after the Great Recession based on figure 1.3. I estimate statistical models with the general form

$$\log F_t = a + \alpha \log \theta_t + \beta_1 \text{trend}_t + \beta_2 \mathbb{1}\,(t > 2009\text{m}6) + \beta_3 \mathbb{1}\,(t > 2009\text{m}6) \times \theta_t + \varepsilon_t.$$

The sample includes data from 2000m12 through 2019m12. Table 1.1 reports the results for the job-finding series uncorrected for time aggregation.

The first column of table 1.1 pools the entire sample, ignoring the shifts depicted in figure 1.3. The coefficient on $\log \theta$ is .694. The estimate implies that a 1 percent increase in tightness predicts that job finding increases by .694 percent. Alternatively, looking at (1.13), the statistical model implies that the elasticity of matching with respect to vacancies is .694 and the elasticity of matching with respect to unemployment is .306.

This estimate is similar to existing estimates. For instance, using a similar specification and similar data through January 2012, Borowczyk-Martins, Jolivet, and Postel-Vinay (2013) estimate the elasticity of matching with respect to vacancies as .842. These numbers are consistent with those in Pissarides (1986), who uses UK data, and in the range of estimates reported in Petrongolo and Pissarides (2001).

Importantly, with an R^2 of .910, this simple matching function is able to explain over 90 percent of the variation over time in job finding. The matching function, in other words, accurately accounts for the movement over time in new hires. This holds for all estimated versions of the statistical model in (1.13) and is yet another statement of the power of the matching models to explain the cyclical behavior of the labor market.

The second column of table 1.1 investigates the deterioration of matching efficiency through time by adding a linear trend. The coefficient on the linear trend is negative and significant at the 1-percent level, suggesting that matching efficiency has decreased over the sample. The third column shows that the significant trend estimate in the second column is well approximated by a shift in matching efficiency that occurred after the Great Recession. The coefficient on the post–Great Recession indicator implies that matching efficiency fell over 20 percent after 2009m6. This number quantifies the fall in efficiency depicted in figure 1.3. The fourth column allows allows for a fall in efficiency after the Great Recession and a shift in the elasticity of matching with respect to vacancies and unemployment. Allowing for these shifts implies that matching efficiency fell 24 percent and the elasticity of matching with respect to vacancies increased. The elasticity of matching with respect to vacancies prior to the Great Recession in the specification of column 4 comes closer to the estimates in Borowczyk-Martins, Jolivet, and Postel-Vinay (2013).

Table 1.2 repeats this analysis for the job-finding series corrected for time-aggregation. The results are meaningfully different. The general time-series pattern is the same—there is a fall in matching efficiency—but the magnitude of that change is less. Columns (3) and (4) of table 1.2 imply matching efficiency fell by 15.8 and 19.1 percent.

But the main takeaway is that the elasticity of matching with respect to unemployment increased meaningfully. For example, using the corrected job-finding series and allowing for a shift in matching technology, the elasticity of matching with respect to vacancies is .536 and the elasticity of matching with respect to unemployment is .464. These estimates are more in line with those in Blanchard and Diamond (1989) who use US data. Importantly, the inverse of the elasticity of matching with respect to unemployment is significantly less. Here the bound is $1/\eta_{M,u} = 1/(1 - .536) = 2.155$; whereas for the uncorrected series, the bound is $1/(1 - .700) = 3.333$. These two values have implications for the bound in (1.1).

Turning to the estimation of the matching technology in (1.5), analogous to the sta-

Table 1.1: Cobb–Douglas matching technology

	Dependent variable:			
	Log(find)			
	(1)	(2)	(3)	(4)
Log(tight)	.694***	.771***	.700***	.738***
	(.015)	(.007)	(.007)	(.015)
Trend		−.002***		
		(.0001)		
Post GR			−.209***	−.243***
			(.007)	(.014)
Post GR X log(tight)				−.046***
				(.017)
Constant	−.081***	.169***	.038***	.066***
	(.013)	(.010)	(.007)	(.012)
Observations	229	229	229	229
R^2	.908	.982	.981	.982
Note:				*p<0.1; **p<0.05; ***p<0.01

Table 1.2: Cobb–Douglas matching technology using adjusted job finding

	Dependent variable:			
	Log(find)			
	(1)	(2)	(3)	(4)
Log(tight)	.531***	.591***	.536***	.573***
	(.012)	(.006)	(.006)	(.014)
Trend		−.001***		
		(.00005)		
Post GR			−.158***	−.191***
			(.007)	(.013)
Post GR X log(tight)				−.046***
				(.016)
Constant	−.464***	−.270***	−.374***	−.347***
	(.010)	(.009)	(.007)	(.011)
Observations	229	229	229	229
R^2	.902	.977	.972	.973
Note:				*p<0.1; **p<0.05; ***p<0.01

tistical model in (1.13), I consider a trend and shift in matching efficiency after the Great Recession. The statistical model is

$$\log F_t = \mathrm{a} + \log \theta_t - \frac{1}{\gamma} \log \left(1 + \Theta_t^{\gamma}\right) + \beta_1 \mathrm{trend}_t + \beta_2 \mathbb{1}\left(t > 2009\mathrm{m}6\right) + \varepsilon_t. \tag{1.12}$$

In general this model can be written $\log F_t = g\left(x_t, \beta\right) + \varepsilon_t$, where

$$x_t = \left(1, \mathrm{trend}_t, \mathbb{1}\left(t > 2009\mathrm{m}6\right), \theta_t, \log \theta_t\right)' \text{ and } \beta = \left(\mathrm{a}, \gamma, \beta_1, \beta_2\right)'.$$

I postulate that $\mathrm{E}\left[\varepsilon_t | x_t\right] = 0$ and $\mathrm{cov}\left[\varepsilon_t \varepsilon_s | x_t\right]$ is σ^2 if $s = t$ and 0 if $s \neq t$. I estimate the nonlinear semiparametric model in (1.12) with nonlinear least squares to minimize the sum of squared errors. I use a Gauss–Newton algorithm to estimate the parameter vector β. Again, the sample uses data from 2000m12 through 2019m12. Table 1.3 reports the results.

Table 1.3: Nonlinear matching technology

	Dependent variable:		
	Log(find)		
	(1)	(2)	(3)
γ	1.183***	1.466***	1.052***
	(.097)	(.076)	(.054)
Trend		−.001***	
		(.0001)	
Post GR			−.175***
			(.009)
Constant	.473***	.527***	.632***
	(.041)	(.020)	(.031)
Observations	229	229	229
Note:	*p<0.1; **p<0.05; ***p<0.01		

The first column of table 1.3 is analogous to the first column of table 1.1, ignoring the shifts in matching efficiency. The second column estimates a trend. The coefficient on the trend variable indicates that matching efficiency fell, on average, by .1 percent each month throughout the sample. The third column, which is analogous to column (3) of table 1.1, implies that matching efficiency fell by 17 percent after 2009m6. This result is comparable to the prediction based on the OLS estimates reported in table 1.1. The estimates for γ in table 1.3 provide a range from 1.052 to 1.466. This range contains the number 1.27, which is used in the linearized simulations reported in den Haan, Ramey, and Watson (2000).

Table 1.4: Nonlinear matching technology using adjusted job finding

	Dependent variable:			
	Log(find)			
	(1)	(2)	(3)	(4)
γ	.301***	.420***	.204***	1.893***
	(.055)	(.029)	(.030)	(.099)
Trend		$-$.001***		
		(.00005)		
Post GR			$-$.150***	$-$.019
			(.006)	(.019)
Constant	1.850***	1.338***	3.018***	
	(.416)	(.112)	(.495)	
Observations	229	229	229	229
Note:		*p<0.1; **p<0.05; ***p<0.01		

Columns 1 through 3 of table 1.4 repeat the nonlinear analysis for the job-finding series corrected for time aggregation. The qualitative picture is similar. For example, the fall in matching efficiency after the Great Recession is estimated to be 15 percent using the adjusted data, whereas the fall is estimated by be 17.3 percent using the unadjusted data. Nevertheless, the adjusted job-finding series implies significantly different levels of $\hat{\gamma}$. Column 4 of table 1.4 specifies a matching technology that requires job-finding and job-filling probabilities to fall within the unit interval. Here $\hat{\gamma} = 1.893$. The implication of these estimates is taken up in the following section.

Table 1.5 compares the estimated statistical models in terms of how well they fit the adjusted data. For each model the table reports the standard error of the regression, which is the square root of the sum of squared error divided by the same size minus 2.[11] Because the models are all estimated using the same sample, the statistic essentially compares the sum of squared errors produced by the model estimates. In the first column of table 1.5, the model labeled "All" allows for no shifts in matching efficiency after the Great Recession or trend component in matching efficiency and corresponds to the first columns of tables 1.2, 1.1, 1.4, and 1.3. Looking at the data in figures 1.2 and 1.3, it is no surprise that these models produce the worst fit. The model labeled "Linear trend" allows for a trend and

[11]That is, for model j,

$$\text{standard error of the regression} = \left[\frac{1}{T-2}\sum_i \left(\hat{\varepsilon}_i^j\right)^2\right]^{1/2}$$

where T is the sample size.

corresponds to the second columns of the tables listed above. The models that include a trend provide a better fit with the Cobb–Douglas specification providing a slightly better fit than the nonlinear specification. On the other hand, for the models that allow for a shift in matching efficiency after the Great Recession, labeled "Great Recession shift," the nonlinear specification provides a better fit. Finally, the $0 - -1$ model, which corresponds to column 4 of table 1.4, reports the sum of squared error for the nonlinear model that specifies job-finding to fall within 0 and 1. Importantly, this number does not adjust for this statistical model having one fewer parameter to match the data. When compared to the nonlinear models with an overall level shift, the $0 - -1$ model increases the sum of squared error by a factor of over 3, with the benefit of specifying a matching technology with desirable theoretical properties.

Table 1.5: Standard error of the regression

Model	Cobb–Douglas	Nonlinear
Linear trend	.0450	.0452
Great Recession shift	.0495	.0481
All	.0927	.0882
0–1 constraint		.1561

Figure 1.4 shows the fit of the models that allow for a shift in matching efficiency after the Great Recession. Both models that estimate over all matching efficiency in addition to the shift fit the data well. The fits are nearly indistinguishable to the eye and both models are precisely estimated. The nonlinear model that constrains matching efficiency to equal 1, labeled "Nonlinear, 0–1", illustrates the error reported in the last row of table 1.5.

The appeal of the Cobb–Douglas specification partly lies in its interpretation. Interpretations of γ in (1.12) are much less apparent. To understand the implications of the nonlinear parameterization, the next section investigates an implication of matching technology for labor–macro models that feature random search. Figure 1.4 foreshadows the discussion. The nonlinear technology that constrains matching efficiency to be 1 increases the variation in job finding and thus the variation in unemployment.

1.4 Implications of Estimates for Fundamental Surplus

This section aims to describe how different parameterization of matching technology influence labor–macro models that feature random search. A key statistic of matching models is the elasticity of tightness with respect to productivity. Shimer (2005), for example, discussed this elasticity in addition to a follow-up investigation in Hagedorn and

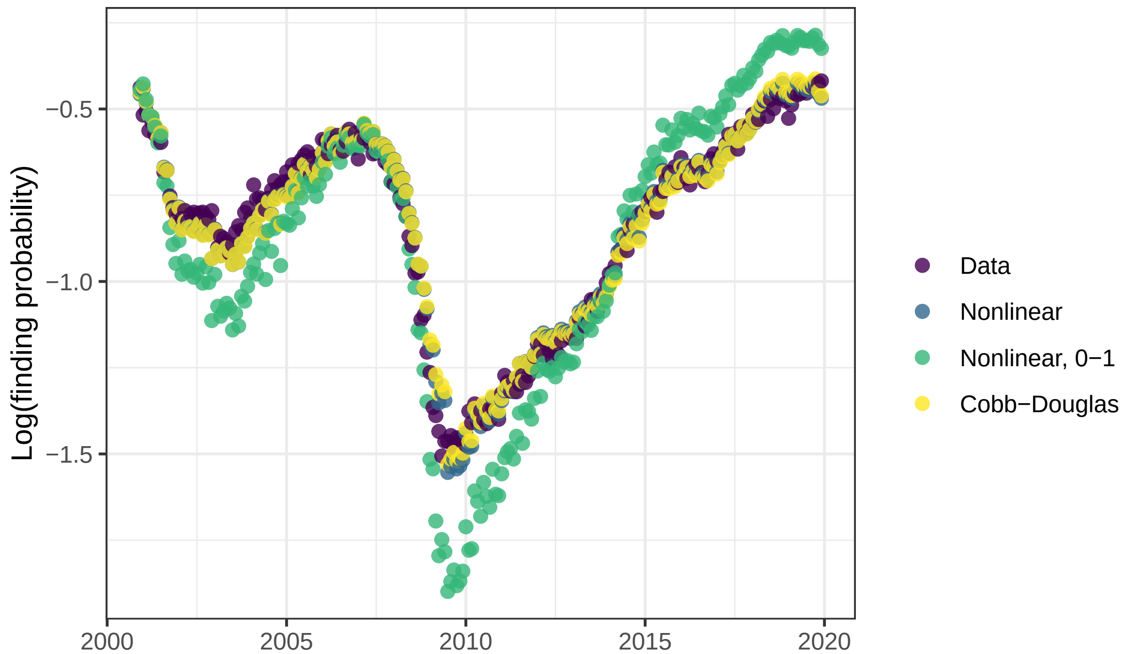


Figure 1.4: Comparison of model fits for data adjusted for time aggregation. Both the nonlinear and Cobb–Douglas models allow for a shift in matching efficiency after the Great Recession.

Manovskii (2008), which, among other things, adds taxes paid by firms and workers. The elasticity has been discussed by many others in the literature. Ljungqvist and Sargent (2017) unify the literature by writing several models in terms of fundamental surplus. They emphasize fundamentals, which include parameters associated with matching technology.

Section 1.4.1 first describes how incredibly cyclical tightness is over the business cycle and then section 1.4.2 discusses how parameters in the matching function affect interpretations of fundamental surplus, using a variation of a canonical search model discussed in Pissarides (1985) and re-examined by Ljungqvist and Sargent (2017).

1.4.1 The Cyclicality of Tightness

This section describes the cyclicality of tightness over the business cycle. Figure 1.5 depicts tightness from December 2000, when the JOLTS program first publishes data, to December 2019. Data from 2020 are significantly affected by COVID-19 and therefore left for future analysis. The average level of tightness over the period is .56, indicating that there were, on average over the period, two people looking for work for every posted vacancy.

Shaded regions in figure 1.5 indicate NBER recession dates.[12] In recessions, tightness falls because both labor demand falls, a drop in v_t, and unemployment increases, an increase in u_t.

To assess how cyclical tightness is over the business cycle, I relate θ_t to measures of business-cycle variability. Measures of the business cycle are indexed by j. I estimate regressions of the form

$$\widetilde{\log \theta}_t = \beta_0 + \beta^j \log\left(\text{measure of the business cycle}\right)_t^j + \varepsilon_t, \tag{1.13}$$

where $\tilde{\log} \theta_t$ is either the series $\log(\theta_t)$ itself or the cyclical component of $\log(\theta_t)$ extracted with a Hodrik–Prescott filter.[13] The estimated $\hat{\beta}^j$ from (1.13) indicates how tightness varies with the business cycle, providing an empirical counterpart of $\eta_{\theta,y}$ in (1.1).

I consider three measures of the business cycle based on series for real GDP, industrial production, and personal consumption expenditures. For each of these series I take their log and then take their cyclical component using a Hodrik–Prescott filter and smoothing parameter 129,600. Consumption and industrial production are both available from FRED at a monthly frequency, whereas real GDP is available at a quarterly frequency. To generate a monthly series for the log of real GDP, I linearly interpolate the values. Real GDP is an empirical counterpart to the elasticity statistic in (1.1) but not the only one.

I consider the series for industrial production, which is an index of real output produced by the manufacturing, mining, electric utilities, and gas utilities because it is available at a monthly frequency. I consider the series for personal consumption expenditures because matching models often feature linear preferences with the implication that all output is consumed. Figure 1.6 depicts the cyclical component of these series.

Figure 1.6 also depicts the cyclical component of tightness in panel A. Panel B depicts the cyclical component of industrial production; panel C depicts the cyclical component of real output; and panel D depicts the cyclical component of real personal consumption expenditures. All series display common significant procyclical patterns.

This common cyclicality is confirmed in the cyclical regressions. Table 1.6 reports the cyclical coefficients in (1.13). The first group of models in columns (1)–(3) use $\log(\theta_t)$ as the dependent variable in (1.13). The first column says indicates that when output is 1 percent above trend, tightness increases 30 percent. This is an important statement of why the elasticity in (1.1) is so important.

[12]See https://www.nber.org/cycles.html for NBER dates.

[13]I use a smoothing parameter of 129,600.

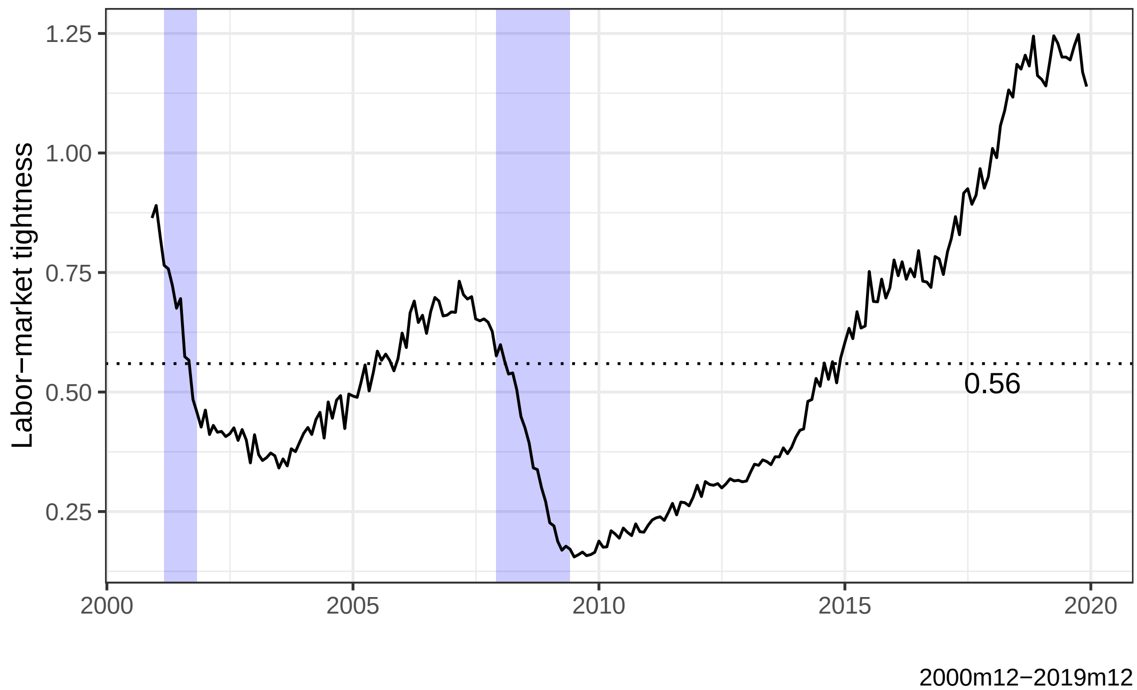


Figure 1.5: Tightness, θ, 2000m12–2019m12.

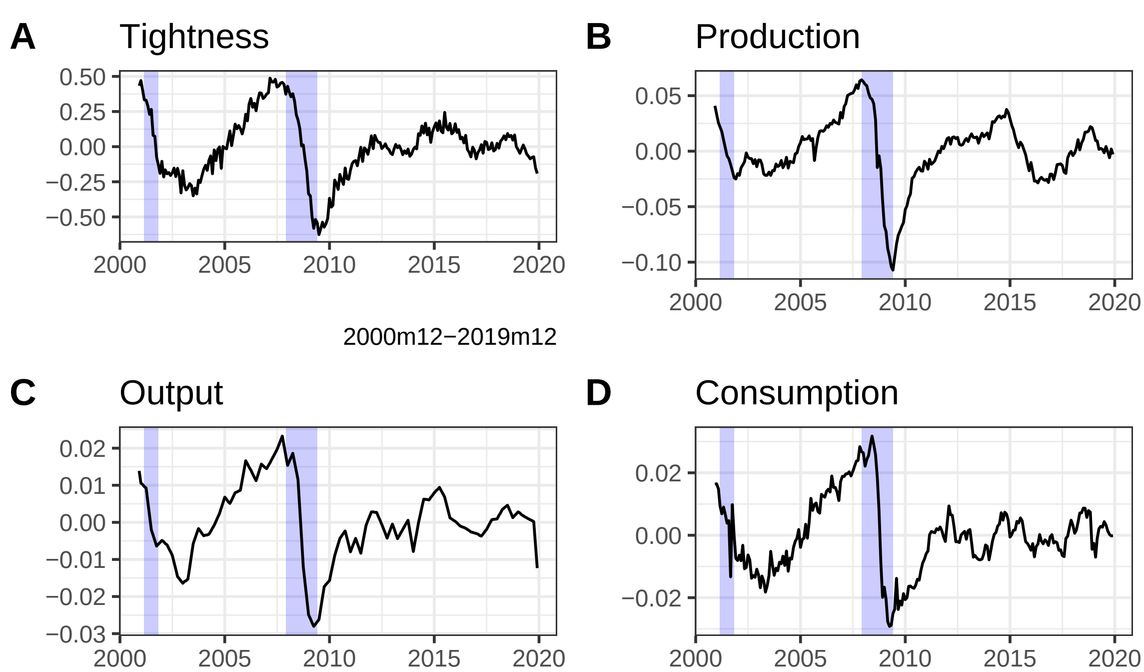


Figure 1.6: Measures of the business cycle, 2000m12–2019m12.

Table 1.6: Cyclicality of tightness

	Dependent variable:					
	log(tightness)			cyclical log(tightness)		
	(1)	(2)	(3)	(4)	(5)	(6)
Output	29.804***			20.711***		
	(2.901)			(.653)		
Industrial production		8.525***			6.580***	
		(1.040)			(.269)	
Consumption			23.299***			17.247***
			(2.543)			(.595)
Observations	229	229	229	229	229	229
R^2	.317	.228	.270	.816	.724	.787
Note:				*p<0.1; **p<0.05; ***p<0.01		

1.4.2 A Model of the Elasticity of Tightness with Respect to Productivity

1.4.2.1 Economic Environment

This section describes a canonical search model that undegirds the analysis in Ljungqvist and Sargent (2017).[14] I show how this framework provides the bound for the elasticity of tightness with respect to productivity. An appendix provides further details and includes conditions for existence and uniqueness of an equilibrium. Existence and uniqueness is not discussed in Ljungqvist and Sargent (2017).

Time is discrete. But because I am interested in a stationary equilibrium, I make no reference to time. A unit measure of identical, infinitey-lived workers populates the economy. Workers are risk neutral with a discount factor of $\beta = (1 + r)^{-1}$. A worker aims to maximize the expected discounted sum of labor income plus leisure. Employed workers earn labor income. Unemployed workers experience the value of leisure $z > 0$ and no labor income.

The economy is also populated my a large measure of firms. Firms are either active or inactive. Inactive firms become active by posting a vacancy. A matching friction, which causes unemployment, prevents a firm from immediately hiring a worker searching for a job. An active firm not in a productive match with a worker incurs a vacancy cost c each period it recruits.

[14] Elsby, Michaels, and Ratner (2015) provide an exposition of a continuous-time version, which simplifies Pissarides (1985) by modeling all firm–worker pairs as having the same level of productivity.

Once matched with a worker, a firm operates a production technology that converts an indivisible unit of labor into y units of output. This production technology exhibits constant returns to scale in labor. Each active firm matched with a worker employs a single worker. While matched with a worker, a firm earns $y - w$, where w is the per-period wage paid to the worker. Wages are set through Nash bargaining.

All matches are exogenously destroyed with per-period probability s. Free entry by the large measure of firms implies that a firm's expected discounted value of a posting a vacancy equals zero.

A matching function $M(u, v)$ determines the measure of successful matches in a period, where u and v are aggregate measures of unemployed workers and vacancies. The function M is increasing in both its arguments because more vacancies for a given level of unemployment leads to more matches and more workers searching for jobs for a given level of vacancies leads to more matches. Additionally, M exhibits constant returns to scale in u and v.

Define tightness, θ, as the ratio of vacancies to unemployed, v/u. The probability that a firm fills a vacancy is given by $q(\theta) = M(u, v)/v = m\left(\theta^{-1}, 1\right)$. The probability that an unemployed worker matches with a firm is given by $\theta q(\theta) = M(u, v)/u = m(1, \theta)$. Each unemployed worker faces the same likelihood of finding a job because firms lack a recruiting technology that allows them to select a particular candidate for a job.

I let $\mathcal{J}$ denote the present discounted value of an active firm in a productive match with an employee and I let $\mathcal{V}$ denote the present discounted value of an active firm engaged in recruiting a worker. The key Bellman equations for firms are

$$\mathcal{J} = y - w + \beta\left[s\mathcal{V} + (1 - s)\mathcal{J}\right] \tag{1.14}$$

$$\mathcal{V} = -c + \beta\left\{q(\theta)\mathcal{J} + \left[1 - q(\theta)\right]\mathcal{V}\right\}. \tag{1.15}$$

Equation (1.14) specifies that the value of a productive match equals the value of output less the wage, $y - w$, plus the discounted future value. The future value equals $\mathcal{J}$ with probability $1 - s$, for the case where the match does not end through separation; plus $\mathcal{V}$ with probability s, for the case where the match ends in separation. Equation (1.15) specifies that the value of recruiting equals the flow cost of posting a vacancy, c, plus the discounted future value. The future value equals $\mathcal{J}$ with probability $q(\theta)$, for the case where the vacancy is filled; plus $\mathcal{V}$ with probability $1 - q(\theta)$, for the case where the vacancy is unfilled. Free entry means $\mathcal{V} = 0$.

I let $\mathcal{E}$ denote the present discounted value of employment for a worker and I let $\mathcal{U}$ denote the present discounted value of unemployment for a worker. The key Bellman equations for workers are

$$\mathcal{E} = w + \beta \left[s\mathcal{U} + (1-s)\,\mathcal{E} \right] \tag{1.16}$$

$$\mathcal{U} = z + \beta \left\{ \theta q(\theta)\,\mathcal{E} + \left[1 - \theta q(\theta)\right]\mathcal{U} \right\}. \tag{1.17}$$

Equation (1.16) specifies that the value of employment equals the earned wage, w, plus the discounted future value. The future value equals $\mathcal{U}$ with probability s, for the case where employment ends in separate; plus $\mathcal{E}$ with probability $1-s$, for the case where employment does not end in separation. Equation (1.17) specifies that the value of unemployment equals the value of non-employment, z, plus the discounted future value. The future value equals $\mathcal{E}$ with probability $\theta q(\theta)$, for the case where the worker finds a job; plus $\mathcal{U}$ with probability $1 - \theta q(\theta)$, for the case where the worker does not find a job.

The unknowns are (1) $\mathcal{J}$, (2) $\mathcal{E}$, (3) $\mathcal{U}$, (4) $\mathcal{V}$, (5) w, and (6) θ. Equations (1.14)–(1.17) along with $\mathcal{V} = 0$ and a Nash bargaining rule for wages allow me to solve for the unknowns. In appendix A.3, proposition 9 establishes the exitence and uniqueness of an equilibrium provided that it is marginally profitable to post an initial vacancy. This equilibrium result allows me to consider the elasticity of tightness with respect to productivity in the next section.

1.4.2.2 How Matching Technology Affects Interpretations of Tightness

Re-arranging the equilibrium expressions and a simple extension of the baseline model discussed in Ljungqvist and Sargent (2017) yields the following result:[15]

$$\eta_{\theta,y} = \Upsilon \frac{y}{y-c} < \frac{1}{\eta_{M,u}} \frac{y}{y-c}.$$

This expression says that in a matching model that features random search and exogenous separations, the elasticity of tightness with respect to productivity is bounded by $1/\eta_{M,u}$.

Under the Cobb–Douglas parameterization, this elasticity is constant with $\eta_{M,u} = 1-\alpha$. Turning to column 3 in table 1.1, which allows for a shift in matching efficiency after the Great Recession, the bound is $1/(1-\hat{\alpha}) = 1/(1-.700) = 3.333$. Using the adjusted job-finding series from table 1.2, the analogous result is $1/(1-.536) = 2.155$.

Under the nonlinear parameterization, the elasticity $\eta_{M,u}$ is nonconstant. This may well be a desirable feature because the percent increase in matches for a given percent increase in unemployment may well depend on whether the economy is near full employment.

[15] These details are covered in the appendix.

But the fact that the elasticity is nonconstant makes a direct comparison with α unfeasible. Instead, I report the inverse of elasticity evaluated at the average level of tightness, .56, using $\hat{\gamma} = 1.064$, which is reported in column 3 of table 1.3. Using these values, the bound is 2.842. Using the analogous estimate based on the time-aggregation-corrected series reported in table 1.4 implies that the bound is 2.2126. Using the nonlinear matching technology that constrains matching efficiency to be 1 and reported in last column of table 1.4, however, implies that the bound rises to 4.003. This suggests there is *some* hope in matching the elasticities reported in section 1.4.1.

My point is that the choice of matching technology may be important. And estimates of the fundamental parameters associated with the choice need to be estimated carefully. The second result of this section is that the choice of matching technology puts less pressure on fundamental surplus to explain the cyclical variation of the labor market. How the choice of matching technology affects all dynamics in nonlinear labor–macro models is a question that I plan to explore in future research.

1.5 Conclusion

The choice of matching technology matters for matching models that aim to capture the cyclical behavior of the labor market. Labor–macro modelers have overwhelmingly used a Cobb–Douglas technology, sometimes calibrated to empirical estimates that use data unadjusted for time aggregation. In this paper I 1) showed how adjusting the data for time aggregation may significantly affect estimates, 2) provided updated estimates of two constant-returns-to-scale matching functions and 2) suggested how parameterizations affect labor–macro modeling.

While the Cobb–Douglas choice is convenient—and fits the data well—it does raise some thorny issues. Namely, the elasticity of matching with respect to unemployment is constant over the business cycle and may easily imply that the probability a firm fills a vacancy is above 1. An alternative parameterization is available. This alternative, nonlinear parameterization allows the elasticity of matching with respect to unemployment to vary over the business cycle; additionally, this parametrization can constrain the probability a worker finds a job within a given month and the probability a firm fills a vacancy within a given month to stay within the unit interval. While the constrained parameterization does not fit the data on job finding equally well because it has one less parameter, the constrained parameterization does increase the cyclicality of job finding. If the interest in a labor–macro model that features employment and unemployment is matching the cyclicality of unemployment, then the nonlinear parameterization is superior.

CHAPTER 2

On the Tension between Maximum Sustainable Yield and Maximum Economic Yield

A typical fishery management plan focuses on the cost–benefit structure of a particular fishery to maximize profits but ignores linkages to the macroeconomy. This creates a tension between the objectives of a hypothetical sole owner and what is preferred economy-wide. To gauge the benefits of recognizing these ignored linkages, I compare harvest patterns targeted by a sole owner to harvest patterns that maximize economy-wide surplus in a two-sector, dynamic general-equilibrium model. I show that maximum economic yield delivers profits to the fishery, but maximum sustainable yield can deliver greater economy-wide surplus when regulation constrains harvest capacity. In a calibration exercise that matches the 2015 Alaskan economy and increases fleet harvest from MEY to MSY, I find that economy-wide employment increases by 1,300 jobs. These findings might explain why real-life fisheries are often managed at MSY as opposed to MEY.

2.1 Introduction

Commercial fishery management is characterized by the decisions of a hypothetical sole owner. The sole owner's goal is profit maximization. They maximize profits by accounting for the externalities of harvesting fish, which include crowding, stock, and ecosystem externalities (Ryan, Holland, and Herrera, 2014). These externalities arise because of the common-property nature of fishing. But the sole owner cannot control all economic and technological features of fishing. In most models of commercial fishing the sole owner focuses on the costs and benefits of a particular harvest pattern for a particular fishery. Additionally, the sole owner views linkages between the fishery and the macroeconomy as beyond their range of control.[1] Ignoring macroeconomic linkages will potentially leave economic benefits unrealized. To gauge these potential benefits, I present and calibrate an operational general-equilibrium model of a two-sector economy where one of the sectors is a commercial fishery regulated by a sole owner.

To frame the issue in this paper, consider that the sole owner might want to account for the entire value chain of fishing. The value chain not only depends on the fleet's harvest, but also includes the processing, distribution, and marketing of fish. By accounting for more and more economic linkages, an argument can be made that the sole owner affects economy-wide welfare. A potential tension, therefore, might exist between the harvest pattern preferred economy-wide and the harvest pattern preferred by a sole owner concerned only with a particular fishery.

The existing literature is divided on the issue of expanding the scope of fishing to include more of the value chain. Christensen (2010) agrees with the above sentiment, saying that "we are missing part of the picture when equating the fisheries sector with fishing boats" (107), emphasizing all the links along the value chain that get fish "from sea to plate" (107). Taking the value-chain perspective, Christensen (2010) finds that, using simulations, maximum economic yield (MEY) is close to maximum sustainable yield (MSY). Sumaila and Hannesson (2010) respond to Christensen (2010) by replacing profits with consumer surplus to represent the benefits of fishing. They show in a static model that the marginal benefits of catching a fish equal the marginal costs at a point where MEY harvest is always less than MSY harvest. The benefits accrue to the consumer, while the costs reflect the entire value chain. Sumaila and Hannesson (2010) take a traditional perspective.[2]

These two papers, though meant to be at loggerheads, mostly agree with one another. Both highlight the need to expand the role that commercial fishing plays in an economy. Doing so, the papers agree, might significantly change the targeted stock levels for certain fisheries.

A further characterization of combined ecosystem and value-change management is provided by Squires and Vestergaard (2016). They extend MEY beyond a particular harvest for a particular species in a dynamic model, emphasizing the role of prices or shadow price, if prices are not directly observable. Their work points toward microfounding the

[1] In fact, fishery management plans typically ignore linkages between interdependent fisheries, like between the lobster and herring fisheries in the Gulf of Maine, where nearly 90 percent of lobster bait is herring and roughly 60 percent of the herring catch is purchased by the lobster fishery (Ryan, Holland, and Herrera, 2010).

[2] Christensen (2010) and (Sumaila and Hannesson, 2010) provide synoptic perspectives and nicely summarize the literature. Squires and Vestergaard (2016) provide another excellent overview.

problem and making prices determined within the model economy.

These papers share a common approach to expanding the role of commercial fishing: profits are replaced with a benefits function or consumer surplus represented by the area under a demand curve. This theory has been nearly universally characterized in partial equilibrium. It has not been fully characterized 1) when acknowledging linkages between fishers and the macroeconomy; and 2) when prices are determined in general equilibrium. To accomplish these two tasks and to explore the gains of recognizing macroeconomic linkages in fishery management plans, I add a commercial fishing sector to a dynamic general-equilibrium economy that features unemployment. The commercial fishing sector uses labor and a stock of fish as inputs to catch fish. The stock is regulated by a sole owner that cannot control all margins of fishing—essentially the commercial fishing sector operates under a formulation of regulated open access (Homans and Wilen, 1997). The second sector uses only labor as an input. These sectors are part of a value chain that produces a homogeneous final good sold to consumers who purchase final goods using labor income.

The microfounded model allows me to compare the economy-wide surplus generated by a profit-maximizing sole owner to the economy-wide surplus generated by a welfare-maximizing social planner. Unlike the sole owner concerned with harvest patterns that maximize fishery profits, the social planner is concerned with harvest patterns that maximize economy-wide surplus, or welfare.

The model describes an equilibrium in which the social planner prefers a constant escapement policy where harvest is maximized, a level associated with MSY, rather than MEY, or the level of harvest associated with maximum fishery profits—but only when the sole owner regulates the fishing capacity of vessels. When vessels are forced to operate below full capacity, increasing harvest to the MSY level acts like a positive productivity innovation. Fishers are allowed to catch more initially, which makes fishing more profitable. As vessels enter the fishery, pushing out the supply of fish, the price of fish falls, which raises the relative price of the good produced by the non-fishery sector. Because it is now more profitable to operate in, the non-fishery sector grows. Both sectors expand and workers take advantage of these employment opportunities. When vessels operate at full capacity, however, the social planner prefers managing the fishery at MEY because they cannot engineer a positive productivity innovation.

The equilibrium result described in the previous paragraph points out the harmony between advocates of MSY and advocates of MEY: When the social planner can engineer a positive innovation to productivity, there are gains to be had from managing the economy at MSY as opposed to MEY. But when vessels operate at maximum capacity, that productivity gain cannot be engineered and consequently MEY produces greater economy-wide surplus. The model I present, in other words, provides grounds for Christensen (2010) and Sumaila and Hannesson (2010) to be right.

While simple, the model I use lies at the center of many richer models. Its features should facilitate normative analysis because the fishery management plan implemented in equilibrium turns out to be a target stock level. This aligns with typical management plans implemented by the National Marine Fisheries Service and regional councils. On the other hand, the analysis abstracts from transition dynamics and stochasticity, which may be relevant for many fishery managers.

Nevertheless, the model incorporates the ready employment opportunities available to fishers. Ready employment opportunities were identified in survey evidence (Wilen, Chen, and Homans, 1991) and are apparent in labor-market data from the Current Population Survey. These data are explored in the following section. Using data matched across months, half the time a fisher leaves unemployment, they transition to a non-fishery job. Importantly, the value of unemployment to a worker reflects labor-market opportunities in both sectors. Strong linkages between sectors mean there are potentially large gains to acknowledging macroeconomic linkages in fishery management plans.

And because most commercial fisheries in the United States operate under significant regulation, the model predicts there are gains to managing a fishery at MSY as opposed to MEY. To explore this prediction, I calibrate the model to match the Alaskan economy in 2015. I consider expanding the total allowable catch by about 18 percent, consistent with management of the groundfish complex in the Bering Sea and Aleutian Islands, which operates at 85 percent of historical MSY estimates. Doing so reduces the unemployment rate by 0.36 percentage points and increases economy-wide employment by 0.4 percent, or roughly 1,300 jobs. The results suggest there are meaningful gains to expanding fishers' productivity in certain cases and explain why fisheries are often managed at MSY as opposed to MEY.

Much is at stake. Arnason, Kelleher, and Willmann (2009) give a headline number of 50 billion dollars lost annually in global fisheries due to "mismanagement" associated with MSY as opposed to MEY. That number rises to 2 trillion over the past 3 decades. But their work starts from the perspective of profit maximization within a representative fishery. The conclusions of this paper, based on a dynamic general-equilibrium approach, suggest a different reading of their result. Evaluating fishery management plans may mean specifying more complete models of fishing within the macroeconomy. Thinking outside the current framework of optimal fishery management serves as good preparation for challenges going forward.

2.2 Facts about the Labor Market of US Fishers

In this section, I look at employment in US commercial fisheries, make comparisons to broad employment categories, and explore the frequency with which unemployed fishers switch occupations when leaving unemployment. Doing so allows me to gauge what labor-market options are available to fishers. Declining opportunities to fish and readily available employment opportunities demand that fishers and fishery managers consider employment outside of commercial fishing.

The data come from the Current Population Survey (CPS), which is the primary source of labor-force statistics for the United States. The CPS is one of two major surveys published by the US Bureau of Labor Statistics concerning labor-market conditions. The other is the Current Employment Statistics survey, which collects data from roughly 160,000 business establishments. The establishment survey, however, varies in its coverage of small firms. For example, in Rhode Island, a crew of less than 10 fishers whose remuneration is a share of the catch is not surveyed. Thus, I use the CPS to look at employment in US commercial fisheries.

Each month the CPS surveys roughly 60,000 households to determine who is employed, unemployed, and not in the labor force. The survey additionally collects information about workers' current occupations. And while the CPS has adopted different occupational classification systems used by the US Census Bureau, a consistent occupational title defines fishers.

Table 2.1 lists the dates associated with occupational classification systems and relevant occupational titles. I use these titles to create a time series of employment in US commercial fisheries. The individual-level data can be used as a representative sample of the civilian population of the United States by using cross-sectional weights provided by the Bureau of Labor Statistics. Fortunately, the data are published beginning in 1976. This offers a description of the fishery since the passage of the Fishery Conservation and Management Act (FCMA).[3]

The CPS interviews an address for four consecutive months, leaves the address unsurveyed the following eight months, and then surveys the address for the next four months. This panel aspect of the survey means people can be matched across months. Additionally, the CPS asks unemployed respondents what their last occupation was. I use this feature of the data to see what jobs unemployed fishers take to gauge their occupation mobility.

[3] Alaska and Hawaii were added to the CPS in January 1960, ensuring that the results are not driven by expansion of the sample (Current Population Survey, 2006).

I next describe employment in US commercials fisheries before turning to the occupational mobility of fishers.

2.2.1 Employment in US Commercial Fisheries

Two facts stand out in the employment series of fishers. The first is the boom–bust pattern in the data, consistent with narratives about the passage of the FCMA. The second is the declining employment opportunities in commercial fisheries.

Figure 2.1 depicts the three-year moving average of the number of fishers employed in the US economy from 1976 through 2018. From the start of the sample, employment increased steadily for roughly a decade. That increase coincides with the passage of the FCMA of 1976, or the Magnuson–Stevens Act, which brought waters within 200 miles of the shore under jurisdiction of the United States and intentions of modernizing the US commercial fishing fleet (Apollonio and Dykstra, 2008; Weber, 2002; Acheson and Gardner, 2011).

Prior to the FCMA of 1976, US fishery management could be described as "nonmanagement rather than mismanagement" (Magnuson, 1977, 428). Foreign fleets dominated catch off the coast of the United States during the 1960s, harvesting three times the amounts taken by domestic fleets (Anthony, 1993). And because the FCMA of 1976 meant excluding foreign fleets, stakeholders felt that a "bonanza" awaited them (Anthony, 1990, 178). Their optimism was buoyed by aberrant record landings (Schrank, 1995) and backed by a favorable tax system (Weber, 2002; Apollonio and Dykstra, 2008; Dewar, 1983).

US fish stocks were overfished. After 1986, employment in US commercial fisheries declined steadily for the next quarter century. This coincides with the "bust" part of the boom–bust narrative.

To gauge the decline in employment opportunities available in commercial fisheries, figure 2.2 shows indexed employment series. The indexed series give an indication of broad trends. The solid black line shows the time series used in figure 2.1, but with 1976 employment indexed to 100. As a comparison, overall employment, similarly indexed with a dotted red line, increased 87 percent from 1976 to 2018. Employment of fishers, on the other hand, decreased 5 percent. That figure would be worse except for the expanded opportunities in commercial fisheries after 2009.

The indexed series for manufacturing employment, depicted in figure 2.2 with a dashed blue line, gives a sense of the decline. Manufacturing employment declined 28 percent from its 1976 level, a decline that has been well documented.[4]

[4]For a recent discussion, see Charles, Hurst, and Schwartz (2018). The decline in manufacturing has also attracted much recent popular attention. For example, in an article for the *New York Times*, Tavernise

The next section uses individual-level longitudinal data to look at fishers' labor-market options.

2.2.2 Occupational Mobility of Fishers

Ready labor-market alternatives are available to fishers. This is best seen in matched data: About half the time someone previously employed as a fisher left unemployment, they took a job outside of commercial fishing. The occupational titles of these jobs were varied. This fact is corroborated with survey data.

The matched data provide a sample of unemployment-to-employment (UE) transitions. I take UE transitions where the unemployed person in the first month of the match reports that their job title was listed in the second column of table 2.1. The second month of the match then provides data on what jobs were taken. The third column of table 2.1 lists the fraction of unemployed fishers that remained fishers upon transitioning to employment.

Over the different periods associated with classification schemes, unemployed fishers were consistently able to find employment inside and outside of commercial fishing. This fact is seen in rows 1–5 of table 2.1. For example, from 1976 to 1982, the fraction of people remaining fishers coming from unemployment was 0.468. From 2011 to 2018 the fraction changes very little and was 0.459. Over the entire 1976–2018 time frame, unemployed fishers found a job outside the fishery roughly half the time. This statistic is listed in the bottom row of table 2.1.

Occupations taken by unemployed fishers are listed in table 2.2. Column 1 of table 2.2 reports the occupational title associated with the employment part of the UE transition. The fraction listed in the second column is calculated by looking at all UE transitions where the person found a job outside the fishery; that is, conditional on taking a non-fishery job. The table lists the top-three occupations taken in each occupational coding scheme.

Unemployed fishers took a wide array of jobs when leaving the fishery. While the majority of transitions involved taking a job that requires manual labor, such as a janitor or construction worker, this is a small fraction of transitions overall. The top occupations in each coding scheme only make up between 6 and 20 percent of transitions.

These facts are consistent with survey evidence. Wilen, Chen, and Homans (1991) conducted a survey of Pacific Coast fishers, reporting that over half earned income from outside the fishery, exclusive of spousal and other family income. When faced with a

quotes an autoworker who lost their job as saying "it's literally in your face — the decline of manufacturing" (Tavernise, 2019).

hypothetical 2-week closure of the fishery, 30.6 percent of respondents said they would switch to a non-fishing job (as opposed to switching to a different fishery or nonwork). For a hypothetical 1-year closure, 46.4 percent would switch to a non-fishing job. Of those 46.4 percent leaving, 68.9 percent would expect to transition to a non-fishery job within a month and 82.2 percent would expect to transition to in a non-fishery job within two months. Respondents also felt they would earn only marginally less if hypothetically forced to leave. For example, skipper/owners who reported mean fishery earnings of $33,225 believed they could either switch to another fishery or take another job and earn $30,000. "Answers to several of [their] questions suggest that [fishers] perceive that they have ready alternatives to fishing, much as one would expect" (Wilen, Chen, and Homans, 1991, 30).

The responses of the Pacific Coast fishers in Wilen, Chen, and Homans's survey corroborate the facts from the CPS. And herein lies the basis of the model economy explored in the following section: The value of unemployment to a worker depends on future employment opportunities. From the perspective of a worker who was previously employed as a fisher, employment opportunities come from an array of occupations—not only opportunities in commercial fisheries. These macroeconomic linkages mean decisions made in either sector will spill over to the other. In the next section, I describe a model economy where increased demand for non-fishers raises wage rates paid by vessel owners. And effort controls set by a sole owner affect economy-wide employment dynamics. I illustrate a potential tension between what is preferred by the sole owner and what is optimal from an economy-wide perspective.

2.3 The Model

2.3.1 Economic Environment

The economic environment is a random search-and-matching model of the labor market. The economy is populated by a unit measure of agents who make up the labor force. Agents are either employed and unemployed. They seek to maximize utility by earning labor income. Employed agents go to work at firms, while unemployed agents look for work. Agents consume a final consumption good whose price is normalized to 1. The final consumption good is produced in a competitive market using two intermediate goods.

Each intermediate good is produced by a particular type of firm. The first type uses only labor as an input in production. They make up the non-fishery sector of the economy. A second type is a vessel in the commercial fishery. They harvest fish and use both labor and a stock of fish regulated by a sole owner as inputs.

The sole owner maximizes profits in the commercial-fishery sector. They are granted exclusive control over vessels' harvest policies. But regulation set by the sole owner cannot control all margins of harvesting fish, resembling Homans and Wilen's (1997) formulation of regulated open access (Smith, 2012). The unaccounted for margins, like wage rates paid to fishers, depend on economy-wide conditions. Thus the harvest patterns preferred by the sole owner in the decentralized economy differ from what is preferred from an economy-wide perspective.

A fundamental part of the economic environment is the value of unemployment, which, based on the previous section, should reflect labor-market options in both sectors. I use a search-and-matching model of the labor market because this framework clearly defines the value of unemployment. By using the structure Acemoglu (2001) built within this framework, I allow job creation to depend on the relative demands between the two sectors.

In the remainder of this section I lay out the basic building blocks of the the model.

2.3.1.1 Preferences

Time is continuous and indexed by t. A continuum of infinitely lived agents populate the economy. At any point in time, agents consume and either go to work or look for work. Agents consume a homogeneous final good and value consumption according to the period utility function $u(c) = c$. They are risk neutral and discount the future at rate r. Each agent is endowed with an indivisible unit of labor, which is applied to either work if employed or looking for work if unemployed.

2.3.1.2 The Value Chain

The value chain consists of firms that produce a final good using the two intermediate goods as inputs. These two intermediate goods are fish, denoted by X_χ, and the good produced in the non-fishery sector, denoted by X_φ. Final-good producers have access to a Cobb–Douglas production technology that bundles X_χ and X_φ to produce the final good consumed by agents. Because both intermediate goods are perishable and the economy does not have access to a technology that allows either good to be stored, each period final-good producers demand levels of fish and the non-fishery good to maximize $X_\chi^\alpha X_\varphi^{1-\alpha} - p_\chi X_\chi - p_\varphi X_\varphi$.

Prices are $p_\varphi(t) = (1-\alpha)X_\chi(t)^\alpha X_\varphi(t)^{-\alpha}$ and $p_\chi(t) = \alpha X_\chi(t)^{\alpha-1} X_\varphi(t)^{1-\alpha}$. Going forward the subscript φ denotes values outside the fishery and the subscript χ denotes values associated with the fishery.

2.3.1.3 Matching

At time t, there are $1 - u(t)$ employed agents and $u(t)$ agents looking for work. These statistics can be interpreted as rates because of the normalization.

The key friction that gives rise to unemployment is the inability of firms looking to hire workers to instantaneously match with agents looking for work. This friction is captured by a matching function defined over the unemployment rate and the vacancy rate, $m(u, v)$, where the vacancy rate is the ratio of open vacancies to the sum of open and filled vacancies. More matches occur the higher the unemployment rate and when there are more open vacancies, making m increasing in both its arguments. It is assumed that m is homogeneous of degree one.

A vacancy is filled at rate $q(\theta) := m(u, v)/v = m(\theta^{-1}, 1)$, where $\theta := v/u$ is the measure of tightness in the labor market and $q(\theta)$ is a differentiable, decreasing function. It is harder for firms to fill vacancies when the number of vacancies increases relative to the number of unemployed persons. An unemployed person becomes employed at rate $f(\theta) := m(u, v)/u = m(1, \theta) = \theta q(\theta)$, which is referred to as the job-finding rate and is a differentiable, increasing function. It is easier to find a job when the number of vacancies increases relative to the number people looking for work. Tightness summarizes the condition of the labor market.

I assume q and f map the positive real numbers onto themselves, with $\lim_{\theta\to 0} q = \lim_{\theta\to\infty} f = \infty$ and $\lim_{\theta\to\infty} q = \lim_{\theta\to 0} f = 0$. Therefore q are f are not truly probabilities. Instead, if there are very few unemployed workers per vacancy, workers find jobs arbitrarily fast and firms find it arbitrarily difficult to hire workers. And when there are very many unemployed workers per vacancy, workers cannot find jobs and firms fill vacancies arbitrarily fast.

Within the owner–worker relationship, occasionally a disruption occurs, forcing workers to separate from jobs. This disruption arrives via a Poisson process with arrival rate λ. A disruption causes the firm to close and the worker becomes unemployed. Job separation and job finding govern the evolution of unemployment:

$$\dot{u}(t) = \lambda(1 - u(t)) - f(\theta(t))u(t). \tag{2.1}$$

At any time t, $1-u(t)$ employed workers separate into unemployment at rate λ and agents leave unemployment at rate $f(\theta(t))$. In steady state there is equilibrium unemployment: $u = \lambda/[\lambda + f(\theta)]$. This relationship traces out the Beveridge curve in v–u space, a downward sloping relationship between vacancies and unemployment. When there are many vacancies, firms are aggressively recruiting workers, which causes unemployment to fall. At the other extreme, when firms find recruiting too costly and do not post vacancies, unemployment rises. Figure 2.3 depicts this relationship.

The Beveridge curve traces out the relationship between labor supply and aggregate labor demand. Aggregate labor demand is the sum of vacancies across both sectors. Relative demand is denoted by $\phi(t)$, the share of vacancies in the fishery. $\phi(t)$ links the two sectors by determining the likelihood that unemployed persons become fishers.

2.3.1.4 Decisions of Workers

Employment is a more productive option than unemployment and therefore agents maintain the match to their employer until they are forced to separate into unemployment.

Let $W_\varphi(t)$ and $W_\chi(t)$ be the present discounted values of being employed outside the fishery and being employed as a fisher. When employed, workers earn wages w_i, $i \in \{\varphi, \chi\}$. Let $U(t)$ be the present discounted value of being unemployed. r is the required rate of return on assets.

Each W_i satisfies the Bellman equation:

$$rW_i(t) = w_i(t) + \lambda\left[U(t) - W_i(t)\right] + \dot{W}_i(t) \text{ for } i \in \{\varphi, \chi\}. \tag{2.2}$$

Having a job is an asset to the worker. The return on the asset W_i, the interest rate times the asset value, equals the current return (or dividends), wages w_i, plus the expected capital gains or the value of switching states in the future weighted by the likelihood of switching plus the change in W_i. The likelihood that a worker transitions from work to unemployment is the job-separation rate, λ.

The focus is on steady-state equilibrium and the time-t steady-state version of equation (2.2) is simply $rW_i = w_i + \lambda(U - W_i)$.

The value of unemployment to the worker hinges on their behavior towards work. Based on the reported ability of fishers to take jobs outside the fishery, workers do not actively seek employment in a particular sector; in other other words, there is no directed search. Unemployed workers accept any available job offer that generates a productive match. Although the assumption is stark, assuming people are only fishers feels equally unappealing and inconsistent with the evidence on the labor market of fishers presented in tables 2.1 and 2.2.

Let z denote the value of nonmarket activity, which includes unemployment benefits and leisure. The value of unemployment is

$$rU = z + f(\theta) \left[\phi W_\chi + (1-\phi) W_\varphi - U \right]. \tag{2.3}$$

While unemployed, a worker receives the flow value of nonwork. They find a job as a fisher with probability $f(\theta)\phi$ and a job outside the fishery with probability $f(\theta)(1-\phi)$. Through unemployment, a worker transitions to either sector depending on the relative demand for labor.

2.3.1.5 Decisions of Firm Owners Outside the Fishery

Firms post vacancies to recruit workers. Recruiting efforts continue until the expected value of posting a vacancy is driven to zero. A filled vacancy is a profitable match and the firm maintains this relationship until a disruption occurs.

Let J_φ and V_φ denote the present discounted values of a productive match and a posted vacancy for firms outside the fishery. Each firm employs a single worker who is paid a wage w_φ. The output of a worker in this sector is normalized to one, making the value of output equal to p_φ. The present discounted value of a working firm equals:

$$rJ_\varphi = p_\varphi - w_\varphi + \lambda(V_\varphi - J_\varphi). \tag{2.4}$$

The firm makes flow profits $p - w_\varphi$ and faces a hazard of having the employment relationship end, which occurs at rate λ.

The present discounted value of a vacancy after the initial startup costs are incurred is

$$rV_\varphi = q(\theta)(J_\varphi - V_\varphi). \tag{2.5}$$

Free entry drives the expected net profits of posting a vacancy to zero, making the value of a vacancy to a firm equal to the startup costs: $V_\varphi = k_\varphi$. The startup costs can be interpreted as purchasing a single unit of capital, which is combined through Leontief production with the worker (Acemoglu, 2001).

2.3.1.6 Decisions of Firm Owners in the Fishery, Regulated by a Sole Owner

The sole owner regulates vessel-level harvest to maximize profits, taking fleet size and prices as given. Additionally, the sole owner does not interfere with wage rates negotiated between vessel owners and fishers. I focus on steady state to get around a richer model with transition dynamics, which would quickly become intractable. Although the sole owner contemplates dynamics, they inherit a fishery that they have optimally managed up to that point in equilibrium.

In steady state, there are $\phi(1-u)$ vessels, each operated by a single fisher engaged in harvesting fish. Each vessel has a capacity to harvest $0 \le h(\tau) \le \overline{h}$ fish per period, where $\overline{h}$ represents maximum capacity. Each fisher earns wage rate w_χ. And each vessel owner in the fishery faces the flow cost of capital equal to rk. Flow profits are $p_\chi \phi(1-u)h(\tau)$ less the flow costs of $w_\chi \phi(1-u) - \phi(1-u)rk$.

Let $s(\tau)$ denote the stock of fish and $g(s(\tau))$ denote density-dependent stock growth, which is concave in $s(\tau)$. The stock of fish evolves according to growth less harvest.[5]

At any time t, the sole owner maximizes profits by regulating vessel-level harvest along the optimal trajectory of the fishery:

$$\max_{\{h(\tau)\}} \int_t^\infty e^{-r(\tau-t)}\phi(1-u)\left[p_\chi h(\tau) - w_\chi - rk\right]\,d\tau \tag{2.6}$$

subject to $ds/d\tau = g(s(\tau)) - \phi(1-u)h(\tau)$ and $h(\tau) \in [0, \overline{h}]$ with $s(t) = s_0$ given.

Letting $\Lambda(\tau)$ be the current-value costate variable associated with the stock of fish, standard arguments in dynamic programming lead to the following characterization of the problem. Linearity of the control implies a bang–bang solution. When $p_\chi > \Lambda(\tau)$, the price of fish is greater than the value of leaving the fish in the sea, directing the sole owner to set $h = \overline{h}$. Conversely, when $p_\chi < \Lambda(\tau)$, the price of fish is less than the value of leaving the fish in the sea, directing the sole owner to set harvest to zero. When $p_\chi = \Lambda(\tau)$, the sole owner is directed to harvest fish at the steady-state level: $\phi(1-u)h = g(s^\star)$.

To maintain $s^\star$, recruitment is divided among the $\phi(1-u)$ vessels. This pattern of harvest is achievable, for example, with a tradable quota system: since the fleet is homogeneous, all vessels will end up harvesting the same amount in equilibrium through cost minimization. In summary, the sole owner moves the fishery as rapidly as possible to $s^\star$ and then sets vessel-level harvest to $\hat{h} = g(s^\star)/[\phi(1-u)]$ to maintain the targeted stock level, assuming the fleet is capable of harvesting $g(s^\star)$.

The target stock level is determined by the necessary condition associated with $s(\tau)$ in the optimal-control problem: $g'(s(\tau)) = r - \frac{d\Lambda}{d\tau}/\Lambda(\tau)$. In steady state, where the shadow value of fish is unchanging, the necessary condition becomes $g'(s^\star) = r$, which provides an implicit characterization of the target stock level, $s^\star$. The sole owner regulates the fishery at the required rate of return of all other assets in the economy, a feature that undergirds the discussion in Clark and Munro (2017).

[5] See Ryan, Holland, and Herrera (2014) for a discussion of the effects harvest can have on density-dependent stock growth.

Figure 2.4 illustrates the sole owner's choice of regulation. To keep the fishery at $s^\star$, total allowable catch (TAC) is set to $g(s^\star)$. The constant escapement policy preferred by the sole owner is less than a constant escapement policy that maximizes catch. That management plan would keep the fishery at s^{MSY}, allowing the fishery to harvest $g(s^{\text{MSY}}) > g(s^\star)$ (see figure 2.4).

In steady state, the sole owner inherits a fishery they have previously managed up to that point, which involves two scenarios. The first is a fishery at $s^\star$ where the equilibrium fleet is capable of harvesting the TAC. The sole owner divides the TAC among the vessel owners. The second is a fleet in equilibrium that is incapable of harvesting the TAC (recall that $s^\star$ depends on r, the required rate of return on assets in the economy, and certain parameter values can make fishing very unprofitable). In this scenario the sole owner directs fishers to harvest $\overline{h}$ and maintains a stock level above s^{MSY}.

The equilibrium vessel-level harvest rule is therefore summarized as $\hat{h} = \min\left\{\overline{h}, \frac{g(s^\star)}{\phi(1-u)}\right\}$, or

$$\hat{h} = \begin{cases} \frac{g(s^\star)}{\phi(1-u)} & \text{if } \phi > \check{\phi}(\theta) \\ \overline{h} & \text{if } \phi \leq \check{\phi}(\theta), \end{cases} \tag{2.7}$$

where $\check{\phi} = g(s^\star)/\overline{h}/(1-u)$ depends on density-dependent growth, vessels' harvest capacity, and employment. The critical value is increasing in density-dependent growth and decreasing in vessels' harvest capacity and employment. Herein lies the notion of regulated open access: the sole owner cannot control fleet size, taking $\phi(1-u)$ as given.

Crowding and stock externalities affect vessel-level harvest or "productivity." Holding θ constant, which fixes employment, vessel-level productivity falls as ϕ increases. For low enough ϕ the fleet is incapable of harvesting $g(s^\star)$, but eventually the fleet is large enough so that it can. Each vessel is allowed to harvest less as the number of fishers increases to maintain the constant escapement policy. Panel (a) of figure 2.5 depicts this dynamic with the solid black line. For values of ϕ below $\check{\phi}^\bullet$ each vessel harvests $\overline{h}$, but as $\phi \to 1$ vessel-level productivity decreases.

Panel (b) of figure 2.5 depicts the effect of increasing employment while holding the share of fishers constant. When θ is near 0, the majority of the labor force is unemployed and the small fleet is directed to harvest $\overline{h}$. As $\theta \to \infty$, the fleet expands through expanded employment and vessel-level harvest must decrease to maintain a constant escapement policy.

Finally, the equations that correspond to equations (2.4) and (2.5) in the fishery are $rJ_\chi = p_\chi \hat{h} - w_\chi + \lambda(V_\chi - J_\chi)$ with $\hat{h}$ given in equation (2.7) and $rV_\chi = q(\theta)(J_\chi - V_\chi)$, where expansion of recruiting effort drives V_χ to equal k_χ as before.

Lastly, I describe a behavioral rule for wage determination.

2.3.1.7 Wage Determination

In search models, where matching frictions give rise to inefficiencies, there are rents to split between firms and employees. But how these rents are split still lacks a unified approach. Following the literature, workers and firm-owners split the surplus from a productive match through Nash wage bargaining. While this may be controversial in macroeconomics, each side taking a fraction of net profits holds precedence in commercial fishing, where it is convention for vessel owners to split net profits with crew under a lay system.[6]

The Nash wage bargain formalizes how the surplus is split. Each person earns an amount commensurate to the other based on the bargaining parameter, β, and their outside option—either unemployment or having a vacancy posted: $(1-\beta)(W_i-U) = \beta(J_i-V_i)$ for $i \in \{\varphi, \chi\}$, assuming that the bargaining parameter is equal throughout the economy. Under Nash bargaining the surplus a worker gains from switching from unemployment to work is proportional to the surplus a firm owner gains from switching from a vacancy to a productive match.

Using expressions for W_i, J_i, V_i, and the fact that recruiting effort expands until $V_i = k_i$, wages are

$$w_\varphi = \beta(p_\varphi - rk_\varphi) + (1-\beta)rU \text{ and } w_\chi = \beta\left(p_\chi \hat{h} - rk_\chi\right) + (1-\beta)rU. \tag{2.8}$$

Workers earn a fraction of the net flow of profits—productivity minus the flow cost of capital—plus a fraction of the flow value of their outside option, rU. Wages, in both sectors, are increasing in own price and the value of unemployment.

2.3.2 Characterization of Equilibrium

An equilibrium consists of prices; wages, determined by Nash bargaining; regulation set by the sole owner; tightness in the labor market, which determines equilibrium unemployment; the share of employment devoted to commercial fishing; and market clearing. I will look for equilibria where the cost of capital is the same in both sectors and it is relatively more productive to fish from an unexploited stock than it is to produce in the non-fishery sector. Positing $k_\chi = k_\varphi = k$ maintains the startup costs are the same in both sectors of the economy. Positing that fishing is more productive than production in the

[6] Studies discussing lay systems include Sutinen (1979); Davis, Gallman, and Hutchins (1990); Hannesson (2000); and McConnell and Price (2006)

non-fishery sector when the stock is near carrying capacity, $\overline{h} > 1$, is based on reports of early American colonists about the abundance of natural resources.

To find an equilibrium, I combine the value functions W_χ, W_φ, and U with expressions for wages to write the equilibrium value of unemployment in terms of prices, θ, and ϕ. I combine the equilibrium value of unemployment with the firms' value functions J_χ, J_φ, V_χ, and V_φ to write two job-creation conditions in terms of prices, θ, and ϕ. To close the model, I use market clearing to write prices in terms of θ and ϕ.

Prices depend on the relative levels of output produced in the two sectors. Output depends on the level of employment and sectoral composition. The equilibrium level of employment, in terms of θ, is determined by the Beveridge-curve relationship in (2.1): $1 - u = f(\theta)/[\lambda + f(\theta)]$. There are $1 - u$ workers engaged in production, of which ϕ are fishers and $1 - \phi$ are employed in the non-fishery sector. Total non-fishery production is $Y_\varphi = (1 - \phi)(1 - u)$. Total catch in the fishery depends on $\check{\phi}(\theta)$. If the $\phi(1 - u)$ fishers maintain the stock level $s^\star$, then total catch in the fishery is $Y_\chi = g(s^\star)$. On the other hand, if the fleet size is relatively small, then total catch is $Y_\chi = \phi(1 - u)\overline{h}$.

Using the two market-clearing conditions, $Y_\chi = X_\chi$ and $Y_\varphi = X_\varphi$, the price of fish in terms of θ and ϕ is

$$p_\chi = \begin{cases} \alpha \left[\frac{(1-\phi)(1-u)}{g(s^\star)}\right]^{1-\alpha} & \text{if } \phi > \check{\phi}(\theta) \\ \alpha \overline{h}^{\alpha-1} \left(\frac{1-\phi}{\phi}\right)^{1-\alpha} & \text{if } \phi \leq \check{\phi}(\theta), \end{cases} \tag{2.9}$$

and the price of non-fishery output in terms of θ and ϕ is

$$p_\varphi = \begin{cases} (1-\alpha) \left[\frac{g(s^\star)}{(1-\phi)(1-u)}\right]^{\alpha} & \text{if } \phi > \check{\phi}(\theta) \\ (1-\alpha) \overline{h}^{\alpha} \left(\frac{\phi}{1-\phi}\right)^{\alpha} & \text{if } \phi \leq \check{\phi}(\theta). \end{cases} \tag{2.10}$$

Existence and uniqueness of an equilibrium then depend on the properties of the two job-creation conditions, written only in terms of θ and ϕ. The remainder of this section explores the value of unemployment and properties of the two job-creation conditions.

2.3.2.1 The Value of Unemployment

Labor-market options in both sectors add value to unemployment. The value of unemployment therefore reflects crowding and stock externalities, depending on regulation set by the sole owner. Regulation affects the value of unemployment in a nonstandard way. When employment is low and there are few fishers, vessels operate at maximum capacity with $\overline{h} > 1$. But as employment expands and the fleet grows, vessels are forced to curtail production to maintain the TAC. The value of unemployment first increases along with a tighter labor market, but can then decrease as crowding and stock become increasingly meaningful.

To see the effects of crowding and stock externalities, write the value of unemployment in terms of θ and ϕ using equation (2.2) in equation (2.11) and the expressions for wages in equation (2.8):

$$\Gamma(\theta, \phi) := rU = \frac{(r+\lambda)z + \beta f(\theta)\left[\phi p_\chi \hat{h} + (1-\phi)p_\varphi - rk\right]}{r+\lambda+\beta f(\theta)}, \tag{2.11}$$

where prices are given in (2.9) and (2.10) and the equilibrium level of unemployment comes from (2.1).

Unlike standard search-and-matching models, the value of unemployment cannot be shown to be everywhere increasing in tightness. Typically this is true: a person can more easily transition from unemployment to work the higher the job-finding rate, which itself is increasing in tightness. The unemployment rate is lower and the worker expects to spend, on average, less time in unemployment; the bargaining position of the worker improves, thereby raising wages that a worker is more likely to earn.

For the same reasons, when crowding and stock externalities have no effect on the value of average productivity the value of unemployment is increasing in tightness. When $\phi < \check{\phi}$, fishers operate at full capacity, harvesting $\overline{h}$ and adding an additional fisher at the margin does not affect vessel-level harvest. But when externalities are meaningful, when $\phi > \check{\phi}$, the value of unemployment may decrease in tightness. While increasing tightness makes it more likely for a worker to find a job, the value of average productivity, $\phi(p_\chi \hat{h} - k_\chi) + (1-\phi)(p_\varphi - k_\varphi)$, might fall—people find jobs more quickly but are more likely to be less productive fishers. As the number of fishers increases, vessels are directed to harvest less through the regulation set in (2.7) and depicted in figure 2.5. Vessel-level harvest is falling in the level of employment, $1-u$.

The relationship between Γ and ϕ is similar: The value of unemployment is first increasing and then decreasing in the share of fishers. When ϕ is small, productivity in the fishery is $\overline{h} > 1$ and shifting work to this sector increases the value of unemployment; but as ϕ increases and the fleet is capable of harvesting $g(s^\star)$, vessels are forced to harvest less according to the regulation in (2.7) and depicted in figure 2.5. This causes the value of unemployment to fall.

The results of this section are summarized in proposition 1.

Proposition 1 *Because of crowding and stock externalities associated with harvesting fish, the value of unemployment given in* (2.11)*, for a given level of tightness, is first increasing* *and then decreasing in the share of fishers. Similarly, for a given share of fishers, the value of unemployment is first increasing in tightness, but is then dragged down by the effect crowding and stock externalities have on average productivity. The value of unemployment can even decrease with tightness if the effect is strong enough.*

Details of proposition 1 are provided in sections B.1.4 and B.1.5 of the appendix.

Because all workers have a chance of experiencing unemployment, proposition 1 establishes that crowding and stock externalities have economy-wide effects. Regulation set by the sole owner affects everyone, foreshadowing the differences in preferred harvest patterns between the sole owner and a social planner. But before these comparisons can be made, I need to establish existence and uniqueness of the equilibrium in the decentralized economy, the subject of the following section.

2.3.2.2 Existence and Uniqueness of the Equilibrium

Job creation in the non-fishery sector is determined by the value functions for firms. Combining the value functions V_φ and J_φ with the expression for Nash-bargained wages in (2.8) leads to the job-creation condition in the non-fishery sector:

$$p_\varphi - \Gamma(\theta, \phi) = rk + \frac{rk(r+\lambda)}{(1-\beta)q(\theta)} \tag{2.12}$$

Combining the value functions V_χ and J_χ with the expression for Nash-bargained wages in (2.8) leads to the job-creation condition in the fishery:

$$p_\chi \hat{h} - \Gamma(\theta, \phi) = rk + \frac{rk(r+\lambda)}{(1-\beta)q(\theta)}. \tag{2.13}$$

A steady-state equilibrium is characterized by the intersection of the two job-creation conditions given in equations (2.12) and (2.13). Investigating the properties of these two loci leads to the following proposition:

Proposition 2 *A unique steady-state equilibrium, $(\phi^\star, \theta^\star)$, always exists in the decentralized economy with $\phi^\star = \alpha \in (0,1)$, where prices are given in equations (2.9) and (2.10), wages are given in equation (2.8), and equilibrium unemployment is $u^\star = \lambda/[\lambda + f(\theta^\star)]$.*

Some intuition for the result is provided in figure 2.6. The curve labeled φ depicts $\theta_\varphi(\phi)$ defined implicitly in equation (2.12) and the curve labeled χ depicts $\theta_\chi(\phi)$, defined implicitly by equation (2.13).

When ϕ is near 0, few fish are caught and it is very profitable to create jobs in the fishery since the price of fish is high. Firms respond by purchasing a vessel and expanding recruiting effort, causing a rise in θ_χ. As $\phi \to 1$, however, more fish are caught and the low price of fish makes it unprofitable to purchase a vessel and post a vacancy with the hope of fishing. By this logic, job creation in the fishery, the χ locus, is downward sloping.

Conversely, the φ locus is upward sloping, reflecting low prices when ϕ is near 0 and high prices when ϕ is near 1. Low prices mean recruiting effort is low, pushing down θ_φ; when prices are high, recruiting effort expands, increasing θ_φ.

Because one curve slopes upward and the other downward, an equilibrium is guaranteed.

To establish the result more formally, combine the two job-creation conditions in equations (2.12) and (2.13). Because the right-hand sides are the same and Γ is common to both, $p_\varphi = p_\chi \hat{h}$. Using this result, straightforward algebra establishes that $\phi^\star = \alpha$. The two job-creation conditions reduce to a single equation in θ alone.

One can write this as $\mathcal{T}(\theta^\star) = 0$, where

$$\mathcal{T}(\theta) := \frac{(p_\varphi(\theta) - rk - z)(r + z)}{r + \lambda + \beta f(\theta)} - \frac{rk(r + \lambda)}{(1 - \beta)q(\theta)} \tag{2.14}$$

with $p_\varphi(\theta)$ given in (2.12), using $\phi = \alpha$. Using the fact that $\mathcal{T}$ is continuous in θ, an application of the intermediate value theorem establishes existence of a $\theta^\star \in (0, \infty)$ such that $\mathcal{T}(\theta^\star) = 0$. Uniqueness follows from $\mathcal{T}$ being monotonically decreasing in θ. Section B.1.9 in the appendix provides the details.

The next section explores how $\mathcal{T}$ varies with respect to biological parameters.

2.3.2.3 The Effect of Biological Parameters on Unemployment

So far no mention has been made of biological parameters other than using density-dependent growth for the evolution of the stock. To explore the effects of biological parameters on macroeconomic variables, parameterize g by K, where g is increasing in K. The parameter K can stand for any number of biological fluctuations. Within the context of logistic growth, for example, it could represent the growth rate of the stock or carrying capacity. While all that matters is that g is increasing in K, to be concrete I consider an increase in carrying capacity.

An increase in carrying capacity directly affects vessel-level harvest. Because $\check{\phi}$ is increasing in K, more vessels can operate at full capacity. Moreover, vessel-level harvest is weakly increasing in K, a result that comes directly from the harvest rule in (2.7). Consider $K^\bullet < K^{\bullet\bullet}$. Panel A of figure 2.5 depicts vessel-level harvest holding θ fixed and increasing ϕ. Initially vessels are directed to harvest $\bar{h}$, but as the fleet expands, the sole owner directs vessels to harvest less in order to maintain $s^\star$. Increasing carrying capacity to $K^{\bullet\bullet}$ shifts the entire profile of vessel-level harvest upward. The upward shift also happens when the share of fishers is held fixed and θ shifts, which is shown in panel B of figure 2.5. The horizontal line in figure 2.5 indicates that a larger carrying capacity expands the region over which nominal productivity in the fishery is above nominal productivity in the non-fishery sector.

Figure 2.5, though, is a description of partial equilibrium—it does not reflect prices. The macroeconomic effects of increased carrying capacity come through $\mathcal{T}$. $\mathcal{T}$ depends positively on p_φ, which depends positively on g and therefore K. Thus $\mathcal{T}$ depends positively on K. Starting from $\mathcal{T}(\theta^\star(K), K) = 0$, for any $\theta^\star \in [0, \infty)$, increasing carrying capacity weakly increases $\mathcal{T}$. Because $\mathcal{T}$ depends negatively on θ, $\theta^\star(K)$ must therefore weakly increase to maintain $\mathcal{T}(\theta^\star(K), K) = 0$. The effect is weak because increasing carrying capacity when fishers are directed to harvest $\overline{h}$ may have no effect on vessel-level harvest.

The result is summarized in proposition 3.

Proposition 3 *Increasing carrying capacity has no effect on the equilibrium share of fishers and lowers equilibrium unemployment; that is, for* $K^\bullet < K^{\bullet\bullet}$, $\theta^\star(K^\bullet) \geq \theta^\star(K^{\bullet\bullet})$.

The effect of increasing carrying capacity from $K^\bullet$ to $K^{\bullet\bullet}$ is depicted in figure 2.3 for the case where the inequality in proposition 3 is sharp. The Beveridge curve relationship that comes from (2.1) is independent of K. The effect of increasing K lowers the level of unemployment and lifts the number of vacancies as the economy moves along the Beveridge curve.

2.4 On the Optimal Composition of Jobs and Socially Preferred Harvest Pattern

The previous section described a benchmark economy that featured commercial fishing regulated by a sole owner and productive options outside fishing. This section asks whether there are potential economy-wide gains from implementing a different harvest policy than the one set by the sole owner in (2.7).

I consider two versions of a hypothetical social planner who wants to maximize economy-wide surplus. The first starts from the decentralized equilibrium and contemplates reallocating labor across the two sectors and implementing a different level of total allowable catch by targeting a different steady-state level of stock. The second is an oracle social planner with control of fishery management and job creation in both sectors. The oracle planner implements the full dynamic program by taking into account stock dynamics and frictions in the labor market. Along the trajectory of the economy the oracle planner chooses the share of fishers in the economy, vessel-level harvest, and the vacancy–unemployment ratio, which is isomorphic to choosing the level of vacancies as u is predetermined.

My main result is that economy-wide surplus can be increased by managing the fishery at MSY as opposed to MEY. This result holds when fishery management plans cannot control job-creation policies. which seems to be case with the most real-world relevancy. When fishery management plans are designed in conjunction with job-creation policies by the oracle planner, economy-wide surplus is maximized at MEY in the stationary equilibrium.

2.4.1 Starting from the Decentralized Equilibrium

The convention in search-and-matching models is to discuss welfare in terms of labor-market tightness. Since wages are determined after a match, the agreed-upon wages do not internalize the effects of matches on workers and firms still searching (Pissarides, 2000, chapter 8). The economy will be suboptimal unless the Hosios (1990) condition is satisfied, which allocates in a socially optimal way the surplus captured by firms, determining the profitability of job creation and thus the socially optimal level of job creation. This condition, however, is unlikely to hold in practice.[7]

Instead of focusing on a labor-market policy that affects wage setting over the business cycle, there are two margins that a social planner might consider adjusting. The first margin is the composition of jobs (Acemoglu, 2001). In this scenario the social planner adjusts ϕ, controlling the entry–exit margin of the fishery subject to economy-wide job creation and destruction to maximize surplus in equilibrium. The sole owner still regulates the fishery, but they inherit the number of fishers from the social planner. The social planner determines number of fishers in a socially optimal way.

The second margin of adjustment worth considering is the choice of $s^\star$ with economy-wide welfare in mind instead of fishery profits. The choice of $s^\star$ determines the constant escapement policy and therefore total harvest or productivity.

2.4.1.1 The Optimal Composition of Jobs

The social planner is interested in maximizing equilibrium surplus. Surplus equals the net value of output plus the flow value of nonwork benefits less the flow costs of job creation in both sectors. Total output is produced by $1-u$ workers. There are $(1-u)(1-\phi)$ workers

[7] See Pissarides (2000, chapter 8) for further discussion.

outside the fishery, producing a good valued at p_φ, where p_φ is given in equation (2.10). The $(1-u)\phi$ fishers catch $\hat{h}$, determined by the sole owner, with value p_χ given in equation (2.9). The flow cost of capital to all firms is rk. The flow value of nonwork experienced by unemployed persons is z. There are $v = \theta u$ firms engaged in job creation, making the flow cost of job creation equal to θurk.

Surplus is $\mathcal{S} = (1-u)[\phi(p_\chi \hat{h} - rk) + (1-\phi)(p_\varphi - rk)] + zu - \theta urk$. Can the social planner, in terms of the composition of jobs, do better than the decentralized equilibrium? Notice that

$$\left.\frac{\partial \mathcal{S}}{\partial \phi}\right|_{\text{dec. eqm.}} = \begin{cases} \frac{f(\theta)}{\lambda+f(\theta)}(p_\chi \overline{h} - p_\varphi) = 0 & \text{if } \phi \leq \check{\phi}(\theta) \\ -\frac{f(\theta)}{\lambda+f(\theta)} p_\varphi < 0 & \text{if } \phi > \check{\phi}(\theta) \end{cases}. \tag{2.15}$$

First consider the case where $\phi < \check{\phi}(\theta)$. In the decentralized equilibrium the value of catching an additional fish, $p_\chi \overline{h}$, equals the value of producing one more unit of the second good, p_φ. This fact can be seen in equations (2.12)) and (2.13. In equilibrium the right-hand sides are equal and since $\Gamma(\theta, \phi)$ is common to both, $p_\chi \overline{h} = p_\varphi$. Starting from a decentralized equilibrium the social planner can do no better—the decentralized equilibrium directs fishers to operate at maximum capacity.

Now consider the case where $\phi > \check{\phi}(\theta)$. In this case the decentralized economy still balances the value of catching an additional fish with the value of producing one more unit of the second good, but this is achieved by reducing the productivity of fishers. When the stock of fish is maintained at $s^\star$ and vessel-level harvest is regulated to be $g(s^\star)/[\phi(1-u)]$, the social planner can increase equilibrium surplus by reducing the number of fishers. The inefficiency arises because once the fleet is capable of catching enough fish to maintain the target stock level, adding fishers only reduces productivity—total catch is the same. The value of the marginal product of these excess workers would be higher outside the fishery; in other words, there is high opportunity cost the sole owner is not considering.

2.4.1.2 The Optimal Target Stock Level

Turning to the target stock level, can the social planner do better than the decentralized equilibrium by targeting a different stock level than the one chosen by the sole owner? The question can be answered by checking whether surplus can be increased:

$$\left.\frac{\partial \mathcal{S}}{\partial s}\right|_{\text{dec. eqm.}} = \begin{cases} 0 & \text{if } \phi \leq \check{\phi}(\theta) \\ p_\chi g'(s^\star) > 0 & \text{if } \phi > \check{\phi}(\theta) \end{cases}. \tag{2.16}$$

The latter term is positive because the resource is being exploited at rate r, making $g'(s^\star) > 0$, depicted in figure 2.4.

When vessels are harvesting $\overline{h}$ there are too few fishers to maintain the stock at $s^\star$. It is just not profitable enough for potential vessel owners to post vacancies and recruit workers into the fishery, despite the generous harvesting policy. Targeting a different stock level would either have no effect or reduce potential profits in the fishery. The social planner, therefore, can do no better than the decentralized equilibrium.

But when the fleet is capable of catching the desired harvest level there is room for the social planner to intervene by pushing g' to 0. From figure 2.4 it can be seen that this is achieved by targeting s^{MSY}. Total harvest increases from $g(s^\star)$ to $g(s^{\mathrm{MSY}})$, which allows each vessel to harvest more (the total number of vessels is fixed in the thought experiment). The economy-wide effect is like a positive productivity shock or an increase in carrying capacity.

The following proposition summarizes these two findings.

Proposition 4 *Let $\phi^{\mathcal{S}}(\theta)$ be the value of ϕ that the social planner would choose at labor market tightness θ and let $s^{\mathcal{S}}(\theta)$ be the stock level the social planner would choose. Comparing these to $\phi^\star(\theta)$ and $s^\star(\theta)$, the values in the decentralized equilibrium,*

$$\phi^{\mathcal{S}}(\theta) = \phi^\star(\theta) \text{ and } s^{\mathcal{S}}(\theta) = s^\star(\theta) \text{ when } \hat{h} = \overline{h}$$

and

$$\phi^{\mathcal{S}}(\theta) < \phi^\star(\theta) \text{ and } s^{\mathcal{S}}(\theta) > s^\star(\theta) \text{ when } \hat{h} = g(s^\star)/\phi^\star(1-u).$$

The algebra that establishes proposition 4 is contained in sections B.1.10 and B.1.11, which are contained in an appendix.

Whenever the fleet is capable of catching more than the desired stock level either reducing the number of fishers or increasing the target stock level will increase equilibrium surplus. Even if the fishery is over-capitalized, in other words, in this economy targeting MSY will increase economy-wide welfare. Either management objective increases steady-state surplus.

Sustainability is not at issue. First, as figure 2.4 illustrates, MSY is associated with a larger stock level. In order to harvest more fish while maintaining a target stock-level, more fish need to be in the sea to increase recruitment, which in turn requires larger harvest levels to maintain the target stock-level. This feature of the model, though, is mechanical and might depend crucially on density-dependent growth.

A second issue is that MSY requires more "effort" and if catch is stochastic, then by chance greater levels of effort might push the stock too low. In unfettered fisheries this might be a major concern, but as Homans and Wilen (1997) emphasize, most fisheries do not fall under this rubric. Which is the case here—holding the fleet size constant, increasing vessel-level harvest leads to higher steady-state surplus. Associated with MSY, in other words, is not a larger fleet with overfishing proportional to fleet size.

What is most important to the conclusions, though, is the classification of workers. In concord with the facts presented in tables 2.1 and 2.2, workers are treated symmetrically, making the goal of the social planner equilibrium surplus. Because of the ready labor-market alternatives and the fluidity with which these outside options are exercised, the value of unemployment is the same for all workers, meaning $\Gamma(\theta, \phi)$ in equation (2.11) does not vary by sector. From equations (2.12) and (2.13), it follows that $p_\varphi = p_\chi \hat{h}$ and therefore the sectoral wages are equal, a result that follows directly from (2.8). Everyone benefits from equilibrium surplus. On the other hand, if fishers are only fishers, then targeting MSY may come at the cost of fishers. From the worker perspective, this strategy lowers the value of unemployment and thus affects wage bargaining. From the perspective of vessel owners, targeting MSY lowers profits.

The gap between Christensen (2010) and Sumaila and Hannesson (2010) reflects the missing pieces between "benefits" generally and what is meant by the fishery sector, a gap that orienting the model in general equilibrium is meant to fill in a small way.

2.4.2 The Oracle Planner

The oracle planner chooses the share of fishers in the economy, a vessel-level harvest policy, and the vacancy–unemployment ratio subject to labor-market frictions and the evolution of the stock of fish. The oracle planner solves

$$\max_{\substack{\{\theta(t)\},\\ \{\phi(t)\}\in[0,1],\\ \{h(t)\}\in[0,\bar{h}]}} \int_0^\infty e^{-rt} \left\{(1-u(t))\left[\phi(t)p_\chi(\phi(t),h(t))h(t) + (1-\phi(t))p_\varphi(\phi(t),h(t)) - rk\right]\right.$$
$$\left. + zu(t) - \theta(t)u(t)rk\right\} \; dt,$$

subject to $\dot{u}(t) = (1-u(t))\lambda - f(\theta(t))u(t)$ and $\dot{s}(t) = g(s(t)) - \phi(t)(1-u(t))h(t)$ with $u(0) = u_0$ and $s(0) = s_0$ given.

I set up the current-value Hamiltonian, using $\sigma_1(t)$ and $\sigma_2(t)$ as the Kuhn–Tucker multipliers for the constraints $h(t) \leq \bar{h}$ and $h(t) \geq 0$. In the stationary equilibrium of this economy, I show that $\sigma_1 > 0$, making it optimal for the oracle planner to set $h = \bar{h}$. The result is summarized in proposition 5.

Proposition 5 *In the stationary equilibrium of the economy regulated by the oracle social planner, the oracle planner maintains the stock level implicitly defined by* $g'(s^{\star\star}) = r$. *If the matching technology is such that* $f(0) = 0$ *and* f *is concave, any interior equilibrium with* $\phi^{\star\star} \in (0,1)$ *requires a positive level of harvest and a positive level of vacancies. Moreover, if the matching technology is Cobb–Douglas, there is a unique level of unemployment in the stationary equilibrium and the oracle planner directs the fleet to harvest at* $\bar{h}$.

Proposition 5 is established in section B.1.12 of an appendix.

Proposition 5 is one reason for the tension between advocates of MEY versus advocates of MSY. The forward-looking oracle planner with total control of the economy chooses $\bar{h}$ in the stationary equilibrium and manages the fishery at a rate equal to the required rate of return on all assets in the economy. In the decentralized equilibrium regulated by a forward-looking sole owner without total control, however, managing the fishery at MSY can lead to greater economy-wide surplus.

2.5 Calibration

The previous section established that managing a fishery at MSY as opposed to MEY, when regulation prevents vessels from operating at maximum capacity, leads to economy-wide gains. In this section, I evaluate the gains by calibrating the steady state of the model to match the 2015 Alaskan labor market.

A meaningful laboratory for studying preferred harvest patterns can be found in Alaska. Alaska's fisheries are, for the most part, regulated by the The North Pacific Fishery Management Council. The Council uses a patchwork of regulation to manage Alaska's fisheries. The clearest statement of their regulation involves the Bering Sea and Aleutian Islands Management Area, which specifies that the groundfish complex operates at 85 percent of the historical estimate of MSY.

I use this target-stock policy to compare steady states of the model economy. Specifically, I match the average vacancy–unemployment ratio that prevailed in Alaska over 2015 to determine $0.85 \times$ MSY and calculate the gains from increasing TAC to MSY. For the calibration exercise, I posit that the Alaskan economy is capable of maintaining the stock of fish at MSY.

2.5.1 Parameters

The time period is one month. I set the required rate of return on assets to match a 5 percent annual rate. The parameter α represents the share of consumers' expenditure on fish.[8] I take α to be a fundamental parameter and use US consumers' 2015 expenditures on fishery products at food service establishments plus retail sales for home consumption. This is divided by 2015 personal consumption expenditures on nondurable goods and services to calculate the share of income spent on fish. This parameter reflects the value of chain of fishing (but not perfectly).[9]

Turning to labor-market parameters, I used Shimer's (2012) procedure to adjust transition rates for temporal aggregation. I used aggregate US data for the calculation. I set the separation rate, λ, equal to the average UE transition rate in 2015. The flow benefit of unemployment comes from Hall (2005) and represents close to 40 percent of the equilibrium wage. The cost of purchasing a unit of capital to begin production, k, is set equal to Hall's flow cost of posting a vacancy multiplied by the expected duration of posting, $1/q\left(\theta^{\star}\right)$. The bargaining parameter, β, is set to 0.5 so that fishers and vessel owners evenly split the gains from their productive match.

The calibration strategy is to match the average vacancy–unemployment ratio for Alaska. Data on Alaskan unemployment are available from the Local Area Unemployment Statistics program. Data on Alaskan vacancies are available from the Conference Board.

Recent vacancy data most often come from the Job Openings and Labor Turnover Survey, which are based on firms' responses. But these data are not disaggregated to the state level. The Conference Board's vacancy data, while available at the state level, are constructed from vacancies advertised online. Since I am unaware of any estimated matching technology based exclusively on vacancies advertised online, I use the Conference Board's data on vacancies to estimate a matching technology for the United States. I parameterize the matching technology as Cobb–Douglas, $m = \omega v^{\xi} u^{1-\xi}$. Based on this estimate, I take $\xi = 0.858$. I use ω in the matching function to match Alaska's 2015 average unemployment rate, which was 7.95 percent. Finally, I set the maximum harvest capacity, $\bar{h}$, to 1.1. The parameters are listed in table 2.3.

2.5.2 Implications of Managing the Fishery at MSY as Opposed to MEY

The implications of managing the fishery at MSY as opposed to MEY increase the TAC by almost 18 percent. Doing so lowers the unemployment rate from 7.95 percent to 7.59

[8]The model economy is isomorphic to one where consumers have Cobb–Douglas utility with the coefficient on fish equal to α.

[9]Personal consumption expenditures on nondurable goods and services is listed in table 1.1.5, lines 5 and 6, in the National Income and Product Accounts.

percent. Figure 2.6 shows the increase in equilibrium labor-market tightness, $\theta^{\star}$.[10]

The equilibrium for the initial calibration of the model is depicted in figure 2.6 as the intersection of the two job-creation curves depicted with solid black lines. The job-creation curves are parameterizations of $\theta_{\chi}(\phi)$ and $\theta_{\varphi}(\phi)$, implicitly given in equations (2.12) and (2.13). As proved in proposition 2, the share of fishers in the economy is α. When TAC increases, both job-creation curves shift upwards. This shift is depicted in the figure with broken red lines. The increase in TAC shifts the vessel-level harvest profile rightward, as depicted in figure 2.6. Catch increases. When catch increases, the relative price of the non-fishery good increases. Firms find it more profitable to recruit workers in the non-fishery sector. The economy expands as the economy moves along the Beveridge curve in figure 2.3. The increase in aggregate demand includes increased demand for fish at the new equilibrium. The expansion is similar to the expansion associated with an increase in carrying capacity described in proposition 3.

For the calibration, the increase in labor-market tightness depicted in figure 2.6 expands Alaskan employment by 1,300 jobs, or 0.4 percent, in *both* sectors of the economy. Unemployment is lower and equilibrium economy-wide surplus increases by 0.13 percent.

Only 11 ($1,300 \times \phi^{\star}$) of the jobs are created in Alaskan commercial fisheries. The remainder represent the part of the economy that gets fish from "sea to plate" (Christensen, 2010) and the associated macroeconomic linkages.

This general-equilibrium effect is unaccounted for by the sole owner but accounted for by the social planner. The tighter labor market in equilibrium means wages are higher. Higher wages increase demand for the homogeneous final good. Faced with increased demand, final-good producers pay more for inputs, both fish and non-fishery goods alike. Overall surplus increases.

2.6 Conclusion

To gauge the benefits of acknowledging linkages between commercial fisheries and the macroeconomy, I presented an operational model economy calibrated to match the 2015 Alaskan economy. The calibration exercise increased the TAC from MEY to MSY and expanded Alaskan employment by 1,300 jobs, but only 11 of those were in commercial fisheries.

This result was guaranteed by propositions 2 and 4, which establish that the share of fishers in equilibrium is α and that equilibrium surplus increases when the fishery is

[10]Figure 2.6 also shows the non-responsiveness of job creation in the non-fishery sector for the small equilibrium share of fishers—the curve is essentially flat in this region.

managed at MSY and harvest policies constrain effort. On the other hand, if the calibration exercise parameterized fishers as fishing near maximum capacity, there would be a much smaller effect.

Proposition 5 establishes that a stationary social policy controlling the number of vacancies posted in the economy, the share of fishers, and a vessel-level harvest policy would have each vessel fish at full capacity and maintain the stock at MEY.

But that level of control is beyond the scope of most fishery management plans. When that is the case, according to proposition 4, there is scope for reallocating labor input away from commercial fisheries or managing the fishery at MSY.

In this paper, I have tried to show how incorporating linkages between commercial fisheries and the larger economy can affect management plans and targeted stock levels. If correct, the theory implies that, in addition to recognizing biological linkages between fisheries and ecosystems, there may be substantial benefits to recognizing macroeconomic linkages as well.

Table 2.1: Fraction remaining fishers coming from unemployment

Occupational classification scheme	Title	Fraction that remain fishers
1976–1982	Fishermen and oystermen	.468
1983–1991	Fishers	.501
1992–2002	Fishers	.532
2003–2010	Fishers and related fishing workers	.486
2011–2018	Fishers and related fishing workers	.459
1976–2018		.508

Sources: Data adapted from Flood et al. (2015); Drew, Flood, and Warren (2014).
Note: Occupational classification scheme refers to the occupational codes used in the Current Population Survey. The fraction that remain fishers is calculated by looking at unemployment-to-employment transitions for unemployed fishers.

Table 2.2: Occupations taken by unemployed fishers

Title	Fraction
1976–1982	
Janitors and sextons	.079
Construction laborers	.075
Garage laborers and car washers and greasers	.058
1983–1991	
Laborers, except construction	.134
Carpenters	.079
Farmers, except horticulture	.064
1992–2002	
Laborers, except construction	.127
Construction laborers	.113
Bus, truck mechanic	.111
2003–2010	
Office clerks, general	.078
Packers and packagers, hand	.072
First-line supervisors/managers of retail sales workers	.068
2011–2018	
Laborers and freight, stock, and material movers	.133
Retail salesperson	.118
Packers and packagers, hand	.066

Sources: Data adapted from Flood et al. (2015); Drew, Flood, and Warren (2014).

Note: The fraction is calculated conditional on switching occupation. Only the top three occupations are listed for each occupational classification scheme.

Table 2.3: Calibration

Parameter	Interpretation	Value
ω	Matching efficiency	.319
ξ	Elasticity of job-finding with respect to θ	.858
z	Flow value of unemployment	.4
$k_\chi = k_\varphi$	Cost of capital	2.983
λ	Separation rate	.022
r	Interest rate	.004 07
β	Bargaining parameter	.5
α	Income share devoted to fish	.0087
$\overline{h}$	Vessel-level harvesting capacity	1.1

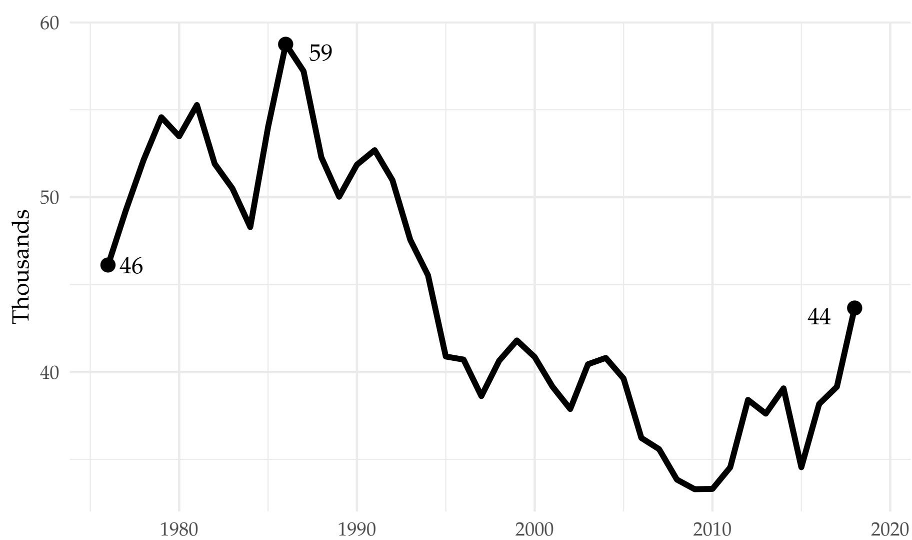


Figure 2.1: Employment of fishers, three-year moving average, 1976–2018.

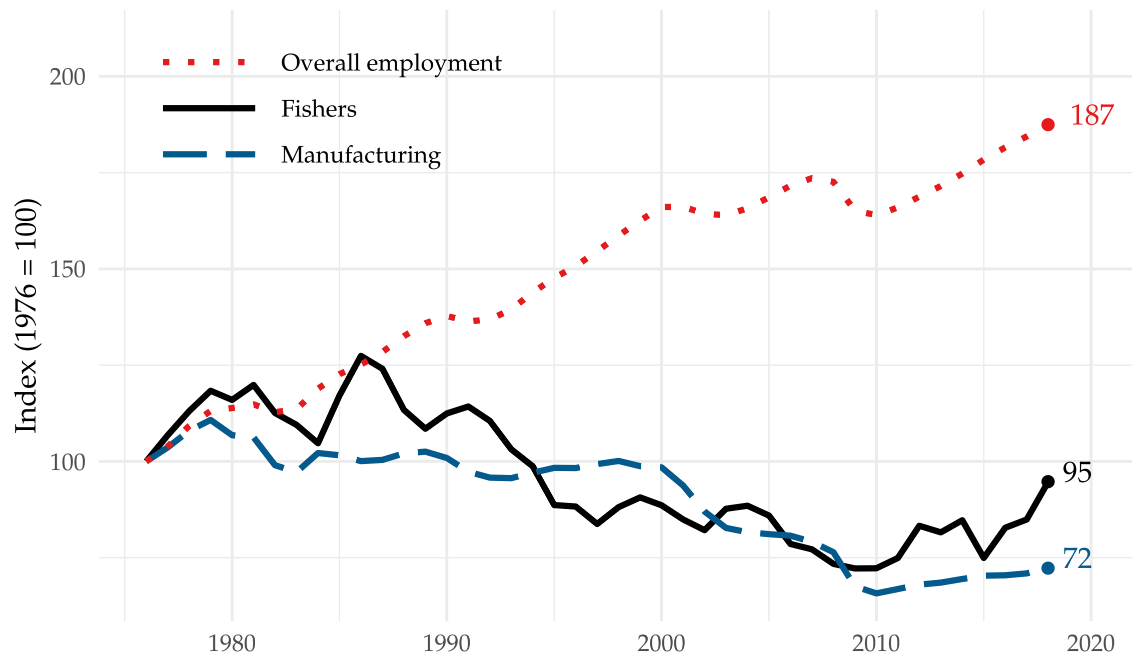


Figure 2.2: Indexes of employment series, 1976–2018.

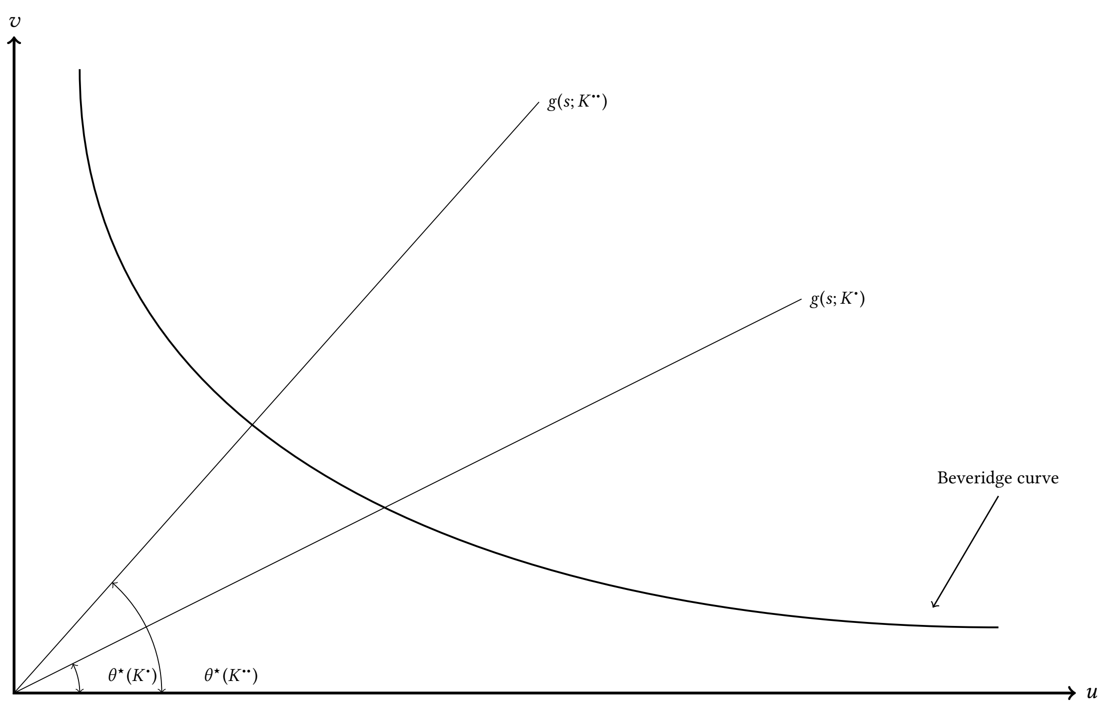


Figure 2.3: Beveridge curve. Increasing stock growth from a low carrying capacity ($K^{\bullet}$) to a high carrying capacity ($K^{\bullet\bullet}$) increases labor-market tightness thereby lowering the unemployment rate.

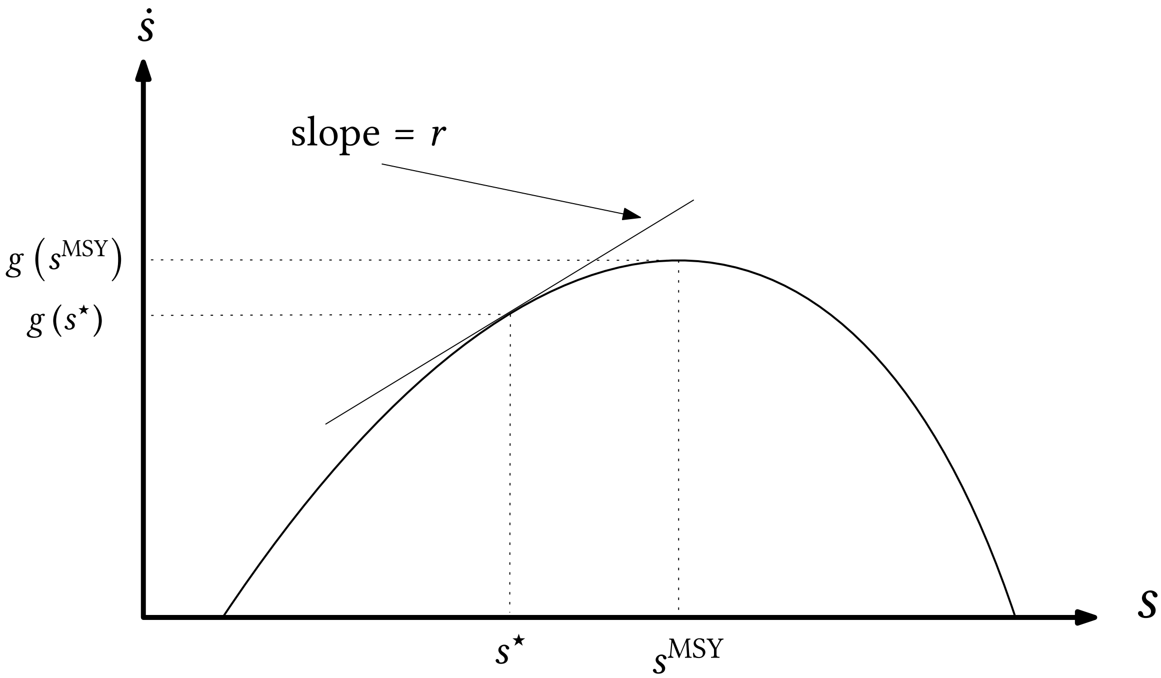
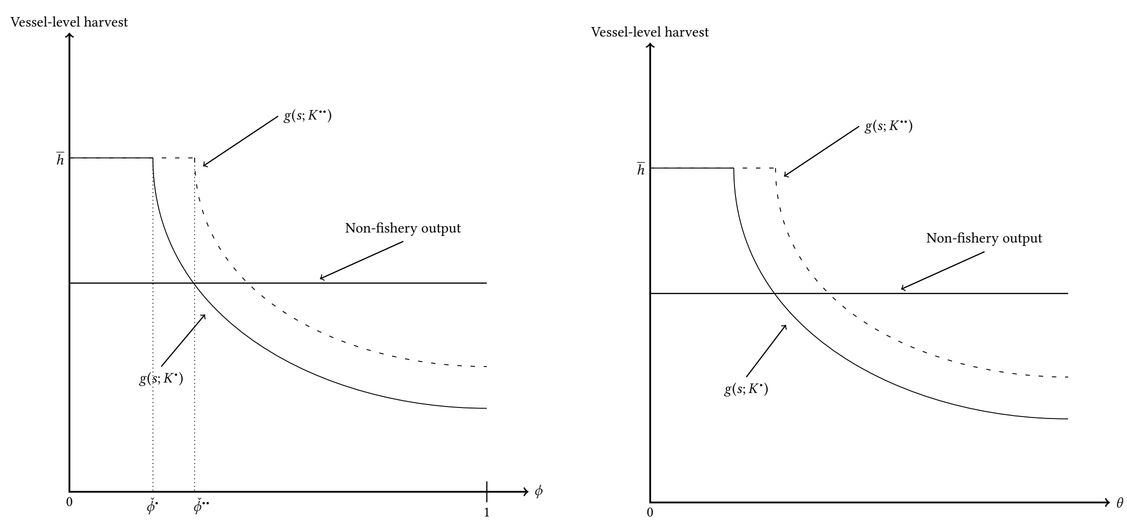


Figure 2.4: Determination of $s^\star$ and comparison of $g\left(s^\star\right)$ versus $g\left(s^{\text{MSY}}\right)$.



A: Holding θ fixed

B: Holding ϕ fixed

Figure 2.5: Partial-equilibrium profile of vessel-level harvest for different biological parameters.

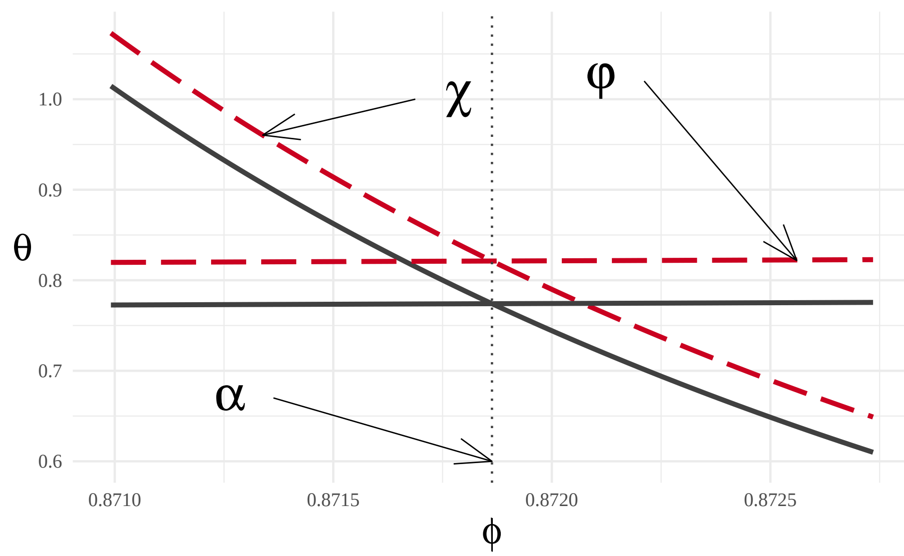


Figure 2.6: Numeric equilibrium, response of managing the fishery at MSY as opposed to MEY. The curve labeled χ depicts the job-creation curve in the fishery. The curve labeled φ depicts the job-creation curve in the non-fishery sector. The black curves match the Alaskan economy and the red curves depict the effect of managing the fishery at MSY as opposed to MEY. The equilibrium share of fishers in both cases is α, measured in percentage points.

CHAPTER 3

Tracking Earnings Ability over the Business Cycle: Implications for Wage Statistics

In data from the National Longitudinal Survey of Youth 1979, individual-level wages exhibit substantially more procyclicality than the average wage. Theories of the labor market disagree on which factors are responsible. I test one explanation: whether in periods of low unemployment, which coincide with rising individual-level wages, employment opportunities available to workers with low earnings ability countercyclically push the average wage downward. Based on individuals' detailed labor-market histories available in the NLSY79, I find that earnings-ability composition explains about 20 percent of the difference in cyclicality between individual-level wages and the average wage. Using data from the NLSY79, Current Population Survey, and Current Employment Statistics program, I find that business-cycle patterns in workers' experience have the largest countercyclical effect on the average wage. Gender, race, educational attainment, union membership, marital status, and industrial composition have little countercyclical effect. Using a simple Mortensen–Pissarides search-and-matching environment, I show how a model that differentiates workers by ability and features idiosyncratic productivity fluctuations can explain the influence earnings ability has on business-cycle variation in the average wage.

3.1 Introduction

Many economists now accept that real wages are procyclical. Their acceptance is based on empirical evidence provided by numerous studies that make use of individual-level data. These studies find that real wages are "substantially procyclical" (Elsby, Shin, and Solon, 2016, S250). Further, they find that real wages are not only procyclical but more procyclical than what is suggested by aggregate wage statistics, a fact attributed to movements of workers into and out of employment over the business cycle.

But the debate over real-wage cyclicality was not always so one-sided. Many convincing models favor real wages being countercyclical or acyclical (Abraham and Haltiwanger, 1995). On one side of the debate, Keynes (1936) expected real wages to exhibit countercyclical patterns because nominal wages were expected to adjust more slowly than prices (Pencavel, 2015). On another side, real wages might exhibit acyclical patterns if wages reflect long-term relationships with employers that insure their employees against bad times. These models are supported empirically by patterns observed in aggregate-level wage statistics, which are found to be mildly procyclical or acyclical (Stock and Watson, 1999).

So why are studies showing wage procyclicality using individual-level data so convincing? Perhaps the best explanation is given by Solon, Barsky, and Parker (1994). According to the narrative, all wages rise during expansions. But expansions provide job opportunities for previously unemployed low-skill, low-paid workers. Adding these workers during expansions pushes the average wage downward, making the average wage appear less cyclical than individual-level wages. The opposite happens during recessions when the economy sheds low-skill, low-paid workers. Shedding these workers during recessions pushes the average wage upward. In summary, Solon, Barsky, and Parker (1994) use individual-level data to establish that real wages are substantially procyclical and they provide an explanation for why the average appears less cyclical.

Solon, Barsky, and Parker's preferred approach dealt with compositional bias in a straightforward way. They constructed a longitudinal dataset of continually employed wage earners to entirely account for movements of workers into and out of employment. In doing so, compositional bias was effectively eliminated. This approach leaves open an interesting question: Who are the people moving into and out of employment and biasing aggregate wage statistics?

To answer this question, I propose a simple measure of a worker's earnings ability that is trackable over the business cycle. Earnings ability is a specific component of a worker's wage. It reflects the difference that persists through time in hourly wages paid to two workers that is not attributable to the business cycle, educational attainment, union affiliation, industry, or experience. Earnings ability represents one measure of what Solon, Barsky, and Parker refer to as skill. The purpose of tracking earnings ability over the business cycle is to characterize the earnings ability of people moving into and out of employment. Doing so provides a measure of how earnings-ability composition affects aggregate wage statistics.

To uncover estimates of earnings ability, I use data from the National Longitudinal Survey of Youth 1979 (NLSY79). NLSY79 respondents not only report their earnings and hours through time, but also answer questions about their employment histories. These data allow me to uncover both earnings ability and the business-cycle component of wages. Detailed employment histories provided by respondents allow me to track wage statistics constructed from earnings ability over the business cycle.

Tracking workers through time also allows me to track the composition of unemployed workers. The composition of unemployment matters for job creation (Mueller, 2017) and has implications for whether workhorse models of macroeconomics can match business-cycle facts (Shimer, 2005).

I find that earnings ability explains 19.6 percent of the difference in cyclicality between individual-level wages and the average wage. This finding is based on two constructed wage statistics. The first wage statistic reflects wage cyclicality holding constant the composition of employment. I find that a 1 percentage point increase in the economy-wide unemployment rate above trend predicts an individual's hourly wage falls 1.192 percent, whereas the average wage falls 0.615 percent using data on compensation per hour in the nonfarm business sector. The difference in cyclicality between individual-level wages and the average wage is therefore 0.577 percentage points. The second wage statistic reflects earning-ability composition holding constant wage cyclicality and the composition of other worker characteristics. I find that a 1 percentage point increase in the economy-wide unemployment rate above trend predicts hourly wages countercyclicaly increase 0.113 percent. The countercyclical compositional effect of earnings ability explains 19.6 percent ($100 \times 0.113/0.577$) of the gap between the cyclicality of individual-level wages and the average wage.

To provide some context for this result, I present a model of the labor market that features a trade-off between earnings ability and idiosyncratic productivity. Fluctuations in idiosyncratic productivity may be meaningful based on the results from panel regressions, which attribute only 50 percent of the variation in wages to characteristics of workers, earnings ability, and the business-cycle. If the remaining component of a worker's wage reflects—at least partially—their performance at work, then employment will reflect a trade-off between earnings ability and idiosyncratic productivity.

Low-ability workers require higher idiosyncratic productivity to remain in a productive match unlike matched high-ability workers who can tolerate lower idiosyncratic productivity. But crucially, at the boundary between productive and nonproductive matches, workers of all abilities generate zero surplus from a match. When aggregate economic shocks shift this boundary, workers of all abilities are affected. Whether aggregate shocks shift the ability composition of employment depends on where workers are distributed along the boundary—in other words, an empirical question. This description of the economy is shown within the context of a simple economic environment that features unemployment because of a random-search friction. Nevertheless, the simple economic environment shares many features of dynamic Mortensen—Pissarides economies.

To explain the remainder of the difference between the cyclicality of individual-level wages and the average wage, I investigate labor-force composition using data from the Current Population Survey and Current Employment Statistics program. These datasets suggest that gender, race, educational attainment, union membership, marital status, and industrial composition exhibit little countercyclical influence on the average wage. Ruling these influences out leaves the composition of workers' experience as the main countercyclical driver of the difference between the cyclicality of individual-level wages and the average wage.

The empirical work provides key elasticities that will be of interest to labor–macro modelers that differentiate workers by productivity. And the simple macroeconomic structure I use to explain the issues at stake contains many features of richer models, which means the general lessons should form the basis for future work on earnings ability.

The remainder of the paper is organized as follows. The next section describes a framework for understanding the issues at stake by comparing two models of the labor market. The following section describes the NLSY79 data used to construct the wage statistics based on earnings ability and time-varying characteristics of workers. These statistics are compared to business-cycle patterns in labor-force composition observed in the Current Population Survey and the Current Employment Statistics program in the following section. Section 3.5 concludes.

3.2 Two Models of the Labor Market

This section compares two models of the labor market to frame the issue in this paper. Section 3.2.1 describes a model of the labor market where workers are essentially organized by "skill" or ability. In this model, lower-ability workers below a cutoff are unemployed and higher-ability workers are employed. Workers transition between employment and unemployment at the cutoff, which produces stark predictions for labor-market composition over the business cycle.

In contrast, section 3.2.2 describes a model of the labor market that adds a trade-off between ability and an idiosyncratic component that affects productivity. Lower-ability workers require higher idiosyncratic productivity to maintain productive matches and higher-ability workers require lower idiosyncratic productivity to maintain productive matches. The boundary between employment and unemployment varies systematically with ability and is determined by productive relationships that produce zero surplus. Importantly, this zero-surplus condition applies to workers of all abilities. And as the zero-surplus condition shifts over the business cycle, workers of all abilities transition between employment and unemployment. The main message is that the employment composition of ability may shift little over the business cycle.

3.2.1 Organized by Earnings Ability

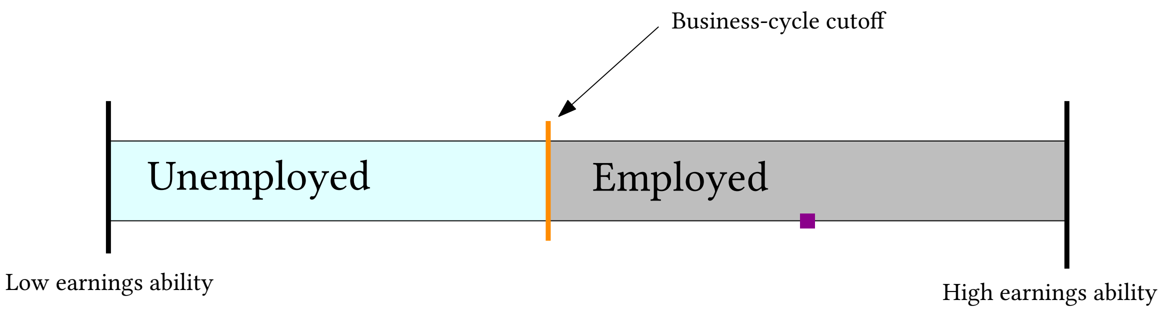
To understand what is meant by earnings ability, consider two workers who work in the same industry at non-union-affiliated jobs. Both have earned the same academic degrees and have worked the same number of years in addition to having worked at their current job the same amount of time. Comparing their hourly wages, the difference could be attributed to differences in common, macroeconomic conditions or temporary, idiosyncratic factors such as health shocks. But if the difference in their wages is not due to macroeconomic conditions and persists through time, then the difference reflects a permanent factor that I refer to as earnings ability.[1]

Now imagine organizing the labor force into a single line ordered by earnings ability. At one end are workers with low earnings ability and at the other end are workers with high earnings ability. If some workers are repeatedly paid less, despite having the same experience and academic degree, then it is reasonable to expect that in periods of high unemployment the lower-paid, lower-skill workers exit employment. In other words, who is unemployed and employed depends on earnings ability and a cutoff determined by the business cycle. Lower-ability workers who are below the cutoff are unemployed and higher-ability workers who are above the cutoff are employed. Panel A of figure 3.1 depicts this model of the labor market for a baseline unemployment rate.

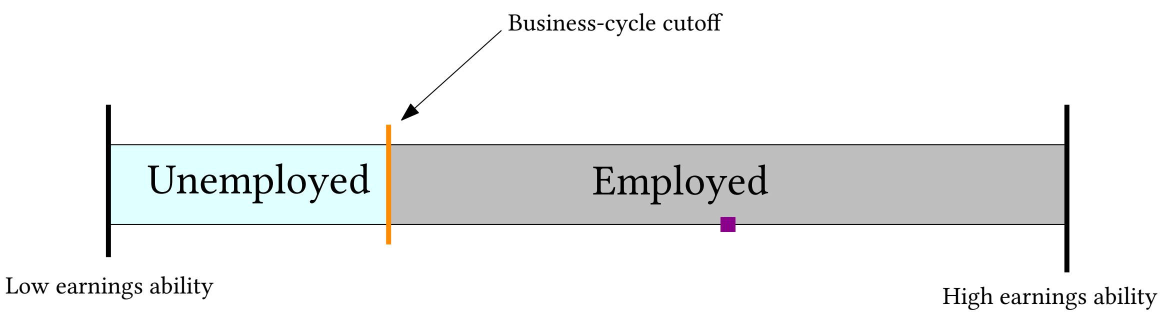
In the model depicted in figure 3.1, wages have two components. The first component reflects earnings ability. A worker with higher earnings ability earns a higher wage than a worker with lower earnings ability. The second component reflects the business cycle. Wages of all workers are positively correlated with the business cycle.

As the economy expands in panel B all wages rise according to the business-cycle component. But the expansion provides job opportunities to previously unemployed, lower-ability workers. As a consequence, the average earnings ability of employed workers falls because of the workers who compose the average. This fall can be seen by comparing

[1] To be clear, earnings ability includes, among other factors, an individual's race and gender. In section 3.4.3 I show that the composition of employment and unemployment varies little over the business cycle in terms of race and gender. In other words, the reason wages are cyclical is that within a gender category, wages are cyclical. This fact is consistent with the idea that what drives wage cyclicality is a component of earnings ability that is unobservable to the researcher and reflects an individual's innate productivity.



A: Baseline unemployment rate.



B: Low unemployment rate.

Figure 3.1: Workers organized by earnings ability to illustrate how the average wage can appear less cyclical than individual-level wages. A business cycle determines unemployment rates. The magenta squares indicate average earnings ability among employed workers.

the magenta squares, which indicate average earnings ability. If the magnitude of the business-cycle component outweighs the compositional effect, then the average wage increases.

The framework is useful because it explains how the ability of workers added to employment over the business cycle cause the average wage to be less cyclical than individual-wages. The framework also establishes how longitudinal data can be used to separate earnings ability from common macroeconomic conditions that affect wages, while taking into account other factors, like on-the-job tenure, in a statistical model of wages.

A statistical model for the real wage paid in period t to person i, $w_{i,t}$, is

$$\log w_{i,t} = \alpha_i + X'_{i,t}\beta + \psi_t + \nu_{i,t}, \tag{3.1}$$

where $\{\alpha_i\}$ are person effects that capture workers' time-invariant earnings ability; $X_{i,t}$ is a vector of time-varying controls that includes years of experience, union affiliation, industry effects, and educational effects for earned academic degrees; $\{\psi_t\}$ are period effects that capture macroeconomic conditions common to all workers; and $\nu_{i,t}$ is an unobserved, time-varying error that captures shocks to human capital, person-specific job matches, and other factors.

Including $\{\alpha_i\}$ in (3.1) is key because the earnings-ability terms capture the compositional effects depicted in figure 3.1 and highlighted by Solon, Barsky, and Parker (1994). To see this, note that the collection $\left\{\hat{\psi}_t\right\}$ is used to summarize composition-adjusted wage cyclicality. These terms trace out the estimated conditional expectation through time controlling for earnings ability and time-varying characteristics of workers.

If person effects are *not included* in (3.1), then an estimate of wage cyclicality will be biased by earnings-ability composition. According to the narrative, a boom in period t' provides jobs for low-ability workers. Low-ability worker $i^{\bullet}$ enters into the sample with a low $\nu_{i^{\bullet}t'}$ without α_i included in the statistical model. Because the period effects are constructed to make residuals have zero mean, $\hat{\psi}_{t'}$ will be pulled down because of composition. Without person fixed effects, real wages appear less cyclical because of compositional bias. This narrative is consistent with the discrepancy between cyclical measures based on aggregate versus individual-level data and the model of employment depicted in figure 3.1.

The organized-by-ability model, however, is missing a major component of wages that features meaningfully in the statistical model for wages in (3.1). In NLSY79 data, the R^2s from the estimated statistical models attribute around 40 percent of the variation in wages to ν. Adding this component to the model lets wages reflect earnings ability, the business cycle, and idiosyncratic productivity fluctuations. The next section considers this feature in the context of a random-search environment.

3.2.2 A Model of Earnings Ability and Idiosyncratic Productivity

This section describes a model where workers of all abilities transition between employment and unemployment over the business cycle. I consider a simple economic environment to be precise about broadly intuitive ideas.

3.2.2.1 Description of the Economic Environment

The environment consists of a single period and is a modified version of a conventional Mortensen–Pissarides search-and-matching economy. The basic features of the environment include a search friction, which prevents firms from instantly hiring workers and leads to unemployment; a homogeneous consumption good; risk-neutral agents and firms; and Nash wage bargaining. Aggregate productivity drives fluctuations in output. I add two features: 1) workers are differentiated by ability to produce the homogeneous consumption good and 2) production is influenced by idiosyncratic fluctuations. I show how these added features together determine the composition of ability over the business cycle.

The time horizon of the economy is a single period. The beginning of the period is indexed by 0 and the end of the period is indexed by 1. The economy is populated by a unit measure of risk-neutral workers and a large measure of risk-neutral firms.

Workers are indexed by their ability, which is denoted by a, and are either employed or unemployed. Unemployed workers search for work. Let $u_0(a) \in (0,1)$ denote the exogenous stock of workers of ability a who begin the period unemployed The total stock of workers looking for jobs is $u_0 = \int_{\underline{a}}^{\overline{a}} u_0(a)\, da$, where $\underline{a}$ and $\overline{a}$ define the support of types.

Employed workers earn wages, which are paid in units of the homogeneous consumption good. Risk-neutral workers consume their wages if employed or the level of home production if unemployed. The home production technology is allowed to depend on ability.

The large measure of firms is divided between active and inactive firms. Inactive firms become active by posting a vacancy and matching with a worker. Once matched with a worker, a firm operates a production technology that turns a worker's indivisible unit of labor into $za\varepsilon$ units of output. The first component of productivity, z, is common to all firms and represents aggregate productivity. The second component of productivity, a, is the worker's ability. The third component of productivity, ε, is specific to a firm–worker pair and represents idiosyncratic productivity.

A matching technology summarizes the complicated process that brings together firms looking to fill vacancies and workers searching for jobs. Given v posted vacancies, $m(u_0, v)$ matches take place within the period. I assume that m is increasing and differentiable in both its arguments; moreover, I assume that m exhibits constant returns to scale. Let $\theta := v/u_0$ denote the "tightness" of the labor market. The probability an unemployed worker finds a job is $f(\theta) := m(u_0, v)/u_0 = m(1, \theta)$ and the probability a vacancy is filled is $q(\theta) := m(u_0, v)/v = m(\theta^{-1}, 1)$. Because finding a job is easier the more vacancies there are relative to the level of unemployment, f is increasing in tightness; likewise, because it is harder for a firm to fill a vacancy the more vacancies there are relative to the level of unemployment, q is decreasing in tightness.

Because firms cannot instantly match with workers looking for work, unemployment persists in the economy. The matching process is random.

Jobs end for two reasons. Some jobs end because of an exogenous disruption. Other jobs end because matched firms and workers bilaterally agree that there are better options than maintaining the match. The decision to end the match depends on aggregate productivity, the worker's ability, and idiosyncratic fluctuations that affect production.

Workers and firms know the level of aggregate productivity in the economy and learn about idiosyncratic productivity fluctuations according to the following timeline:

1. The economy starts with u_0 unemployed and $n_0 = 1 - u_0$ employed workers.

2. Firms post vacancies to recruit workers.

3. Unemployed workers match with firms that had posted a vacancy. Newly matched workers and the n_0 employed workers travel to work.

4. Worker–firm matches end exogenously with likelihood s_x, with $s_x \in (0, 1)$.

5. At this point, a firm knows the ability of the worker they are matched with. Idiosyncratic productivity is then observed, determining potential output. Reservation productivity is determined and endogenous separations occur.

6. The remaining workers engage in production.

3.2.2.2 Region of Production

I solve the model by analyzing a collection of functions.

On the worker side of the economy, I denote the value of unemployment as U and the value of employment as W. The value of employment to a worker of ability a having drawn idiosyncratic productivity ε is the earned wage, $w(a, \varepsilon)$: $W(a, \varepsilon) = w(a, \varepsilon)$. The value of unemployment for a worker of ability a is the level of home production, denoted by $b(a)$: $U(a) = b(a)$. The level of home production, which includes unemployment benefits, is in units of the homogeneous consumption good and is allowed to depend on ability, with $b'(a) > 0$ for reasons discussed below.

On the firm side of the economy, I denote the value of a filled vacancy by J and the value of a posted vacancy by V. Firms are either active or inactive. Inactive firms expand recruiting effort until the expected benefits of posting a vacancy are driven to zero: $V = 0$. The value of an active firm matched with a worker of ability a engaged in production with idiosyncratic productivity draw ε is the value of production less the wage paid to the worker: $J(a, \varepsilon) = za\varepsilon - w(a, \varepsilon)$.

Given the value of a filled vacancy, the value of a posted vacancy, the value of being employed, and the value of being unemployed it is straightforward to determine the surplus generated from a match, $\mathcal{S}$. The surplus generated from a match between a worker with ability a having idiosyncratic productivity ε and a firm is the sum of the surplus contributed by the firm, $J - V$, plus the surplus contributed by the worker, $W - U$:

$$\mathcal{S}(a, \varepsilon) = \underbrace{[J(a, \varepsilon) - V]}_{\text{Firm contribution}} + \underbrace{[W(a, \varepsilon) - U(a)]}_{\text{Worker contribution}} = za\varepsilon - b(a). \tag{3.2}$$

For a given level of ability, $\mathcal{S}$ is increasing in ε. The unique reservation idiosyncratic productivity level below which the firm and worker no longer find it profitable to maintain the match is determined implicitly by $\mathcal{S} = 0$, or $\hat{\varepsilon}(a) = b(a)/(za)$.

Reservation idiosyncratic productivity determines employment and the region of production. For ability level $a_\bullet$, all workers who travel to work and receive an idiosyncratic productivity draw of at least $\hat{\varepsilon}(a_\bullet)$ engage in production. A trade-off exists between a and ε, where low- and high-ability workers require different levels of idiosyncratic productivity to remain employed. Low-ability workers require higher idiosyncratic productivity draws and high-ability workers require lower idiosyncratic productivity draws. This stipulation amounts to low-ability workers experiencing higher unemployment rates on average and high-ability workers experiencing lower unemployment rates on average. For production to be characterized according to this trade-off, it must be that $\partial\hat{\varepsilon}/\partial a < 0$. This condition requires that $b'(a)a/b(a) < 1$, which says that the elasticity of the flow value of nonwork with respect to ability is less than one. (More will be said about this dependence below.)

Figure 3.2 depicts the region of production and the trade-off between ability and idiosyncratic productivity. For a given aggregate state of the economy, a worker of ability $a_\bullet \in [\underline{a}, \overline{a}]$ who travels to work engages in production as long as they receive an idiosyncratic productivity draw of at least $b(a_\bullet)/(za_\bullet)$. The region of production is therefore given by $\{(a, \varepsilon) : \mathcal{S}(a, \varepsilon) \geq 0\} = \left\{(a, \varepsilon) : \underline{a} \leq a \leq \overline{a}, b(a)/(za) \leq \varepsilon \leq \overline{\varepsilon}\right\}$.

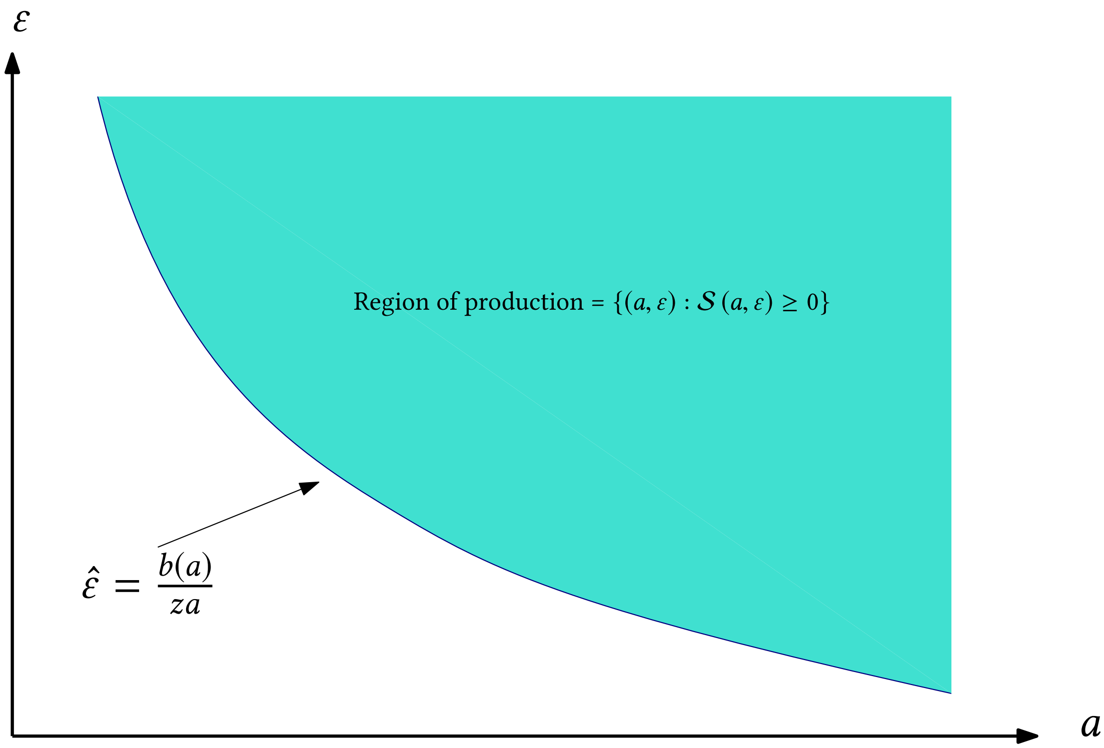


Figure 3.2: Region of production. Workers who travel to work and do not separate from employment for an exogenous reason or due to a low idiosyncratic productivity draw engage in production. Productive workers are characterized by ordered pairs (a, ε) that generate non-negative surplus. These productive pairs are colored green.

Characterizing the region of production depends on whether surplus is generated from a match, not on how a firm and worker split the surplus. Any surplus is split through wage payments made by firms to workers. I follow convention by having wages determined through Nash wage bargaining. Doing so provides a characterization of the productive region in terms of reservation wages.

Let η be the relative bargaining strength of a worker and let $1 - \eta$ be the relative bargaining strength of a firm. Consider a match between a worker of ability a who has drawn idiosyncratic productivity ε and a firm. The outcome of Nash bargaining requires that $w(a, \varepsilon)$ be chosen so that $(1 - \eta)\,[W(a, \varepsilon) - U(a)] = \eta\,[J(a, \varepsilon) - V]$. This requirement sets the worker's gain, $W(a, \varepsilon) - U(a)$, proportional to the firm's gain, $J(a, \varepsilon) - V$. Developing the outcome of Nash bargaining leads to an expression for wages:

$$w(a, \varepsilon) = \eta z a \varepsilon + (1 - \eta) b(a). \tag{3.3}$$

The worker gets their η-share of production plus $1-\eta$ of their outside option. A worker's outside option is their value of unemployment, which equals $b(a)$.

At the reservation idiosyncratic productivity level, using the expression for wages in (3.3), a worker of ability a earns $w_r(a, \hat{\varepsilon}(a)) = b(a)$. At the reservation wage the worker is indifferent between work and nonwork and the firm is indifferent between maintaining the match and foregoing the opportunity of posting a vacancy, which has a value of zero. The reservation wage is increasing in a because $b'(a) > 0$.

This result is summarized in proposition 6:

Proposition 6 *For a worker of ability a, the reservation wage, $w_r(a)$, is the wage paid when $\mathcal{S} = 0$ and equals $w(a, \hat{\varepsilon}(a)) = b(a)$, the worker's outside option. Reservation wages are increasing in ability: $\partial w_r(a)/\partial a > 0$.*

3.2.2.3 Equilibrium

This section establishes the existence and uniqueness of an equilibrium and investigates how ability affects labor-market outcomes.

It will be convenient to change the order of integration. Define $\zeta(a) := a/b(a)$. Under the elasticity assumption, $b'(a)a/b(a) < 1$, ζ is strictly increasing and therefore its inverse exists. Changing the order of integration, the productive region is given by $\underline{\varepsilon} \leq \varepsilon \leq \overline{\varepsilon}$ and $\zeta^{-1}(1/(z\varepsilon)) =: \chi(1/(z\varepsilon)) \leq a \leq \overline{a}$. Because ζ is strictly increasing, so is χ. And therefore both $\partial\chi/\partial z$ and $\partial\chi/\partial\varepsilon$ are negative, which says that cutoff ability is decreasing in both the aggregate productivity level and the level of idiosyncratic productivity.

Inactive firms become active by paying a cost $c > 0$ to post a vacancy. Any inactive firm can choose to post a vacancy, which drives the value of posting a vacancy to zero. As firms expand recruiting effort the likelihood of filling a vacancy falls, causing the value of posting to fall as well. This free-entry margin guarantees an interior equilibrium.

In addition to the number of other vacancies posted in the economy, the value of a posted vacancy depends on the composition of the unemployment pool. Firms post vacancies prior to knowing who they will match with and what the match's idiosyncratic productivity will be.

Let $H(a)$ denote the cumulative distribution function that describes the ability composition of agents in the unemployment pool and let $h(a)$ denote the associated probability density function, where $h(a) = u(a)/u_0$. Idiosyncratic productivity is drawn from a distribution G with associated probability density function g. The value of posting a vacancy is

$$V = -c + [1 - q(\theta) + q(\theta)s_x] \times 0 + q(\theta)(1 - s_x) \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \int_{\underline{a}}^{\chi(1/(z\varepsilon'))} 0 \times h(a')h(\varepsilon')\, da'd\varepsilon'$$

$$+ q(\theta)(1 - s_x) \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \int_{\chi(1/(z\varepsilon'))}^{\overline{a}} J(a', \varepsilon')h(a')g(\varepsilon')\, da'd\varepsilon'. \tag{3.4}$$

The vacancy costs c to post. With probability the $1 - q(\theta)$ the vacancy is unfilled, yielding zero payoff for the posted vacancy. With probability $q(\theta)s_x$ the vacancy is filled and the worker travels to work, but the match terminates due to an exogenous separation, yielding zero payoff for the posted vacancy. With probability $q(\theta)(1 - s_x)$ the vacancy is filled, the worker travels to work, and the match does not end exogenously. The firm–worker pair then learn their level of idiosyncratic productivity. For a given level of idiosyncratic productivity ε', a worker must be of ability $\chi(1/(z\varepsilon'))$ in order for surplus to be non-negative. An idiosyncratic productivity draw less than ε' yields zero payoff for the posted vacancy. An idiosyncratic productivity draw greater than ε' yields the value of a filled vacancy. Only if the (a, ε) pair falls within the shaded region in figure 3.2 does the vacancy yield $J(a, \varepsilon)$.

The willingness of firms to post vacancies depends on the ability composition in the unemployment pool. The likelihood of encountering higher-ability workers shows up in h. And because there are no complementarities among workers engaged in production, the willingness of firms to post vacancies does not depend on the ability composition of the employment pool.

An equilibrium in the economy consists of a list, $\langle \theta^\star, \hat{\varepsilon}(a), n \rangle$, such that vacancy posting is optimal, making V equal zero in (3.4), and wages are set through Nash bargaining. The first element of the list is a unique level of labor-market tightness, $\theta^\star$, which determines $v^\star$ and therefore the number of matches. The second element is a function that describes the reservation level of productivity to generate a positive surplus, which determines the matches that survive and therefore the level of employment. The third element is a function $n : [\underline{a}, \overline{a}] \rightarrow [0, 1]$, with $n(a)$ denoting the stock of employed workers of ability a.

To characterize the economy, I parameterize the job-filling rate as $q(\theta) = 1/(1 + \theta)$ before relaxing the parameterization below. Parameterizing q this way guarantees that q and f are constrained within zero and one (den Haan, Ramey, and Watson, 2000). Using the free-entry condition in (3.4), the equilibrium level of tightness is given by

$$\theta^{\star} = \frac{(1-s^{x})(1-\eta)}{c} \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \int_{\chi(1/(z\varepsilon'))}^{\bar{a}} \left[za'\varepsilon' - b(a)\right] h(a')g(\varepsilon')\, da'd\varepsilon' - 1. \tag{3.5}$$

The cutoff rule for idiosyncratic productivity comes from $\mathcal{S}(a, \hat{\varepsilon}(a)) = 0$, which uniquely determines $\hat{\varepsilon} = b\,(a)\,/(za)$.

Equation (3.5) makes clear that in order for vacancy creation to be optimal, a firm's properly discounted share of the expected surplus must be greater than the cost of posting a vacancy. From (3.2), recall that the surplus generated from a worker with ability a and idiosyncratic productivity draw ε is $za\varepsilon - b(a)$. The firm's share of the surplus is $1-\eta$, but this amount must be further discounted by the likelihood that the match ends exogenously. Therefore $(1-s_x)(1-\eta)$ times the expected surplus must be larger than c for $\theta^{\star} > 0$.[2] This characterizes optimal vacancy posting.

The expression for the equilibrium level of labor-market tightness in (3.5) shows that labor-market tightness is decreasing in the exogenous separation rate, the relative bargaining strength of workers, and the cost of posting a vacancy. These parameters all make posting a vacancy less attractive to a firm. Increasing aggregate productivity increases labor-market tightness: $\partial\theta^{\star}/\partial z$. The result follows from the fact that $za\varepsilon - b(a)$ evaluated at $a = \chi(1/(z\varepsilon))$ is zero, making part of Leibniz's rule zero. All workers added at the employment margin, in other words, generate zero surplus.

How labor-market tightness responds to parameters does not depend on the parameterization of the job-filling rate. For q strictly decreasing in labor-market tightness, the equilibrium level of tightness is given by

$$\theta_q^{\star} = q^{-1}\left[c\left\{\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \int_{\chi(1/(z\varepsilon'))}^{\bar{a}} (1-s_x)\,(1-\eta)\left[za'\varepsilon' - b(a)\right] h(a')g(\varepsilon')\, da'd\varepsilon'\right\}^{-1}\right].$$

Straightforward calculations establish the comparative statics in proposition 7 hold. In general the subsequent analysis holds for a general matching technology; however, the algebra is significantly less transparent and is therefore avoided.

To determine n, it is useful to derive the transition probabilities for workers of ability a. The economy starts with $n_0\,(a) \in (0,1)$ employed workers. After posting vacancies,

[2] Otherwise, $\theta^{\star} = 0$. The expression for the equilibrium level of tightness in (3.5) therefore excludes certain combinations of parameters. A general expression for the equilibrium level of tightness is $\max\left\{\theta^{\star}, 0\right\}$. For the case when $\theta^{\star} = 0$, the functions $\hat{\varepsilon}$ and n are inconsequential.

$f\left(\theta^{\star}\right)u_0(a)$ matches occur and $n_0(a)+f\left(\theta^{\star}\right)u_0(a)$ workers travel to work. Of the workers who travel to work, a fraction s_x separate exogenously. Of those who do not separate exogenously, a fraction $G(\hat{\varepsilon}(a))$ have ability levels too low, given their idiosyncratic productivity draws, to generate a positive surplus. The composite separation rate, which captures both exogenous and endogenous separations, is $s = s_x + (1-s_x)G(\hat{\varepsilon}(a))$. These transition probabilities determine n. To see that $n : \left[\underline{a}, \overline{a}\right] \rightarrow [0,1]$, it is true that

$$\begin{aligned} n(a) &= (1-s)\left[n_0(a) + f\left(\theta^{\star}\right)u_0(a)\right] \\ &\leq n_0(a) + u_0(a) \leq n_0 + u_0 = 1, \end{aligned}$$

where the first inequality uses the fact that $s = s_x + (1-s_x)G(\hat{\varepsilon}(a)) \leq s_x + (1-s_x) = 1$ and $s \geq s_x$; $f\left(\theta^{\star}\right) \in (0,1)$; and the fact that $u_0 + n_0 = 1$ by construction.

These following proposition summarizes the results of this section:

Proposition 7 *Suppose that $b'(a)a/b(a) < 1$. Then, a unique equilibrium $\langle\theta^{\star}, \hat{\varepsilon}(a), n\rangle$ exists. The equilibrium level of labor-market tightness is decreasing in the exogenous separation rate, the relative bargaining strength of workers, and the cost of posting a vacancy. Equilibrium labor-market tightness is increasing in aggregate productivity.*

3.2.2.4 How the Region of Production Shifts in Response to an Aggregate Productivity Shock

Given a unique equilibrium, consider the economy's response to an increase in aggregate productivity from z' to $z'' > z'$. When the economy initially has access to aggregate productivity z', the productive boundary is given by $\hat{\varepsilon}' = b(a)/(z'a)$. The region of production at z' in figure 3.3 is colored blue.

When aggregate productivity shifts to $z'' > z'$, the productive boundary is given by $\hat{\varepsilon}'' = b(a)/(z''a)$. Higher productivity benefits workers of all abilities because lower idiosyncratic productivities now manage to generate a positive surplus. The economy's region of production shifts downward. In addition to the blue region, the economy's region of production also includes the green region after the productivity shift.

Figure 3.3 does not answer how the ability composition of employment responds to an aggregate productivity shock. The answer depends on where workers are distributed along the boundary between employment and unemployment. Denote the probability density function that describes the ability stock of employed workers as λ, where $\lambda(a,z) = n(a)/\int_{\underline{a}}^{\overline{a}} n(a)\ da$. The average ability is given by $\int_{\underline{a}}^{\overline{a}} \lambda(a,z)\ da$. In general the relationship between λ and z is complicated and requires parameterizing both n_0 and u_0, making the relationship between average ability and z hard to quantify. In a dynamic model both n_0 and u_0 are endogenous objects and, most likely, both would be matched to empirical statistics, making this an empirical question. I turn to this question in sections 3.3 and 3.4.

The main message I want to convey is that, unlike the model presented in section 3.2.1, composition of ability is not necessarily guaranteed to shift in a predictable way.

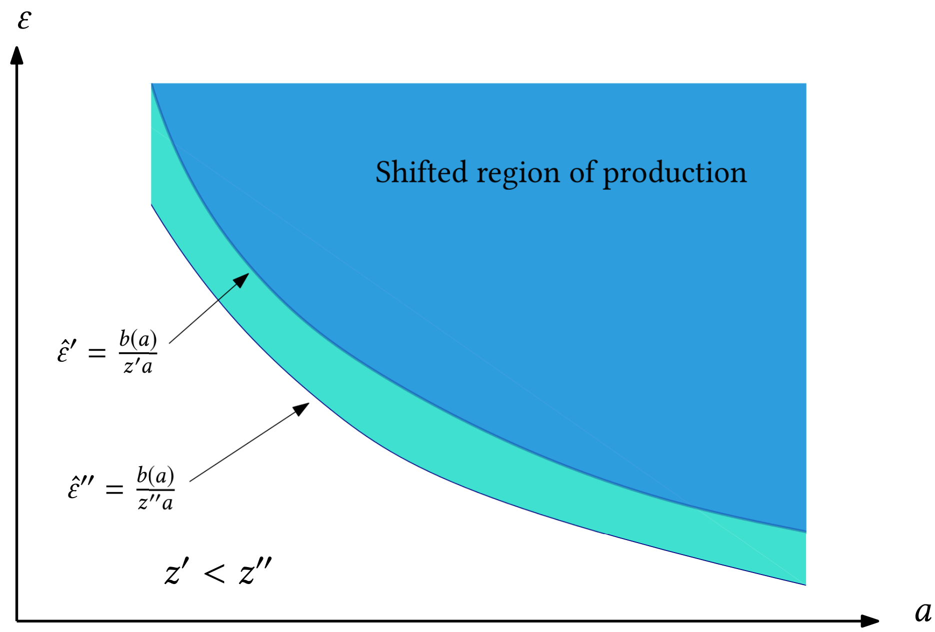


Figure 3.3: Compositional response to an increase in aggregate productivity from z' to $z'' > z'$. Initially the productive boundary is $\hat{\varepsilon}'$ and the region of production equals the blue region. Increasing productivity shifts the region of production to include the green region in addition to the blue region.

3.2.2.5 Reinterpreting the Level of Home Production and Ability

Many other factors affect production besides an agent's ability. These factors include work experience, on-the-job tenure, and education. There are at least a few ways to interpret these factors within the context of the model.

The easiest way would be to consider these factors fixed over the duration of a business cycle. Letting x denote an additional factor of production common to all agents, a firm's output would be given be $zax\varepsilon$. In the above analysis, this interpretation would amount to relabeling aggregate productivity. The worry with this interpretation is that factors like schooling or experience may depend on ability; that is, x depends on a. When this is the case, the surplus generated from a productive match is $zax(a)\varepsilon - b(a)$ and the surplus will be non-negative as long as $\underline{\varepsilon} \leq \varepsilon \leq \overline{\varepsilon}$ and $ax(a)/b(a) \geq 1/(z\varepsilon)$. The above analysis will go through as long as

$$b'(a)\frac{a}{b(a)} < 1 + x'(a)\frac{a}{x(a)}.$$

The left-hand side of the above expression is the elasticity of home production with respect to ability. The right-hand side of the above expression is one plus the elasticity of the factor of production with respect to ability.

With linear utility, b and x capture the gains from transitioning from unemployment to employment. Unlike in neoclassical models of labor supply determined at an intensive margin, in this economy labor is determined entirely at the extensive margin. If b is rapidly increasing in a, then an agent with higher ability may have less to gain from transitioning from unemployment to employment than a lower-ability agent. This effect, however, may be negated by the effect a has on x. If x increases with a, which would be the case if higher-ability agents earn higher educational degrees, then a higher-ability agent stands more to gain from transitioning to employment from unemployment because they will earn more with a higher x. While these features of the labor market are not explicitly modeled in this paper, endogenizing labor supply for different ability levels is an interesting way forward.[3]

In summary, the simple economic environment provides a precise description of worker flows at different ability levels. In contrast to the organized-by-ability model presented in section 3.2.1, the ability composition of the employment and unemployment pools is not guarenteed to be countercyclical. This is an empirical question. The remainder of the paper provides evidence that earnings-ability composition is countercyclical.

3.3 Earnings Ability

This section describes the data used to estimate the statistical model of wages and the estimation of earnings ability, including the dispersion of earnings ability.[4]

3.3.1 Data

Uncovering earnings ability and tracking it over the business cycle requires data to have two features. First, the data need to be longitudinal. Using a sample of wage earners over time allows me to estimate a worker's person effect in (3.1) or their earnings ability. Second, tracking earnings ability over the business cycle requires detailed labor-market

[3]This extension could be done along the lines of Garín and Lester (2019).

[4]Section C.2, in an appendix, depicts how the entire distribution of earnings ability varies over the business cycle for the pools of employed and unemployed workers.

histories of wage earners. Data like these are available in the National Longitudinal Survey of Youth 1979 (NLSY79).

The NLSY79 is a survey of men and women born in the years 1957 to 1964. NLSY79 respondents were interviewed over 26 rounds. Respondents were interviewed annually from 1979 to 1994 and biennially afterwards. Each respondent answered questions about jobs held at the time of the survey as well as questions about previously held jobs, including beginning and ending dates associated with each job. This feature of the survey is key because, by tracking wage earners over time, it provides a week-by-week work record of each respondent's labor-force status.

The NLSY79 initially surveyed 12,686 people in 1979 who comprised three subsamples. A supplemental subsample surveyed non-institutionalized, civilian Black, Hispanic or Latino, and economically disadvantaged non-Black, Non-Hispanic, or Latino youths and a military subsample surveyed the population enlisted in the Army, Air Force, Navy, or Marine Corps. I put aside these two subsamples, which were eventually dropped from the survey, and focus on the cross-sectional subsample. The cross-sectional subsample was designed to represent the noninstitutionalized civilian segment of people living in the United States. It initially surveyed 6,111 people.

The hourly wage statistics I use are those computed by the NLSY79. These statistics are computed using respondents' answers to usual earnings and hours worked. Wages include earnings from tips, overtime pay, and bonuses but are based on usual-earnings data before deductions.[5] I deflate wages using the implicit price deflator for personal consumption expenditures. Using 1979 dollars, wages greater than \$100 per hour and less than \$1 per hour are excluded from the analysis because, in the words of the Bureau of Labor Statistics (BLS), "the reported earnings levels were almost certainly in error."[6] The BLS uses the same requirement for its publication titled "Number of Jobs Held, Labor Market Activity, and Earnings Growth Among the Youngest Baby Boomers."

In addition to information on usual earnings and hours worked, respondents provide information on educational attainment, whether they were in a union or employee association, and the industry in which they worked. These controls are included as a system of dummy variables. Because the panel regression includes a person effect, the effects of education, union status, and industrial composition are estimated by respondents completing more schooling across survey rounds, taking jobs covered and not covered by a union, and taking jobs in different industries.

[5]Beginning in 1988, respondents' hours worked from home were included in the calculation for respondents who reported that they worked from home but the hours worked from home were not included in the usual hours worked per week.

[6]See https://www.bls.gov/news.release/pdf/nlsoy.pdf.

The dummies for educational attainment are primarily identified by young workers earning academic degrees early in the sample. To see this, figure 3.4 depicts the shares of NLSY79 respondents with a college degree, a high-school degree, and some college experience by year. In every year, the shares add to 1. The vertical line indicates the year 1986. Before 1986 the average worker in the sample is younger than 25 years old. Young workers who previously earned high-school degrees steadily earned college degrees from 1976 through 1985, indicated in figure 3.4 by a fall in the high-school share and a rise in the college share before 1986. By 1986, however, there is little aggregate evidence that workers continued their schooling, which offers little variation to identify the effect of a college degree. Therefore the early period when workers earned college degrees is dropped in parts of the subsequent analysis along with the educational effects specified in (3.1).

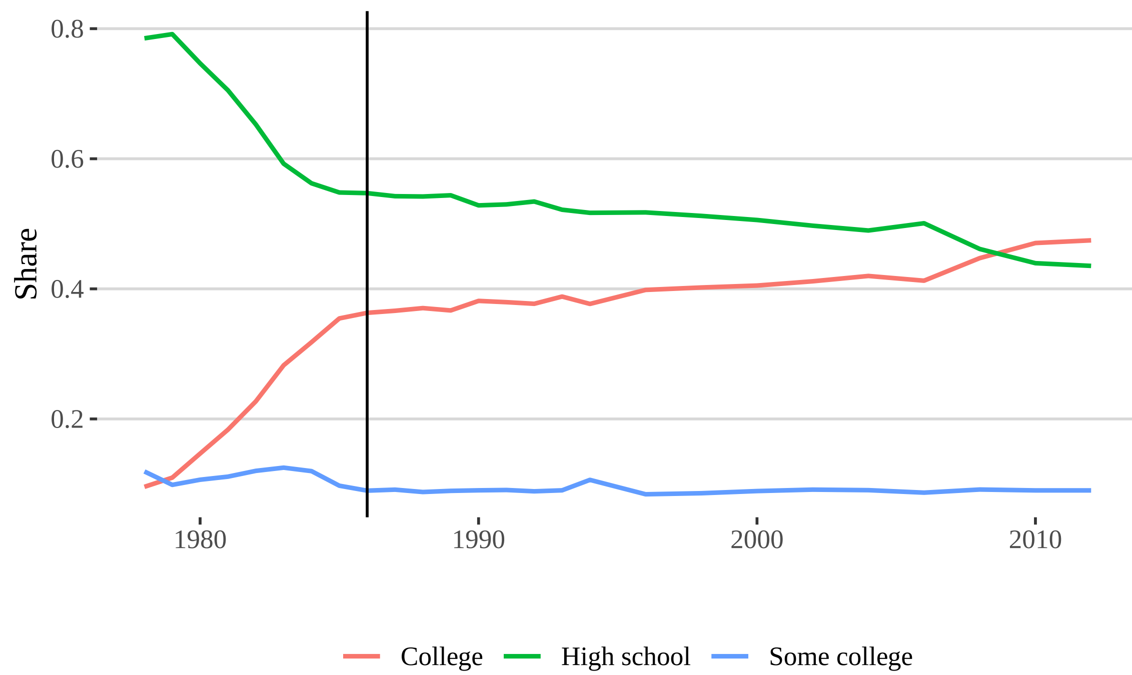


Figure 3.4: Educational attainment in the NLSY79 by year. The vertical line indicates 1986 when the average age in the sample reaches 25.

In addition to these controls, I include controls based on respondents' detailed work histories. I control for cumulative work experience and tenure at a respondent's current job.

Finally, I make two sample restrictions. Wages reported in non-NLSY79 survey years are dropped from the sample because average wages in those years exhibit an unusual pattern identified by Basu and House (2016). And I require that respondents be observed in the sample at least five times in order to estimate the person effect.[7] The end result is a sample of 5,293 people who reported usual earnings and hours worked between 1978 and 2012. This sample is used to estimate the regression specification in (3.1).

To be clear, earnings ability has a less transparent interpretation than the one given so far. The person effects in (3.1), for example, reflect firm fixed effects, a point made transparently by Abowd, Kramarz, and Margolis (1999). Firm dynamics no doubt affect labor-market dynamics and these are embodied in what I am calling earnings ability.[8] Earnings ability also reflects time-invariant characteristics of workers like their gender and race. These components of wages are analyzed subsequently using data from the Current Population Survey. The basic message from that analysis is that race and gender explain very little of the business-cycle variation in CPS wages, suggesting that the business-cycle variation in earnings ability identified below is not driven by these factors.

3.3.2 Estimated Earnings Ability

Estimates of earnings ability are obtained by estimating equation (3.1) using $100 \times \log w_{i,t}$ as the dependent variable. Figure 3.5 depicts the estimates $\{\widehat{\alpha}_i\}$, where the histogram is constructed to have a zero mean. The level of earnings ability is not identified, only the dispersion is identified. To get a sense of the estimated dispersion, a wage earner at the 75th percentile of earnings ability earns approximately 20.6 times a wage earner at the 50th percentile of earnings ability when the two wage earners have the same observable characteristics. The difference between a wage earner at the 95th percentile and a wage earner at the 50th percentile is 53.9 percent.[9]

3.4 Wage Cyclicality

Within the context of the statistical model of wages in (3.1), wages are determined by a worker's earnings ability, the value of their time-varying characteristics, and a common business-cycle component that measures real-wage cyclicality. These are included in the

[7] Requiring respondents to be observed in the sample at least ten times has little effect.

[8] The admixture is not ideal. But datasets that allow for separate identification of person and firm effects do not have high-frequency data on workers' labor-force status, which is a concern given the high rate of job finding in the United States.

[9] For example, to be precise, the constructed wages are $\widehat{100 \times \log w}_{75,t} = \widehat{\alpha}_{75} + \overline{X}'\widehat{\beta} + \hat{\psi}_t$ and $\widehat{100 \times \log w}_{50,t} = \widehat{\alpha}_{50} + \overline{X}'\widehat{\beta} + \hat{\psi}_t$. The difference in constructed wages is

$$\widehat{100 \times \log w}_{75,t} - \widehat{100 \times \log w}_{50,t} = \widehat{\alpha}_{75} - \widehat{\alpha}_{50} = 20.6.$$

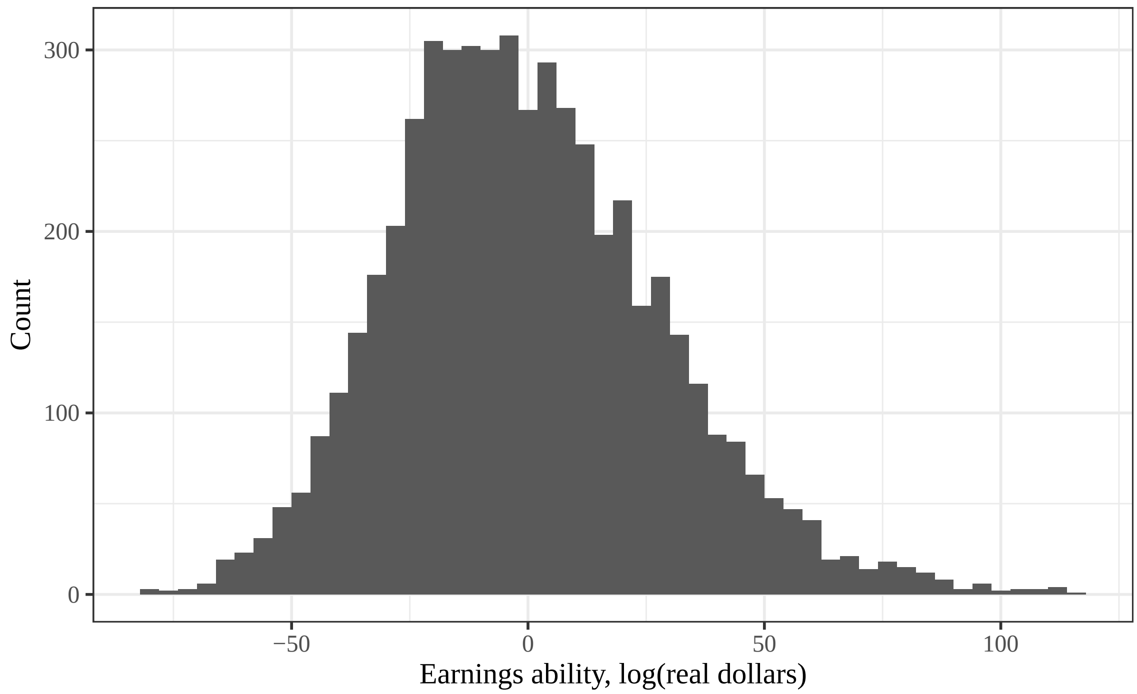


Figure 3.5: Histogram of estimated earnings ability constructed to have zero mean.

model as α_i, $X'_{i,t}\beta$, and ψ. To assess how these factors contribute to wage cyclicality, I construct wage statistics that reflect each factor in isolation and compare them to the simple cross-sectional average wage.

In light of figure 3.4 and the lack of variation in educational attainment in the NLSY79 sample after 1985, I restrict my discussion to estimates based on the sample of prime-age workers and do not include the system of educational dummies in (3.1). The prime-age sample NLSY79 begins in 1986 and runs through 2012. I use data from NLSY79 survey years.[10] Section C.3, in an appendix, contains a broader discussion alternative samples.

The relationships between the constructed statistics and the business cycle are assessed through regressions of the form

$$\overline{w}_{j,t} = \varsigma_j + \beta_j \left(\text{economy-wide unemployment rate} - \text{CBO trend}\right)_t + g_j\left(t\right) + \varepsilon_{j,t}, \tag{3.6}$$

where j indexes the wage statistic $\overline{w}_{j,t}$ constructed for period t, $g_j\left(t\right)$ is a nonlinear trend, and the difference between the economy-wide unemployment rate and the Congressional Budget Office's estimate of the natural rate is a measure of the business cycle. The cyclical component of the unemployment rate reflects fluctuations in aggregate demand. The

[10]There are 18 years in the sample. The interval 1986–1994 consists of 9 years and the even years in interval 1996–2012 consists of another 9 years. The NLSY79 was administered biennially after 1994.

nonlinear trend term reflects wage growth along the economy's balanced growth path and aging among respondents over time. Both of these wage components are estimated in (3.1) through the period fixed effects. I parameterize g_j with a quadratic trend going forward. The term ς_j reflects the fact that levels of wage statistics are in general not identified.

3.4.1 Major Components of Wage Cyclicality

The cross-sectional average wage, $\overline{w}_{\text{avg},t}$, is constructed as the weighted average of wages observed in year t of NLSY79 data using cross-sectional weights provided by the NLSY79. How the average wage varies with the business cycle is reported in column 1 of table 3.1. The average-wage column reports $\widehat{\beta}_{\text{avg}}$ estimated from the regression specified in (3.6). The estimate predicts that when the cyclical unemployment rate is 1 percentage point above trend, $\overline{w}_{\text{avg}}$ procyclically falls 0.176 percent.

As Solon, Barsky, and Parker (1994) point out, this statistic masks individual wage cyclicality because different workers are employed over the business cycle, which affects the average. A wage statistic that controls for characteristics of workers and reflects only real-wage cyclicality can be constructed from $\hat{\psi}_t$. I construct $\overline{w}_{\text{cyclical},t}$ as the average predicted cross-sectional wage in the sample:

$$\overline{w}_{\text{cyclical},t} = \sum_{i \in \mathcal{E}_t} \left(\overline{\alpha} + \overline{X}' \widehat{\beta} + \widehat{\psi}_t \right) \Big/ |\mathcal{E}_t| = \varrho_{\text{cyclical}} + \widehat{\psi}_t,$$

where $\mathcal{E}_t$ indexes the set of respondents in the NLSY79 sample used to estimate (3.1), $|\mathcal{E}_t|$ represents the cardinality of this set, and $\varrho_{\text{cyclical}}$ is a constant.[11] Essentially $\overline{w}_{\text{cyclical},t}$ traces out the estimated conditional expectation function through time, holding constant earnings ability and time-varying characteristics of the sample. Because the second-step regression in (3.6) contains a constant, there is no need to specify an "average person" through $\overline{X}$ and $\overline{\alpha}$ as these are included in $\varrho_{\text{cyclical}}$ and estimated as part of the constant in (3.6).

The cyclicality column of table 3.1 reports $\widehat{\beta}_{\text{cyclical}}$. When the cyclical unemployment rate is 1 percentage point above trend, $\overline{w}_{\text{cyclical},t}$ procyclically falls 1.192 percent. This estimate indicates that individual-level wages are 6.773(1.192/0.176) times as cyclical as the average wage.[12]

[11] The presentation does not include the use of weights even though weights are used to construct wage statistics unless explicitly mentioned.

[12] This factor is larger than what is found in Basu and House (2016). The reason is the fall in cyclicality of the average wage later in the sample. Using data from 1979–2012 in the second-step cyclical regression,

Table 3.1: Components of wage cyclicality

	1986–2012			
	Avg wage	Cyclicality	Characteristics	Earnings ability
Cyclical UR	−0.176	-1.192^{*}	0.576^{**}	0.439^{**}
	(0.627)	(0.597)	(0.229)	(0.165)
N	18	18	18	18

Standard errors in parentheses

$^{+}$ $p < 0.15$, * $p < 0.1$, ** $p < 0.05$

The difference in cyclicality between individual-level wages and the average wage is due to the types of workers who compose employment over the business cycle. Workers differ in terms of their earnings ability and their time-varying characteristics. A wage statistic that reflects only time-varying characteristics in $X_{i,t}$ is

$$\overline{w}_{X,t} = \sum_{i \in \mathcal{E}_t} \left(\overline{\alpha} + X'_{i,t}\widehat{\beta} + \overline{\psi} \right) \Big/ |\mathcal{E}_t| = \varrho_X + \overline{X}'_t \widehat{\beta},$$

where $\overline{X}_t$ is the cross-sectional average respondent, holding constant earnings ability, and ϱ_X is a constant. The time-varying characteristics column of table C.1 reports how $\overline{w}_{X,t}$ varies with the business cycle. When the cyclical unemployment rate is 1 percentage point above trend, $\overline{w}_{X,t}$ countercyclically increases by 0.576 percent.

Finally, a wage statistic that reflects only earnings ability is

$$\overline{w}_{\text{earnings ability},t} = \sum_{i \in \mathcal{E}_t} \left(\widehat{\alpha}_i + \overline{X}'\widehat{\beta} + \overline{\psi} \right) \Big/ |\mathcal{E}_t| = \varrho_{\text{earnings ability}} + \overline{\widehat{\alpha}}_t,$$

where $\overline{\widehat{\alpha}}_t$ is the time-t average of earnings ability in the sample and $\varrho_{\text{earnings ability}}$ is a constant. The earnings-ability column of table C.1 reports how $\overline{w}_{\text{earnings ability},t}$ varies with the business cycle. When the cyclical unemployment rate is 1 percentage point above trend, $\overline{w}_{\text{earnings ability},t}$ countercyclically increases by 0.439 percent.

The countercyclical coefficients for $\overline{w}_{X,t}$ and $\overline{w}_{\text{earnings ability},t}$ indicate that workers with characteristics that predict higher earnings are disproportionately retained in recessions and workers with characteristics that predict lower earnings are disproportionately let go in recessions. Mechanically, because the period fixed effects are constructed so that the cross-sectional average wage matches the predicted average wage and wages are

the cyclicality coefficient for the average wage is −0.624. The magnitude of this coefficient implies that individual-level wages are relatively less cyclical than the average wage.

modeled as log-linear, the sum of the countercyclical effects of time-varying characteristics and earnings ability, $0.576 + 0.439 = 1.015$, equals the difference in cyclicality between individual-level wages and the average wage. These numbers imply 43.3 percent ($100 \times 0.439/1.105$) of the difference is explained by procyclical selection on earnings ability.

The next set of regressions, reported in table 3.2, investigates the individual components used to construct $\overline{w}_X$. The components of $\overline{X}_t$ used to construct $\overline{w}_X$ include dummies for educational attainment, major industries, and union status as well as controls for cumulative work experience and tenure at a respondent's current job. For each of these components I construct $\overline{w}_X$ by leaving the other components unchanged. The constructed $\overline{w}_X$s are then used to estimate regressions specified in (3.6). As a comparison, the first column of table 3.2 reports the overall cyclicality of time-varying characteristics. This entry is copied over from column 3 of table 3.1.

The second column of table 3.2 indicates that variation in the industrial composition of jobs held by NLSY79 respondents does not affect overall average-wage variation.[13] The third and fourth columns of table 3.2 indicate that variation in union affiliation and experience explain the variation in wages. The coefficient on union affiliation is 0.130 and the coefficient on experience is 0.448. Both are statistically significant at the 0.15 level. The positive signs indicate that both effects are countercylical. Again, because the statistical model for wages in (3.1) is linear, the individual contributions approximately sum to the overall cyclicality of time-varying characteristics reported in column 1 of table 3.2. [The sum is not exact because the trends in (3.6) are estimated separately.][14]

The estimates suggest that fluctuations in experience and earnings ability explain over 40 percent of average-wage cyclicality. These estimates, though, only rely on respondents' wages used in the regression sample. Respondents in the NLSY79 also provide detailed work histories that indicate the weeks they were employed and unemployed throughout the sample period. Joining the monthly labor-force status of respondents to their earnings ability offers another measure of earnings ability over the business cycle. Importantly, this feature of the NLSY79 sample characterizes the unemployment pool in addition to the employment pool.

[13]This finding is consistent with evidence reported in McLaughlin and Bils (2001) and evidence from the BLS's Current Employment Statistics program that I report in figure 3.7.

[14]Cyclical variation in education is section C.3. Table C.2 indicates that variation in educational attainment explains much of the variation in the average wage before 1986, which is consistent with shares of educational attainment depicted in figure 3.4. After 1985, the cyclical coefficient for $\overline{w}_X$ constructed only from variation in educational attainment is 0.0895.

Table 3.2: Time-varying components of wage cyclicality

	Characteristics	Industry	Union	Experience
Cyclical UR	0.576^{**}	−0.000949	0.130^{+}	0.448^{**}
	(0.229)	(0.0345)	(0.0809)	(0.179)
N	18	18	18	18

Standard errors in parentheses

$^{+}$ $p < 0.15$, * $p < 0.1$, ** $p < 0.05$

3.4.2 Average Earnings Ability in the Employment and Unemployment Pools over the Business Cycle

Using a respondent's reported weekly labor-force status, I take their monthly status to be their status in whatever week contains the 12th day of the month, corresponding to the reference week in the CPS.[15] I use a respondent's status in January, April, July, and October to get a quarterly series, the frequency at which the Congressional Budget Office publishes their series on the natural rate of unemployment. I match a respondent's labor-force status to their earnings ability. The measure of earnings ability comes from estimating the panel regression in (3.1), using data that begin in 1985 and omitting educational effects in $X_{i,t}$. The result is a quarterly panel of earnings ability.

A quarterly series of average earnings ability is constructed by averaging over respondents in a given quarter by labor-force status. This is analogous to $\overline{w}_{\text{earnings ability},t}$ for employed workers and represents a shadow-wage statistic for unemployed workers.

To see how these quarterly statistics vary over the business cycle, I use the series to estimate regressions like those in (3.6).[16] The estimated coefficients on the cyclical unemployment rate are reported in table 3.3. A total of 72 observations compose the sample. These years of the sample include 1986–1994 and the even years after 1994 through 2012. They correspond to the NLSY79 survey rounds and total 18 years and 72 quarters.[17]

Table 3.3 reports the results. The employed column indicates that when the cyclical unemployment rate is 1 percentage point above trend, average earnings ability increases 0.113 percent among employed workers. The unemployed column indicates that when the cyclical unemployment rate is 1 percentage point above trend, average earnings ability increases 0.518 percent among unemployed workers. Both of these estimates are precisely

[15] Although the BLS has occasional exceptions to this rule. See https://www.bls.gov/cps/definitions.htm#refweek.

[16] In addition I include quarterly dummies to control for seasonality.

[17] Including non-NLSY79 years makes little difference and no qualitative difference. The results using these years are reported in table C.4. Not including these years is motivated by distinct patterns in respondents' wages in non-NLSY79 years (Basu and House, 2016).

Table 3.3: Cyclicality of average earnings ability for employed and unemployed

	1986q1–2012q4	
	Employed	Unemployed
Cyclical UR	0.113**	0.518**
	(0.0167)	(0.200)
N	72	72

Standard errors in parentheses
$^{+}$ $p < 0.15$, * $p < 0.1$, ** $p < 0.05$

estimated.

The cyclicality of average earnings ability in table 3.3 indicates that average earnings ability is less cyclical when measured using respondents' labor-force histories than when using only respondents who report earnings. This difference is partly attributable to more observations used to construct average earnings ability from the weekly status arrays. A respondent's weekly labor-force status is asked retrospectively. So even if the respondent missed reporting their earnings in a survey round, they are asked about about when they started their job. Their answer updates their labor-force status through their current status.

Putting aside differences in samples, the labor-force status arrays characterize the ability composition of the unemployment pool. Average earnings ability varies more in the unemployment pool than in the employment pool. This is not surprising given that the number of unemployed workers is less than one tenth the number of employed workers. These are no unemployed workers in the regression sample, so table 3.3 provides the only measure of earnings ability in the unemployment pool. The positive coefficients in table 3.3 indicate that, on average, workers with lower earnings ability are disproportionately let go in periods of high unemployment. As low-ability workers exit employment, they increase average ability in the unemployment pool. This pattern is consistent with the organized-by-ability model described in figure 3.1 and informs the model described in figures 3.2 and 3.3. Improving the incentive to post vacancies in periods of high unemployment will exacerbate the ability of heterogeneous-agent labor–macro models to replicate sustained periods of high unemployment observed in the data.

Earnings ability includes the earnings effect associated with a respondent's gender and race. The next section uses CPS data to make the case that these factors matter little for business-cycle variation in wages. The composition of employment in terms of educational attainment, marital status, age, and industry is also investigated. I argue that these factors do not drive the gap between the cyclicality of individual-level wages and the average wage. Eliminating these factors, leaves experience as the driving force.

In summary, using detailed labor-market histories of NLSY79 respondents in my preferred specification, earnings ability explains 19.6 percent of the gap between individual-level wages and the average wage. I arrive at this number through the following calculation.

Wage cyclicality is remarkably consistent in the NLSY79 data. But the NLSY79 cohort, which ages over time, does not provide a good measure of the average wage. To get a measure of the average wage I follow Basu and House (2016) and use compensation per hour in the nonfarm business sector. Using this series to estimate the cyclical-regression specification in (3.6) for the 1986–2012 period, the cyclicality coefficient is -0.615. The difference between -0.615 and individual-level wage cyclicality in table 3.1 is 0.577 $[(-0.615)-(-1.192)]$. The countercyclical effect of earnings ability over this period, reported in the bottom panel of table 3.3, is 0.113. Using these numbers, earnings ability countercyclically accounts for 19.6 percent ($100 \times 0.113/0.577$) of the difference between individual-level wages and the average wage.

3.4.3 Evidence from the Current Population Survey and Current Employment Statistics Program

This section is motivated by figure 3.6, which depicts shares of employment by educational attainment. These data come from the Current Population Survey.[18] Over time workers have earned higher degrees. For example, the college share more than doubled—college graduates made up around 12.5 percent of employment in early 1992 and made up over 25 percent by 2018.

But what stands out in the figure is the lack of cyclical variation in the shares. Even in the teeth of the Great Recession, the employment shares seemed to be on trend. The lack of cyclicality in employment shares offers little scope for the story that low-skill workers with low education comprise employment exists in recessions, making the average wage appear less cyclical.[19] Instead, the average wage is less cyclical because, within each broad educational category, workers with low earnings ability exit employment.

The remainder of this section looks at labor-force composition of prime-aged individuals over the business cycle. I find that the composition of employment, unemployment, and non-employment varies little with the business cycle in terms of education, sex, race, marital status, and age. The lack of variability in composition for different demographic

[18]The data come from the Bureau of Labor Statistics' LN series in the LABSTAT database. The series are seasonally adjusted by the BLS and count workers 25 years and older.

[19]Which is one reading of Solon, Barsky, and Parker (1994).

and industrial groups means variation in average wages is driven by wage cyclicality within each group. A wage-decomposition exercise corroborates this idea. Within each group, in other words, the evidence is consistent with firms retaining workers with more experience and higher earnings ability when the unemployment rate is high. Workers with lower ability who exit employment when the unemployment rate is high are not concentrated in any particular demographic group or industry.

3.4.3.1 Data

I use data from the Current Population Survey. Each month the CPS interviews a sample of roughly 60,000 households to determine characteristics of the civilian noninstitutional population. The civilian noninstitutional population comprises two groups—people in the labor force and people not in the labor force. The labor force in turn comprises two groups—the employed and unemployed. Additionally I consider the non-employed—people either unemployed or not in the labor force.

In addition to labor-force status, the CPS collects information on a respondents' demographic information. Respondents' answers to these questions can be used as a representative sample of the civilian population of the United States by using cross-sectional weights provided by the Bureau of Labor Statistics.

I restrict the population to respondents between 25 and 64 years of age. This restriction allows me to focus on the prime years of employment that are largely unaffected by other life transitions, like schooling and retirement. Additionally, focusing on respondents of this age group allows me to compare the cross-sectional data to longitudinal NLSY79 data, which follows respondents continually over the prime ages of their careers.

Respondents answers to the monthly surveys are used to construct aggregate series. I aggregate series to the quarterly level by using the average of monthly values in each quarter. All aggregated series are seasonally adjusted using the Census Bureau's X-13ARIMA-SEATS automatic procedures. The series cover 1976q1–2019q1.

Many of the series of interest represent the composition of a particular group. For example, CPS respondents report their gender as either male or female in the survey. The unemployment pool can be therefore written as the sum of unemployed men (M) and unemployed women (W): $u_t = u_{\mathrm{M}t} + u_{\mathrm{W}t}$. In general, the unemployment pool can be divided into groups listed as $\{g_1, \ldots, g_n\} = G$. For the case of men and women, $G = \{\mathrm{M}, \mathrm{W}\}$. Then

$$u_t = \sum_{g \in G} u_{gt} \implies 1 = \sum_{g \in G} \frac{u_{gt}}{u_t} = \sum_{g \in G} \text{share}_{gt},$$

where share_{gt} denotes the share of group g in the unemployment pool in period t. I similarly construct share series for the employed and non-employed and for people not in the labor force. I take the cyclical component of the share of workers in each group. The cyclical component of each share series is computed as described in Hamilton (2018).

I am interested in how the composition of the employment, unemployment, non-employment, and not-in-the-labor-force pools vary with the business cycle. Each pool is broken down by education, sex, race, marital status, and age to get a sense of how demographic composition varies over the business cycle. The industrial composition of employment is then investigated using data from the Current Employment Statistics (CES) program of the BLS, which provides estimates of employment and wage information for major industrial sectors of the economy.

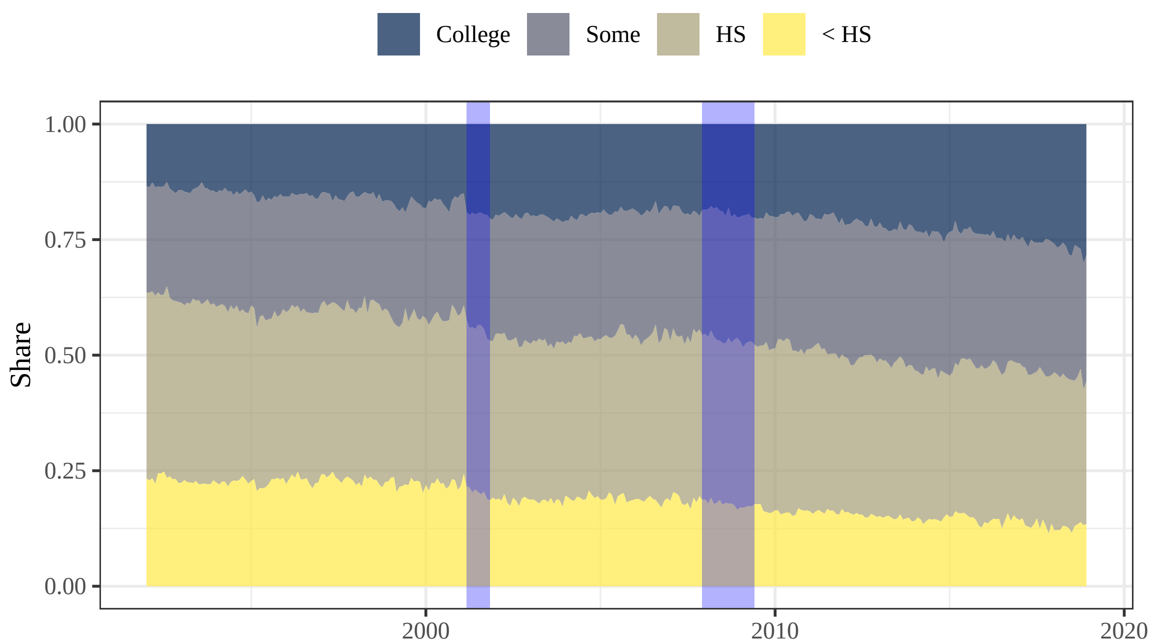


Figure 3.6: Shares of employment by educational attainment, 1992m1–2018m12.

3.4.3.2 Cyclicality of Shares

To assess how composition varies over the business cycle, I take a demographic like educational attainment and construct the shares of college-educated people, people with some college, and people with a high-school degree or less. For each of these share series

Table 3.4: Cyclical response of shares to fluctuations in the aggregate unemployment rate.

Series	E	U	NILF	N
College	0.084	−0.111	−0.172	−0.180
	(0.051)	(0.125)	(0.044)	(0.046)
Some college	0.036	0.194	0.099	0.171
	(0.049)	(0.117)	(0.051)	(0.058)
HS or less	−0.119	−0.093	0.061	−0.001
	(0.045)	(0.163)	(0.053)	(0.061)
Female	0.079	−1.255	−0.176	−0.837
	(0.027)	(0.119)	(0.061)	(0.111)
Nonwhite	−0.096	−0.483	0.064	0.129
	(0.035)	(0.109)	(0.045)	(0.045)
Unmarried	−0.041	−0.711	0.192	0.372
	(0.049)	(0.103)	(0.082)	(0.089)
Under 30	−0.035	−0.205	0.139	0.323
	(0.052)	(0.131)	(0.038)	(0.048)

Standard errors in parentheses.

I report $\hat{\beta}_g$ estimated from a regression of the form

$$\widetilde{\text{share}}_{gt} = \alpha_g + \beta_g \widetilde{\text{UR}}_t + \varepsilon_{gt}, \tag{3.7}$$

where $\widetilde{\text{UR}}_t$ is the cyclical component of the unemployment rate.[20] The cyclical coefficients, $\left\{\hat{\beta}_g\right\}$, are reported in table 3.4.

When there are only two classifications within a group, like married and unmarried people, I report only one of the two regression coefficients estimated in (3.4). This is because the unreported regression coefficient equals the negative of the reported coefficient. While this isn't guaranteed by the empirical exercise—I don't impose that the shares sum to one and the trends are estimated separately for each series—the estimated coefficients do in fact nearly sum to one in response to a business-cycle fluctuation.

Table 3.4 shows that business-cycle variation in labor-force shares is small. Turning

[20]The cyclical component is extracted using the method described in Hamilton (2018).

to the employment pool, which is reported in the first column of table 3.4, the first three rows—"College", "Some college", and "HS or less"—report the response of the employment pool delineated by educational attainment to the cyclical component of the aggregate unemployment rate. Table 3.4 indicates that a 1-percentage-point increase in the aggregate unemployment rate above trend predicts that the share of employed, college-educated workers increases by 0.08 percentage points; the share of employed workers with some college increases by 0.04 percentage points; and the share of employed workers with at least a high-school degree decreased by 0.12 percentage points. In other words, the the composition of the employment pool barely responds to the business cycle.

The same holds for gender. The share of men in the employment pool decreases by 0.08 percentage points when the unemployment rate is 1 percentage point above trend, according to the linear prediction. For the same fluctuation in unemployment, the share of women increases by 0.08 percentage points.

The white share in the employment pool, according to the linear regression, increases by 0.1 percentage points. The nonwhite share decreases by 0.1 percentage points. The married share increases slightly and the under-30 group decreases slightly.

Together, as indicated by the first column of table 3.4, the composition of the employment pool responds little to the business cycle.

This lack of business-cycle compositional variation may not be too surprising. This type of variation requires moving the composition of the stock of employed workers, which is large. Unemployment, although also a stock, is much smaller. Yet the composition of the unemployment pool responds only mildly more than the employment pool. The largest coefficient in the second column of table 3.4 is 1.25 for men. This number represents the prediction that the share of men in the unemployment pool increases by 1.25 percentage points when the aggregate unemployment rate is 1 percentage point above its trend.[21]

Columns 3 and 4 of table 3.4 indicate that the not-in-the-labor-force pool and the non-employment pool are predicted to barely budge when the aggregate unemployment rate is 1 percentage point above trend.

A similar business-cycle pattern holds for employment shares by major industrial sector. To see this, I repeat the exercise using data from the BLS's Current Employment Statistics program. The CES program publishes employment data for consistent industrial sectors beginning in 1972. There are 15 industrial sectors, including government.

[21]To be clear, the nonwhite unemployment rate is more volatile, even though the share of nonwhites in the unemployment pool falls relative to trend. The not-in-the-labor-force share of nonwhite rises as does the non-employment share. If these transitions happen because workers are discouraged, then this is further evidence of that this group "suffers disproportionately" (Aaronson et al., 2019, 370).

Figure 3.7 shows the $\hat{\beta}_g$s associated with estimating equation (3.4) for the 15 industrial sectors. The dot next to each sector represents the estimated coefficient and the black bar shows the 95-percent confidence interval. The "Government" coefficient reported in the first row of figure 3.7 indicates that that a 1-percentage-point increase in the aggregate unemployment rate above trend predicts that the share of workers working for the government increases by about 0.3 percentage points. This share is counter-cyclical but not large.

Looking down the row in figure 3.7, there are no large effects. For example, the share of workers engaged in durables manufacturing is predicted to decrease by less than 0.2 percentage points. Of the 15 sectors, 11 respond to a 1-percentage-point increase in the aggregate unemployment rate above trend by less than 0.1 percent.

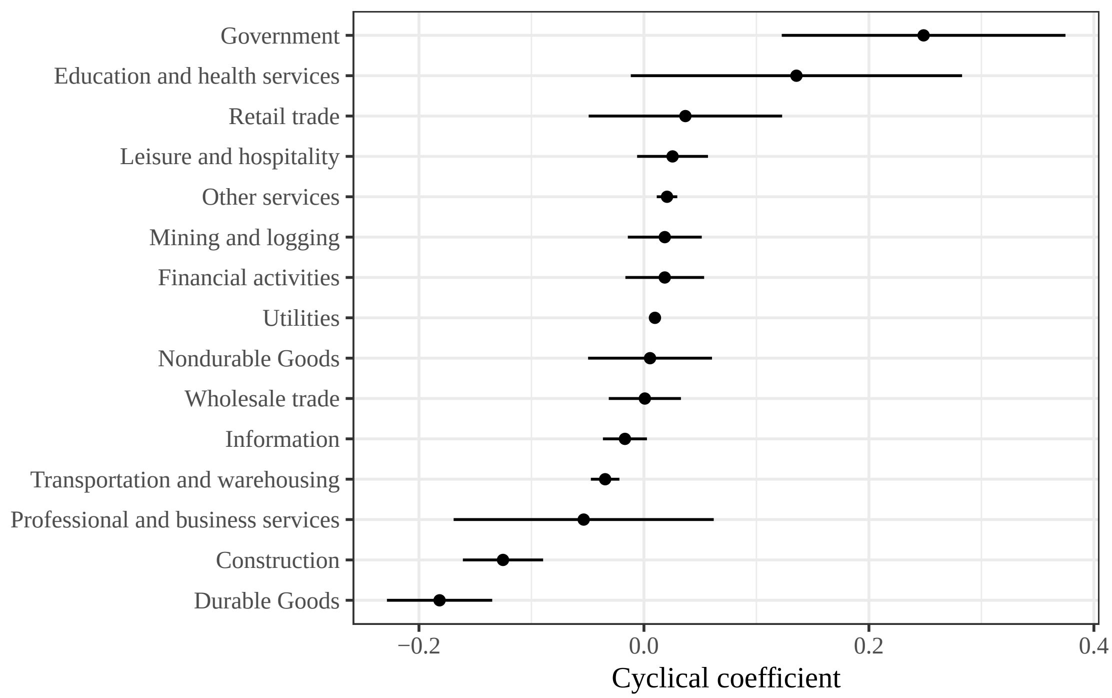


Figure 3.7: Predicted change in share when the cyclical unemployment rate is 1 percentage point above trend for major industrial sectors, 1972m1–2019m6.

3.4.3.3 Wage Decompositions

This section decomposes the variation in wages into a compositional effect and a wage-cyclicality effect. The decomposition is based on writing the average wage as a share-weighted average of different groups. The economy-wide average wage can therefore fluctuate for two reasons. Either the shares can fluctuate or average wages for different groups can fluctuate. I call fluctuations in the economy-wide average wage due to fluctuations in shares *compositional.* I call fluctuations in the economy-wide average wage due to fluctuation in a group's average wage *wage-cyclicality.* The main message of this section is that variation in the average wage is nearly entirely due to real-wage cyclicality. This pattern holds for decompositions based on the industrial composition of employment and employment delineated by different demographic groups.

To formalize the decomposition, let t denote the period of time. Let $\overline{W}_t$ denote the average wage in period t. In period t, the employment pool comprises different groups of workers. If there are only two groups, for example, then the average wage can be written as

$$\overline{W}_t = S_{1t}\overline{W}_{1t} + S_{2t}\overline{W}_{2t}, \tag{3.8}$$

where $\overline{W}_{jt}$ is the period-t average wage of group j.

For the average wage in period t, $\overline{W}_t$, let $\overline{w}_t$ denote its logarithm. Let the bold term $\overline{w}_t^{\top}$ denote the trend component of $\overline{w}_t$. Let $\widetilde{\overline{w}}_t$ denote $\overline{w}_t - \overline{w}_t^{\top}$, which represents the percent away $\overline{w}_t$ is from trend in period t, or the cyclical component of the average wage. The trend and cyclical components of the S_{jt}s are defined similarly, although $\widetilde{S}_{jt}$ represents the percentage points S_{jt} is from trend.

A first-order approximation of the average wage in equation (3.8) around trend is

$$\widetilde{\overline{w}}_t \approx \frac{\overline{W}^{\top}_{1t}}{\overline{W}^{\top}_t}\widetilde{S}_{1t} + \frac{\overline{W}^{\top}_{2t}}{\overline{W}^{\top}_t}\widetilde{S}_{2t} + S^{\top}_{1t}\frac{\overline{W}^{\top}_{1t}}{\overline{W}^{\top}_t}\widetilde{\overline{w}}_{1t} + S^{\top}_{2t}\frac{\overline{W}^{\top}_{2t}}{\overline{W}^{\top}_t}\widetilde{\overline{w}}_{2t}. \tag{3.9}$$

According to (3.9), how far the average wage is away from its trend in percentage terms depends on both fluctuations in group employment shares and how far group wages are away from trend in percentage terms.

The difference between a share and its trend in percentage points, $\widetilde{S}_{jt}$, is weighted by a group's relative-wage trend, $\overline{W}^{\top}_{jt}/\overline{W}^{\top}_t$. While it is true that $\widetilde{S}_{1t} + \widetilde{S}_{2t} = 0$, inducing a negative correlation between the two shares, the weights provide scope for share variation to explain fluctuations in the average wage. For example, if the share of group j fluctuates meaningfully and wages in group j are not insignificant relative to the average, then the effect on $\widetilde{\overline{w}}_t$ will be small.

The percent away a group's average wage is from its trend, $\widetilde{\overline{w}}_{jt}$, is weighted by the trend component of a group's employment share, $S^{\top}_{jt}$, and a group's relative-wage trend, $\overline{W}^{\top}_{jt}/\overline{W}^{\top}_t$. Fluctuations in a group's wage will be meaningful if the group's wage is high relative to average, but only if the employment share of group j is also meaningful.

One way to make the decomposition operational is by investigating the variance of average-wage fluctuations and attributing it to either compositional effects or wage cyclicality. The variance of average wage fluctuations can be written as $\text{var}\left(\widetilde{\overline{w}}_t\right) = \text{cov}\left(\widetilde{\overline{w}}_t, \widetilde{\overline{w}}_t\right)$. The two-sector analysis carries over directly to the n-sector case. Developing the expression for the variation in the average wage yields

$$1 \approx \underbrace{\sum_{j=1}^{n} \frac{\text{cov}\left(\overline{w}_t, \frac{\overline{W}_{jt}^{\intercal}}{\overline{W}_t^{\intercal}} \widetilde{S}_{jt}\right)}{\text{var}\left(\widetilde{\overline{w}}_t\right)}}_{\text{share contributions}} + \underbrace{\sum_{j=1}^{n} \frac{\text{cov}\left(\overline{w}_t, S_{jt}^{\intercal} \frac{\overline{W}_{jt}^{\intercal}}{\overline{W}_t^{\intercal}} \widetilde{w}_{jt}\right)}{\text{var}\left(\widetilde{\overline{w}}_t\right)}}_{\text{wage-cyclicality contributions}} . \tag{3.10}$$

The terms in (3.10) are convenient because each of the right-hand-side terms represents a regression coefficient. For example, the term $\text{cov}\left(\widetilde{\overline{w}}_t, \frac{\overline{W}_{jt}^{\intercal}}{\overline{W}_t^{\intercal}} \widetilde{S}_{jt}\right) / \text{var}\left(\widetilde{\overline{w}}_t\right)$ is the coefficient on $\widetilde{\overline{w}}_t$ in a regression of $\frac{\overline{W}_{jt}^{\intercal}}{\overline{W}_t^{\intercal}} \widetilde{S}_{jt}$ on $\widetilde{\overline{w}}_t$.

The sum of the share contributions represents the composition effect and the sum of the wage contributions represents the wage-cyclicality effect. The two effects should approximately sum to 1.

I use the decomposition in equation (3.10) to decompose fluctuations in the average wage into composition and real-wage-cyclicality effects for different demographic groups. Wages come from respondents answers in months 4 and 8 to their hourly earnings in the CPS. All series are seasonally adjusted using the Census X-13 program.

Table 3.5 lists separate decompositions for demographic groups delineated by education, sex, race, marital status, and age. Fluctuations in wages explain nearly all of the variation in the average wage.

Shares, though, do explain a small portion of the variation in the average wage for the education demographic. The educational composition of employment explains about 2.5 percent of the variation in the average wage. And marital status explains about 1 percent of the variation in the average wage.

While wage variation explains nearly all the variation in the average wage, wage variation differs across demographic groups. For example, looking at the first block of table 3.5, the wages of college-educated workers explain 48.8 percent of the variation of the average wage, the wages of workers with some college education explain 19.1 percent, and the wages of workers with a high-school degree or less explain 29.6 percent. In the second block of table 3.5, the wages of men explain 63.1 percent and the wages of women explain 37.7 percent of the variation in the average wage. The same pattern holds for workers delineated by race, marital status, and age; namely, variation in the majority group's wage

Table 3.5: Decompositions of average wages for various demographic groups.

Series	Wage	Share	Wage + share
College	0.488	0.040	0.528
Some college	0.191	0.059	0.250
HS or less	0.296	−0.075	0.221
Total	0.975	0.024	1.000
Female	0.377	0.047	0.424
Male	0.631	−0.054	0.577
Total	1.008	−0.007	1.001
Nonwhite	0.200	−0.034	0.166
White	0.797	0.037	0.834
Total	0.997	0.003	1.000
Unmarried	0.371	−0.072	0.299
Married	0.618	0.083	0.701
Total	0.990	0.010	1.000
Under 30	0.179	−0.019	0.160
Over 30	0.818	0.023	0.841
Total	0.997	0.003	1.001

explains the most significant share of the variation in the average wage.

While not reported, the patterns uncovered in the decomposition based on demographics continues to hold for industrial sectors in the CES data. This idea was foreshadowed in figure 3.7 by the lack of cyclical variability.

Data from the CPS and CES program established that both the industrial and demographic *compositions* of employment have little effect on the business-cycle variation in wages. Fluctuations in real wages are not explained by people moving in and out of employment for these demographic groups and industrial sectors, but rather by fluctuations in real wages at the industrial-sector or demographic-group level.

These facts are consistent with the analysis using NLSY79 data: The main drivers of wage cyclicality are experience and, to a lesser extent, earnings ability.

3.5 Conclusion

A worker's wage, on average, varies more with the business cycle than what would be expected by just looking at the average wage. This has to do with workers moving in and out of employment over the business cycle. Using data from the NLSY79, I have shown that the countercyclical compositional effect of earnings ability explains around 20 percent of the gap using respondents' detailed labor-market histories. The rest of the gap is primarily driven by a worker's cumulative work experience and on-the-job tenure. It is not driven by gender, race, educational attainment, union membership, marital status, and industrial composition.

I hope these facts will be of interest to macro modelers that differentiate workers by productivity. The simple macroeconomic structure I use to explain the issues at stake contains many features of richer models, which means the general lessons should form the basis for future work on earnings ability.

APPENDIX A

Appendix to Estimates of Matching Technology under Constant Returns to Scale with Implications for Fundamental Surplus

A.1 Derivation of Time-Aggregation Adjustment

This section provides a fuller derivation of the time-aggregation adjustment.

Workers are either employed or unemployed, transitioning between labor-market states. Time is continuous, while data are available only at discrete dates. Following Shimer (2012), refer to the interval $[t, t+1)$, for $t \in \{0, 1, 2, \ldots\}$, as "period t." Within the interval $[t, t+1)$ workers neither exit or enter the laborforce.

Define the following quantities:

- $F_t \in [0, 1]$ is the job-finding probability in period t,
- $S_t \in [0, 1]$ is the separation probability in period t,
- $f_t \equiv -\log(1 - F_t) \geq 0$ is the arrival rate of the Poisson process that changes a worker's state from unemployment to employment, and
- $s_t \equiv -\log(1 - S_t) \geq 0$ is the arrival rate of the Poisson process that changes a worker's state from employment to unemployment.

To be clear, F_t and S_t are probabilities, while f_t and s_t are rates. F_t is the probability that a worker who begins period t unemployed finds a job during period t; S_t is the probability that a worker who begins period t employed loses a job during period t. I am interested in uncovering S_t and F_t.

For any $t \in \{0, 1, 2, \ldots\}$ I let $\tau \in [0, 1]$ be the elapsed time since the start of the period. Define the following stocks:

- $e_{t+\tau}$ is the number of employed workers at time $t + \tau$,
- $u_{t+\tau}$ is the number of unemployed workers at time $t + \tau$, and
- $e_t^h(\tau)$ is the number of workers who are employed at time $t+\tau$ but were unemployed at some time $t' \in [t, t+\tau)$; that is, the number of new hires or matches.

The number of new hires is zero at the start of the period:

$$e_t^h(0) = 0 \text{ for all } t.$$

The number of new hires at the end of the period is defined as

$$e_{t+1}^h \equiv e_t^h(1)$$

as the number of new hires at the end of the period, as measured by the JOLTS program.

Employment within the period evolves according to

$$\dot{e}_{t+\tau} = f_t u_{t+\tau} - s_t e_{t+\tau} \tag{A.1}$$

$$\dot{e}^h_{t+\tau} = f_t u_{t+\tau} - s_t e^h_{t+\tau}. \tag{A.2}$$

Solving equation (A.2) for $f_t u_{t+h}$ and substituting this into equation (A.1) yields

$$\begin{aligned}\dot{e}_{t+\tau} &= f_t u_{t+\tau} - s_t e_{t+\tau} \\ &= \left(\dot{e}^h_{t+\tau} + s_t e^h_{t+\tau}\right) - s_t e_{t+\tau} \\ &= \dot{e}^h_{t+\tau} + s_t e^h_{t+\tau} - s_t e_{t+\tau} \text{ for } \tau \in [0,1).\end{aligned}$$

This is a linear first-order equation for $\dot{e}_{t+\tau}$ with $\tau \in [0,1)$.

The generation solution is

$$\begin{aligned}e_{t+\tau} &= \left[c + \int_0^\tau \left(\dot{e}^h_{t+z} + s_t e^h_{t+z}\right) e^{-\int_0^z -s_t dv} dz\right] e^{\int_0^\tau -s_t dz} \\ &= \left[c + \int_0^\tau \left(\dot{e}^h_{t+z} + s_t e^h_{t+z}\right) e^{s_t z} dz\right] e^{-s_t \tau}.\end{aligned} \tag{A.3}$$

which uses the fact that $a(\tau)$ is constant and thus independent of τ. Expanding the integral in the bracketed term in (A.3) yields

$$\begin{aligned}\int_0^\tau \left(\dot{e}^h_{t+z} + s_t e^h_{t+z}\right) e^{s_t z} dz &= \int_0^\tau e^{s_t z} \dot{e}^h_{t+z} dz + \int_0^\tau e^{s_t z} s_t e^h_{t+z} dz \\ &= \int_0^\tau e^{s_t z} \dot{e}^h_{t+z} dz + s_t \int_0^\tau e^{s_t z} e^h_{t+z} dz.\end{aligned} \tag{A.4}$$

The first integral on the right-hand size of the latter equation can be integrated by parts:

$$\begin{aligned}\int_0^\tau e^{s_t z} \dot{e}^h_{t+z} dz &= e^{s_t z} e^h_{t+z}\Big|_{z=0}^{z=\tau} - \int_0^\tau s_t e^{s_t z} e^h_{t+z} dz \\ &= \left(e^{s_t \tau} e^h_{t+\tau} - e^{s_t 0} e^h_t\right) - s_t \int_0^\tau e^{s_t z} e^h_{t+z} dz \\ &= e^{s_t \tau} e^h_{t+\tau} - s_t \int_0^\tau e^{s_t z} e^h_{t+z} dz,\end{aligned}$$

where the last equality uses the fact that $e^h_t = e^h_t(0) = 0$. Using this result in equation

(A.4) yields

$$\int_0^\tau \left(\dot{e}^h_{t+z} + s_t e^h_{t+z}\right) e^{s_t z} dz = e^{s_t \tau} e^h_{t+\tau} - s_t \int_0^\tau e^{s_t z} e^h_{t+z} dz + s_t \int_0^\tau e^{s_t z} e^h_{t+z} dz$$
$$= e^{s_t \tau} e^h_{t+\tau}.$$

Substituting this result into equation (A.3) yields

$$\begin{aligned} e_{t+\tau} &= \left[c + \int_0^\tau \left(\dot{e}^h_{t+z} + s_t e^h_{t+z}\right) e^{s_t z} dz\right] e^{-s_t \tau} \\ &= \left[c + e^{s_t \tau} e^h_{t+\tau}\right] e^{-s_t \tau} \\ &= c e^{-s_t \tau} + e^h_{t+\tau}. \end{aligned}$$

The determination of c comes from evaluating the latter at $\tau = 0$:

$$e_t = c + e^h_t (0) = c,$$

as there are no new hires at the beginning of the period and $e^h_t (0) = 0$. Therefore

$$e_{t+\tau} = e_t e^{-s_t \tau} + e^h_{t+\tau}.$$

Evaluating the latter at $\tau = 1$ yields

$$\begin{aligned} e_{t+1} &= e_t e^{-s_t} + e^h_{t+1} \\ &= e_t (1 - S_t) + e^h_{t+1}, \end{aligned} \tag{A.5}$$

which indicates that the level of employment in the following survey period equals the employed who do not separate from their jobs plus new hires. Solving this expression for S_t yields

$$\begin{aligned} e_{t+1} &= e_t (1 - S_t) + e^h_{t+1} \\ \therefore\ e_{t+1} - e_t &= -e_t S_t + e^h_{t+1} \\ \therefore\ e_t S_t &= e^h_{t+1} - e_{t+1} + e_t \\ \therefore\ S_t &= 1 - \frac{e_{t+1} - e^h_{t+1}}{e_t}. \end{aligned} \tag{A.6}$$

To solve for the finding rate, which is the point of interest, solve equation (A.1) forward. Define $l_t = u_t + e_t$ as the labor force. The labor force is assumed to be constant during that period; that is, $u_{t+\tau} + e_{t+\tau} = l_t$ for all $\tau \in [0, 1)$. Using this assumption implies

$$\begin{aligned} \dot{e}_{t+\tau} &= f_t u_{t+\tau} - s_t e_{t+\tau} \\ &= f_t \left(l_t - e_{t+\tau}\right) - s_t e_{t+\tau} \\ &= f_t l_t - \left(s_t + f_t\right) e_{t+\tau}. \end{aligned}$$

This is a linear differential equation for $\dot{e}_{t+\tau}$ with constant coefficients. The solution to this differential equation is

$$e_{t+\tau} = \frac{f_t l_t}{(s_t + f_t)} + c_2 e^{-(s_t+f_t)\tau}, \tag{A.7}$$

where c_2 is a constant. The value of c_2 comes from evaluating the latter expression at $\tau = 0$:

$$c_2 = e_t - \frac{f_t l_t}{s_t + f_t}.$$

Using the expression for c_2 in equation (A.7) implies

$$\begin{aligned} e_{t+\tau} &= \frac{f_t l_t}{(s_t + f_t)} + \left(e_t - \frac{f_t l_t}{s_t + f_t}\right) e^{-(s_t+f_t)\tau} \\ &= \frac{f_t l_t}{(s_t + f_t)} + e_t e^{-(s_t+f_t)\tau} - \frac{f_t l_t}{s_t + f_t} e^{-(s_t+f_t)\tau} \\ &= \frac{f_t l_t}{(s_t + f_t)} \left[1 - e^{-(s_t+f_t)\tau}\right] + e_t e^{-(s_t+f_t)\tau}. \end{aligned}$$

Evaluating this expression at $\tau = 1$ yields

$$e_{t+1} = \frac{f_t l_t}{(s_t + f_t)} \left[1 - e^{-(s_t+f_t)}\right] + e_t e^{-(s_t+f_t)}$$

or

$$e_{t+1} = \frac{\left(1 - e^{-s_t - f_t}\right) f_t}{(s_t + f_t)} l_t + e^{-(s_t+f_t)} e_t. \tag{A.8}$$

I am interested in uncovering F_t and S_t. The separation probability, S_t, is calculated using (A.6). The finding probability F_t is defined implicitly in equation (A.8). I solve equation A.8 using a bisection method to uncover f_t and then compute $F_t = 1 - e^{-f_t}$.

To see how a bisection method can be used to solve for f_t using (A.8), a simple application of the intermediate-value theorem establishes that there exists a solution. Define

$$\mathcal{T}(x) = e_{t+1} - \frac{\left(1 - e^{-s_t - x}\right) x}{(s_t + x)} l_t - e^{-(s_t + x)} e_t.$$

Then

$$\begin{aligned}
\mathcal{T}(0) &= e_{t+1} - e^{-s_t} e_t \\
&= e_{t+1} - (1 - S_t)\, e_t \\
&= e_{t+1} - \left(e_{t+1} - e^h_{t+1}\right) \\
&= e^h_{t+1} > 0,
\end{aligned}$$

where the second-to-last equation uses the expression for $(1 - s_t)\, e_t$ in (A.5). Additionally

$$\begin{aligned}
\lim_{x\to\infty} \mathcal{T}(x) &= e_{t+1} - \lim_{x\to\infty} \left[\frac{\left(1 - e^{-s_t - x}\right) x}{(s_t + x)} l_t\right] \\
&= e_{t+1} - \lim_{x\to\infty} \left[1 - e^{-s_t - x}\right] \lim_{x\to\infty} \left[\frac{x}{s_t + x} l_t\right] \\
&= e_{t+1} - l_t \\
&< 0,
\end{aligned}$$

where the last line uses L'Hôpital's rule and the fact that the limit of a product of functions is the product of the limits, provided that both limits exist. The inequality holds as long as next period's level of employment is less than the size of the current period's laborforce. An application of the intermediate-value theorem establishes the existence of an f_t that solves $\mathcal{T}(f_t) = 0$.

Uniqueness is harder to establish. The function $\mathcal{T}$ can be written

$$\begin{aligned}
\mathcal{T}(x) &= e_{t+1} - \frac{\left(1 - e^{-s_t - x}\right) x}{(s_t + x)} l_t - e^{-(s_t + x)} e_t \\
&= e_{t+1} - \frac{\left(1 - e^{-s_t} e^{-x}\right) x}{(s_t + x)} l_t - e^{-s_t} e^{-x} e_t \\
&= e_{t+1} - \frac{\left[1 - (1 - S_t)\, e^{-x}\right] x}{(s_t + x)} l_t - (1 - S_t)\, e^{-x} e_t \\
&= e_{t+1} - \frac{\left[x - (1 - S_t)\, x e^{-x}\right]}{(s_t + x)} l_t - (1 - S_t)\, e^{-x} e_t.
\end{aligned}$$

Developing this expression further yields

$$\begin{aligned}\mathcal{T}(x) &= e_{t+1} - \frac{x}{s_t + x} l_t + \frac{(1-S_t)\, x e^{-x}}{s_t + x} l_t - (1-S_t)\, e^{-x} e_t \\ &= e_{t+1} - \frac{x}{s_t + x} l_t + (1-S_t)\, e^{-x} \left(\frac{x}{s_t + x} l_t - e_t \right) \\ &= e_{t+1} - \frac{x}{s_t + x} l_t + (1-S_t)\, e^{-x} \left[\frac{x}{s_t + x} (e_t + u_t) - e_t \right] \\ &= e_{t+1} - \frac{x}{s_t + x} l_t + (1-S_t)\, e^{-x} \left[\frac{x}{s_t + x} u_t - \frac{s_t}{s_t + x} e_t \right].\end{aligned}$$

Then

$$\mathcal{T}'(x) = -\frac{s_t}{(s_t + x)^2} l_t + \frac{d}{dx} \left\{ (1-S_t)\, e^{-x} \left[\frac{x}{s_t + x} u_t - \frac{s_t}{s_t + x} e_t \right] \right\}.$$

Looking at the term in curly brackets

$$\begin{aligned}&\frac{d}{dx} \left\{ (1-S_t)\, e^{-x} \left[\frac{x}{s_t + x} u_t - \frac{s_t}{s_t + x} e_t \right] \right\} \\ &\quad = -(1-S_t)\, e^{-x} \left[\frac{x}{s_t + x} u_t - \frac{s_t}{s_t + x} e_t \right] + (1-S_t)\, e^{-x} \frac{d}{dx} \left[\frac{x}{s_t + x} u_t - \frac{s_t}{s_t + x} e_t \right] \\ &\quad = -(1-S_t)\, e^{-x} \left[\frac{x}{s_t + x} u_t - \frac{s_t}{s_t + x} e_t \right] + (1-S_t)\, e^{-x} \left[\frac{s_t}{(s_t + x)^2} u_t + \frac{s_t}{(s_t + x)^2} e_t \right] \\ &\quad = -(1-S_t)\, e^{-x} \left[\frac{x}{s_t + x} u_t - \frac{s_t}{s_t + x} e_t \right] + (1-S_t)\, e^{-x} \frac{s_t}{(s_t + x)^2} l_t.\end{aligned}$$

Note that

$$\frac{x}{s_t + x} u_t - \frac{s_t}{s_t + x} e_t = \frac{x u_t - s_t e_t}{s_t + x}.$$

Combining these two expressions yields

$$\begin{aligned}\mathcal{T}'(x) &= -\frac{s_t}{(s_t + x)^2} l_t + \frac{d}{dx} \left\{ (1-S_t)\, e^{-x} \left[\frac{x}{s_t + x} u_t - \frac{s_t}{s_t + x} e_t \right] \right\} \\ &= \frac{s_t}{(s_t + x)^2} l_t \left[(1-S_t)\, e^{-x} - 1 \right] - (1-S_t)\, e^{-x} \left[\frac{x}{s_t + x} u_t - \frac{s_t}{s_t + x} e_t \right] \\ &= \frac{s_t}{(s_t + x)^2} l_t \left[(1-S_t)\, e^{-x} - 1 \right] - \frac{(1-S_t)\, e^{-x}}{s_t + x} \dot{e}_t,\end{aligned}$$

where the last line uses the definition of $\dot{e}_{t+\tau}$ in (A.1) evaluated at $\tau = 0$. The first term is negative because

$$(1 - S_t)\, e^{-x} - 1 < 0$$

as $1 - S_t$ and e^{-x} are both in $[0, 1]$. If $\dot{e}_t > 0$, then there is a unique solution. If $\dot{e}_t < 0$, then there is a unique solutions as long as the magnitude of $\dot{e}_t$ is not too large. In general, this does not appear to be true. None of the empirical work ran into the problem of multiple solutions.

A.2 Properties of Matching Technologies

In this section I collect a few well known properties of two constant-returns-to-scale matching technologies. The first is the familiar Cobb–Douglas parameterization. The second is the matching technology suggested in den Haan, Ramey, and Watson (2000). The main takeaway is that the Cobb–Douglas parameterization implies that within a given period the probability that a worker finds a job and the probability that a firm fills a vacancy can fall outside of the unit interval, $[0, 1]$; whereas, the nonlinear parameterization in den Haan, Ramey, and Watson (2000) constrains the job-finding and job-filling probabilities to fall inside of the unit interval.

A.2.1 Elasticity of Matching with Respect to Unemployment

Before turning to the particular parameterization, I state and prove a well known result about the elasticity of matching with respect to unemployment. This result will be used in the discussion about the decomposition of the elasticity of tightness.

In general, a matching technology computes the number of new matches or new hires produced when u workers are searching for jobs and v vacancies are posted. A matching technology, in other words, maps unemployment and vacancies into matches. It is increasing in both its arguments and exhibits constant returns to scale in u and v.

I use the following notation:

- M denotes the number of new matches.
- u denotes the measure of unemployed workers searching for a job.
- v denotes the measure of vacancies posted by firms recruiting workers.
- $\theta = v/u$, the ratio of vacancies to unemployment, denotes labor-market tightness.

- $q(\theta) = M/v$ denotes the probability that a vacancy is filled.

- $\theta q(\theta) = M/u$ denotes the probability that a worker finds a job.

That M/v denotes the probability that a posted vacancy is filled follows from the assumption that search is random, meaning each vacancy faces the same likelihood of being filled.

The elasticity of matching with respect to unemployment is

$$\eta_{M,u} = \frac{dM}{du}\frac{u}{M} = -\frac{\theta q'(\theta)}{q(\theta)}. \tag{A.9}$$

The expression in (A.9) comes from direct computation. Indeed, from the definition of job filling, $q(\theta) = M/v$,

$$\begin{aligned}\frac{dM}{du} &= \frac{d}{du}\left[q(\theta)\, v\right] \\ &= q'(\theta)\frac{-v}{u^2}v = -q'(\theta)\,\theta^2.\end{aligned}$$

Thus

$$\begin{aligned}\frac{dM}{du}\frac{u}{M} &= -\frac{q'(\theta)\,\theta^2}{M/u} = -\frac{q'(\theta)\,\theta^2}{\theta q(\theta)} \\ &= -\frac{\theta q'(\theta)}{q(\theta)} > 0,\end{aligned}$$

where the last equality in the first line uses the definition of job finding; that is, $\theta q(\theta) = M/u$. The inquality uses the property that $q' < 0$. The fact that $\eta_{M,u} > 0$ says that, for a given level of labor demand, an increase in workers searching for jobs increases the number of new hires.

Moreover $\eta_{M,u}$ lies in the interval $(0, 1)$. It has already been established that $\eta_{M,u} > 0$. The fact that $\eta_{M,u} < 1$ can be established by differentiation $\theta q(\theta)$ with respect to v:

$$\begin{aligned}\frac{d}{dv}\left[\theta q(\theta)\right] &= \{[1 \times q(\theta)] + \theta q'(\theta)\}\frac{1}{u} \\ &= \left[q(\theta) + \theta q'(\theta)\right]\frac{1}{u}.\end{aligned}$$

Because $\theta q(\theta)$ can be written $\theta M(u,v)/v$, it also follows that

$$\begin{aligned}\frac{d}{dv}[\theta q(\theta)] &= \frac{1}{u}\frac{M}{v} + \theta\frac{M_v v - M}{v^2} \\ &= \frac{1}{u}\frac{M}{v} + \theta\left(\frac{M_v}{v} - \frac{q(\theta)}{v}\right) \\ &= \frac{1}{v}\left\{\theta q(\theta) + \theta\left[M_v - q(\theta)\right]\right\}\end{aligned}$$

where M_v is the derivative of the matching function with respect to vacancies. Combining these two expressions yields

$$\begin{aligned}\left[q(\theta) + \theta q'(\theta)\right]\frac{1}{u} &= \frac{1}{v}\left\{\theta q(\theta) + \theta\left[M_v - q(\theta)\right]\right\} \\ \therefore\ \left[q(\theta) + \theta q'(\theta)\right]\theta &= \theta q(\theta) + \theta\left[M_v - q(\theta)\right] \\ \therefore\ \theta q'(\theta) &= M_v - q(\theta) \\ \therefore\ \frac{\theta q'(\theta)}{q(\theta)} &= \frac{M_v}{q(\theta)} - 1 \\ \therefore\ 1 - \left(-\frac{\theta q'(\theta)}{q(\theta)}\right) &= \frac{M_v}{q(\theta)} \\ \therefore\ 1 - \eta_{M,u} &= \frac{M_v}{q(\theta)}.\end{aligned}$$

Therefore $1 - \eta_{M,u}$ is positive since M is increasing in both its arguments. Re-arranging establishes that $\eta_{M,u} < 1$. Hence $\eta_{M,u} \in (0,1)$. These results are collected in proposition 8.

Proposition 8 *Given a constant-returns to scale matching function, $M(u,v)$, that is increasing in both its arguments, the elaticity of matching with respect to unemployment, $\eta_{M,u}$=-$\theta q'(\theta)/q(\theta)$, lies in the interval* $(0,1)$.

A.2.2 Parameterizations

One parameterization of M is

$$\mathsf{M}(u,v) = Au^{1-\alpha}v^{\alpha},$$

which imposes constant returns to scale in u and v. This is the Cobb–Douglas parameterization. Because M is increasing in both unemployment and vacancies, $1-\alpha > 0$ and $\alpha > 0$. These two inequalities imply $0 < \alpha < 1$. Under random search, the probability that a vacancy is filled is M/v.

Under the Cobb–Douglas parameterization, the probability that a vacancy is filled is

$$q(\theta) = \frac{M}{v} = Au^{1-\alpha}v^{\alpha-1} = A\theta^{\alpha-1} = A\left(\frac{1}{\theta}\right)^{1-\alpha}.$$

A direct computation establishes that the job-filling probability is decreasing in tightness. It is harder, in other words, for an individual firm to fill a vacancy the more vacancies there are for a given level of unemployment. From the expression for $q(\theta)$, it is straightforward to see how q can be larger than 1. For example, if $A = 1$, $\theta = .25$, and $\alpha = .5$, then

$$\left(\frac{1}{.25}\right)^{.5} = 4^{.5} = 2.$$

This value of α is well within the range of empirical estimates and the value of θ was experienced in the US economy around 2010.

The probability that a worker finds a job is

$$\theta q(\theta) = \frac{M}{u}\frac{v}{v} = A\theta^{\alpha}.$$

The job-finding probability is increasing in θ. It is easier, in other words, for an individual worker to find a job the more vacancies there are for a given level of unemployment. When evaluated at the parameters from the example above, job-finding is

$$.25^{.5} = \left(\frac{1}{4}\right)^{.5} = .5.$$

These simple numerical examples suggest that a reasonable job-finding probability may easily lead to an unreasonable job-filling rate.

The elasticity of matching with respect to unemployment is

$$\begin{aligned}\eta_{M,u} &= -\frac{\theta q'(\theta)}{q(\theta)} \\ &= -\frac{\theta(\alpha-1)A\theta^{\alpha-2}}{A\theta^{\alpha-1}} \\ &= 1-\alpha.\end{aligned}$$

Another parameterization of the matching technology, suggested in den Haan, Ramey, and Watson (2000), is

$$\mathcal{M}(u,v) = \mathcal{A}\frac{uv}{[u^{\gamma}+v^{\gamma}]^{1/\gamma}}, \quad \gamma > 0.$$

The function $\mathcal{M}$ is increasing in both its arguments. Indeed,

$$\begin{aligned}
\frac{\partial \mathcal{M}}{\partial u} &= \mathcal{A}\frac{v\left(u^{\gamma}+v^{\gamma}\right)^{1/\gamma}-\frac{1}{\gamma}\left(u^{\gamma}+v^{\gamma}\right)^{1/\gamma-1}\gamma u^{\gamma-1}uv}{\left(u^{\gamma}+v^{\gamma}\right)^{\frac{2}{\gamma}}} \\
&= \mathcal{A}\frac{v\left[u^{\gamma}+v^{\gamma}\right]^{1/\gamma}-\left(u^{\gamma}+v^{\gamma}\right)^{1/\gamma-1}u^{\gamma}v}{\left(u^{\gamma}+v^{\gamma}\right)^{\frac{2}{\gamma}}} \\
&= \mathcal{A}\frac{u\left[u^{\gamma}+v^{\gamma}\right]^{1/\gamma}}{\left(u^{\gamma}+v^{\gamma}\right)^{\frac{2}{\gamma}}}\left(1-\frac{u^{\gamma}}{u^{\gamma}+v^{\gamma}}\right) \\
&> 0.
\end{aligned}$$

A symmetric argument establishes that $\mathcal{M}$ is increasing in v as well.

Under the nonlinear parameterization, the probability that a vacancy is filled is

$$\begin{aligned}
q(\theta) = \frac{\mathcal{M}}{v} &= \mathcal{A}\frac{u}{\left[u^{\gamma}+v^{\gamma}\right]^{1/\gamma}}\frac{1/u}{1/u} \\
&= \mathcal{A}\frac{1}{\left[1+(v/u)^{\gamma}\right]^{1/\gamma}} \\
&= \mathcal{A}\frac{1}{\left(1+\theta^{\gamma}\right)^{1/\gamma}}.
\end{aligned}$$

A direct computation establishes that the job-filling probability is decreasing in tightness:

$$\frac{dq(\theta)}{d\theta} = -\frac{\mathcal{A}}{\gamma}\frac{1}{\left(1+\theta^{\gamma}\right)^{1/\gamma-1}}\gamma\theta^{\gamma-1} < 0.$$

The probability a worker finds a job is

$$\theta q(\theta) = \frac{\mathcal{M}}{u} = \mathcal{A}\frac{\theta}{\left(1+\theta^{\gamma}\right)^{1/\gamma}}.$$

The job-finding probability under the nonlinear parameterization is increasing in θ:

$$\begin{aligned}
\frac{d}{d\theta}\left[\theta q(\theta)\right] &= \mathcal{A}\frac{\left(1+\theta^{\gamma}\right)^{1/\gamma}-\theta\frac{1}{\gamma}\left(1+\theta^{\gamma}\right)^{1/\gamma-1}\gamma\theta^{\gamma-1}}{\left(1+\theta^{\gamma}\right)^{2/\gamma}} \\
&= \mathcal{A}\frac{\left(1+\theta^{\gamma}\right)^{1/\gamma}-\left(1+\theta^{\gamma}\right)^{1/\gamma-1}\theta^{\gamma}}{\left(1+\theta^{\gamma}\right)^{2/\gamma}} \\
&= \mathcal{A}\frac{\left(1+\theta^{\gamma}\right)^{1/\gamma}}{\left(1+\theta^{\gamma}\right)^{2/\gamma}}\left(1-\frac{\theta^{\gamma}}{1+\theta^{\gamma}}\right) \\
&> 0.
\end{aligned}$$

Importantly, for the nonlinear matching technology, when $\mathcal{A} = \infty$, the job-finding probability is between 0 and 1. Indeed,

$$\lim_{\theta \to 0} \theta q(\theta) = \lim_{\theta \to 0} \mathcal{A} \frac{\theta}{(1+\theta^{\gamma})^{1/\gamma}} = 0$$

and

$$\begin{aligned}\lim_{\theta \to \infty} \theta q(\theta) &= \lim_{\Theta \to \infty} \mathcal{A} \frac{\theta}{(1+\theta^{\gamma})^{1/\gamma}} = \lim_{\theta \to \infty} \mathcal{A} \frac{1}{(1+\Theta^{\gamma})^{1/\gamma - 1} \Theta^{\gamma-1}} \\ &= \mathcal{A},\end{aligned}$$

where the second-to-last equality uses L'Hôpital's rule and the fact that

$$\begin{aligned}(1+\theta^{\gamma})^{1/\gamma-1}\theta^{\gamma-1} &= (1+\theta^{\gamma})^{\frac{1-\gamma}{\gamma}} \left(\frac{1}{\theta}\right)^{1-\gamma} \\ &= (1+\theta^{\gamma})^{\frac{1-\gamma}{\gamma}} \left(\frac{1}{\theta}\right)^{1-\gamma} \\ &= (1+\theta^{\gamma})^{\frac{1-\gamma}{\gamma}} \left(\left[\frac{1}{\theta}\right]^{\gamma}\right)^{\frac{1-\gamma}{\gamma}} \\ &= (1+\theta^{\gamma})^{\frac{1-\gamma}{\gamma}} \left(\frac{1}{\theta^{\gamma}}\right)^{\frac{1-\gamma}{\gamma}} \\ &= \left[\frac{1}{\theta^{\gamma}}(1+\theta^{\gamma})\right]^{\frac{1-\gamma}{\gamma}} = \left[1+\frac{1}{\theta^{\gamma}}\right]^{\frac{1-\gamma}{\gamma}}\end{aligned}$$

and therefore

$$\lim_{\theta \to \infty} (1+\theta^{\gamma})^{1/\gamma-1}\theta^{\gamma-1} = \lim_{\theta \to \infty} \left[1+\frac{1}{\theta^{\gamma}}\right]^{\frac{1-\gamma}{\gamma}} = 1.$$

Additionally, the fact that the job-finding probability is increasing everywhere implies that the probability a worker finds a job lies between 0 and 1 when $\mathcal{A} = 1$.

Similarly, the job-filling probability for the nonlinear parameterization falls between 0 and 1. Indeed,

$$\lim_{\theta \to \infty} q(\theta) = \lim_{\theta \to \infty} \mathcal{A} \frac{1}{(1+\theta^{\gamma})^{1/\gamma}} = 0$$

and

$$\lim_{\theta \to 0} q(\theta) = \lim_{\theta \to \infty} \mathcal{A} \frac{1}{(1+\theta^{\gamma})^{1/\gamma}} = \mathcal{A}.$$

The fact that the job-filling probability is decreasing everywhere implies that the probability a job is filled falls between 0 and 1 when $\mathcal{A} = 1$.

The elasticity of matching with respect to unemployment for the nonlinear parameterization is

$$
\begin{aligned}
\eta_{M,u} &= -\frac{\theta q'(\theta)}{q(\theta)} \\
&= \frac{\theta \frac{1}{\gamma} \mathcal{A} (1+\theta^{\gamma})^{-1/\gamma - 1} \gamma \theta^{\gamma-1}}{\mathcal{A} (1+\theta^{\gamma})^{-1/\gamma}} \\
&= \frac{(1+\theta^{\gamma})^{-1/\gamma-1} \theta^{\gamma}}{(1+\theta^{\gamma})^{-1/\gamma}} \\
&= (1+\theta^{\gamma})^{-1} \theta^{\gamma} = \frac{\theta^{\gamma}}{1+\theta^{\gamma}}.
\end{aligned}
$$

As implied by proposition 8, the elasticity falls inside the unit interval.

A.3 Canonical Search Model

This section provides a detailed analysis of the model described in section 1.4.2.

A.3.1 Key Bellman Equations and a Unique Equilibrium

Here I repeat the economy's key Bellman equations described in the text for ease of reference. The key Bellman equations for firms are

$$\mathcal{J} = y - w + \beta \left[s\mathcal{V} + (1-s)\mathcal{J} \right], \tag{A.10}$$

$$\mathcal{V} = -c + \beta \left\{ q(\theta)\mathcal{J} + [1 - q(\theta)]\mathcal{V} \right\}. \tag{A.11}$$

Imposing the zero-profit condition in equation (A.11) implies

$$
\begin{aligned}
0 &= -c + \beta \left\{ q(\theta)\mathcal{J} + [1 - q(\theta)\,0] \right\} \\
\therefore\ c &= \beta q(\theta)\mathcal{J}
\end{aligned}
$$

or

$$\mathcal{J} = \frac{c}{\beta q(\theta)}. \tag{A.12}$$

Substituting this result into equation (A.10) and imposing the zero-profit condition implies

$$\begin{aligned}
\mathcal{J} &= y - w + \beta \left[s\mathcal{V} + (1-s)\, \mathcal{J} \right] \\
\therefore \frac{c}{\beta q(\theta)} &= y - w + \beta (1-s) \frac{c}{\beta q(\theta)} \\
\therefore \frac{c}{\beta q(\theta)} &= y - w + (1-s) \frac{c}{q(\theta)} \\
\therefore w &= y + (1-s) \frac{c}{q(\theta)} - \frac{c}{\beta q(\theta)} \\
\therefore w &= y + \frac{c}{q(\theta)} \left(1 - s - \frac{1}{\beta} \right) \\
\therefore w &= y + \frac{c}{q(\theta)} (-s - r)\,.
\end{aligned}$$

This simplifies to

$$w = y - \frac{r+s}{q(\theta)} c. \tag{A.13}$$

The key Bellman equations for workers are

$$\mathcal{E} = w + \beta \left[s\mathcal{U} + (1-s)\, \mathcal{E} \right] \tag{A.14}$$

$$\mathcal{U} = z + \beta \left\{ \theta q(\theta)\, \mathcal{E} + [1 - \theta q(\theta)]\, \mathcal{U} \right\}. \tag{A.15}$$

In the canonical matching model, the match surplus, $\mathcal{S} = (\mathcal{J} - \mathcal{V}) + (\mathcal{E} - \mathcal{U})$, is split between a matched firm and worker. The outcome of Nash bargaining specifies

$$\mathcal{E} - \mathcal{U} = \phi \mathcal{S} \text{ and } \mathcal{J} = (1-\phi)\, \mathcal{S}, \tag{A.16}$$

where $\phi \in [0, 1)$ measures the worker's bargaining power.

Solving equation (A.10) for $\mathcal{J}$ yields

$$\begin{aligned}
\mathcal{J} &= y - w + \beta (1-s)\, \mathcal{J} \\
\therefore \mathcal{J} \left[1 - \beta (1-s) \right] &= y - w \\
\therefore \mathcal{J} &= \frac{y - w}{1 - \beta (1-s)}.
\end{aligned}$$

And solving (A.14) for $\mathcal{E}$ yields

$$\begin{aligned}
\mathcal{E} &= w + \beta s \mathcal{U} + \beta (1-s) \mathcal{E} \\
\therefore \mathcal{E} [1 - \beta (1-s)] &= w + \beta s \mathcal{U} \\
\therefore \mathcal{E} &= \frac{w + \beta s \mathcal{U}}{1 - \beta (1-s)} = \frac{w}{1 - \beta (1-s)} + \frac{\beta s \mathcal{U}}{1 - \beta (1-s)}.
\end{aligned}$$

Developing the expressions in (A.16) yields

$$\begin{aligned}
\mathcal{E} - \mathcal{U} &= \phi \mathcal{S} \\
&= \phi \frac{\mathcal{J}}{1 - \phi}
\end{aligned}$$

and using the just-derived expressions for $\mathcal{J}$ and $\mathcal{E}$ yields

$$\underbrace{\left[\frac{w}{1 - \beta (1-s)} + \frac{\beta s \mathcal{U}}{1 - \beta (1-s)}\right]}_{\mathcal{E}} - \mathcal{U} = \frac{\phi}{1 - \phi} \underbrace{\left[\frac{y - w}{1 - \beta (1-s)}\right]}_{\mathcal{J}}.$$

Developing this expression yields

$$\begin{aligned}
w + \beta s \mathcal{U} - [1 - \beta (1-s)] \mathcal{U} &= \frac{\phi}{1 - \phi} (y - w) \\
\therefore w + \beta s \mathcal{U} - \mathcal{U} + \beta \mathcal{U} - s \beta \mathcal{U} &= \frac{\phi}{1 - \phi} (y - w) \\
\therefore w &= \frac{\phi}{1 - \phi} (y - w) + (1 - \beta) \mathcal{U} \\
\therefore (1 - \phi) w &= \phi (y - w) + (1 - \phi)(1 - \beta) \mathcal{U} \\
\therefore w &= \phi y + (1 - \beta) \mathcal{U} - \phi (1 - \beta) \mathcal{U}.
\end{aligned}$$

Using the fact that

$$1 - \beta = 1 - \frac{1}{1 + r} = \frac{1 + r - 1}{1 + r} = \frac{r}{1 + r},$$

the latter expression can be written as

$$w = \frac{r}{1 + r} \mathcal{U} + \phi \left(y - \frac{r}{1 + r} \mathcal{U} \right), \tag{A.17}$$

which is equation (12) in Ljungqvist and Sargent (2017). The annuity value of being unemployed is $r\mathcal{U}/(1 + r)$. To get an expression for the annuity value, solve equation (A.15) for $\mathcal{E} - \mathcal{U}$ and substitute this expression and equation (A.12)) into equations (A.16. Turning to equation (A.15):

$$\begin{aligned}
\mathcal{U} &= z + \beta \left\{ \theta q(\theta) \mathcal{E} + [1 - \theta q(\theta)] \mathcal{U} \right\} \\
\therefore \mathcal{U} &= z + \beta \theta q(\theta) \mathcal{E} + \beta \mathcal{U} - \beta \theta q(\theta) \mathcal{U} \\
\therefore \mathcal{U} &= z + [\beta \theta q(\theta)] (\mathcal{E} - \mathcal{U}) + \beta \mathcal{U} \\
\mathcal{U}(1 - \beta) - z &= [\beta \theta q(\theta)] (\mathcal{E} - \mathcal{U}) \\
\therefore \mathcal{E} - \mathcal{U} &= \frac{1}{\beta \theta q(\theta)} [(1 - \beta) \mathcal{U} - z] \\
&= \frac{1 + r}{\theta q(\theta)} [(1 - \beta) \mathcal{U} - z] \\
&= \frac{r}{\theta q(\theta)} \mathcal{U} - \frac{1 + r}{\theta q(\theta)} z.
\end{aligned}$$

Using this expression for $\mathcal{E} - \mathcal{U}$ in (A.16) yields

$$\begin{aligned}
\mathcal{E} - \mathcal{U} &= \phi \mathcal{S} \\
\therefore \frac{r}{\theta q(\theta)} \mathcal{U} - \frac{1 + r}{\theta q(\theta)} z &= \phi \mathcal{S} \\
&= \phi \left(\frac{\mathcal{J}}{1 - \phi} \right) \\
&= \frac{\phi}{1 - \phi} \frac{c}{\beta q(\theta)},
\end{aligned}$$

where the last equality uses the expression for $\mathcal{J}$ in equation (A.12). Developing this expression yields

$$\begin{aligned}
\frac{r}{\theta q(\theta)} \mathcal{U} - \frac{1 + r}{\theta q(\theta)} z &= \frac{\phi}{1 - \phi} \frac{c}{\beta q(\theta)} \\
\therefore r\mathcal{U} - (1 + r) z &= \frac{\phi}{1 - \phi} \frac{1}{\beta} c\theta \\
\therefore r\mathcal{U} - (1 + r) z &= \frac{\phi}{1 - \phi} (1 + r) c\theta \\
\therefore \frac{r}{1 + r} \mathcal{U} &= z + \frac{\phi c\theta}{1 - \phi}, \qquad \text{(A.18)}
\end{aligned}$$

which is equation (10) in Ljungqvist and Sargent (2017). Substituting equation (A.18)) into equation (A.17 yields

$$\begin{aligned}
w &= \frac{r}{1+r}\mathcal{U} + \phi\left(y - \frac{r}{1+r}\mathcal{U}\right) \\
&= z + \frac{\phi c\theta}{1-\phi} + \phi\left(y - z - \frac{\phi c\theta}{1-\phi}\right) \\
&= (1-\phi)\,z + (1-\phi)\left(\frac{\phi c\theta}{1-\phi}\right) + \phi y \\
&= (1-\phi)\,z + \phi c\theta + \phi y \\
&= z + \phi\,(y - z + \theta c)\,. \qquad \text{(A.19)}
\end{aligned}$$

The two expression in (A.13)) and (A.19 for the wage rate jointly determine the equilibrium value of θ:

$$y - \frac{r+s}{q(\theta)}c = z + \phi\,(y - z + \theta c)\,.$$

Developing this expression yields

$$\begin{aligned}
y - \frac{r+s}{q(\theta)}c &= z + \phi\,(y - z + \theta c) \\
\therefore\ y - z &= \phi\,(y - z + \theta c) + \frac{r+s}{q(\theta)}c \\
\therefore\ (1-\phi)\,(y-z) &= \phi\theta c + \frac{r+s}{q(\theta)}c \\
&= \left[\frac{\phi\theta q(\theta)}{q(\theta)} + \frac{r+s}{q(\theta)}\right]c \\
\therefore\ y - z &= \frac{r+s+\phi\theta q(\theta)}{(1-\phi)\,q(\theta)}c \qquad \text{(A.20)}
\end{aligned}$$

Equation A.20 implicitly defines an equilibrium level of tightness. Existence and uniqueness of equilibrium tightness is established by proposition 9.

Proposition 9 *Suppose $y > z$, which says that workers produce more of the homogeneous consumption good at work than at home, and suppose that $(1-\phi)(y-z)/(r+s) > c$, a unique $\theta > 0$ solves the relationship in (A.20). The condition that $(1-\phi)(y-z)/(r+s) > c$ requires that the marginal value of the first vacancy is positive.*

To establish the existence part of proposition 9, I define the function

$$\begin{aligned}\mathcal{T}\left(\tilde{\theta}\right) &= \frac{y-z}{c} - \frac{r+s+\phi\tilde{\theta}q\left(\tilde{\theta}\right)}{(1-\phi)\,q\left(\tilde{\theta}\right)} \\ &= \frac{y-z}{c} - \frac{r+s}{(1-\phi)\,q\left(\tilde{\theta}\right)} - \frac{\phi}{1-\phi}\tilde{\theta}.\end{aligned}$$

Then, using the fact that $\lim_{\theta\to 0} q(\theta) = 1$,

$$\lim_{\tilde{\theta}\to 0}\mathcal{T}\left(\tilde{\theta}\right) = \frac{y-z}{c} - \frac{r+s}{1-\phi} > 0,$$

where the inequality uses the assumption that $(1-\phi)(y-z)/(r+s) > 0$. Additionally, I define

$$\tilde{\theta}^{\bullet} = \frac{1-\phi}{\phi}\frac{y-z}{c} > 0,$$

where the inequality comes from the fact that $y > z$ and $\phi \in (0,1)$ by assumption. Then

$$\begin{aligned}\mathcal{T}\left(\tilde{\theta}^{\bullet}\right) &= \frac{y-z}{c} - \frac{r+s}{(1-\phi)\,q\left(\tilde{\theta}^{\bullet}\right)} - \frac{\phi}{1-\phi}\tilde{\theta}^{\bullet} \\ &= \frac{y-z}{c} - \frac{r+s}{(1-\phi)\,q\left(\tilde{\theta}^{\bullet}\right)} - \frac{\phi}{1-\phi}\frac{1-\phi}{\phi}\frac{y-z}{c} \\ &= -\frac{r+s}{(1-\phi)\,q\left(\tilde{\theta}^{\bullet}\right)} \\ &< 0,\end{aligned}$$

where the inequality comes from the fact that $q\left(\tilde{\theta}^{\bullet}\right) > 0$. Because $\mathcal{T}$ is a combination of continuous functions, it is also continuous. Therefore, an application of the intermediate value theorem establishes that there exists $\theta \in \left(0, \frac{1-\phi}{\phi}\frac{y-z}{c}\right)$ such that $\mathcal{T}(\theta) = 0$.

The uniqueness part of proposition 9 comes from the fact that $\mathcal{T}$ is decreasing. Indeed,

$$\mathcal{T}'\left(\tilde{\theta}\right) = \frac{r+s}{(1-\phi)\left[q\left(\tilde{\theta}\right)\right]^2}q'\left(\tilde{\theta}\right) - \frac{\phi}{(1-\phi)} < 0,$$

and the inequality comes from the fact that $q' < 0$.

In proposition 9, the condition that $(1-\phi)(y-z)/(r+s) > c$ requires that the marginal value of posting an initial vacancy is profitable. The following thought experiment illustres why.

Starting from a given level of unemployment, which is guarenteed with exogenous separations, the marginal value of posting an initial vacancy is computed as $\lim_{\theta\to 0}\mathcal{V}$. In this thought experiment, the probability that the vacancy is filled is 1; that is, $\lim_{\theta\to 0} q(\theta) = 1$. The following period the firm earns the value of a productive match, which equals the flow payoff $y - w$ plus the value of a productive match discounted by $\beta(1-s)$; that is, $y - w + \beta(1-s)\mathcal{J}$. Solving this expression for $\mathcal{J}$ yields

$$\begin{aligned}
\mathcal{J} &= y - w + \beta(1-s)\mathcal{J} \\
\therefore \mathcal{J}[1-\beta(1-s)] &= y - w \\
\therefore \mathcal{J} &= \frac{y-w}{1-\beta(1-s)}.
\end{aligned}$$

The wage rate paid by the firm, looking at the expression in (A.19), is

$$\begin{aligned}
\lim_{\theta\to 0} w &= \lim_{\theta\to 0} z + \phi(y - z + \theta c) \\
&= \phi y + (1-\phi) z,
\end{aligned}$$

making

$$\mathcal{J} = \frac{(1-\phi)(y-z)}{1-\beta(1-s)}.$$

Using this expression in the value of an initial vacancy

$$\begin{aligned}
\lim_{\theta\to 0}\mathcal{V} &= \lim_{\theta\to 0}\langle -c + \beta\{q(\theta)\mathcal{J} + [1-q(\theta)]\mathcal{V}\}\rangle \\
&= -c + \beta\frac{(1-\phi)(y-z)}{1-\beta(1-s)} \\
&> 0.
\end{aligned}$$

The inequality stipulations that in order to start the process of posting vacancies, the first needs to be profitable. Developing this inequality yields

$$\begin{aligned}
\beta\frac{(1-\phi)(y-z)}{1-\beta(1-s)} &> c \\
\therefore \frac{1}{1+r}\frac{(1-\phi)(y-z)}{1-\frac{1}{1+r}(1-s)} &> c \\
\therefore \frac{(1-\phi)(y-z)}{1+r-1+s} &> c
\end{aligned}$$

or

$$\frac{(1-\phi)(y-z)}{r+s} > c.$$

Which is the condition listed in proposition 9.

Finally, for thoroughness, the equilibrium level of unemployment is

$$u = \frac{s}{s+\theta q(\theta)}. \tag{A.21}$$

A.3.2 A Eecomposition of the Elasticity of Market Tightness and Fundamental Surplus

The elasticity of tightness with respect to productivity is

$$\eta_{\theta,y} = \frac{d\theta}{dy}\frac{\theta}{y}.$$

Folliwng Ljungqvist and Sargent (2017), in this section I decompose this key elasticity into two terms the first is a constant and the second is the inverse of fundamental surplus. I show that the constant depends is bounded below by 1 and above the inverse of the elasticity of matching with respect to unemployment.

To uncover $\eta_{\theta,y}$, note that equation (A.20) can be written

$$\begin{aligned} y - z &= \frac{r+s+\phi\theta q(\theta)}{(1-\phi)\,q(\theta)}c \\ \therefore\ \frac{1-\phi}{c}(y-z) &= \frac{r+s+\phi\theta q(\theta)}{q(\theta)} \\ &= \frac{r+s}{q(\theta)} + \phi\theta. \end{aligned} \tag{A.22}$$

Define

$$F(\theta, y) \equiv \frac{1-\phi}{c}(y-z) - \frac{r+s}{q(\theta)} - \phi\theta.$$

Then implicit differentiation implies

$$\begin{aligned} \frac{d\theta}{dy} = -\frac{\partial F/\partial y}{\partial F/\partial \theta} &= -\frac{\frac{1-\phi}{c}}{\frac{r+s}{[q(\theta)]^2}q'(\theta) - \phi} \\ &= -\frac{\left[\frac{r+s}{q(\theta)} + \phi\theta\right]\frac{1}{y-z}}{\frac{r+s}{[q(\theta)]^2}q'(\theta) - \phi}, \end{aligned}$$

where the last equality uses the equality in (A.22).

Developing this expression yields

$$\begin{aligned}\frac{d\theta}{dy} &= -\frac{\left[\frac{r+s}{q(\theta)}+\phi\theta\right]}{\frac{r+s}{[q(\theta)]^2}q'(\theta)-\phi}\frac{1}{y-z}\times\frac{\theta q(\theta)}{\theta q(\theta)}\\ &= -\frac{[r+s+\phi\theta q(\theta)]}{(r+s)\frac{q'(\theta)\theta}{q(\theta)}-\phi\theta q(\theta)}\frac{\theta}{y-z}.\end{aligned} \tag{A.23}$$

The expression in the denominator is related to the elasticity of matching with respect to unemployment.

Using the expression for $\eta_{M,u}$ in (A.9) in the developing expression for $d\theta/dy$ yields

$$\begin{aligned}\frac{d\theta}{dy} &= -\frac{[r+s+\phi\theta q(\theta)]}{(r+s)\left(\frac{q'(\theta)\theta}{q(\theta)}\right)-\phi\theta q(\theta)}\frac{\theta}{y-z}\\ &= \frac{[r+s+\phi\theta q(\theta)]}{(r+s)\eta_{M,u}+\phi\theta q(\theta)}\frac{\theta}{y-z}.\end{aligned}$$

Further developing this expression yields

$$\begin{aligned}\frac{d\theta}{dy} &= \frac{r+s+\phi\theta q(\theta)}{(r+s)\eta_{M,u}+\phi\theta q(\theta)}\frac{\theta}{y-z}\\ &= \frac{r+s+(r+s)\eta_{M,u}-(r+s)\eta_{M,u}+\phi\theta q(\theta)}{(r+s)\eta_{M,u}+\phi\theta q(\theta)}\frac{\theta}{y-z}\\ &= \left[1+\frac{r+s-(r+s)\eta_{M,u}}{(r+s)\eta_{M,u}+\phi\theta q(\theta)}\right]\frac{\theta}{y-z}\\ &= \left[1+\frac{(r+s)\left(1-\eta_{M,u}\right)}{(r+s)\eta_{M,u}+\phi\theta q(\theta)}\right]\frac{\theta}{y-z}.\end{aligned}$$

This expression implies

$$\begin{aligned}\eta_{\theta,y} &= \frac{d\theta}{dy}\frac{y}{\theta}\\ &= \left[1+\frac{(r+s)\left(1-\eta_{M,u}\right)}{(r+s)\eta_{M,u}+\phi\theta q(\theta)}\right]\frac{y}{y-z}\\ &= \Upsilon\frac{y}{y-z},\end{aligned} \tag{A.24}$$

which is a fundamental result in Ljungqvist and Sargent (2017). The expression in (A.24) decomposes the elasticity of tightess with respect to productivity into two factors. Ljungqvist and Sargent (2017) focus on the second factor because the first factor, Υ, "has an upper bound coming from a consensus about values of the elasticity of matching with respect to unemployment."

As long as the conditions for an interior equilibrium in proposition 9 hold, Υ is bounded above by $1/\eta_{M,u}$. Ljungqvist and Sargent (2017) establish this fact by noting that Υ can be viewed as a function of ϕ. The evaluating Υ at $\phi = 0$ implies

$$\begin{aligned}\Upsilon\Big|_{\phi=0} &= 1 + \frac{(r+s)\left(1-\eta_{M,u}\right)}{(r+s)\,\eta_{M,u}} = 1 + \frac{1-\eta_{M,u}}{\eta_{M,u}} \\ &= \frac{1}{\eta_{M,u}} > 1,\end{aligned}$$

where the inequality is established in proposition 8. Moreover, Υ is decreasing in ϕ:

$$\frac{\partial \Upsilon}{\partial \phi} = -\frac{(r+s)\left(1-\eta_{M,u}\right)}{\left[(r+s)\,\eta_{M,u} + \phi\theta q(\theta)\right]^2}\theta q(\theta) < 0.$$

These two facts establish that Υ is bounded above by $1/\eta_{M,u}$. Moreover, the expression for Υ in (A.24) establishes that Υ is bounded below by 1. These results collected in proposition 10.

Proposition 10 *In the canonical search model, which features random search, exogenous separations, and no disturbances in aggregate productivity, the elasticity of market tightness with respect to productivity can be decomposed as*

$$\eta_{\theta,y} = \Upsilon \frac{y}{y-z},$$

where the second factor is the inverse of fundamental surplus (Ljungqvist and Sargent, 2017) and the first factor is bounded below by 1 *and above by* $1/\eta_{M,u}$*; that is,*

$$1 < \Upsilon < \frac{1}{\eta_{M,u}}.$$

Proposition 10 suggests the importance of $\eta_{M,u}$ in the data.

APPENDIX B

Appendix to On the Tension between Maximum Sustainable Yield and Maximum Economic Yield

B.1 Proofs and derivations for analysis in Tension between Maximum Sustainable Yield and Maximum Economic Yield

B.1.1 The fishery manager's problem

The unit mass of people that make up the household is divided between employment and unemployment: $1 = e(t) + u(t)$. Employment is further divided between people who work in commercial fishing and those that don't: $e(t) = e_\chi + e_\varpi$. Employment in the fishery evolves according to

$$\dot{e}_\chi = -\lambda e_\chi(t) + \phi(t) f(\theta(t)) u(t),$$

where $\phi(t)$ is the relative number of vacancies posted by vessel owners. Commercial fishing loses vessels at rate λ and gains workers from unemployment at rate $\phi(t) f(\theta(t)) u(t)$. Employment outside the fishery evolves similarly:

$$\dot{e}_\chi = -\lambda e_\chi(t) + \phi(t) f(\theta(t)) u(t).$$

Steady-state employment in the two sectors is given by

$$e_\chi = \frac{\phi f(\theta) u}{\lambda}$$
$$e_\varphi = \frac{(1-\phi) f(\theta) u}{\lambda}.$$

Using the Beveridge curve relationship that comes out of (2.1), $u = \lambda/(\lambda + f(\theta))$, the two above expressions can be written as

$$e_\chi = \frac{\phi f(\theta) u}{\lambda} = \frac{\phi f(\theta)}{\lambda + f(\theta)}$$
$$e_\varphi = \frac{(1-\phi) f(\theta) u}{\lambda} = \frac{(1-\phi) f(\theta)}{\lambda + f(\theta)}.$$

As a check:

$$e_\chi + e_\varphi = \frac{\phi f(\theta) + (1-\phi) f(\theta)}{\lambda + f(\theta)} = \frac{f(\theta)}{\lambda + f(\theta)}$$
$$= 1 - u(\theta) = 1 - \frac{\lambda}{\lambda + f(\theta)} = \frac{f(\theta)}{\lambda + f(\theta)},$$

which simply confirms that steady-state employment is one minus the unemployment.

Steady-state employment in the fishery is

$$e_\chi = \phi \frac{f(\theta)}{\lambda + f(\theta)} = \phi e = \phi(1-u);$$

that is, the employment share in the fishery is ϕ and the employment share outside the fishery is $1 - \phi$.

As in the text, at time t, the regulator chooses an optimal path by regulating vessel-level harvest:

$$\max_{\{h(\tau)\}} \int_{\tau=t}^{\infty} e^{-r(\tau-t)} \left[p_\chi(t)\phi(t)(1-u(t))h(\tau) - w_\chi(t)\phi(t)(1-u(t)) - \phi(t)(1-u(t))rk \right] \, d\tau$$

subject to

$$\frac{ds}{d\tau} = g(s(\tau)) - \phi(t)(1-u(t))h(\tau),$$

with $s(t) = s_0$ given. The regulator takes $p_\chi(t)$, $\phi(t)$, $u(t)$, and $w_\chi(t)$ as fixed. The current-value Hamiltonian, with current-value costate variable $\Lambda(\tau)$, is

$$\begin{aligned} H(h(\tau), s(\tau), \Lambda(\tau)) = p_\chi(t)\phi(t)(1-u(t))h(\tau) - w_\chi(t)\phi(t)(1-u(t)) - \phi(t)(1-u(t))rk \\ + \Lambda(\tau)\left[g(s(\tau)) - \phi(t)(1-u(t))h(\tau)\right]. \end{aligned} \tag{B.1}$$

Linearity of the control implies a bang–bang solution. When $p_\chi > \Lambda$, the price of fish is greater than the value of leaving the fish in the sea, directing the regulator to set $h = \overline{h}$. Conversely, when $p_\chi < \Lambda$, $h = 0$. When $p_\chi = \Lambda$, the regulator is directed to harvest fish at the steady-state level: $\phi(1-u)h = g(s^\star)$. To maintain $s^\star$, recruitment is divided among the $\phi(1-u)$ vessels. The regulator, in summary, moves the fishery as rapidly as possible to $s^\star$ and then sets vessel-level harvest to $\hat{h} = g(s^\star)/[\phi(1-u)]$ in order to maintain the targeted stock level.

The level $s^\star$ is determined from the two necessary conditions from the optimal-control problem:

$$\frac{\partial H}{\partial h} = (1-u(t))\phi(t)p_\chi(t) - \Lambda(\tau)(1-u(t))\phi(t) = 0 \tag{B.2}$$

$$\frac{\partial H}{\partial s} = \Lambda(\tau)g_s(s(\tau)) = r\Lambda(\tau) - \frac{d\Lambda}{d\tau}. \tag{B.3}$$

Equation (B.2) implies

$$p_\chi = \Lambda(\tau), \quad \Rightarrow \frac{d\Lambda}{d\tau} = 0,$$

as price is assumed to be fixed. Using this result in equation (B.3) yields

$$\begin{aligned} \Lambda g_s(s^\star) &= r\Lambda \\ g_s(s^\star) &= r. \end{aligned}$$

Therefore the fishery manager chooses $s^\star$ so that $g_s(s^\star) = r$. This choice is depicted in figure 2.4.

B.1.2 Derivation of the wage rule in the two sectors under Nash bargaining

The following is the derivation of expressions for wages given in (2.8). Using (2.2) we have

$$\begin{aligned}(r+\lambda)W_i &= w_i + \lambda U \\ W_i &= \frac{w_i + \lambda U}{r+\lambda}.\end{aligned} \tag{B.4}$$

From equation (2.4), we have

$$\begin{aligned}(r+\lambda)J_\varphi &= p - w_\varphi + \lambda V_\varphi \\ J_\varphi &= \frac{p - w_\varphi + \lambda V_\varphi}{r+\lambda}\end{aligned}$$

Nash wage bargaining in the nonresource sector means the surplus a worker gains from going from unemployment to nonresource employment, $W_\varphi - U$, equals a fraction of the the surpluse a nonresource firm gains by filling a vacancy, $J_\varphi - V_\varphi$: $(1-\beta)(W_\varphi - U) = \beta(J_\varphi - V_\varphi)$; and it follows that

$$\begin{aligned}(1-\beta)(W_\varphi - U) &= \beta(J_\varphi - V_\varphi) \\ (1-\beta)((r+\lambda)^{-1}(w_\varphi + \lambda U) - U) &= \beta((r+\lambda)^{-1}(p - w_\varphi + \lambda V_\varphi) - V_\varphi) \\ (1-\beta)(w_\varphi + \lambda U - (r+\lambda)U) &= \beta(p - w_\varphi + \lambda V_\varphi - (r+\lambda)V_\varphi) \\ (1-\beta)(w_\varphi - rU) &= \beta(p - w_\varphi - rV_\varphi)\end{aligned}$$

$(V_\varphi = k_\varphi)$

$$\begin{aligned}(1-\beta)(w_\varphi - rU) &= \beta(p - w_\varphi - rk_\varphi) \\ (1-\beta)w_\varphi - (1-\beta)rU &= \beta(p - rk_\varphi) - \beta w_\varphi \\ w_\varphi &= \beta(p - rk_\varphi) + (1-\beta)rU,\end{aligned}$$

which is the expression found in (2.8) of the text.

Similarly in the resource sector, from the corresponding equation (2.4), we have

$$\begin{aligned}(r+\lambda)J_\chi &= p_\chi \hat{h}_\chi - w_\chi + \lambda V_\chi \\ J_\chi &= \frac{p_\chi \hat{h}_\chi - w_\chi + \lambda V_\chi}{r+\lambda}.\end{aligned}$$

Nash wage bargaining means $(1-\beta)(W_\chi - U) = \beta(J_\chi - V_\chi)$. Using this fact gives

$$(1-\beta)((r+\lambda)^{-1}(w_\chi + \lambda U) - U) = \beta((r+\lambda)^{-1}(p_\chi \hat{h} - w_\chi + \lambda V_\chi) - V_\chi)$$

(using the expression for W_χ in equation (B.4))

$$\begin{aligned}(1-\beta)(w_\chi + \lambda U - (r+\lambda)U) &= \beta(p_\chi \hat{h} - w_\chi + \lambda V_\chi - (r+\lambda)V_\chi)\\ (1-\beta)(w_\chi - rU) &= \beta(p_\chi \hat{h} - w_\chi - rV_\chi)\end{aligned}$$

$(V_\chi = k_\chi)$

$$\begin{aligned}(1-\beta)(w_\chi - rU) &= \beta(p_\chi \hat{h} - w_\chi - rk_\chi)\\ (1-\beta)w_\chi - (1-\beta)rU &= \beta(p_\chi \hat{h} - rk_\chi) - \beta w_\chi\\ w_\chi &= \beta(p_\chi \hat{h} - rk_\chi) + (1-\beta)rU,\end{aligned}$$

which is the expression found in (2.8) of the text.

B.1.3 Derivation of the value of being unemployed

This section derives equation (2.11). Using equation (2.2), we have:

$$\begin{aligned}rW_\varphi &= w_\varphi + \lambda(U - W_\varphi)\\ \Rightarrow W_\varphi &= \frac{w_\varphi + \lambda U}{r+\lambda}\end{aligned}$$

and

$$\begin{aligned}rW_\chi &= w_\chi + \lambda(U - W_\chi)\\ \Rightarrow W_\chi &= \frac{w_\chi + \lambda U}{r+\lambda}.\end{aligned}$$

Using the expressions for wages given in (2.8), the latter two expressions become

$$\begin{aligned}W_\varphi &= \frac{\beta(p - k_\varphi) + (1-\beta)rU + \lambda U}{r+\lambda}\\ &= \frac{\beta(p - k_\varphi) + [(1-\beta)r + \lambda]U}{r+\lambda}\end{aligned}$$

and

$$W_\chi = \frac{\beta(p_\chi \hat{h} - rk_\chi) + (1-\beta)rU + \lambda U}{r+\lambda}$$
$$= \frac{\beta(p_\chi \hat{h} - rk_\chi) + [(1-\beta)r + \lambda]U}{r+\lambda}$$

Using the latter expressions for the value of work in equation (2.11) gives

$$\begin{aligned}
rU &= z + f(\theta)\left[\phi W_\chi + (1-\phi)W_\varphi - U\right] \\
&= z + f(\theta)\Bigg\{\phi\frac{\beta(p_\chi \hat{h} - rk_\chi) + [(1-\beta)r+\lambda]U}{r+\lambda} \\
&\qquad +(1-\phi)\frac{\beta(p_\varphi - rk_\varphi) + [(1-\beta)r+\lambda]U}{r+\lambda} - U\Bigg\} \\
&= z + f(\theta)\Bigg\{\phi\frac{\beta(p_\chi \hat{h} - rk_\chi) + [(1-\beta)r+\lambda]U}{r+\lambda} \\
&\qquad +(1-\phi)\frac{\beta(p_\varphi - rk_\varphi) + [(1-\beta)r+\lambda]U}{r+\lambda} - \frac{r+\lambda}{r+\lambda}U\Bigg\} \\
&= z + f(\theta)\Bigg[\phi\frac{\beta(p_\chi \hat{h} - rk_\chi)}{r+\lambda} + (1-\phi)\frac{\beta(p_\varphi - rk_\varphi)}{r+\lambda} \\
&\qquad + \frac{(1-\beta)r+\lambda}{r+\lambda}U - \frac{r+\lambda}{r+\lambda}U\Bigg] \\
&= z + f(\theta)\Bigg[\phi\frac{\beta(p_\chi \hat{h} - rk_\chi)}{r+\lambda} + (1-\phi)\frac{\beta(p_\varphi - rk_\varphi)}{r+\lambda} \\
&\qquad + \frac{(1-\beta)r+\lambda-r-\lambda}{r+\lambda}U\Bigg]
\end{aligned}$$

$((1-\beta)r + \lambda - r - \lambda = r - r\beta + \lambda - r - \lambda = -r\beta)$

$$= z + f(\theta)\left[\phi\frac{\beta(p_\chi \hat{h} - rk_\chi)}{r+\lambda} + (1-\phi)\frac{\beta(p_\varphi - rk_\varphi)}{r+\lambda} - \frac{\beta r}{r+\lambda}U\right].$$

$$\Rightarrow (r+\lambda)rU = (r+\lambda)z + f(\theta)\left[\phi\beta(p_\chi \hat{h} - rk_\chi) + (1-\phi)\beta(p_\varphi - rk_\varphi) - \beta rU\right]$$
$$\Rightarrow rU = \frac{(r+\lambda)z + \beta f(\theta)\left[\phi(p_\chi \hat{h} - rk_\chi) + (1-\phi)(p_\varphi - rk_\varphi)\right]}{r+\lambda+\beta\theta q(\theta)},$$

which is equation (2.11) in the text.

B.1.4 How the value of unemployment changes with respect to tightness

The value of unemployment has an ambiguous relationship with tightness due to declining productivity in the renewable resource sector. To be sure, the value of unemployment is increasing in tightness for small values of ϕ; that is, when productivity in the fishery is $\bar{h} > 1$, the value of unemployment is increasing in tightness:

B.1.4.1 Case: $\phi < \check{\phi}$

When $\phi < \check{\phi}$, assuming the costs associated with posting a vacancy are the same across the two sectors, $k_\chi = k_\varphi = k$, the value of unemployment, from equation (2.11), is

$$\Gamma = \frac{(r+\lambda)z + \beta f(\theta)\left[\phi p_\chi \hat{h} + (1-\phi)p_\varphi - rk\right]}{r+\lambda+\beta f(\theta)}, \tag{B.5}$$

where, using equation (2.9),

$$\phi p_\chi \hat{h} = \phi \alpha \bar{h}^\alpha \left(\frac{1-\phi}{\phi}\right)^{1-\alpha} \tag{B.6}$$

and, from equation (2.10),

$$(1-\phi)p_\varphi = (1-\phi)(1-\alpha)\bar{h}^\alpha \left(\frac{\phi}{1-\phi}\right)^\alpha . \tag{B.7}$$

To determine $\partial\Gamma/\partial\theta$, define the following terms:

$$\begin{aligned} T_1 &:= (r+\lambda)z + \beta f(\theta)\left[\phi p_\chi \hat{h} + (1-\phi)p_\varphi - rk\right] \\ T_2 &:= \phi p_\chi \hat{h} + (1-\phi)p_\varphi - rk \\ T_3 &:= r+\lambda+\beta f(\theta). \end{aligned}$$

The term T_2 does not depend on θ. Then

$$\frac{\partial\Gamma}{\partial\theta} = \frac{\frac{\partial T_1}{\partial\theta}T_3 - \frac{\partial T_3}{\partial\theta}T_1}{T_3^2} \tag{B.8}$$

and

$$\text{sgn}\,\frac{\partial\Gamma}{\partial\theta} \propto \frac{\partial T_1}{\partial\theta}T_3 - \frac{\partial T_3}{\partial\theta}T_1 \tag{B.9}$$

as the denominator is positive. Evaluating these expressions gives

$$\begin{aligned}\frac{\partial T_1}{\partial\theta}T_3 - \frac{\partial T_3}{\partial\theta}T_1 &= \beta f'(\theta)T_2T_3 - \beta f'(\theta)T_1 \\ &= \beta f'(\theta)\left[T_2T_3 - T_1\right] \\ &= \beta f'(\theta)\left\{T_2\left[r + \lambda + \beta f(\theta) - (r+\lambda)z - \beta f(\theta)T_2\right]\right\} \\ &= \beta f'(\theta)\left[T_2(r+\lambda) - (r+\lambda)z\right] \\ &= \beta f'(\theta)(r+\lambda)(T_2 - z) \\ &= \beta f'(\theta)(r+\lambda)\left[\phi p_\chi \hat{h} + (1-\phi)p_\varphi - rk - z\right].\end{aligned}$$

The latter gives

$$\text{sgn}\,\frac{\partial\Gamma}{\partial\theta} \propto \beta f'(\theta)(r+\lambda)(\phi p_\chi \sigma^* h + (1-\phi)p_\varphi - rk - z) > 0. \tag{B.10}$$

The inequality comes from the fact that $\phi p_\chi \hat{h} + (1-\phi)p_\varphi - rk$ is the net expected flow of output from all jobs (the value of output less the rental cost of capital) and this is greater than the flow benefit of nonwork (otherwise there would be no incentive to work). This result yields

$$\frac{\partial\Gamma}{\partial\theta} > 0. \tag{B.11}$$

B.1.4.2 Case: $\phi > \check{\phi}$

On the other hand, when $\phi > \check{\phi}$, the value of unemployment has an ambiguous relationship with tightness. Still, from equation (2.11),

$$\Gamma = \frac{(r+\lambda)z + \beta f(\theta)\left[\phi p_\chi \hat{h} + (1-\phi)p_\varphi - rk\right]}{r + \lambda + \beta f(\theta)}, \tag{B.12}$$

but now, using price from equation (2.9) and harvest from equation (2.7),

$$\phi p_\chi \hat{h} = \phi\alpha \left[\frac{(1-\phi)(1-u)}{g(s^\star)}\right]^{1-\alpha} \frac{g(s^\star)}{\phi(1-u)} \tag{B.13}$$

$$= \alpha(1-\phi)^{1-\alpha}\left(\frac{g(s^\star)}{1-u}\right)^{\alpha} \tag{B.14}$$

$$= \alpha(1-\phi)^{1-\alpha} g(s^\star)^\alpha (1-u)^{-\alpha} \tag{B.15}$$

and

$$(1-\phi)p_\varphi = (1-\phi)(1-\alpha)\left[\frac{g(s^\star)}{(1-\phi)(1-u)}\right]^{\alpha} \tag{B.16}$$

$$= (1-\alpha)(1-\phi)^{1-\alpha} g(s^\star)^\alpha (1-u)^{-\alpha}. \tag{B.17}$$

It will be useful to know

$$\frac{\partial}{\partial\theta}\left[\phi p_\chi \hat{h}\right] = \alpha^2 g(s^\star)^\alpha (1-\phi)^{1-\alpha}\left(\frac{1}{1-u}\right)^{1+\alpha}\frac{\partial u}{\partial\theta} < 0, \tag{B.18}$$

as unemployment is decreasing in tightness. And that

$$\frac{\partial}{\partial\theta}\left[(1-\phi)p_\varphi\right] = \alpha(1-\alpha)g(s^*)^\alpha(1-\phi)^{1-\alpha}\left(\frac{1}{1-u}\right)^{1+\alpha}\frac{\partial u}{\partial\theta} < 0, \tag{B.19}$$

as, again, unemployment is decreasing in tightness.

Define the terms

$$T_4 := \phi p_\chi \hat{h} + (1-\phi)p_\varphi \tag{B.20}$$

$$T_5 := (r+\lambda)z + \beta f(\theta) T_4 \tag{B.21}$$

$$T_6 := r + \lambda + \beta f(\theta). \tag{B.22}$$

From (B.18) and (B.19)

$$\frac{\partial T_4}{\partial\theta} < 0. \tag{B.23}$$

Using the above terms,

$$\Gamma = \frac{(r+\lambda)z + \beta f(\theta)T_4}{T_6}.$$

Differentiating Γ with respect to θ yields:

$$\frac{\partial \Gamma}{\partial \theta} = \frac{\left[\beta f'(\theta)T_4 + \frac{\partial T_4}{\partial \theta}\beta f(\theta)\right] T_6 - \beta f'(\theta)T_5}{T_6^2}. \tag{B.24}$$

Evaluating the latter yields

$$\begin{aligned} \operatorname{sgn} \frac{\partial \Gamma}{\partial \theta} &\propto \beta f'(\theta)T_4T_6 + \frac{\partial T_4}{\partial \theta}\beta f(\theta)T_6 - \beta f'(\theta)T_5 \\ &= \beta f'(\theta)T_4[r + \lambda + \beta f(\theta)] + \frac{\partial T_4}{\partial \theta}\beta f(\theta)[r + \lambda + \beta f(\theta)] - \beta f'(\theta)[(r + \lambda)z + \beta f(\theta)T_4] \\ &= \beta f'(\theta)(r + \lambda)(T_4 - z) + \frac{\partial T_4}{\partial \theta}\beta f(\theta)[r + \lambda + \beta f(\theta)]. \end{aligned}$$

The latter equation gives

$$\begin{aligned} \operatorname{sgn} \frac{\partial \Gamma}{\partial \theta} \propto \beta f'(\theta)(r + \lambda)[\phi p_\chi \hat{h} + (1 - \phi)p_\varphi - rk - z] \\ + \frac{\partial T_4}{\partial \theta}\beta f(\theta)[r + \lambda + \beta f(\theta)]. \end{aligned} \tag{B.25}$$

The expression in (B.25) is the same as the expression for $\partial \Gamma / \partial \theta$ in (B.10) except for the effects of crowding and stock externalities: $\partial T_4 / \partial \theta$, which is the effect of tightness on the value of average productivity. Since $\partial T_4 / \partial \theta < 0$, established in equation (B.23), crowding and stock externalities have a negative effect on the value of average productivity.

B.1.5 How the value of unemployment changes with respect to the share of fishers

The value of unemployment first increases and then decreases at the share of jobs in the fishery increases. The value of unemployment is given in equation (2.11). Assuming the cost of posting vacancies is the same across both sectors, this expression can be simplified to

$$\Gamma(\theta, \phi) = \frac{(r + \lambda)z + \beta f(\theta)\left[\phi p_\chi \hat{h} + (1 - \phi)p_\varphi\right]}{r + \lambda + \beta f(\theta)}, \tag{B.26}$$

with prices given in equations (2.9) and (2.10). Then

$$\operatorname{sgn} \frac{\partial \Gamma}{\partial \phi} = \operatorname{sgn} \frac{\partial}{\partial \phi}\left[\phi p_\chi \hat{h} + (1 - \phi)p_\varphi\right]. \tag{B.27}$$

When ϕ is nearly zero [see (2.7)], the latter expression evaluates to

$$\phi\alpha\bar{h}^{\alpha-1}\left(\frac{1-\phi}{\phi}\right)^{1-\alpha}\bar{h}+(1-\phi)(1-\alpha)\bar{h}^{\alpha}\left(\frac{\phi}{1-\phi}\right)^{\alpha},$$

which simplifies to

$$\begin{gathered}\alpha\bar{h}^{\alpha}\phi\left(\frac{1-\phi}{\phi}\right)^{1-\alpha}+(1-\alpha)\bar{h}^{\alpha}(1-\phi)\left(\frac{\phi}{1-\phi}\right)^{\alpha}\\ \bar{h}^{\alpha}\left[\alpha\phi\left(\frac{1-\phi}{\phi}\right)^{1-\alpha}+(1-\alpha)(1-\phi)\left(\frac{\phi}{1-\phi}\right)^{\alpha}\right]\\ \bar{h}^{\alpha}\left[\alpha\phi^{\alpha}(1-\phi)^{1-\alpha}+(1-\alpha)\phi^{\alpha}(1-\phi)^{1-\alpha}\right]\\ \bar{h}^{\alpha}\phi^{\alpha}(1-\phi)^{1-\alpha}.\end{gathered}$$

Differentiating $\phi^{\alpha}(1-\phi)^{1-\alpha}$ with respect to ϕ gives

$$\begin{gathered}\alpha\phi^{\alpha-1}(1-\phi)^{1-\alpha}-(1-\alpha)\phi^{\alpha}(1-\phi)^{-\alpha}\\ \alpha\left(\frac{1-\phi}{\phi}\right)^{1-\alpha}-(1-\alpha)\left(\frac{\phi}{1-\phi}\right)^{\alpha}\\ \alpha\left(\frac{\phi}{1-\phi}\right)^{-1+\alpha}-(1-\alpha)\left(\frac{\phi}{1-\phi}\right)^{\alpha}\\ \left(\frac{\phi}{1-\phi}\right)^{\alpha}\left(\alpha\frac{1-\phi}{\phi}-(1-\alpha)\right)\\ \left(\frac{\phi}{1-\phi}\right)^{\alpha}\frac{\alpha-\phi}{\phi}.\end{gathered}\tag{B.28}$$

Therefore, for an interior equilibrium,

$$\operatorname{sgn}\frac{\partial\Gamma}{\partial\phi}=\operatorname{sgn}\frac{\alpha-\phi}{\phi}.\tag{B.29}$$

The expression in equation (B.29) is positive for small values of ϕ and negative for large values of ϕ. Preference parameterization determines the sign.

When ϕ is away from 0; that is, when the fishery is able to harvest an amount equal to recruitment, $\phi p_\chi \hat{h} + (1-\phi)p_\varphi$ evaluates to

$$\begin{gathered}\phi p_\chi \sigma^* h + (1-\phi)p_\varphi \\ \phi\alpha(1-\phi)^{1-\alpha}(1-u)^{1-\alpha} g(s^*)^{-1+\alpha}\frac{g(s^*)}{\phi(1-u)} + (1-\phi)(1-\alpha)\left[\frac{g(s^*)}{(1-\phi)(1-u)}\right]^\alpha \\ \alpha g(s^*)^\alpha(1-u)^{-\alpha}(1-\phi)^{1-\alpha} + (1-\alpha)g(s^*)^\alpha(1-u)^{-\alpha}(1-\phi)^{1-\alpha} \\ g(s^*)^\alpha(1-u)^{-\alpha}(1-\phi)^{1-\alpha}\end{gathered} \tag{B.30}$$

Differentiating the latter with respect to ϕ yields

$$-\alpha(1-\alpha)\left[\frac{g(s^*)}{1-u}\right]^\alpha \frac{1}{(1-\phi)^2} - (1-\alpha)^2\left[\frac{g(s^*)}{(1-\phi)(1-u)}\right]^\alpha < 0. \tag{B.31}$$

Here $\operatorname{sgn} \partial\Gamma/\partial\phi < 0$: The value of unemployment is decreasing once the fishery is capable of harvesting a level equal to recruitment. The manager of the fishery sets a target harvest level and then divides this level evenly among the fishers. Adding more fishers at this point decreases catch per vessel.

B.1.6 The value of unemployment is finite, given a level of tightness

From equation (2.11),

$$\begin{aligned}\Gamma(\theta,\phi) &= \frac{(r+\lambda)z + \beta f(\theta)\left[\phi\left(p_\chi\hat{h} - rk_\chi\right) + (1-\phi)\left(p_\varphi - rk_\varphi\right)\right]}{r+\lambda+\beta f(\theta)} \\ &= \frac{(r+\lambda)z + \beta f(\theta)\left[\phi p_\chi \hat{h} + (1-\phi)p_\varphi - rk\right]}{r+\lambda+\beta f(\theta)},\end{aligned} \tag{B.32}$$

where the second equality comes from the maintained assumption that $k_\chi = k_\varphi = k$. The only terms in (B.32) that depend on ϕ are contained in the bracket. And because the other terms of finite, focus on the bracketed term.

B.1.6.1 When the fishery manager directs the fleet to harvest at full capacity

When $\hat{h} = \overline{h}$, the term in brackets (ignoring the cost of capital) becomes, using the expressions for prices in equations (2.9) and (2.10),

$$\begin{aligned}
&\phi p_\chi \hat{h} + (1-\phi)p_\varphi \\
&\qquad = \phi\alpha\overline{h}^{\alpha-1}\left(\frac{1-\phi}{\phi}\right)^{1-\alpha}\overline{h} + (1-\phi)(1-\alpha)\overline{h}^{\alpha}\left(\frac{\phi}{1-\phi}\right)^{\alpha} \\
&\qquad = \alpha\overline{h}^{\alpha}\phi^{\alpha}(1-\phi)^{1-\alpha} + (1-\alpha)\overline{h}^{\alpha}\phi^{\alpha}(1-\phi)^{1-\alpha} \\
&\qquad = \overline{h}^{\alpha}\phi^{\alpha}(1-\phi)^{1-\alpha}.
\end{aligned}$$

The term $\overline{h}^{\alpha}$ is a finite, positive number. The dependence on ϕ will therefore be qualitatively determined by $\rho(\phi) := \phi^{\alpha}(1-\phi)^{1-\alpha}$. As $\phi \to 0$, this term goes to 0. As $\phi \to 0$, this term goes to 0.

The maximum of $\rho(\phi)$ on $[0,1]$ is determined by setting the first derivative equal to 0:

$$\rho'(\phi) = \alpha\phi^{\alpha-1}(1-\phi)^{1-\alpha} - (1-\alpha)\phi^{\alpha}(1-\phi)^{-\alpha} = 0.$$

Developing this expression by multiplying both sides by $\phi^{-\alpha}$ yields

$$0 = \alpha\phi^{-1}(1-\phi)^{1-\alpha} - (1-\alpha)(1-\phi)^{-\alpha} = \alpha\phi^{-1}(1-\phi) - (1-\alpha),$$

where the last equality comes from multiplying both sides by $(1-\phi)^{\alpha}$. Re-arranging the latter yields:

$$\frac{\phi}{1-\phi} = \frac{\alpha}{1-\alpha} \iff \phi = \alpha.$$

The critical point $\phi = \alpha$ is a maximum. Indeed:

$$\begin{aligned}
\rho''(\phi) &= \alpha(\alpha-1)\phi^{\alpha-2}(1-\phi)^{1-\alpha} + \alpha(1-\alpha)\phi^{\alpha-1}(1-\phi)^{-\alpha}(-1) - \\
&\quad \left[\alpha(1-\alpha)\phi^{\alpha-1}(1-\phi)^{-\alpha} - \alpha(1-\alpha)\phi^{\alpha}(1-\phi)^{-\alpha-1}(-1)\right] \\
&= \alpha - (1-\alpha)\phi^{\alpha-2}(1-\phi)^{1-\alpha} - \alpha(1-\alpha)\phi^{\alpha-1}(1-\phi)^{-\alpha} \\
&\quad - \alpha(1-\alpha)\phi^{\alpha-1}(1-\phi)^{-\alpha} - \alpha(1-\alpha)\phi^{\alpha}(1-\phi)^{-\alpha-1} \\
&< 0.
\end{aligned}$$

Because $|\alpha^{\alpha}(1-\alpha)^{1-\alpha}| < +\infty$, the value of unemployment, given $\theta \in (0,\infty)$, takes on only finite values as ϕ varies.

B.1.6.2 When the fishery manager implements a tradable quota scheme

When the fishery is capable of harvesting $g(s^\star)$ the fishery manager implements a tradable quota scheme, leading to each vessel harvesting $g(s^\star)/[\phi(1-u)]$. As with the case where the fishery is directed to harvest at full capacity, how the value of unemployment depends on ϕ is determined by the bracketed terms in (B.32). Using prices in equations (2.9) and (2.10), the term in brackets, ignoring the cost of capital, becomes

$$
\begin{aligned}
&\phi p_\chi \hat{h} + (1-\phi)p_\varphi \\
&\quad = \phi\alpha\left[\frac{(1-\phi)(1-u)}{g(s^\star)}\right]^{1-\alpha}\frac{g(s^\star)}{\phi(1-u)} + (1-\phi)(1-\alpha)\left[\frac{g(s^\star)}{(1-\phi)(1-u)}\right]^{\alpha} \\
&\quad = \alpha(1-\phi)^{1-\alpha}(1-u)^{-\alpha}g(s^\star)^{1-\alpha} + (1-\alpha)(1-\phi)^{1-\alpha}g(s^\star)^{\alpha}(1-u)^{-\alpha} \\
&\quad = (1-\phi)^{1-\alpha}g(s^\star)^{\alpha}(1-u)^{-\alpha}.
\end{aligned}
$$

This term is finite for all $\phi \in [0,1]$.

B.1.7 Equilibrium Job-creation Equations for Firms

Equation (2.12) comes from equations (2.4) and (2.5). These two equations generate a single equation in terms of θ and U:

$$
\begin{aligned}
rJ_\varphi - rV_\varphi &= r(J_\varphi - V_\varphi) \\
&= p_\varphi - w_\varphi + \lambda(V_\varphi - J_\varphi) - q(\theta)(J_\varphi - V_\varphi) \\
&= p_\varphi - w_\varphi - \lambda(J_\varphi - V_\varphi) - q(\theta)(J_\varphi - V_\varphi).
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow [r + \lambda + q(\theta)](J_\varphi - V_\varphi) &= p_\varphi - w_\varphi \\
J_\varphi - V_\varphi &= \frac{p_\varphi - w_\varphi}{r + \lambda + q(\theta)}
\end{aligned}
$$

(using the expression for wages in equation (2.8))

$$
J_\varphi - V_\varphi = \frac{p_\varphi - \beta(p_\varphi - rk_\varphi) - (1-\beta)rU}{r + \lambda + q(\theta)}.
$$

From equation (2.5), $rV_\varphi = rk_\varphi = q(\theta)(J_\varphi - V_\varphi)$, we have

$$\begin{aligned}
\frac{rk_\varphi}{q(\theta)} &= \frac{p_\varphi - \beta(p_\varphi - rk_\varphi) - (1-\beta)rU}{r+\lambda+q(\theta)} \\
rk_\varphi &= q(\theta)\frac{p_\varphi - \beta(p_\varphi - rk_\varphi) - (1-\beta)rU}{r+\lambda+q(\theta)} \\
rk_\varphi[r+\lambda+q(\theta)] &= q(\theta)[p_\varphi - \beta(p_\varphi - rk_\varphi) - (1-\beta)rU] \\
rk_\varphi[r+\lambda+(1-\beta)q(\theta)] &= (1-\beta)q(\theta)(p_\varphi - rU).
\end{aligned}$$

The latter gives,

$$p_\varphi - rU = \frac{rk_\varphi[r+\lambda+(1-\beta)q(\theta)]}{(1-\beta)q(\theta)},$$

which gives equation (2.12) in the text.

Equation (2.13) comes from using the expressions for J_χ and $V_\chi = 0$, which correspond to equations (2.4) and (2.5). These expressions generate an equation in terms of θ and U:

$$\begin{aligned}
rJ_\chi - rV_\chi &= r(J_\chi - V_\chi) \\
&= p_\chi\hat{h} - w_\chi + \lambda(V_\chi - J_\chi) - q(\theta)(J_\chi - V_\chi) \\
&= p_\chi\hat{h} - w_\chi - \lambda(J_\chi - V_\chi) - q(\theta)(J_\chi - V_\chi).
\end{aligned}$$

$$\begin{aligned}
\Rightarrow [r+\lambda+q(\theta)](J_\chi - V_\chi) &= p_\chi\hat{h} - w_\chi \\
J_\chi - V_\chi &= \frac{p_\chi\hat{h} - w_\chi}{r+\lambda+q(\theta)}
\end{aligned}$$

(using the expression for wages in equation (2.8))

$$J_\chi - V_\chi = \frac{p_\chi\hat{h} - \beta(p_\chi\hat{h} - rk_\chi) - (1-\beta)rU}{r+\lambda+q(\theta)}$$

Using the latter and the fact that $rV_\chi = rk_\chi = q(\theta)(J_\chi - V_\chi)$, we have

$$\begin{aligned}
\frac{rk_\chi}{q(\theta)} &= \frac{p_\chi\hat{h} - \beta(p_\chi\hat{h} - rk_\chi) - (1-\beta)rU}{r+\lambda+q(\theta)} \\
rk_\chi &= q(\theta)\frac{p_\chi\hat{h} - \beta(p_\chi\hat{h} - rk_\chi) - (1-\beta)rU}{r+\lambda+q(\theta)} \\
rk_\chi[r+\lambda+q(\theta)] &= q(\theta)[p_\chi\hat{h} - \beta(p_\chi\hat{h} - rk_\chi) - (1-\beta)rU] \\
rk_\chi[r+\lambda+(1-\beta)q(\theta)] &= (1-\beta)q(\theta)(p_\chi\hat{h} - rU).
\end{aligned}$$

The latter gives

$$p_\chi\hat{h} - rU = \frac{rk_\chi[r+\lambda+(1-\beta)q(\theta)]}{(1-\beta)q(\theta)},$$

which is equation (2.13) in the text.

B.1.8 Equilibrium share of fishers

Looking at the two job-creation equations in (2.12) and (2.13), the right-hand sides are both the same. The two left-hand sides therefore must be equal. Thus, in equilibrium, the value of output must be the same in both sectors:

$$p_\chi\hat{h} = p_\varphi. \tag{B.33}$$

Regardless of the fishery manager's regulation, the share of equilibrium fishers is α. Indeed, when fishers are directed to harvest $\bar{h}$, equation (B.33) evaluates to

$$\alpha\bar{h}^{\alpha-1}\left(\frac{1-\phi}{\phi}\right)^{1-\alpha}\bar{h} = (1-\alpha)\bar{h}^{\alpha}\left(\frac{\phi}{1-\phi}\right)^{\alpha},$$

using prices in (2.9) and (2.10). Developing this expression yields

$$\begin{aligned}
\alpha\bar{h}^{\alpha}\left(\frac{1-\phi}{\phi}\right)^{1-\alpha} &= (1-\alpha)\bar{h}^{\alpha}\left(\frac{\phi}{1-\phi}\right)^{\alpha} \\
\therefore\ \alpha\left(\frac{1-\phi}{\phi}\right)^{1-\alpha} &= (1-\alpha)\left(\frac{\phi}{1-\phi}\right)^{\alpha} \\
\therefore\ \alpha(1-\phi)^{1-\alpha}\phi^{-1+\alpha} &= (1-\alpha)\phi^{\alpha}(1-\phi)^{-\alpha} \\
\therefore\ \alpha(1-\phi)\phi^{-1} &= (1-\alpha) \\
\therefore\ \frac{\alpha}{1-\alpha} &= \frac{\phi}{1-\phi}.
\end{aligned}$$

So $\alpha(1-\phi) = (1-\alpha)\phi$, or $\alpha = \phi$, establishing part of proposition 2.

The same result holds when the fleet is capable of harvesting $g(s^\star)$. In this case, equation (B.33) becomes

$$\alpha \left[\frac{(1-\phi)(1-u)}{g(s^\star)} \right]^{1-\alpha} \frac{g(s^\star)}{\phi(1-u)} = (1-\alpha) \left[\frac{g(s^\star)}{(1-\phi)(1-u)} \right]^{\alpha}.$$

Therefore

$$\alpha(1-\phi)^{1-\alpha}(1-u)^{1-\alpha} g(s^\star)^{-1+\alpha} g(s^\star)\phi^{-1}(1-u)^{-1} = \\ (1-\alpha)g(s^\star)^{\alpha}(1-\phi)^{-\alpha}(1-u)^{-\alpha}.$$

Canceling terms yields

$$\begin{aligned} \alpha(1-\phi)^{1-\alpha}(1-u)^{-\alpha} g(s^\star)^{\alpha}\phi^{-1} &= (1-\alpha)g(s^\star)^{\alpha}(1-\phi)^{-\alpha}(1-u)^{-\alpha} \\ \therefore \alpha(1-\phi)\phi^{-1} &= (1-\alpha) \\ \therefore \frac{\alpha}{1-\alpha} &= \frac{\phi}{1-\phi}. \end{aligned}$$

The same algebra from above establishes that $\phi = \alpha$.

B.1.9 Existence and uniqueness of equilibrium

To get the existence and uniqueness result in proposition 2, use the fact that $p_\varphi = p_\chi \hat{h}$ in equilibrium to write the value of unemployment in equilibrium as

$$\begin{aligned} \Gamma(\theta, \phi) &= \frac{(r+\lambda)z + \beta f(\theta)\left[\phi p_\chi \hat{h} + (1-\phi)p_\varphi - rk\right]}{r+\lambda+\beta f(\theta)} \\ &= \frac{(r+\lambda)z + \beta f(\theta)\left[p_\varphi - rk\right]}{r+\lambda+\beta f(\theta)}. \end{aligned}$$

Use this result in the equilibrium job-creation curve in the nonresource sector, given in equation (2.12):

$$p_\varphi - \Gamma(\theta, \phi) = rk + \frac{rk(r+\lambda)}{(1-\beta)q(\theta)},$$

yielding

$$
\begin{aligned}
\frac{rk(r+\lambda)}{(1-\beta)q(\theta)} &= p_\varphi - rk - \frac{(r+\lambda)z + \beta f(\theta)\left[p_\varphi - rk\right]}{r+\lambda+\beta f(\theta)} \\
\therefore\ \frac{rk(r+\lambda)}{(1-\beta)q(\theta)} &= \frac{(p_\varphi - rk)\left[r+\lambda+\beta f(\theta) - \beta f(\theta)\right]}{r+\lambda+\beta f(\theta)} - \frac{(r+\lambda)z}{r+\lambda+\beta f(\theta)} \\
\therefore\ \frac{rk(r+\lambda)}{(1-\beta)q(\theta)} &= \frac{(p_\varphi - rk)(r+\lambda)}{r+\lambda+\beta f(\theta)} - \frac{(r+\lambda)z}{r+\lambda+\beta f(\theta)} \\
\therefore\ \frac{rk(r+\lambda)}{(1-\beta)q(\theta)} &= \frac{(p_\varphi - z - rk)(r+\lambda)}{r+\lambda+\beta f(\theta)}.
\end{aligned}
$$

When the equilibrium result $\phi^\star = \alpha$ is used in the above expression, the latter is an expression only in terms of θ. One can write $\mathcal{T}(\theta^\star) = 0$, where

$$
\mathcal{T}(\theta) := \frac{(p_\varphi - rk - z)(r+\lambda)}{r+\lambda+\beta f(\theta)} - \frac{rk(r+\lambda)}{(1-\beta)q(\theta)} \tag{B.34}
$$

and p_φ is given in (2.10). Equilibrium tightness is defined implicitly as $T(\theta^\star) = 0$.

There are two cases to consider, depending on the form of p_φ. The form of p_φ depends on whether or not the fleet is capable of harvesting $g(s^\star)$. For low values of θ, the fleet is incapable of harvesting $g(s^\star)$. As θ becomes arbitrarily large, however, the fleet is capable of harvesting $g(s^\star)$. The critical value of θ comes from the harvest rule in (2.7).

The critical value of θ, denoted by $\hat{\theta}$, is defined by

$$
\overline{h} = \frac{g(s^\star)}{\alpha \frac{f(\hat{\theta})}{\lambda + f(\hat{\theta})}},
$$

where ϕ in (2.7) has been replaced by its equilibrium value α and $(1-u)$ has been replaced by the equilibrium level of employment. Rearranging yields

$$
\begin{aligned}
\alpha\overline{h} f(\hat{\theta}) &= \left[\lambda + f(\hat{\theta})\right] g(s^\star) \\
\therefore\ \left[\alpha\overline{h} - g(s^\star)\right] f(\hat{\theta}) &= \lambda g(s^\star) \\
\therefore\ f(\hat{\theta}) &= \frac{\lambda g(s^\star)}{\alpha\overline{h} - g(s^\star)}.
\end{aligned}
$$

In order for the right-hand side to be positive in the above expression $\alpha\overline{h} > g(s^\star)$. Which is equivalent to requiring that the equilibrium fleet for the case when the entire economy is working is capable of harvesting the desired stock level. This is not a restriction on parameters for models of modern commercial fishing.

The critical value $\hat{\theta}$ is then defined as

$$\hat{\theta} = f^{-1}\left(\frac{\lambda g(s^\star)}{\alpha\overline{h} - g(s^\star)}\right).$$

Note that, other than being a function of biological parameters, $s^\star$ is an implicit function of r. So the expression for $\hat{\theta}$ is written entirely in terms of deep parameters.

For values of $\theta < \hat{\theta}$, $\overline{h} < g(s^\star)/\alpha/(1-u)$ and the fishery manager directs the fleet to harvest at full capacity. For values of $\theta > \hat{\theta}$, $\overline{h} > g(s^\star)/\alpha/(1-u)$ and the fishery manager implements a tradable quota scheme to target the desired stock level.

B.1.9.1 Existence

To prove that an equilibrium exists, observe that $\mathcal{T}$ is continuous.

For $\theta < \hat{\theta}$, because $\phi = \alpha$, $p_\varphi = (1-\alpha)\overline{h}^\alpha \alpha^\alpha (1-\alpha)^{-\alpha}$. The price of the good produced in the nonresource sector is independent of θ. And because $\lim_{\theta\to 0} f(\theta) = 0$ and $\lim_{\theta\to 0} q(\theta) = \infty$,

$$\mathcal{T}(0) = p_\varphi - z - rk > 0.$$

The inequality follows from the fact that $p_\varphi - rk$ is the net product of a productive match and, in equilibrium, this is greater than the flow value of nonwork.

On the other hand, when $\theta > \hat{\theta}$ and $\theta \to \infty$, p_φ depends on θ. From equation (2.10)

$$\begin{aligned} p_\varphi &= (1-\alpha)\left[\frac{g(s^\star)}{(1-\alpha)(1-u)}\right]^\alpha \\ &= (1-\alpha)g(s^\star)^\alpha(1-\alpha)^{-\alpha}(1-u)^{-\alpha} \\ &= (1-\alpha)^{1-\alpha} g(s^\star)^\alpha \left(\frac{f(\theta)}{\lambda + f(\theta)}\right)^{-\alpha}, \end{aligned}$$

where $f(\theta)/[\lambda + f(\theta)]$ is the equilibrium level of employment.

The expression for $\mathcal{T}$ becomes

$$\mathcal{T}(\theta) := \frac{\left[(1-\alpha)^{1-\alpha} g(s^\star)^\alpha \left(\frac{f(\theta)}{\lambda+f(\theta)}\right)^{-\alpha} - rk - z\right](r+\lambda)}{r + \lambda + \beta f(\theta)} - \frac{rk(r+\lambda)}{(1-\beta)q(\theta)}.$$

Define $\tilde{\theta}$ implicitly as

$$\frac{(p_\varphi - rk - z)(r+\lambda)}{r+\lambda+\beta f(\tilde{\theta})} = \frac{1}{2}\frac{rk(r+\lambda)}{(1-\beta)q(\tilde{\theta})}. \tag{B.35}$$

and put aside whether $\tilde{\theta}$ exists for the moment. Then

$$\mathcal{T}(\tilde{\theta}) = \frac{rk(r+\lambda)}{(1-\beta)q(\tilde{\theta})}\left(\frac{1}{2}-1\right) < 0.$$

By the intermediate-value theorem, there exists $\theta^\star \in (\theta, \tilde{\theta})$ such that $\mathcal{T}(\theta^\star) = 0$.

The existence of such a $\tilde{\theta}$ in (B.35) comes from another application of the intermediate value theorem. Write (B.35) as

$$q(\tilde{\theta})\frac{p_\varphi - rk - z}{r+\lambda+\beta f(\tilde{\theta})} = \frac{1}{2}\frac{rk}{(1-\beta)}.$$

Using the equilibrium expression for p_φ the latter evaluates to

$$\frac{q(\tilde{\theta})\left[\frac{(1-\alpha)^{1-\alpha}g(s^\star)^\alpha}{\left(\frac{f(\tilde{\theta})}{\lambda+f(\tilde{\theta})}\right)^\alpha} - rk - z\right]}{r+\lambda+\beta f(\tilde{\theta})} = \frac{1}{2}\frac{rk}{(1-\beta)}.$$

The question is whether the left-hand side can equal the positive number $rk/(1-\beta)/2$. The answer is yes.

Define the function

$$\Psi(\theta) := \frac{q(\theta)\left[\frac{(1-\alpha)^{1-\alpha}g(s^\star)^\alpha}{\left(\frac{f(\theta)}{\lambda+f(\theta)}\right)^\alpha} - rk - z\right]}{r+\lambda+\beta f(\theta)}. \tag{B.36}$$

First note that Ψ is a continuous function of θ. Moreover, evaluating Ψ at $\hat{\theta}$ leads to a positive number. To see this, use the fact that

$$\frac{f(\hat{\theta})}{\lambda+f(\hat{\theta})} = \frac{g(s^\star)}{\alpha\bar{h}}.$$

Use this expression in (B.36), yielding

$$\Psi(\hat{\theta}) = \frac{q(\hat{\theta})\left[\frac{(1-\alpha)^{1-\alpha}g(s^\star)^\alpha}{\left(\frac{g(s^\star)}{\alpha\overline{h}}\right)^\alpha - rk - z}\right]}{r+\lambda+\beta f(\hat{\theta})}.$$

Simplifying yields the expression

$$\begin{aligned}\Psi(\hat{\theta}) &= \frac{q(\hat{\theta})}{r+\lambda+\beta f(\hat{\theta})}\left\{(1-\alpha)^{1-\alpha}\alpha^\alpha\overline{h}^\alpha - rk - z\right\}\\ &= \frac{q(\hat{\theta})}{r+\lambda+\beta f(\hat{\theta})}\left\{p_\varphi - rk - z\right\}\\ &> 0,\end{aligned}$$

where the second equality uses the expression for p_φ in (2.10) with $\phi = \alpha$ and the inequality follows from $p_\varphi - rk > z$ in equilibrium.

To get a negative value, use the fact that $\lim_{\theta\to\infty} q(\theta) = 0$ and $\lim_{\theta\to\infty} f(\theta) = \infty$ to see that $\Psi(\theta) \to 0$ as $\theta \to \infty$. The intermediate-value theorem then establishes the existence of $\tilde{\theta} \in (\hat{\theta}, \infty)$ such that $\Psi(\tilde{\theta}) = rk/(1-\beta)/2$. This result establishes the existence of $\tilde{\theta}$ defined in (B.35).

B.1.9.2 Uniqueness

Uniqueness of the equilibrium follows from the fact that $\mathcal{T}$ is a decreasing function of θ. When $\theta < \hat{\theta}$, the derivative of $\mathcal{T}$ with respect to θ evaluates to:

$$\begin{aligned}\left.\frac{\partial\mathcal{T}}{\partial\theta}\right|_{\theta>\hat{\theta}} &= -\frac{(p_\varphi - rk - z)(r+\lambda)}{(r+\lambda+\beta f(\theta))^2}\beta f'(\theta) + \frac{rk(r+\lambda)}{(1-\beta)[q(\theta)]^2}q'(\theta)\\ &< 0,\end{aligned}$$

where the inequality comes from the fact that $f' > 0$ and $q' < 0$.

When the case when $\theta > \hat{\theta}$, Define

$$x(\theta) := (1-\alpha)^{1-\alpha}g(s^\star)^\alpha\left(\frac{f(\theta)}{\lambda+f(\theta)}\right)^{-\alpha}. \tag{B.37}$$

And note that

$$
\begin{aligned}
\frac{\partial x(\theta)}{\partial \theta} &= -(1-\alpha)^{1-\alpha} g(s^{\star})^{\alpha} \alpha \left(\frac{f(\theta)}{\lambda + f(\theta)}\right)^{-\alpha-1} \frac{f'(\theta)\left[\lambda + f(\theta)\right] - f'(\theta) f(\theta)}{\left[\lambda + f(\theta)\right]^2} \\
&= -(1-\alpha)^{1-\alpha} g(s^{\star})^{\alpha} \alpha \left(\frac{f(\theta)}{\lambda + f(\theta)}\right)^{-\alpha-1} \frac{f'(\theta)\lambda}{\left[\lambda + f(\theta)\right]^2} \\
&< 0.
\end{aligned}
\tag{B.38}
$$

The inequality follows from the fact that $f'(\theta) > 0$.

Using $x(\theta)$ defined in (B.37), write $\mathcal{T}(\theta)$ as

$$
\mathcal{T}(\theta)\Big|_{\theta > \hat{\theta}} = \frac{\left[x(\theta) - rk - z\right](r+\lambda)}{r + \lambda + \beta f(\theta)} - \frac{rk(r+\lambda)}{(1-\beta) q(\theta)}.
$$

Then

$$
\begin{aligned}
\frac{\partial \mathcal{T}}{\partial \theta}\Big|_{\theta < \hat{\theta}} &= \frac{x'(\theta)(r+\lambda)\left[r + \lambda + \beta f(\theta)\right] - \beta f'(\theta)\left[x(\theta) - rk - z\right]}{\left[r + \lambda + \beta f(\theta)\right]^2} + \frac{rk(r+\lambda)}{(1-\beta)\left[q(\theta)\right]^2} \\
&= \frac{x'(\theta)(r+\lambda)\left[r + \lambda + \beta f(\theta)\right] - \beta f'(\theta)\left[p_{\phi} - rk - z\right]}{\left[r + \lambda + \beta f(\theta)\right]^2} + \frac{rk(r+\lambda)}{(1-\beta)\left[q(\theta)\right]^2} q'(\theta) \\
&< 0,
\end{aligned}
$$

where the inequality follows from the fact that $p_{\varphi} - rk > z$ because equilibrium net profit is greater than the flow value of nonwork, $x'(\theta) < 0$ from (B.38), $f'(\theta) > 0$, and $q'(\theta) < 0$.

Thus $\mathcal{T}$ is monotonically decreasing in θ and therefore must equal zero exactly once at $\mathcal{T}(\theta^{\star})$, establishing uniqueness.

B.1.10 Optimal composition of jobs

This section derives results on the optimal composition of jobs. There are two cases to consider. Equilibrium surplus is

$$
\mathcal{S} = (1-u)[\phi(p_{\chi}\hat{h} - rk) + (1-\phi)(p_{\varphi} - rk)] + zu - \theta urk.
$$

How equilibrium surplus changes with respect to the composition of jobs is given by

$$
\frac{\partial \mathcal{S}}{\partial \phi} = (1-u)\frac{\partial}{\partial \phi}\left[\phi p_{\chi}\hat{h} + (1-\phi)p_{\varphi}\right]. \tag{B.39}
$$

Our interest lies in evaluating the derivative in equation (B.39) at the equilibrium levels.

B.1.10.1 Case: $\phi < \check{\phi}$

When $\phi < \check{\phi}$, the derivative in equation (B.39) evaluated at equilibrium levels, using the expressions for equilibrium prices in equations (2.10)) and (2.9) and harvest in equation (2.7, is:

$$\begin{aligned}
\frac{\partial \mathcal{S}}{\partial \phi} &= (1-u)\frac{\partial}{\partial \phi}\left[\phi p_\chi \hat{h} + (1-\phi)p_\varphi\right] \\
\left.\frac{\partial \mathcal{S}}{\partial \phi}\right|_{\text{dec. eqm.}} &= (1-u^\star)\frac{\partial}{\partial \phi}\left[\phi\alpha\overline{h}^{\alpha-1}\left(\frac{\phi}{1-\phi}\right)^\alpha \overline{h} + (1-\phi)(1-\alpha)\overline{h}^\alpha\left(\frac{\phi}{1-\phi}\right)\right] \\
&= (1-u^\star)\frac{\partial}{\partial \phi}\left[\alpha\overline{h}^\alpha\phi\left(\frac{1-\phi}{\phi}\right)^{1-\alpha} + (1-\alpha)\overline{h}^\alpha(1-\phi)\left(\frac{\phi}{1-\phi}\right)^\alpha\right] \\
&= (1-u^\star)\overline{h}^\alpha\frac{\partial}{\partial \phi}\left[\alpha\phi^\alpha(1-\phi)^{1-\alpha} + (1-\alpha)\phi^\alpha(1-\phi)^{1-\alpha}\right] \\
&= (1-u^\star)\overline{h}^\alpha\frac{\partial}{\partial \phi}\left[\phi^\alpha(1-\phi)^{1-\alpha}\right].
\end{aligned}$$

Differentiating the latter gives

$$\begin{aligned}
\left.\frac{\partial \mathcal{S}}{\partial \phi}\right|_{\text{dec. eqm.}} &= (1-u^\star)\overline{h}^\alpha\left[\alpha\phi^{\alpha-1} - (1-\alpha)\phi^\alpha(1-\phi)^{-\alpha}\right] \\
&= (1-u^\star)\overline{h}^\alpha\left[\alpha\left(\frac{1-\phi}{\phi}\right)^{1-\alpha} - (1-\alpha)\left(\frac{\phi}{1-\phi}\right)^\alpha\right].
\end{aligned}$$

Using the expressions for prices in equations (2.9)) and (2.10, and the fact that $u^\star = \lambda/[\lambda + f(\theta)]$,

$$\begin{aligned}
\left.\frac{\partial \mathcal{S}}{\partial \phi}\right|_{\text{dec. eqm.}} &= \frac{f(\theta)}{\lambda + f(\theta)}\overline{h}^\alpha\left(p_\chi\overline{h}^{-\alpha+1} - p_\varphi\overline{h}^{-\alpha}\right) \\
&= \frac{f(\theta)}{\lambda + f(\theta)}\left(p_\chi\overline{h} - p_\varphi\right).
\end{aligned} \tag{B.40}$$

B.1.10.2 Case: $\phi > \check{\phi}$

When $\phi > \check{\phi}$, the derivative in equation (B.39) evaluated at equilibrium levels, using the expressions for equilibrium prices in equations (2.10)) and (2.9) and harvest in equation

(2.7, is:

$$
\begin{aligned}
\frac{\partial \mathcal{S}}{\partial \phi} &= (1-u)\frac{\partial}{\partial \phi}\left[\phi p_\chi \hat{h} + (1-\phi)p_\varphi\right] \\
\left.\frac{\partial \mathcal{S}}{\partial \phi}\right|_{\text{dec. eqm.}} &= (1-u^\star)\frac{\partial}{\partial \phi}\left\{\phi\alpha\left[\frac{(1-\phi)(1-\alpha)}{g(s^\star)}\right]^{1-\alpha}\frac{g(s^\star)}{\phi(1-u^\star)}\right. \\
&\quad\left. + (1-\phi)(1-\alpha)\left[\frac{g(s^\star)}{(1-\phi)(1-u^\star)}\right]^{\alpha}\right\} \\
&= (1-u^\star)\frac{\partial}{\partial \phi}\left\{\alpha(1-\phi)^{1-\alpha}(1-u^\star)^{-\alpha}g(s^\star)^{\alpha}\right. \\
&\quad \left. +(1-\alpha)(1-\phi)^{1-\alpha}(1-u^\star)^{-\alpha}g(s^\star)^{\alpha}\right\} \\
&= (1-u^\star)\frac{\partial}{\partial \phi}\left\{(1-\phi)^{1-\alpha}(1-u^\star)^{-\alpha}g(s^\star)^{\alpha}\right\} \\
&= -(1-\alpha)(1-u^\star)^{1-\alpha}g(s^\star)^{\alpha}(1-\phi)^{-\alpha} \\
&= -(1-\alpha)(1-u^\star)\left[\frac{g(s^\star)}{(1-\phi)(1-u^\star)}\right]^{\alpha}.
\end{aligned}
$$

Using the expression for the price of the second good in equation (2.10) and the expression for equilibrium unemployment, the latter evaluates to

$$
\left.\frac{\partial \mathcal{S}}{\partial \phi}\right|_{\text{dec. eqm.}} = -\frac{f(\theta)}{\lambda + f(\theta)}p_\varphi. \tag{B.41}
$$

B.1.11 Optimal stock level

This section derives results on the optimal stock level. There are two cases to consider. Equilibrium surplus is

$$
\mathcal{S} = (1-u)[\phi(p_\chi \hat{h} - rk) + (1-\phi)(p_\varphi - rk)] + zu - \theta urk.
$$

How equilibrium surplus changes with respect to the stock of fish is given by

$$
\frac{\partial \mathcal{S}}{\partial s} = (1-u)\frac{\partial}{\partial s}\left[\phi p_\chi \hat{h} + (1-\phi)p_\varphi\right]. \tag{B.42}
$$

B.1.11.1 Case: $\phi < \check{\phi}$

When $\phi < \check{\phi}$, the derivative in equation (B.42) evaluated at equilibrium levels, using the expressions for equilibrium prices in equations (2.10)) and (2.9) and harvest in equation

(2.7, is:

$$\left.\frac{\partial \mathcal{S}}{\partial s}\right|_{\text{dec. eqm.}} = \frac{f(\theta)}{\lambda + f(\theta)}\frac{\partial}{\partial s}\left[\phi\alpha\overline{h}^{\alpha-1}\left(\frac{1-\phi}{\phi}\right)^{1-\alpha}\overline{h} + (1-\phi)(1-\alpha)\overline{h}^{\alpha}\left(\frac{\phi}{1-\phi}\right)^{\alpha}\right] \tag{B.43}$$
$$= 0,$$

as the term on the right-hand side is independent of $s^\star$. In this scenario the fisher manager is directing the fishery to fish at $\overline{h}$ and still cannot get the fishery to $s^\star$ in equilibrium, which follows from the fact that the economy does not want that many fish.

B.1.11.2 Case: $\phi > \check{\phi}$

When $\phi > \check{\phi}$, the derivative in (B.42) evaluated at equilibrium levels, using the expressions for equilibrium prices in equations (2.10)) and (2.9) and harvest in equation (2.7, is

$$\frac{\partial \mathcal{S}}{\partial s} = (1-u)\frac{\partial}{\partial \phi}\left[\phi p_\chi \hat{h} + (1-\phi)p_\varphi\right]$$

$$\begin{aligned}
\left.\frac{\partial \mathcal{S}}{\partial s}\right|_{\text{dec. eqm.}} &= (1-u^\star)\frac{\partial}{\partial s}\Bigg\{\phi\alpha\left[\frac{(1-\phi)(1-\alpha)}{g(s^\star)}\right]^{1-\alpha}\frac{g(s^\star)}{\phi(1-u^\star)} \\
&\quad + (1-\phi)(1-\alpha)\left[\frac{g(s^\star)}{(1-\phi)(1-u^\star)}\right]^{\alpha}\Bigg\} \\
&= (1-u^\star)\frac{\partial}{\partial s}\left\{\alpha(1-\phi)^{1-\alpha}(1-u^\star)^{-\alpha}g(s^\star)^{\alpha}\right. \\
&\quad \left.+(1-\alpha)(1-\phi)^{1-\alpha}(1-u^\star)^{-\alpha}g(s^\star)^{\alpha}\right\} \\
&= (1-u^\star)\frac{\partial}{\partial s}\left\{(1-\phi)^{1-\alpha}(1-u^\star)^{-\alpha}g(s^\star)^{\alpha}\right\} \\
&= \alpha(1-u^\star)^{1-\alpha}(1-\phi)^{1-\alpha}g(s^\star)^{\alpha-1}g'(s^\star) \\
&= \alpha\left[\frac{(1-\phi)(1-u^\star)}{g(s^\star)}\right]^{1-\alpha}g'(s^\star).
\end{aligned}$$

From the equilibrium price of fish in equation (2.9), the latter evaluates to

$$\left.\frac{\partial \mathcal{S}}{\partial s}\right|_{\text{dec. eqm.}} = p_\chi g'(s^\star). \tag{B.44}$$

B.1.12 The oracle planner's full dynamic program

This section considers the full dynamic program from the perspective of a social planner who maximizes the value of surplus subject to stock dynamics and frictions in the labor market. Unlike the fishery manager the social planner maximizes the *economy-wide* value of surplus, not simply the value of output in the fishery.

Note that prices do not depend on unemployment per say, rather they depend on the composition of employment. Indeed, from the expressions for prices and the two market-clearing conditions:

$$\begin{aligned} p_\varphi &= (1-\alpha)\left(\frac{Y_\chi}{Y_\varphi}\right)^\alpha = (1-\alpha)\left(\frac{\phi(1-u)h}{(1-\phi)(1-u)}\right)^\alpha = (1-\alpha)\left(\frac{\phi h}{1-\phi}\right)^\alpha \\ p_\chi &= \alpha\left(\frac{Y_\varphi}{Y_\chi}\right)^{1-\alpha} = \alpha\left(\frac{(1-\phi)(1-u)}{\phi h(1-u)}\right) = \alpha\left(\frac{1-\phi}{\phi h}\right)^{1-\alpha} = \alpha\left(\frac{\phi h}{1-\phi}\right)^{\alpha-1}. \end{aligned} \tag{B.45}$$

I look for an equilibrium where startup costs are the same across sectors: $k_\chi = k_\varphi = k$. In addition, I think of the planner as choosing the vacancy–unemployment ratio, which is isomorphic to choosing the level of vacancies as u is predetermined.

The flow objective of the social planner consists of the net flow value of output produced by fishers and nonfishers, plus the flow value of home production, minus the flow cost of job creation. In the fishery there are $(1-u(t))\phi(t)$ fishers who each catch $h(t)$ valued at $p_\chi(\phi(t), h(t))$. The flow cost of capital is rk. The net flow value of output in the fishery is therefore equal to

$$(1-u(t))\phi(t)[p_\chi(\phi(t), h(t))h(t) - rk].$$

The net flow value of output produced by the nonresource sector equals the total number of nonfishers, $(1-u(t))(1-\phi(t))$, times the net value of their output, $[p_\varphi(\phi(t), h(t)) - rk]$, making the net flow-value of output in the nonresource sector equal to

$$(1-u(t))(1-\phi(t))\left[p_\varphi(\phi(t), h(t)) - rk\right].$$

The flow value of output produced at home equals $u(t)z$. The flow cost of job creation equals the number of vacancies, $\theta(t)u(t)$, times their flow cost, rk.

The oracle planner solves

$$\max_{\substack{\{\theta(t)\},\\ \{\phi(t)\}\in[0,1],\\ \{h(t)\}\in[0,\bar{h}]}} \int_0^\infty e^{-rt}\left\{(1-u(t))\left[\phi(t)p_\chi(\phi(t), h(t))h(t) + (1-\phi(t))p_\varphi(\phi(t), h(t)) - rk\right]\right.$$
$$\left. + zu(t) - \theta(t)u(t)rk\right\}\, dt,$$

subject to frictions in the labor market

$$\dot{u}(t) = (1 - u(t))\lambda - f(\theta(t))u(t) \tag{B.46}$$

and the evolution of the stock of fish:

$$\dot{s}(t) = g(s(t)) - (1 - u(t))\phi(t)h(t), \tag{B.47}$$

with $s(0) = s_0$ and $u(0) = u_0$ given.

The Lagrangian for the social planner [the current-value Hamiltonian augmented with the constraints in (B.46) and (B.47)] is

$$\begin{aligned} L = {} & (1 - u(t)) \left[\phi(t)p_\chi(\phi(t), h(t))h(t) + (1 - \phi(t))p_\varphi(\phi(t), h(t)) - rk\right] \\ & + zu(t) - \theta(t)u(t)rk \\ & + \mu(t) \left[(1 - u(t))\lambda - f(\theta(t))u(t)\right] + \gamma(t) \left[g(s(t)) - (1 - u(t))\phi(t)h(t)\right] \\ & + \sigma_1(t)(\overline{h} - h(t)) + \sigma_2(t)h(t) \\ & + \zeta_1(t)(1 - \phi(t)) + \zeta_2(t)\phi(t), \end{aligned}$$

where μ and γ are the costate variables associated with the constraints in (B.46) and (B.47); σ_1, σ_2 are the Kuhn–Tucker multipliers for the constraint $h(t) \in [0, \overline{h}]$; and ζ_1, ζ_2 are the Kuhn–Tucker multipliers for the constraint $\phi(t) \in [0, 1]$.

B.1.12.1 Necessary conditions

The first-order necessary condition for θ is:

$$L_\theta = -u(t)rk - \mu(t)f'(\theta(t))u(t) = 0. \tag{B.48}$$

The constrained-maximum first-order necessary condition for ϕ is

$$\begin{aligned} L_\phi = {} & (1 - u(t)) \left\{h(t) \left[p_\chi(t) + p_{\chi\phi}(t)\phi(t)\right] - p_\varphi(t) + (1 - \phi(t))p_{\varphi\phi}(t)\right\} \\ & - \gamma(t)(1 - u(t))h(t) - \zeta_1(t) + \zeta_2(t) = 0, \end{aligned} \tag{B.49}$$

along with the complimentary slackness conditions

$$\zeta_1(t) \geq 0 \qquad \zeta_1(t) \left[1 - \phi(t)\right] = 0 \tag{B.50}$$

$$\zeta_2(t) \geq 0 \qquad \zeta_2(t)\phi(t) = 0. \tag{B.51}$$

The constrained-maximum first-order condition for h is

$$\begin{aligned} L_h = (1 - u(t)) \left\{ \phi(t) \left[p_{\chi h}(t) h(t) + p_\chi(t) \right] + (1 - \phi(t)) p_{\varphi h}(t) \right\} \\ - \gamma(t)(1 - u(t))\phi(t) - \sigma_1(t) + \sigma_2(t) = 0, \end{aligned} \tag{B.52}$$

along with the complimentary slackness conditions

$$\sigma_1(t) \geq 0 \qquad \sigma_1(t) \left[\overline{h} - h(t) \right] = 0 \tag{B.53}$$

$$\sigma_2(t) \geq 0 \qquad \sigma_2(t) h(t) = 0. \tag{B.54}$$

The first-order necessary condition for u is

$$\begin{aligned} L_u = r\mu(t) - \dot{\mu}(t) = - \left[\phi(t) p_\chi(t) h(t) + (1 - \phi(t)) p_\varphi(t) - rk \right] + z - \theta(t) rk \\ - \mu(t) \left[\lambda + f(\theta(t)) \right] + \gamma(t)\phi(t)h(t). \end{aligned} \tag{B.55}$$

The first-order necessary condition for s is

$$L_s = r\gamma(t) - \dot{\gamma}(t) = \gamma(t) g'(s(t)). \tag{B.56}$$

B.1.12.2 The system of equations

I look for a stationary equilibrium. The system of stationary equations under analysis is

$$0 = -rk - \mu f'(\theta) \tag{B.57}$$

$$0 = (1 - u) \left[h(p_\chi + p_{\chi\phi}\phi) - p_\varphi + (1 - \phi) p_{\varphi\phi} \right] - \gamma(1 - u)h - \zeta_1 + \zeta_2 \tag{B.58}$$

$$0 = (1 - u) \left[\phi(p_{\chi h} h + p_\chi) + (1 - \phi) p_{\varphi h} \right] - \gamma(1 - u)\phi - \sigma_1 + \sigma_2 \tag{B.59}$$

$$r\mu = - \left[\phi p_\chi h + (1 - \phi) p_\varphi - rk \right] + z - \theta rk - \mu \left[\lambda + f(\theta) \right] + \gamma\phi h \tag{B.60}$$

$$r\gamma = \gamma g'(s) \tag{B.61}$$

along with the state equations and the complimentary slackness conditions

$$0 = (1 - u)\lambda - f(\theta) u \tag{B.62}$$

$$0 = g(s) - (1 - u)\phi h \tag{B.63}$$

$$0 = \zeta_1 (1 - \phi) \tag{B.64}$$

$$0 = \zeta_2 \phi \tag{B.65}$$

$$0 = \sigma_1 (\overline{h} - h) \tag{B.66}$$

$$0 = \sigma_2 h, \tag{B.67}$$

with $\sigma_1, \sigma_2 \geq 0$; $\zeta_1, \zeta_2 \geq 0$; and prices defined in (B.45). Equation (B.57) corresponds to (B.48); equation (B.58) corresponds to (B.49); equation (B.59) corresponds to (B.52); equation (B.60) corresponds to (B.55); equation (B.61) corresponds to (B.56). Equation (B.62) is the stationary version of the job-destruction constraint given in (B.46) and equation (B.63) is the stationary version of the stock-evolution constraint given in (B.47). Equations (B.64)–(B.67) are the complimentary slackness conditions given by the Kuhn–Tucker theorem.

I am interested in the following equilibrium objects 1) s, 2) θ, 3) u, 4) h, 5) γ, 6) ϕ, 7) μ, 8) σ_1, 9) σ_2, 10) ζ_1, 11) ζ_2. Equations (B.57)–(B.67) are the 11 corresponding nonlinear, stationary equations.

The next section uses the necessary conditions to rule out canidate solutions to the stationary equilibrium.

B.1.12.3 Establishing that $\zeta_1 = \zeta_2 = 0$

Equation (B.57) requires

$$\mu = -\frac{rk}{f'(\theta)}.$$

μ represents the shadow value of unemployment to the oracle planner, which is negative because adding an unemployed worker means taking away a more productive employed worker.

Use the expression for μ in (B.60):

$$\begin{aligned}
-r\frac{rk}{f'(\theta)} &= -\left[\phi p_\chi h + (1-\phi)p_\varphi - rk\right] + z - \theta rk + \frac{rk}{f'(\theta)}\left[\lambda + f(\theta)\right] + \gamma\phi h \\
\therefore\ r\frac{rk}{f'(\theta)} &= \phi p_\chi h + (1-\phi)p_\varphi - rk - z + \theta rk - \frac{rk}{f'(\theta)}\left[\lambda + f(\theta)\right] - \gamma\phi h.
\end{aligned}$$

Combining terms yields

$$z + rk + \frac{rk}{f'(\theta)}\left[r + \lambda + f(\theta) - \theta f'(\theta)\right] = \phi p_\chi h + (1-\phi)p_\varphi - \gamma\phi h. \tag{B.68}$$

Develop equation (B.58) to yield

$$\begin{aligned}
\gamma h &= p_\chi h + \phi h p_{\chi\phi} - p_\varphi + (1-\phi)p_{\varphi\phi} - \frac{\zeta_1}{1-u} + \frac{\zeta_2}{1-u} \\
\therefore\ \gamma\phi h &= \phi p_\chi h + \phi^2 h p_{\chi\phi} - \phi p_\varphi + \phi(1-\phi)p_{\varphi\phi} - \frac{\phi}{1-u}\zeta_1 + \frac{\phi}{1-u}\zeta_2.
\end{aligned}$$

Using the expression for $\gamma\phi h$ in (B.68) yields

$$
\begin{aligned}
z + rk + \frac{rk}{f'(\theta)}\left[r + \lambda + f(\theta) - \theta f'(\theta)\right] &= \phi p_\chi h + (1-\phi)p_\varphi \\
&\quad - \phi p_\chi h - \phi^2 h p_{\chi\phi} + \phi p_\varphi - \phi(1-\phi)p_{\varphi\phi} \\
&\quad + \frac{\phi}{1-u}\zeta_1 - \frac{\phi}{1-u}\zeta_2 \\
&= p_\varphi - \phi^2 h p_{\chi\phi} - \phi(1-\phi)p_{\varphi\phi} + \frac{\phi}{1-u}\zeta_1 - \frac{\phi}{1-u}\zeta_2 \\
&= p_\varphi + \frac{\phi}{1-u}\zeta_1 - \frac{\phi}{1-u}\zeta_2,
\end{aligned}
$$

where the last line uses the algebra in fact 1.

Write this expression as

$$
\begin{aligned}
p_\varphi - rk - z = {} & \frac{rk}{f'(\theta)}\left[r + \lambda + f(\theta) - \theta f'(\theta)\right] \\
& - \frac{\phi}{1-u}\zeta_1 + \frac{\phi}{1-u}\zeta_2.
\end{aligned}
\tag{B.69}
$$

Equation (B.69) requires that $\phi^{\star\star} \in (0,1)$. To see this, define

$$
\Psi(\theta) := \frac{rk}{f'(\theta)}\left[r + \lambda + f(\theta) - \theta f'(\theta)\right]. \tag{B.70}
$$

Using the definition of Ψ in (B.70), the expression for p_φ in (B.69) can be written

$$
p_\varphi = rk + z + \Psi(\theta) - \frac{\phi}{1-u}\zeta_1 + \frac{\phi}{1-u}\zeta_2. \tag{B.71}
$$

Suppose $\zeta_2 > 0$. The complimentary slackness condition in (B.65) then requires $\phi = 0$. The expression for p_φ, using (B.45), evaluates to $(1-\alpha)[(0 \times h)/1]^\alpha = 0$. Proposition 11 establishes that when the matching technology is parameterized as Cobb–Douglas, $\lim_{\theta \to 0} \Psi(\theta) = 0$ and $\lambda_{\theta \to \infty} \Psi(\theta) = \infty$. Proposition 12 establishes that Ψ is increasing in θ. These two facts together with the fact that $\phi\zeta_1/(1-u) = 0$ when $\phi = 0$ establish that the right-hand side of (B.71) is strictly positive while the left-hand side is zero when $\zeta_2 > 0$. This is a contradiction. Hence $\zeta_2 = 0$.

Now suppose $\zeta_1 > 0$. The complimentary slackness condition in (B.64) then requires $\phi = 1$. When $\phi = 1$, using (B.45) p_φ evaluates to ∞. In order to balance, the right-hand side of (B.71) must also become arbitrarily large, which requires $\theta = \infty$ as established in proposition 11. Is this a stationary equilibrium? The Hamiltonian function evaluated at $\phi = 1$ and $\theta = \infty$, however, is arbitrarily negative, violating the Maximum Principle;

namely, that the Hamiltonian is maximized by the choice variable.

To see this, note that $u = 0$ when $\theta = \infty$. The algebra in fact 2 establishes that average productivity evaluates to zero in this case. But the economy faces the burden of carrying around the vacancies at a flow cost of rk, making the cost of job creation arbitrarily large. The oracle planner could do better by producing nothing and having the economy engage entirely in home production. This violates the Maximum Principle. Therefore $\zeta_1 = 0$.

A stationary steady state necessarily requires $\zeta_1 = \zeta_2 = 0$. The same logic also rules out $\phi = 0$ and $\phi = 1$ as solutions. Therefore $\phi^{\star\star} \in (0, 1)$.

B.1.12.4 Establishing that $\sigma_2 = 0$

Suppose that $\sigma_2 > 0$. The complimentary slackness condition in (B.67) then requires $h = 0$. Equation (B.63) then requires $g(s) = 0$, which says that the stock will be at carrying capacity in order for there to be no growth. This violates the planner's choice in (B.61) to exploit the stock at the same rate of return as other assets in the economy. The humped-shaped dynamics of stock growth require $g' < 0$ when evaluated at carrying capacity as depicted in figure 2.4. This outcome contradicts the oracle planner's choice, establishing that $\sigma_2 = 0$.

B.1.12.5 The reduced system of equations

These requirements reduce the list of equilibrium objects that I am interested in: 1) s, 2) θ, 3) u, 4) h, 5) γ, 6) ϕ, 7) μ, 8) σ_1.

The corresponding system of equations is

$$0 = -rk - \mu f'(\theta) \tag{B.72}$$

$$0 = h(p_\chi + p_{\chi\phi}\phi) - p_\varphi + (1-\phi)p_{\varphi\phi} - \gamma h \tag{B.73}$$

$$0 = (1-u)\left[\phi(p_{\chi h}h + p_\chi) + (1-\phi)p_{\varphi h}\right] - \gamma(1-u)\phi - \sigma_1 \tag{B.74}$$

$$r\mu = -\left[\phi p_\chi h + (1-\phi)p_\varphi - rk\right] + z - \theta rk - \mu\left[\lambda + f(\theta)\right] + \gamma\phi h \tag{B.75}$$

$$r = g'(s), \tag{B.76}$$

along with the state equations and complimentary slackness condition for σ_1:

$$0 = (1-u)\lambda - f(\theta)u \tag{B.77}$$

$$0 = g(s) - (1-u)\phi h \tag{B.78}$$

$$0 = \sigma_1(\overline{h} - h), \tag{B.79}$$

with $\sigma_1 \geq 0$. There are eight equations and eight unknowns.

B.1.12.6 Reducing the system of equations further

Use equation (B.72) to eliminate μ from the system by writing μ in terms of θ. Use equation (B.77) to eliminate u from the system by writing u in terms of θ (recall that prices depend on relative output and therefore do not have u in them). Use equation (B.76) to pin down $s^{\star\star} = (g')^{-1}(r)$. The system becomes

$$
\begin{aligned}
0 &= h(p_\chi + p_{\chi\phi}\phi) - p_\varphi + (1-\phi)p_{\varphi\phi} - \gamma h && \text{(B.80)}\\
0 &= \frac{f(\theta)}{\lambda + f(\theta)}\left[\phi(p_{\chi h}h + p_\chi) + (1-\phi)p_{\varphi h}\right] - \gamma\phi\frac{f(\theta)}{\lambda + f(\theta)} - \sigma_1 && \text{(B.81)}\\
r\frac{-rk}{f'(\theta)} &= -\left[\phi p_\chi h + (1-\phi)p_\varphi - rk\right] + z - \theta rk + \frac{rk}{f'(\theta)}\left[\lambda + f(\theta)\right] + \gamma\phi h, && \text{(B.82)}
\end{aligned}
$$

along with the state equations and Kuhn–Tucker conditions

$$
\begin{aligned}
0 &= g(s) - \phi h\frac{f(\theta)}{\lambda + f(\theta)} && \text{(B.83)}\\
0 &= \sigma_1(\overline{h} - h), && \text{(B.84)}
\end{aligned}
$$

with $\sigma_1 \geq 0$.

The unknowns are 1) θ, 2) h, 3) γ, 4) ϕ, 5) σ_1.

B.1.12.7 Simplifying and expanding expressions

Equation (B.82) can be simplified:

$$
\begin{aligned}
r\frac{-rk}{f'(\theta)} &= -\left[\phi p_\chi h + (1-\phi)p_\varphi - rk\right] + z - \theta rk + \frac{rk}{f'(\theta)}\left[\lambda + f(\theta)\right] + \gamma\phi h\\
\therefore\ r\frac{rk}{f'(\theta)} &= \left[\phi p_\chi h + (1-\phi)p_\varphi - rk\right] - z + \theta rk - \frac{rk}{f'(\theta)}\left[\lambda + f(\theta)\right] - \gamma\phi h.
\end{aligned}
$$

Collecting terms on the left-hand side yields

$$
\frac{rk}{f'(\theta)}\left[r + \lambda + f(\theta) - \theta f'(\theta)\right] = \phi p_\chi h + (1-\phi)p_\varphi - rk - z - \gamma\phi h.
$$

With the simplification, the system of equations is

$$0 = h(p_\chi + p_{\chi\phi}\phi) - p_\varphi + (1-\phi)p_{\varphi\phi} - \gamma h \tag{B.85}$$

$$0 = \frac{f(\theta)}{\lambda + f(\theta)}\left[\phi(p_{\chi h}h + p_\chi) + (1-\phi)p_{\varphi h}\right] - \gamma\phi\frac{f(\theta)}{\lambda + f(\theta)} - \sigma_1 \tag{B.86}$$

$$\begin{aligned} 0 = {} & \phi p_\chi h + (1-\phi)p_\varphi - rk - z - \gamma\phi h \\ & - \frac{rk}{f'(\theta)}\left[r + \lambda + f(\theta) - \theta f'(\theta)\right] \end{aligned} \tag{B.87}$$

$$0 = g(s) - \phi h\frac{f(\theta)}{\lambda + f(\theta)} \tag{B.88}$$

$$0 = \sigma_1(\overline{h} - h). \tag{B.89}$$

B.1.12.8 Eliminating γ

To eliminate γ from the system described by (B.85)–(B.89) use equation (B.85) to write $\phi\gamma h$ as

$$\phi\gamma h = \phi p_\chi h + \phi^2 h p_{\chi\phi} - \phi p_\varphi + \phi(1-\phi)p_{\varphi\phi}. \tag{B.90}$$

Use equation (B.90) in equation (B.87):

$$
\begin{aligned}
\frac{rk}{f'(\theta)}\left[r+\lambda+f(\theta)-\theta f'(\theta)\right] &= \phi p_\chi h + (1-\phi)p_\varphi - rk - z - \gamma\phi h \\
&= \phi p_\chi h + (1-\phi)p_\varphi - rk - z - \phi p_\chi h - \phi^2 h p_{\chi\phi} + \phi p_\varphi - \phi(1-\phi)p_{\varphi\phi} \\
&= p_\varphi - rk - z - \phi^2 h p_{\chi\phi} - \phi(1-\phi)p_{\varphi\phi} \\
&= p_\varphi - rk - z \\
&\quad - \phi^2 h(\alpha-1)\alpha\left(\frac{\phi h}{1-\phi}\right)^{\alpha-2} h\frac{(1)(1-\phi)-(-1)(\phi)}{(1-\phi)^2} \\
&\quad - \phi(1-\phi)\alpha(1-\alpha)\left(\frac{\phi h}{1-\phi}\right)^{\alpha-1} h\frac{(1)(1-\phi)-(-1)(\phi)}{(1-\phi)^2} \\
&= p_\varphi - rk - z \\
&\quad - \phi^2 h(\alpha-1)\alpha\left(\frac{\phi h}{1-\phi}\right)^{\alpha-2} h\frac{1}{(1-\phi)^2} \\
&\quad - \phi(1-\phi)\alpha(1-\alpha)\left(\frac{\phi h}{1-\phi}\right)^{\alpha-1} h\frac{1}{(1-\phi)^2} \\
&= p_\varphi - rk - z + \alpha(1-\alpha)\frac{\phi^2 h^2}{(1-\phi)^2}\left(\frac{\phi h}{1-\phi}\right)^{\alpha-2} \\
&\quad - \alpha(1-\alpha)\frac{\phi h(1-\phi)}{(1-\phi)^2}\left(\frac{\phi h}{1-\phi}\right)^{\alpha-1} \\
&= p_\varphi - rk - z.
\end{aligned}
$$

Use equation (B.85) to write

$$
\begin{aligned}
\gamma h &= p_\chi h + p_{\chi\phi}\phi h - p_\varphi + (1-\phi)p_{\varphi\phi} \\
\therefore\ \gamma &= p_\chi + p_{\chi\phi}\phi - \frac{p_\varphi}{h} + (1-\phi)\frac{p_{\varphi\phi}}{h} \\
\therefore\ \gamma\phi &= \phi p_\chi + p_{\chi\phi}\phi^2 - \frac{\phi p_\varphi}{h} + \phi(1-\phi)\frac{p_{\varphi\phi}}{h}.
\end{aligned}
\tag{B.91}
$$

Use the expression for $\gamma\phi$ in (B.86):

$$
\begin{aligned}
\sigma_1 &= \frac{f(\theta)}{\lambda+f(\theta)}\left[\phi(p_{\chi h}h + p_\chi) + (1-\phi)p_{\varphi h}\right] - \gamma\phi\frac{f(\theta)}{\lambda+f(\theta)} \\
&= \frac{f(\theta)}{\lambda+f(\theta)}\left[\phi(p_{\chi h}h + p_\chi) + (1-\phi)p_{\varphi h} - \gamma\phi\right] \\
&= \frac{f(\theta)}{\lambda+f(\theta)}\left[\phi(p_{\chi h}h + p_\chi) + (1-\phi)p_{\varphi h} - \phi p_\chi - p_{\chi\phi}\phi^2 + \frac{\phi p_\varphi}{h} - \phi(1-\phi)\frac{p_{\varphi\phi}}{h}\right],
\end{aligned}
$$

where the last line uses the expression for $\gamma\phi$ given in (B.91). Developing this equation yields

$$\begin{aligned}
\sigma_1 &= \frac{f(\theta)}{\lambda+f(\theta)}\left[\phi(p_{\chi h}h+p_\chi)+(1-\phi)p_{\varphi h}-\phi p_\chi-p_{\chi\phi}\phi^2+\frac{\phi p_\varphi}{h}-\phi(1-\phi)\frac{p_{\varphi\phi}}{h}\right]\\
&= \frac{f(\theta)}{\lambda+f(\theta)}\left\{\phi h p_{\chi h}+\phi p_\chi+(1-\phi)p_{\varphi h}-\phi p_\chi-p_{\chi\phi}\phi^2+\frac{\phi p_\varphi}{h}-\phi(1-\phi)\frac{p_{\varphi\phi}}{h}\right\}\\
&= \frac{f(\theta)}{\lambda+f(\theta)}\left\{\phi h p_{\chi h}+(1-\phi)p_{\varphi h}-p_{\chi\phi}\phi^2+\frac{\phi p_\varphi}{h}-\phi(1-\phi)\frac{p_{\varphi\phi}}{h}\right\}\\
&= \frac{f(\theta)}{\lambda+f(\theta)}\left\{\phi h\alpha(\alpha-1)\left(\frac{\phi h}{1-\phi}\right)^{\alpha-2}\frac{\phi}{1-\phi}+(1-\phi)\alpha(1-\alpha)\left(\frac{\phi h}{1-\phi}\right)^{\alpha-1}\frac{\phi}{1-\phi}\right.\\
&\quad-\phi^2\alpha(\alpha-1)\left(\frac{\phi h}{1-\phi}\right)^{\alpha-2}h\frac{(1)(1-\phi)-(-1)\phi}{(1-\phi)^2}+\frac{\phi}{h}(1-\alpha)\left(\frac{\phi h}{1-\phi}\right)^{\alpha}\\
&\quad\left.-\frac{\phi(1-\phi)}{h}\alpha(1-\alpha)\left(\frac{\phi h}{1-\phi}\right)^{\alpha-1}h\frac{(1)(1-\phi)-(-1)\phi}{(1-\phi)^2}\right\}\\
&= \frac{f(\theta)}{\lambda+f(\theta)}\left\{\phi\alpha(\alpha-1)\left(\frac{\phi h}{1-\phi}\right)^{\alpha-1}+\phi\alpha(1-\alpha)\left(\frac{\phi h}{1-\phi}\right)^{\alpha-1}\right.\\
&\quad-\phi^2\alpha(\alpha-1)\left(\frac{\phi h}{1-\phi}\right)^{\alpha-2}h\frac{1}{(1-\phi)^2}+\frac{\phi}{h}(1-\alpha)\left(\frac{\phi h}{1-\phi}\right)^{\alpha}\\
&\quad\left.-\frac{\phi(1-\phi)}{h}\alpha(1-\alpha)\left(\frac{\phi h}{1-\phi}\right)^{\alpha-1}h\frac{1}{(1-\phi)^2}\right\}\\
&= \frac{f(\theta)}{\lambda+f(\theta)}\left\{-\alpha(\alpha-1)\left(\frac{\phi h}{1-\phi}\right)^{\alpha-1}\frac{\phi}{1-\phi}+\frac{\phi}{h}(1-\alpha)\left(\frac{\phi h}{1-\phi}\right)^{\alpha}\right.\\
&\quad\left.-\alpha(1-\alpha)\left(\frac{\phi h}{1-\phi}\right)^{\alpha-1}\frac{\phi}{1-\phi}\right\}\\
&= \frac{f(\theta)}{\lambda+f(\theta)}\left\{\frac{\phi}{h}(1-\alpha)\left(\frac{\phi h}{1-\phi}\right)^{\alpha}\right\}.
\end{aligned}$$

Developing the latter equation further yields

$$\begin{aligned}
h\sigma_1 &= \frac{f(\theta)}{\lambda+f(\theta)}\phi(1-\alpha)\left(\frac{\phi h}{1-\phi}\right)^{\alpha}\\
&= \underbrace{\frac{f(\theta)}{\lambda+f(\theta)}}_{e}\phi p_\varphi.
\end{aligned} \tag{B.92}$$

To the oracle planner, $h\sigma_1$ is the shadow value of increasing the harvesting capacity of the fleet, as σ_1 is the dollar value per harvest units and h represents units of harvest. The units of $h\sigma_1$ are therefore dollars in the stationary equilibrium. The shadow value of increasing the harvesting capacity of the fleet in the stationary equilibrium is set equal to value of harvest, which equals $\phi e p_\varphi$, the fleet size, ϕe, times the value of catch p_φ. Equation (B.92) says that the oracle planner puts no constraints on vessel-level harvest. The shadow value of increasing vessel-level harvest in the stationary equilibrium reflects the added catch of the entire fleet.

The system of equations without γ is

$$h\sigma_1 = \frac{f(\theta)}{\lambda + f(\theta)}\phi p_\varphi \tag{B.93}$$

$$p_\varphi - rk - z = \frac{rk}{f'(\theta)}\left[r + \lambda + f(\theta) - \theta f'(\theta)\right] \tag{B.94}$$

$$0 = g(s) - \phi h \frac{f(\theta)}{\lambda + f(\theta)} \tag{B.95}$$

$$0 = \sigma_1(\overline{h} - h). \tag{B.96}$$

The unknowns are now 1) θ, 2) h, 3) ϕ, 4) σ_1.

B.1.12.9 Establishing that $\sigma_1 > 0$

What if $\sigma_1 = 0$? If $\sigma_1 = 0$, then equation (B.93) becomes

$$\begin{aligned} 0 &= \frac{f(\theta)}{\lambda + f(\theta)}\phi(1-\alpha)\left(\frac{\phi h}{1-\phi}\right)^\alpha \\ &= \frac{f(\theta)}{\lambda + f(\theta)}\phi p_\varphi. \end{aligned}$$

$f(\theta)/[\lambda + f(\theta)]$ is employment in the stationary equilibrium, which is positive. The share of fishers in the stationary equilibrium is $\phi \in (0,1)$. The above condition therefore requires $p_\varphi = 0$. Equation (B.94) then evaluates to

$$-rk - z = \frac{rk}{f'(\theta)}\left[r + \lambda + f(\theta) - \theta f'(\theta)\right].$$

The left-hand side of this equation is strictly negative while the right-hand side is non-negative. This can't happen. It must be the case that $\sigma_1 > 0$. Equation (B.96) then requires

$$h = \overline{h}. \tag{B.97}$$

Equation (B.97) reduces the list of equilibrium objects by one. The corresponding system to (B.93)–(B.96) with (B.97) imposed is

$$\bar{h}\sigma_1 = \frac{f(\theta)}{\lambda + f(\theta)}\phi(1-\alpha)\left(\frac{\phi\bar{h}}{1-\phi}\right)^{\alpha} \tag{B.98}$$

$$(1-\alpha)\left(\frac{\phi\bar{h}}{1-\phi}\right)^{\alpha} - rk - z = \frac{rk}{f'(\theta)}\left[r + \lambda + f(\theta) - \theta f'(\theta)\right] \tag{B.99}$$

$$0 = g(s) - \phi\bar{h}\frac{f(\theta)}{\lambda + f(\theta)}. \tag{B.100}$$

The unknowns are now 1) θ, 2) ϕ, 3) σ_1.

B.1.12.10 The unique level of tightness in the stationary equilibrium

Use equation (B.100) to solve for ϕ:

$$\phi = \frac{g(s)}{\bar{h}}\frac{\lambda + f(\theta)}{f(\theta)}. \tag{B.101}$$

The oracle planner's choice of labor share in the fishery is interior. Equation (B.101) therefore requires

$$0 < \frac{g(s)}{\bar{h}}\frac{\lambda + f(\theta)}{f(\theta)} < 1.$$

Rearranging yields

$$\begin{aligned} 0 &< \frac{g(s)}{\bar{h}} < \frac{f(\theta)}{\lambda + f(\theta)} \leq 1 \\ \therefore 0 &< g(s) < \frac{f(\theta)}{\lambda + f(\theta)}\bar{h} \leq \bar{h}, \end{aligned} \tag{B.102}$$

where the inequality uses the fact that the stationary employment rate, $f(\theta)/[\lambda + f(\theta)]$, is constrained by 1. Equation (B.102) states that the oracle planner chooses vacancies so that all workers operating at full capacity are capable of harvesting enough fish so that the stock is maintained at $s^{\star\star}$.

Use the expression for ϕ in (B.101) in (B.99) with p_φ given in (B.45):

$$(1-\alpha)\overline{h}^{\alpha}\left(\frac{\phi}{1-\phi}\right)^{\alpha} - rk - z = \frac{rk}{f'(\theta)}\left[r+\lambda+f(\theta)-\theta f'(\theta)\right]$$
$$\therefore (1-\alpha)\overline{h}^{\alpha}\left(\frac{\frac{g(s)}{\overline{h}}\frac{\lambda+f(\theta)}{f(\theta)}}{1-\frac{g(s)}{\overline{h}}\frac{\lambda+f(\theta)}{f(\theta)}}\right)^{\alpha} - rk - z = \frac{rk}{f'(\theta)}\left[r+\lambda+f(\theta)-\theta f'(\theta)\right].$$

Which can be simplified to

$$(1-\alpha)\overline{h}^{\alpha}\left(\frac{g(s)/\overline{h}}{\frac{f(\theta)}{\lambda+f(\theta)}-\frac{g(s)}{\overline{h}}}\right)^{\alpha} - rk - z = \frac{rk}{f'(\theta)}\left[r+\lambda+f(\theta)-\theta f'(\theta)\right]. \tag{B.103}$$

Equation (B.103) is in terms of the oracle planner's choice θ alone. The oracle planner's choice of θ is implicitly defined by $\mathcal{T}(\theta^{\star\star}) = 0$, where

$$\begin{aligned}\mathcal{T}(\theta) &= (1-\alpha)\overline{h}^{\alpha}\left(\frac{g(s)/\overline{h}}{\frac{f(\theta)}{\lambda+f(\theta)}-\frac{g(s)}{\overline{h}}}\right)^{\alpha} - rk - z \\ &\quad - \frac{rk}{f'(\theta)}\left[r+\lambda+f(\theta)-\theta f'(\theta)\right] \\ &= (1-\alpha)\overline{h}^{\alpha}\left(\frac{g(s)/\overline{h}}{\frac{f(\theta)}{\lambda+f(\theta)}-\frac{g(s)}{\overline{h}}}\right)^{\alpha} - rk - z - \Psi(\theta),\end{aligned}$$

using equation (B.103) and the definition of Ψ in (B.70).

Since the first term and Ψ are both continuous in θ, $\mathcal{T}$ is continuous in θ. As θ approaches 0, stationary employment approaches 0. Before employment approaches 0, however, the oracle planner's choice of θ is constrained by (B.102). At the constraint, the first term evaluates to ∞ and Ψ evaluates to a finite number using proposition 11. Therefore $\mathcal{T}$ evaluates to ∞. At $\theta = \infty$ stationary employment is 1. The first term evaluates to a finite number. Proposition 11 establishes that $\Psi(\infty) = \infty$. Therefore $\mathcal{T}$ evalutes to $-\infty$ a $\theta = \infty$.

Lastly, $\mathcal{T}$ is decreasing in θ. To see this note that ϕ defined in (B.101) is decreasing in θ:

$$\begin{aligned}\frac{\partial\phi}{\partial\theta} &= \frac{g(s)}{\overline{h}}\frac{f'(\theta)f(\theta)-f'(\theta)\left[\lambda+f(\theta)\right]}{\left[f(\theta)\right]^2} \\ &= \frac{g(s)}{\overline{h}}\frac{-f'(\theta)\lambda}{\left[f(\theta)\right]^2} \\ &< 0.\end{aligned}$$

The derivative of p_φ with respect to θ is

$$
\begin{aligned}
\frac{\partial p_\varphi}{\partial \theta} &= \alpha(1-\alpha)\left(\frac{\phi(\theta)\bar{h}}{1-\phi(\theta)}\right)^{\alpha-1}\bar{h}\frac{(1)[1-\phi(\theta)]-(-1)\phi(\theta)}{[1-\phi(\theta)]^2}\frac{\partial\phi(\theta)}{\partial\theta} \\
&= \alpha(1-\alpha)\left(\frac{\phi(\theta)\bar{h}}{1-\phi(\theta)}\right)^{\alpha-1}\bar{h}\frac{1}{[1-\phi(\theta)]^2}\frac{\partial\phi(\theta)}{\partial\theta} \\
&< 0,
\end{aligned}
$$

using the fact that $\partial\phi(\theta)/\partial\theta < 0$. Moreover, proposition 12 establishes that Ψ is increasing in θ, making $-\Psi$ decreasing in θ.

Together, these properties of $\mathcal{T}$ establish the existence and uniqueness of a $\theta^{\star\star}$ that solves $\mathcal{T}(\theta^{\star\star}) = 0$.

I have shown that equations (B.99) and (B.100) lead to a unique choice of $\theta^{\star\star}$. Equation (B.98) provides an expression for σ_1. At this point the stationary equilibrium of the oracle planner's problem is completely characterized.

B.1.12.11 Useful properties and derivations

Proposition 11 *Paremeterizing the matching technology as $m(u,v) \equiv \omega\theta^\xi$, it is true that*

$$\lim_{\theta\to 0}\Psi(\theta) = \lim_{\theta\to 0}\frac{rk}{f'(\theta)}\left[r+\lambda+f(\theta)-\theta f'(\theta)\right] = 0 \tag{B.104}$$

and

$$\lim_{\theta\to \infty}\Psi(\theta) = \lim_{\theta\to \infty}\frac{rk}{f'(\theta)}\left[r+\lambda+f(\theta)-\theta f'(\theta)\right] = \infty. \tag{B.105}$$

Proof 1 *Under the parameterization of the matching technology the job-finding rate is as given in* (B.108)*:*

$$f(\theta) = \omega\theta^\xi \textit{ with } \xi \in (0,1).$$

First note that

$$
\begin{aligned}
\lim_{\theta\to 0}\frac{rk(r+\lambda)}{f'(\theta)} &= \lim_{\theta\to 0}\frac{rk(r+\lambda)}{\xi\omega\,(\theta)^{\xi-1}} \\
&= \lim_{\theta\to 0}\frac{rk(r+\lambda)}{\xi\omega}\theta^{1-\xi} \\
&= 0.
\end{aligned}
\tag{B.106}
$$

Furthermore,

$$\frac{f(\theta) - \theta f'(\theta)}{f'(\theta)} = \frac{\omega\theta^{\xi} - \theta\xi\omega\theta^{\xi-1}}{\xi\omega\theta^{\xi-1}} = \frac{(1-\xi)\omega\theta^{\xi}}{\xi\omega\theta^{\xi-1}}$$
$$= \frac{1-\xi}{\xi}\theta.$$

Using this bit of algebra, it is true that

$$\lim_{\theta\to 0} \frac{rk\left[f(\theta) - \theta f'(\theta)\right]}{f'(\theta)} = 0. \tag{B.107}$$

The two limit results in (B.106) *and* (B.107) *together establish* (B.104) *by showing* $\lim_{\theta\to 0}\Psi(\theta) = 0$*. Moreover, through* (B.106) *and* (B.107) *it is easy to see that* $\lim_{\theta\to\infty}\Psi(\theta) = \infty$.

Proposition 12 *When the matching technology is parameterized as Cobb–Douglas, making* $f(\theta) = \omega\theta^{\xi}$ *with* $\xi \in (0,1)$*, the function* $\Psi(\theta)$ *defined in* (B.70) *is increasing in* θ*.*

Proof 2 *Using the parameterization,*

$$\Psi(\theta) := \frac{rk(1+\lambda)}{\omega\xi}\theta^{1-\xi} + rk\frac{1-\xi}{\xi}\theta.$$

The expression on the right-hand side uses the algebra in the proof of proposition 11. Then

$$\frac{\partial\Psi}{\partial\theta} = \frac{(rk)(1+\lambda)(1-\xi)}{\omega\xi}\theta^{-\xi} + rk\frac{1-\xi}{\xi} > 0,$$

where the inequality comes from the fact that $\xi \in (0,1)$.

Fact 1 *In equilibrium,* $-\phi^2 h p_{\chi\phi} - \phi(1-\phi)p_{\varphi\phi} = 0$.

Proof 3 *Indeed:*

$$\begin{aligned}
-\phi^2 h p_{\chi\phi} - \phi(1-\phi)p_{\varphi\phi} &= \phi^2 h\alpha(1-\alpha)\left(\frac{\phi h}{1-\phi}\right)^{\alpha}\frac{1}{h\phi^2} \\
&\quad - \phi(1-\phi)\alpha(1-\alpha)\left(\frac{\phi h}{1-\phi}\right)^{\alpha-1}\frac{h}{(1-\phi)^2} \\
&= \alpha(1-\alpha)\left(\frac{\phi h}{1-\phi}\right)^{\alpha} - \alpha(1-\alpha)\left(\frac{\phi h}{1-\phi}\right)^{\alpha-1}\frac{\phi h}{1-\phi} \\
&= 0.
\end{aligned}$$

Fact 2 *The value of productivity (the value of ouput divided by the number of workers) is*

$$\phi p_\chi h + (1-\phi)p_\varphi = (h\phi)^\alpha(1-\phi)^{1-\alpha}. \tag{B.108}$$

Proof 4 *Use expressions for prices in* (B.45)*:*

$$\begin{aligned}
\phi p_\chi h + (1-\phi)p_\varphi &= \phi\alpha\left(\frac{\phi h}{1-\phi}\right)^{\alpha-1} h + (1-\phi)(1-\alpha)\left(\frac{\phi h}{1-\phi}\right)^{\alpha} \\
&= \phi\alpha\phi^{\alpha-1}h^{\alpha-1}h(1-\phi)^{1-\alpha} + (1-\phi)(1-\alpha)\phi^\alpha h^\alpha(1-\phi)^{-\alpha} \\
&= \alpha\phi^\alpha h^\alpha(1-\phi)^{1-\alpha} + (1-\alpha)\phi^\alpha h^\alpha(1-\phi)^{1-\alpha} \\
&= (\phi h)^\alpha(1-\alpha),
\end{aligned}$$

establishing (B.108).

Fact 3 *As long as f is concave with $f(0)=0$, then $f(\theta)-\theta f'(\theta)\geq 0$.*

Proof 5 *$f : [0,\infty) \longrightarrow [0,\infty)$. Take any $a, b \in [0,\infty)$. Because f is concave it is bounded above by its first-order Taylor approximation (Varian, 1992):*

$$f(b) \leq f(a) + f'(a)\,(b-a)\,.$$

Let 0 play the role of b and θ play the role of a. Because $f(0)=0$, we have

$$0 \leq f(\theta) - \theta f'(\theta),$$

establishing what we want to show.

The following derivatives will be used in the analysis below. The following expressions are true:

$$\begin{aligned}
p_{\chi\phi} &= \alpha(\alpha-1)\left(\frac{\phi h}{1-\phi}\right)^{\alpha-2} h\frac{(1)(1-\phi)-(-1)\phi}{(1-\phi)^2} \\
&= -\alpha(1-\alpha)\left(\frac{\phi h}{1-\phi}\right)^{\alpha-2}\frac{h}{(1-\phi)^2} \\
&= -\alpha(1-\alpha)\left(\frac{\phi h}{1-\phi}\right)^{\alpha}\left(\frac{1-\phi}{\phi h}\right)^{2}\frac{h}{(1-\phi)^2} \\
&= -\alpha(1-\alpha)\left(\frac{\phi h}{1-\phi}\right)^{\alpha}\frac{1}{h\phi^2} \\
&= -(1-\alpha)p_\chi\frac{1}{\phi(1-\phi)}.
\end{aligned} \tag{B.109}$$

$$
\begin{aligned}
p_{\varphi\phi} &= \alpha(1-\alpha)\left(\frac{\phi h}{1-\phi}\right)^{\alpha-1} h\frac{(1)(1-\phi)-(-1)(\phi)}{(1-\phi)^2} \\
&= \alpha(1-\alpha)\left(\frac{\phi h}{1-\phi}\right)^{\alpha-1} \frac{h}{(1-\phi)^2} \\
&= \alpha p_{\varphi}\frac{1-\phi}{\phi h}h\frac{1}{(1-\phi)^2} \\
&= \alpha p_{\varphi}\frac{1}{\phi(1-\phi)}.
\end{aligned}
\tag{B.110}
$$

$$
\begin{aligned}
p_{\varphi h} &= \alpha(1-\alpha)\left(\frac{\phi h}{1-\phi}\right)^{\alpha-1} \frac{\phi}{1-\phi} \\
&= \alpha p_{\varphi}\frac{1}{h}.
\end{aligned}
\tag{B.111}
$$

$$
\begin{aligned}
p_{\chi h} &= -\alpha(1-\alpha)\left(\frac{\phi h}{1-\phi}\right)^{\alpha-2} \frac{\phi}{1-\phi} \\
&= -\alpha p_{\chi}\frac{1-\phi}{\phi h}\frac{\phi}{1-\phi} \\
&= -\alpha p_{\chi}\frac{1}{h}.
\end{aligned}
\tag{B.112}
$$

APPENDIX C

Appendix to Tracking Earnings Ability over the Business Cycle: Implications for Wage Statistics

C.1 Aggregate Implications of Ability Composition

C.1.1 Introduction

The simple economic environment provides a precise description of worker flows. Worker flows determine ability composition. This section shows how composition affects labor-market aggregates through equilibrium tightness.

The economics environment is described in sectrion 3.2.2. Section 3.2.2 establishes the following results: 1) There is a unique equilibrium allocation. See proposition 7. 2) The distribution of ability among the employed does not affect the unemployment rate. While a result of the one-firm-one-worker production technology in the conventional Mortensen–Pissarides environment, this result motivates looking at the unemployment pool in addition to the employment pool over the business cycle. 3) A worker's reservation wage depends on the flow benefit of unemployment. The flow benefit of unemployment is modeled as weakly increasing in ability and therefore reservation wages are weakly increasing in ability. See proposition 6.

In this section, I establish the following results: 1) Improving the ability composition of the unemployment pool lowers the unemployment rate. When firms expect the likelihood of meeting a productive worker to increase, the value of posting a vacancy increases and firms expand recruiting effort to exhaust these opportunities. Specifically, a first-order stochastic dominance shift in the ability distribution of unemployed workers lowers the unemployment rate. 2) A mean-preserving spread of the ability distribution of unemployed workers lowers the unemployment rate. Because there is a reservation productivity level in the economy below which workers and firms agree to separate, a mean-preserving spread increases the likelihood that a firm posting a vacancy matches with a high-ability worker, but does not increase the likelihood of meeting a low-ability worker who would be below the reservation productivity level. 3) The responsiveness of the unemployment rate to aggregate productivity depends on the ability distribution of unemployed workers, a form of state dependence.

These intuitive results are summarized in propositions 13–15. The propositions are stated and proved in the next section.

C.1.2 Propositions and Proofs

Section 3.2.2.3 established uniqueness of the equilibrium. The equilibrium also characterized the ability composition of employment. This section investigates how the ability composition of the unemployment pool affects labor-market aggregates through equilibrium tightness.

Tightness, or the number of posted vacancies given u_0, determines employment and unemployment in the economy. To see how ability composition affects tightness, consider two distributions for ability in the unemployment pool, H_1 and H_2, where $H_2(a) \le H_1(a)$ for all $a \in [\underline{a}, \overline{a}]$; that is, H_2 dominates H_1 in the first-order stochastic sense.

Then

$$\theta_2^\star - \theta_1^\star = \frac{(1-s_x)(1-\eta)}{c} \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \left\{ \int_{\chi(1/(z\varepsilon'))}^{\overline{a}} ([za'\varepsilon' - b(a)]\,[h_2(a) - h_1(a)]\;da')\,g(\varepsilon')\,d\varepsilon' \right\}. \tag{C.1}$$

Integrating the integral in curly brackets by parts yields

$$\begin{aligned}
&[za'\varepsilon' - b(a)]\,[H_2(a') - H_1(a')]\Big|_{a'=\chi(1/(\varepsilon' z))}^{a'=\overline{a}} - \int_{\chi(1/(z\varepsilon'))}^{\overline{a}} [H_2(a') - H_1(a')]\,z\varepsilon'\,da' \\
&\qquad = -\int_{\chi(1/(z\varepsilon'))}^{\overline{a}} [H_2(a') - H_1(a')]\,z\varepsilon'\,da',
\end{aligned}$$

where the equality uses the fact that $z\chi(1/(z\varepsilon'))\varepsilon' - b(a) = 0$ and $H_1(\overline{a}) = H_2(\overline{a}) = 1$. Using this result in (C.1) along with the fact that $H_2(a) \le H_1(a)$ for all $a \in [\underline{a}, \overline{a}]$ establishes $\theta_2^\star \ge \theta_1^\star$. Improving the composition of the unemployment pool in terms of ability leads to a tighter labor market and a lower unemployment rate. When the composition of the unemployment pool improves, a posted vacancy has a better chance of matching with a higher-ability worker, which makes posting a vacancy more profitable. Firms expand recruiting effort until these gains are exhausted, which lowers the unemployment rate. The result is summarized in proposition 13.

Proposition 13 *Suppose $b'(a)a/b(a) < 1$. Consider two distribution functions, H_1 and H_2, that describe the composition of ability in the unemployment pool. If H_2 dominates H_1 in the first-order stochastic sense, then $\theta_2^\star \geq \theta_1^\star$.*

A mean-preserving spread in the composition of the unemployment pool causes equilibrium labor-market tightness to increase. Consider two parameterizations of the ability distribution of unemployed agents, H_1 and H_1, where H_2 and H_1 have the same mean and H_2 dominates H_1 in the second-order stochastic sense. Continuing to develop the expression behind proposition 13 yields

$$\begin{aligned}\theta_2^\star - \theta_1^\star &= -\frac{(1-s_x)(1-\eta)}{c}\int_{\underline{\varepsilon}}^{\overline{\varepsilon}} z\varepsilon' \left\{\int_{\chi(1/(z\varepsilon'))}^{\overline{a}} [H_2(a') - H_1(a')]\,da'\right\} g(\varepsilon')d\varepsilon' \\ &= -\frac{(1-s_x)(1-\eta)}{c}\int_{\underline{\varepsilon}}^{\overline{\varepsilon}} z\varepsilon' \left\{\int_{\underline{a}}^{\overline{a}} [H_2(a') - H_1(a')]\,da' \right. \\ &\qquad \left. - \int_{\underline{a}}^{\chi(1/(z\varepsilon'))} [H_2(a') - H_1(a')]\,da'\right\} g(\varepsilon')d\varepsilon' \\ &= \frac{(1-s_x)(1-\eta)}{c}\int_{\underline{\varepsilon}}^{\overline{\varepsilon}} z\varepsilon' \left\{\int_{\underline{a}}^{\chi(1/(z\varepsilon'))} [H_2(a') - H_1(a')]\,da'\right\} g(\varepsilon')d\varepsilon',\end{aligned}$$

where the last equality uses the fact that the integral evaluated over the entire domain is zero. The result that $\theta_2^\star - \theta_1^\star \geq 0$ follows from the definition of mean-preserving spread; namely, $\int_{\underline{a}}^{y} [H_2(x) - H_1(x)]\ dx \geq 0$ for all $y \in [\underline{a}, \overline{a}]$.

An increase in "risk," in other words, increases a firm's incentive to post a vacancy. In the words of Diamond and Stiglitz (1974, 338), "H_2 is derived from H_1 by taking weight from the center of the probability distribution and shifting it to the tails, while keeping the mean of the distribution constant." The presence of the reservation productivity level means a firm considering posting a vacancy gets to ignore shifts in weight below the reservation productivity level while increasing the likelihood that the firm matches with a high-ability worker due to the shift in weight to the upper tail. The result is expanded recruiting effort, which is summarized in proposition 14.

Proposition 14 *Suppose $b'(a)a/b(a) < 1$. Consider two distribution functions, H_1 and H_2, that describe the composition of ability in the unemployment pool. If H_2 is a mean-preserving spread of H_1, then $\theta_2^\star \geq \theta_1^\star$.*

The composition of the unemployment pool matters for how the labor market responds to aggregate productivity, a form of state dependence. Consider again two parameterizations of the unemployment pool, H_1 and H_2, where H_2 dominates H_1 in the first order stochastic sense. Then

$$\frac{\partial}{\partial z}\left[\theta_2^\star\right] - \frac{\partial}{\partial z}\left[\theta_1^\star\right] = \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \left\{ \int_{\chi(1/(z\varepsilon'))}^{\bar{a}} a'\left[h_2(a') - h_1(a')\right]\, da' \right\} \varepsilon' g\left(\varepsilon'\right)\, d\varepsilon'.$$

Integrating the bracketed integral by parts yields the expression

$$-\chi\left(1/(z\varepsilon')\right)\left[H_2\left(\chi(1/(z\varepsilon'))\right) - H_1\left(\chi(1/(z\varepsilon'))\right)\right] - \int_{\chi(1/(z\varepsilon'))}^{\bar{a}} \left[H_2(a') - H_1(a')\right]\, da'.$$

Both of the terms in brackets are negative as $H_2(x) \leq H_1(x)$ for all $x \in \left[\underline{a}, \bar{a}\right]$. Therefore $\partial\theta_2^\star/\partial z \geq \partial\theta_1^\star/\partial z$.

Proposition 15 *Suppose $b'(a)a/b(a) < 1$. Consider two parameterizations of the unemployment pool, H_1 and H_2, where H_2 dominates H_1 in the first-order stochastic sense. Then the response of labor-market tightness to aggregate productivity is greater when the unemployment pool is characterized by H_2 rather than H_1.*

C.2 Densities of Earnings Ability over the Business Cycle

This section provides a picture of how earnings ability varies over the business cycle. I match a worker's estimated earnings ability to their labor-market history. The result is a dataset of repeated cross sections of earnings ability for employed and unemployed workers.

Figure C.1 depicts estimated densities of earnings ability of employed workers by year. The estimated densities are constructed using the May employment pools in each year over the 90s. Using the May samples eliminates seasonal effects. The color of the densities corresponds to the aggregate unemployment rate that prevailed during the month. The mean earnings ability for each month is shown in the figure with a magenta square.

According to the organized-by-earnings-ability model, these densities should exhibit cyclical patterns. In periods of high unemployment the earnings-ability composition of the employment pool should improve as lower-ability workers flow to unemployment. In the context of figure C.1, the densities and their associated averages should shift to the right when the aggregate unemployment rate is high. Yellow densities should be to the right of navy densities.[1] It is difficult to discern this pattern using the data on employed workers.

The employment pool, however, represents a much larger stock than the pool of unemployed workers and it may be easier to observe the pattern among the unemployed.[2] Figure C.2 shows similarly estimated densities for the May pools of unemployed workers. According to the organized-by-ability model, in periods of high unemployment, the earnings ability composition of the unemployment pool should improve as previously employed high-ability workers flow to unemployment. In the context of figure C.2, the densities and their associated averages should shift to the right when the aggregate unemployment rate is high.

There may be some support for this countercyclical relationship, although the visual evidence is not overwhelming. Section 3.4.3 provides a more formal analysis within the broader context of wage cyclicality predicted by the statistical model.

C.3 Robustness of Wage Cyclicality

This section considers alternative samples to investigate wage cyclicality. I use the samples to estimate the first-step regression specified in (3.1). The estimates are then used to construct the wage statistics. These wage statistics are used in the second-step regressions specified in (3.6). Results are reported in tables C.1 through C.4.

The first sample I consider uses all data available from NLSY79 respondents. The sample uses data from NLSY79 survey years starting in 1979 and going through 2012. The cross-sectional average wage, $\overline{w}_{\text{avg},t}$, is constructed as the weighted average of wages observed in year t of NLSY79 data. How the average wage varies with the business cycle is reported in table C.1.

The average-wage column reports $\widehat{\beta}_{\text{avg}}$ from (3.6). The estimate predicts that when

[1] Or, at U-M, the maize densities should be to the right of the blue densities.

[2] Not observing the pattern in the employment pools still matters for interpretations of wage statistics.

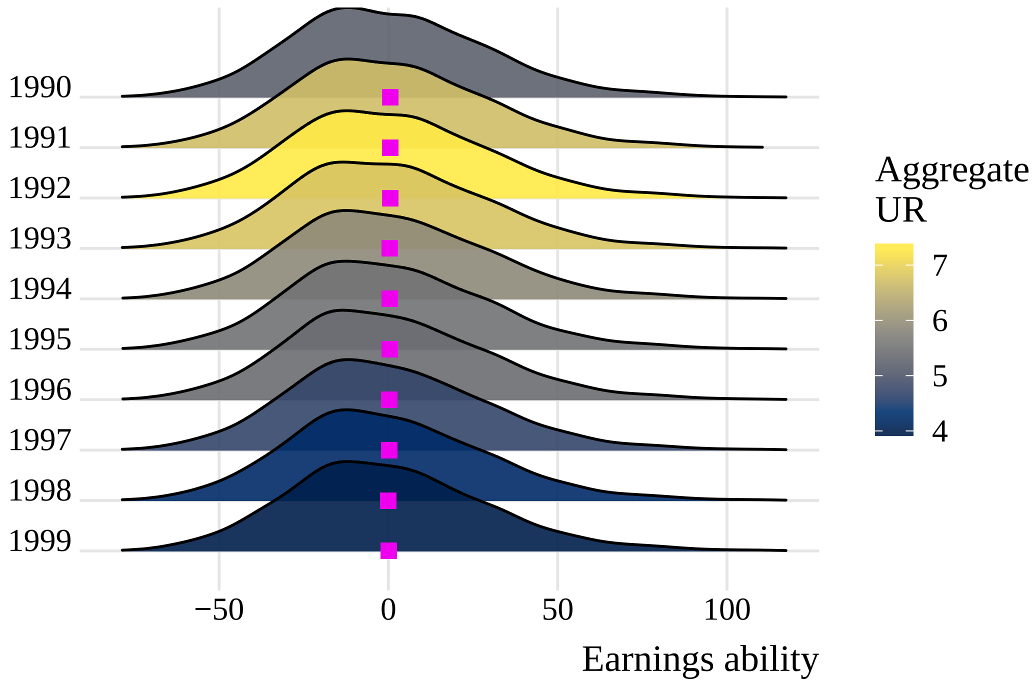


Figure C.1: Estimated earnings-ability densities of employed workers by year, 1990–1999. The color of the density corresponds to the aggregate unemployment rate.

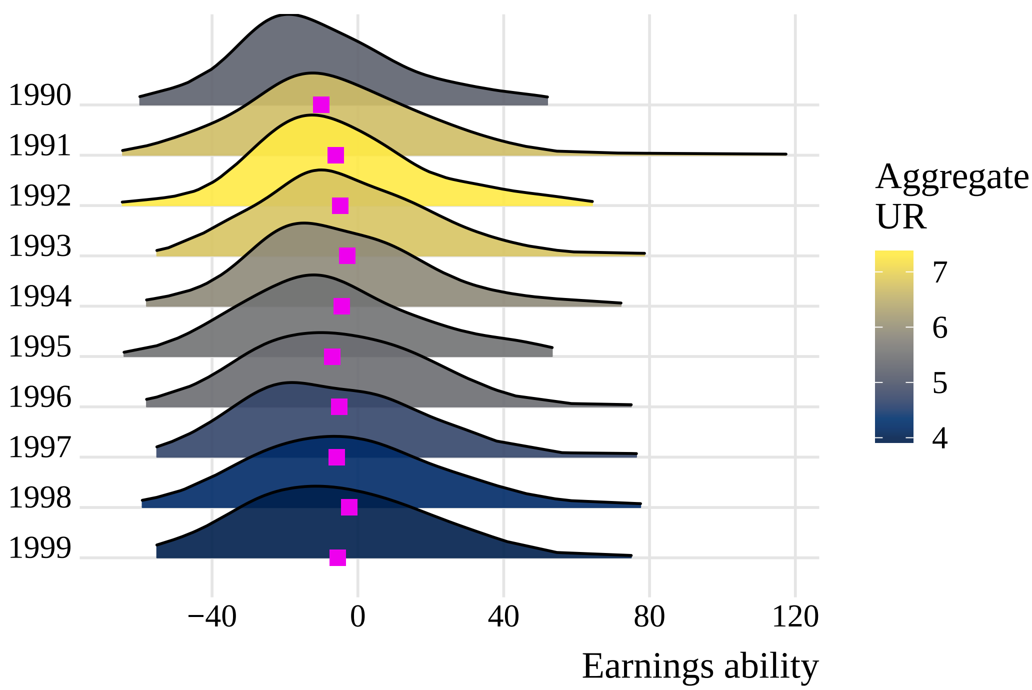


Figure C.2: Estimated earning-ability densities of unemployed workers by year, 1990–1999. The color of the density corresponds to the aggregate unemployment rate.

the cyclical unemployment rate is 1 percentage point above trend, $\overline{w}_{\text{avg}}$ procyclically falls 0.6 percent. The cyclicality column of table C.1 reports $\widehat{\beta}_{\text{cyclical}}$. When the cyclical unemployment rate is 1 percentage point above trend, $\overline{w}_{\text{cyclical},t}$ procyclically falls 1.162 percent. This estimate indicates that individual-level wages are about twice as cyclical as the average wage.

This estimate masks individual wage cyclicality because different workers are employed over the business cycle, which affects the average. The cyclicality column of table 3.1 reports $\widehat{\beta}_{\text{cyclical}}$. When the cyclical unemployment rate is 1 percentage point above trend, $\overline{w}_{\text{cyclical},t}$ procyclically falls 1.192 percent. This estimate indicates that individual-level wages are about twice as cyclical as the average wage.

The difference in cyclicality between individual-level wages and the average wage is due to the types of workers who compose employment over the business cycle. Workers differ in terms of their earnings ability and their time-varying characteristics. The time-varying characteristics column of table C.1 reports how $\overline{w}_{X,t}$ varies with the business cycle. When the cyclical unemployment rate is 1 percentage point above trend, $\overline{w}_{X,t}$ countercyclically increases by 0.401 percent.

Finally, a wage statistic that reflects only earnings ability is $\overline{w}_{\text{earnings ability},t}$. The earnings-ability column of table C.1 reports how $\overline{w}_{\text{earnings ability},t}$ varies with the business cycle. When the cyclical unemployment rate is 1 percentage point above trend, $\overline{w}_{\text{earnings ability},t}$ countercyclically increases by 0.137 percent.

The countercyclical coefficients for $\overline{w}_{X,t}$ and $\overline{w}_{\text{earnings ability},t}$ indicate that workers with characteristics that predict higher earnings are disproportionately retained in recessions and workers with characteristics that predict lower earnings are disproportionately let go in recessions. Mechanically, because the period fixed effects are constructed so that the cross-sectional average wage hits the predicted average wage, the sum of the countercyclical effects of time-varying characteristics and earnings ability, $0.401 + 0.137 = 0.538$, equals the difference in cyclicality between individual-level wages and the average wage.

The top panel of table C.2 reports how the individual components of $\overline{w}_X$ vary over the business cycle for the same period. The components of $\overline{X}_t$ include dummies for educational attainment, major industries, and union status as well as controls for cumulative work experience and tenure at a respondent's current job. The first column of the top panel of table C.2 reports the cyclicality of time-varying characteristics. This entry is copied over from table C.1.

The next column indicates that the cyclicality of time-varying characteristics is mostly accounted for educational attainment. And the rest is accounted for by cyclical variation in union status. Cyclical variation in industrial composition and experience have a small effect. Because the statistical model for wages in (3.1) is linear, the individual contributions approximately sum to the cyclicality of time-varying characteristics. [The sum is not exact because the trends in (3.6) are estimated separately.]

Cyclical variation in education, however, may be entirely driven by workers earning college degrees in the early part of the sample, which also coincides with the 1980 and 1981–1982 recessions when the economy experienced high rates of unemployment. To see if this is the case, the bottom panel of table C.1 restricts the sample to the period 1986–2012.

Over the 1986–2012 period, the average wage in the sample was much less cyclical. As indicated in the first column of the bottom panel of table C.1, when the cyclical unemployment rate is 1 percentage point above trend, $\overline{w}_{\text{avg},t}$ procyclically falls 0.181 percent. The amount of wage cyclicality, though, remains stable. When the cyclical unemployment rate is 1 percentage point above trend over 1986–2012, $\overline{w}_{\text{cyclical},t}$ procyclically falls on average by 1.176 percent. This is nearly the same magnitude exhibited over the entire sample.

Looking at the last two columns in table C.1, in the later period, $\overline{w}_{X,t}$ and $\overline{w}_{\text{earnings ability},t}$ contribute more to wage cyclicality. When the cyclical unemployment rate is 1 percentage point above trend over 1986–2012, $\overline{w}_{X,t}$ countercyclically increases by 0.684 percent and $\overline{w}_{\text{earnings ability},t}$ increases by 0.311 percent.

Importantly, however, the components of $\overline{w}_{X,t}$ exhibit different cyclical patterns in the later period. The middle panel of table C.2 reports the cyclical contribution of the individual components. In the later sample, the cyclical contribution of experience is the largest component. When the cyclical unemployment rate is 1 percentage point above trend over 1986–2012, $\overline{w}_{X,t}$ countercyclically increases by 0.444 percent due to experience alone. The increased effect of experience, in some sense, is mechanical. As the cohort ages, a larger and larger share of earnings is a function of experience. Similarly, cyclicality in educational attainment is small, suggesting that the earlier period was dominated by workers earning college degrees while younger.

The other two components, union membership and industrial composition, exhibit patterns in the later sample that are similar to patterns observed over the entire sample.

Because of the marked change in educational attainment, I estimate the statistical model for wages in (3.1) using NLSY79 survey data from 1985 onwards,[3] but do not include dummies for educational attainment. Using these estimates, I construct similar wage statistics. The estimated cyclicalities of these statistics are reported in table 3.1. For the most part the cyclicality measures are similar to those reported in the bottom panel of

[3]Which provides an estimate of period fixed effects beginning in 1986.

table C.1. Substantial wage cyclicality exists in the data, despite the average wage appearing much less cyclical. The components of time-varying characteristics are reported in the bottom panel of table C.2. The contributions of industrial composition, union status, and experience are remarkably similar to the counterparts constructed from estimates of the panel regression in (3.1) using controls for educational attainment and estimating the cyclical regressions in (3.6) over the later period.

Together, the cyclical measures in tables 3.1, C.1, and C.2 show that real wages of respondents in the NLSY79 are substantially procyclical. Additionally, the average wage in the sample is much less procyclical. The difference between the cyclical component of wages and the average wage can be attributed to countercyclical compositional effects. The compositional effects are due to earnings ability and time-varying characteristics. And the influence of time-varying characteristics is mostly due to fluctuations in experience relative to a quadratic trend over the business cycle.

Table C.1: Components of wage cyclicality, full sample with educational attainment

	1979–2012			
	Avg wage	Cyclicality	Characteristics	Earnings ability
Cyclical UR	−0.624	-1.162^{**}	0.401^{+}	0.137
	(0.492)	(0.371)	(0.237)	(0.0964)
N	25	25	25	25
	1986–2012			
	Avg wage	Cyclicality	Characteristics	Earnings ability
Cyclical UR	−0.181	-1.176^{*}	0.684^{**}	0.311^{**}
	(0.640)	(0.636)	(0.234)	(0.132)
N	18	18	18	18

Standard errors in parentheses

$^{+}$ $p < 0.15$, * $p < 0.1$, ** $p < 0.05$

All these statistics rely on respondents' wages used in the regression samples. Respondents in the NLSY79 also provide detailed work histories that indicate whether a respondent was employed or unemployed in any given month. Joining the monthly labor-force status of respondents to their earnings ability offers another measure of earnings ability over the business cycle.

In many instances, respondents report their labor-force status even though their is no wage recorded for them in the sample. Average earnings ability according to labor-force status is less cyclical. Tables C.3 and C.4 corroborate this claim. The bottom panel of table C.3 repeats the contents of table 3.3 for comparison.

Table C.2: Time-varying components of wage cyclicality

	Characteristics	Education	Industry	Union	Experience
			1979–2012, $N = 25$		
Full sample	0.401^{+}	0.331^{**}	0.00988	0.110^{+}	−0.0494
	(0.237)	(0.117)	(0.0537)	(0.0668)	(0.192)
			1986–2012, $N = 18$		
Full sample, after 1985	0.684^{**}	0.0895^{*}	−0.00666	0.157^{+}	0.444^{**}
	(0.234)	(0.0489)	(0.0364)	(0.0985)	(0.168)
			1986–2012, $N = 18$		
1985 sample	0.576^{**}		−0.000949	0.130^{+}	0.448^{**}
	(0.229)		(0.0345)	(0.0809)	(0.179)

Robust standard errors in parentheses. $^{+}$ $p < 0.15$, * $p < 0.1$, ** $p < 0.05$. The column labeled characteristics refers to the total value of time-varying controls, $\overline{X}_t'\widehat{\beta}$. The value of individual components in $\overline{X}_t'\widehat{\beta}$ are Education, Industry, Union, and Experience. Experience includes a worker's tenure at their current job and total years worked at any job.

The employed column of table C.3 indicates that, over the 1979q1–2012q4 period, when the cyclical unemployment rate is 1 percentage point above trend, average earnings ability increases 0.0438 percent among employed workers. The unemployed column indicates that when the cyclical unemployment rate is 1 percentage point above trend, average earnings ability increases 0.126 percent among unemployed workers. The second panel of table C.3 drops non-NLSY79 survey years. Dropping these years does not substantially change the average earnings ability observed in the employment and unemployment pools.

Finally, the last panel of table C.3 uses data from 1985 onwards and drops educational dummies when estimating (3.1). These results repeat the results presented in table 3.3. This sample produces a different estimate. The employment column indicates that when the cyclical unemployment rate is 1 percentage point above trend over 1986q1–2012q4, average earnings ability increases 0.113 percent among employed workers. The unemployment column indicates that when the cyclical unemployment rate is 1 percentage point above trend over 1986q1–2012q4, average earnings ability increases 0.518 percent among unemployed workers. Both of these estimates are precisely estimated.

Table C.4 includes non-NLSY79 survey years. There are 108 observations in this sample. The sample comprises 27 years, 1986–2012, and there are 4 quarters per year for a total of 108 observations (27×4). Adding these years does not change much the conclusions from the bottom pabel of table C.3.

Table C.3: Cyclicality of average earnings ability for employed and unemployed

	1979q1–2012q4	
	Employed	Unemployed
Cyclical UR	0.0435**	0.126
	(0.0212)	(0.123)
N	136	136
	Non-NLSY79	
	Employed	Unemployed
Cyclical UR	0.0305	0.142
	(0.0257)	(0.145)
N	100	100
	1986q1–2012q4	
	Employed	Unemployed
Cyclical UR	0.113**	0.518**
	(0.0167)	(0.200)
N	72	72

Robust standard errors in parentheses. $^{+}$ $p < 0.15$, * $p < 0.1$, ** $p < 0.05$. Cross-sectional sample weights are used in the estimation of earnings ability.

Table C.4: Cyclicality of average earnings ability for employed and unemployed with non-NLSY79 years

	1986q1–2012q4	
	Employed	Unemployed
Cyclical UR	0.160**	0.410**
	(0.0241)	(0.191)
N	108	108

Standard errors in parentheses

$^{+}$ $p < 0.15$, * $p < 0.1$, ** $p < 0.05$

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