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Anti-windup controller design for singularly perturbed systems subject to actuator saturation

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Abstract: This study considers the problem of anti-windup (AW) controller design for singularly perturbed systems with actuator saturation. The AW controller consists of a dynamic state feedback (DSF) controller and an AW compensator. A convex optimisation problem in terms of linear matrix inequalities is formulated to simultaneously design both the DSF controller and the AW compensator. The resulting AW controller depends on the singular perturbation parameter ε and is shown to be well-conditioned for any ε of interest. Furthermore, a two-stage design method is proposed to handle the case that ε is unknown. An ε-independent DSF controller is designed at the first stage, and then an ε-independent AW compensator is constructed by solving a convex optimisation problem. Both of the methods can achieve a desired stability bound and enlarge the basin of attraction at the same time. Finally, examples are given to show the advantages and effectiveness of the obtained results.

1 Introduction

Actuator saturation is a common phenomenon in practical control systems and may lead to system performance's degradation or even instability if not properly accounted for in control design [1–4]. Thus intensive research efforts have been devoted to control systems subject to actuator saturation [5–9]. There are mainly two design strategies: direct design and indirect design. The former considers the actuation saturation at the outset of controller design [10–12], while the latter consists of an anti-windup (AW) compensator and a pre-designed controller achieving desired system performance when saturation does not occur. The indirect design approach attracts more attention of engineers because of its intuitive approach and abroad applicability. Since global stability of control systems with actuator saturation is difficult to achieve when the system is open-loop unstable [6], expanding the basin of attraction is still an open problem. Many of the indirect methods reported in the literature follow the two-step approach [5, 13]: first design a nominal controller without considering actuator saturation, then design the add-on AW compensator. This approach, however, often leads to conservatism in terms of performance and basin of attraction. Recently, the simultaneous design of the nominal controller and the AW compensator has been explored by several research groups, which brought a large basin of attraction and/or better performance [14, 15].

Singularly perturbed systems (SPSs), with a singular perturbation‘ parameter ε determining the degree of separation between the slow and fast modes of the systems, often arise in engineered systems, such as aerospace systems, chemical processes, power systems and so on [16, 17]. Stability problem of SPSs, which is different from that for normal systems, is known as the problem of determining a bound ε_0, such that stability of a given SPS is guaranteed for all $\varepsilon \in (0, \varepsilon_0]$ [18–21]. A key issue for stability analysis and control design of SPSs is to address the possible ill-conditioned numerical challenges resulted from the existence of singular perturbation parameter ε. In standard singular perturbation theory, the SPSs are decomposed into fast and slow reduced-order subsystems and then the analysis and design of the original system are reduced to the corresponding problems for the reduced-order systems. As a result, the ill-conditioned numerical issues are avoided and the computational burden is reduced [16, 17].

SPSs subject to actuator saturation are non-smooth systems, which violates one of the essential assumptions of the standard singular perturbation theory. Thus most available analysis and design methods based on reduced-order subsystems require additional conditions when actuator saturation is involved [22–25]. In [22], the saturation non-linearity is assumed to only rely on the slow state variables, while in [23, 24], the saturation non-linearity is required to only depend on the fast state variables. In [25], the reduced-order systems are constructed without taking saturation into account and a composite controller which can force the closed-loop system to have a linear behaviour in a region is designed. To alleviate these limits, alternative approaches not relying on system decomposition were proposed in [26, 27]. These methods successfully avoid the possible ill-conditioned numerical issues by choosing appropriate Lyapunov functions at the cost of added computational burden. However, all of the preceding results are under the framework of the direct design. To the best knowledge of the authors, AW design (indirect design approach) is still an open problem for SPSs subject to actuator saturation.

In this paper, we will focus on the problem of AW controller design for SPSs subject to actuator saturation. An AW controller is constructed by combining a DSF controller and an AW compensator. We first consider the situation that the singular perturbation parameter ε is known. By an ε-dependent Lyapunov function, simultaneous design of both parts of the AW controller is reduced to solving a convex optimisation problem in the form of LMIs. For the resulting AW controller, the DSF controller gains depend on ε, while the AW compensator gain does not. Since the controller takes advantage of the knowledge of the singular perturbation parameter ε, it can result in a desired stability bound and enlarge the basin of attraction at the same time. Then we pay attention to the case that the singular perturbation parameter ε is unknown. A set of linear matrix inequalities (LMIs) are proposed for designing an ε-independent DSF controller. Based on the resulting DSF controller, an optimisation problem is formulated to design the AW compensator to achieve a desired stability bound while enlarging the basin of attraction. Finally, examples are presented to illustrate the proposed methods.

The rest of this paper is organised as follows: in Section 2, the problem under consideration is formulated and preliminaries are presented. In Sections 3 and 4, approaches to designing

ε-dependent and ε-independent AW controllers are proposed, respectively. The designs are transformed into solving convex optimisation problems. Examples are given in Section 4 to illustrate various features of the proposed methods. Section 5 concludes the paper.

Notation: The superscript T stands for matrix transposition and the notation $M^{-\mathrm{T}}$ denotes the transpose of the inverse matrix of M. For vectors $v, w \in R^p$, $v \preceq w$ means that the inequalities between the vectors are componentwise. $\star$ denotes the block induced by symmetry. For a matrix M, $M_{(i)}$ denotes the ith row of M.

2 Problem formulation and preliminaries

Consider the control system depicted in Fig. 1. The plant is a linear SPS represented by

$$E(\varepsilon)\dot{x}(t) = Ax(t) + Bu_m(t), \tag{1}$$

where $x \in R^n$ is the state, $u_m \in R^q$ is the control input,

$$E(\varepsilon) = \begin{bmatrix} I_{n_1} & 0 \\ 0 & \varepsilon I_{n_2} \end{bmatrix} \in R^{n\times n},$$

$A \in R^{n\times n}$, and $B \in R^{n\times q}$ are constant matrices. The actuator is subject to saturation and thus the control input is generated by $u_m = \mathrm{sat}(u)$ with u being the desired control input and $\mathrm{sat}(\cdot)$ being a componentwise saturation map $R^q \mapsto R^q$ defined as

$$\mathrm{sat}(u_i(t)) = \mathrm{sign}(u_i(t))\min\{1, |u_i(t)|\}, \quad i = 1, 2, \ldots, q. \tag{2}$$

The goal of this paper is the design of an AW controller consisting of a DSF controller

$$\dot{u}(t) = Fx(t) + Gu(t) + \zeta, \tag{3}$$

and an AW compensator

$$\zeta = E_c(\mathrm{sat}(u(t)) - u(t)), \tag{4}$$

where $u(t)$ is the controller state. Matrices $E_c \in R^{q\times q}$, $F \in R^{q\times n}$ and $G \in R^{q\times q}$ are controller gains to be determined.

When the control input is not saturated, i.e. $\mathrm{sat}(u(t)) = u(t)$, the AW controller will serve as a normal controller. The matrix E_c is AW compensator gain, and the matrices F and G are DSF controller gains.

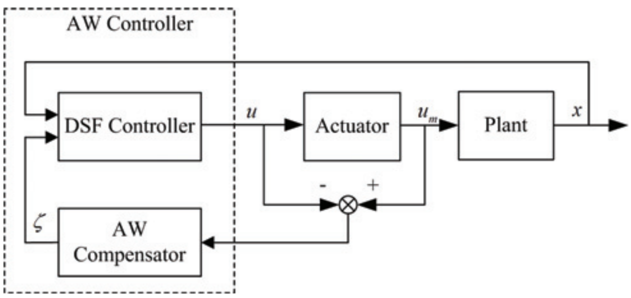

Fig. 1 *AW close-loop system*

The resulting closed-loop (1)–(4) is described as

$$\tilde{E}(\varepsilon)\dot{\eta}(t) = (\hat{A} + I_R K)\eta + (\hat{B} + I_R E_c)\psi(\hat{C}\eta), \tag{5}$$

where $\psi(u(t)) = \mathrm{sat}(u(t)) - u(t)$ is a decentralised dead-zone non-linearity, and

$$\eta = \begin{bmatrix} u \\ x \end{bmatrix}, \quad \tilde{E}(\varepsilon) = \begin{bmatrix} I & 0 \\ 0 & E(\varepsilon) \end{bmatrix}, \quad \hat{A} = \begin{bmatrix} 0 & 0 \\ B & A \end{bmatrix}, \quad \hat{B} = \begin{bmatrix} 0 \\ B \end{bmatrix},$$

$$I_R = \begin{bmatrix} I \\ 0 \end{bmatrix}, \quad \hat{C} = \begin{bmatrix} I & 0 \end{bmatrix}, \quad K = \begin{bmatrix} F & G \end{bmatrix}.$$

The problem under consideration is expressed as:

Problem 1: Given a desired stability bound $\varepsilon_0 > 0$, determine an AW controller (3) and (4) and an ellipsoid $\Omega \subseteq R^n$, as large as possible, such that for any $\varepsilon \in (0, \varepsilon_0]$ the closed-loop system (5) is locally asymptotically stable with Ω contained in the basin of attraction.

The following lemmas will be used in the sequel:

Lemma 1 [28]: For any diagonal positive definite matrix $\Gamma \in R^{q\times q}$, the non-linearity $\psi(v) = sat(v) - v$ satisfies the following inequality

$$\psi^{\mathrm{T}}(v)\Gamma(\psi(v) + w) \le 0, \quad \forall v, w \in S(v_0), \tag{6}$$

where $S(v_0) = \{v, w \in R^q | -v_0 \preceq v - w \preceq v_0\}$ and $v_0 \in R^q$ is given.

Lemma 2 [29]: For a positive scalar ε_0 and symmetric matrices S_1, S_2 and S_3 with appropriate dimensions, if

$$S_1 \ge 0, \tag{7}$$

$$S_1 + \varepsilon_0 S_2 > 0, \tag{8}$$

$$S_1 + \varepsilon_0 S_2 + \varepsilon_0^2 S_3 > 0, \tag{9}$$

hold, then

$$S_1 + \varepsilon S_2 + \varepsilon^2 S_3 > 0, \quad \forall \varepsilon \in (0, \varepsilon_0]. \tag{10}$$

Lemma 3 [29]: If there exist matrices Z_i $(i = 1, 2, \ldots, 5)$ with $Z_i = Z_i^{\mathrm{T}}$ $(i = 1, 2, 3, 4)$ satisfying

$$Z_1 > 0, \tag{11}$$

$$\begin{bmatrix} Z_1 + \varepsilon_0 Z_3 & \varepsilon_0 Z_5^{\mathrm{T}} \\ \varepsilon_0 Z_5 & \varepsilon_0 Z_2 \end{bmatrix} > 0, \tag{12}$$

$$\begin{bmatrix} Z_1 + \varepsilon_0 Z_3 & \varepsilon_0 Z_5^{\mathrm{T}} \\ \varepsilon_0 Z_5 & \varepsilon_0 Z_2 + \varepsilon_0^2 Z_4 \end{bmatrix} > 0, \tag{13}$$

then

$$\tilde{E}(\varepsilon)Z(\varepsilon) = Z^{\mathrm{T}}(\varepsilon)\tilde{E}(\varepsilon) > 0, \quad \forall \varepsilon \in (0, \varepsilon_0], \tag{14}$$

where

$$Z(\varepsilon) = \begin{bmatrix} Z_1 + \varepsilon Z_3 & \varepsilon Z_5^{\mathrm{T}} \\ Z_5 & Z_2 + \varepsilon Z_4 \end{bmatrix}.$$

3 Main results

In this section, we will present two methods to solve Problem 1. First, we consider the case that the singular perturbation parameter ε is known and can be used for controller design. An ε-dependent AW controller is designed. Then we come to the situation that ε is unknown, and propose an ε-independent AW controller design.

3.1 Design of ε-dependent AW controller

In this subsection, we present a convex optimisation problem to simultaneously design the DSF controller gains and the AW compensator gain. The following theorem establishes sufficient conditions for the existence of AW controller (3) and (4), under which the closed-loop system (5) is locally asymptotically stable for any singular perturbation parameter ε of interest with a specified ellipsoid contained in the basin of attraction.

Theorem 1: Given a scalar $\varepsilon_0 > 0$, if there exist a diagonal positive definite matrix $S \in R^{q\times q}$, matrices $Q \in R^{q\times q}$, $M_1 \in R^{q\times(n_1+q)}$, $M_2 \in R^{q\times n_2}$, $Y \in R^{q\times n}$, $Z_1 \in R^{(n_1+q)\times(n_1+q)}$, $Z_2 \in R^{n_2\times n_2}$, $Z_3 \in R^{(n_1+q)\times(n_1+q)}$, $Z_4 \in R^{n_2\times n_2}$, $Z_5 \in R^{n_2\times(n_1+q)}$ with $Z_i = Z_i^{\mathrm{T}}$ $(i = 1, 2, 3, 4)$, such that LMIs

$$\begin{bmatrix} \Phi & \star \\ S\hat{B}^{\mathrm{T}} + QI_R^{\mathrm{T}} - M\tilde{E}(0) - \hat{C}U_1 & -2S \end{bmatrix} < 0, \tag{15}$$

$$\begin{bmatrix} \Theta & \star \\ S\hat{B}^{\mathrm{T}} + QI_R^{\mathrm{T}} - M\tilde{E}(\varepsilon_0) - \hat{C}(U_1 + \varepsilon_0 U_2) & -2S \end{bmatrix} < 0, \tag{16}$$

$$\begin{bmatrix} Z_1 & \star \\ M_{1(i)} & 1 \end{bmatrix} > 0, \quad i = 1, 2, \ldots, q, \tag{17}$$

$$\begin{bmatrix} Z_1 + \varepsilon_0 Z_3 & \star & \star \\ \varepsilon_0 Z_5 & \varepsilon_0 Z_2 & \star \\ M_{1(i)} & \varepsilon_0 M_{2(i)} & 1 \end{bmatrix} > 0, \quad i = 1, 2, \ldots, q, \tag{18}$$

$$\begin{bmatrix} Z_1 + \varepsilon_0 Z_3 & \star & \star \\ \varepsilon_0 Z_5 & \varepsilon_0 Z_2 + \varepsilon_0^2 Z_4 & \star \\ M_{1(i)} & \varepsilon_0 M_{2(i)} & 1 \end{bmatrix} > 0, \quad i = 1, 2, \ldots, q, \tag{19}$$

hold, where

$$U_1 = \begin{bmatrix} Z_1 & 0 \\ Z_5 & Z_2 \end{bmatrix}, \quad U_2 = \begin{bmatrix} Z_3 & Z_5^{\mathrm{T}} \\ 0 & Z_4 \end{bmatrix},$$

$M = \begin{bmatrix} M_1 & M_2 \end{bmatrix}$, $\Phi = U_1^{\mathrm{T}}\hat{A}^{\mathrm{T}} + \hat{A}U_1 + Y^{\mathrm{T}}I_R^{\mathrm{T}} + I_R Y$ and $\Theta = (U_1 + \varepsilon_0 U_2)^{\mathrm{T}}\hat{A}^{\mathrm{T}} + \hat{A}(U_1 + \varepsilon_0 U_2) + Y^{\mathrm{T}}I_R^{\mathrm{T}} + I_R Y$.

Then the AW controller (3) and (4) with $E_c = Q^{\mathrm{T}}S^{-1}, K(\varepsilon) = YZ^{-1}(\varepsilon)$, $Z(\varepsilon) = U_1 + \varepsilon U_2$ stabilises the system (5) for any $\varepsilon \in (0, \varepsilon_0]$. And, the ellipsoid $\Omega(\varepsilon) = \{\eta | \eta^{\mathrm{T}}Z^{-\mathrm{T}}(\varepsilon)\tilde{E}(\varepsilon)\eta \le 1\}$ is within the basin of attraction of the closed-loop system.

Proof: Let $v = \hat{C}\eta$ and $w = M\tilde{E}(\varepsilon)Z^{-1}(\varepsilon)\eta + \hat{C}\eta = (M\tilde{E}(\varepsilon)Z^{-1}(\varepsilon) + \hat{C})\eta$. Then from Lemma 1, the non-linearity $\psi(\hat{C}\eta)$ satisfies

$$\psi^{\mathrm{T}}(\hat{C}\eta)\Gamma(\psi(\hat{C}\eta) + (M\tilde{E}(\varepsilon)Z^{-1}(\varepsilon) + \hat{C})\eta) \le 0, \quad \forall \eta \in S(\rho), \tag{20}$$

where Γ is an arbitrary diagonal positive definite matrix, $S(\rho) = \{\eta | -\rho \preceq M\tilde{E}(\varepsilon)Z^{-1}(\varepsilon)\eta \preceq \rho\}$, and $\rho = \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}^{\mathrm{T}}$.

From Lemma 2, LMIs (17)–(19) imply that

$$\begin{bmatrix} Z^{\mathrm{T}}(\varepsilon)\tilde{E}(\varepsilon) & \star \\ M_{(i)}\tilde{E}(\varepsilon) & 1 \end{bmatrix} > 0, \quad i = 1, 2, \ldots, q, \quad \forall \varepsilon \in (0, \varepsilon_0], \tag{21}$$

which is equivalent to

$$\begin{bmatrix} \tilde{E}^{-1}(\varepsilon)Z^{\mathrm{T}}(\varepsilon) & \star \\ M_{(i)} & 1 \end{bmatrix} > 0, \quad i = 1, 2, \ldots, q, \quad \forall \varepsilon \in (0, \varepsilon_0]. \tag{22}$$

Pre- and post-multiplying (22) by

$$\mathrm{diag}([\tilde{E}^{-1}(\varepsilon)Z^{\mathrm{T}}(\varepsilon)]^{-1}, I)$$

and its transpose, respectively, we have

$$\begin{bmatrix} \tilde{E}(\varepsilon)Z^{-1}(\varepsilon) & \star \\ M_{(i)}\tilde{E}(\varepsilon)Z^{-1}(\varepsilon) & 1 \end{bmatrix} > 0, \quad i = 1, 2, \ldots, q,$$

which implies

$$\tilde{E}(\varepsilon)Z^{-1}(\varepsilon) > Z^{-\mathrm{T}}(\varepsilon)\tilde{E}(\varepsilon)M_{(i)}^{\mathrm{T}}M_{(i)}\tilde{E}(\varepsilon)Z^{-1}(\varepsilon).$$

Then for any $\eta \in \Omega(\varepsilon)$, it holds that

$$\eta^{\mathrm{T}}Z^{-\mathrm{T}}(\varepsilon)\tilde{E}(\varepsilon)M_{(i)}^{\mathrm{T}}M_{(i)}\tilde{E}(\varepsilon)Z^{-1}(\varepsilon)\eta < 1,$$

which implies that $\Omega(\varepsilon) \subseteq S(\rho)$.

LMIs (15) and (16) imply

$$\begin{bmatrix} \Lambda & \star \\ S\hat{B}^{\mathrm{T}} + QI_R^{\mathrm{T}} - M\tilde{E}(\varepsilon) - \hat{C}(U_1 + \varepsilon U_2) & -2S \end{bmatrix} < 0, \quad \forall \varepsilon \in (0, \varepsilon_0], \tag{23}$$

which can be rewritten as

$$\begin{bmatrix} \Lambda & \star \\ S\hat{B}^{\mathrm{T}} + QI_R^{\mathrm{T}} - M\tilde{E}(\varepsilon) - \hat{C}Z(\varepsilon) & -2S \end{bmatrix} < 0, \quad \forall \varepsilon \in (0, \varepsilon_0], \tag{24}$$

where $\Lambda = (U_1 + \varepsilon U_2)^{\mathrm{T}}\hat{A}^{\mathrm{T}} + \hat{A}(U_1 + \varepsilon U_2) + Y^{\mathrm{T}}I_R^{\mathrm{T}} + I_R Y = Z^{\mathrm{T}}(\varepsilon)\hat{A}^{\mathrm{T}} + \hat{A}Z(\varepsilon) + Y^{\mathrm{T}}I_R^{\mathrm{T}} + I_R Y$.

Pre- and post-multiplying (24) by

$$\mathrm{diag}(Z^{-\mathrm{T}}(\varepsilon), S^{-1})$$

and its transpose, respectively, we have

$$\begin{bmatrix} \Phi_1 & \star \\ \Phi_2 & -2S^{-1} \end{bmatrix} < 0, \quad \forall \varepsilon \in (0, \varepsilon_0], \tag{25}$$

where $\Phi_1 = \hat{A}^{\mathrm{T}}Z^{-1}(\varepsilon) + Z^{-\mathrm{T}}(\varepsilon)Y^{\mathrm{T}}I_R^{\mathrm{T}}Z^{-1}(\varepsilon) + Z^{-\mathrm{T}}(\varepsilon)\hat{A} + Z^{-\mathrm{T}}(\varepsilon)I_R YZ^{-1}(\varepsilon)$ and $\Phi_2 = \hat{B}^{\mathrm{T}}Z^{-1}(\varepsilon) + S^{-1}QI_R^{\mathrm{T}}Z^{-1}(\varepsilon) - S^{-1}M\tilde{E}(\varepsilon)Z^{-1}(\varepsilon) - S^{-1}\hat{C}$.

Letting $K(\varepsilon) = YZ^{-1}(\varepsilon), \Gamma = S^{-1}, P(\varepsilon) = Z^{-1}(\varepsilon)$ and $E_c = Q^{\mathrm{T}}S^{-1}$ in (25), we have

$$\Pi \triangleq \begin{bmatrix} \Pi_1 & \star \\ (\hat{B} + I_R E_c)^{\mathrm{T}}P(\varepsilon) - \Gamma M\tilde{E}(\varepsilon)P(\varepsilon) - \Gamma\hat{C} & -2\Gamma \end{bmatrix} < 0, \quad \forall \varepsilon \in (0, \varepsilon_0], \tag{26}$$

where $\Pi_1 = (\hat{A} + I_R K(\varepsilon))^{\mathrm{T}}P(\varepsilon) + P^{\mathrm{T}}(\varepsilon)(\hat{A} + I_R K(\varepsilon))$.

By Lemma 3, LMIs (17)–(19) guarantee that

$$\tilde{E}(\varepsilon)Z(\varepsilon) = Z^{\mathrm{T}}(\varepsilon)\tilde{E}(\varepsilon) > 0, \quad \forall \varepsilon \in (0, \varepsilon_0],$$

which implies

$$\tilde{E}(\varepsilon)P(\varepsilon) = P^{\mathrm{T}}(\varepsilon)\tilde{E}(\varepsilon) > 0, \quad \forall \varepsilon \in (0, \varepsilon_0]. \tag{27}$$

Define an ε-dependent Lyapunov function

$$V(\eta) = \eta^{\mathrm{T}}\tilde{E}(\varepsilon)P(\varepsilon)\eta. \tag{28}$$

Computing the derivative of $V(\eta)$ along the trajectories of system (5) and taking into account (20) and (26), we have

$$\begin{aligned} \dot{V}|_{(5)} &= (\tilde{E}(\varepsilon)\dot{\eta})^{\mathrm{T}}P(\varepsilon)\eta + \eta^{\mathrm{T}}P^{\mathrm{T}}(\varepsilon)\tilde{E}(\varepsilon)\dot{\eta} \\ &= \eta^{\mathrm{T}}((\hat{A} + I_R K)^{\mathrm{T}}P(\varepsilon) + P^{\mathrm{T}}(\varepsilon)(\hat{A} + I_R K))\eta \\ &\quad + 2\psi^{\mathrm{T}}(\hat{C}\eta)(\hat{B} + I_R E_c)^{\mathrm{T}}P(\varepsilon)\eta \\ &\le \eta^{\mathrm{T}}((\hat{A} + I_R K)^{\mathrm{T}}P(\varepsilon) + P^{\mathrm{T}}(\varepsilon)(\hat{A} + I_R K))\eta \\ &\quad + 2\psi^{\mathrm{T}}(\hat{C}\eta)(\hat{B} + I_R E_c)^{\mathrm{T}}P(\varepsilon)\eta \\ &\quad - 2\psi^{\mathrm{T}}(\hat{C}\eta)\Gamma(\psi(\hat{C}\eta) + (M\tilde{E}(\varepsilon)P(\varepsilon) + \hat{C})\eta) \\ &= \begin{bmatrix} \eta \\ \psi \end{bmatrix}^{\mathrm{T}} \Pi \begin{bmatrix} \eta \\ \psi \end{bmatrix} < 0, \quad \forall \varepsilon \in (0, \varepsilon_0], \eta \in \Omega(\varepsilon), \eta \ne 0. \end{aligned} \tag{29}$$

Therefore, the closed-loop system is locally asymptotically stable for any $\varepsilon \in (0, \varepsilon_0]$. Moreover, the ellipsoid $\Omega(\varepsilon)$ is within the basin of attraction of the closed-loop system. □

Remark 1: For many SPSs, the singular perturbation parameter ε has a physical meaning and its value is available for controller design. Thus various ε-dependent controllers have been designed for different kinds of SPSs [27, 29–31]. For such design problems, a key task is to ensure the design procedure and the obtained controller to be well-defined for any allowable singular perturbation parameter. Theorem 1 proposes sufficient conditions for the existence of ε-dependent AW controller (3) and (4). LMIs (15)–(19) depends on the stability bound ε_0, but not the singular perturbation parameter ε. It can be seen that LMI (16) becomes (15), while (18) and (19) are reduced to (17) when ε_0 is small enough. Thus LMIs (15)–(19) are well-conditioned.

Remark 2: From LMIs (17) and (18), it follows that $Z_1 > 0$ and $Z_2 > 0$, which implies that matrix

$$U_1 = \begin{bmatrix} Z_1 & 0 \\ Z_5 & Z_2 \end{bmatrix}$$

is non-singular. In addition, from the proof for Theorem 1, it can be seen that $Z(\varepsilon) = U_1 + \varepsilon U_2$ is non-singular for all $\varepsilon \in (0, \varepsilon_0]$. Therefore, $K(\varepsilon) = Y(U_1 + \varepsilon U_2)^{-1}$ is well-defined for all $\varepsilon \in (0, \varepsilon_0]$ and robust with respect to ε.

Remark 3: On analysis and design of control system with actuator saturation, there are two main approaches for dealing with saturation non-linearity, namely, the sector bound approach and the convex hull approach [6]. The convex hull approach is less conservative than the sector bound approach at the cost of computational complexity. When it comes to anti-windup design, the convex hull approach usually leads to bilinear matrix inequality problem, while the sector bound approach can be reduced to LMI problem which is much easier to solve [6, 32]. Theorem 1 is derived by Lemma 1 which represents a modified sector bound approach. The convex hull approach can be applied to design the AW controller (3) and (4) by a similar way in [32, 33].

With the LMI conditions of Theorem 1, we are interested in obtaining the best estimate of the basin of attraction of the closed-loop system. Following the line of [6], we try to maximise the volume of ellipsoid $\Omega(\varepsilon)$ by solving the following optimisation problem

$$\begin{aligned} &\min_{S,M,Y,U_1,U_2} \lambda \\ &\text{s.t. } (15)-(19), \\ &\qquad \lambda > 0 \text{ and } Z^{-\mathrm{T}}(\varepsilon)\tilde{E}(\varepsilon) < \lambda I. \end{aligned} \tag{30}$$

It can be seen that $Z^{-\mathrm{T}}(\varepsilon)\tilde{E}(\varepsilon) < \lambda I$ with $\lambda > 0$ is equivalent to

$$\begin{bmatrix} Z^{\mathrm{T}}(\varepsilon)\tilde{E}(\varepsilon) & \tilde{E}(\varepsilon) \\ \tilde{E}(\varepsilon) & \lambda I \end{bmatrix} < 0. \tag{31}$$

By Lemma 2, inequality (31) is guaranteed by

$$\begin{bmatrix} Z_1 & \star & \star & \star \\ 0 & 0 & \star & \star \\ I & 0 & \lambda I & \star \\ 0 & 0 & 0 & \lambda I \end{bmatrix} \geq 0, \tag{32}$$

$$\begin{bmatrix} Z_1 + \varepsilon_0 Z_3 & \star & \star & \star \\ \varepsilon_0 Z_5 & \varepsilon_0 Z_2 & \star & \star \\ I & 0 & \lambda I & \star \\ 0 & \varepsilon_0 I & 0 & \lambda I \end{bmatrix} > 0, \tag{33}$$

and

$$\begin{bmatrix} Z_1 + \varepsilon_0 Z_3 & \star & \star & \star \\ \varepsilon_0 Z_5 & \varepsilon_0 Z_2 + \varepsilon_0^2 Z_4 & \star & \star \\ I & 0 & \lambda I & \star \\ 0 & \varepsilon_0 I & 0 & \lambda I \end{bmatrix} > 0. \tag{34}$$

It is easy to see that inequality (32) is equivalent to

$$\begin{bmatrix} Z_1 & \star \\ I & \lambda I \end{bmatrix} > 0. \tag{35}$$

Then the optimisation problem (30) can be reformulated to the following convex optimisation problem

$$\begin{aligned} &\min_{S,M,Y,U_1,U_2} \lambda \\ &\text{s.t. } (15)-(19) \text{ and } (33)-(35). \end{aligned} \tag{36}$$

Remark 4: The convex optimisation problem (36) can be solved by numerical algorithms in polynomial time [34]. With the aid of LMI Toolbox in Matlab, we can solve the convex optimisation problem (36) efficiently.

3.2 Design of ε-independent AW controller

In this subsection, we first present a method for designing ε-independent DSF controller gains, and then formulate a convex optimisation problem to maximise the estimate of the basin of attraction by constructing an appropriate ε-independent anti-windup gain.

Theorem 2: If there exist a diagonal positive definite matrix $S \in R^{q\times q}$, matrices $Q \in R^{q\times q}$, $M_1 \in R^{q\times(n_1+q)}$, $M_2 \in R^{q\times n_2}$, $Y \in R^{q\times n}$, $Z_1 \in R^{(n_1+q)\times(n_1+q)}$, $Z_2 \in R^{n_2\times n_2}$, $Z_5 \in R^{n_2\times(n_1+q)}$ with $Z_i = Z_i^{\mathrm{T}}$ $(i = 1, 2)$, such that LMIs

$$\begin{bmatrix} \Phi & \star \\ S\hat{B}^{\mathrm{T}} + QI_R^{\mathrm{T}} - M\tilde{E}(0) - \hat{C}U_1 & -2S \end{bmatrix} < 0, \tag{37}$$

$$\begin{bmatrix} Z_1 & \star \\ M_{1(i)} & 1 \end{bmatrix} > 0, \quad i = 1, 2, \ldots, q, \tag{38}$$

$$\begin{bmatrix} Z_1 & \star & \star \\ \alpha Z_5 & \alpha Z_2 & \star \\ M_{1(i)} & \alpha M_{2(i)} & 1 \end{bmatrix} > 0, \quad i = 1, 2, \ldots, q, \tag{39}$$

hold, where α is a pre-defined positive scalar,

$$U_1 = \begin{bmatrix} Z_1 & 0 \\ Z_5 & Z_2 \end{bmatrix},$$

$M = \begin{bmatrix} M_1 & M_2 \end{bmatrix}$ and $\Phi = U_1^{\mathrm{T}}\hat{A}^{\mathrm{T}} + \hat{A}U_1 + Y^{\mathrm{T}}I_R^{\mathrm{T}} + I_R Y$.

Then there exists a positive scalar $\varepsilon_0 \leq \alpha$, such that the closed-loop system (5) with $K = \begin{bmatrix} F & G \end{bmatrix} = YU_1^{-1}, E_c = Q^{\mathrm{T}}S^{-1}$ is locally asymptotically stable for any $\varepsilon \in (0, \varepsilon_0]$. Moreover, the ellipsoid $\Omega(\varepsilon_0) = \{\eta | \eta^{\mathrm{T}} P(\varepsilon_0)\eta \leq 1\}$ with

$$P(\varepsilon_0) = \begin{bmatrix} Z_1 & Z_5^{\mathrm{T}} \\ Z_5 & \frac{1}{\varepsilon_0} Z_2 \end{bmatrix}^{-1}$$

is within the basin of attraction of the closed-loop system.

Proof: LMI (39) implies that $Z_2 > 0$. Then by LMIs (37)–(39), there exist a positive scalar $\varepsilon_0 \leq \alpha$, such that

$$\begin{bmatrix} \Theta & \star \\ S\hat{B}^{\mathrm{T}} + QI_R^{\mathrm{T}} - M\tilde{E}(\varepsilon_0) - \hat{C}(U_1 + \varepsilon_0 U_2) & -2S \end{bmatrix} < 0, \tag{40}$$

$$\begin{bmatrix} Z_1 & \star & \star \\ \varepsilon_0 Z_5 & \varepsilon_0 Z_2 & \star \\ M_{1(i)} & \varepsilon_0 M_{2(i)} & 1 \end{bmatrix} > 0, \quad i = 1, 2, \ldots, q, \tag{41}$$

where

$$U_2 = \begin{bmatrix} 0 & Z_5^{\mathrm{T}} \\ 0 & 0 \end{bmatrix},$$

and $\Theta = (U_1 + \varepsilon_0 U_2)^{\mathrm{T}}\hat{A}^{\mathrm{T}} + \hat{A}(U_1 + \varepsilon_0 U_2) + Y^{\mathrm{T}}I_R^{\mathrm{T}} + I_R Y$.

Then using Theorem 1 by setting $Z_4 = 0$, the inequalities (37), (38), (40) and (41) yield that the closed-loop system (5) with $K = \begin{bmatrix} F & G \end{bmatrix} = YU_1^{-1}, E_c = Q^T S^{-1}$ is locally asymptotically stable for any $\varepsilon \in (0, \varepsilon_0]$. Moreover, the ellipsoid $\Omega(\varepsilon) = \{\eta | \eta^T P(\varepsilon)\eta \le 1\}$ with

$$P(\varepsilon) = \begin{bmatrix} Z_1 & Z_5^T \\ Z_5 & \frac{1}{\varepsilon} Z_2 \end{bmatrix}^{-1}$$

is an estimate of the basin of attraction of the closed-loop system.

In addition, it follows from (41) that

$$\begin{bmatrix} Z_1 & Z_5^T \\ Z_5 & \frac{1}{\varepsilon_0} Z_2 \end{bmatrix} > 0. \tag{42}$$

Inequality (42) implies that

$$\begin{bmatrix} Z_1 & Z_5^T \\ Z_5 & \frac{1}{\varepsilon} Z_2 \end{bmatrix} \ge \begin{bmatrix} Z_1 & Z_5^T \\ Z_5 & \frac{1}{\varepsilon_0} Z_2 \end{bmatrix}, \quad \forall \varepsilon \in (0, \varepsilon_0],$$

which is equivalent to

$$\begin{bmatrix} Z_1 & Z_5^T \\ Z_5 & \frac{1}{\varepsilon} Z_2 \end{bmatrix}^{-1} \le \begin{bmatrix} Z_1 & Z_5^T \\ Z_5 & \frac{1}{\varepsilon_0} Z_2 \end{bmatrix}^{-1}, \quad \forall \varepsilon \in (0, \varepsilon_0].$$

Then we have $\Omega(\varepsilon_0) \subseteq \Omega(\varepsilon)$, $\forall \varepsilon \in (0, \varepsilon_0]$. That is, the ellipsoid $\Omega(\varepsilon_0)$ is within the basin of attraction of the closed-loop system for any $\varepsilon \in (0, \varepsilon_0]$. This completes the proof. □

Remark 5: In the proof of Theorem 2, LMI (39) is not involved, which means that the conclusion of Theorem 2 still hold even if LMI (39) is omitted. We notice that LMIs (37) and (38) with $Z_2 > 0$ are sufficient conditions for the existence of AW controllers, but do not take into account the enlargement of the stability bound and basin of attraction. To get satisfactory stability bound and basin of attraction, we introduce LMI (39) which is closely related with the stability bound and the structure of basin of attraction.

By Theorem 2, we can get an ε-independent AW controller, under which the closed-loop system is locally asymptotically stable for small enough ε. With the obtained ε-independent gains F and G, the following theorem presents a method for constructing a new AW compensator gain E_c to achieve a desired stability bound and enlarge the basin of attraction.

Theorem 3: Given a pre-defined stability bound $\varepsilon_0 > 0$, and DSF controller gains F and G, if there exist a diagonal positive definite matrix $S \in R^{q\times q}$, matrices $Q \in R^{q\times q}$, $M_1 \in R^{q\times(n_1+q)}$, $M_2 \in R^{q\times n_2}$, $Y \in R^{q\times n}$, $Z_1 \in R^{(n_1+q)\times(n_1+q)}, Z_2 \in R^{n_2\times n_2}$, $Z_5 \in R^{n_2\times(n_1+q)}$ with $Z_i = Z_i^T$ $(i = 1, 2)$, such that LMIs

$$\begin{bmatrix} \Phi & \star \\ S\hat{B}^T + QI_R^T - M\tilde{E}(0) - \hat{C}U_1 & -2S \end{bmatrix} < 0, \tag{43}$$

$$\begin{bmatrix} \Theta & \star \\ S\hat{B}^T + QI_R^T - M\tilde{E}(\varepsilon_0) - \hat{C}(U_1 + \varepsilon_0 U_2) & -2S \end{bmatrix} < 0, \tag{44}$$

$$\begin{bmatrix} Z_1 & \star \\ M_{1(i)} & 1 \end{bmatrix} > 0, \quad i = 1, 2, \ldots, q, \tag{45}$$

$$\begin{bmatrix} Z_1 & \star & \star \\ \varepsilon_0 Z_5 & \varepsilon_0 Z_2 & \star \\ M_{1(i)} & \varepsilon_0 M_{2(i)} & 1 \end{bmatrix} > 0, \quad i = 1, 2, \ldots, q, \tag{46}$$

hold, where

$$U_1 = \begin{bmatrix} Z_1 & 0 \\ Z_5 & Z_2 \end{bmatrix}, \quad U_2 = \begin{bmatrix} 0 & Z_5^T \\ 0 & 0 \end{bmatrix},$$

$M = \begin{bmatrix} M_1 & M_2 \end{bmatrix}$ and $\Phi = U_1^T(\hat{A} + I_R K)^T + (\hat{A} + I_R K)U_1$ and $\Theta = (U_1 + \varepsilon_0 U_2)^T(\hat{A} + I_R K)^T + (\hat{A} + I_R K)(U_1 + \varepsilon_0 U_2)$.

Then the closed-loop system (5) with $K = \begin{bmatrix} F & G \end{bmatrix}, E_c = Q^T S^{-1}$ is locally asymptotically stable for any $\varepsilon \in (0, \varepsilon_0]$. Moreover, the ellipsoid $\Omega(\varepsilon_0) = \{\eta | \eta^T P(\varepsilon_0)\eta \le 1\}$ with

$$P(\varepsilon_0) = \begin{bmatrix} Z_1 & Z_5^T \\ Z_5 & \frac{1}{\varepsilon_0} Z_2 \end{bmatrix}^{-1}$$

is within the basin of attraction of the closed-loop system.

Using Theorem 3 and similar to the last subsection, the following convex optimisation problem can be used to maximise the estimate of the basin of attraction

$$\begin{aligned} &\min_{S,M,Y,U_1,U_2} \lambda \\ &\text{s.t. } (43)-(46) \text{ and} \\ &\begin{bmatrix} Z_1 & \star \\ I & \lambda I \end{bmatrix} > 0, \\ &\begin{bmatrix} Z_1 & \star & \star & \star \\ \varepsilon_0 Z_5 & \varepsilon_0 Z_2 & \star & \star \\ I & 0 & \lambda I & \star \\ 0 & \varepsilon_0 I & 0 & \lambda I \end{bmatrix} > 0. \end{aligned} \tag{47}$$

Remark 6: The approach proposed in the previous subsection takes advantage of the knowledge of the singular perturbation parameter ε and designs the DSF controller and AW compensator simultaneously. This subsection presents a two-stage method for designing ε-independent AW controller. The ε-dependent AW controller can result in larger stability bound and estimate of basin of attraction while ε-independent AW controller can be applied to the case that ε is unknown.

4 Examples

In this section, two examples are given to illustrate various features of the proposed methods and show their advantages over the existing results.

Example 1: This example will demonstrate the advantage of the ε-dependent AW controller over the existing results. Consider an inverted pendulum system controlled by a DC motor via a gear train. The model, which was first established in [35], is described by

$$\begin{cases} \dot{x}_1(t) = x_2(t), \\ \dot{x}_2(t) = \frac{g}{l}\sin(x_1(t)) + \frac{NK_m}{ml^2}x_3(t), \\ L_a\dot{x}_3(t) = -K_b N x_2(t) - R_a x_3(t) + u(t), \end{cases} \tag{48}$$

where $x_1(t) = \theta_p(t)$ denotes the angle (rad) of the pendulum from the vertical upward, $x_2(t) = \dot{\theta}_p(t), x_3(t) = I_a(t)$ denotes the current of the motor, $u(t)$ is the control input voltage, K_m is the motor torque constant, K_b is the back emf constant, N is the gear ratio and L_a is the inductance which is usually a small positive constant.

The parameters for the plant are as follows: $g = 9.8\,\text{m/s}^2$, $N = 50$, $l = 1\,\text{m}$, $m = 1\,\text{kg}$, $K_m = 0.1\,\text{Nm/A}$, $K_b = 0.1\,\text{Vs/rad}$, $R_a = 1\ \Omega$ and $L_a = 0.05\,\text{H}$ and the input voltage is required to satisfy $|u| \le 1$. Note that L_a represents the singular perturbation parameter of the system. With these parameters, the linearised system of (48) is as follows

$$\begin{cases} \dot{x}_1(t) = x_2(t), \\ \dot{x}_2(t) = 9.8\,x_1(t) + x_3(t), \\ \varepsilon\dot{x}_3(t) = -x_2(t) - x_3(t) + u, \end{cases} \tag{49}$$

where $\varepsilon = L_a$.

The equilibrium point of system (49), that is, $x_e = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}$ corresponds to the upright rest position of the inverted pendulum. We will design a controller to balance the pendulum around its upright rest position.

For system (49), the following static state feedback controller was proposed in [27]

$$u = \begin{bmatrix} -2920.1 & -813.0 & -44.6 \end{bmatrix} x$$

and the obtained estimate of the basin of attraction is $\Omega_1 = \{x \in R^3 | x^{\mathrm{T}} P_1 x \le 1\}$, with

$$P_1 = \begin{bmatrix} 153.7649 & 42.9750 & 1.8181 \\ 42.9750 & 12.0363 & 0.5066 \\ 1.8181 & 0.5066 & 0.0291 \end{bmatrix}.$$

We now use Theorem 1 to design an AW controller (3) and (4) for system (49). To do this, we first augment system (49) into the form of (5). The obtained system matrices are as follows

$$\tilde{E}(\varepsilon) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \varepsilon \end{bmatrix}, \quad \hat{A} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 9.8 & 0 & 1 \\ 1 & 0 & -1 & -1 \end{bmatrix},$$

$$\hat{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad I_R = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \hat{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}.$$

Solving the optimisation problem (36) with $\varepsilon_0 = 0.1$, we have

$$Z_1 = \begin{bmatrix} 1079.1000 & -1.0000 & 2.5000 \\ -1.0000 & 4.2000 & -15.2000 \\ 2.5000 & -15.2000 & 55.0000 \end{bmatrix},$$

$$Z_3 = \begin{bmatrix} 4.8711 & -2.7063 & 8.7456 \\ -2.7063 & 7.2440 & -19.5834 \\ 8.7456 & -19.5834 & 52.6286 \end{bmatrix},$$

$$Z_2 = 7.3379, \quad Z_4 = -0.6264, \quad Z_5 = [1.6524 \; -4.3844 \; 9.4953],$$

$$Y = \begin{bmatrix} -427.9000 & 0.8000 & -1.6000 & -1077.8000 \end{bmatrix},$$

$$M_1 = \begin{bmatrix} -3.0688 & 0.0152 & 0.2274 \end{bmatrix}, \quad M_2 = -0.0895,$$

$$S = 0.7323, Q = 1076.1000, \lambda = 181.6076.$$

Taking into account $\varepsilon = 0.05$, the AW controller gains are as follows

$$K = \begin{bmatrix} F & G \end{bmatrix}$$
$$= \begin{bmatrix} -9.0000 & -39419.0000 & -10988.0000 & -616.0000 \end{bmatrix},$$
$$E_c = 1469.4000.$$

An estimate of the basin of attraction of the augmented closed-loop system is $\Omega = \{\eta \in R^4 | \eta^{\mathrm{T}} P \eta \le 1\}$, where

$$P = \begin{bmatrix} 0.0009 & 0.0358 & 0.0100 & 0.0004 \\ 0.0358 & 154.5060 & 43.1649 & 1.8306 \\ 0.0100 & 43.1649 & 12.0767 & 0.5103 \\ 0.0004 & 1.8306 & 0.5103 & 0.0286 \end{bmatrix}.$$

To compare the ellipsoids Ω_1 and Ω intuitively, we make the projection of Ω on the coordinates x, which leads to an ellipsoid $\Omega_2 = \{x \in R^3 | x^{\mathrm{T}} P_2 x \le 1\}$. According to Lemma 5 of [24], $P_2 = P_{22} - P_{12}^{\mathrm{T}} P_{11}^{-1} P_{12}$ with

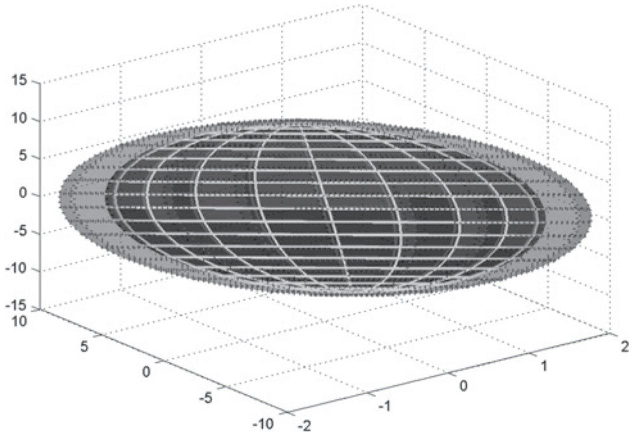

Fig. 2 *Comparison of the ellipsoids Ω_1 and Ω_2*

$$P_{11} = 0.0009, \quad P_{12} = \begin{bmatrix} 0.0358 & 0.0100 & 0.0004 \end{bmatrix},$$

$$P_{22} = \begin{bmatrix} 154.5060 & 43.1649 & 1.8306 \\ 43.1649 & 12.0767 & 0.5103 \\ 1.8306 & 0.5103 & 0.0286 \end{bmatrix}.$$

By simple calculation, we have

$$P_2 = \begin{bmatrix} 153.0820 & 42.7671 & 1.8147 \\ 42.7671 & 11.9656 & 0.5059 \\ 1.8147 & 0.5059 & 0.0284 \end{bmatrix}.$$

Fig. 2 depicts the ellipsoids Ω_1 and Ω_2, which shows that the estimate of the basin of attraction obtained by Theorem 1 is larger than that given by Yang *et al.* [27].

Example 2: This example will show how the two-stage method in Section 3.2 is applied to the inverted pendulum system (49). When the singular perturbation parameter ε is not known, the AW controller designed by Theorem 1 cannot be used to stabilise the system. The two-stage method in Section 3.2 is useful in this case.

First, Theorem 2 is used to construct ε-independent DSF controller gains. Solving LMIs (37)–(39)with $\alpha = 0.1$ leads to ε-independent DSF controller gains

$$K = \begin{bmatrix} F & G \end{bmatrix} = \begin{bmatrix} -3.0067 & -1718.1357 & -489.5623 & -34.1096 \end{bmatrix}.$$

Then, with the obtained gain matrix K, the optimisation problem (47) is used to design ε-independent AW compensator gain to achieve a desired stability bound and enlarge the estimate of basin of attraction. Solving the optimisation problem (47) with the obtained matrix K and $\varepsilon_0 = 0.05$ results in the AW compensator gain $E_c = 149.8265$ and an estimate of basin of attraction $\hat{\Omega} = \{\eta \in R^4 | \eta^{\mathrm{T}} \hat{P} \eta \le 1\}$, where

$$\hat{P} = \begin{bmatrix} 0.0874 & 1.6460 & 0.4349 & -0.0431 \\ 1.6460 & 419.1652 & 119.2192 & 3.3353 \\ 0.4349 & 119.2192 & 34.3254 & 0.9759 \\ -0.0431 & 3.3353 & 0.9759 & 0.1750 \end{bmatrix}.$$

According to Theorem 3, the closed-loop system is locally asymptotically stable for any $\varepsilon \in (0, 0.05]$ and the ellipsoid $\hat{\Omega}$ is within the basin of attraction. It is easy to show that $\eta_0 = \begin{bmatrix} 0 & -0.43 & 1.5 & -0.75 \end{bmatrix}^{\mathrm{T}} \in \hat{\Omega}$. Figs. 3 and 4 depict the state response and control signal starting from η_0 for $\varepsilon = 0.03$ and $\varepsilon = 0.05$, respectively. It can be seen that the trajectories converge to the origin in both cases.

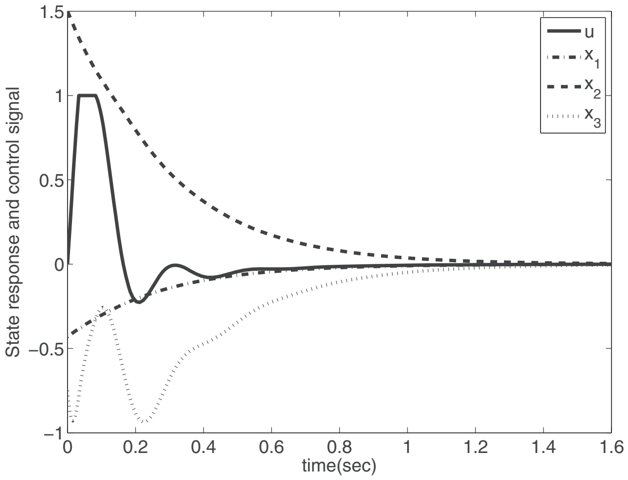

Fig. 3 *State response and control signal with $\varepsilon = 0.03$*

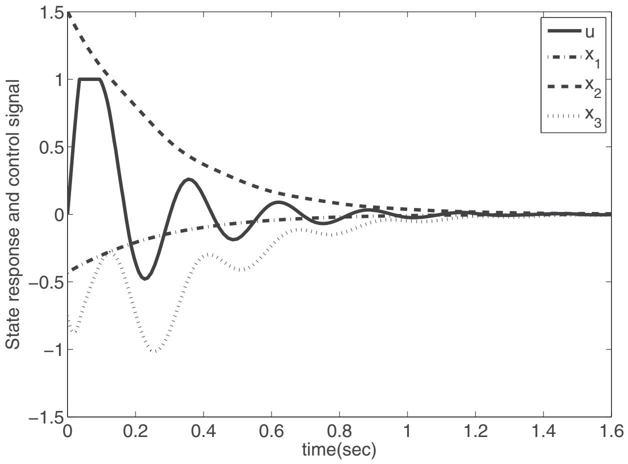

Fig. 4 *State response and control signal with $\varepsilon = 0.05$*

When we disregard LMI condition (39) in the first stage, we get an estimate of the basin of attraction $\bar{\Omega} = \{\eta \in R^4 | \eta^{\mathrm{T}} \bar{P} \eta \le 1\}$ corresponding to stability bound $\varepsilon_0 = 0.05$, where

$$\bar{P} = \begin{bmatrix} 0.2824 & 12.8211 & 3.6714 & -0.0682 \\ 12.8211 & 2893.1759 & 793.6079 & 6.3982 \\ 3.6714 & 793.6079 & 223.4684 & 1.7982 \\ -0.0682 & 6.3982 & 1.7982 & 0.3969 \end{bmatrix}.$$

It is easy to see that the ellipsoid $\bar{\Omega}$ is much smaller than $\hat{\Omega}$. Thus LMI condition (39) plays a key role in the two-stage design.

5 Conclusion

This paper presents approaches to design AW controllers consisting of a dynamic state feedback controller and an AW compensator for SPSs subject to actuator saturation. When the singular perturbation parameter ε is known, the two parts of the AW controller can be designed simultaneously by solving a convex optimisation problem. When ε is unknown, the dynamic state feedback controller and AW compensator can be designed sequentially. Both designs can lead to a desired stability bound and enlarge the basin of attraction of the closed-loop systems. The presented examples have illustrated the proposed methods.

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