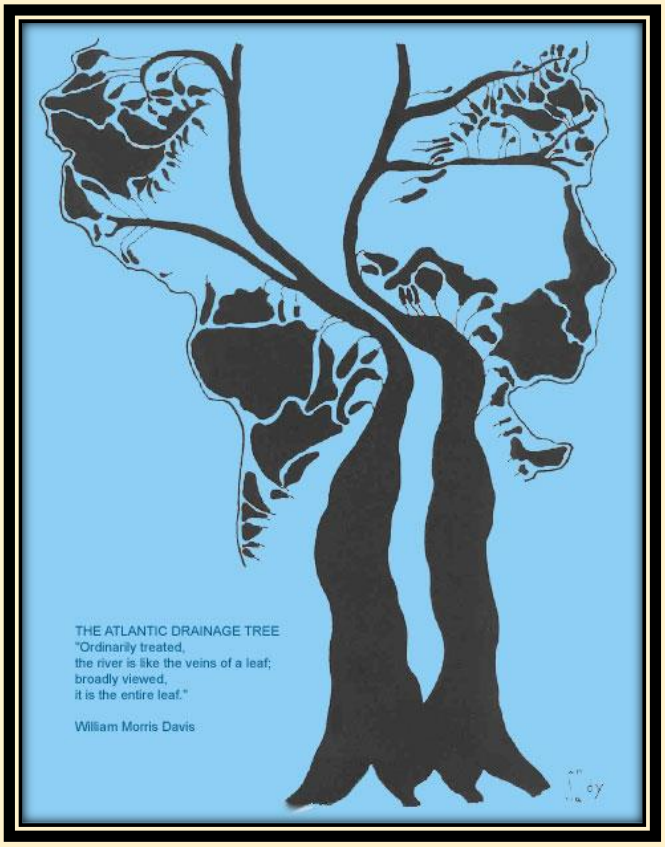
ISSN: 1059-5325

Solstice: An Electronic Journal of Geography and Mathematics

Volume XXXI, Number 2
December 21, 2020



Founded: 1990
Persistent archive: Deep Blue, University of Michigan
Mathematical Geography (Institute of);
Non-persistent user-friendly archive:
Institute of Mathematical Geography

31st year (1990-2020) of publication of SOLSTICE:
An Electronic Journal of Geography and Mathematics
Volume XXXI, Number 2; Monday, December 21, 2020; 4:02 am, Central Time

SOLSTICE:

An Electronic Journal of Geography and Mathematics

31 YEARS OF PUBLICATION!

Persistent URL: **http://deepblue.lib.umich.edu/handle/2027.42/58219**

Cite articles as: Author name(s), Year. Title of article, *Solstice: An Electronic Journal of Geography and Mathematics*, Vol. YY, No. ZZ. Ann Arbor: Institute of Mathematical Geography.

Virtual reality of The University of Michigan "Diag", below. Note the observatory dome atop Angell Hall, in the foreground.

Congratulations to the University of Chicago
as they mark the 50th Anniversary of the
Joseph Regenstein Library!

31st year (1990-2020) of publication of SOLSTICE:
An Electronic Journal of Geography and Mathematics
Volume XXXI, Number 2; Monday, December 21, 2020; 4:02 am, Central Time

In Memoriam
Jeffrey A. Nystuen, 1957-2020
Solstice Author
Obituary

31st year (1990-2020) of publication of SOLSTICE:
An Electronic Journal of Geography and Mathematics
Volume XXXI, Number 2; Monday, December 21, 2020; 4:02 am, Central Time

Table of Contents

31st year (1990-2020) of publication of SOLSTICE:
An Electronic Journal of Geography and Mathematics
Volume XXXI, Number 2; Monday, December 21, 2020; 4:02 am, Central Time

Frontmatter

Solstice was born digital in 1990, before the advent of the Internet. Early volumes were typeset using the digital typesetting language, TeX. The digital files were sent to subscribers via email and the receiver printed out the TeX code, if desired, to produce a typeset-quality journal, on-demand. Selected monographs in the IMaGe Monograph series contain typeset versions of *Solstice*, printed from the code transmitted as the original version of that issue of *Solstice*. Later, when the Internet became available, *Solstice* switched to the Internet as the platform for transmission, writing documents in html rather than TeX.

Early in *Solstice's* production history, some authors worried that their electronic files could be maliciously altered by random readers and uploaded to replace their own writings. Of course, that could not have happened (because everything was passworded). However, as reassurance to prospective authors not yet familiar with the mechanics of servers and such, early documents were edited to introduce deliberate errors in spacing, inserted by hand, that a random word-processed document would fail to automatically duplicate. Hence, a bogus copy could be detected simply by overlaying a 'new' printout on the 'old' printout on a light table. The hand-insertion of erroneous spaces motivated the oriental rug motif, photographed from a Bokhara rug from the 1964 New York City World's Fair; that symbol is carried forward (although the practice itself is not) in *Solstice* today, as a subtle reminder of one element of the journal's history.

Over the years, *Solstice* has gained media attention from a variety of sectors: from *Science* (AAAS) and *Science News* early on. A bit later with interaction with a museum, the Exploratorium (San Francisco), and the TV show, *Nova*. For all these notices, as well as for those in more conventional academic arenas, our primary thanks go to our contributors, volunteers, and readers who have been with us for so many years. Best wishes to all!

31st year (1990-2020) of publication of SOLSTICE:
An Electronic Journal of Geography and Mathematics
Volume XXXI, Number 2; Monday, December 21, 2020; 4:02 am, Central Time

Persistent URL, University of Michigan Deep Blue Archive (housed under 'Communities & Collections', 'Mathematical Geography (Institute of)': **http://deepblue.lib.umich.edu/handle/2027.42/58219**

Transient URLs that point to the persistent URL: imagenet.org; mathematicalgeography.org; mathematicalgeography.com; instituteofmathematicalgeography.org; instituteofmathematicalgeography.com

AUTHORS OF ORIGINAL MATERIALS APPEARING IN SOLSTICE, 1990-present:
Ainslie, V. | Albert, D. | Ard, K. | Arlinghaus, D. E. | Arlinghaus, S. L. | Arlinghaus, W. C. | Arlinghaus, W. E. | Austin, R. F. | Baird, N. D. | Bandyopadhyay, G. | Barmore, F. E. | Barr, D. | Batty, M. | Beal, F. J. | Beier, K.-P. | Bhatia, S. S. | Blake, B. | Borgers, A. W. J. | Burkhalter, B. R. | Chattopadhyay, S. | Chaudhuri, S. | Childers, P. E. | Chirapiwat, T. | Crane, J. | Crawford, T. C. | De la Sierra, R. | Derudder, B. | Dison, A. C. | Drake, W. D. | Eagle, K. A. | Earl, E. | Favro, R. | Frank, A. I. | Gober, C. | Goodman, F. L. | Gorstein, J. | Griffith, D. A. | Haidar, S. | Hall, B. | Hall, D.| Hamilton, R. C. | Han, S.-H. | Harary, F. | Haug, R. | Haynes, D. | Hoffman, L. L. | Hogh, G. | Iyer, S. D. | Jacobs, D. A. | Kaur, K. | Kelbaugh, D. S. | Kerski, J. | Kim, E-Y. | Koenig, R. | Kos, S. | Kwon, T. | Lach, A. | Larimore, A. E. | Laug, A. | Lazzaro, A. A. | Lee, H. | Licate, J. A. | Lindemann, W. | Maeda, J. | Martin, P. | McCloskey, J. C. | Metzler, D. | Mokray, G. | Muhilal | Naud, M. | Noguchi, M. | Nystuen, J. A. | Nystuen, J. D. | Onstott, B. | Oppenheim, P. J. | Oswalt, K. S. | Pak, S. | Phillips, L. R. | Pogany, T. | Ponce de Leon, J. S. | Pothukuchi, K. | Purvis, E. M. | Ram, B. | Rayle, R. | Robinson, Craig | Rosenblum, A. | Ross, S. | Rushing, D. | Ryznar, R. | Sammataro, D. | Schlossberg, M. | Schumann, L. | Stern, H. L. | Stuckman, P. | Tarwotjo, I. | Tilden, R. | Timmermans, H. J. P. | Tobler, W. R. | Van Acker, V. | Wagner, T. | Wallace, R. | Walton, A. | Ward, M. | Wilson, M. L. | Witlox, F. J. A. | Woldenberg, M. J. | Zander, R. H.

31st year (1990-2020) of publication of SOLSTICE:
An Electronic Journal of Geography and Mathematics
Volume XXXI, Number 2; Monday, December 21, 2020; 4:02 am, Central Time

Update on Activities…

The ongoing pandemic has changed life for everyone. Fortunately, it has been a good season for organizing materials and making them available to others, at least online. Thus, we are pleased to announce the publication of two new monographs in the IMaGe monograph series. The last persistent monograph in the series, Monograph #23 was published in 1998. There was also a new Monograph #24 published shortly after that, but a conventional publisher agreed to publish it as their first eBook and asked that we not continue with Monograph #24 (fortunately, the process of publication had not proceeded to the final stage of 'persistence'). That work remains available from Wiley.

- **IMaGe Monograph #24.** Fast-forward a few years. In 2015, William C. Arlinghaus created a new Monograph #24 based on workshops designed to aid students studying Calculus 2. Somehow, over the course of the next few years, that manuscript draft got misplaced. Now, it has been found, and edited, and it is available as persistent IMaGe Monograph #24, *Calculus 2 Workshops*!
- **IMaGe Monograph #25.** More recently, a small group has been interacting about precollegiate education--its merits and drawbacks. The thoughts of some of us are presented in the brand new IMaGe Monograph #25, *Pre-Collegiate Education in Utopia*. Look for reports of elements of its implementation in forthcoming issues of *Solstice*.
- **Meridian History Garden Pilot Project—A Garden of Unintended Consequences.**

  An auxiliary project, in support of the broader kudzu project (written about in recent issues of Solstice) is one that I (S. Arlinghaus) imagine is simple to create and to implement without involving many people in public or crowded meetings, at least in a 'pilot' stage. It involves the idea of 'archiving' plants...of preserving specimen plants of kudzu in a sort of 'history garden'. That way prospective developers, and others, can see what they look like as time goes on; Kudzu varieties might be planted in containers to contain its spread. Also, other plants that had one originally intended use, but later had other issues arise around them, might also be planted in such a container garden. We might consider kudzu as originally for erosion control; but then it got out of hand and became a problem in various ways, including with carbon footprint problems. Cotton is

31st year (1990-2020) of publication of SOLSTICE:
An Electronic Journal of Geography and Mathematics
Volume XXXI, Number 2; Monday, December 21, 2020; 4:02 am, Central Time

surely a useful plant for its fiber but perhaps not for the consequences of the group of people who picked the cotton. Tobacco might also find a place in a History Garden as a useful cash crop with hazardous health consequences not known at the time. Bahia grass, which apparently grows quickly but is a real problem in lateral coverage (not as spectacular as kudzu) to cemetery owners. No doubt the list goes on, but there should be plenty of types of plant material to make a 'History Garden' that is also a ‘Garden of Unintended Consequences’.

A full-blown History Garden might be planted, in containers, on the lawn of a prominent building (public or private). However, in advance of such consideration, it seems prudent to plant a trial, pilot, garden, not only to test capability to manage such plants, but also to gauge public reaction to such a garden. A successful pilot project might help to convince City officials that such a garden has merit.

Currently, we are developing such a pilot garden in the Courtyard of the Brickhaus Brewtique in downtown Meridian (I am co-owner of that parcel along with William C. Arlinghaus and William E. Arlinghaus). The players in the program are, in addition to the three parcel owners, Chris Vallot (Bartender at the Brickhaus Brewtique) and Pam Stuckman (landscape architect). To date, one planter has a very healthy tobacco plant growing in it that Chris started on his own and has carefully tended as his pet plant that he raised from a seed. Pam has offered advice in moving forward come spring. I have made a QR code for the tobacco plant’s container, so folks in the Courtyard who are curious about the plant can scan the QR code and link to online material about tobacco plants. There will be more coming in the spring, as a mild (from a Northern viewpoint) winter sets in now. In the meantime, we gather information and input. In the those regards, I would like to thank my colleagues, Bob Grese and Rosina Bierbaum, both of the School for Environment and Sustainability, at The University of Michigan in Ann Arbor. They have been a great help! Stay tuned…

31st year (1990-2020) of publication of SOLSTICE:
An Electronic Journal of Geography and Mathematics
Volume XXXI, Number 2; Monday, December 21, 2020; 4:02 am, Central Time

- **Give a Pint, Get a Pint: Blood for Beer**

One way to help shore up valued resources during a national medical emergency (such as during the pandemic of 2020), or at other times, is to donate blood. However, not everyone is eligible to do so even though they might wish to. Here's what one set of folks (Arlinghaus family) in Meridian MS is doing to contribute in a different way.

Originally, we thought that we might offer a pint of free craft beer to folks who had just recently contributed a pint of blood. We were, however, reminded that such activity could be viewed as an 'incentive' to donate and that any incentive offered should be offered to all, lest some wishing to capture the free incentive distort their eligibility to donate blood. Having less than fine blood introduced into the system was certainly not the intention!

Thus, it was time to craft a different plan. Past donors are issued donor cards. Instead, we will create a 'donor appreciation night' at the Brickhaus Brewtique. We will encourage, using Facebook and other social media, local blood recipients as well as local card-carrying donors, to come to this special night. Appreciative blood recipients donate cash to the Brickhaus in support of supplying free pints to past donors…to as many who come until the cash runs out. This form of 'crowdsourcing' might provide pints for every donor; or for a limited number, drawn by chance. The program promises to offer winning opportunities all around—the blood recipients will give back different pints to those who might previously have given them pints…a big-hearted springtime event!

31st year (1990-2020) of publication of SOLSTICE:
An Electronic Journal of Geography and Mathematics
Volume XXXI, Number 2; Monday, December 21, 2020; 4:02 am, Central Time

Planning Units: Kudzu Opportunity Zones?

Sandra L. Arlinghaus, William E. Arlinghaus, and Weston Lindemann

General Background

Work has been ongoing for over a year to develop a project that we believe will assist in supporting the infant Meridian Renaissance. There are big projects, development-oriented, that are keystones in this process: the recent completion of the state-funded museum in downtown Meridian, The Mississippi Arts and Entertainment Experience (The MAX), the forthcoming completion of a Children's Museum (also downtown), the forthcoming completion of a renovated national historic landmark, The Threefoot Building, as a Courtyard by Marriott. The latter project has taken some time to move forward; the building had fallen into a sad state of disrepair; it is an Art Deco building, the only tall building in Meridian (17 stories) reminiscent of some of the fine vintage buildings, such as the Fisher Building, in downtown/New Center area of Detroit. It is with considerable excitement that we look forward to a revival of Meridian as the once proud city it was. From 1890-1930, Meridian was the most populous city in Mississippi; it was home to railroad executives as it served as a rail hub for the transmission of tall straight timber harvested from the majestic stands of trees in the surrounding southern pine belt…timber to serve in constructing early New York City skyscrapers. Rail executives from around the country built large homes in Tudor and other styles from elsewhere (in addition to beautiful, classic Southern architecture). The City prospered. Decades of disrepair have dampened the buildings, but not the spirit. Meridian is still home to arts and music. I recall a particularly nice event when the Moscow Ballet came to the USA and came to Meridian (as one stop) and included on stage local Mississippi children dancing with the graceful Russian ballerinas. What a thrill for all. I also recall going to the Jim Henson exhibition held at the MAX, as one of four stops (others in Seattle, Chicago, and New York I think) in the US for this memorial to a Mississippi native. These are a few points of light; there are others, to be sure, but they remain discrete points.

We seek a continuous experience to throw into the mix; one factor that can provide continuity is the environment in which we are all embedded. The project we have been working to develop was

31st year (1990-2020) of publication of SOLSTICE:
An Electronic Journal of Geography and Mathematics
Volume XXXI, Number 2; Monday, December 21, 2020; 4:02 am, Central Time

discussed in the last issue of *Solstice*. One arm of our project has a working title of *"Benefit Swaps—Developmental for Environmental Resource Protection."*

Idea Background: Kudzu and Carbon

Here is the idea, in brief: An article in USA Today, entitled 'Toxic 'forever chemicals' found in drinking water throughout US' notes the following:

> ***The report, published by the Environmental Working Group, found that 20 cities and regions nationwide—including Washington D.C., Philadelphia, Miami, and Louisville, Kentucky—contained PFAS levels of at least 10 parts per trillion. Forty-three areas, including New York City, Nashville, Las Vegas and Sacramento had detectable PFAS at least 1 part per trillion.***
>
> ***Only one city, Meridian, Mississippi, which uses well water 700 feet below the surface, found no PFAS, while Tuscaloosa, Alabama and Seattle had levels lower than the 1 part per trillion limit advised by the EWG.*** **Joshua Bote, Jan. 23, 2020.**

According to local municipal authorities in Meridian, one reason that Meridian boasts this top water resource ranking is because the City draws its drinking water directly from deep wells (at least 700 feet below the surface) in the aquifer. The other cities in this study all draw their drinking water from sources closer to the surface. Surface waters become contaminated by runoff containing PFAs from various sources. The aquifer under Meridian and much of East Central Mississippi apparently does not have great exposure to PFAs, at least not near where the water was tested.

Thus, Meridian needs to protect its unique resource and keep it free from such contaminants if this benefit is to accrue substantial economic and environmental interest over time. Rainwater offers much to aquifer recharge. It is important for rainwater to infiltrate the soil vertically, directed downward toward the aquifer, rather than directed laterally across surface lands laden with possible contaminants. There may be a variety of ways to ensure such directed vertical water movement. Clearly, what has happened in the past has worked. But, as new development is encouraged to enter

31st year (1990-2020) of publication of SOLSTICE:
An Electronic Journal of Geography and Mathematics
Volume XXXI, Number 2; Monday, December 21, 2020; 4:02 am, Central Time

the picture, it needs to do so in a way that will protect Meridian's precious resource and preserve the existing favorable environmental balance.

Swap Concept

Meridian has a systematic plan review in place for new and revised project development. The idea here is simply to offer to a prospective developer, extra opportunity for development in exchange for extra input on his/her part in regard to preservation and enhancement of environmental amenities, focused particularly on the one we have that is unique: water free of PFAs. For example, on any plan that triggers site development review, a business might receive an extra benefit that does not harm the environment, such as accelerated project approval, a tax advantage, permission for increased building height or density, or variances involving setbacks or open space coverage. In exchange, the business offers increased environmental amenities beyond current requirements, that enhance the local environment by implementing such improvements either on the proposed site or offsite on public (or possibly private) lands. We assume that City of Meridian Planning staff will know about zoning opportunities for projects, including PUD (Planned Unit Development) possibilities, as a basic form in which to execute a swap of extra development benefit for extra environmental amenities.

The Environmental End of the Swap: Kudzu

Planning Staff already has ways available to implement encouragement of increasing vertical flow of rainfall runoff, through use of various hydrological tools, such as (but not limited to) requiring onsite retention of water, curb and gutter positioning, use of rain gardens or barrels, requiring reduction of impervious surface or increase of pervious surface, and so forth. Thus, we focus on what we see as more out-of-the-conventional environmental amenities that might come from systematic urban development.

Kudzu, often viewed as 'the vine that ate the South' is a well-known invasive plant that kills the plants it drapes over. Any advantage it offers is apparently offset by the plants it kills as it drapes itself over trees, shrubs, and more. So, we assume, it has no net physical advantage. Until recently, folks might have thought of kudzu only as a nuisance. However, scientists at various universities have

31st year (1990-2020) of publication of SOLSTICE:
An Electronic Journal of Geography and Mathematics
Volume XXXI, Number 2; Monday, December 21, 2020; 4:02 am, Central Time

seen it as far more (Georgia Tech, University of Virginia, Clemson University). It is of particular importance in part because, in addition to its obvious physical problems, it is a carbon emitter rather than, what one might assume, a carbon capturer (as a green plant). The logic is explained by scientists at Clemson University.

> "Clemson University scientists are shedding new light on how invasion by exotic plant species affects the ability of soil to store greenhouse gases. The research could have far-reaching implications for how we manage agricultural land and native ecosystems.
>
> In a paper published in the scientific journal *New Phytologist*, plant ecologist Nishanth Tharayil and graduate student Mioko Tamura show that invasive plants can accelerate the greenhouse effect by releasing carbon stored in soil into the atmosphere.
>
> Since soil stores more carbon than both the atmosphere and terrestrial vegetation combined, the repercussions for how we manage agricultural land and ecosystems to facilitate the storage of carbon could be dramatic.
>
> In their study, Tamura and Tharayil examined the impact of encroachment of Japanese knotweed and kudzu, two of North America's most widespread invasive plants, on the soil carbon storage in native ecosystems.
>
> They found that kudzu invasion released carbon that was stored in native soils, while the carbon amassed in soils invaded by knotweed is more prone to oxidation and is subsequently lost to the atmosphere.
>
> The key seems to be how plant litter chemistry regulates the soil biological activity that facilitates the buildup, composition and stability of carbon-trapping organic matter in soil.
>
> This layer of decomposing knotweed will eventually form soil organic matter in invaded ecosystems.
>
> 'Our findings highlight the capacity of invasive plants to effect climate change by destabilizing the carbon pool in soil and shows that invasive plants can have profound influence on our understanding to manage land in a way that mitigates carbon emissions,' Tharayil said.
>
> Clemson University, July1,2014, https://phys.org/news/2014-07-kudzu-soil-carbon-global.html

So, how much kudzu is there in the US? The estimated US load of kudzu is 7.4 million acres. Those 7.4 million acres emit about 4.8 metric tons of carbon. That is roughly equivalent to the amount of carbon emitted by consuming 540 million gallons of gasoline or burning 5.1 billion pounds of coal. At a national level, that translates to being equivalent to the annual carbon footprint for a city of one million people (https://arstechnica.com/science/2014/07/invasive-kudzu-drives-carbon-out-of-the-soil-into-the-atmosphere/). Kudzu clearly adds an important negative element to the annual urban carbon footprint. When removed, therefore, it obviously has the potential to reduce the carbon footprint.

31st year (1990-2020) of publication of SOLSTICE:
An Electronic Journal of Geography and Mathematics
Volume XXXI, Number 2; Monday, December 21, 2020; 4:02 am, Central Time

Thus, we focus on creating an environmental swap involving kudzu removal for increased development benefit—that is, we adopt a simple underlying fundamental philosophy.

> *IN A REGION ALREADY BOASTING CONSISTENTLY GOOD AIR QUALITY, WHATEVER CARBON YOU TAKE OUT THROUGH KUDZU REMOVAL, YOU CAN PUT BACK SOME PORTION THROUGH LEGAL COMMERCIAL/INDUSTRIAL ACTIVITY. THE EXISTING CARBON FOOTPRINT IS NOT INCREASED (HOPEFULLY EVEN REDUCED) ALTHOUGH THE REGIONAL COMMERCIAL/INDUSTRIAL CAPABILITY IS INCREASED.*

So, subtract the carbon that kudzu creates in Lauderdale County, and allow businesses to put some back, given that the air quality is already reasonable with the present kudzu load. Trade kudzu for business, jobs, and things people want while giving the developer some sort of extra benefit or tax break related to their kudzu removal rate, perhaps as some sort of renewable energy certificate.

That is, we seek a win-win situation that will foster increased urban activity in support of the forthcoming Renaissance and will do so while also fostering preservation of existing fine environmental amenities and simultaneous carbon footprint reduction. We view Meridian as an ideal location for a pilot study to implement these ideas. The three of us are prepared to do the necessary work with municipal government and business communities, structured within an academic environmental context. With a successful pilot project in place, we would then wish to move forward to other communities.

Steps to Pilot Project Development

Scientific Basis

The general rationale for the suitability of promoting the idea that removal of kudzu improves the carbon footprint was explained above. The links below build the scientific base, on the work of other scientists, as a foundation from which to move forward.

All issues of *Solstice: An Electronic Journal of Geography and Mathematics* are archived in the permanent digital archive of The University of Michigan. The link http://www.imagenet.org/ will direct

31st year (1990-2020) of publication of SOLSTICE:
An Electronic Journal of Geography and Mathematics
Volume XXXI, Number 2; Monday, December 21, 2020; 4:02 am, Central Time

readers to this digital archive. These articles contain many references to scientific works done elsewhere, particularly at great universities in the southeastern and southern United States.

- References, including those associated with various kudzu use possibilities (as a view of the 'whole') as well as those based on science.
    - Scientific Orientation
        - Campbell, M. July 16, 2014. Invasive kudzu drives carbon out of the soil, into the atmosphere.
        - Clemson University, 2014. Kudzu can release soil carbon, accelerate global warming. *Science Daily*. www.sciencedaily.com/releases/2014/07/140701145720.htm
        - "Controlling Kudzu With Naturally Occurring Fungus". *Science Daily*. July 20, 2009. https://www.sciencedaily.com/releases/2009/07/090719185107.htm Last accessed December 10, 2018.
        - Government of Canada. 2017. Why Invasive Alien Species are a Problem. Last accessed, November 29, 2019, https://www.canada.ca/en/environment-climatechange/services/biodiversity/why-invasive-alien-species-are-problem.html
        - Hickman, E. H.; Wu,Shilian; Mickley, L. J.; and, Lerdau, M. Proceedings of the National Academy of Sciences of the United States of America. Last accessed November 30. 2019. https://www.pnas.org/content/107/22/10115
        - Hill, J. February 23, 2015. Invasive Species: How They Affect the Environment. Environmental Science. Last accessed, November 29, 2019: https://www.environmentalscience.org/invasive-species
        - Mississippi Encyclopedia: https://mississippiencyclopedia.org/entries/kudzu-plant/
        - Moseman, A. 2010. Discover Magazine reprint. CBS News presentation.
        - Science.gov. https://www.science.gov/topicpages/n/nitrogen-fixing+legume+kudzu Last accessed December 10, 2018.
        - Tamura, M. and Tharayil, N. 2014. Plant litter chemistry and microbial piming regulate the accrual, composition and stability of soil carbon in invaded ecosystems.*New Phytologist* Last accessed November 30, 2019: https://nph.onlinelibrary.wiley.com/doi/full/10.1111/nph.12795
        - U.S. Fish & Wildlife Service, National Wildlife Refuge System. Volunteers and Invasive Plants. 2008. Last accessed December 10, 2018.
    - Popular Orientation
        - Finch, B. 2015. The True Story of Kudzu, the Vine That Never Truly Ate the South. *Smithsonian Magazine*
        - McKee, A. July 16, 2010. Got Kudzu? *Meridian Star*. Last accessed December 10, 2018.

- Miller, James H. Kudzu Eradication and Management. Chapter 28 in Hoots, D., Baldwin, J. Eds. 1996. *Kudzu the Vine to Love or Hate*. Kodak, TN: Suntop Press, 137-149. Last accessed December 10, 2018.
- Nature Conservancy. 2018. Kudzu: Stories in Indiana. Last accessed December 10, 2018.
- Orvell, T. December 6, 2009. Bhutan to Atlanta: Baskets Full of Ambition. *Atlanta Journal Constitution*. Last accessed December 10, 2018.
- Swearingen, J., Reshetiloff, K., Slattery, B., Zwicker, S. 2002. Plant Invaders of Mid-Atlantic Natural Areas. *The Bugwood Network*. Last accessed December 10, 2018.
- Treadwell, D. April 23, 1988. Southerners Once Hated Kudzu but It Grew on Them. *Los Angeles Times*.

- Issues of *Solstice* in which Kudzu and science basis for project are developed:
  - Arlinghaus, S. L. 2018. Cascade: Observation and Challenge, Solstice: An Electronic Journal of Geography and Mathematics. Vol. XXIX, No. 2. https://deepblue.lib.umich.edu/bitstream/handle/2027.42/146743/SolsticeDecember2018Final.pdf?sequence=1&isAllowed=y
  - http://www.mylovedone.com/image/solstice/SolsticeVolXXXNo2/SolsticeVolumeXXXNumber2.pdf
  - http://www.mylovedone.com/image/solstice/SolsticeVolXXXINo1/SolsticeVolumeXXXINumber1.pdf
- Meridian Renaissance, Special Issues of *Solstice* and related item
  - http://www.mylovedone.com/image/solstice/sum17/index.html
  - http://www.mylovedone.com/image/solstice/SolsticeVolXXIXNo1/SolsticeJune2018Final.pdf
  - Bonus Use website: http://www.BonusUse.com
- Correspondence. Comments and input, in varying nature and amount, regarding this project, and suggestions for names of other scientists, were derived from correspondence of the principal author with:
  - Rosina Bierbaum (Ph.D.); Douglas Kelbaugh (Ph.D.); Matthew Naud (M.S.; M.P.P); Mitchell J. Rycus (Ph.D.); Donna K. Tope (J.D., M.S.). There is to be no implication of any form of 'endorsement'; simply reaction and helpful suggestion or commentary. All interpretation and remaining blunders are solely those of the principal author.
  - An article, based on correspondence, publicizing linkage of kudzu removal with urban development and with use of kudzu in food products appeared in the *Quarterly Magazine* of Les Dames d'Escoffier International: p. 28, Green Tables section: http://www.ldei.org/uploads/archives/96.pdf

31st year (1990-2020) of publication of SOLSTICE:
An Electronic Journal of Geography and Mathematics
Volume XXXI, Number 2; Monday, December 21, 2020; 4:02 am, Central Time

Opportunity Zones: One Rationale for Geographical Boundary Decisions

Boundary Criteria and Mapping

There are a number of boundaries that can be used for planning purposes. There are ZIP code boundaries that the postal service uses. There are school district boundaries that public schools use for student assignment purposes. Neither of these meshes with the other one. Nor do they mesh with federal or county boundaries. They are single purpose boundaries. As long as they remain fixed over time, they are useful; one can track patterns over time.

Often, however, local ward boundaries and such change frequently to meet varying political, or other agendas. This kind of frequent boundary change can cause serious problems because there is a loss of a consistent logic and associated institutional memory. The population may shift within fixed boundaries; but, because the boundaries are fixed, the inventory of past actions, good or bad, is not lost. The history is a guide to what not to do again; more important, it is a benchmark from which to move forward, and as such should be preserved, as a guide to tracking progress and propelling it into the future.

There is probably no point to changing ZIP code or school district boundaries; they are entrenched and they do serve a function. Perhaps they are not optimal, but they work, and change would be extremely disruptive. Where creating new boundaries for zones is concerned, it does seem appropriate to build a stable platform of boundaries that integrate seamlessly, and in a nested fashion, with established boundaries already in use for data collection. These should not change even when the underlying population shifts. They are a stable platform for tracking change over time.

In terms of regional planning for kudzu opportunity zones to come in the future, consider that kudzu does not know city boundaries, so make the planning effort a county-wide one (to abut other counties seamlessly) even though there might be city planning processes that differ from rural ones. Thus, encourage using boundaries that (Figures attached) implement the followings strategies.

31st year (1990-2020) of publication of SOLSTICE:
An Electronic Journal of Geography and Mathematics
Volume XXXI, Number 2; Monday, December 21, 2020; 4:02 am, Central Time

1. Smaller polygons mesh with the existing County boundary, without overlap or underlap: they nest inside the rectangular county boundary with no gaps or overlap between boundaries.. The US Bureau of the Census has such boundaries, Census Tract boundaries, that nest within each county in the US. Figure 1 shows those boundaries for 19 tracts within Lauderdale County.

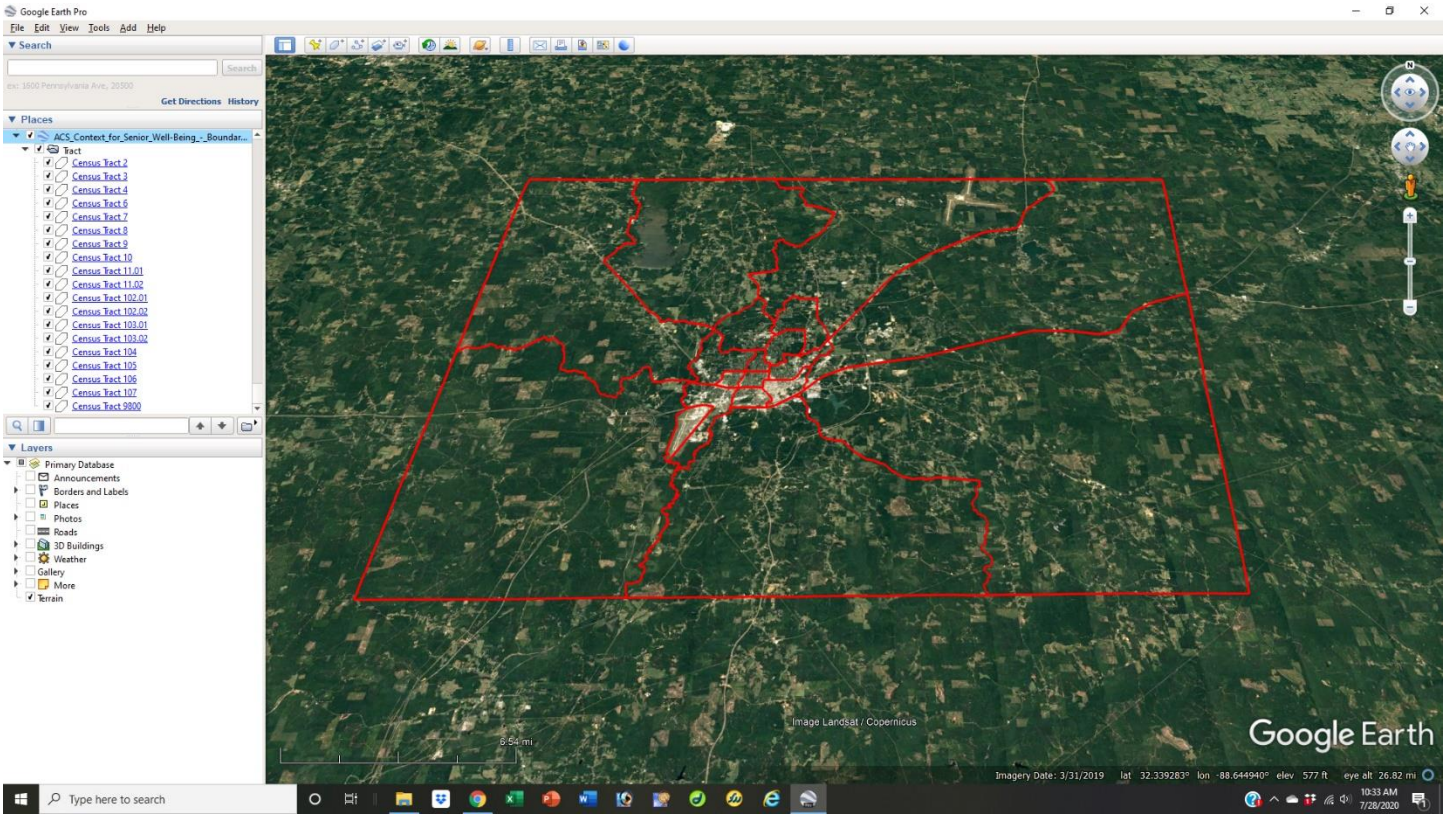


Figure 1. 19 Census Tract Boundaries (outlined in red) within Lauderdale County, Mississippi, centered on the City of Meridian. Census boundary file is projected onto Google Earth.

2. The Census provides files that can be opened directly in Google Earth. They can also be altered there, visually, as well. And, one can employ the default set of features already present in Google Earth. Figure 2 shows the image from Figure 1 with the default Google Earth road network turned on.
3. 3. Census data is linked, in the background, to the Tract boundaries. Click on a tract and the associated data set pops up. Figure 3 shows an example. Note the scroll bar on the right;

31st year (1990-2020) of publication of SOLSTICE:
An Electronic Journal of Geography and Mathematics
Volume XXXI, Number 2; Monday, December 21, 2020; 4:02 am, Central Time

there is a lot of data here! But, one does need to unscramble what it means. The Census provides such a template, as another separated file.

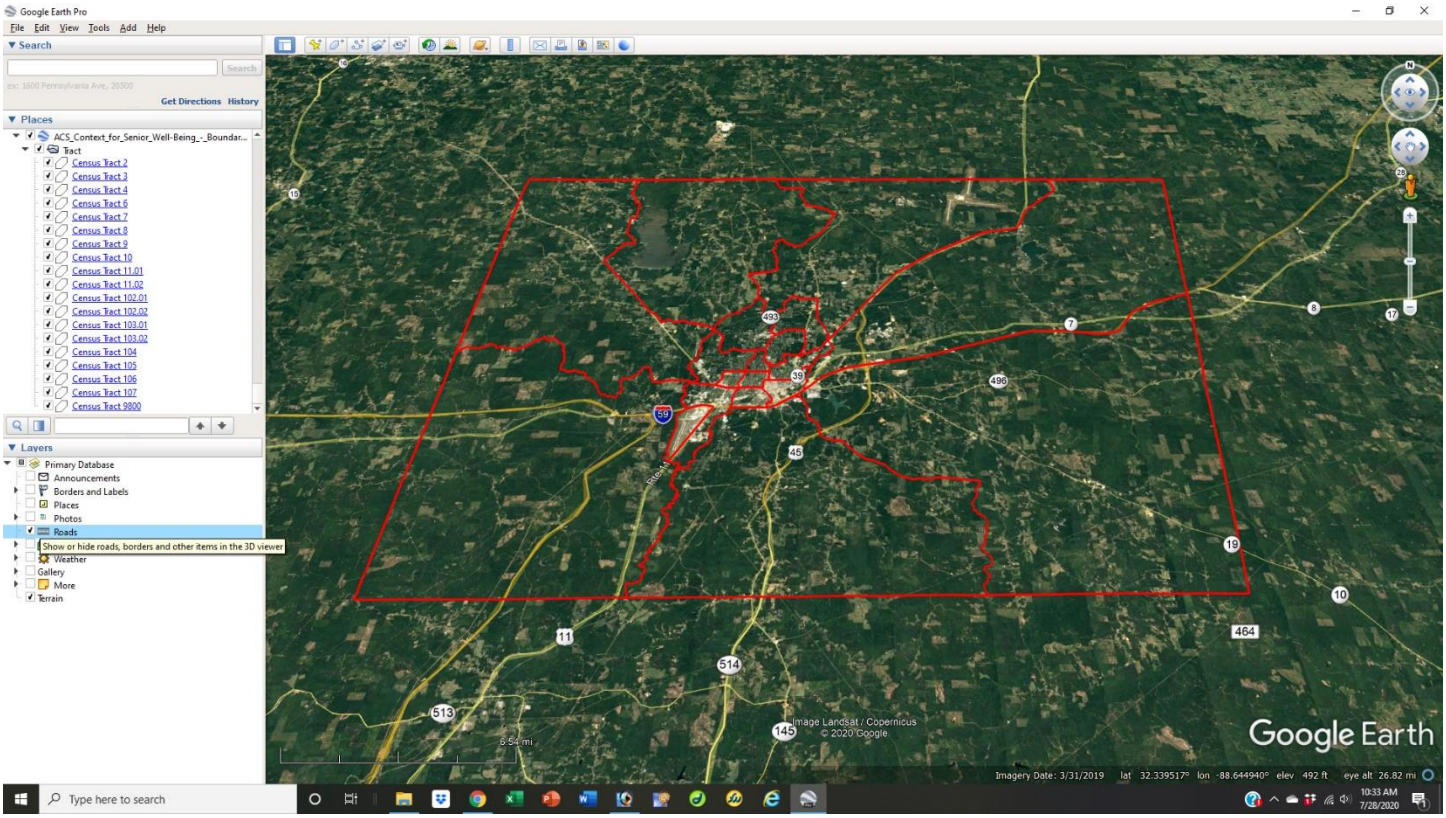


Figure 2. Figure 1 with default Google Earth road networks included.

4. One can zoom around and turn on other layers. Figure 4 shows the 3-D buildings (done a while ago) layer turned on. Note the red lines in the background--they are Census Tract boundaries. That is, the imported boundaries remain present independent of manipulating the default map and associated layers.

31st year (1990-2020) of publication of SOLSTICE:
An Electronic Journal of Geography and Mathematics
Volume XXXI, Number 2; Monday, December 21, 2020; 4:02 am, Central Time

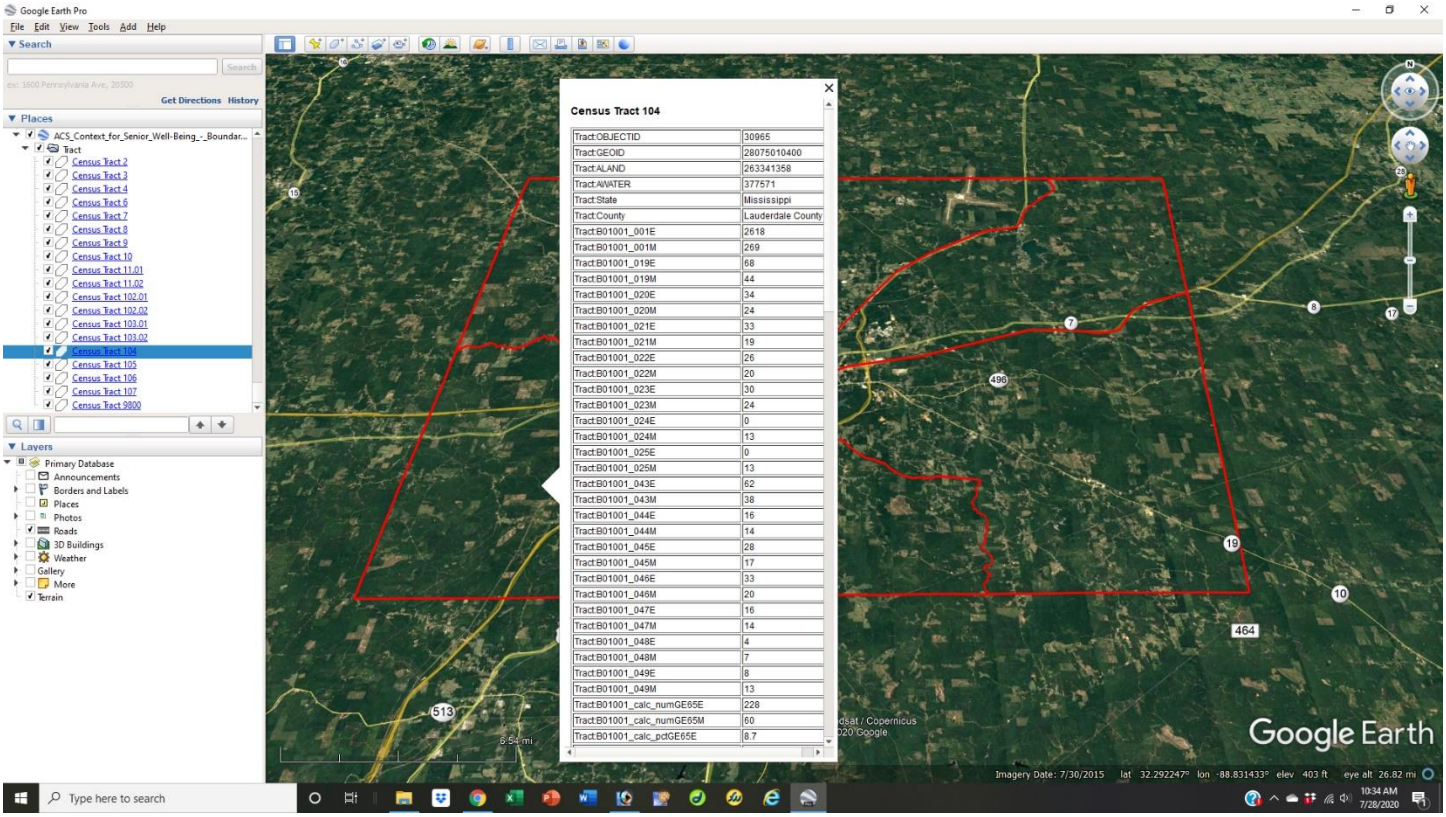


Figure 3. Figure 2 with associated underlying database entries for Tract in the southwest of Lauderdale County.

5. The imported graphics can be manipulated directly in Google Earth. Here are a couple of ways...a fence is put up around each tract (so it is still easy to look inside). The lower part is shaded yellow, representing percentage of senior citizens in the tract as part of the total red fence height representing total tract population. One tract is shaded a solid, but semitransparent color (as another way to look through to the bottom). Or, simple balloons, showing percentage of 65+ population can easily be inserted at the centroid of each tract polygon. There is lots of variety as to what one can do within Google Earth--free publicly available software. There is even more that can be done within expensive purchased high-end (and high price) mapping software. But what can be done for free is quite good. Figure 5 shows an example.

31st year (1990-2020) of publication of SOLSTICE:
An Electronic Journal of Geography and Mathematics
Volume XXXI, Number 2; Monday, December 21, 2020; 4:02 am, Central Time

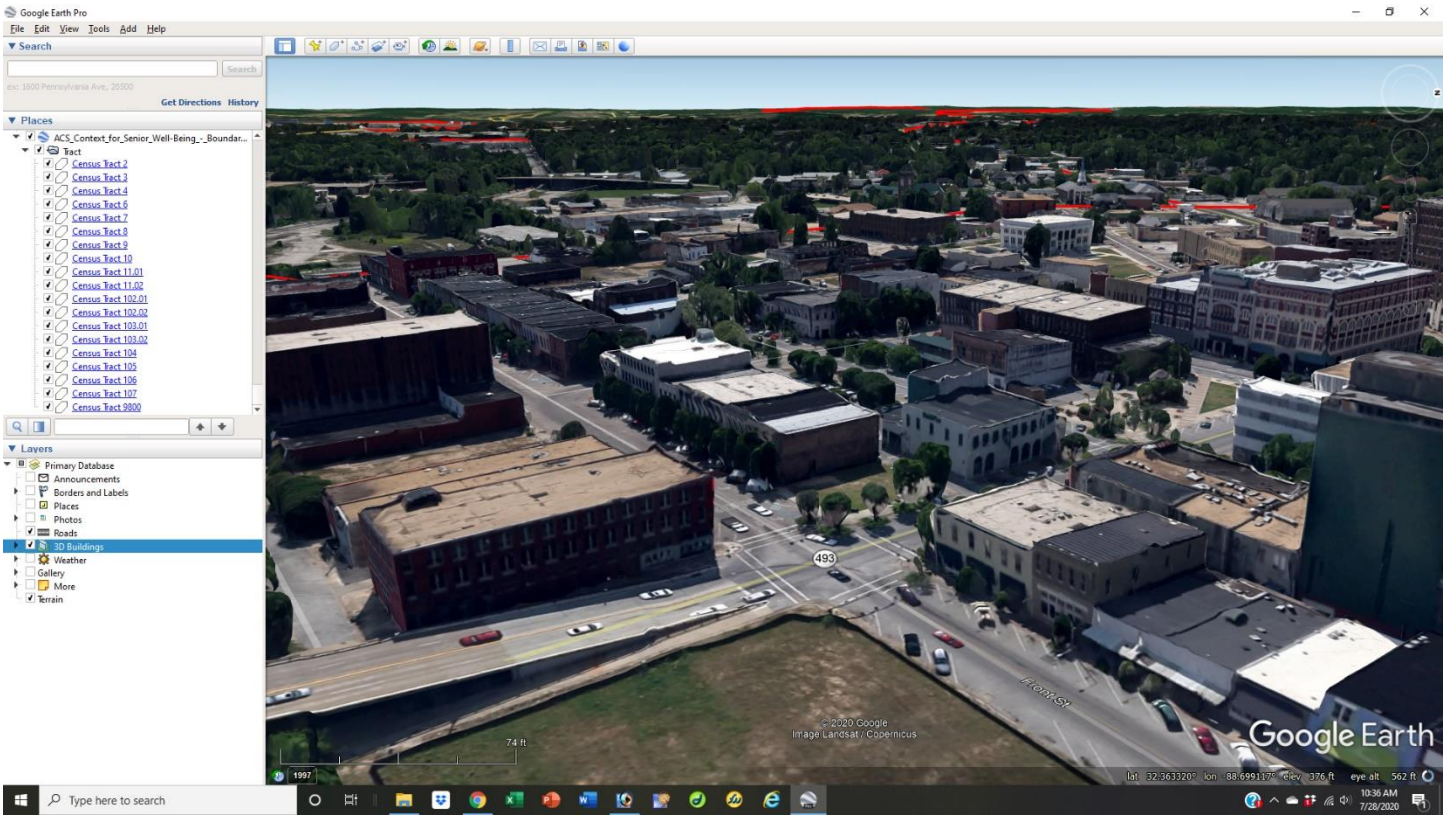


Figure 4. Default 3-D Google Earth imagery inserted. Added red Census Tract boundaries remain (toward the back of the image) even when zooming in to take a closer look at downtown Meridian.

6. And, independent of manipulation of the imported file within Google Earth, the underlying Census data base still remains correctly aligned with the altered polygons--Figure 6. Notice that the database has been edited so it is sensible and still it aligns with the tracts. Someone looking at Covid-19 patterns might appreciate the particular selection chosen from the vast Census database.

31st year (1990-2020) of publication of SOLSTICE:
An Electronic Journal of Geography and Mathematics
Volume XXXI, Number 2; Monday, December 21, 2020; 4:02 am, Central Time

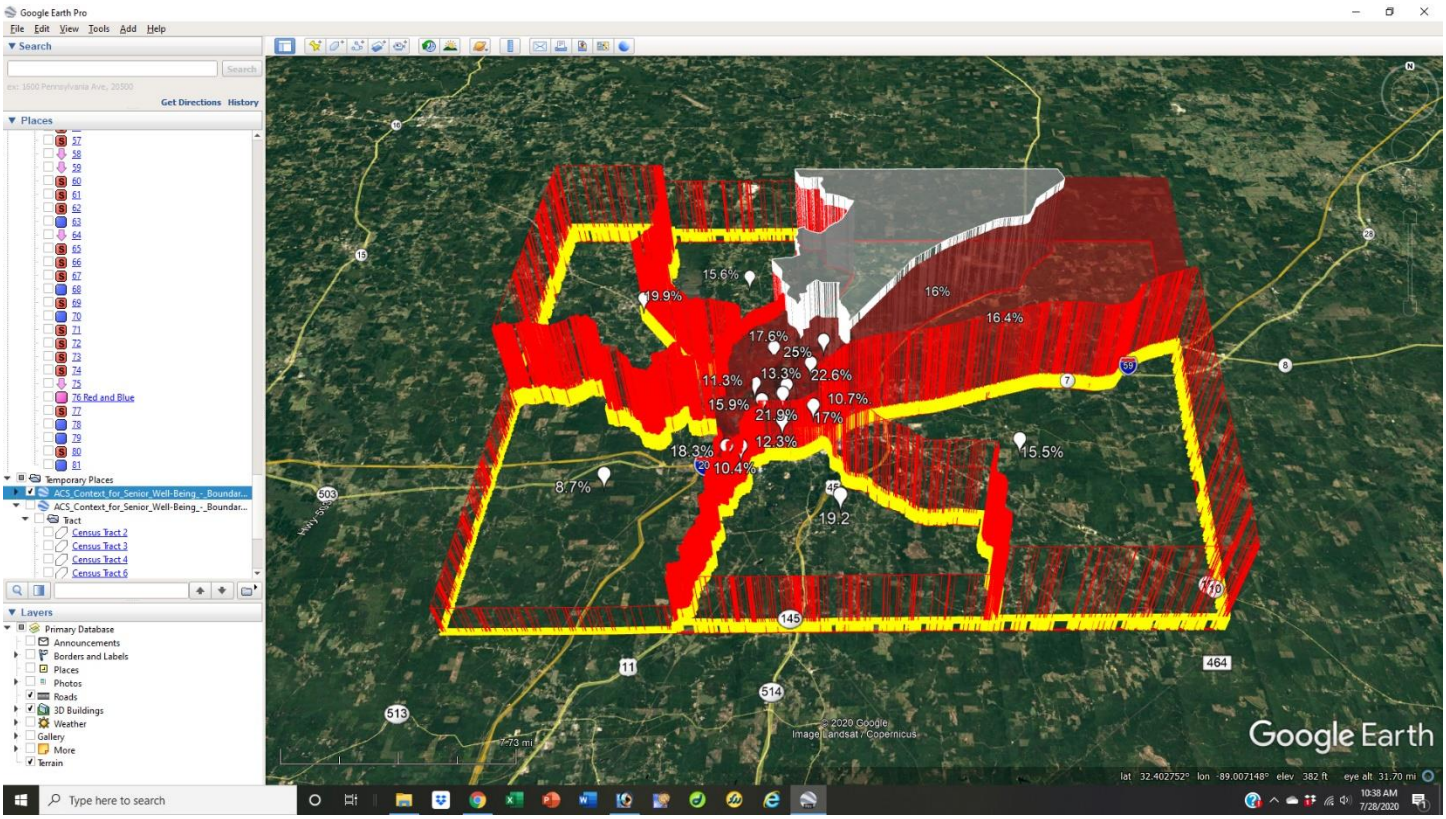


Figure 5. Enhancement of Census Tract boundaries, in relation to data, using only tools within Google Earth.

31st year (1990-2020) of publication of SOLSTICE:
An Electronic Journal of Geography and Mathematics
Volume XXXI, Number 2; Monday, December 21, 2020; 4:02 am, Central Time

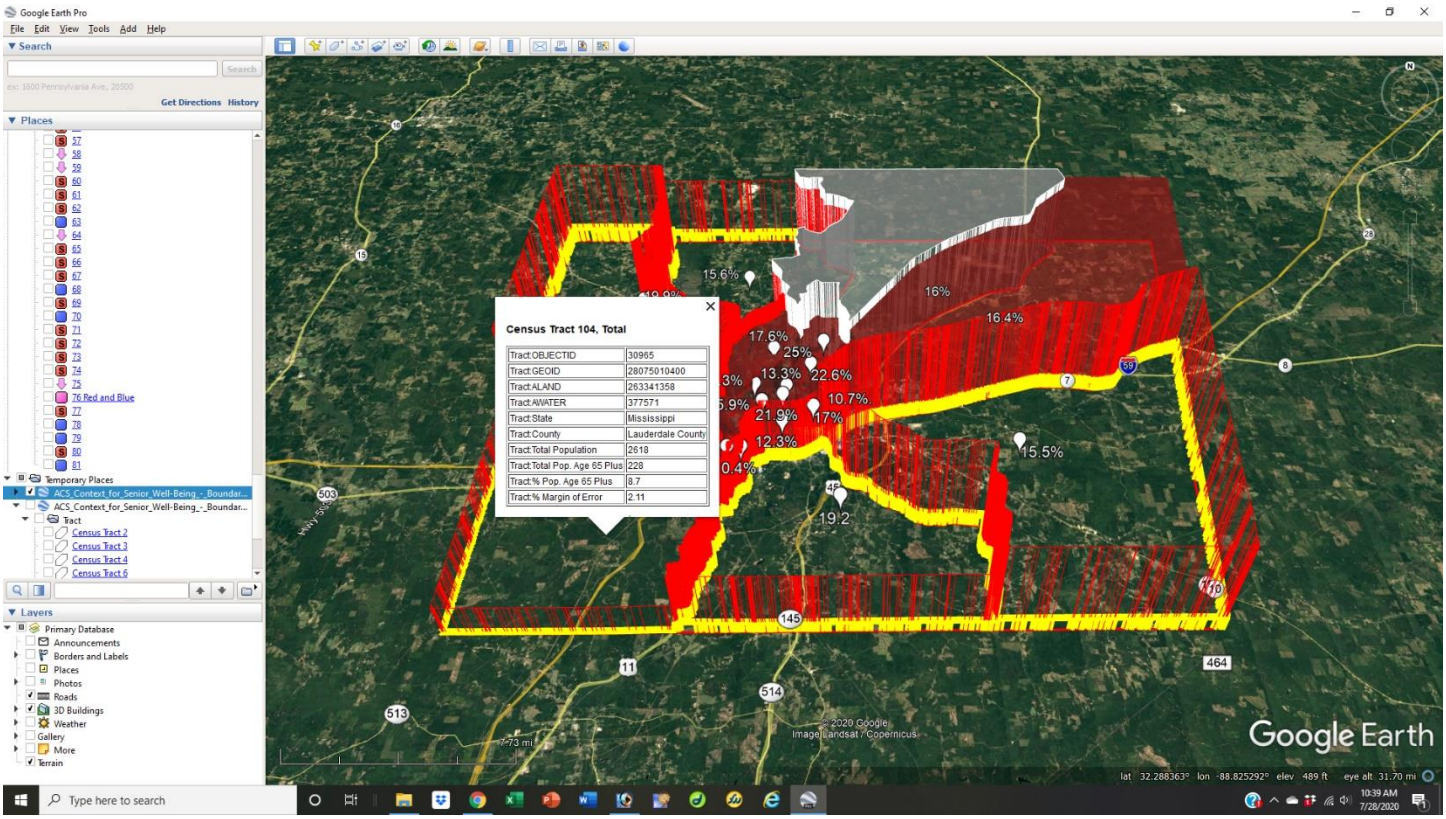


Figure 6. Underlying database edited in NotePad ++. Association with correct polygon is unchanged.

31st year (1990-2020) of publication of SOLSTICE:
An Electronic Journal of Geography and Mathematics
Volume XXXI, Number 2; Monday, December 21, 2020; 4:02 am, Central Time

Boundary Observations: Usage

Census Tract boundaries have been fixed for a long period of time. Originally, their geographic area was intended to embrace a fixed population size, perhaps a quantity between 1,000 and 8,000, so typically 4,000 or 5,000. Thus, more densely populated areas would have tracts of smaller geographic area. In Lauderdale County what we see is that the size of the tract is small in the center of Meridian and increases as one moves away from the center toward county boundaries...from downtown to the suburbs and rural. At the time the tracts were created, that is how the population was distributed. Now we may all 'know', anecdotally, that the population has been moving outward from downtown. But anecdotal evidence leaves room for disagreement and negative feelings in charged political contexts. The visual evidence of the Census Tract pattern (Figure 1) provides logical support, coupled with easy to read digital proof (Figure 6), that in fact the population is moving outward. Logic prevails.

When are Census Tracts useful? In Regional Planning? Where there are no other boundary units otherwise established? Yes. Here are some examples.

1. As downtown Meridian enlarges, the road network may become overburdened by individual cars as newcomers move into the area, both in the City and the suburbs. Increased business opportunity in the downtown will focus traffic in that direction (see Figure 2 for the road network). Once the differential between inbound and outbound traffic becomes large (typically one direction (inbound) in the morning and the opposite direction in the evening) then public mass transit (buses) become sensible (from an economic and planning perspective). At that point, planning might be done at a County level, using Census Tracts to capture existing federal and other population data. And, data that gets created should also use tract boundaries so that information nests from the local to the county to the state to the federal scale. The approach is a 'surgical' one. There are also nesting census units smaller than tracts if that seemed useful. So, regional transportation, where there is none present, could be one useful application of Census Tracts as managing units.

31st year (1990-2020) of publication of SOLSTICE:
An Electronic Journal of Geography and Mathematics
Volume XXXI, Number 2; Monday, December 21, 2020; 4:02 am, Central Time

2. Kudzu project.

a. There are currently no units present for this project. Again, it appears to be a county project and would benefit from Census Tracts as data management units..both in terms of existing data in the Census and in creating its own data base involving kudzu collection and so forth, particularly as we extend this project beyond the county boundaries across the entire south (or elsewhere) all of which already have Census Tracts.

b. Another reason to employ federally-known units, such as Census Tracts, in a brand new regional project could center on funding opportunity. Federal grant application, an arduous process by any standard, might be made more comprehensible to readers/reviewer and proposal writers alike by employing standard units and using data, such as Census data, associated with those units. This usage is certainly appropriate for any style of grant proposal and might be especially useful if some sort of bootstrapping process is involved in securing funding—that is, let the prospective target monitor early project progress and then decide to invest as the project progresses. There is advantage to the surgical process of being able to sincle out kudzu opportunity zones by Census Tract. Thus, kudzu opportunity zones become entire Census Tracts, or other smaller units that nest within Trancts, such as Census Block Groups or Census Blocks: whatever is appropriate.

c. Use existing boundaries from the Census, tracts and the finer subdivisions of tracts, such as block groups and blocks, as ways to accumulate community-oriented data that can fuel a community-oriented policing effort to work seamlessly as a layer in a regional and state network already present (City police, county, and state—with city and county coordinating so census tracts can be used in regional policing). When incorporated as part of the kudzu project, one source of labor for kudzu removal might come from juveniles (who have had a minor problem with the law). They might be assigned to kudzu removal as community-service in fulfillment of making good to a judicial order. Indeed, the kudzu opportunity zone would offer

relief (or opportunity) to these young people from beginning their adult lives with a prison record.

With a strategy such as this one, everything fits together neatly to develop an historical record from which to move forward and monitor need for continuing kudzu treatment as well as monitor progress of teenagers (who hopefully fall out of this database—but if not, their path can be tracked to try to get them directed along a better path). Once plans for geographic boundaries and database management of kudzu opportunity zones are linked with Planning Department possibilities for development swaps and Police Department opportunities for labor swaps, then it is time to move forward and complete the process, with procuring business opportunities to move in and support the project effort in exchange for favorable locational or other consideration. Currently, there are two local business targets; one a shoe company and the other an IT connection (Lindemann, 2020). The local shoe company (Avrio, 2020) is an example of biofabric business orientation, present in a number of communities across the nation. Elsewhere, kudzu fiber has been successfully integrated into fabric (Murai and Murai, 2014). Avrio claims, for example, a carbon offset of 437 hours of smart phone use for each pair of shoes sold. Do they already use kudzu as one of their materials (biofabrics)? Might they do so? Might integrating kudzu into their product increase, even more, their carbon offset? The opportunity of having a new biofabrics store locate in Meridian, gives extra impetus to the logic of creating planning units based on biological supply chains: hence, Kudzu Opportunity Zones. Stay tuned to see how these might play out, first locally, and then more broadly!

References

- Avrio website: https://avriofootwear.com/
- Lindemann, Weston. 2020. Personal Communications with a Google Employee, with expertise in supply chains, who is moving to Meridian.
- Murai, T. and Murai, R. 2014. Changing of Kudzu Textiles in the Japanese Culture. Textile Society of America Symposium Proceedings. http://digitalcommons.unl.edu/cgi/viewcontent.cgi?article=1916&context=tsaconf

31st year (1990-2020) of publication of SOLSTICE:
An Electronic Journal of Geography and Mathematics
Volume XXXI, Number 2; Monday, December 21, 2020; 4:02 am, Central Time

Kudzu, Carbon Footprint, and Bioplastics

Sandra L. Arlinghaus, William E. Arlinghaus, and Weston Lindemann

For Millennia, human beings have used plants as materials from which to create objects. Plants have served as basket weaving materials; they have served as food; they have served as heat sources. The list goes on and on. And, they continue to fulfill all those functions as well as many more. What is of particular interest where kudzu is concerned is that it adds to, rather than detracts from, the carbon footprint. It is counterintuitive to consider that fact; plants soak up carbon dioxide. They reduce greenhouse gases. But, kudzu does not, even though it has broad leafy green surfaces, still it is a net emitter, not a reducer, of greenhouse gases. The science is there to establish that. The annual kudzu carbon footprint is roughly equivalent to the annual carbon footprint for a city of one million people (https://arstechnica.com/science/2014/07/invasive-kudzu-drives-carbon-out-of-the-soil-into-the-atmosphere/). Kudzu clearly adds an important negative element to the annual urban carbon footprint. When removed, therefore, it obviously has the potential to reduce the carbon footprint (see numerous references in the previous article).

That fact, that kudzu contributes to greenhouse gas load, elevates it to a unique position in terms of 'need to use'. Anywhere that kudzu can replace other plants, is a plus. Replace a plant that is a greenhouse gas reducer with one that is an emitter. There is obvious reduction in greenhouse gas for each one-to-one swap—use the 'bad' plant; save the 'good' plant.

So, let's look at the existing biomaterials and ethanol production industries and businesses with an eye to encouraging them to use kudzu instead of green plants they already use or in addition to green plants they already use (or both). Below are some helpful links to existing literature:

- "Bioplastics Global Renaissance" Agricultural Utilization Research Institute. 2011. https://www.auri.org/ag-innovation-news/apr-jun-2011/bioplastics-global-renaissance/
- "Kudzu Bioplastics" https://kudzubioplastics.weebly.com
- Sage, et. Al..2009. "Kudzu [Pueraria montana (Lour.) Merr. Variety lobata]: A new source of carbohydrate for bioethanol production." *Biomass and Bioenergy* 33(1): 57-61.

31st year (1990-2020) of publication of SOLSTICE:
An Electronic Journal of Geography and Mathematics
Volume XXXI, Number 2; Monday, December 21, 2020; 4:02 am, Central Time

https://www.researchgate.net/publication/222427546_Kudzu_Pueraria_montana_Lour_Merr_Variety_lobata_A_new_source_of_carbohydrate_for_bioethanol_production

- https://www.natureworksllc.com NatureWorks manufactures its own brand of polylactic acid and lactides, greenhouse gas-based materials used to manufacture plastics and fibers.

To bring businesses to Meridian MS, one locale with many advantages enumerated elsewhere, but including a sizeable kudzu resource base, we will reach out to local expertise in supply-chain logistics to contact companies that might make good use of East Mississippi's kudzu base. At a long-range planning view, it makes sense to find companies who might take advantage of our unique water supply with no PFAs. At a more immediate view, it makes sense to contact companies already considering coming to Meridian. The goal in either case would be to have them locate a distribution center, manufacturing plant, or both, in Meridian. The biobased shoe company, Avrio (https://avriofootwear.com/), already looking at Meridian as a potential site for a distribution center seems like a logical place to start.

31st year (1990-2020) of publication of SOLSTICE:
An Electronic Journal of Geography and Mathematics
Volume XXXI, Number 2; Monday, December 21, 2020; 4:02 am, Central Time

Coloring Maps: In the Plane and on the Surface of a Sphere*

William Charles Arlinghaus

(Use the Appendix at the end to assist with technical matters, as needed.)

In order to solve problems, mathematicians often change their forms to ones they find easier to analyze. Such is the case in trying to find the minimum number of colors necessary to color a plane map and the relation of that problem to coloring a map on a sphere.

Coloring a map in the plane.

Definition. Let *G* be a plane map. The dual graph *G'* is obtained from *G* as follows: place a node inside the interior of each region of the map. Draw an edge between nodes if they lie in adjacent regions.

In the following example, the map *G* is colored blue, green, and yellow, while the dual graph *G'* is in red (Figure 1).

It is easy to see that a planar map leads to a planar graph (without loops), and vice versa. The problem of coloring the regions in *G* so that no two adjacent regions have the same color is equivalent to the problem of coloring the nodes of *G'* so that no two adjacent nodes have the same color. In the example of Figure 1, a coloring with three colors is shown. In Figure 2 the map and its graphical representation require four colors.

31st year (1990-2020) of publication of SOLSTICE:
An Electronic Journal of Geography and Mathematics
Volume XXXI, Number 2; Monday, December 21, 2020; 4:02 am, Central Time

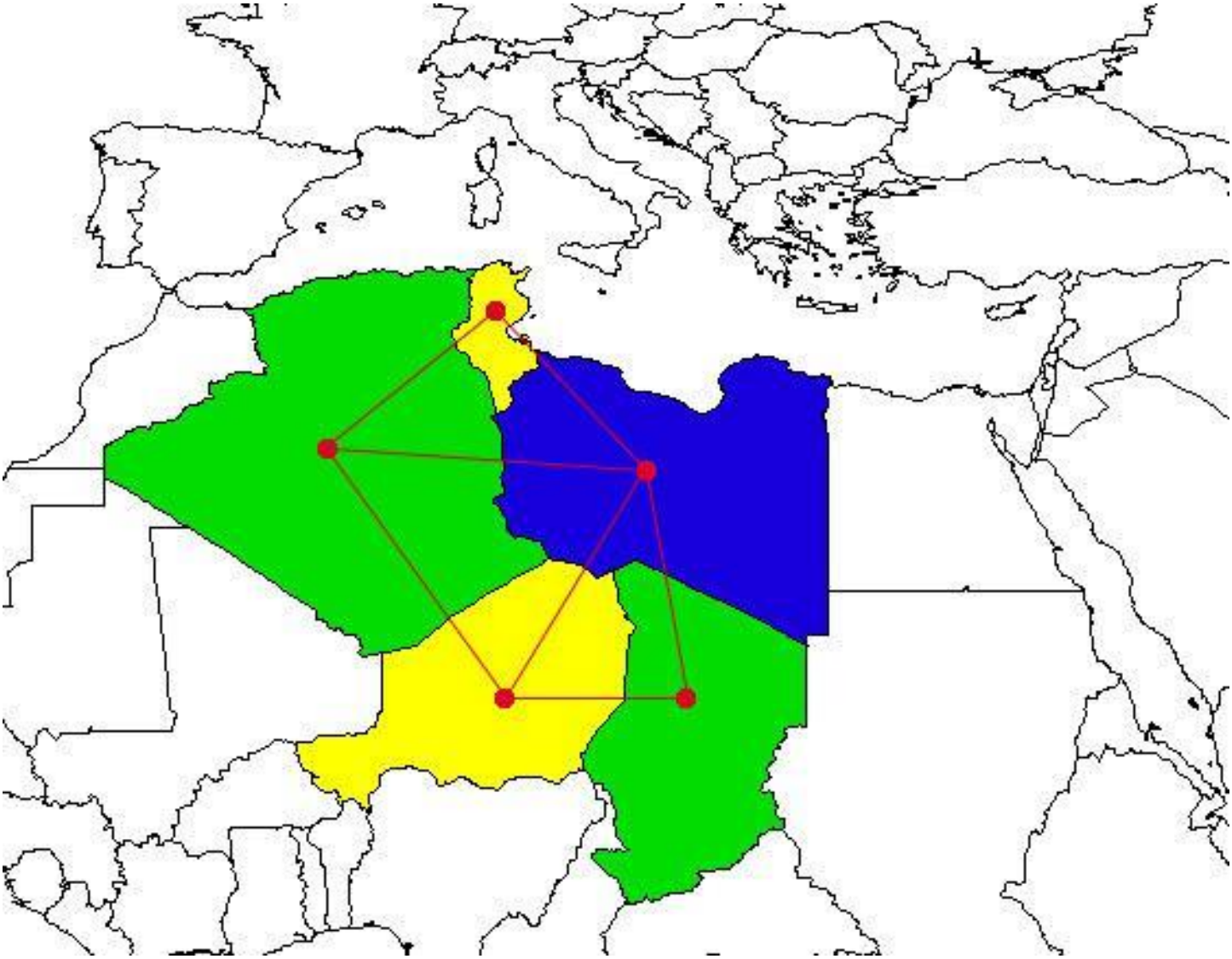


Figure 1.

31st year (1990-2020) of publication of SOLSTICE:
An Electronic Journal of Geography and Mathematics
Volume XXXI, Number 2; Monday, December 21, 2020; 4:02 am, Central Time

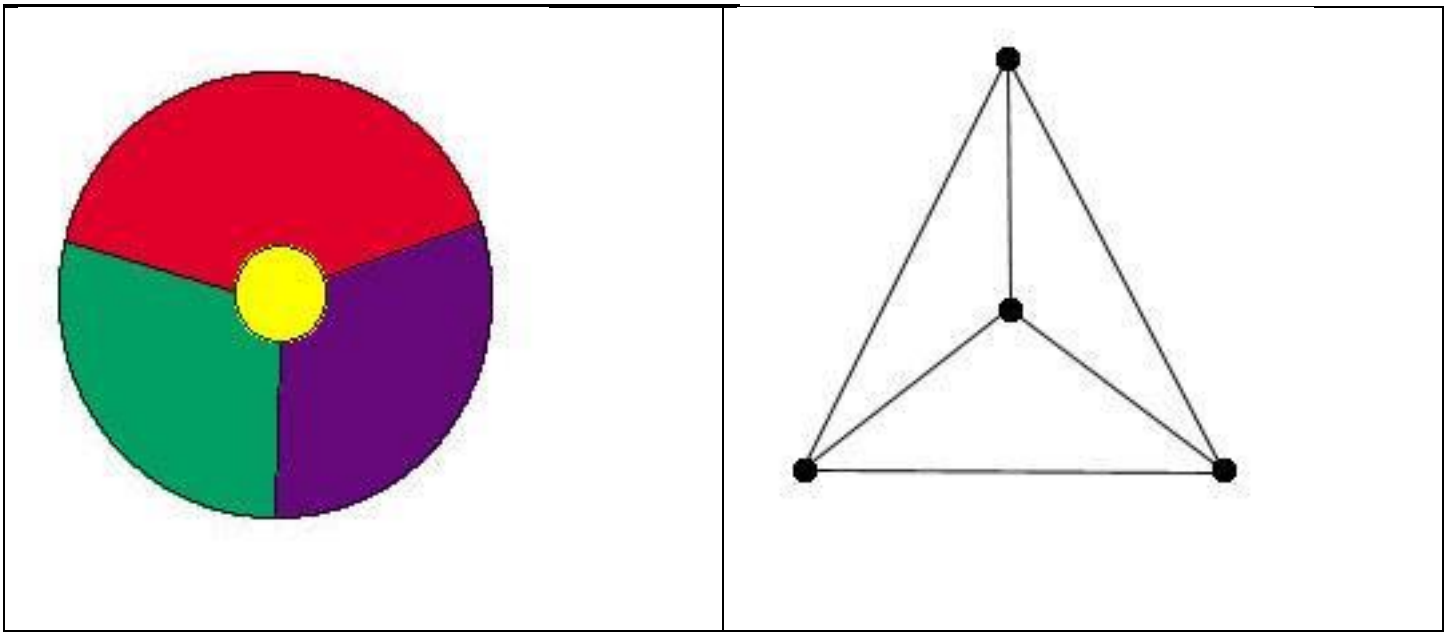


Figure 2. a. Map with four colored regions. b. Top node is red; central node is yellow, lower right node is purple, lower left node is green.

While many people have drawn planar maps that appeared to need five or six colors, it has always been possible to re-assign the colors to get by with four colors.

Again, in trying to prove a theorem, it is often instructive to prove a simpler theorem first. Here, we shall first prove that six colors are enough, then five, and finally we will discuss the proof that four colors are enough.

We start with an elementary fact that can be deduced from a Corollary to Euler's formula (Theorem App1), both in the Appendix.

Lemma 1. Every planar graph has a node of degree *5* or less.

31st year (1990-2020) of publication of SOLSTICE:
An Electronic Journal of Geography and Mathematics
Volume XXXI, Number 2; Monday, December 21, 2020; 4:02 am, Central Time

Proof. Suppose *G* were a planar graph with each node of degree at least 6. Then, since the sum of the degrees of the nodes is twice the number of edges, we would have $6v \leq 2e$ or $3v \leq 3$. However, by Corollary App1 to Euler's formula (Theorem App1) the following inequality holds: $e \leq 3v - 6$. This yields the contradiction $3v \leq 3v - 6$.

Theorem 2. Six-color Theorem. Every planar graph can be colored with no more than six colors so that adjacent nodes have different colors. (If *G* is planar, $\chi(G) \leq 6$.)

Proof. The proof is by mathematical induction on the number of nodes in *G*. The Theorem is clearly true for all graphs with six or fewer nodes, since each node can have its own color. Now suppose the Theorem is true for all graphs with fewer than *n* nodes, and let *G* be a graph with *n* nodes. Pick a node *v* of *G* of degree 5 or less (this is possible by the lemma). Color *v* with a color different from those assigned to its adjacent vertices in a 6-coloring of $G - \{v\}$; this assignment gives a 6-coloring of *G*.

Next, we hope to modify the proof of this theorem to show any planar graph can be colored with only five colors. This modification will clearly be more difficult, since there will not be a color to spare for *v*, necessarily. We shall have to modify the coloring of $G - \{v\}$.

Theorem 3. Five-color Theorem. Every planar graph can be colored with no more than five colors so that adjacent nodes have different colors.

Proof: The proof again proceeds by induction on the number of nodes in *G*. This time the theorem is clearly true for all graphs with five or fewer nodes, since again each node can have its own color.

Now suppose the theorem is true for all graphs with fewer than *n* nodes, and let *G* be a graph with *n* nodes. As before, pick a node *v* of *G* of degree 5 or less.

a. If deg $(v)<5$, look at a 5-coloring of $G - \{v\}$, and color *v* with a color different from that of any of its (four or fewer) adjacent nodes.

31st year (1990-2020) of publication of SOLSTICE:
An Electronic Journal of Geography and Mathematics
Volume XXXI, Number 2; Monday, December 21, 2020; 4:02 am, Central Time

b. Suppose deg (v) = 5, but in a 5-coloring of G - {v}, no more than four different colors are used on the five nodes adjacent to v. Then again, v may be given a fifth color.
c. The difficulty arises when deg (v) = 5, and a 5-coloring of G - {v} colors the five nodes v_1, v_2, v_3, v_4, v_5 adjacent to v with five different colors 1, 2, 3, 4, 5 as illustrated below (Figure 3).

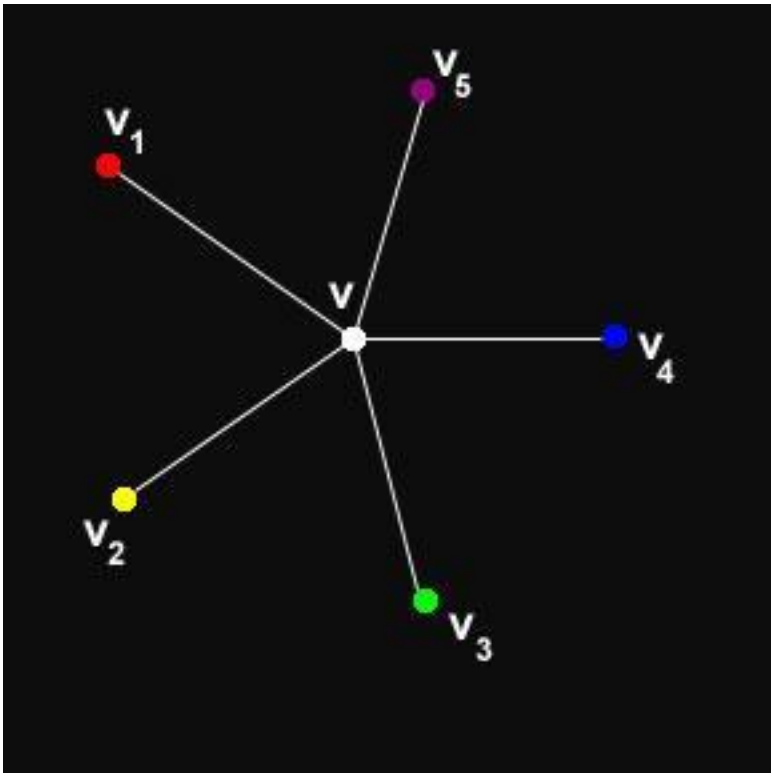


Figure 3.

Portions of G - {v} (G without v) must be re-colored so that {v_1, v_2, v_3, v_4, v_5} uses only four colors.

Consider the bipartite subgraph $H_{1,3}$ of G consisting of those nodes of G colored with colors 1 and 3 and those edges of G that join a node of color 1 with a node of color 3.

Case 1 Suppose v_1 and v_3 are in different components of $H_{1,3}$. Then interchange the colorings of the component of $H_{1,3}$ containing v_3 (i.e. color those that had color 1 with color 3 and vice-versa). Now v_3 has color 1, and color 3 is available for v.

31st year (1990-2020) of publication of SOLSTICE:
An Electronic Journal of Geography and Mathematics
Volume XXXI, Number 2; Monday, December 21, 2020; 4:02 am, Central Time

Case 2 Suppose v_1 and v_3 are in the same component of $H_{1,3}$; hence there is a path from v_1 to v_3 in which the nodes alternate colors (dotted line in diagram below). Adding the edges v_1v and vv_3 completes a circuit, as indicated below (Figure 4)

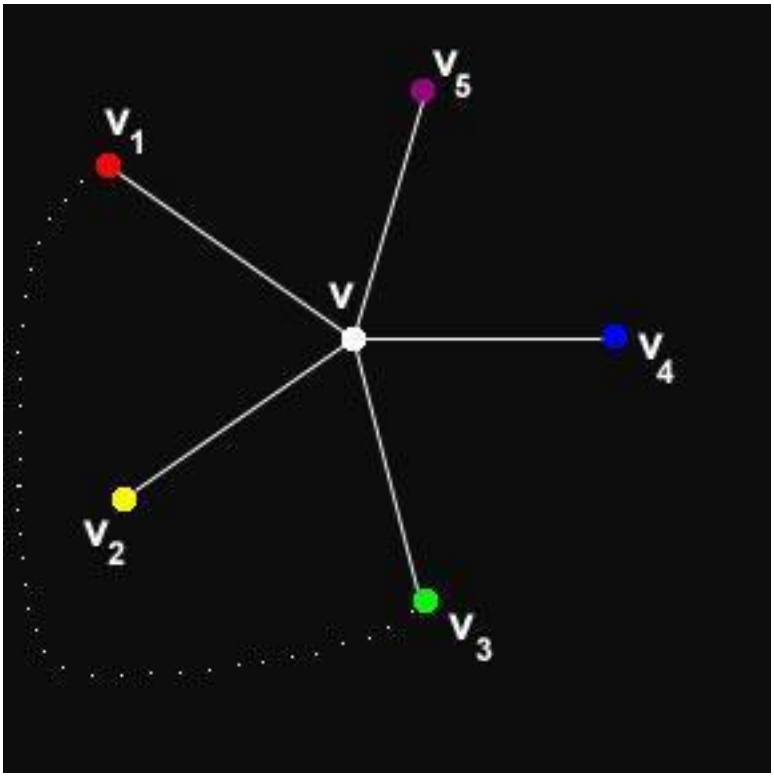


Figure 4.

Here, the Jordan Curve Theorem comes into play (Theorem App2, in the Appendix). Consider the bipartite subgraph $H_{2,4}$ of G defined analogously to $H_{1,3}$. Since v_2 lies inside the circuit and v_4 outside it, there can be no path from v_2 to v_4 lying entirely in $H_{2,4}$ (since G is planar, any path from v_2 to v_4 must include v or a node colored 1 or 3).

Now the component of $H_{2,4}$ that contains v_4 (and hence not v_2) can have its colors interchanged, as happened in case 1 to a component of $H_{1,3}$. This interchange gives v_4 color 2, allowing v to be given color 4.

31st year (1990-2020) of publication of SOLSTICE:
An Electronic Journal of Geography and Mathematics
Volume XXXI, Number 2; Monday, December 21, 2020; 4:02 am, Central Time

One might notice that things became more difficult as we progressed from six colors to five colors. As might therefore be expected, the situation is much more complex for four colors.

Many people believe they have proved the four-color theorem when they discover for themselves, again using the Jordan Curve Theorem (1869), a result first proved by Augustus De Morgan (1852).

Theorem 4. De Morgan's Theorem. No five regions in the plane can be mutually adjacent.

This fact does not suffice to prove the four-color theorem, however. For example, we can find a map in which no more than three regions are mutually adjacent but that nonetheless needs four colors (Figure 5).

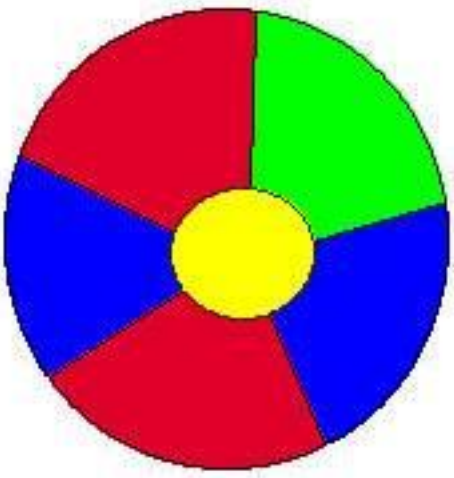


Figure 5.

In the nineteenth century, Kempe knew the basic ideas that led to the proof of the four-color theorem in the late twentieth century. However, the details were far too complicated to pursue.

What Kempe sought was called an unavoidable set of reducible configurations. That is, he supposed that a map requiring 5 colors existed. By various arguments, he hoped to show that this map must

31st year (1990-2020) of publication of SOLSTICE:
An Electronic Journal of Geography and Mathematics
Volume XXXI, Number 2; Monday, December 21, 2020; 4:02 am, Central Time

contain one of a number of smaller submaps, a so-called unavoidable set. If he could then show each of these submaps could be colored with one fewer color (hence reducible), then this map requiring five colors could not exist.

Unfortunately, the number of configurations in an unavoidable set turned out to be very large, and showing even one was reducible could be tedious. Even using a method of Heesch (1969) called discharging, the prospects were daunting.

Eventually, Kenneth Appel and Wolfgang Haken, working at the University of Illinois, were able to devise a computer program to help analyze reducible configurations. It took 1000 hours of computer time on an IBM 360 computer to analyze an unavoidable set of 1500 configurations. This was one of the first theorems to rely on computer methods; since others rarely had the computer time or expertise to check the results, it was some time before the mathematical community came to accept this new type of proof. But they have, and hence we now have the following theorem.

Theorem 5. The Four-Color Theorem (Appel and Haken, 1976). Every planar graph can be colored with four (or fewer) colors so that no two adjacent nodes have the same color.

Coloring a Map on the Surface of a Sphere

How many colors does it take then to color a map on the surface of a sphere? The answer once again is four. Given any map on the surface of a sphere. Place the sphere on a plane in such a way that the south pole of the sphere touches the plane and the north pole of the sphere lies within (NOT on the boundary of) a region of the given map on the surface of the sphere. Now use stereographic projection to project the given map on the surface of the sphere, projecting from the north pole, into the plane. Only the north pole fails to project into the plane. All elements of the map on the surface of the sphere, other than the north pole, project into the plane. Color the map in the plane using four colors (which we know can be done). Now, invert the stereographic projection and pull the colored plane map back to the surface of the sphere. All points of the map on the surface of the sphere, EXCEPT the north pole, are colored (using four colors). Because the north pole was contained within

31st year (1990-2020) of publication of SOLSTICE:
An Electronic Journal of Geography and Mathematics
Volume XXXI, Number 2; Monday, December 21, 2020; 4:02 am, Central Time

a region of the map (not on a boundary), the north pole inherits the color of its surrounding region. Hence, no more than four colors were required to color the map on the sphere.

References

- Appel, K., and Haken, W., "A proof of the 4-color theorem," *Discrete Mathematics* **16** (1976).
- Appel, K. and Haken, W., "Every planar map is four colorable," *Bulletin of the American Mathematical Society* **82**, (1976), pp. 711–712.
- Appel, K. and Haken, W., " The Four Color Problem," in *Mathematics Today* (L. Steen, ed.), Springer-Verlag, New York, 1976.
- Arlinghaus, S. L.; Arlinghaus, W. C.; and Harary, F. 2002. Graph Theory in Geography: An Interactive View. Ebook. John Wiley and Sons. https://www.wiley.com/en-us/exportProduct/pdf/9780471411895 Archived in Deep Blue, persistent archive of The University of Michigan. Pirelli Finalist: Link
- De Morgan, A. Letter to W. Hamilton. 1852. Referred to in: http://assets.press.princeton.edu/chapters/s10116.pdf
- Heesch, Heinrich, *Untersuchungen zum Vierfarbenproblem*, Bibliographisches Institut, Mannheim 1969.
- Hilbert, D., and Cohn-Vossen, S.. *Geometry and the Imagination,* Chelsea, New York, 1952.
- Jordan, C., "Sur les assemblages de lignes," *J. Reine Angew. Math.* **70**(1869), pp. 185–190.
- Kempe, A. B., "On the geographical problem of four colors," *Amer. J. Math.* **2** (1879), pp. 193–204.

31st year (1990-2020) of publication of SOLSTICE:
An Electronic Journal of Geography and Mathematics
Volume XXXI, Number 2; Monday, December 21, 2020; 4:02 am, Central Time

Appendix: Graph Theory Background

Definitions

- **Adjacent**
  - Two nodes in a graph are adjacent if there is an edge joining them.
  - Two nodes in a digraph are adjacent if there is an arc joining them. If there is an arc from node *u* to node *v*, *u* is adjacent to *v* and *v* is adjacent from *u*.
  - Two edges are adjacent if they are incident with a common node.
- **bipartite graph**

  A graph whose set of nodes can be divided into 2 subsets such that each edge has one node in each of the subsets**.**
- **coloring**

  An assignment of different colors to the vertices of a graph so that edges which are adjacent have different colors. If different geographic regions are viewed as single nodes, and regions are adjacent if they have a common border of more than one point, assigning colors to the graph thus defined colors the regions, also.
- **complete graph with n nodes (K_n)**

  The graph with n nodes containing all of the $n(n-1)/2$ possible edges; that is, every node is adjacent to every node and is of degree $n - 1$.
- **component**

  A maximal connected subgraph of a graph.
- **connected**

  A graph is connected if there is a walk between any two nodes. A connected graph has only one component.
- **degree (in-degree, out-degree) of a node**
  - In a graph, the degree is the number of nodes adjacent with the node.
  - In a digraph, the indegree is the number of nodes adjacent to the node, and the outdegree is the number of nodes adjacent from the node.

31st year (1990-2020) of publication of SOLSTICE:
An Electronic Journal of Geography and Mathematics
Volume XXXI, Number 2; Monday, December 21, 2020; 4:02 am, Central Time

- **dual graph**
  A graph obtained from a plane map by placing a node inside each region and joining nodes which lie in adjacent regions.
- **graph**
  A set of nodes *V* and edges *E* such that each edge is an undirected connection between two nodes.
- **loop**
  An edge joining a node to itself.
- **planar**
  A graph is called planar if it can be drawn in the plane without any edges crossing except at nodes with which they are incident.
- **Schlegel diagram**
  A planar representation of a regular polyhedron.
- **subgraph**
  A subgraph *H* of a graph *G* is a graph having all of its nodes and edges in *G*.
- **tetrahedron**
  A four-sided polyhedron with four triangular faces.
- **Walk**
  - An alternating sequence of nodes and edges, beginning and ending with nodes, such that an edge in the sequence is in fact an edge between the nodes it is listed between. Given this convention, one often lists only a sequence of edges, with the proviso that consecutive edges are incident.
  - A walk is called closed if the first and last node are the same. Otherwise it is open.
  - A walk is called a trail if all the edges are distinct.
  - If all the nodes are distinct (except perhaps the first being also the last), the walk is called a path.

In digraphs, walks are directed; that is, each arc must be *from* the node preceding it in the sequence *to* the node following it. A sequence without this added restriction is called a semiwalk.

31st year (1990-2020) of publication of SOLSTICE:
An Electronic Journal of Geography and Mathematics
Volume XXXI, Number 2; Monday, December 21, 2020; 4:02 am, Central Time

Theorem

Theorem App1. (Euler's Formula). Let *G* be a connected planar graph (loops and multiple edges allowed). Suppose *G* has *v* nodes, *e* edges, and *f* faces. Then $v = e - f + 2$.

This theorem is essential to the study of planarity and in fact was known in its classical context long before graph theory existed as a separate discipline of mathematics (Hilbert and Cohn-Vossen).

Corollary App1. If *G* has no loops, then the following inequality holds: $e \leq 3v - 6$.

Proof: Since *G* has no loops, each face is at least a triangle and so bounded by at least 3 edges. Since each edge is on 2 faces, this guarantees $2e \geq 3f$. (Recall that in the tetrahedron, octahedron, and icosahedron, which had all faces triangular, we had $2e = 3f$.) Since

$f = e - v + 2$

$3f = 3e - 3v + 6 \leq 2e$

Thus, $e \leq 3v - 6$.

Theorem App2. Jordan Curve Theorem. A simple closed curve *J* in the plane separates the plane into two distinct domains, each with boundary *J*.

*Materials in this article selected from Arlinghaus, Arlinghaus, and Harary (2002) and modified for the more general audience reader while retaining the technical detail necessary to execute proof.

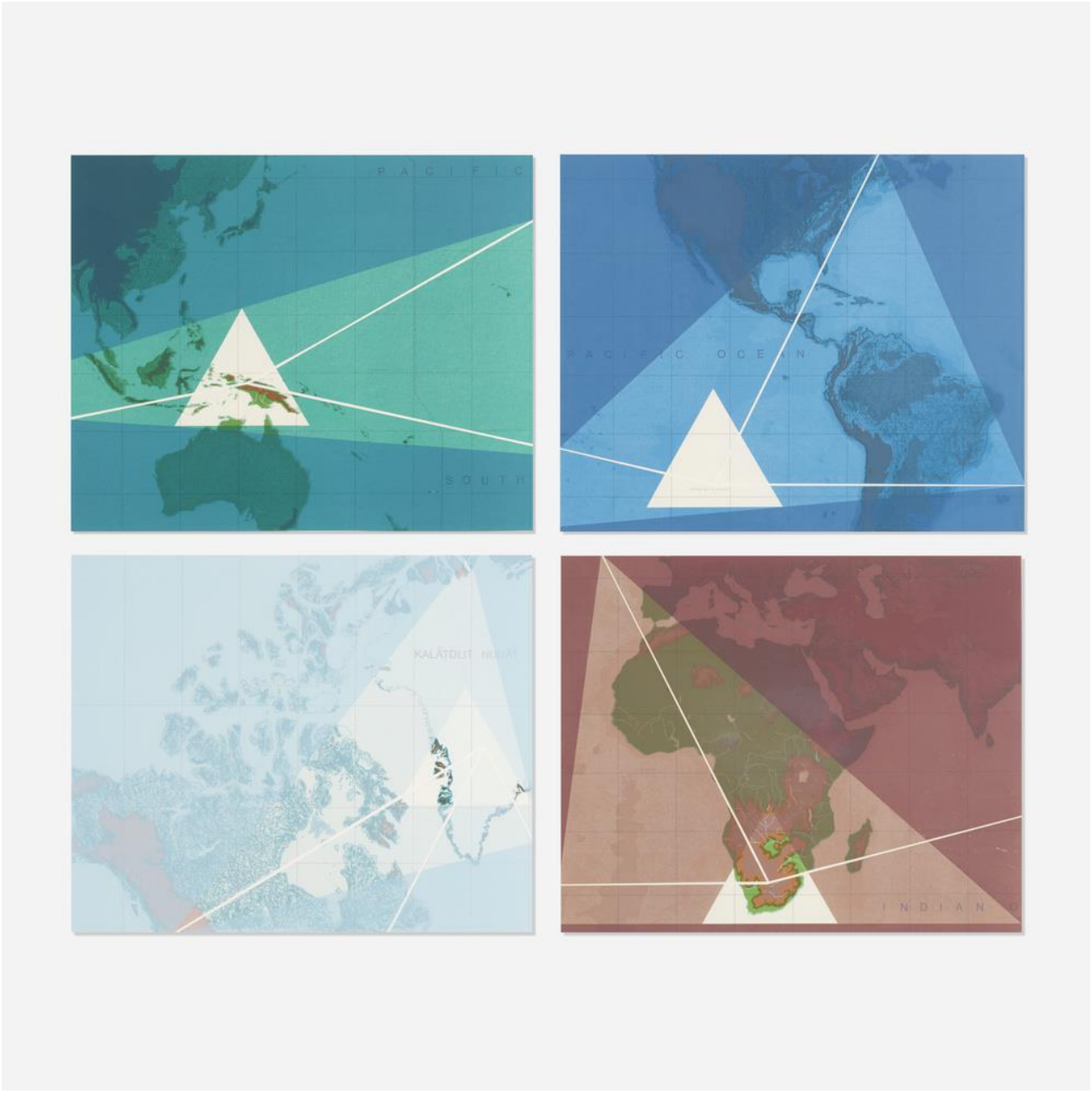
31st year (1990-2020) of publication of SOLSTICE:
An Electronic Journal of Geography and Mathematics
Volume XXXI, Number 2; Monday, December 21, 2020; 4:02 am, Central Time

Selections: From the IMaGe Archive—Global Art*

1. **The "Four Corners" and "SunSweep" Projects by Michigan Sculptor David Barr (1940-2015).**
   - ***Four Corners Project***: Link to filmed project description. "In Celebration: The Four Corners Project by David Barr." An Archives of American Art, Smithsonian Institution, Production.
   - Suite of impressionistic lithographs (by Barr) suggestive of actual physical vertex placement on each of four corners in Irian Jaya, Easter Island, Greenland, and Kalahari Desert. Source of image: https://www.wright20.com/auctions/2016/04/structurist-the-art-of-david-barr/145
   - Associated Institute of Mathematical Geography Publications:
     - Arlinghaus, Sandra L. and Nystuen, John D. 1986. The Mathematics of David Barr's 'Four Corners Project.' Institute of Mathematical Geography, Monograph #1. https://deepblue.lib.umich.edu/handle/2027.42/58275
     - Arlinghaus, Sandra L. 2009. "The Platonic Solids: Earth-sculpture Anchored at Easter Island (Barr's Condition). *Solstice: An Electronic Journal of Geography and Mathematics*, Volume XX, Number 1, June, 2009. http://www.mylovedone.com/IMaGe/Monograph1/PlatonicSolids.html Persistent URL: The University of Michigan, Deep Blue, Institute of Mathematical Geography, http://deepblue.lib.umich.edu/handle/2027.42/58219
     - Arlinghaus, Sandra L. 2016. "Global Singularity: In Memory of David Barr." *Solstice: An Electronic Journal of Geography and Mathematics*, Volume XXVII, Number 1, June, 2016. http://www.mylovedone.com/image/solstice/sum16/GoogleGlobeSingularity/ArlinghausSingularitySolstice2016.html Persistent URL: The University of Michigan, Deep Blue, Institute of Mathematical Geography, http://deepblue.lib.umich.edu/handle/2027.42/58219
     - Arlinghaus, Sandra L., Barr, David, and Nystuen, John D. The Spatial Shadow: Light and Dark---Whole and Part. *Solstice: An Electronic Journal of Geography and Mathematics*, Volume II, Number 1, June, 1991. Original TeX file: http://www-

31st year (1990-2020) of publication of SOLSTICE:
An Electronic Journal of Geography and Mathematics
Volume XXXI, Number 2; Monday, December 21, 2020; 4:02 am, Central Time

personal.umich.edu/~copyrght/image/solstice/sols191.html Scanned image of TeX printout appears in: *Solstice: An Electronic Journal of Geography and Mathematics*, Volume II, 1991. http://www-personal.umich.edu/~copyrght/image/monog15/fulltext.pdf Persistent URL: The University of Michigan, Deep Blue, Institute of Mathematical Geography, http://deepblue.lib.umich.edu/handle/2027.42/58219



- ***SunSweep Project***: link to project installation. "Time Is No Object," produced by Double Visions. 1985.
- Impressionistic image of SunSweep project. Lithograph by David Barr, 1985. Source of image: https://artcollection.wayne.edu/picture-of-the-week/david-barr-sunsweep-1985-lithograph-62105

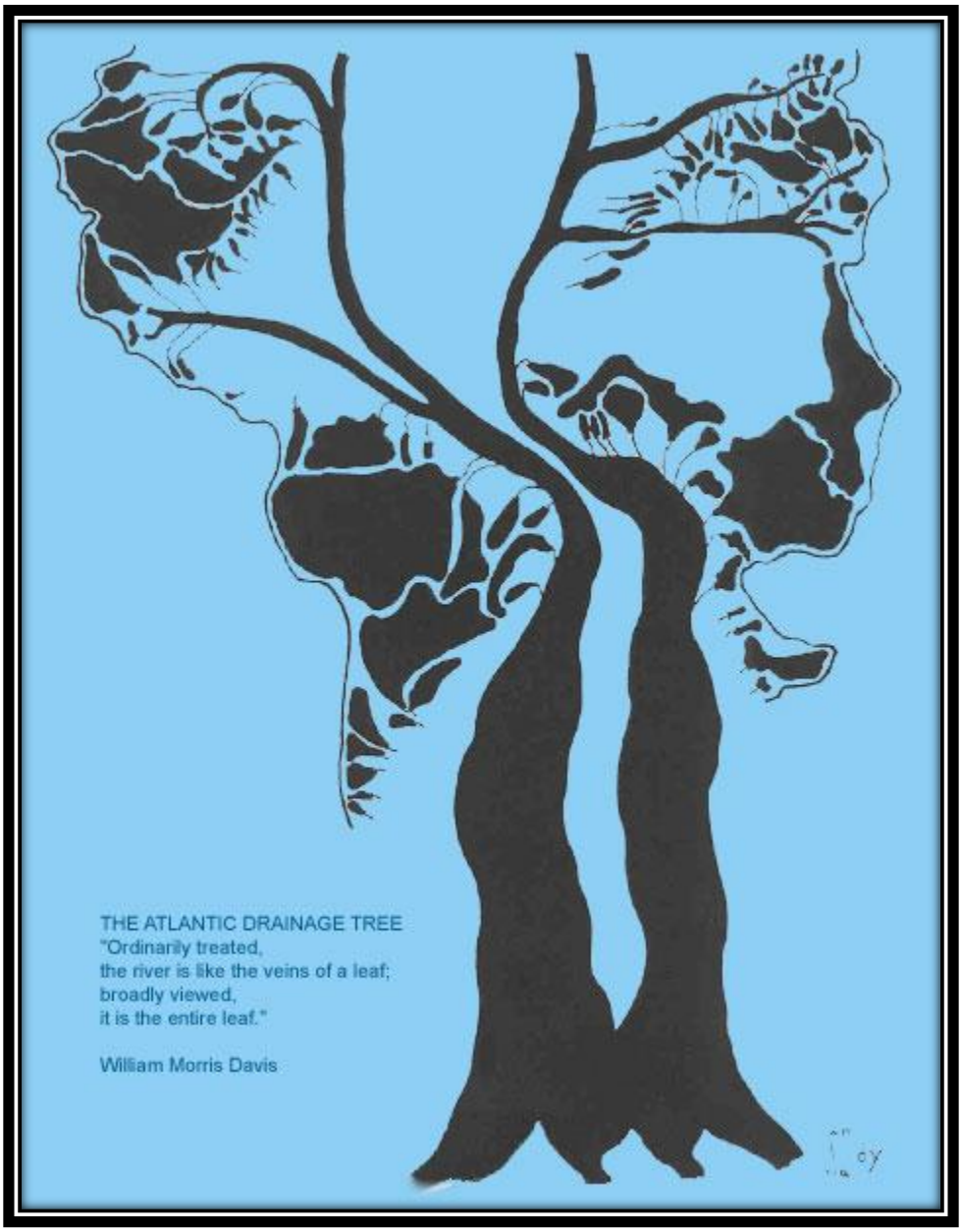
- Associated Institute of Mathematical Geography Publications:
    - Arlinghaus, Sandra L., Barr, David, and Nystuen, John D. The Spatial Shadow: Light and Dark---Whole and Part. *Solstice: An Electronic Journal of Geography and Mathematics*, Volume II, Number 1, June, 1991. Original TeX file: http://www-personal.umich.edu/~copyrght/image/solstice/sols191.html Scanned image of TeX printout appears in: *Solstice: An Electronic Journal of Geography and Mathematics*,

Volume II, 1991. http://www-personal.umich.edu/~copyrght/image/monog15/fulltext.pdf Persistent URL: The University of Michigan, Deep Blue, Institute of Mathematical Geography, http://deepblue.lib.umich.edu/handle/2027.42/58219

- Nystuen, John D. The Photographic Record. SunSweep: A Visit on the Summer Solstice. *Solstice: An Electronic Journal of Geography and Mathematics*, Volume VIII, Number 2, December, 1997. http://www-personal.umich.edu/~copyrght/image/solstice/win97/solsa297.html Persistent URL: The University of Michigan, Deep Blue, Institute of Mathematical Geography, http://deepblue.lib.umich.edu/handle/2027.42/58219

31st year (1990-2020) of publication of SOLSTICE:
An Electronic Journal of Geography and Mathematics
Volume XXXI, Number 2; Monday, December 21, 2020; 4:02 am, Central Time

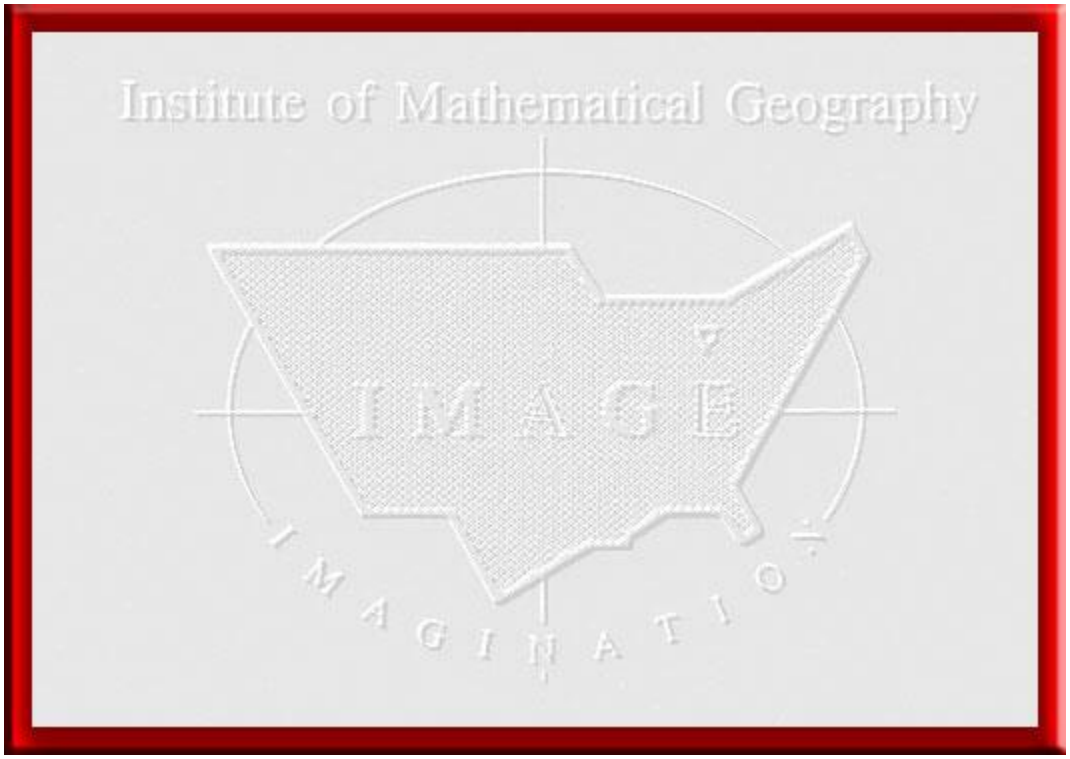
2. **The Atlantic Drainage Tree**; Sandra L. Arlinghaus. Hand-drawn using Indian Ink pen on vellum, circa 1973. Source of an early published version: *Essays on Mathematical Geography*, IMaGe Monograph #5. http://hdl.handle.net/2027.42/58267



*"Selections" curated by Sandra L. Arlinghaus.

31st year (1990-2020) of publication of SOLSTICE:
An Electronic Journal of Geography and Mathematics
Volume XXXI, Number 2; Monday, December 21, 2020; 4:02 am, Central Time

Endmatter



Original logo designed by Sandra L. Arlinghaus and stylized and redrafted by Allen K. Philbrick.

AWARDS AND SELECTED COMMENTS

- ***Solstice* page translated into Belorussian, April, 2016; many thanks to Valerie Bastiaan.**
- ***Solstice* cover materials translated into Ukranian, August 25, 2011; many thanks to Galina Miklosic.**
- ***Solstice* was a Pirelli INTERNETional Award Semi-Finalist, 2001 (top 80 out of over 1000 entries worldwide)**
- **One article in Solstice was a Pirelli INTERNETional Award Semi-Finalist, 2003 (Spatial Synthesis Sampler).**
- ***American Mathematical Monthly*, September 1992, in Telegraphic Reviews section notes *Solstice* as "one of the world's first electronic journals using TeX." L. A. Steen.**
- ***Science News*, 25 January, 1992. Article about *Solstice*.**
- ***Science*, AAAS, 29 November, 1991. Article about *Solstice*.**

31st year (1990-2020) of publication of SOLSTICE:
An Electronic Journal of Geography and Mathematics
Volume XXXI, Number 2; Monday, December 21, 2020; 4:02 am, Central Time

LISTINGS IN DIRECTORIES AND DATABASES

- **IMaGe is listed as a "Collection" in the persistent online archive, DeepBlue, of The University of Michigan library. It is listed under "Mathematical Geography" on the Collections link.**
- ***Solstice* has been listed in the Directory of Open Access Journals (for its first 28 years) maintained by the University of Lund.**
- ***Solstice* is listed on the journals section of the website of the American Mathematical Society, http://www.ams.org/**
- ***Solstice* has been listed in the EBSCO database.**
- **IMaGe has been listed on the website of the Numerical Cartography Lab of The Ohio State University, with thanks to Harold Moellering.**
- ***Solstice* was listed in Geoscience e-Journals, with thanks to Bruno Granier.**

Sandra L. Arlinghaus, celebrating 30 full years of archived *Solstice* publication in 2020.

Moving into a new decade…