

Understanding Complex Design Models through Bayesian Networks and Network Theory

by

Claire Marie Wincott

A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
(Naval Architecture and Marine Engineering)
in The University of Michigan
2021

Doctoral Committee:

Associate Professor Matthew D. Collette, Chair
Assistant Professor Jean-Baptiste Jeannin
Associate Professor David J. Singer
Professor of Practice Donald Winter

Claire Marie Wincott

cwincott@umich.edu

ORCID iD: [0000-0002-4039-8813](https://orcid.org/0000-0002-4039-8813)

© Claire Marie Wincott 2021

ACKNOWLEDGEMENTS

When first deciding what to study as an incoming freshman, I could be heard declaring I would be an engineer because I could complete just four more years of school and never be in a classroom again. As that naive freshman I failed to understand the power that a community could have on my desire to learn. I sort of happened upon the Naval Architecture and Marine Engineering department as a freshman and had no idea that the decision to set up an informational meeting with Warren Noone would change my life. I immediately found a community within NA&ME: professors that valued the how and why of an approach to a problem over a perfect final solution; classmates that became more of a family and support system than I could ever imagine and a staff that has supported me far beyond advising what classes to take or paperwork to fill out. All of this came with the ability to study under world-class experts, to travel the world and to explore passions far beyond engineering; for that, I can never express enough gratitude.

I would like to thank chair of my committee and advisor, Prof. Matthew Collette. Prof. Collette has a distinct skill in teaching and advising, and I am so grateful to work under someone so equally dedicated to both research and education. I'd also like to thank the rest of my committee, Dr. David Singer, Dr. Jean-Baptiste Jeannin and Dr. Donald Winter for their support academically and professionally.

I would not have made it sanely through the last five years, and especially 2020, without the friendship and support of Dr. Sam Taylordean, Dr. Matt Schirmann, Dr. Sam Edwards, Dan Sinnott, James Coller and Sammi Meister. Dr. Lauren Claus

and Dr. Kaihua Zhang provided great guidance as I started my journey into graduate school.

Finally, I would like to thank the support from my family. They have always ensured I could pursue any path I dreamed of, even when that path changed frequently. For that, I will be forever grateful.

TABLE OF CONTENTS

ACKNOWLEDGEMENTS	ii
LIST OF FIGURES	vi
LIST OF TABLES	x
LIST OF APPENDICES	xiv
ABSTRACT	xv
CHAPTER	
I. Introduction	1
1.1 Computer Aided Design of Physically Large and Complex Sys- tems	2
1.1.1 Bayesian Networks in Design	3
1.1.2 Network Analysis in Design	4
1.2 Research Objectives	7
II. Bayesian Networks	8
2.1 Observations and Updating	10
2.2 Learning BN	12
2.3 Methods Used	15
III. Network Analysis	16
3.1 Network Science Tools	16
3.2 Comparison for Consistency	20
IV. Developed Methodology	23
4.1 Converting Conditional Probabilities to Weighted Edges	23

4.2	Identifying and Chunking Redundant Nodes of the Bayesian Network	25
4.3	Complete Framework for Analyzing Computer Aided Design Model with Network Analysis	29
V. Osyczka and Kundu Design Problem		32
5.1	The Design Problem	32
5.2	Methodology	33
5.3	Consistency of Results	35
5.4	Accuracy of Results	38
5.5	Lessons Learned	53
VI. Sen and Yang Bulk Carrier Design Problem		54
6.1	The Design Problem	54
6.2	Initial Methodology	55
6.2.1	Initial Results	58
6.3	Chunking	64
6.3.1	Methodology	64
6.3.2	Chunking Thresholds and Consistency	65
6.3.3	Chunking Thresholds and Accuracy	72
6.4	Final Accuracy	76
6.4.1	Community Detection	81
6.4.2	Betweenness Centrality	84
6.4.3	Closeness Centrality	88
6.4.4	Eigenvector Centrality	92
6.5	Lessons Learned	95
VII. Conclusions and Future Work		97
7.1	Meeting the Research Objectives	97
7.2	Contributions	100
7.3	Future Work	100
7.3.1	Observations as Constraints	100
7.3.2	Integration of Chunking into Bayesian Network Learning	101
7.3.3	Exploration of Different Edge Weight Scoring	101
APPENDIX		102
BIBLIOGRAPHY		139

LIST OF FIGURES

Figure

2.1	Bayesian Network of Recreational Boat Characteristics	10
2.2	Updated Bayesian Network of Recreational Boat Characteristics with Information Propagation	11
3.1	Weighted Network and Adjacency Matrix	19
4.1	Redundant Nodes Example	27
4.2	Process of transforming complex design data into simple weighted edge networks and defining network metrics	31
5.1	K2 Scored Weighted Edge Network Modeling the Full Design Dataset of the Osyczka and Kundu Problem	39
5.2	Match Distance Scored Weighted Edge Network Modeling the Full Design Dataset of the Osyczka and Kundu Problem	39
5.3	K2 Scored Weighted Edge Network Modeling the Feasible Design Dataset of the Osyczka and Kundu Problem	40
5.4	Match Distance Scored Weighted Edge Network Modeling the Feasible Design Dataset of the Osyczka and Kundu Problem	40
5.5	Feasible Region and Optimal Points for x_1 and x_2	41
5.6	Feasible Region and Optimal Points for x_3 and x_4	41
5.7	Feasible Region and Optimal Points for x_5 and x_6	42
5.8	K2 Scored Weighted Edge Network Modeling the Pareto Design Dataset of the Osyczka and Kundu Problem	43
5.9	Match Distance Scored Weighted Edge Network Modeling the Pareto Design Dataset of the Osyczka and Kundu Problem	43
5.10	Maximum Modularity Communities of K2 Scored Weighted Edge Network Modeling the Full Design Dataset of the Osyczka and Kundu Problem	45
5.11	Maximum Modularity Communities of K2 Scored Weighted Edge Network Modeling the Feasible Design Dataset of the Osyczka and Kundu Problem	45
5.12	Maximum Modularity Communities of K2 Scored Weighted Edge Network Modeling the Pareto Design Dataset of the Osyczka and Kundu Problem	45

5.13	Maximum Modularity Communities of Match Distance Scored Weighted Edge Network Modeling the Full Design Dataset of the Osyzcka and Kundu Problem	46
5.14	Maximum Modularity Communities of Match Distance Scored Weighted Edge Network Modeling the Feasible Design Dataset of the Osyzcka and Kundu Problem	46
5.15	Maximum Modularity Communities of Match Distance Scored Weighted Edge Network Modeling the Pareto Design Dataset of the Osyzcka and Kundu Problem	46
5.16	Betweenness Centrality of K2 Scored Weighted Edge Network Modeling the Full Design Dataset of the Osyzcka and Kundu Problem	48
5.17	Betweenness Centrality of K2 Scored Weighted Edge Network Modeling the Feasible Design Dataset of the Osyzcka and Kundu Problem	48
5.18	Betweenness Centrality of K2 Scored Weighted Edge Network Modeling the Pareto Design Dataset of the Osyzcka and Kundu Problem	48
5.19	Closeness Centrality of K2 Scored Weighted Edge Network Modeling the Full Design Dataset of the Osyzcka and Kundu Problem	49
5.20	Closeness Centrality of K2 Scored Weighted Edge Network Modeling the Feasible Design Dataset of the Osyzcka and Kundu Problem	49
5.21	Closeness Centrality of K2 Scored Weighted Edge Network Modeling the Pareto Design Dataset of the Osyzcka and Kundu Problem	49
5.22	Eigenvector Centrality of K2 Scored Weighted Edge Network Modeling the Full Design Dataset of the Osyzcka and Kundu Problem	52
5.23	Eigenvector Centrality of K2 Scored Weighted Edge Network Modeling the Feasible Design Dataset of the Osyzcka and Kundu Problem	52
5.24	Eigenvector Centrality of K2 Scored Weighted Edge Network Modeling the Pareto Design Dataset of the Osyzcka and Kundu Problem	52
6.1	K2 Scored Weighted Edge Network Modeling the Full Design Dataset of the Bulk Carrier Design Problem, Trial 0	61
6.2	K2 Scored Weighted Edge Network Modeling the Full Design Dataset of the Bulk Carrier Design Problem, Trial 1	61
6.3	Betweenness Centrality of K2 Scored Weighted Edge Network Modeling the Full Design Dataset of the Bulk Carrier Design Problem, Trial 0	63
6.4	Betweenness Centrality of K2 Scored Weighted Edge Network Modeling the Full Design Dataset of the Bulk Carrier Design Problem, Trial 1	63
6.5	Weighted Edge Network Modeling the Full Design Dataset of the Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.99, Trial 1	73
6.6	Weighted Edge Network Modeling the Full Design Dataset of the Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.98, Trial 1	73

6.7	Weighted Edge Network Modeling the Full Design Dataset of the Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.975, Trial 1	73
6.8	Weighted Edge Network Modeling the Feasible Design Dataset of the Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.99, Trial 1	74
6.9	Weighted Edge Network Modeling the Feasible Design Dataset of the Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.98, Trial 1	74
6.10	Weighted Edge Network Modeling the Feasible Design Dataset of the Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.975, Trial 1	74
6.11	Weighted Edge Network Modeling the Pareto Design Dataset of the Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.99, Trial 1	75
6.12	Weighted Edge Network Modeling the Pareto Design Dataset of the Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.98, Trial 1	75
6.13	Weighted Edge Network Modeling the Pareto Design Dataset of the Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.975, Trial 1	75
6.14	Weighted Edge Network Modeling the Full Design Dataset of the Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.99, Trial 1	78
6.15	Weighted Edge Network Modeling the Feasible Design Dataset of the Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.99, Trial 1	78
6.16	Weighted Edge Network Modeling the Pareto Design Dataset of the Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.99, Trial 1	79
6.17	Maximum Modularity Communitites of Weighted Edge Network Modeling the Full Design Dataset of the Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.99, Trial 1	82
6.18	Maximum Modularity Communitites of Weighted Edge Network Modeling the Feasible Design Dataset of the Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.99, Trial 1	83
6.19	Maximum Modularity Communitites of Weighted Edge Network Modeling the Pareto Design Dataset of the Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.99, Trial 1	83
6.20	Betweenness Centrality of Weighted Edge Network Modeling the Full Design Dataset of the Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.99, Trial 1	85
6.21	Betweenness Centrality of Weighted Edge Network Modeling the Feasible Design Dataset of the Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.99, Trial 1	86

6.22	Betweenness Centrality of Weighted Edge Network Modeling the Pareto Design Dataset of the Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.99, Trial 1	86
6.23	Closeness Centrality of Weighted Edge Network Modeling the Full Design Dataset of the Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.99, Trial 1	88
6.24	Closeness Centrality of Weighted Edge Network Modeling the Feasible Design Dataset of the Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.99, Trial 1	89
6.25	Closeness Centrality of Weighted Edge Network Modeling the Pareto Design Dataset of the Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.99, Trial 1	90
6.26	Eigenvector Centrality of Weighted Edge Network Modeling the Full Design Dataset of the Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.99, Trial 1	92
6.27	Eigenvector Centrality of Weighted Edge Network Modeling the Feasible Design Dataset of the Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.99, Trial 1	93
6.28	Eigenvector Centrality of Weighted Edge Network Modeling the Pareto Design Dataset of the Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.99, Trial 1	94

LIST OF TABLES

Table

3.1	Example Variable Centrality Ranks	21
3.2	Sum of Concordant and Discordant Pairs for Kendall Tau Correlation	21
3.3	Sum of Squared Rank Differences for Spearman Correlation	21
5.1	Osyczka and Kundu Benchmark Problem Definition <i>Osyczka and Kundu</i> (1995)	33
5.2	Number of Osyczka and Kundu design data points in each subset .	34
5.3	Average Kendall Tau Correlation of Adjacency Matrices of Osyczka and Kundu Problem	35
5.4	Average Spearman Correlation of Adjacency Matrices of Osyczka and Kundu Problem	36
5.5	Average Cell-wise Standard Deviation of Adjacency Matrices of Osyczka and Kundu Problem	36
5.6	Average Kendall Tau Correlation of Shortest Path Lengths of Osyczka and Kundu Problem	37
5.7	Average Spearman Correlation of Shortest Path Lengths of Osyczka and Kundu Problem	37
5.8	Average Cell-wise Standard Deviation of Shortest Path Lengths of Osyczka and Kundu Problem	37
5.9	Average Kendall Tau Correlation of Centralities of Osyczka and Kundu Problem	38
5.10	Average Spearman Correlation of Centralities of Osyczka and Kundu Problem	38
5.11	Average Centralities of Osyczka and Kundu Problem	44
5.12	Average Ranked Centrality Measures of K2 Scored Osyczka and Kundu Problem	50
5.13	Average Ranked Centrality Measures of MD Scored Osyczka and Kundu Problem	51
6.1	Sen Bulk Carrier Design Model Summary	56
6.2	Upper and limits of six independent variables	57
6.3	Number of Sen Bulker design data points in each subset	57
6.4	Average Kendall Tau Correlation of Adjacency Matrices of Bulk Carrier Design Problem	58

6.5	Average Spearman Correlation of Adjacency Matrices of Bulk Carrier Design Problem	58
6.6	Average Cell-wise Standard Deviation of Adjacency Matrices of Bulk Carrier Design Problem	58
6.7	Average Kendall Tau Correlation of Shortest Path Lengths of Bulk Carrier Design Problem	59
6.8	Average Spearman Correlation of Shortest Path Lengths of Bulk Carrier Design Problem	59
6.9	Average Cell-wise Standard Deviation of Shortest Path Lengths of Bulk Carrier Design Problem	59
6.10	Average Kendall Tau Correlation of Centralities of Bulk Carrier Design Problem	59
6.11	Average Spearman Correlation of Centralities of Bulk Carrier Design Problem	60
6.12	Average Centralities of Bulk Carrier Design Problem	62
6.13	Average Kendall Tau Correlation of Adjacency Matrices of Bulk Carrier Design Problem	66
6.14	Average Spearman Correlation of Adjacency Matrices of Bulk Carrier Design Problem	66
6.15	Average Cell-wise Standard Deviation of Adjacency Matrices of Bulk Carrier Design Problem	67
6.16	Average Kendall Tau Correlation of Shortest Path Lengths of Bulk Carrier Design Problem	68
6.17	Average Spearman Correlation of Shortest Path Lengths of Bulk Carrier Design Problem	68
6.18	Average Cell-wise Standard Deviation of Shortest Path Lengths of Bulk Carrier Design Problem	69
6.19	Average Kendall Tau Correlation of Centralities of Bulk Carrier Design Problem	70
6.20	Average Spearman Correlation of Centralities of Bulk Carrier Design Problem	71
6.21	Average Ranked Degree Centrality of K2 Scored Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.99	80
6.22	Average Ranked Betweenness Centrality of K2 Scored Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.99	87
6.23	Average Ranked Closeness Centrality of K2 Scored Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.99	91
6.24	Average Ranked Eigenvector Centrality of K2 Scored Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.99	95
A.1	Independent Design Variables	103
A.2	Model Parameters	103
A.3	Model Constants	104
B.1	Average K2 Score Adjacency Matrix of Full Dataset Network of Oszka and Kundu Problem	106

B.2	Average K2 Score Adjacency Matrix of Feasible Dataset Network of Osyczka and Kundu Problem	107
B.3	Average K2 Score Adjacency Matrix of Pareto Dataset Network of Osyczka and Kundu Problem	107
B.4	Average MD Score Adjacency Matrix of Full Dataset Network of Osyczka and Kundu Problem	108
B.5	Average MD Score Adjacency Matrix of Feasible Dataset Network of Osyczka and Kundu Problem	108
B.6	Average MD Score Adjacency Matrix of Pareto Dataset Network of Osyczka and Kundu Problem	109
B.7	Average K2 Score Shortest Path Lengths of Full K2 Scores of Osyczka and Kundu Problem	109
B.8	Average K2 Score Shortest Path Lengths of Feasible K2 Scores of Osyczka and Kundu Problem	110
B.9	Average K2 Score Shortest Path Lengths of Pareto K2 Scores of Osyczka and Kundu Problem	110
B.10	Average MD Score Shortest Path Lengths of Full MD Scores of Osyczka and Kundu Problem	111
B.11	Average MD Score Shortest Path Lengths of Feasible MD Scores of Osyczka and Kundu Problem	111
B.12	Average MD Score Shortest Path Lengths of Pareto MD Scores of Osyczka and Kundu Problem	112
C.1	Average K2 Score Adjacency Matrix of Full Dataset Network of Unchunked Bulk Carrier Design Problem	114
C.2	Average K2 Score Adjacency Matrix of Feasible Dataset Network of Unchunked Bulk Carrier Design Problem	115
C.3	Average K2 Score Adjacency Matrix of Pareto Dataset Network of Unchunked Bulk Carrier Design Problem	116
C.4	Average MD Score Adjacency Matrix of Full Dataset Network of Unchunked Bulk Carrier Design Problem	117
C.5	Average MD Score Adjacency Matrix of Feasible Dataset Network of Unchunked Bulk Carrier Design Problem	118
C.6	Average MD Score Adjacency Matrix of Pareto Dataset Network of Unchunked Bulk Carrier Design Problem	119
C.7	Average K2 Score Shortest Path Lengths of Full K2 Scores of Bulk Carrier Design Problem	120
C.8	Average K2 Score Shortest Path Lengths of Feasible K2 Scores of Bulk Carrier Design Problem	121
C.9	Average K2 Score Shortest Path Lengths of Pareto K2 Scores of Bulk Carrier Design Problem	122
C.10	Average MD Score Shortest Path Lengths of Full MD Scores of Bulk Carrier Design Problem	123
C.11	Average MD Score Shortest Path Lengths of Feasible MD Scores of Bulk Carrier Design Problem	124

C.12	Average MD Score Shortest Path Lengths of Pareto MD Scores of Bulk Carrier Design Problem	125
D.1	Average K2 Score Adjacency Matrix of Full Dataset Network of Unchunked Bulk Carrier Design Problem	127
D.2	Average K2 Score Adjacency Matrix of Feasible Dataset Network of Unchunked Bulk Carrier Design Problem	128
D.3	Average K2 Score Adjacency Matrix of Pareto Dataset Network of Unchunked Bulk Carrier Design Problem	129
D.4	Average MD Score Adjacency Matrix of Full Dataset Network of Unchunked Bulk Carrier Design Problem	130
D.5	Average MD Score Adjacency Matrix of Feasible Dataset Network of Unchunked Bulk Carrier Design Problem	131
D.6	Average MD Score Adjacency Matrix of Pareto Dataset Network of Unchunked Bulk Carrier Design Problem	132
D.7	Average K2 Score Shortest Path Lengths of Full K2 Scores of Bulk Carrier Design Problem	133
D.8	Average K2 Score Shortest Path Lengths of Feasible K2 Scores of Bulk Carrier Design Problem	134
D.9	Average K2 Score Shortest Path Lengths of Pareto K2 Scores of Bulk Carrier Design Problem	135
D.10	Average MD Score Shortest Path Lengths of Full MD Scores of Bulk Carrier Design Problem	136
D.11	Average MD Score Shortest Path Lengths of Feasible MD Scores of Bulk Carrier Design Problem	137
D.12	Average MD Score Shortest Path Lengths of Pareto MD Scores of Bulk Carrier Design Problem	138

LIST OF APPENDICES

Appendix

- A. Bulk Carrier Design Model by Sen and Yang 103
- B. Oyszka and Kundu Adjacency Matrices and Shortest Path Lengths . 106
- C. Unchunked Bulk Carrier Design Model Adjacency Matrices and Shortest Path Lengths 113
- D. Bulk Carrier Design Model Chunked with Threshold of 0.99 Adjacency Matrices and Shortest Path Lengths 126

ABSTRACT

Like the design of many large and complex systems, modern ship design often involves the automated creation of thousands of viable design alternatives developed through computer driven design models and optimizations. The models used to develop these designs are often multi-disciplinary and contain highly interconnected engineering systems. Consequently, even the most experienced designer has a limited ability to develop complete mental models for a large number of complex and varied design alternatives. Furthermore, when design decisions are made and need to be communicated to non-technical stakeholders, the complex relationships driving the design model become even more difficult to effectively communicate.

Automated learning of Bayesian networks can offer designers an opportunity to quickly analyze a large set of designs with the efficiency with which they are created. As sets of nodes, edges and conditional probabilities, Bayesian networks can identify and quantify the influential relationships between design parameters. Transforming the learned Bayesian networks into simpler weighted edge networks further aids communication of the driving factors of a complex design model to all stakeholders by presenting the learned information visually and through simple to understand network metrics.

This dissertation presents a framework for learning Bayesian networks, transforming them into weighted edge networks and analyzing those networks with metrics from network science. Additionally, an algorithm for identifying and chunking redundant variables is presented. Two case studies, a simple multi-objective function from Osyczka and Kundu and more complex ship design model from Sen and Yang,

are presented and analyzed with the proposed framework. Each is sampled with a Latin hypercube to develop ten design trials of 100,000 design alternatives each. The variables of the more complex ship design model are analyzed for redundancy and chunked using the proposed chunking algorithm. Bayesian networks are learned from each design database and transformed into weighted edge networks using two scoring methods, derived from log gamma K2 and match distance. Finally the weighted edge networks are analyzed to identify communities and compute degree, betweenness, closeness and Eigenvector centralities. These metrics identify disciplines and driving factors of the design space at progressive stages of design.

CHAPTER I

Introduction

Modern ship design practices often involve the creation and evaluation of thousands of design alternatives through automated analysis. These automated designs are developed through a connected set of multi-disciplinary models, which use the independent and dependent design variables of a problem to return objective function values. As most ship design problems are multi-objective, there is often no single, best solution but rather a set of Pareto-optimal solutions; designers must rely on their own knowledge of best practices to differentiate between solutions. For simple, explicit design models the relationships between these variables and their effects on one another are easily understood by designers. However, as models become larger, more complex and more interconnected the relationships driving the overall design model become clouded. With a blurred understanding of the design space relationships that lead to the final design set, designers may have difficulty differentiating between a set of Pareto-optimal designs and understanding exactly what has led to a diverse set of similarly optimal designs.

Furthermore, the stakeholders in the design of ships and other physically large and complex systems tend to include groups beyond just the technical designers. Engineers must often relay the set of design, their differences and their optimalities to nontechnical stakeholders in a clear fashion so that an informed decision can be

made. For a set of thousands of theoretical designs, this can easily become a daunting task for designers.

In order to make an informed decision about which designs are truly optimal for a large set of often conflicting objectives and to communicate the rationale for these decisions to non-technical stakeholders, a clear definition of the inner relationships must be obtained. Machine learning allows designers to analyze and assess the large design spaces created through automated design to assist in this task. One such technique is the use of Bayesian networks to organize large sets of design data and quantify their relationships. From a concept developed following work using the Bayesian Optimization Algorithm for early stage marine structure design, this work seeks to use machine learned Bayesian networks to identify influential design relationships and use network analysis tools to clearly define and communicate the strength of those relationships.

1.1 Computer Aided Design of Physically Large and Complex Systems

Computer-aided design and machine learning have transformed design processes for the design of ships and other physically large and complex systems. While initially used for more efficient computation within traditional design methods, computer-aided design also promotes novel approaches to the design process, like multi-disciplinary optimizations and set-based design. Machine learning goes further, supporting tools for engineering design like fuzzy logic, heuristic optimization algorithms and artificial neural networks *Saridakis and Dentsoras (2008)*. These tools enhance the engineering design process and can lead to more robust design solutions.

In ship design, computer-aided design allows for geometric modeling of arrangements and hull forms, analysis of wider design spaces, like in set-based design, and

detailed structural analyses *Nowacki (2010)*. But in addition to providing more robust analyses and faster computations, machine learning and computer-aided design can aid human designers in understanding ever more complex and multidisciplinary designs, rather than acting as standalone tools for design problem solving. When used as standalone solutions to a design problem, computer-aided design methods can turn into black boxes, accepting inputs of independent variables and constraints and returning some solved objective function as an output without disclosing the design factors that determine the solution. If designers can extract the information about the design space and relationships learned by computer-aided design methods on their way to the overall solutions, they can reap the advantages of computer-aided design without losing intuition about what drives it. This research applies machine learning through Bayesian networks to support designers in understanding the inner workings of advanced design methods and the design problems they are used to solve.

1.1.1 Bayesian Networks in Design

Bayesian networks are probabilistic networks used to capture influential relationships *Kjærulff and Madsen (2008)*. While they are widely used for reliability analyses, diagnostics and decision making under uncertainty, they have been applied as support tools in design applications as well.

Shahan and Seepersad used Bayesian networks in the design of unmanned aerial vehicles to map promising designs in to satisfactory and unsatisfactory regions *Shahan and Seepersad (2012)*. Nannapaneni et al. applied Bayesian networks to cover areas of uncertainty in the multi-objective optimization of an airplane wing *Nannapaneni et al. (2017)*. Within ship design, Bayesian networks have been applied to identify failure points and system sensitivities for survivability in naval design *Friebe et al. (2019)*.

Pelikan pioneered a specialized use of Bayesian networks to support heuristic opti-

mization. His optimization algorithm, the Bayesian Optimization Algorithm (BOA), is similar to a genetic algorithm but uses Bayesian networks to probabilistically determine new generations with the greatest likelihood of having higher fitness levels *Pelikan* (2002). In his dissertation work, Devine applied the BOA to the design of marine structures. This work noted the potential wealth of information about the design space being held in the intermediate networks used by the BOA *Devine* (2016).

These uses of Bayesian networks for design fall in one of two categories: the networks are explicitly defined and used to better understand the design or they are learned automatically but used to aid the computer's design process not the designer's understanding. When explicitly defined, the Bayesian networks are useful tools to organize the flow of causality and capture uncertainty but cannot bring new understanding of relationships beyond those defined by the builder. Conversely, using Bayesian networks to aid optimizers will help with efficiency and covering uncertain regions, but the wealth of information learned is only known to the computers, which have not been used to output that learned information beyond their solutions. The research presented in this dissertation uses network theory to capture and communicate the abundance of knowledge that can be gleaned from Bayesian networks learned from design data.

1.1.2 Network Analysis in Design

Network theory has been applied broadly for use in design methods. Networks can hold a huge amount of information as a simple set of nodes and edges. Additionally, network analysis can be applied to these design networks to develop knowledge about the structure and relationships of the design problems they model .

Many uses of networks and their structure in design come from design structure matrix (DSM) methods. While typically conveyed as matrices, this method tracks the interactions of physical components, designers and design activities, connecting

all three, effectively, as a set of nodes and edges *Steward* (1981). This method has been expanded several times: Eppinger and Browning added process architecture and organization *Steven D. Eppinger and Tyson R. Browning* (2012), Bartolomei integrated quantitative methods from social science *Jason E. Bartolomei* (2007), and Maurer added business project management policies to the structure *Maurer* (2007).

Other, more dynamic approaches use networks to track flow of information and design changes rather than represent physical interactions or connections. Pasqual and de Weck analyzed change propagation in engineering design through multi-layer networks representing product, change and social networks *Pasqual and de Weck* (2012). Parraguez et al. tracked information flow through dynamic networks modeling designers and their activities *Parraguez et al.* (2015).

Network analysis has also been used extensively in naval architecture. MacCallum first introduced the use of networks in the marine industry with his paper presenting the use of networks to track dependencies between design variables in the early introduction of computerized design calculations. The work presented manually developed networks to keep track of which disciplines had overlapping variables; when iterating through design changes, the engineer could track when updating a value in one discipline's calculation would affect the other disciplines and their variables' values *KJ MacCallum* (1982). Parker similarly used network theory to understand the structure of naval architecture design, tracking changes across more complex design tools *Parker* (2014). In his analysis Parker completed more complex network analysis and introduced new metrics for design analysis, path influence and Winston centrality *Parker and Singer* (2015). Work by Gillespie, Rigterink and Shields used networks to analyze physical connections and configurations in ship design. Gillespie used ideas similar to architecture's facility layout planning, using networks in general arrangement planning for early stage design. Edges in these networks were defined as physical connections between compartments represented as nodes *Gillespie* (2012).

Rigterink applied a similar process for distributed ship systems *Rigterink* (2014), and Shields combined these methods to develop networks representing probabilistic ship arrangements and distributed systems *Shields and Singer* (2017). These networks of required physical connections on a ship allow for the analysis of probable arrangements and distributed systems in early stage design before they can be fully laid out and developed. This allows for an understanding of functionality of the ship much earlier in the design process. Most recently Goodrum used a multi-layer network to map how information sources are translated into knowledge in order to understand the generation of design knowledge *Goodrum* (2020). Goodrum’s work follows the idea that networks can be used to aid the understanding of design models and how their results are developed.

These uses of networks to describe design and ship architecture are useful models but are all defined manually, either with expert knowledge or after cumbersome studying of the design problem. These frameworks can then become prohibitively difficult in complex design problems, and many design problems are prohibitively complex. Ship design models, for example, are often actually several sub-models developed by engineers from vastly different disciplines, from structures and survivability to hydrodynamics and response predictions. The network used in MacCallum’s work used 13 simple variables to represent the ship in preliminary design *KJ MacCallum* (1982). Modern design models, even those used in preliminary design, may share much larger and more complex information, like complete hull geometries instead of length, beam and block coefficient or extremely large hotel and surge electrical loads and power requirements in addition to the traditional mechanical powering loads. With this additional complexity comes additional opacity—engineers may be able to build a complete mental model of the interactions and variable connections within their discipline’s model, but the complete mental model needed to build a network of dependencies manually, with understanding of all dependencies across all disciplines,

becomes extremely difficult to capture even for experienced naval architects. Additionally, even when all dependencies are thought to be understood, a manually built network will always represent the designer’s knowledge about the design problem rather than how the design model has actually been built and is producing solutions. As a solution, this dissertation offers a framework to learn Bayesian networks from design data, without the need for intervention with expert knowledge. An algorithm is proposed for the automated preparation of design data for Bayesian network learning by identifying and chunking redundant output variables from the design model. From this prepared design data, networks are learned automatically. The resulting network would represent the design space and design variable dependencies, as evaluated by a complex design model. Finally, network analysis methods are used to determine the driving factors of a design at progressive design stages.

1.2 Research Objectives

This research seeks to develop a framework to use Bayesian networks and network theory to understand and communicate complex design models, answering three main questions:

1. Can Bayesian network learning methods produce networks that consistently and accurately depict a design model’s structure from a database of its outputs?
2. Can the influence between nodes captured as conditional probability tables in a Bayesian network be represented with single edge weights to allow the use of traditional network science tools?
3. Are the yielded networks’ structures and network metrics informative about the design space and can they be easily communicated to stakeholders?

CHAPTER II

Bayesian Networks

Bayesian networks are probabilistic models used to represent a set of related variables and their conditional probabilities. Bayesian networks are defined by two parts: a qualitative network structure and quantitative joint probability distribution. The structure of a Bayesian network is an acyclic directed graph consisting of nodes and directed edges. The nodes of a graph represent variable of the modeled problem, while the directed edges depict the causal relationships between those variables. Discrete variables, like those used in this work, have a finite set of states representing possible values of the node. These states can be binary, numbered, intervalled or labelled but must encompass an exhaustive set of mutually exclusive values the represented variable could take *Kjærulff and Madsen (2008)*. The nodes are connected by a set of directed edges. The directed edges of a Bayesian network represent conditional dependencies and point from a parent node to a child node, where the state of the parent node has some influence on the state of the child node. For each child node, there is a conditional probability distribution , the quantitative side of Bayesian networks. These distributions, held in conditional probability tables, quantify the influence the state of each parent node has on the state of the child node. For nodes without parent nodes, likelihood is represented by simple probability distributions. Formally,

- A set of nodes V and a set of directed edges $E \subseteq V \times V$ between nodes

- The nodes together with the directed edges form an acyclic, directed graph (DAG) $G = (V, E)$.
- Each node $X \in V$ has a finite set of states, e.g. $||X|| = n$.
- Attached to each node X with parents Y_1, \dots, Y_n there is a conditional probability table $P(X|Y_1, \dots, Y_n)$.

$$P(V) = \prod_{X \in V} P(X|pa(X))$$

Kjærulff and Madsen (2008)

Bayesian networks are governed by Bayes' Rule, relating conditional and prior probabilities.

$$P(X|Y) = P(Y|X)P(X)/P(Y)$$

where $P(X)$ and $P(Y)$ are the marginal, or prior, probabilities of two variables X and Y and $P(Y|X)$ and $P(X|Y)$ are the conditional probabilities. Through Bayes' Rule the marginal probability can be determined for any given node from its conditional probability tables.

The example in Figure 2.1 shows Bayesian network defining some basic design characteristics of recreational boats, including length (L), whether or not it is mechanically powered (powered), maximum passengers (passengers) and whether it can be stored at home (home storage). In this example, L is the only parent node and has a simple distribution defining its prior probability, representing the fraction of recreational craft less than 25 feet long, between 25 and 50 feet long and greater than 50 feet. The probability of home storage is defined by conditional probability table because it is dependent on the length of the craft; the longer a boat is, the less likely it is to be able to be stored at the owner's home. The prior probability of home storage is then defined by Bayes rule, giving an overall likelihood that a recreational boat could be stored at home of 68%.

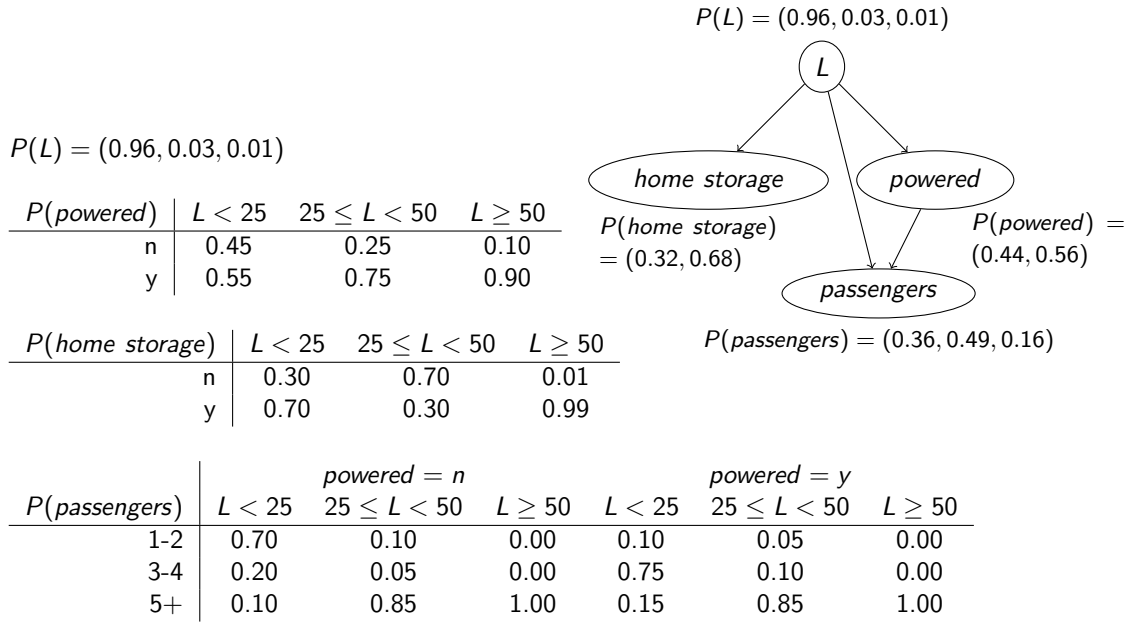


Figure 2.1: Bayesian Network of Recreational Boat Characteristics

2.1 Observations and Updating

Bayes' Theorem can also be used to update the prior probabilities of a Bayesian network as new evidence or observations are introduced. For known state y of node Y ,

$$P(X|Y = y) = P(Y = y|X)P(X)/P(Y = y).$$

In the previous example, if an owner requires the boat must be stored at home, we can provide that as an observation and update the remaining nodes' marginal probabilities. In Bayesian networks an observation is some known information about the state of a node. In this example if home storage becomes an owner's requirement, we know the state of *home storage* will be true. As depicted in Figure 2.2, the probability distribution of the boat's length will change, as a requirement for home storage will greatly decrease the likelihood of a boat over 50 feet in length. However, because the probability distribution of length has changed, the marginal probability of whether the craft is powered will be updated to reflect the change in length probabilities and

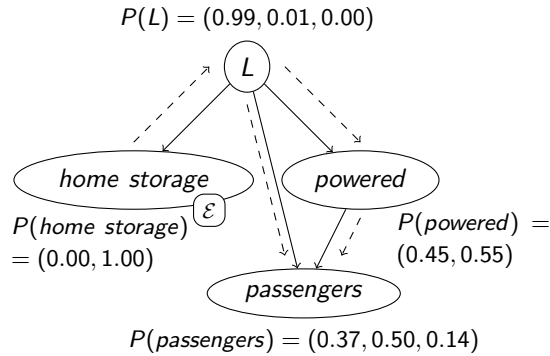


Figure 2.2: Updated Bayesian Network of Recreational Boat Characteristics with Information Propagation

the marginal probability of the maximum number of passengers will be updated to reflect the changes in probabilities of both length and powering.

Additional evidence could also be introduced that would update the prior probabilities of the model. In this example that would be something like new information on the number of powered craft owned by individuals. Incorporation of new evidence may change the likelihood of nodes' states, but not reflect a certainty like an observation does.

This cascade of updated likelihoods is the information flow of the Bayesian network. The flow of information travels differently based on the structure of the connections in a Bayesian network. In serial connections, $A \rightarrow B \rightarrow C$, information in any one node will update all other nodes. Similarly, in a diverging connection, $A \leftarrow B \rightarrow C$, information about any one node will update the marginal probabilities of both other nodes. The exception in both these cases is if information is known about B and one other node, the third node will only be updated according to the marginal probability of B . This is because known information about a node will supersede any update to that nodes distribution by the of information.

However, information flows differently in a converging connection, where a child node has two parents, $A \rightarrow B \leftarrow C$. While information about child node B will update the marginal probabilities of both parent nodes A and C , known information

about one parent will update the only the child node and not any additional parents. This is because the updated belief about child node B is known to be the effect of information about one parent node, for example A , so it cannot be counted an effect of another parent node, for example C , as well *Kjærulff and Madsen* (2008). These properties rely on the modeling of causality in a Bayesian network. Changing the direction of an edge, effectively reversing its causality, could change a group of three nodes from a series to a converging connection and interrupt the flow of information.

While causality and interruption of information flow is important for Bayesian networks modeling causal events, it is less important when modeling design. Because of the iterations through and variety of approaches to design, cause and effect relationships are less clear. In our example, length and powering are clearly correlated, but one cannot be definitively determined to be the cause of the other. Length can be said to require more power at longer lengths, but installing power can also allow us to build longer boats. This flexibility in causality is taken advantage of in this work when converting and analyzing the design Bayesian networks as traditional weighted edge networks.

2.2 Learning BN

The structure and distribution of Bayesian networks can be constructed manually using input from expert knowledge, semi-automatically using a database of representative cases, or by some combination of expert knowledge and data. When developed manually, the structure of a Bayesian network is first constructed by identifying known variables of the modeled problem and the flow of influence between them. A set of rules is then constructed to define and quantify the influences of parent nodes on their child nodes. This process requires deep knowledge of the complexities of the problem and confidence that the assumed causalities are correct, so this method is generally suitable for well studied problems or those governed by innate rules that de-

fine the relationships of the variables. Learning a Bayesian network from a database of known cases requires the same steps of defining the structure then populating conditional probability tables to define the joint probability distribution. To rely on a data-driven approach for defining a Bayesian network, you must assume the database of cases consists of independent and randomly distributed cases and that the data can adequately be represented by an underlying probability distribution *Kjærulff and Madsen* (2008). In design, this means we need databases of design alternatives adequately spread across the design space. The design alternatives should also be developed independently of each other. If the design alternatives are created by spiral design methods for example, the design set would likely only cover a local design space; additionally, even if enough changes were made to cover the full possible design space, the design alternatives would be developed from and, therefore, dependent on each other. For this reason a set of design alternatives learned from methods like set-based design would create a much stronger Bayesian network than one learned through spiral design methods.

There are several methods for learning Bayesian networks, but this work relies on the greedy search and score algorithm. This algorithm scores the goodness of a candidate network via some defined scoring function. The greedy search and score algorithm searches all possible networks by adding, reversing and removing edges and evaluating the resulting change in the goodness score. These scores are generally decomposable, so with every structure change the algorithm only needs to recalculate the score for the affected edge *A/S* (2018). This property of decomposition will be used later in this work. The scoring functions used to evaluate candidate networks can be broken down into two categories: those based on log-likelihood and those based on Bayesian posterior probability. The log-likelihood (LL) function is

$$LL(B|T) = \sum_{i=1}^n \sum_{j=1}^{q_i} \sum_{k=1}^{r_i} N_{ijk} \log \left(\frac{N_{ijk}}{N_{ij}} \right)$$

where B is the network, T is the database, n is the number of variables, q_i is the number of parent states for the parent of variable i , r_i is the number of states of the variable i , N_{ijk} is the number of data points in the database with parent state j and child state k , and N_{ij} is the number of data points in the database with parent state j . This function has no penalty for complexity, so using it as the scoring function in a search and score algorithm would result in a network that is over fit to the data provided. Essentially, any possible node connection with a conditional relationship between the nodes would result in an edge. This would create so many edges that very little uncertainty would be considered. Any observation would produce the exact probabilities of evidence datapoints already used to learn the network with states matching that observation.

Instead, the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) functions each add penalty functions to the log-likelihood reduce complexity of the learned network. The AIC penalizes complexity of the number of the states in candidate parent and child nodes *Akaike* (1974); the BIC, also known as minimum description length (MDL), uses a similar penalty but also factors the number of cases of the database so that the penalty is relatively smaller for large, more accurately representative databases *Schwarz* (1978). AIC and BIC are defined, respectively, as

$$AIC = LL(B|T) - |B|$$

$$BIC = LL(B|T) - \frac{1}{2} \log(N)|B|$$

where

$$|B| = \sum_{i=1}^n (r_i - 1)q_i$$

and N is the total number of instances in the data and n , r_i and q_i are the same as above.

The other type of scoring functions are based on Bayesian statistics and look to

maximize the posterior probability of the network. The foundation of these scoring functions is the Bayesian Dirchlet (BD) score,

$$BD(B, T) = \log(P(B)) + \sum_{i=1}^n \sum_{j=1}^{q_i} \left(\log \left(\frac{\Gamma(N'_{ij})}{\Gamma(N_{ij} + N'_{ij})} \right) + \sum_{k=1}^{r_i} \log \left(\frac{\Gamma(N_{ijk} + N'_{ijk})}{\Gamma(N'_{ijk})} \right) \right)$$

where $P(B)$ is the prior probability of network B and N_{ij} , N_{ijk} , n , q_i and r_i are the same as above. $N'_{ijk} = N' \times P(X_i = x_{ik}, \Pi_{X_i} = w_{ij}|G)$ where N' is an equivalent sample size for some graph G and scale with our belief in the prior distribution *Heckerman* (1995). N' is extremely difficult to calculate in practice, so the simplest handling is to set $N' = 1$, which produces the K2 score,

$$K2(B, T) = \log(P(B)) + \sum_{i=1}^n \sum_{j=1}^{q_i} \left(\log \left(\frac{(r_i - 1)!}{(N_{ij} + r_i - 1)!} \right) + \sum_{k=1}^{r_i} \log(N_{ijk}!) \right)$$

where $P(B)$, N_{ij} , N_{ijk} , n , q_i and r_i are the same as above *Cooper and Herskovits* (1992).

2.3 Methods Used

This work uses the commercial software HUGIN to learn and analyze Bayesian networks modeling design data *A/S* (2018). Within HUGIN, the networks structures will be learned through a greedy search and score algorithm. HUGIN uses only the AIC and BIC for its search and score algorithm, so the BIC score was chosen for its ability to consider validity of the database size in scoring. For each design dataset, the database is uploaded to HUGIN through the programs *Learning Wizard*, which facilitates network structure and probability learning. The structure is learned with HUGIN's greedy search and score algorithm and conditional probability tables are defined using the database of design points as evidence sorted into appropriate cells.

CHAPTER III

Network Analysis

Networks as they appear for use in network theory are defined by their nodes and edges, without additional data like Bayesian networks. Their edges may be directed, showing some path of information flow, or undirected, simply linking two nodes. These edges can also be weighted or unweighted: unweighted edges signify uniform or unquantified connections and weighted edges having some value of strength or length attached to them. Weighted edges whose values represent strength have higher values for more strongly connected nodes; however, if the weighted edge is representative of length, a higher value will generally refer to a more distant connection, opposite of the strength value. In this work, all edge weight values will refer to strength of connection. When path distance is required for analysis, the cost definition of path distance as an inverse of the weighted value is used *Newman* (2001).

3.1 Network Science Tools

Network graphs are often represented as adjacency matrices, \mathbf{A} . In an adjacency matrix each node of a network is represented by a row and a column. For unweighted networks, each cell A_{ij} is populated with 0 or 1, designating whether nodes i and j share an edge, 1, or do not share an edge, 0. For weighted networks, like those used in this work, the weight of an edge populates the cell A_{ij} rather than a value of 1, as

shown in Figure 3.1.

For directed networks, the network is represented by an asymmetrical adjacency matrix. An edge from node j to node i is populated in A_{ij} of the adjacency matrix but A_{ji} remains equal to zero, unless there is a second edge connecting the nodes. However, for undirected networks, like those used in this work, the adjacency matrices are symmetrical, so an undirected edge between nodes i and j populates both A_{ij} and A_{ji} of the network's adjacency matrix.

Degree centrality is the simplest measure of centrality for networks. It measures just the number of edges connected to a node. In this work degree centrality will be normalized by the number of all possible edges to aid comparison,

$$C_D(v) = \frac{deg(v)}{n - 1}$$

Closeness centrality is the reciprocal of the average shortest path distance to all other reachable nodes.

$$C_C(v) = \sum_{u=1}^{n-1} \frac{n - 1}{d(u, v)}$$

where $d(u, v)$ is the shortest path distance between nodes u and v for all n nodes u that can reach v .

Betweenness centrality is the sum of the fraction of all shortest paths between two nodes that pass through the node.

$$C_B(v) = \sum_{s, t \in V} \frac{\sigma(s, t|v)}{\sigma(s, t)}$$

where V is the total set of nodes, $\sigma(s, t)$ is the number of shortest paths between node pairs, and $\sigma(s, t|v)$ is the number of those shortest paths that pass through v .

Eigenvector centrality is a centrality measure which accounts for the centrality of

a node's neighbors in its measurement. Defined as Eigenvector \mathbf{x} in

$$\mathbf{A}\mathbf{x} = \kappa_1\mathbf{x}$$

where κ_1 is the largest eigenvalue of adjacency matrix \mathbf{A} .

In addition to centrality measurements, community detection can be used to find groupings of similar nodes within a network. One of the simplest methods of community detection is modularity maximization. Modularity is a network property measuring the extent to which like node are connected to like nodes, defined as

$$Q = \frac{1}{2m} \sum_{ij} \left(A_{ij} - \frac{k_i k_j}{2m} \right) \delta(c_i, c_j)$$

where m is the number of edges in a network, k_i and k_j are the degrees of nodes i and j , respectively, c_i and c_j are the types nodes i and j , and $\delta(c_i, c_j)$ is 1 when c_i are c_j are the same and 0 when they are different. The modularity of a network increases when there are more edges between nodes of the same type than we would expect by chance *Newman* (2018).

As noted above, while this work uses edge weight to represent strength of connection, some metrics like closeness and betweenness centrality use shortest path distance rather than weight values. For each edge weight, path distance is

$$d(u, v) = \frac{1}{A_{u,v}}$$

where $d(u, v)$ is the distance between nodes u and v connected by an edge with weight $A_{u,v}$ *Newman* (2001). This includes the 0 values of an adjacency matrix, whose inverse results in an infinite path length for nodes without a connection.

Shortest path lengths is also a useful measure of connectedness for networks in this work because of the flow of information and updating in Bayesian networks.

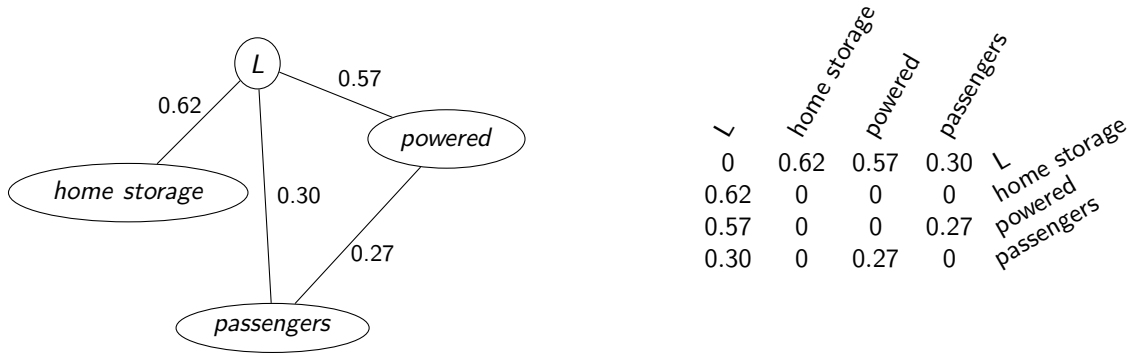


Figure 3.1: Weighted Network and Adjacency Matrix

Information about one node affects not only the node directly connected to it, but also any nodes connected to those surrounding nodes and so on. This work uses the Dijkstra algorithm to identify shortest path lengths between pairs *Dijkstra* (1959).

For example, in a weighted network representing the same design of recreational boats used in the discussion of Bayesian networks, the four connecting edges each carry a weight between 0 and 1. The path distance between length and home storage would equal 0.62^{-1} , or 1.61. While passengers and home storage are not directly connected by an edge, their connection is represented by the shortest path distance, passing through the length node, $0.62^{-1} + 0.30^{-1}$, or 4.94.

The Pearson coefficient of an adjacency matrix is a measure of similarity between two vertices of a network; it is the actual number of common neighbors that have minus the expected number that they would have if they chose neighbors at random, normalized by the maximum possible value when the networks are exactly the same,

$$r_{ij} = \frac{cov(A_i, A_j)}{\sigma_i * \sigma_j}$$

where A_i and A_j are the i th and j th rows of matrix A , and σ_i and σ_j are the standard deviations. The Pearson coefficient can be used to assess the structural similarity between two nodes.

3.2 Comparison for Consistency

Much of network theory focuses on the comparison of networks with no known node correspondence; these are networks like social networks or networks of portions of the internet, where it is expected that the number of nodes occurring in both networks is unknown and likely somewhat small. Because this work, on contrary, looks to compare networks with the same nodes to look for consistency of structure, simple distance- or rank-based methods can be used. The simplest measures of difference between matrices are based on entry wise (or cell-wise) distances, $\mathbf{A} - \mathbf{B}$, and their total magnitudes determined by the Euclidean, norm of that distance matrix. Because we are comparing 10 matrices rather than just two we will use the cell-wise standard deviation of the ten adjacency matrices to determine overall consistency.

A better measure of similarity is to compare the results of network analysis like shortest path lengths and centralities. Because slight shifts in edge weights have much less effect on centralities' and shortest paths' ranks, especially when they are proportional across the network, rank-based correlations are most useful to compare a group of networks for consistency.

Kendall Tau correlation and Spearman correlation are two correlation measures that compare rank rather than absolute value. Kendall Tau correlation is defined as

$$\tau = \frac{n_c - n_d}{\frac{1}{2}n(n-1)}$$

where n is the number of observations and n_c and n_d are the numbers of concordant and discordant pairs, respectively *Kendall* (1948). Spearman correlation is defined as

$$\rho = 1 - \frac{6 \sum (d_i^2)}{n^3 - n}$$

where d_i is the difference between ranks of corresponding variables i and n is the

number of observations *Kendall* (1948). Kendall Tau correlation accounts only for the binary, higher or lower rank, while Spearman correlation measures the distance between ranks. Due to this difference, Spearman correlation generally returns larger values but is more susceptible to large errors between observations.

Table 3.1 provides a small example of centrality ranks for three trials of a network with nodes p, q, r, s and t . Table 3.2 delineates whether each pair of variables have concordant (+1) or discordant (-1) ranks between trials I and II and between trials I and III, and Table 3.3 delineates the difference in ranks for each variable between the same networks.

Table 3.1: Example Variable Centrality Ranks

	I	II	III
p	1	2	1
q	2	3	2
r	3	4	4
s	4	1	3
t	5	5	5

Table 3.2: Sum of Concordant and Discordant Pairs for Kendall Tau Correlation

	I-II	I-III
pq	+1	+1
pr	+1	+1
ps	-1	+1
pt	+1	+1
qr	+1	+1
qs	-1	+1
qt	+1	+1
rs	-1	-1
rt	+1	+1
st	+1	+1
$n_c - n_d$	4	8

Table 3.3: Sum of Squared Rank Differences for Spearman Correlation

	I-II		I-III	
	d_i	d_i^2	d_i	d_i^2
p	1	1	0	0
q	1	1	0	0
r	1	1	1	1
s	-3	9	-1	1
t	0	0	0	0
$\sum(d_i^2)$	12		2	

For these rankings, with $n = 5$ variables, networks I and II have a Kendall Tau correlation of 0.4 and a Spearman correlation of 0.4. Networks I and III have a Kendall Tau correlation of 0.8 and a Spearman correlation of 0.9. In network II variable s jumps to the highest rank, and the rest move down one rank, while in network III s jumps just one rank and only r 's rank is effected. For the small change between networks I and III, the Spearman correlation is higher than Kendall Tau, but for the larger jump of s between networks I and II Spearman penalizes the change as much

as Kendall Tau does. Practically, this means small flips between nodes on a branch will have a smaller effect on Spearman correlation than Kendall Tau correlation, but large changes in centrality will have similar penalties to both correlations.

CHAPTER IV

Developed Methodology

4.1 Converting Conditional Probabilities to Weighted Edges

The first challenge of this work is discerning a method for transforming Bayesian networks into traditional networks for analysis. The method must be able to simplify but maintain the knowledge of the conditional probability tables in addition to the structure of the network. To succinctly capture the relationship in formation in these tables, edge weights measuring the amount of information transferred between nodes were captured using two scoring methods: K2 and match distance. Log gamma K2 is a scoring method derived from the Bayesian Dirchlet, introduced in Chapter III. While it is generally used as a score to assess the goodness of an entire network, it can be decomposed into individual components for each edge of a network. This work uses the log gamma K2 of each edge as

$$\text{loggammaK2} = \sum_{i=1}^q \left(\log \left(\frac{\Gamma(r)}{\Gamma(N_i + r)} \right) + \sum_{j=1}^r \log(\alpha_{ij} \Gamma(\alpha_{ij})) \right)$$

where q is the number of parent states, r is the number of child states, N_i is the number of observed data points with parent state i and α_{ij} is the number of observed data points with parent state i and child state j . This score produces negative values from about $O(-10^5)$ to $O(-10^2)$ where values closer to zero indicate a stronger influence

and transfer of more information. To normalize these scores we use the log gamma K2 score between two uniform distributions with no correlation. Since all of our nodes have the same number of states, this normalization factor is constant across all nodes of the same network but varies between networks trained with different sized data sets or different numbers of node states. The normalization factor then equals

$$loggammaK2_{normfactor} = \sum_{i=1}^q \left(\log \left(\frac{\Gamma(r)}{\Gamma(\frac{N}{q} + r)} \right) + \sum_{j=1}^r \log \left(\frac{N}{qr} \Gamma \left(\frac{N}{qr} \right) \right) \right)$$

where q and r are the same as above, and N is the total number of observed data points Devine (2016). The edge weight for each nonzero network edge is

$$K2_{edge} = 1 - \frac{\sum_{i=1}^q \left(\log \left(\frac{\Gamma(r)}{\Gamma(N_i+r)} \right) + \sum_{j=1}^r \log(\alpha_{ij} \Gamma(\alpha_{ij})) \right)}{\sum_{i=1}^q \left(\log \left(\frac{\Gamma(r)}{\Gamma(\frac{N}{q}+r)} \right) + \sum_{j=1}^r \log \left(\frac{N}{qr} \Gamma \left(\frac{N}{qr} \right) \right) \right)}$$

This $K2_{edge}$ score produces an edge weight value between 0 and 1: zero signifies no connection and 1 signifies a complete connection, where all parent nodes states match child node states perfectly with no uncertainty.

In addition to K2, match distance was tested as a simpler measure for edge weight scoring. Match distance is a measure of difference between two cumulative histograms and was chosen for its simplicity and clarity. The match distance between two histograms H, K is defined as

$$d_M(H, K) = \sum_i |\hat{h}_i - \hat{k}_i|$$

where $\hat{h}_i = \sum_{j \leq i} h_j$ is the cumulative histogram of h_i and \hat{k}_i is the same Rubner *et al.*. To determine an edge weight between two nodes this distance was computed between the prior distribution of the child node and the distribution after each state of the

parent node is selected and normalized,

$$MD_{edge} = \frac{\sum_{i=1}^q \left(\sum_{j=1}^r |P(X \leq x_j) - P(X \leq x_j | Y = y_i)| \right)}{q}$$

where q is the number of states y_i in parent state Y , r is the number of states x_j in child state X , and $P(X \leq x_j)$ is the cumulative probability of X in state x_j . This new edge weight score considers the average impact of the knowledge about the state of one node on the knowledge about the state of the connected node. Match distance has not been applied for use as a scoring function for Bayesian networks but was chosen to evaluate whether the simple measurement could be effective in describing influence between nodes with conditional probability.

Of the K2 and match distance scores evaluated in this work, K2 is the stronger metric for measuring influence. For both the Osyczka and Kundu problem and Sen and Yang’s bulk carrier design model, match distance produced scores that were generally low and had little variation between its maximum and minimum edge weights within a network. Scoring edges with the K2 score produced wider ranges of edge weights that accurately represented differences in the strengths of influence between variables. Both the Osyczka and Kundu and bulk carrier design problems were analyzed with both K2 and match distance scores, but due to match distance’s weaker performance most discussion and analysis in this work centers on the networks learned with the K2 score.

4.2 Identifying and Chunking Redundant Nodes of the Bayesian Network

A second challenge in developing networks automatically from design databases is the preparation the databases for learning as a Bayesian network. As noted in Chapter II, nodes should be chosen such that the states of nodes are mutually exclusive.

When nodes' states are not mutually exclusive, consistency of the learned networks is affected. When considering all variables of a complex design model it is likely there are variables that whose states are not mutually exclusive. These variables arise from simple regressions or linear relationships between variables. For example, if you were to design a car, the number of seat belts and number of overhead lights may both relate directly to the number of passengers the car can hold; these direct relationships would render the states of those variables to not be mutually exclusive, even though they are distinct variables whose values are needed for its design. It is likely that a complex design model contains an unknown number of variables with similar relationships that render their states not mutually exclusive, as design models are designed to output practical design information not perfect databases for Bayesian networks.

The example of the networks in Figure 4.1 demonstrates the lack of consistency that can arise when non-mutually exclusive nodes are included in a network. In this example, nodes E and D represent two nodes which are non-mutually exclusive because one is linearly defined by the other. This means the state of one directly predicts the state of the other with very little to no complexity or uncertainty in the relationship. Because the greedy search and score algorithm will favor network structure with the most information delivered by the fewest edges, when there are redundant pairs of nodes like E and D the example only node of the nodes will share an edge with any other nodes, even if both are highly influenced. This results in networks like the two shown in Figure 4.1; these networks are functionally the same but could result from two similar databases with just slight variations in data points. However, when measuring centrality these networks would have very different results for nodes D and E in either network.

To avoid this phenomenon's effect on the consistency with which the networks can be learned and the accuracy of their analysis, the redundant nodes need to be

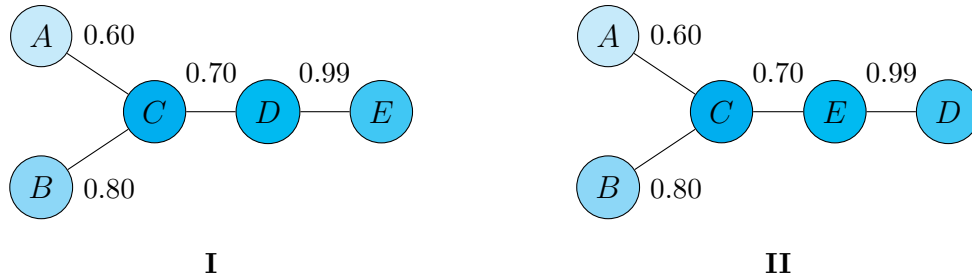


Figure 4.1: Redundant Nodes Example

excluded so that all nodes provide unique information. The lack of mutual exclusivity of these variables can result from variables that are simply too tightly related, when one defines the other with no complexity or uncertainty, or from binning into intervals that make the variables effectively not mutually exclusive because additional factors don't have a large enough effect to overcome bin size. While identifying redundant nodes can be done through expert knowledge, in order to automate the process the same scores used for edge weights can be used to determine whether the variables are too tightly correlated. This automation also ensures nodes whose correlations are unknown to the designers are also managed; while some models may have easily identifiable redundancies, others, like those resulting from assumptions or simple regressions, may be unknown to the designer. Automation of the identification of redundant variables also allows the Bayesian networks to continue to be learned without the intervention of expert knowledge. When variables are too tightly correlated and would result in redundant nodes, they can be chunked into one larger node for learning the Bayesian networks and for network analysis.

An algorithm to automatically identify and chunk redundant nodes was developed. To capture nodes that are functionally but not exactly mutually exclusive a process was developed to identify variables that would have the same or very similar structures in a learned networks. These similar structures are the result of similar levels of influence on all other variables in the model. Because we have already used scoring methods to represent the influence of variables on each other, we can assess those

scores for the relationships between all variable pairs and assess the similarity of nodes' potential structures. The following process was developed to identify and chunk redundant variable of a design database:

1. Score all possible pairs of variables with K2 or MD score and populate adjacency-like matrix
2. Compute provisional centrality for each node by summing across row/column of each variable
3. Compute Pearson correlation between matrix rows
4. In order of descending centrality combine variable with next most central node with Pearson correlation of scores over threshold into a chunked node
5. Continue to add any variable with a correlation between all chunked variables of the node over threshold
6. When all highly correlated variables have been chunked into the node, continue with next most central variable and new chunked node
7. Use only the value of design data for most central variable of each node to learn Bayesian network

This process can be used to identify redundant variables in any design model with a large number of intermediate variables that may be too dependent on each other to provide unique information in the structure of a Bayesian network.

The Pearson coefficient between rows is used to measure similarity of the relationships of the variables the rows represent. As the matrix of K2 scores are representative of a hypothetical network, the Pearson coefficient becomes a measurement of structural equivalence. Cosine similarity and jacquard coefficient are somewhat more common measures of structural equivalence in network but are more appropriate for unweighted networks. For weighted networks, they penalize the edge weight

between the two considered nodes for any value below 1, even if the edge weights for every other node connection are exactly equal. The Pearson coefficient, explained in Chapter III, returns a similarity metric more aligned with the needs of the chunking algorithm. With the Pearson coefficient, variables with similar influences on all other nodes of the design model can be identified and chunked together to eliminate redundant nodes of the network that cause inconsistency and inaccuracy. In this work, five thresholds for Pearson coefficient were assessed to determine the proper threshold for similarity: 0.975, 0.98, 0.985, 0.99 and 0.995. As a coefficient of 1 would designate perfect structure similarity and exact mutual-inexclusivity of states, higher thresholds require much stronger correlations of similarity to chunk nodes together while lower thresholds allow less similar potential structures to be assessed as the same. As the higher thresholds require much stronger correlation, there is less risk that they will chunk too many variables that should be considered unique, but can miss redundancies due to small but trivial differences in distributions due to the randomness of the database sampling. Lower thresholds are less effected by differences in sampling but can result in too many variables with similar but indeed unique structures chunking into a single node. However, because the lower thresholds are less effected by small differences, they may result in more consistent, though less accurate, networks. The selection of a chunking threshold is, therefore, a trade-off between accuracy and consistency. The accuracy and consistency of networks learned from databases chunked with each of these thresholds is discussed further in Section 6.3.

4.3 Complete Framework for Analyzing Computer Aided Design Model with Network Analysis

The complete methodology used in this framework combines established methods for learning Bayesian networks and completing network analysis with the novel meth-

ods described in this chapter. Figure 4.2 summarizes the overall methodology used to complete the analysis in this work. This work uses two design models to demonstrate the strength of the proposed analysis. Starting with design models the first step is to develop a database of design alternatives from which to learn the Bayesian networks. This work samples the design models using a Latin hypercube sampling to define the independent variables of 100,000 design alternatives for ten design trials. This work uses the *pyDOE* package in python to complete the sampling. In a real world application, any database of design alternatives which are independent of each other, describe the entire design space and are numerous enough to sufficiently populate the conditional probability tables of the Bayesian networks to come. Subsets of each design database are identified: the feasible subset with all design alternatives that do not violate any constraints of the design model and the Pareto subset with any feasible alternatives that are non-dominated with respect to the objective functions of the design model.

Once the design data is developed any continuous numerical data describing the state of variables are binned into a finite number of intervals based on the amount of evidence available. The final preparation of the design data is chunking any variables whose States are not mutually exclusive. This step is done using the process described in the previous section.

Once the data has been prepared, Bayesian networks are learned from each database using a greedy search and score algorithm with the BIC scoring method. The commercial software HUGIN was used for this step.

Next, weighted edge networks are developed from the Bayesian networks by scoring each parent-child node pair; this work scores all of the Bayesian networks with both the K2 and match distance edge scores described in the first section of this chapter.

Finally, network analysis can be completed using the weighted edge networks. First, shortest path -length between all pairs of nodes using Dijkstra's method and the

algorithm of this method in the python package *networkx*. This work uses *networkx*'s greedy modularity maximization algorithm to detect communities in the networks. Additionally, degree centrality, betweenness centrality, closeness centrality and Eigenvector centrality are computed for all nodes of each network, again using *networkx*. These metrics will highlight variable groups within the model and nodes that drive feasible and optimal designs.

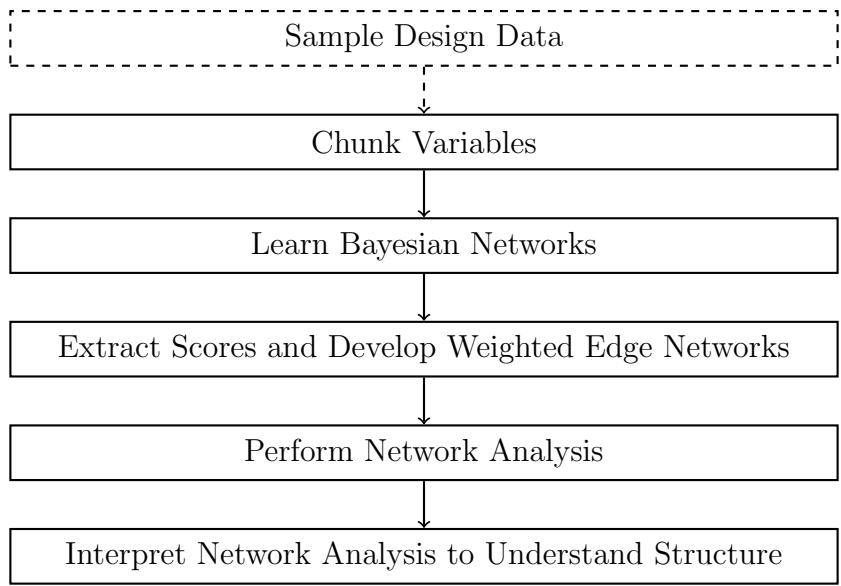


Figure 4.2: Process of transforming complex design data into simple weighted edge networks and defining network metrics

CHAPTER V

Osyczka and Kundu Design Problem

To determine the accuracy of the networks built by the proposed framework, a simple but explicit design model was analyzed first. A model like this can be manually directed to determine the relationships between variables and objective functions to assess the validity of the network structure and calculated edge weights. This simple problem is also used to evaluate the consistency with which the Bayesian networks learn the network structure with variables we know to have mutually exclusive states.

5.1 The Design Problem

In their paper applying multi-objective optimization techniques to genetic algorithms, Osyczka and Kundu present a six variable, two objective design optimization problem that this work used to refine and test the framework for network learning. The design problem presents a simple but multi-objective, multi-variable problem to test the proposed development and analysis of networks. The design problem is defined in Table 5.1.

Table 5.1: Osyczka and Kundu Benchmark Problem Definition *Osyczka and Kundu (1995)*

minimize	$f_1(\mathbf{x}) = -(25(x_1 - 2)^2 + (x_2 - 2)^2 + (x_3 - 1)^2 + (x_4)^2 + (x_5 - 1)^2)$
	$f_2(\mathbf{x}) = x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 + x_6^2$
 w.r.t.	 $0 \leq x_1, x_2, x_6 \leq 10$
	$1 \leq x_3, x_5 \leq 5$
	$0 \leq x_4 \leq 6$
 subject to	 $x_1 + x_2 - 2 \geq 0$
	$6 - x_1 - x_2 \geq 0$
	$2 - x_2 + x_1 \geq 0$
	$2 - x_1 + 3x_2 \geq 0$
	$4 - (x_3 - 3)^2 - x_4 \geq 0$
	$(x_5 - 3)^2 + x_6 - 4 \geq 0$

5.2 Methodology

To replicate a database of design alternatives, the Osyczka and Kundu problem was sampled using a Latin hypercube sampling to generate ten design trial databases of 100,000 data points each. Ten design trials were developed to evaluate the consistency of this method for learning Bayesian networks from design data, supporting this work’s objective to consistently learn accurate networks. The design alternatives which violate none of the constraints delineated by the problem definition are identified as the feasible subset for each design trial; these comprise of about 3% or 3000 data points in each design trial. Additionally the design alternatives which are feasible and Pareto optimal, non-dominated in terms of either objective, are identified as the Pareto subset; these points generally comprise just less than 1% or 1000 points in each design trial. The number of design alternatives in each set and subset of each trial are delineated in Table 5.2.

The values of the variables for each design data point were intervalled into five uniform bins each. Because the number of cells in a conditional probability table in Bayesian networks scales the number of bins exponentially the number of parents of a

Table 5.2: Number of Osyczka and Kundu design data points in each subset

Design Trial, D_t	Total Points	Feasible Points	Pareto Points
D_1	100,000	3271	81
D_2	100,000	3236	75
D_3	100,000	3285	84
D_4	100,000	3300	92
D_5	100,000	3332	83
D_6	100,000	3282	103
D_7	100,000	3245	126
D_8	100,000	3253	39
D_9	100,000	3233	90
D_{10}	100,000	3354	96

node, $r^{1+n_{parents}}$, a node with five states and just two parents with five states creates 125 cells in its conditional probability table. Five bins allow the Osyczka and Kundu problem to have sufficient accuracy when modeled while still being able to sufficiently populate the conditional probability tables of each node.

A network for each full dataset, feasible subset and Pareto subset of the ten design trials was learned using a greedy search and score algorithm with BIC scoring as described in Chapter II. The networks modeling the Osyczka and Kundu problem were learned from the variable data directly from the problem definition; the six independent variables are, by definition, not dependent on each other or any other shared input, so they do not need to be chunked. Additionally, f_1 and f_2 have different enough functions such that the state of one nodes doesn't directly translate to the state of the other; all variables have mutually exclusive states as defined in the problem. Each edge in the Bayesian networks was then scored with the K2 and MD scores defined in Chapter IV and transformed into a weighted edge network and accompanying adjacency matrix. Finally the shortest path distances, degree, betweenness, closeness and Eigenvector centralities and consistency measures were calculated; additionally, communities of nodes were identified using maximum modularity.

5.3 Consistency of Results

The networks learned by all ten design trials were fairly consistent in identification and weighting of edges.

The adjacency matrices and path lengths were evaluated for consistency by comparing each cell, representing the connection between each pair of variables using Kendall Tau and Spearman correlation and averaging the correlations between all pairs of design trials and; the centrality values for each node were also evaluated with the same correlations and averaged in the same manner. Additionally, the cell-wise standard deviation across all ten design trials was found and averaged across the matrix, excluding the diagonal of self-connections. Match distance scores are not more or less more consistent in rank correlations but do have much lower standard deviations. This consistency is due to the lack in overall variation when learning edge scores. Whereas K2 scores range from 0.003 to 0.990, all match distance scores are between 0.1 and 0.3.

The adjacency matrices are most consistent in the full and feasible networks, though all Kendall Tau and Spearman correlations average greater than 0.9 in the full, feasible and Pareto networks. Spearman correlation is slightly higher than Kendall Tau for the adjacency matrices signifying the differences between matrices are small as Spearman correlation is highly effected by large errors in rank. Kendall Tau and Spearman correlations are listed in Table 5.3 and Table 5.4, respectively.

Table 5.3: Average Kendall Tau Correlation of Adjacency Matrices of Osyczka and Kundu Problem

	Full	Feasible	Pareto
K2	0.956	0.948	0.900
MD	0.945	0.939	0.903

The average cell-wise standard deviation, listed in Table 5.5, seem small but are difficult to consider without context. The average standard deviation is also much

Table 5.4: Average Spearman Correlation of Adjacency Matrices of Osyczka and Kundu Problem

	Full	Feasible	Pareto
K2	0.974	0.977	0.916
MD	0.968	0.968	0.915

lower for sparse networks where the majority of the adjacency matrix is zero because cells that are always zero will have zero standard deviation, while even consistently learned edges with small variation will have standard deviations greater than zero.

Table 5.5: Average Cell-wise Standard Deviation of Adjacency Matrices of Osyczka and Kundu Problem

	Full	Feasible	Pareto
K2	0.033	0.040	0.021
MD	0.010	0.011	0.015

The shortest path lengths have slightly more variation than the adjacency matrices, though still have very high Kendall Tau and Spearman correlations, listed in Table 5.6 and Table 5.7. Interestingly, the Pareto networks seem to have the lowest variation, but that is due to a similar reason as the low standard deviation for adjacency matrices; when nodes consistently disconnect completely from the network, their path lengths to other nodes are always the same: infinite. Unlike the adjacency matrices this only occurs when the nodes are completely disconnected because an edge to any other connected node will create a finite path length. The larger difference between higher Spearman correlation and lower Kendall Tau correlation signifies that the slightly larger variation is still due to small differences not large outliers.

The cell-wise standard deviations for full and feasible K2 networks is noticeably high, Table 5.8; as path lengths are the inverse of the edge weights, the very small edge weights, on the order of 10^{-3} , create very large path lengths with variations disproportionately large, especially when compared to similar size differences to higher

Table 5.6: Average Kendall Tau Correlation of Shortest Path Lengths of Osyczka and Kundu Problem

	Full	Feasible	Pareto
K2	0.874	0.897	0.950
MD	0.815	0.862	0.999

Table 5.7: Average Spearman Correlation of Shortest Path Lengths of Osyczka and Kundu Problem

	Full	Feasible	Pareto
K2	0.949	0.962	0.984
MD	0.909	0.947	1.000

Table 5.8: Average Cell-wise Standard Deviation of Shortest Path Lengths of Osyczka and Kundu Problem

	Full	Feasible	Pareto
K2	4.737	2.081	0.503
MD	0.604	0.945	0.634

edge weights.

Finally the centrality correlations are highest for the full dataset networks, but lowest for the feasible networks. Kendall Tau correlations of all centrality ranks are listed in Table 5.9 and Spearman correlations are listed in 5.10. The betweenness centrality rankings are notably least consistent for both K2 and match distance scores feasible networks. For betweenness centrality in the feasible dataset networks, the Kendall Tau correlations are 0.552 and 0.593 for K2 and match distance networks while Spearman correlations are 0.562 and 0.602. Overall, the learned networks are consistent, but cell-wise standard deviations are weak measures of correlation.

Table 5.9: Average Kendall Tau Correlation of Centralities of Osyczka and Kundu Problem

		Full	Feasible	Pareto
Degree	K2	0.921	0.859	0.866
	MD	0.921	0.859	0.866
Betweenness	K2	0.905	0.552	0.774
	MD	1.000	0.593	0.774
Closeness	K2	0.835	0.619	0.804
	MD	0.735	0.611	0.841
Eigenvector	K2	0.875	0.631	0.785
	MD	0.719	0.689	0.841

Table 5.10: Average Spearman Correlation of Centralities of Osyczka and Kundu Problem

		Full	Feasible	Pareto
Degree	K2	0.933	0.876	0.907
	MD	0.933	0.876	0.907
Betweenness	K2	0.913	0.562	0.785
	MD	1.000	0.602	0.785
Closeness	K2	0.917	0.747	0.877
	MD	0.860	0.688	0.908
Eigenvector	K2	0.953	0.746	0.871
	MD	0.811	0.739	0.892

5.4 Accuracy of Results

The both the structure and edge weights of the learned networks are very representative of the Osyczka and Kundu problem.

In the images above of the K2- and MD-scored networks of a full dataset, you can see the edges reflect the functions of the problem. The second function of the problem, $f_2 = x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 + x_6^2$ includes all independent variables equally

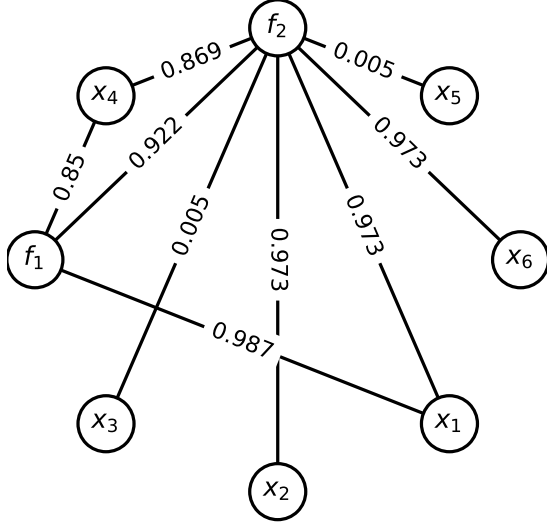


Figure 5.1: K2 Scored Weighted Edge Network Modeling the Full Design Dataset of the Osyzcka and Kundu Problem

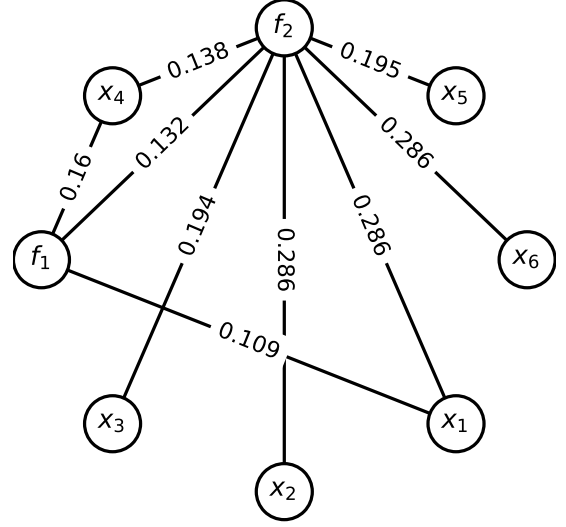


Figure 5.2: Match Distance Scored Weighted Edge Network Modeling the Full Design Dataset of the Osyzcka and Kundu Problem

weighted, so they are all connected to f_2 by an edge. The boundaries of the problem produce the varying edge weights, presented in Table B.1, for these full networks; in the K2-scored network, x_1 , x_2 and x_6 have the largest range of possible values and as such have the highest edge weights, x_3 , and x_5 have the smallest range of possible values, which translates to less influence on the overall value of f_2 and the smallest edge weights connected to it. In the MD-scored network, x_4 has a smaller edge weight connecting it to f_2 than both x_3 and x_5 , an early indication that K2 is a stronger scoring method for capturing influence. The edges connecting f_1 to other variables are also representative of the problem functions. While all independent variables are present in the function of f_1 , x_1 is the only independent variable with a coefficient greater than 1 (its coefficient is 25). This results in a strong influence and resulting edge weight between x_1 and f_1 . The second strongest influence on the value of f_1 is x_4 , as it is the only independent variable not modified by a linear term. As the other variables have both linear terms and a coefficient of only 1, they have little individual influence on the value of f_1 ; instead, f_2 connects to f_1 as a representative

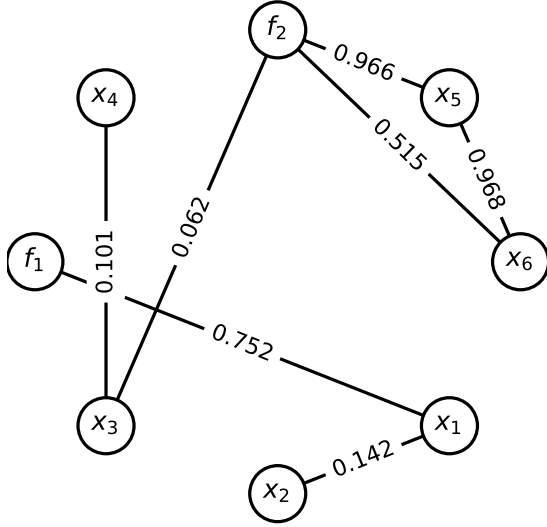


Figure 5.3: K2 Scored Weighted Edge Network Modeling the Feasible Design Dataset of the Osyczka and Kundu Problem

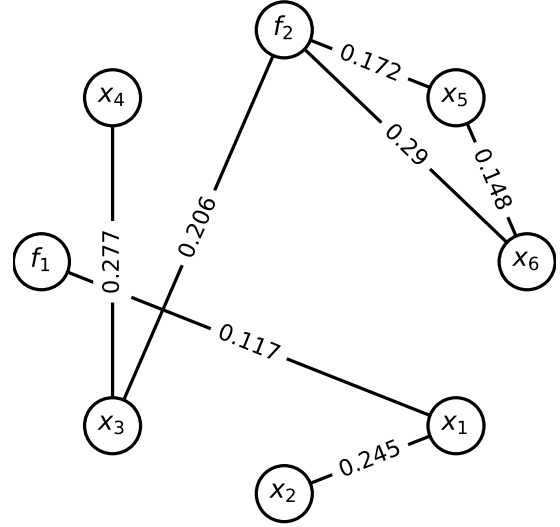


Figure 5.4: Match Distance Scored Weighted Edge Network Modeling the Feasible Design Dataset of the Osyczka and Kundu Problem

of all squared values of the independent variables.

The changes to the structure of the network from the full to feasible subset reflect the constraints of the Osyczka and Kundu problem. The independent variables are constrained in pairs with all constraints relating x_1 and x_2 , x_3 and x_4 or x_5 and x_6 . Because of the simplicity of this problem, we can dissect the constraints manually and show their effects on the design spaces of the independent variables graphically in Figure 5.5, Figure 5.6 and Figure 5.7.

The constraints restrict the independent variables to much smaller fractions of their full design space: 10% of previous spaces for x_1 and x_2 , 44% for x_3 and x_4 and 76% for x_5 and x_6 . These overall restrictions have an effect on the possible values of f_1 and f_2 and the contributions of each variable to the objective functions' values. For example, x_1 and x_2 lose some influence on f_1 because the value and variation in value of their terms in f_1 are particularly limited. Conversely, x_3 gains some influence on f_2 because its range of possible values are not limited by x_4 . Variables x_5 and x_6 lose the least design space to due to their constraint of each other; as such their

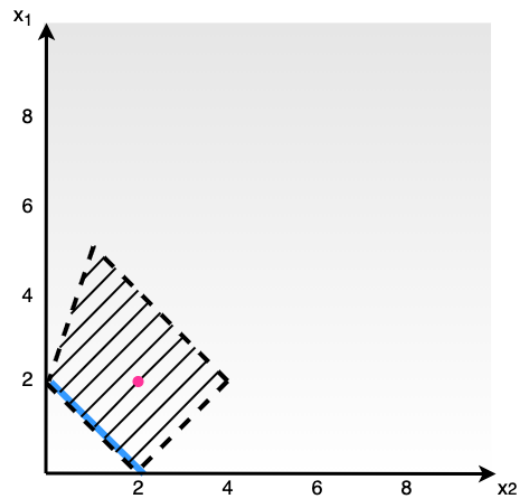


Figure 5.5: Feasible Region and Optimal Points for x_1 and x_2

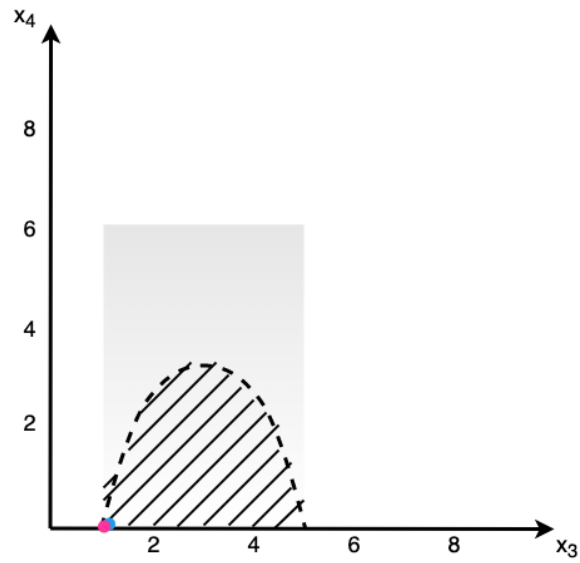


Figure 5.6: Feasible Region and Optimal Points for x_3 and x_4

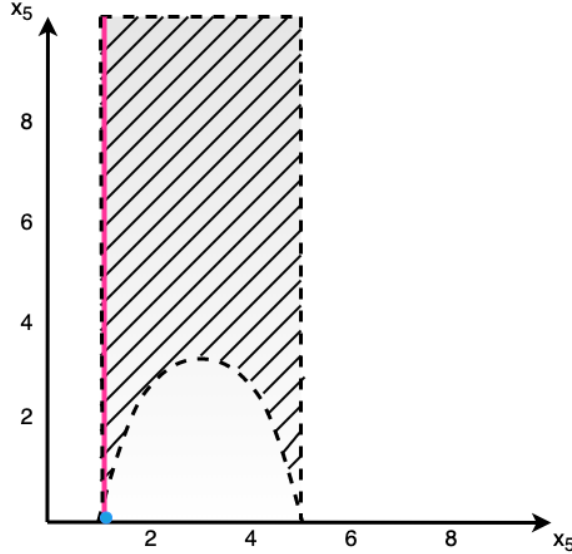


Figure 5.7: Feasible Region and Optimal Points for x_5 and x_6

terms' values and variation of values make up a larger portion of f_2 than in the full design sets.

Finally, reducing the design dataset to only non-dominated data points simplifies the network to connect only the variable which influence the values of the objective functions at their minimum values, marked in blue and pink in Figures 5.5, 5.6 and 5.7. The Pareto optimal networks are displayed in Figure 5.8 and Figure 5.9. The values of x_3 , x_4 and x_5 which contribute minimum term values for f_1 and f_2 are the same, 1, 0 and 1 respectively; as such these values do not contribute to any non-dominated objective function values and have no effect on the Pareto design points. The Pareto networks of the Osyczka and Kundu problem reflect this, dropping all edge connecting these three nodes to any other. Variable x_6 is equal to 0 for the global optimum of f_2 ; however, because it is not a component of f_1 , any feasible value of x_6 produces a value for f_2 which is Pareto-optimal because it cannot be dominated by a changing f_1 value. The optimal values of x_1 and x_2 are different for f_1 and f_2 , so those variables have the most impact on the development of the Pareto front. Objective function f_1 is optimal when $x_1 = 2$ and $x_2 = 2$ (within the feasible design space),

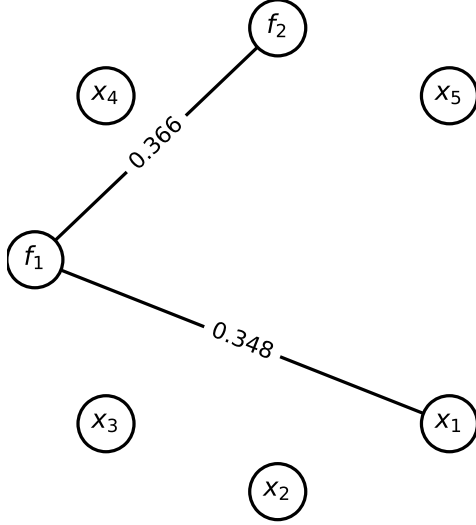


Figure 5.8: K2 Scored Weighted Edge Network Modeling the Pareto Design Dataset of the Osyczka and Kundu Problem

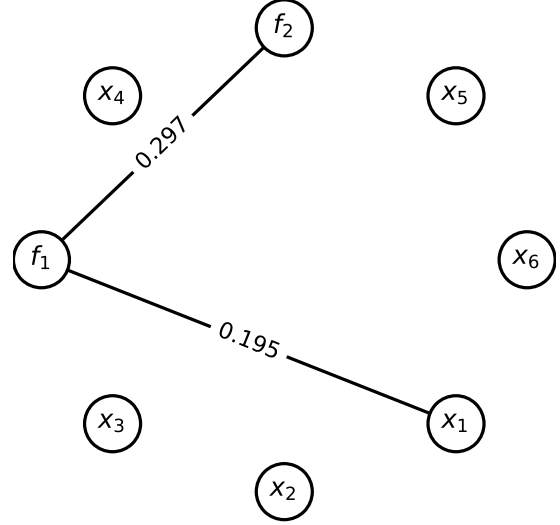


Figure 5.9: Match Distance Scored Weighted Edge Network Modeling the Pareto Design Dataset of the Osyczka and Kundu Problem

but f_2 is optimal when $x_1 + x_2 = 2$; the values of x_1 and x_2 between these points dictate whether a data point will have Pareto-optimality. As such, these variables remain strongly connected to the objective functions' nodes in the networks learned from Pareto subsets. Finally, while comparatively similar between the full networks, the K2 score of the x_6 - f_2 edge is more accurate than the extremely low MD score for the same edge. Overall, the network structures and edge weights, especially when scored with the K2 score, accurately represent the design model and the effects of its constraints and objectives.

Community detection using the greedy modularity maximization algorithm identified the separate objective functions and their largest contributing independent variables in the full dataset networks weighted with K2 scoring, shown in Figure 5.10. Additionally, the feasible communities correctly identify the pairs of independent variables constraining each other. These communities, while obvious for this simple problem, signify that the algorithm can be successful in detecting groups of variables in a design model from Bayesian networks learned solely from output data from the

model. The consistently low and less-varied match distance edge weights make identifying communities within the network more difficult, shown in Figure 5.13, Figure 5.14 and Figure 5.15. In fact, no communities are detected in the match distance scored network of the full dataset because edge weights are so low and homogeneous, Figure 5.13.

The average centrality values depicting trends in overall centrality are delineated in Table 5.11. Degree Centrality is highest on average in the full dataset network and decrease in progressive stages of design. Betweenness Centrality also has the highest average value in the full dataset network. Closeness Centrality peaks in the feasible networks, where nodes are most tied together by the constraints. The overall values of closeness centrality are generally low due to the weak connections to x_3 and x_5 in the full dataset networks and the disconnection of nodes and partial networks in the feasible and Pareto subset networks. These extremely low edge weights have disproportionately high path lengths and drive up the overall sums of path lengths. Eigenvector Centrality is the most consistent centrality measurement, but does decrease in average value in later design stage networks.

Table 5.11: Average Centralities of Osyczka and Kundu Problem

		Full	Feasible	Pareto
Degree	K2	0.321	0.232	0.093
	MD	0.321	0.232	0.093
Betweenness	K2	0.113	0.029	0.023
	MD	0.126	0.027	0.023
Closeness	K2	0.013	0.075	0.043
	MD	0.128	0.053	0.027
Eigenvector	K2	0.29	0.217	0.247
	MD	0.322	0.242	0.246

The values of each node's centralities, ranked in descending order are listed in

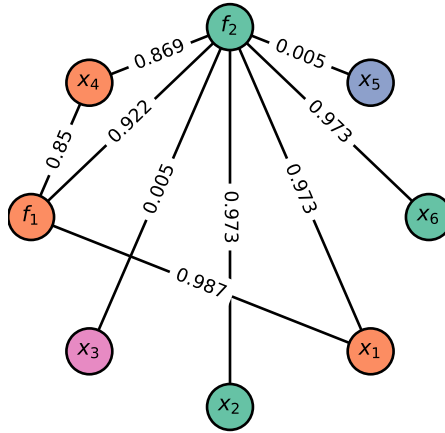


Figure 5.10: Maximum Modularity Communities of K2 Scored Weighted Edge Network Modeling the Full Design Dataset of the Oszzcka and Kundu Problem

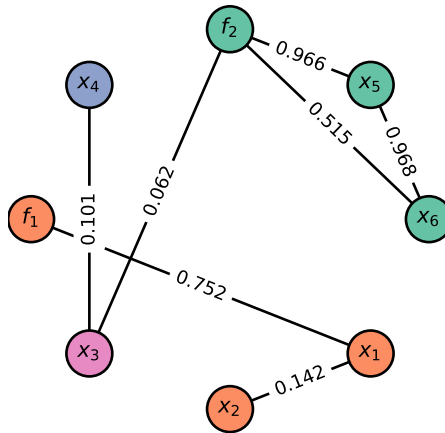


Figure 5.11: Maximum Modularity Communities of K2 Scored Weighted Edge Network Modeling the Feasible Design Dataset of the Oszzcka and Kundu Problem

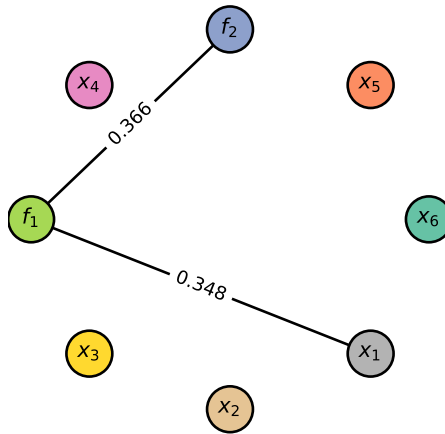


Figure 5.12: Maximum Modularity Communities of K2 Scored Weighted Edge Network Modeling the Pareto Design Dataset of the Oszzcka and Kundu Problem

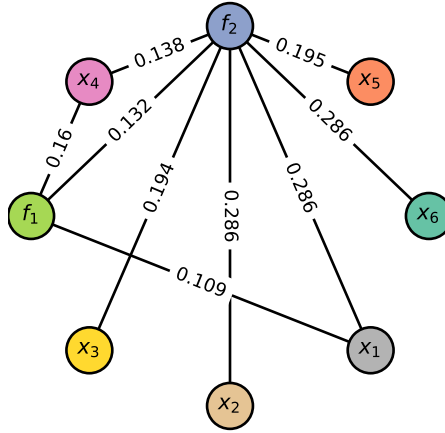


Figure 5.13: Maximum Modularity Communities of Match Distance Scored Weighted Edge Network Modeling the Full Design Dataset of the Osyzcka and Kundu Problem

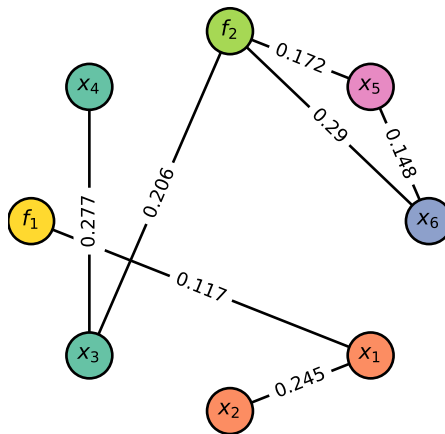


Figure 5.14: Maximum Modularity Communities of Match Distance Scored Weighted Edge Network Modeling the Feasible Design Dataset of the Osyzcka and Kundu Problem

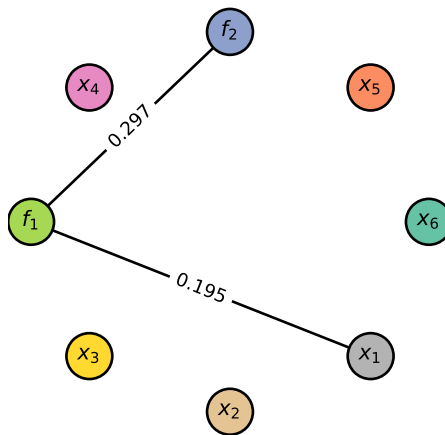


Figure 5.15: Maximum Modularity Communities of Match Distance Scored Weighted Edge Network Modeling the Pareto Design Dataset of the Osyzcka and Kundu Problem

Table 5.12. Overall centralities measured for the Osyczka and Kundu problem are not immensely insightful because the networks are small and relatively sparse. Objective function f_2 has the highest degree, betweenness, closeness and Eigenvector centralities in the full and feasible networks simply because of the nodes position as the catchall of equally weighted independent variables. However, f_1 has the greatest centrality across all measures in the Pareto networks because it remains dependent on x_1 and f_2 , while f_2 is connected only to f_1 . The change in the most central node is interesting, but because all four centrality measures have the same trends and there are only eight total nodes the Osyczka and Kundu problem is not ideal for understanding the specific differences between centrality measures.

Betweenness centrality provides little additional information about the network, as it simply highlights the most connected node. Betweenness is a measure of the fraction of shortest path pairs that pass through a node, so having edges to four nodes with no other edges connecting them drives the centrality automatically for f_2 in the full network, shown in Figure 5.16. In the feasible, Figure 5.17, and Pareto, Figure 5.18, networks have pretty uniformly low betweenness centralities because of the loss of connection to parts of the network. Match distance and K2 scores do not create much difference in the calculation of betweenness centrality because its calculation involves the sums of a binary rather than the edge weight values themselves.

Closeness centrality is uniformly low for all three design stages of networks scored with K2 edge weights. The very low edge weights connecting x_3 and x_5 in the full network have disproportionately high path lengths and drive the closeness centralities of all nodes down, seen in Figure 5.19. The disconnection of parts of the network in the feasible and Pareto networks has the same effect, seen in Figure 5.20 and Figure 5.21. The networks scored with match distance have larger, but still generally closeness centrality; the consistent, mid-range edge weights of the match distance networks do not cause the same issues with path length, especially in the full network.

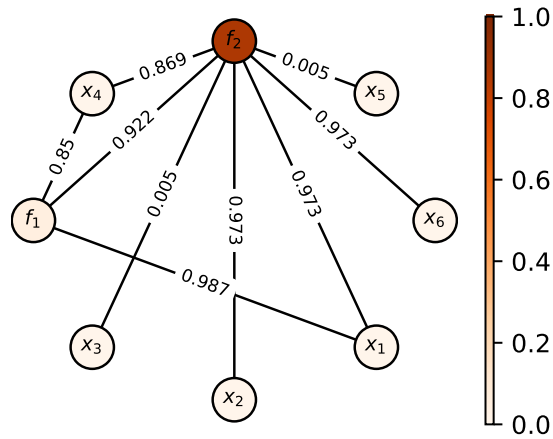


Figure 5.16: Betweenness Centrality of K2 Scored Weighted Edge Network Modeling the Full Design Dataset of the Osyzcka and Kundu Problem

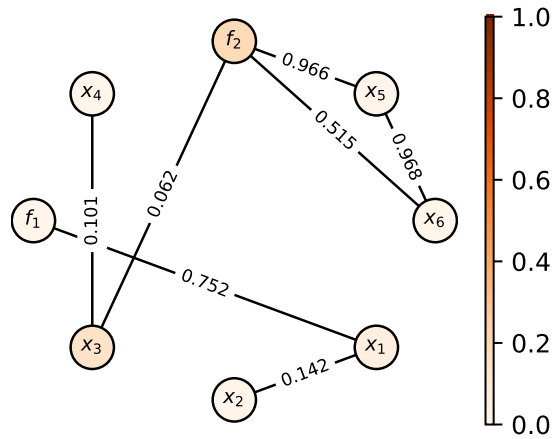


Figure 5.17: Betweenness Centrality of K2 Scored Weighted Edge Network Modeling the Feasible Design Dataset of the Osyzcka and Kundu Problem

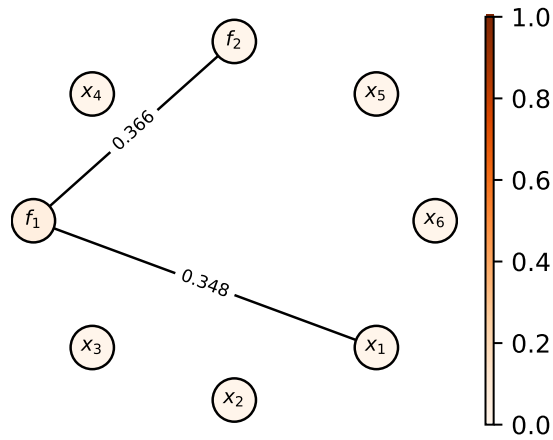


Figure 5.18: Betweenness Centrality of K2 Scored Weighted Edge Network Modeling the Pareto Design Dataset of the Osyzcka and Kundu Problem

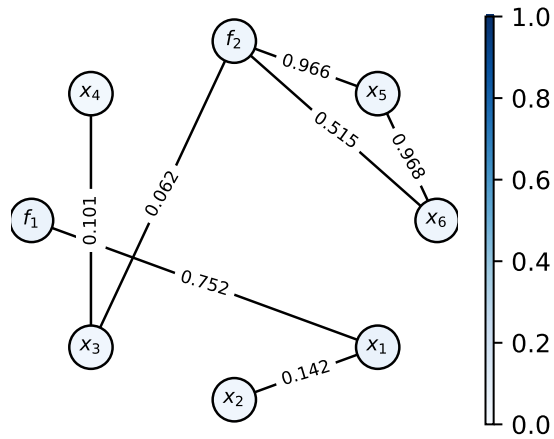


Figure 5.19: Closeness Centrality of K2 Scored Weighted Edge Network Modeling the Full Design Dataset of the Osyczka and Kundu Problem

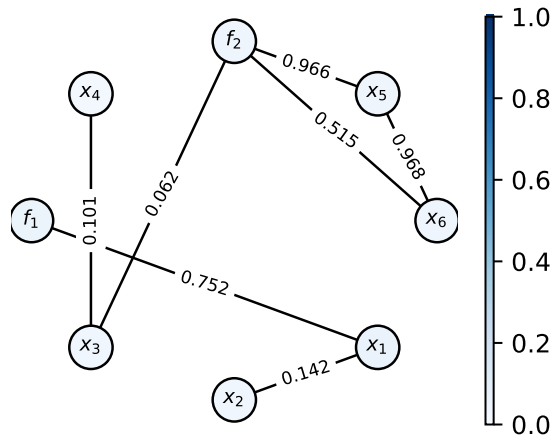


Figure 5.20: Closeness Centrality of K2 Scored Weighted Edge Network Modeling the Feasible Design Dataset of the Osyczka and Kundu Problem

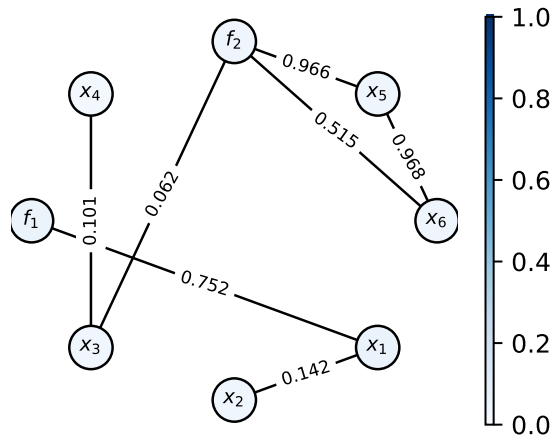


Figure 5.21: Closeness Centrality of K2 Scored Weighted Edge Network Modeling the Pareto Design Dataset of the Osyczka and Kundu Problem

Table 5.12: Average Ranked Centrality Measures of K2 Scored Osyczka and Kundu Problem

	Degree Centrality		Betweenness Centrality		Closeness Centrality		Eigenvector Centrality	
Full	f_2	1.0	f_2	0.867	f_2	0.017	f_2	0.613
	f_1	0.4	f_1	0.038	f_1	0.016	f_1	0.465
	x_4	0.286	x_6	0.0	x_1	0.016	x_1	0.406
	x_1	0.286	x_5	0.0	x_4	0.016	x_4	0.351
	x_6	0.157	x_4	0.0	x_2	0.016	x_2	0.241
	x_2	0.157	x_3	0.0	x_6	0.016	x_6	0.24
	x_5	0.143	x_2	0.0	x_3	0.004	x_3	0.001
	x_3	0.143	x_1	0.0	x_5	0.004	x_5	0.001
Feasible	f_2	0.357	f_2	0.095	f_2	0.133	f_2	0.587
	x_6	0.286	x_3	0.071	x_5	0.132	x_5	0.583
	x_5	0.286	x_1	0.048	x_6	0.121	x_6	0.551
	x_1	0.286	x_5	0.014	x_1	0.07	x_3	0.014
	x_3	0.214	x_6	0.0	f_1	0.058	x_4	0.001
	x_4	0.143	x_4	0.0	x_2	0.039	x_1	0.0
	x_2	0.143	x_2	0.0	x_3	0.029	f_1	0.0
	f_1	0.143	f_1	0.0	x_4	0.021	x_2	0.0
Pareto	f_1	0.257	f_1	0.09	f_1	0.109	f_1	0.625
	x_1	0.214	x_1	0.067	x_1	0.093	x_1	0.48
	f_2	0.157	f_2	0.029	f_2	0.081	f_2	0.478
	x_2	0.086	x_6	0.0	x_2	0.046	x_2	0.195
	x_6	0.029	x_5	0.0	x_6	0.016	x_6	0.09
	x_5	0.0	x_4	0.0	x_5	0.0	x_5	0.035
	x_4	0.0	x_3	0.0	x_4	0.0	x_4	0.035
	x_3	0.0	x_2	0.0	x_3	0.0	x_3	0.035

Eigenvector centrality does highlight more than just the most connected node for the Osyczka and Kundu problem. Eigenvector centrality considers the centrality of a node's closest connections in addition to its own closeness. This means, nodes like x_1 and x_4 have high Eigenvector centrality in the full network because they are so closely tied to the central objective function nodes, shown in Figure 5.22. Match distance and K2 scored networks have very similar Eigenvector centrality values due to the scaled nature of Eigenvectors. The average ranked centrality measures of the

K2 scored networks can be found in Table 5.12 and those of the match distance scored networks can be found in Table 5.13.

Table 5.13: Average Ranked Centrality Measures of MD Scored Osyczka and Kundu Problem

	Degree Centrality		Closeness Centrality		Betweenness Centrality		Eigenvector Centrality	
Full	f_2	1.0	f_2	0.867	f_2	0.207	f_2	0.676
	f_1	0.4	f_1	0.143	x_1	0.133	x_1	0.358
	x_4	0.286	x_6	0.0	x_2	0.129	x_2	0.307
	x_1	0.286	x_5	0.0	x_6	0.128	x_6	0.305
	x_6	0.157	x_4	0.0	f_1	0.111	f_1	0.287
	x_2	0.157	x_3	0.0	x_5	0.108	x_4	0.241
	x_5	0.143	x_2	0.0	x_3	0.108	x_5	0.202
	x_3	0.143	x_1	0.0	x_4	0.103	x_3	0.201
Feasible	f_2	0.357	f_2	0.095	f_2	0.081	f_2	0.616
	x_6	0.286	x_3	0.071	x_6	0.063	x_6	0.529
	x_5	0.286	x_1	0.048	x_3	0.058	x_5	0.47
	x_1	0.286	x_6	0.0	x_5	0.056	x_3	0.204
	x_3	0.214	x_5	0.0	x_1	0.052	x_4	0.114
	x_4	0.143	x_4	0.0	x_4	0.046	x_1	0.0
	x_2	0.143	x_2	0.0	x_2	0.038	x_2	0.0
	f_1	0.143	f_1	0.0	f_1	0.032	f_1	0.0
Pareto	f_1	0.257	f_1	0.09	f_1	0.067	f_1	0.624
	x_1	0.214	x_1	0.067	x_1	0.056	f_2	0.559
	f_2	0.157	f_2	0.029	f_2	0.053	x_1	0.389
	x_2	0.086	x_6	0.0	x_2	0.03	x_2	0.183
	x_6	0.029	x_5	0.0	x_6	0.012	x_6	0.106
	x_5	0.0	x_4	0.0	x_5	0.0	x_5	0.035
	x_4	0.0	x_3	0.0	x_4	0.0	x_4	0.035
	x_3	0.0	x_2	0.0	x_3	0.0	x_3	0.035

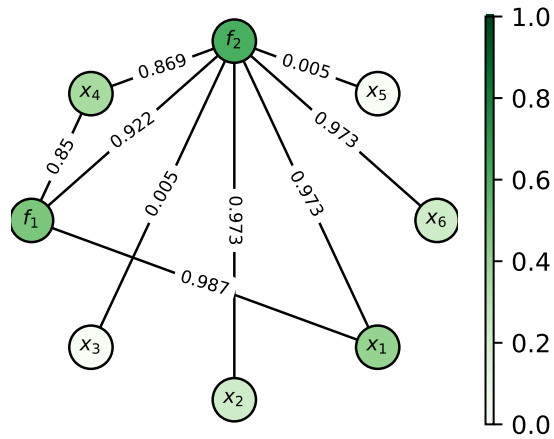


Figure 5.22: Eigenvector Centrality of K2 Scored Weighted Edge Network Modeling the Full Design Dataset of the Osyzcka and Kundu Problem

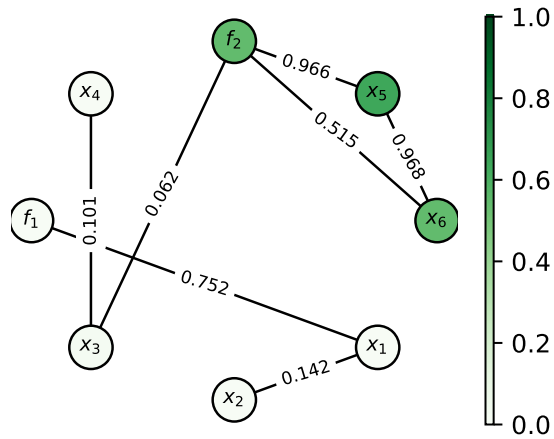


Figure 5.23: Eigenvector Centrality of K2 Scored Weighted Edge Network Modeling the Feasible Design Dataset of the Osyzcka and Kundu Problem

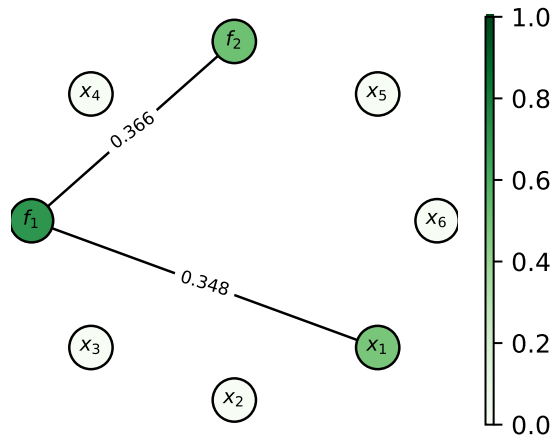


Figure 5.24: Eigenvector Centrality of K2 Scored Weighted Edge Network Modeling the Pareto Design Dataset of the Osyzcka and Kundu Problem

5.5 Lessons Learned

This analysis of the Osyczka and Kundu design problem demonstrates the effectiveness of the proposed framework. While the design problem is not complex enough to require and demonstrate the algorithm for chunking superfluous nodes or clearly differentiate between centrality measures, it does show how accurately the learned Bayesian networks can identify the model's relationships. These networks are learned consistently across the ten design trials of different design alternatives sampled from the problem definition.

The edge weights scored with the K2 edge score accurately represent the influences the design variables have on each other and the objective functions at each stage of the design process. While the effects of the constraints and objective optimization could be manually identified for this simple problem, the accuracy with which these effects were captured by the networks is promising for the methodology's use on more complex problems where manual dissection of the problem is not possible. The differing results of K2 and match distance scores begin to indicate that K2 is a better scoring method for calculating edge weights. This trend is continued and confirmed in the next case study.

CHAPTER VI

Sen and Yang Bulk Carrier Design Problem

After demonstrating capability in representing a simple multi-objective design model, the framework tested in this work should be applied to a more complex model. A larger, more complex model for a bulk carrier was analyzed to demonstrate capability in consistently learning a representative structure and computing network metrics that identify the variables driving solutions at each stage of design. This bulk carrier design model is ideal for demonstrating the strength of the framework because it is explicit enough to understand the understand at the initial design stage but complex enough to produce information in the networks modeling the feasible and Pareto-optimal design spaces that would be difficult to understand through standard analysis.

6.1 The Design Problem

In Sen and Yang's book on decision making in design, the chapter on multiple objective decision making provides a simple and useful ship design model for bulk carriers *Sen and Yang* (1998). As an example of multi-objective ship design the example has three objective functions: minimization of transportation cost, minimization of light ship mass and maximization of annual cargo capacity. These three objective functions are explicitly computed from six independent variables and a series of 27

intermediate variables. The model additionally defines 13 constraints on both the independent and intermediate variables. The function’s variables, constraints and objectives are delineated in Table 6.1. The complete bulk carrier model from Sen and Yang can be found in Appendix B.

6.2 Initial Methodology

Like the Osyczka and Kundu problem, the Sen and Yang bulk carrier design model was sampled to develop a design database to learn a Bayesian network from. Ten design trials were again developed using a Latin hypercube sampling of 100,000 data points each. To avoid too small feasible and Pareto subsets, constraints on the independent variables of the problem were incorporated into the sampling. A normalized 100,000 point Latin hypercube sampling for 6 independent variables produced and denormalized progressively to the acceptable ranges of the independent variables, as defined by the model and simplified in Table 6.2. Independent variables C_B and V we denormalized first to a fixed range of values. Then L was fit using a minimum value defined by the V at each data point and a fixed upper limit. This process was continued for D , T and B , each with maximum or minimum limits dependent on earlier-defined variables.

The resulting 100,000 data points do not represent a true Latin hypercube sampling in the full input design space but greatly increase the number of feasible points while still covering the design space.

The feasible subset is then defined by the points which did not violate any of the remaining constraints; for each of the ten trials this includes about 12% of the full dataset. The Pareto subset includes all points that are feasible and non-dominated with respect to the three objective functions; most trials had Pareto subsets that included just under 1%, or 1000, of the full data points. The exact number of design alternatives in each dataset are delineated in Table 6.3.

Table 6.1: Sen Bulk Carrier Design Model Summary

minimize	transportation cost, TC light ship mass, Δ_{LS}
maximize	annual cargo, $\Delta_{C,ann}$
w.r.t.	length, L draft, T depth, D block coefficient, C_B beam, B speed, V Froude number, Fn displacement, Δ steel mass, Δ_S outfit mass, Δ_O $a(C_B)$ $b(C_B)$ sea days, t_s BM KG KB $P(\Delta, V, a, b, L)$ GM ship cost, SC machinery mass, Δ_M daily consumption, Δ_{DC} fuel cost, FC capital charges, CC fuel carried, Δ_F deadweight, Δ_{DW} running costs, RC port costs, PC stores, water and crew, Δ_{SWC} voyage cost, VC cargo weight, Δ_C port days, t_p round trip per annum, $RTPA$ annual cost, AC
subject to	$L/B > 6$ $L/D < 15$ $L/T < 19$ $T < 0.45 \times \Delta_{DW}^{0.31}$ $T < 0.7D + 0.7$ $\Delta_{DW} > 3000$ $\Delta_{DW} < 500,000$ $C_B > 0.63$ $C_B < 0.75$ $V > 14$ $V < 18$ $Fn < 0.32$ $0.07B < GM$

Table 6.2: Upper and limits of six independent variables

Variable	Lower Limit	Upper Limit
C_B	0.63	0.75
V	14	18
L	$(\frac{V}{0.32})^2/g$	362
D	$L/15$	30
T	$L/19$	$0.7D + 0.7$
B	5	$L/6$

Table 6.3: Number of Sen Bulker design data points in each subset

Design Trial, D_t	Total Points	Feasible Points	Pareto Points
D_1	100,000	11,936	903
D_2	100,000	11,950	863
D_3	100,000	11,659	800
D_4	100,000	11,627	886
D_5	100,000	11,910	906
D_6	100,000	11,776	1053
D_7	100,000	11,846	908
D_8	100,000	11,792	915
D_9	100,000	11,773	835
D_{10}	100,000	11,670	926

The values of each variable within datasets were binned into ten uniform intervals, each bin as a node state. These datasets and subsets were then used originally to learn 30 Bayesian networks using a greedy search and score algorithm with BIC scoring as before. The resulting networks were analyzed for consistency and accuracy using community detection, centrality measurements and correlation measures. The results of this process, which produced inconsistent networks due to redundant node pairs in the bulk carrier design model, are discussed below; the methodology of chunking variables into combined nodes designed to reduce this issue and its results are detailed later in the Chapter.

6.2.1 Initial Results

The consistency of these networks learned from the complete, unchunked model data was much lower than those modeling the Osyczka and Kundu problem on all measures of consistency. The Kendall Tau correlations of both the adjacency matrices, Table 6.4, shortest path lengths, Table 6.7, and the centrality measurements, Table 6.10, were all worse than those measured for the Osyczka and Kundu problem, including some centrality values almost 50% lower for the full datasets, whose size should make the networks most consistent. However, the Spearman correlations for shortest path length, Table 6.8, and centralities, Table ??, are still higher than their Kendall Tau counterparts. This indicates that at least part of the variation between trials are small changes in the order of nodes on a branch rather than large substantial changes to the structure.

Table 6.4: Average Kendall Tau Correlation of Adjacency Matrices of Bulk Carrier Design Problem

	Full	Feasible	Pareto
K2	0.745	0.863	0.863
MD	0.745	0.857	0.857

Table 6.5: Average Spearman Correlation of Adjacency Matrices of Bulk Carrier Design Problem

	Full	Feasible	Pareto
K2	0.764	0.869	0.869
MD	0.761	0.865	0.865

Table 6.6: Average Cell-wise Standard Deviation of Adjacency Matrices of Bulk Carrier Design Problem

	Full	Feasible	Pareto
K2	0.059	0.021	0.018
MD	0.038	0.016	0.016

Table 6.7: Average Kendall Tau Correlation of Shortest Path Lengths of Bulk Carrier Design Problem

	Full	Feasible	Pareto
K2	0.492	0.633	0.63
MD	0.494	0.600	0.598

Table 6.8: Average Spearman Correlation of Shortest Path Lengths of Bulk Carrier Design Problem

	Full	Feasible	Pareto
K2	0.656	0.793	0.793
MD	0.656	0.762	0.764

Table 6.9: Average Cell-wise Standard Deviation of Shortest Path Lengths of Bulk Carrier Design Problem

	Full	Feasible	Pareto
K2	0.825	1.234	1.277
MD	1.324	1.874	1.750

Table 6.10: Average Kendall Tau Correlation of Centralities of Bulk Carrier Design Problem

		Full	Feasible	Pareto
Degree	K2	0.520	0.706	0.722
	MD	0.520	0.706	0.722
Betweenness	K2	0.356	0.594	0.704
	MD	0.373	0.603	0.704
Closeness	K2	0.465	0.653	0.712
	MD	0.442	0.613	0.690
Eigenvector	K2	0.425	0.622	0.680
	MD	0.408	0.598	0.666

Figure 6.1 and Figure 6.2 show networks of the the full datasets of trials 0 and 1, respectively, grouped into communities detected using maximum modularity. It is apparent from these networks that while generally nodes form similar groups of tight

Table 6.11: Average Spearman Correlation of Centralities of Bulk Carrier Design Problem

		Full	Feasible	Pareto
Degree	K2	0.609	0.744	0.752
	MD	0.609	0.744	0.752
Betweenness	K2	0.464	0.679	0.773
	MD	0.493	0.691	0.773
Closeness	K2	0.631	0.801	0.852
	MD	0.594	0.765	0.834
Eigenvector	K2	0.575	0.768	0.820
	MD	0.548	0.749	0.810

connections, their structures are very different overall. Many nodes jump between communities or move inward or outward along network branches, completely changing their centralities.

As mentioned in Chapter IV, changing orders of nodes, while not hugely impactful on the predictive results of the Bayesian network, create large differences in measurements of centrality. For example, Figure 6.3 and Figure 6.4 show the betweenness centralities for these networks from Trials 0 and 1. Variables C_B , a and b are highly connected, as a and b are both quadratic functions of C_B . In Trial 0 node a connects to the rest of the network and in Trial 1 C_B does. This is the result of the greedy search and score algorithm used to learn the networks and the overlap of influence of these nodes. The three nodes are so highly connected to each other, their influences on other variables are all effectively the same; for computational efficiency, the greedy search and score algorithm will only keep one edge if multiple edge would all provide the same information. This results in a single edge connecting this group to the rest of the network, and that edge connecting to whichever of the group has the cleanest data for that trial; the connection becomes reliant on small differences in binning rather than one having a stronger influence than the other. Following this somewhat

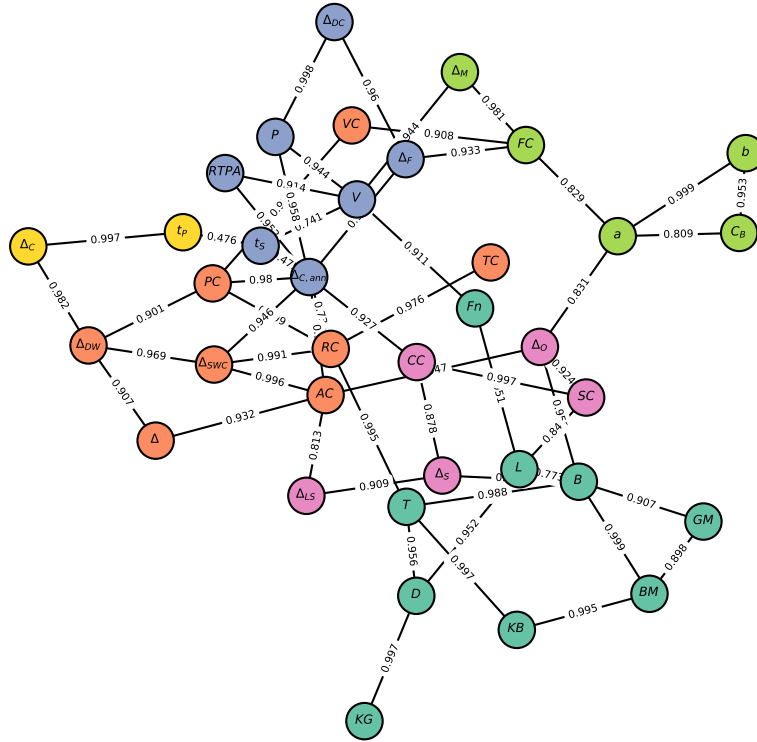


Figure 6.1: K2 Scored Weighted Edge Network Modeling the Full Design Dataset of the Bulk Carrier Design Problem, Trial 0

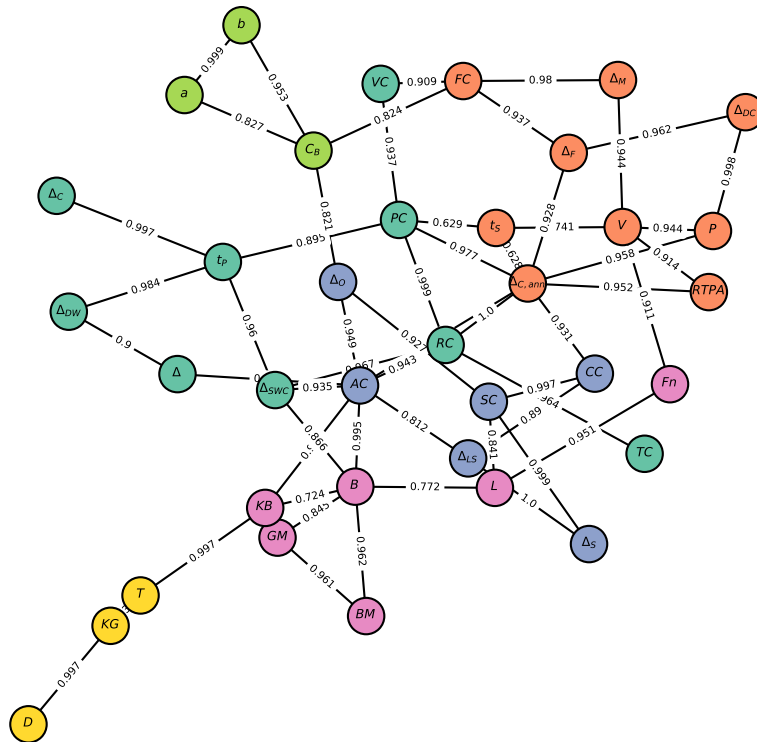


Figure 6.2: K2 Scored Weighted Edge Network Modeling the Full Design Dataset of the Bulk Carrier Design Problem, Trial 1

arbitrary assignment of edges, the centrality of those nodes becomes highly dependent on the same chance differences. In Trial 0, Figure 6.3, a has much higher betweenness centrality than C_B and b because shares edges with other nodes, while C_B is highest in Trial 1, Figure 6.4, for the same reasons. If we can eliminate these changing orders of nodes and arbitrary connections and misconnections, the networks will be learned much more consistently and their network metrics will have much more meaning.

Table 6.12: Average Centralities of Bulk Carrier Design Problem

		Full	Feasible	Pareto
Degree	K2	0.089	0.058	0.052
	MD	0.089	0.058	0.052
Betweenness	K2	0.067	0.095	0.088
	MD	0.067	0.095	0.088
Closeness	K2	0.287	0.145	0.114
	MD	0.179	0.102	0.091
Eigenvector	K2	0.130	0.122	0.111
	MD	0.127	0.120	0.106

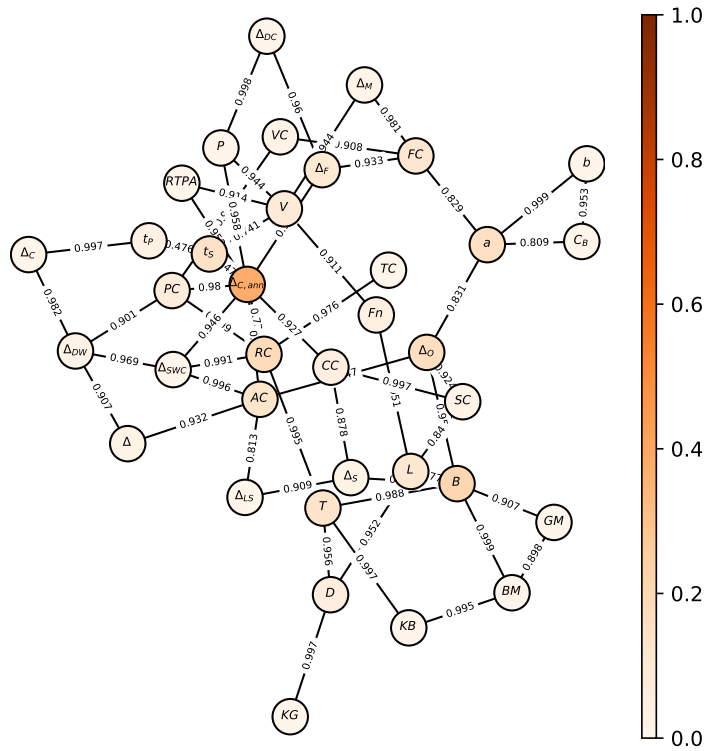


Figure 6.3: Betweenness Centrality of K2 Scored Weighted Edge Network Modeling the Full Design Dataset of the Bulk Carrier Design Problem, Trial 0

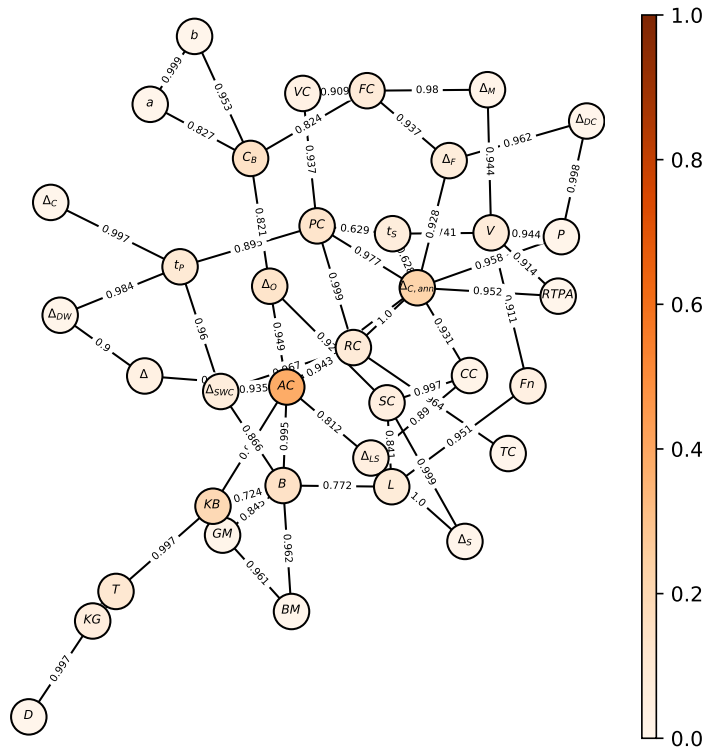


Figure 6.4: Betweenness Centrality of K2 Scored Weighted Edge Network Modeling the Full Design Dataset of the Bulk Carrier Design Problem, Trial 1

6.3 Chunking

6.3.1 Methodology

To avoid the inconsistencies redundant variables, those whose states are not mutually exclusive, produce, the variables of the bulk carrier design problem must be analyzed to identify which variables should be chunked into a single node. The process described in Chapter IV for eliminating redundant nodes is applied to the intervalled bulk carrier design data. That process requires chunking together any nodes whose scores with all other nodes have a Pearson correlation coefficient over some threshold. To determine which threshold would be most useful in this work, five were tested for both the K2 and match distance scores: 0.975, 0.98, 0.985, 0.99 and 0.995. A total of 300 additional networks were learned in HUGIN with varying numbers of nodes and variables per node from the differing correlation thresholds. These 300 networks were assigned edge weights from their K2 and match distance scores for conversion to weighted edge networks. Finally the networks chunked by all ten thresholds were analyzed for path length, centralities and detected communities. To compare the consistency of these networks with possible different numbers of nodes from chunking, the adjacency matrices, path distances and centralities were expanded back and assigned to all variables rather than just the nodes. In the expanded adjacency matrix, all variables within a chunked node were assigned edge weight values of 1 and the calculated edge weight for all variables in a connected node. A similar approach was taken for the shortest path length, but all variables within a node were assigned a path length of zero. The centrality measurements were calculated with the original adjacency matrices and path distances rather than the expanded ones to avoid an effectively higher weight on nodes with more chunked variables, as the number of redundant nodes is an arbitrary result of the calculation methods and output of the design problem rather than an indicator of importance. The resulting centrality

measurements for each node were simply assigned to all variables within the node for comparison.

6.3.2 Chunking Thresholds and Consistency

The chunked bulk carrier networks have overall greater consistency than their unchunked counterparts. Generally, the highest consistencies were observed at a K2 correlation threshold of 0.99 for full dataset networks and 0.98 for feasible subset networks and 0.98 or 0.975 for Pareto subset networks. This trend is especially prevalent in the Kendall Tau and Spearman correlations of the centrality measurements, shown in Table 6.19 and Table 6.20. The match distance scored and chunked networks also perform more consistently than their unchunked counterparts, but not better than the K2 scored and chunked networks. Because of their similar performance here and lower accuracy representing the Osyzcka and Kundu problem, this work will focus on the results from the K2 scored networks for the remainder of the analysis.

The adjacency matrices have higher Kendall Tau and Spearman correlations for the full and feasible networks with thresholds of 0.99, 0.90 and 0.975 than without any chunking, evidenced in Table 6.13 and Table 6.14. The adjacency matrix correlations for the Pareto networks, however, are lower when chunked than when not; the biggest issue for chunking the Pareto networks is the large, most central nodes. Because edge weights are smaller at the Pareto design stage, when a variable is not always chunked into one of these nodes its edge weights shared with the variables within the node vary from 1, when it is chunked, to values between 0.5 and 0.7 when it is not. In the full and feasible networks the same change to a chunked variable would be from 1 when chunked to between 0.6 and 1.0 when not; similarly the nodes in the unchunked networks change order but have somewhat consistent edge weight values. This means the same changes to a network structure could result in a larger calculated deviation. The average cell-wise standard deviations, delineated in Table 6.15, are worse for all

chunking thresholds than the unchunked versions; like demonstrated in the Osyczka and Kundu problem, the cell-wise standard deviations highly favor sparse nodes with consistent zeros in their adjacency matrices.

Table 6.13: Average Kendall Tau Correlation of Adjacency Matrices of Bulk Carrier Design Problem

		Full	Feasible	Pareto
K2	0.975	0.816	0.836	0.755
	0.98	0.835	0.906	0.772
	0.985	0.819	0.790	0.753
	0.99	0.839	0.877	0.719
	0.995	0.793	0.860	0.740
	Unchunked	0.745	0.863	0.863
MD	0.975	0.821	0.847	0.655
	0.98	0.824	0.863	0.626
	0.985	0.814	0.869	0.628
	0.99	0.829	0.784	0.786
	0.995	0.822	0.840	0.866
	Unchunked	0.745	0.857	0.857

Table 6.14: Average Spearman Correlation of Adjacency Matrices of Bulk Carrier Design Problem

		Full	Feasible	Pareto
K2	0.975	0.898	0.863	0.786
	0.98	0.913	0.921	0.800
	0.985	0.863	0.810	0.781
	0.99	0.883	0.891	0.745
	0.995	0.833	0.871	0.758
	Unchunked	0.764	0.869	0.869
MD	0.975	0.857	0.894	0.695
	0.98	0.852	0.896	0.665
	0.985	0.843	0.900	0.661
	0.99	0.851	0.816	0.806
	0.995	0.842	0.855	0.881
	Unchunked	0.761	0.865	0.865

The consistency measurements for shortest path length and centrality ranks per-

Table 6.15: Average Cell-wise Standard Deviation of Adjacency Matrices of Bulk Carrier Design Problem

		Full	Feasible	Pareto
K2	0.975	0.096	0.068	0.080
	0.98	0.088	0.038	0.071
	0.985	0.094	0.067	0.073
	0.99	0.082	0.035	0.071
	0.995	0.088	0.035	0.051
	Unchunked	0.059	0.021	0.018
MD	0.975	0.063	0.06	0.119
	0.98	0.058	0.048	0.123
	0.985	0.059	0.046	0.100
	0.99	0.047	0.057	0.045
	0.995	0.045	0.029	0.025
	Unchunked	0.038	0.016	0.016

form better across all chunking thresholds than those for the unchunked networks, with one exception. Kendall Tau correlation in Table 6.16, Spearman correlation in Table 6.17 and the cell-wise standard deviation in Table 6.18 for path lengths all perform consistently better than the unchunked networks; additionally, like before, the Spearman correlations are higher than the Kendall Tan correlations, signifying that the variations are small and not due to major differences in path length. While the standard deviations of the shortest path lengths are also smaller for chunked networks, it is still not a great metric for consistency without a lot of context. Due to chunking the networks have fewer nodes and, therefore, fewer edges between the nodes resulting in shorter path lengths overall. From the results of this analysis and those of the Osyczka and Kundu problem, it is evident that standard deviation has too many factors other than structure consistency and is the weakest measure of network learning consistency used in this work.

The Kendall Tau and Spearman correlations of the centrality measurements were better for the networks chunked with thresholds of 0.99, 0.98 and 0.975, with the exception of betweenness centrality of the 0.99-chunked Pareto networks, shown in

Table 6.16: Average Kendall Tau Correlation of Shortest Path Lengths of Bulk Carrier Design Problem

		Full	Feasible	Pareto
K2	0.975	0.680	0.746	0.748
	0.98	0.707	0.815	0.734
	0.985	0.731	0.672	0.695
	0.99	0.744	0.701	0.657
	0.995	0.663	0.706	0.623
	Unchunked	0.492	0.633	0.630
MD	0.975	0.678	0.730	0.515
	0.98	0.719	0.747	0.524
	0.985	0.663	0.756	0.460
	0.99	0.678	0.670	0.570
	0.995	0.589	0.722	0.665
	Unchunked	0.494	0.600	0.598

Table 6.17: Average Spearman Correlation of Shortest Path Lengths of Bulk Carrier Design Problem

		Full	Feasible	Pareto
K2	0.975	0.836	0.859	0.885
	0.98	0.855	0.905	0.884
	0.985	0.865	0.814	0.847
	0.99	0.876	0.817	0.828
	0.995	0.809	0.837	0.794
	Unchunked	0.656	0.793	0.793
MD	0.975	0.817	0.871	0.650
	0.98	0.854	0.879	0.665
	0.985	0.807	0.884	0.614
	0.99	0.813	0.819	0.721
	0.995	0.742	0.875	0.815
	Unchunked	0.656	0.762	0.764

Table 6.19 and Table 6.20. Betweenness centrality has the same issue with inconsistency of the largest chunked nodes that was prevalent in the adjacency matrices. The large, least consistently chunked nodes of the Pareto networks have much higher betweenness centrality than the other nodes in the work; as a result, variables that

Table 6.18: Average Cell-wise Standard Deviation of Shortest Path Lengths of Bulk Carrier Design Problem

		Full	Feasible	Pareto
K2	0.975	0.543	0.474	0.529
	0.98	0.517	0.337	0.626
	0.985	0.422	0.567	0.761
	0.99	0.349	0.537	0.853
	0.995	0.466	0.623	1.002
	Unchunked	0.825	1.234	1.277
MD	0.975	0.69	0.631	0.897
	0.98	0.664	0.661	0.940
	0.985	0.712	0.664	1.183
	0.99	0.716	0.889	1.244
	0.995	0.918	1.281	1.129
	Unchunked	1.324	1.874	1.750

are sometimes but not always chunked into the central nodes have large variation in their betweenness centralities and ranks. For the remainder of the centrality measurements, both Kendall Tau and Spearman correlations are greater for the chunked networks than those for the unchunked networks. Additionally, Spearman correlation is higher than Kendall Tau correlation across all the centrality measurements. To determine the best correlation threshold for chunking, the accuracy of the chunking must be analyzed in addition to the consistency measures.

Table 6.19: Average Kendall Tau Correlation of Centralities of Bulk Carrier Design Problem

			Full	Feasible	Pareto
Degree	K2	0.975	0.799	0.868	0.934
		0.98	0.811	0.936	0.848
		0.985	0.828	0.800	0.796
		0.99	0.889	0.804	0.750
		0.995	0.829	0.809	0.732
		Unchunked	0.520	0.706	0.722
	MD	0.975	0.789	0.829	0.722
		0.98	0.788	0.889	0.704
		0.985	0.752	0.889	0.769
		0.99	0.811	0.783	0.817
		0.995	0.751	0.821	0.701
		Unchunked	0.520	0.706	0.722
Betweenness	K2	0.975	0.524	0.853	0.918
		0.98	0.533	0.928	0.788
		0.985	0.660	0.742	0.699
		0.99	0.809	0.795	0.668
		0.995	0.725	0.773	0.642
		Unchunked	0.356	0.594	0.704
	MD	0.975	0.677	0.781	0.534
		0.98	0.523	0.836	0.540
		0.985	0.584	0.839	0.682
		0.99	0.668	0.727	0.822
		0.995	0.617	0.754	0.712
		Unchunked	0.373	0.603	0.704
Closeness	K2	0.975	0.628	0.773	0.878
		0.98	0.637	0.817	0.821
		0.985	0.684	0.669	0.745
		0.99	0.749	0.705	0.714
		0.995	0.669	0.692	0.682
		Unchunked	0.465	0.653	0.712
	MD	0.975	0.558	0.792	0.553
		0.98	0.616	0.769	0.627
		0.985	0.548	0.775	0.546
		0.99	0.594	0.651	0.658
		0.995	0.545	0.651	0.735
		Unchunked	0.442	0.613	0.690
Eigenvector	K2	0.975	0.585	0.741	0.865
		0.98	0.595	0.796	0.825
		0.985	0.703	0.660	0.761
		0.99	0.711	0.685	0.724
		0.995	0.62	0.711	0.686
		Unchunked	0.425	0.622	0.680
	MD	0.975	0.595	0.664	0.516
		0.98	0.495	0.654	0.584
		0.985	0.460	0.660	0.514
		0.99	0.566	0.595	0.643
		0.995	0.457	0.588	0.740
		Unchunked	0.408	0.598	0.666

Table 6.20: Average Spearman Correlation of Centralities of Bulk Carrier Design Problem

			Full	Feasible	Pareto	
Degree	K2	0.975	0.857	0.910	0.960	
		0.98	0.868	0.968	0.891	
		0.985	0.868	0.856	0.836	
		0.99	0.925	0.858	0.798	
		0.995	0.883	0.863	0.778	
		Unchunked	0.609	0.744	0.752	
	MD	0.975	0.827	0.878	0.760	
		0.98	0.828	0.932	0.746	
		0.985	0.798	0.935	0.816	
		0.99	0.867	0.849	0.860	
		0.995	0.838	0.870	0.743	
		Unchunked	0.609	0.744	0.752	
	Betweenness	K2	0.975	0.631	0.899	0.951
			0.98	0.639	0.973	0.839
0.985			0.746	0.813	0.738	
0.99			0.886	0.853	0.716	
0.995			0.812	0.834	0.689	
Unchunked			0.464	0.679	0.773	
MD		0.975	0.736	0.847	0.565	
		0.98	0.619	0.924	0.579	
		0.985	0.679	0.926	0.732	
		0.99	0.777	0.821	0.861	
		0.995	0.743	0.814	0.752	
		Unchunked	0.493	0.691	0.773	
Closeness		K2	0.975	0.746	0.864	0.955
			0.98	0.752	0.901	0.930
	0.985		0.806	0.794	0.868	
	0.99		0.856	0.793	0.851	
	0.995		0.805	0.800	0.827	
	Unchunked		0.631	0.801	0.852	
	MD	0.975	0.670	0.887	0.676	
		0.98	0.715	0.890	0.753	
		0.985	0.672	0.895	0.673	
		0.99	0.722	0.791	0.798	
		0.995	0.705	0.776	0.879	
		Unchunked	0.594	0.765	0.834	
	Eigenvector	K2	0.975	0.716	0.839	0.947
			0.98	0.720	0.884	0.925
0.985			0.812	0.796	0.878	
0.99			0.833	0.790	0.864	
0.995			0.764	0.829	0.830	
Unchunked			0.575	0.768	0.820	
MD		0.975	0.700	0.784	0.631	
		0.98	0.612	0.782	0.713	
		0.985	0.585	0.790	0.641	
		0.99	0.688	0.731	0.784	
		0.995	0.581	0.710	0.877	
		Unchunked	0.548	0.749	0.810	

6.3.3 Chunking Thresholds and Accuracy

As the 0.99, 0.98 and 0.975 correlation thresholds had the most consistent networks at progressive stages of design, their structures should be analyzed to determine which produces the most accurate chunked nodes. For the full dataset a threshold of 0.98 produces the most consistent networks. Figure 6.5, Figure 6.6 and Figure 6.7 show the full dataset networks chunked with correlation thresholds of 0.99, 0.98 and 0.975, respectively. The some nodes of the 0.98 and 0.975 networks contain more variables, as they have chunked nodes that are connected but are not so redundant that they should be a single node. For example, in the 0.99 threshold network there is a chunked node containing variables related to deadweight and cargo weight: Δ , Δ_{DW} , Δ_C and t_p . This is an acceptable chunk; Δ_C and t_p are linearly related, and the simplicity of weight estimations in the model make Δ , Δ_{DW} and Δ_C all scale together, with only small additional inputs. However, when chunked with thresholds 0.98 and 0.975, this node also included $RTPA$, round trips per annum. Because $RTPA$ is a function highly dependent on both t_p and t_s , it should not be combined with that node because $RTPA$ and t_p do have truly mutually exclusive states.

The networks modeling the feasible and Pareto subsets follow a similar pattern. While the feasible networks are most consistent when chunked with a threshold of 0.98, their nodes also chunk too many mutually exclusive variables at the lower thresholds. Figure 6.8, Figure 6.9 and Figure 6.10 show examples of feasible subset networks chunked with correlation thresholds of 0.99, 0.98 and 0.975, respectively. In the 0.98 and 0.975 threshold networks, one single node has eight chunked variables. While some of these variables like Δ_O and Δ_{SWC} have similar regressions leading to their results and are likely not mutually exclusive, variables like L should definitely be excluded. This shows that while the networks chunked with a threshold of 0.98 are most consistent, they are consistent because they consistently chunk too many nodes together and don't allow for different edge weights or structures.

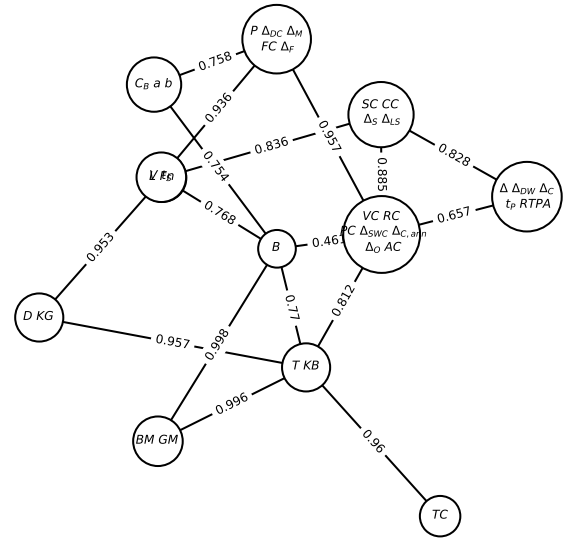
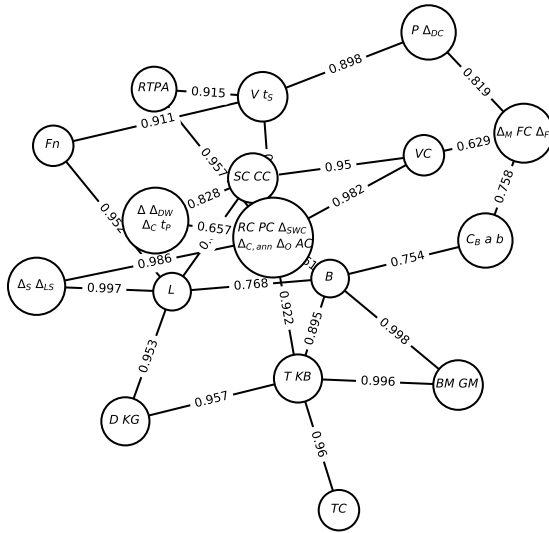


Figure 6.5: Weighted Edge Network Modeling the Full Design Dataset of the Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.99, Trial 1

Figure 6.6: Weighted Edge Network Modeling the Full Design Dataset of the Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.98, Trial 1

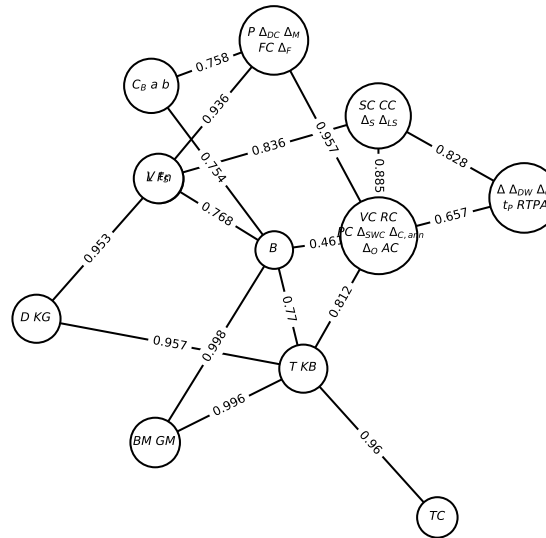


Figure 6.7: Weighted Edge Network Modeling the Full Design Dataset of the Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.975, Trial 1

The Pareto networks have the same issues as the feasible issues, and due to the small database size for learning the Bayesian networks even begin to see more illogical chunks in the 0.99 threshold networks. Figure 6.11, Figure 6.12 and Figure 6.13 show

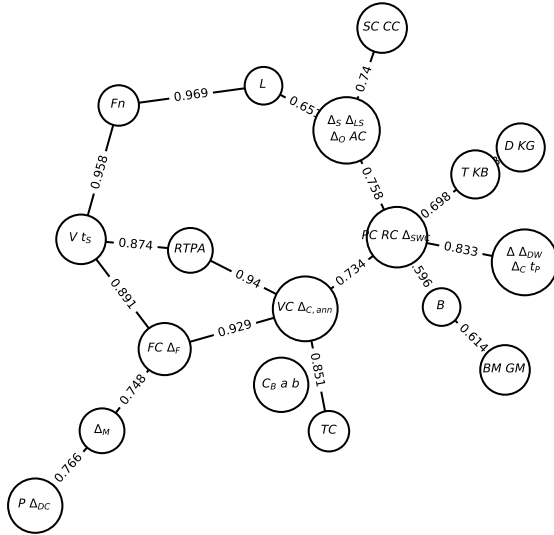


Figure 6.8: Weighted Edge Network Modeling the Feasible Design Dataset of the Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.99, Trial 1

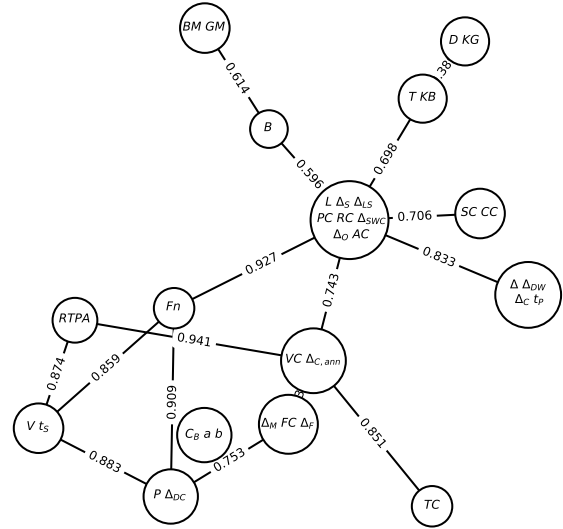


Figure 6.9: Weighted Edge Network Modeling the Feasible Design Dataset of the Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.98, Trial 1

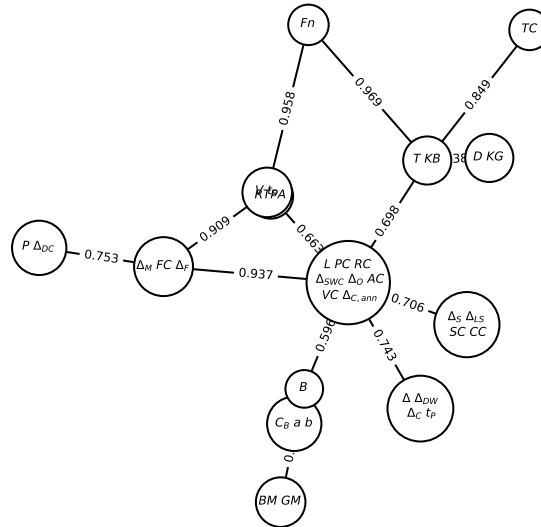


Figure 6.10: Weighted Edge Network Modeling the Feasible Design Dataset of the Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.975, Trial 1

examples of Pareto subset networks chunked with correlation thresholds of 0.99, 0.98 and 0.975, respectively. In the 0.99 chunked network for design trial 1, you can

see L and RC are chunked into a singular node, though its not likely running cost is completely dependent on ship length, even at a Pareto optimal stage. Overall,

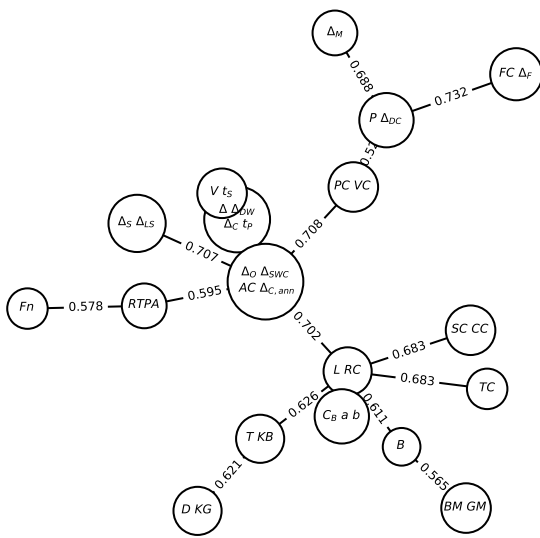


Figure 6.11: Weighted Edge Network Modeling the Pareto Design Dataset of the Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.99, Trial 1

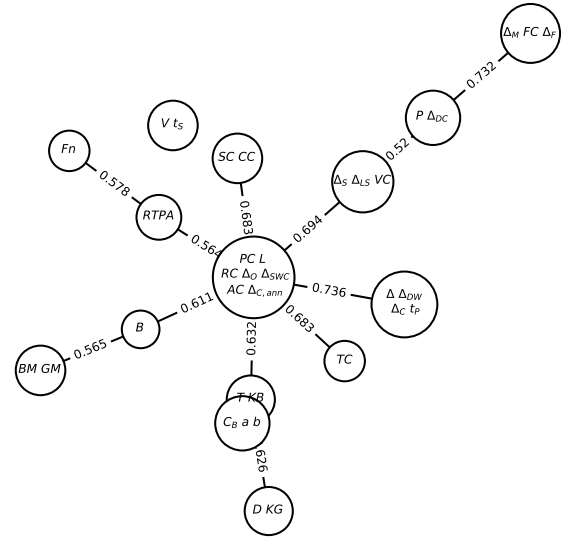


Figure 6.12: Weighted Edge Network Modeling the Pareto Design Dataset of the Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.98, Trial 1

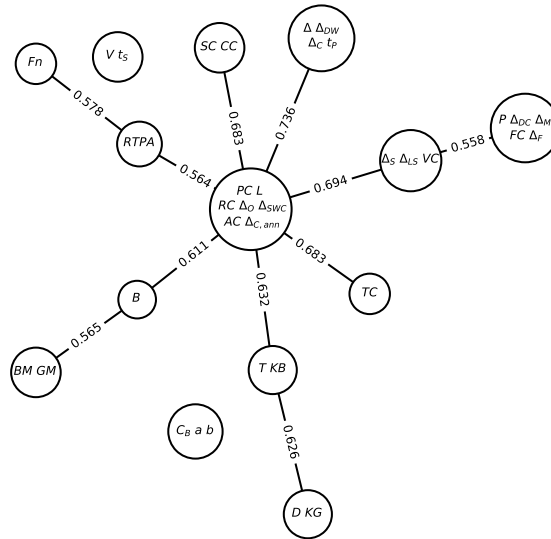


Figure 6.13: Weighted Edge Network Modeling the Pareto Design Dataset of the Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.975, Trial 1

because thresholds of 0.98 and 0.975 create too large of chunks including variables that have mutually exclusive states, a correlation threshold of 0.99 will be used for analysis of the model as it is the most accurate. While 0.99 is the most accurate of those tested, it should be understood that it is less consistent at the feasible and Pareto subsets and can still chunk some variables too liberally.

6.4 Final Accuracy

The chunked networks modeling Sen and Yang’s bulk carrier problem successfully represent the model; community detection, centralities and the chunked variables all describe the relationships of the model’s variables within the design space. The result of chunking variables into nodes itself reveals information about the design model: both where efficiencies in computation could come and where the model could be improved with the fewer assumptions or higher fidelity. For example, variables like a and b are dependent only on C_B and are used in only one intermediate function, so could be streamlined and don’t need to be output. The same could be done for capital charges which is a very simple 20% of ship charges and could be eliminated as a standalone variable. Chunked variables can also highlight functions with low fidelity or large assumptions. Figure 6.14, which depicts the full dataset network of Trial 1, has several examples of this. For example, one node includes PC , RC , Δ_{SWC} , $\Delta_{C,ann}$, Δ_O and AC . Even when chunked with the most conservative threshold of 0.995, four of these variables are still chunked within the same node. This indicates that the regressions and functions defining these nodes are likely too simple and similar:

$$\Delta_O = 1.0 \times L^{0.8} \times B^{0.6} \times D^{0.3} \times C_B^{0.1}$$

$$RC = 40,000 \times \Delta_{DW}^{0.3}$$

$$PCs = 6.3 \times \Delta_{DW}^{0.8}$$

$$\Delta_{SWC} = 2.0 \times \Delta_{DW}^{0.5}$$

$$\Delta_{C,ann} = \Delta_C \times RTPA$$

$$AC = CC + RC + (VC \times RTPA)$$

From these variable definitions its clear that while none of these variables different influences on the network, only RC and AC are directly dependent on each other. The node containing Δ_M , Δ_F and FC is an example of a node whose assumptions within the design model are adequate, but would require updating for wider applications; while fuel mass is likely to relate directly to machinery mass with traditional powering, new powering alternatives could require more complexity that break that assumption.

Conversely, the node of BM and GM highlights a weakness of this methodology. GM is a function of KB , BM , and KG but due to the interval sizing for the Bayesian networks, the states of GM and BM are not mutually exclusive because the impact of KB and KG cannot overcome the size of the bins. To accurately represent the impact of KB and KG the nodes in the Bayesian network would need more states to allow for narrower intervals; this would require more states for every node or prior knowledge an intervention to identify the variables that require higher fidelity in how they are binned.

The degree centralities represent the number of direct relationships impacting nodes. Generally the independent variables have high degree centrality in the full dataset networks but decrease as the datasets are pared down for later design stages. Interestingly, independent variable C_B has a degree centrality of zero (is fully disconnected from the rest of the network) in the feasible and Pareto networks, seen in Figure 6.15 and Figure 6.16. This signifies that its influence is so constrained by at least one of the design constraints that it no longer effects the state of any other nodes. Another independent variable V has zero degree centrality in the Pareto subset network, Figure 6.16. Like the nodes that lost all edges in the Pareto subset of

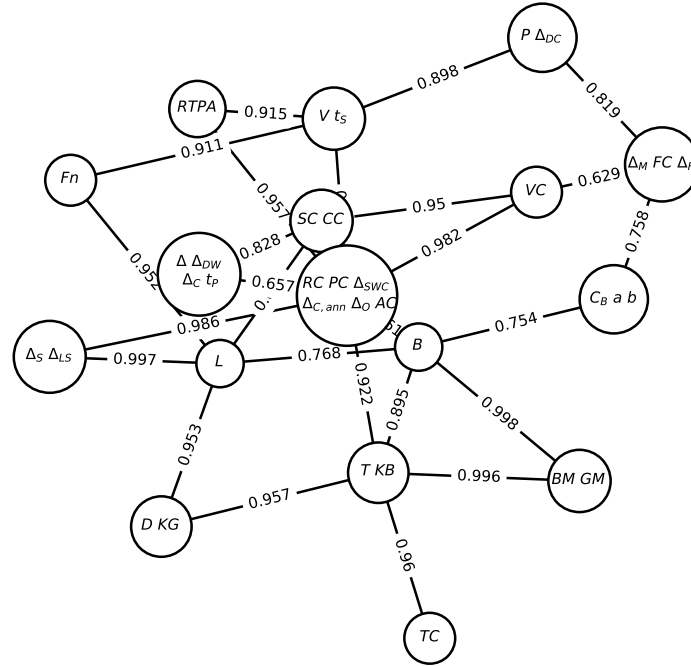


Figure 6.14: Weighted Edge Network Modeling the Full Design Dataset of the Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.99, Trial 1

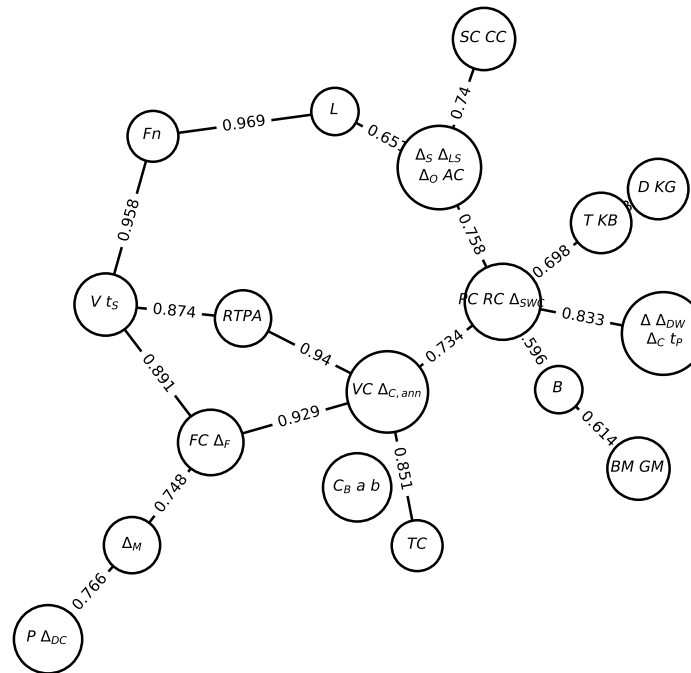


Figure 6.15: Weighted Edge Network Modeling the Feasible Design Dataset of the Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.99, Trial 1

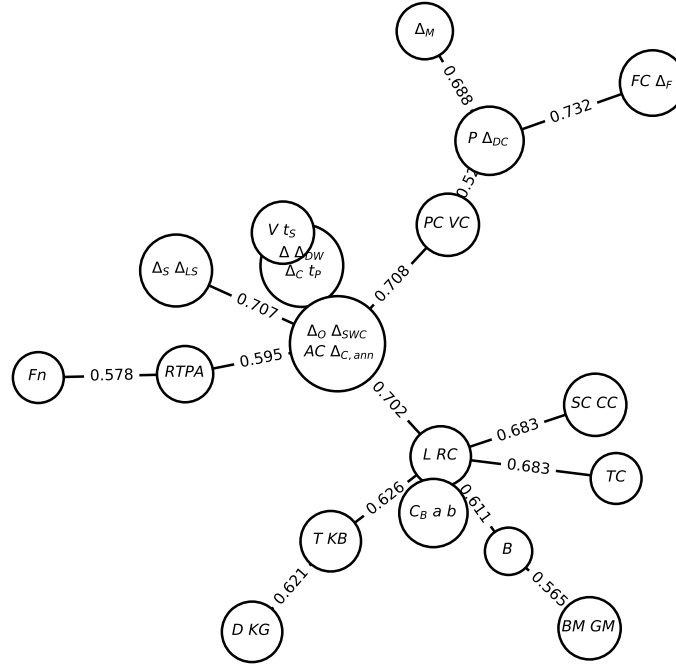


Figure 6.16: Weighted Edge Network Modeling the Pareto Design Dataset of the Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.99, Trial 1

Osyczka and Kundu problem, V is highly constrained within the optimal subset; in the Pareto subset, its marginal probability highly favors high speeds rather than its uniform distribution of the full and feasible subsets. Interestingly, Fn and $RTPA$ which are highly dependent on V do remain influential in the Pareto network, showing the optimal design alternatives are effected by relative speed much more than absolute speed. The average degree centralities ranked by subset are delineated in Table 6.21. The average adjacency matrices and pair path lengths for the chunked bulk carrier problem networks can be found in Appendix D.

It’s important to note that the network model produced by the Bayesian network and edge weight calculation is a reflection of the design model, not the underlying physics that the model is trying to capture. Therefore, while ship speed may actually be significant in a Pareto-optimal set of bulk carrier designs, the model as written finds other variables to be more influential on the design solution. This quality of the learned networks is beneficial because it provides an opportunity to reassess

Table 6.21: Average Ranked Degree Centrality of K2 Scored Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.99

Full Dataset		Feasible Subset		Pareto Subset	
<i>AC</i>	0.525	<i>PC</i>	0.3	<i>RC</i>	0.342
Δ_{SWC}	0.525	<i>RC</i>	0.3	$\Delta_{C,ann}$	0.341
<i>RC</i>	0.525	Δ_{SWC}	0.3	Δ_{SWC}	0.336
<i>PC</i>	0.525	<i>VC</i>	0.194	<i>AC</i>	0.225
Δ_O	0.525	$\Delta_{C,ann}$	0.194	Δ_O	0.225
$\Delta_{C,ann}$	0.525	<i>KB</i>	0.188	<i>L</i>	0.191
<i>L</i>	0.344	<i>T</i>	0.188	<i>KB</i>	0.126
<i>KB</i>	0.312	<i>t_S</i>	0.182	<i>T</i>	0.126
<i>T</i>	0.312	<i>V</i>	0.182	<i>CC</i>	0.125
<i>t_S</i>	0.256	Δ_S	0.147	<i>SC</i>	0.125
<i>V</i>	0.256	<i>AC</i>	0.147	<i>B</i>	0.125
<i>B</i>	0.25	Δ_O	0.147	<i>PC</i>	0.123
<i>CC</i>	0.231	Δ_{LS}	0.147	<i>RTPA</i>	0.119
<i>SC</i>	0.231	Δ_M	0.129	Δ_F	0.114
Δ_F	0.2	<i>Fn</i>	0.118	<i>FC</i>	0.114
<i>FC</i>	0.2	<i>L</i>	0.118	<i>VC</i>	0.113
Δ_M	0.2	<i>RTPA</i>	0.118	Δ_M	0.097
<i>RTPA</i>	0.162	<i>B</i>	0.118	Δ_{DC}	0.088
Δ	0.156	<i>FC</i>	0.118	<i>P</i>	0.088
<i>t_P</i>	0.156	Δ_F	0.118	<i>D</i>	0.065
Δ_C	0.156	Δ_{DC}	0.106	<i>KG</i>	0.065
Δ_{DW}	0.156	<i>P</i>	0.106	<i>BM</i>	0.06
<i>VC</i>	0.144	<i>KG</i>	0.059	<i>Fn</i>	0.06
Δ_{LS}	0.131	<i>BM</i>	0.059	<i>GM</i>	0.06
Δ_S	0.131	<i>D</i>	0.059	Δ_S	0.06
<i>b</i>	0.125	<i>SC</i>	0.059	Δ_{LS}	0.06
<i>a</i>	0.125	<i>GM</i>	0.059	<i>t_P</i>	0.06
Δ_{DC}	0.125	<i>t_P</i>	0.059	<i>TC</i>	0.06
<i>P</i>	0.125	<i>TC</i>	0.059	Δ_C	0.06
<i>KG</i>	0.125	Δ	0.059	Δ_{DW}	0.06
<i>GM</i>	0.125	Δ_{DW}	0.059	Δ	0.06
<i>Fn</i>	0.125	Δ_C	0.059	<i>t_S</i>	0
<i>D</i>	0.125	<i>CC</i>	0.059	<i>a</i>	0
<i>C_B</i>	0.125	<i>b</i>	0	<i>b</i>	0
<i>BM</i>	0.125	<i>a</i>	0	<i>C_B</i>	0
<i>TC</i>	0.062	<i>C_B</i>	0	<i>V</i>	0

our understanding either of the design model or the design itself. If the learned network structure differs from our previous understanding of the design, either on

understanding of the design, the assumptions in our mental model, are incorrect, or the assumptions in the design model are incorrect. This could be a benefit of using the framework presented over the manual development of a network model, because a network developed from a designers understanding of a design model can only reflect what is already understood; a network learned from design data can highlight what may be incorrect about either the model or a designer’s knowledge.

6.4.1 Community Detection

Identified by the greedy maximum modularity algorithm, communities of highly connected nodes reflect the separation of disciplines within the bulk carrier design model. Figure 6.17 shows the four communities identified in the full dataset network of Design Trial 1. The community colored orange in the left side of the graphic collects the overall size components; the blue in the lower right side of the graphic collects stability factors; the pink in the upper right collect required power variables; finally, the remaining green nodes collect factors driving cargo amounts and cost. While these groups could be identified manually with expert knowledge or examination of the design model, the algorithmic detection of the communities is promising for use in more complex or opaque design models. It also sorts some nodes into groups that may not be the immediate designation even for those with a good understanding of the model but indicate the actual structure of the computerized model. For example, D is grouped with size rather than stability and F_n and V are split between size and the cargo/cost communities.

Figure 6.18 shows the communities identified in the feasible networks. In this network there is a further separation of costs based on cargo and costs based on build size. Of course, C_B loses its position in the required power community as it disconnects from the core network.

The Pareto network, shown in Figure 6.19, breaks down into an additional com-

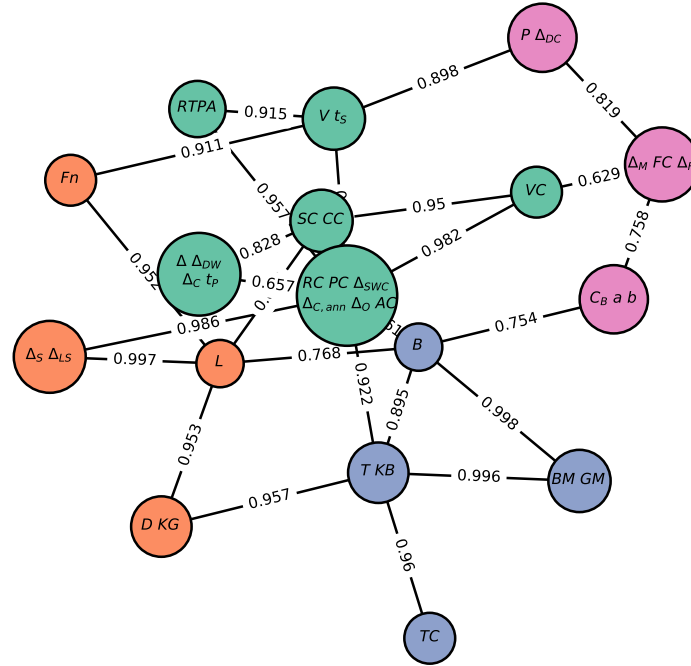


Figure 6.17: Maximum Modularity Communities of Weighted Edge Network Modeling the Full Design Dataset of the Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.99, Trial 1

munity within its connected nodes. This occurs as the branches of the network break apart into single branches that do not interconnect except through the center nodes. In Trial 1, the stability branches are separated from overall displacement and cargo size, base costs from size remain grouped separately from cargo costs and relative speed is fully separated from required power. These separations reflect the interrupted paths of influence resulting from restricting to optimal objective functions.

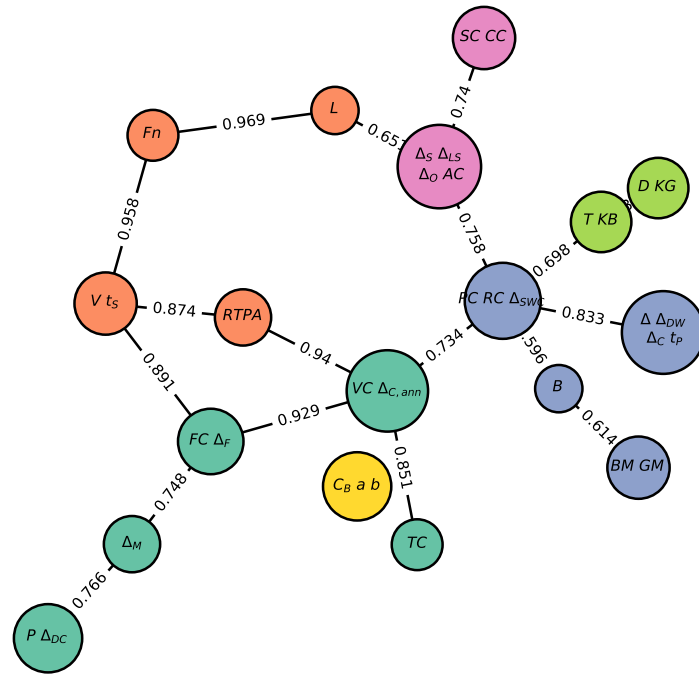


Figure 6.18: Maximum Modularity Communités of Weighted Edge Network Modeling the Feasible Design Dataset of the Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.99, Trial 1

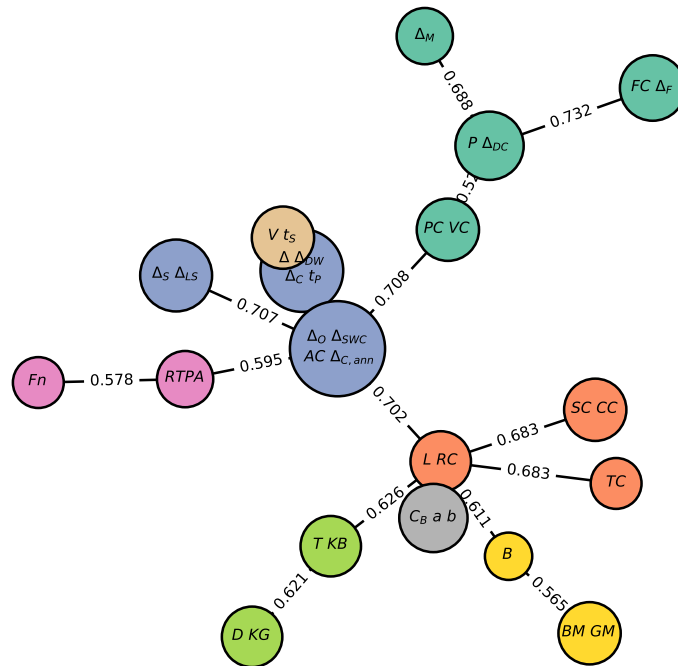


Figure 6.19: Maximum Modularity Communités of Weighted Edge Network Modeling the Pareto Design Dataset of the Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.99, Trial 1

6.4.2 Betweenness Centrality

Betweenness centrality, the measure of the fraction of shortest length between node pairs a node falls on, tends to highlight the nodes that connect their communities to the rest of the network. Node with high betweenness centrality lie on the paths of greatest influence. The independent variables generally have high betweenness centralities in the full networks, an example for which is shown in Figure 6.20. In the full networks these variables have a high impact on the intermediate variables while there is large variation in their possible states. Before constraints or optimalities are integrated, the independent variables are the foundations of the first intermediate variables calculated in each discipline. For example, in the full network, B is part of the community of stability nodes but is also connected to L and build size, C_B and required power and cargo and cost nodes. However, the betweenness centralities of the independent variables vary more in the feasible and Pareto networks as their distributions become more polarized due to constraints and optimization. Independent variable B , which has high betweenness centrality in the full network, is much less central in the feasible and Pareto networks because it has lost its direct connections to other disciplines' communities in the network.

In the feasible networks, betweenness centrality continues to highlight the nodes that connect their communities, depicted in Figure 6.18, to others in the network. In Figure 6.21, Δ_S , Δ_{LS} , Δ_O and AC connect build size and costs to overall size and costs, Δ_F and FC connect required power to speed and VC connects required power to other cost measures. The most central node containing PC , RC and Δ_{SWC} bridges multiple disciplines' communities: build size and cost, stability, cargo and required power.

The highest betweenness centralities in the Pareto networks, darkest orange in Figure 6.22, are the center points between disciplines. The node containing Δ_O , Δ_{SWC} , AC and $\Delta_{C,ann}$ connects relative speed, required power and size and cargo

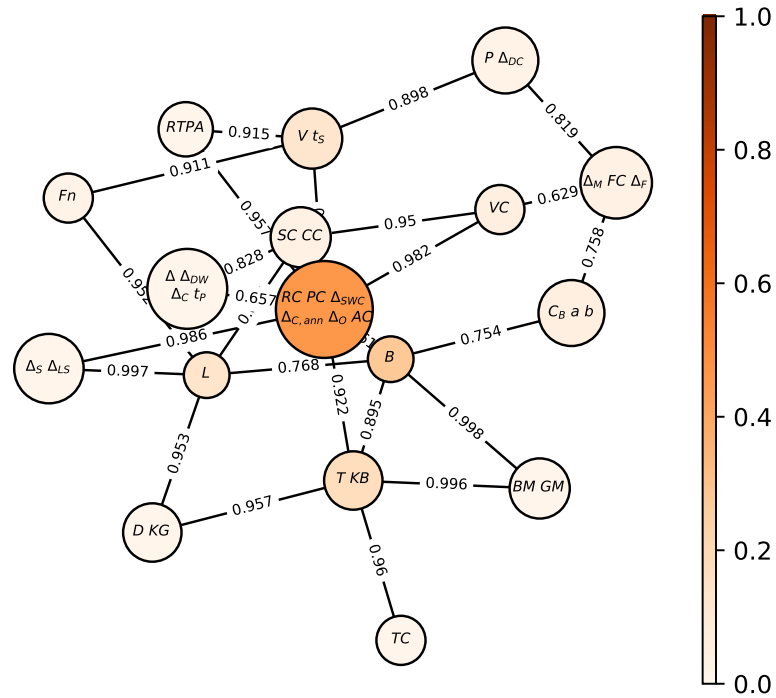


Figure 6.20: Betweenness Centrality of Weighted Edge Network Modeling the Full Design Dataset of the Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.99, Trial 1

while the node containing L and RC connects stability and build-size based costs. The betweenness centralities ranked in each subset are delineated in Table 6.22.

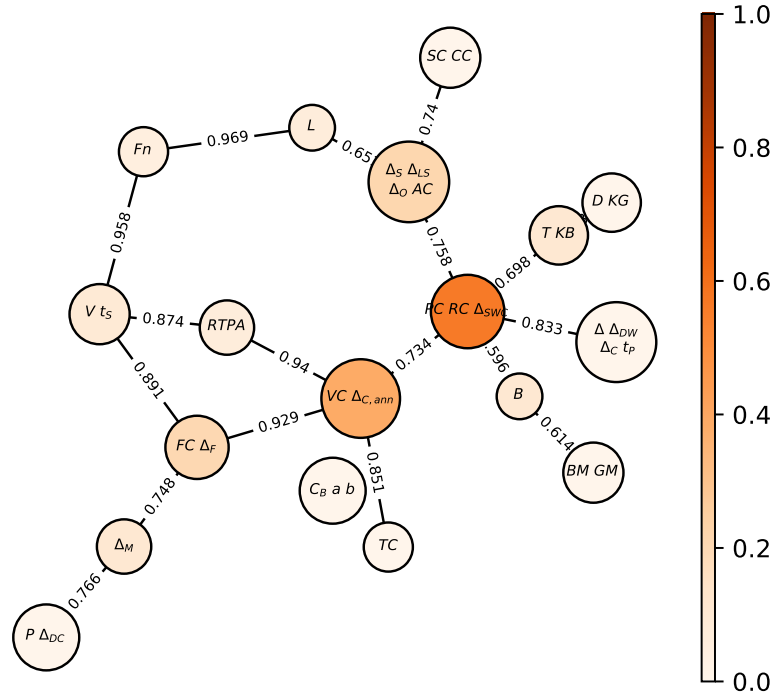


Figure 6.21: Betweenness Centrality of Weighted Edge Network Modeling the Feasible Design Dataset of the Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.99, Trial 1

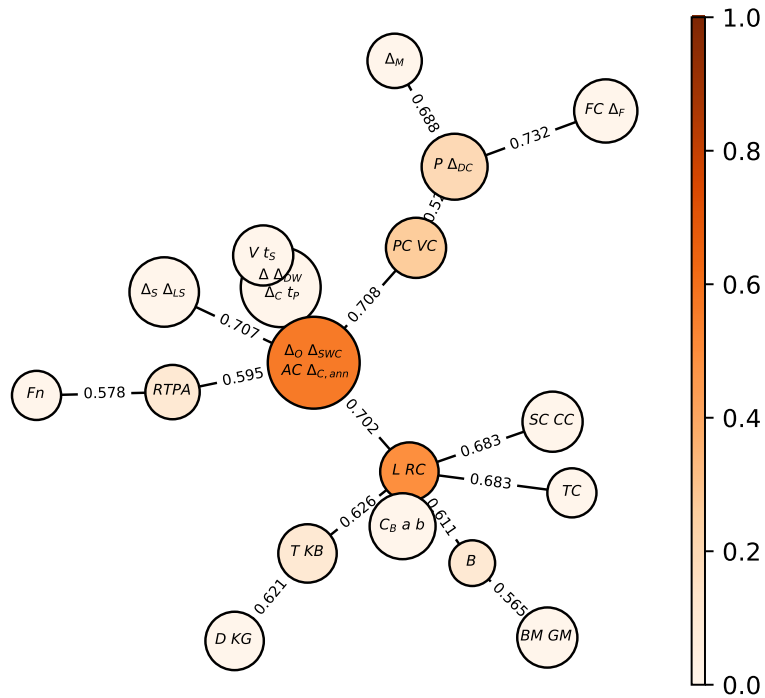


Figure 6.22: Betweenness Centrality of Weighted Edge Network Modeling the Pareto Design Dataset of the Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.99, Trial 1

Table 6.22: Average Ranked Betweenness Centrality of K2 Scored Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.99

Full Dataset		Feasible Subset		Pareto Subset	
<i>AC</i>	0.513	Δ_{SWC}	0.574	$\Delta_{C,ann}$	0.593
Δ_{SWC}	0.513	<i>RC</i>	0.574	<i>RC</i>	0.581
<i>RC</i>	0.513	<i>PC</i>	0.574	Δ_{SWC}	0.565
<i>PC</i>	0.513	<i>VC</i>	0.29	<i>AC</i>	0.336
Δ_O	0.513	$\Delta_{C,ann}$	0.29	Δ_O	0.336
$\Delta_{C,ann}$	0.513	<i>KB</i>	0.235	<i>L</i>	0.253
<i>B</i>	0.212	<i>T</i>	0.235	<i>VC</i>	0.201
<i>KB</i>	0.176	<i>AC</i>	0.19	<i>PC</i>	0.19
<i>T</i>	0.176	Δ_S	0.19	<i>CC</i>	0.132
<i>L</i>	0.15	Δ_O	0.19	<i>SC</i>	0.132
t_S	0.142	Δ_{LS}	0.19	<i>KB</i>	0.114
<i>V</i>	0.142	Δ_M	0.121	<i>T</i>	0.114
<i>VC</i>	0.07	<i>FC</i>	0.112	<i>B</i>	0.113
<i>CC</i>	0.042	Δ_F	0.112	<i>RTPA</i>	0.104
<i>SC</i>	0.042	<i>B</i>	0.11	Δ_F	0.102
<i>FC</i>	0.033	<i>V</i>	0.085	<i>FC</i>	0.102
Δ_M	0.033	t_S	0.085	Δ_M	0.065
Δ_F	0.033	<i>L</i>	0.084	Δ_{DC}	0.049
Δ_{DC}	0.022	Δ_{DC}	0.082	<i>P</i>	0.049
<i>P</i>	0.022	<i>P</i>	0.082	<i>D</i>	0.01
<i>Fn</i>	0.022	<i>RTPA</i>	0.051	<i>KG</i>	0.01
Δ_{DW}	0.017	<i>Fn</i>	0.051	<i>C_B</i>	0
Δ	0.017	<i>KG</i>	0	<i>GM</i>	0
t_P	0.017	<i>BM</i>	0	Δ_S	0
Δ_C	0.017	<i>C_B</i>	0	<i>BM</i>	0
<i>a</i>	0.014	<i>D</i>	0	<i>Fn</i>	0
<i>b</i>	0.014	<i>GM</i>	0	t_P	0
<i>C_B</i>	0.014	<i>SC</i>	0	<i>V</i>	0
Δ_{LS}	0.001	Δ_{DW}	0	t_S	0
Δ_S	0.001	<i>a</i>	0	<i>b</i>	0
<i>RTPA</i>	0	t_P	0	<i>TC</i>	0
<i>KG</i>	0	Δ	0	Δ_C	0
<i>GM</i>	0	<i>b</i>	0	Δ_{DW}	0
<i>D</i>	0	<i>TC</i>	0	Δ	0
<i>BM</i>	0	Δ_C	0	Δ_{LS}	0
<i>TC</i>	0	<i>CC</i>	0	<i>a</i>	0

6.4.3 Closeness Centrality

Closeness centrality measures how strongly a node is connected to the rest of the network as a whole. Variables with high closeness centrality are most effected by the states of other variables, and their states have the greatest effect on other variables. The distribution of closeness centralities for all nodes is much more uniform across a network than it is for betweenness centrality, especially for the full dataset networks. In the full networks, like that in Figure 6.23, variables that are inputs to many functions of the design model have the highest centralities. These are generally cost variables, which remain highly central at later design stages. Independent variables like T , L , V and B also have high closeness centrality in the full network due to their broad impact across the design.

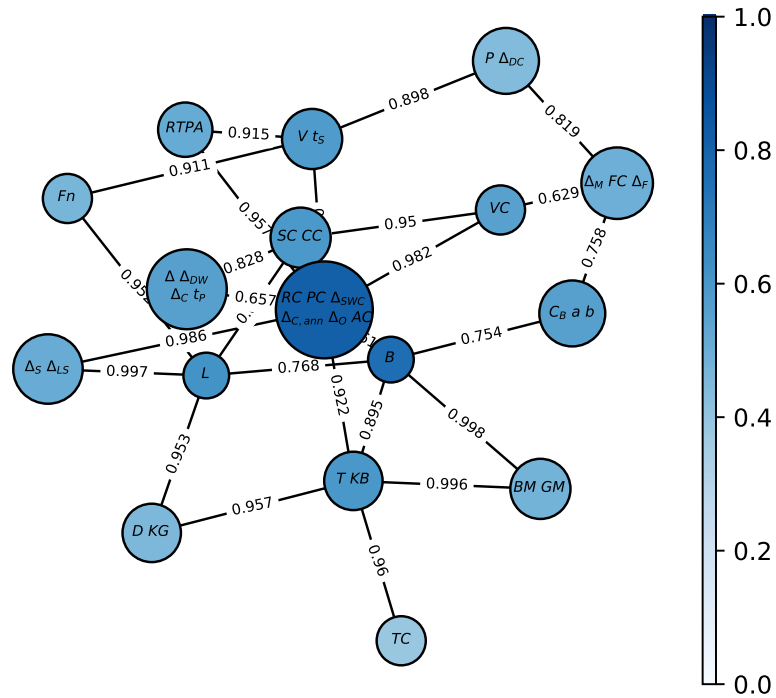


Figure 6.23: Closeness Centrality of Weighted Edge Network Modeling the Full Design Dataset of the Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.99, Trial 1

In the feasible subset networks the most impactful variables, like the independent variables are slightly less central, but variables that are impacted by the most variables

maintain high closeness centrality. Costs and annual cargo are the most central, most influenced nodes, as seen in Figure 6.24.

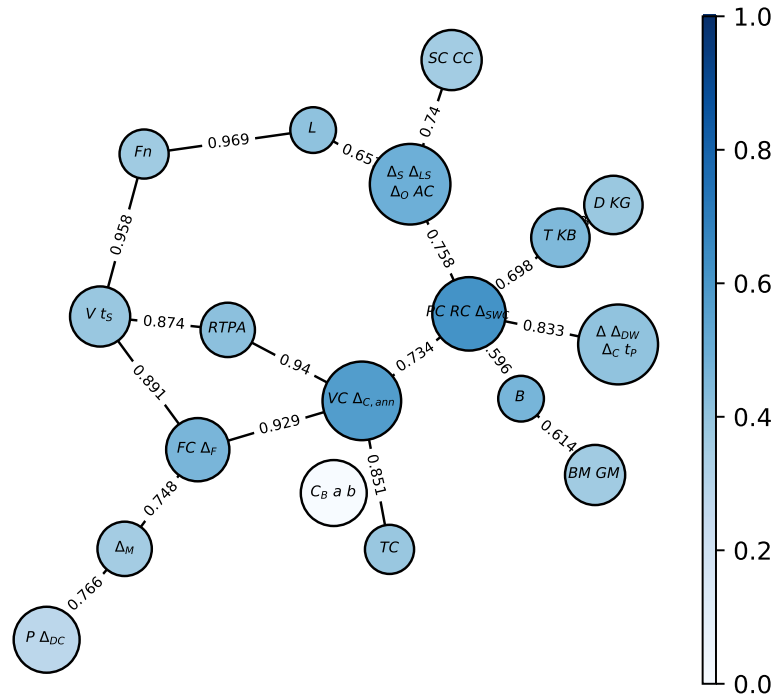


Figure 6.24: Closeness Centrality of Weighted Edge Network Modeling the Feasible Design Dataset of the Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.99, Trial 1

The Pareto subset networks, an example of which is in Figure 6.25, follow the same trend: $\Delta_{C,ann}$ is the most varied based on the influences of other nodes and is most central. Meanwhile, the other two objective functions, TC and Δ_{LS} have relatively low closeness centrality and are much more stable in the optimal stage of design. This indicates that $\Delta_{C,ann}$ likely drives the differences in design alternatives along the Pareto-front. Average closeness centralities ranked by subset are delineated in Table 6.23.

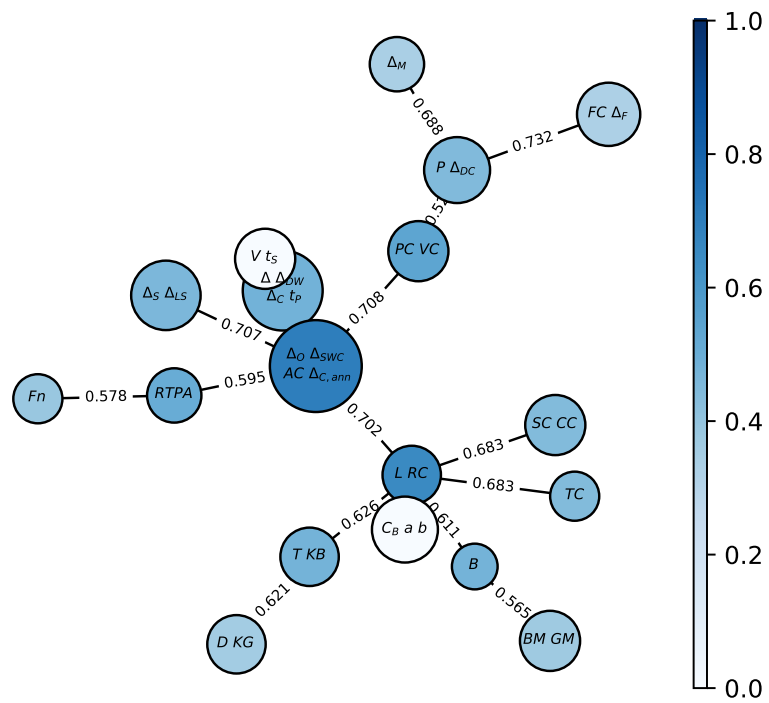


Figure 6.25: Closeness Centrality of Weighted Edge Network Modeling the Pareto Design Dataset of the Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.99, Trial 1

Table 6.23: Average Ranked Closeness Centrality of K2 Scored Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.99

Full Dataset		Feasible Subset		Pareto Subset	
<i>AC</i>	0.566	<i>RC</i>	0.351	$\Delta_{C,ann}$	0.313
Δ_{SWC}	0.566	Δ_{SWC}	0.351	<i>RC</i>	0.31
<i>RC</i>	0.566	<i>PC</i>	0.351	Δ_{SWC}	0.308
<i>PC</i>	0.566	<i>VC</i>	0.312	<i>AC</i>	0.267
Δ_O	0.566	$\Delta_{C,ann}$	0.312	Δ_O	0.267
$\Delta_{C,ann}$	0.566	<i>T</i>	0.274	<i>L</i>	0.249
<i>KB</i>	0.454	<i>KB</i>	0.274	<i>PC</i>	0.241
<i>T</i>	0.454	Δ_{LS}	0.269	<i>VC</i>	0.236
<i>L</i>	0.445	<i>AC</i>	0.269	<i>CC</i>	0.228
<i>CC</i>	0.44	Δ_S	0.269	<i>SC</i>	0.228
<i>SC</i>	0.44	Δ_O	0.269	<i>B</i>	0.214
Δ_S	0.421	<i>t_S</i>	0.268	<i>RTPA</i>	0.208
Δ_{LS}	0.421	<i>V</i>	0.268	<i>KB</i>	0.207
<i>t_S</i>	0.42	<i>RTPA</i>	0.267	<i>T</i>	0.207
<i>V</i>	0.42	Δ	0.246	<i>TC</i>	0.194
<i>B</i>	0.408	Δ_{DW}	0.246	Δ_{LS}	0.193
<i>RTPA</i>	0.405	Δ_C	0.246	Δ_S	0.193
Δ_C	0.39	<i>t_P</i>	0.246	<i>t_P</i>	0.188
<i>t_P</i>	0.39	<i>L</i>	0.246	Δ_C	0.188
Δ_{DW}	0.39	<i>Fn</i>	0.241	Δ_{DW}	0.188
Δ	0.39	<i>FC</i>	0.241	Δ	0.188
<i>Fn</i>	0.382	Δ_F	0.241	<i>FC</i>	0.171
<i>VC</i>	0.38	<i>B</i>	0.233	Δ_F	0.171
<i>KG</i>	0.364	Δ_M	0.229	<i>KG</i>	0.161
<i>D</i>	0.364	<i>TC</i>	0.225	<i>D</i>	0.161
<i>a</i>	0.355	<i>P</i>	0.224	Δ_M	0.161
<i>b</i>	0.355	Δ_{DC}	0.224	<i>GM</i>	0.153
<i>C_B</i>	0.355	<i>SC</i>	0.198	<i>BM</i>	0.153
<i>BM</i>	0.346	<i>CC</i>	0.198	<i>Fn</i>	0.149
<i>GM</i>	0.346	<i>GM</i>	0.169	Δ_{DC}	0.148
<i>TC</i>	0.315	<i>BM</i>	0.169	<i>P</i>	0.148
Δ_F	0.314	<i>KG</i>	0.162	<i>V</i>	0
<i>FC</i>	0.314	<i>D</i>	0.162	<i>b</i>	0
Δ_M	0.314	<i>a</i>	0	<i>t_S</i>	0
Δ_{DC}	0.307	<i>b</i>	0	<i>C_B</i>	0
<i>P</i>	0.307	<i>C_B</i>	0	<i>a</i>	0

6.4.4 Eigenvector Centrality

Eigenvector centrality is somewhat similar to closeness centrality but favors nodes that are highly connected to other highly connected nodes. Outside of the most central and connected node, independent variables L , T and B have very high closeness centralities in the full networks; an example of a full network with Eigenvector centrality is shown in Figure 6.26. These variables have the widest impact on overall results as inputs. Independent variable B is an example of a node with a higher ranked Eigenvector centrality than closeness centrality; while it doesn't have the highest edge weights, its edges connect to nodes outside its community in addition to just like nodes and the most central node.

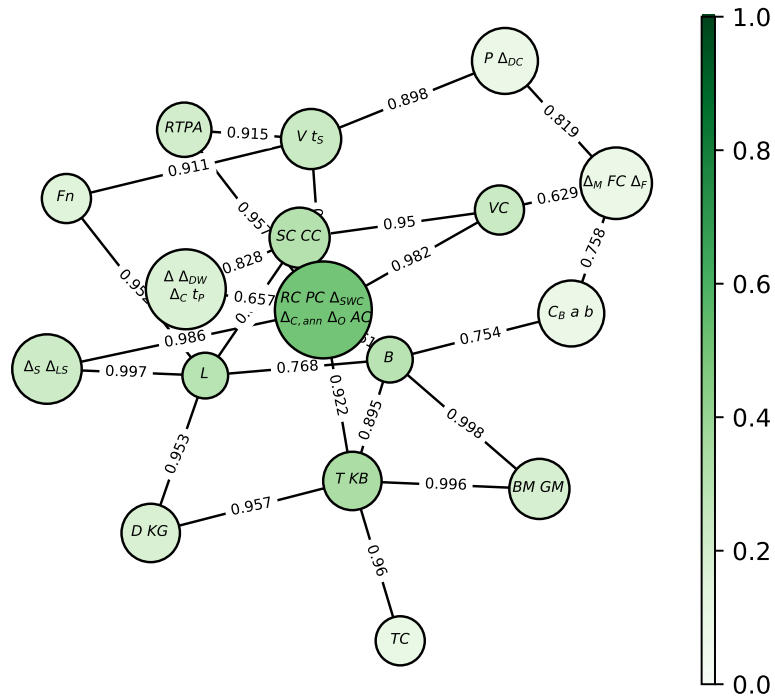


Figure 6.26: Eigenvector Centrality of Weighted Edge Network Modeling the Full Design Dataset of the Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.99, Trial 1

In the feasible networks, like shown in Figure 6.27, FC and V both have higher ranked Eigenvector centrality than closeness centrality as well. These nodes connect both inwards to the most central node and to outer nodes that are well connected.

These nodes are the factors driving the distributions of the most central variables.

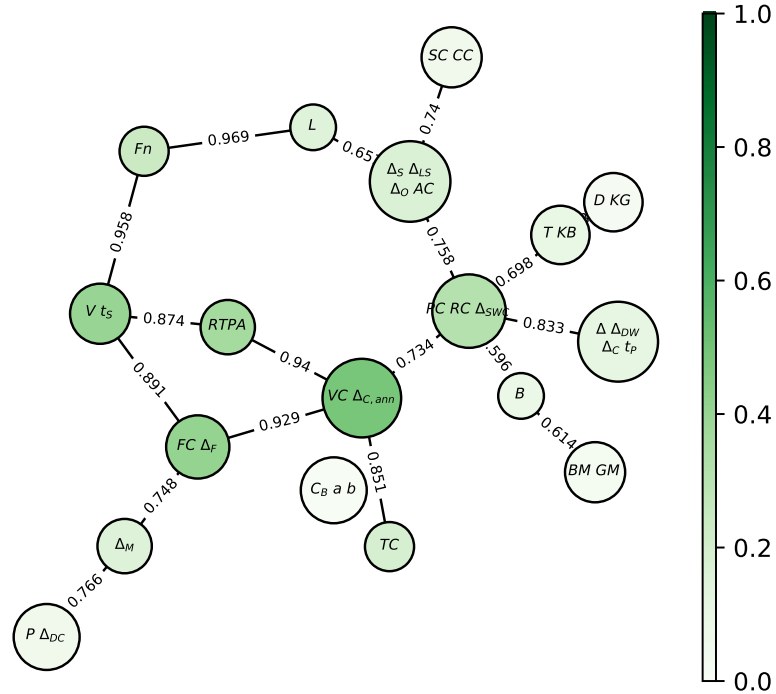


Figure 6.27: Eigenvector Centrality of Weighted Edge Network Modeling the Feasible Design Dataset of the Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.99, Trial 1

In the Pareto networks, like Figure 6.28, Eigenvector centrality identifies the variables that drive the variation in Pareto-optimal design alternatives. Variables with high closeness centrality are the varying attributes of the alternatives, and variables with high Eigenvector centrality drive the variation. Average Eigenvector centralities ranked by subset are delineated in Table 6.24.

Of the centralities evaluated, most have the same most central node but distinct differences in which nodes beyond the absolute center have high centrality values. Betweenness identifies the nodes with the most impact beyond their discipline is communities. Closeness highlights the nodes most impacted by the other nodes of the network; these contain the variables with variation among design alternatives of the feasible and Pareto subsets. As the centrality measure that considers the influence of a node's neighbors, Eigenvector centrality is high for nodes that drive the variation

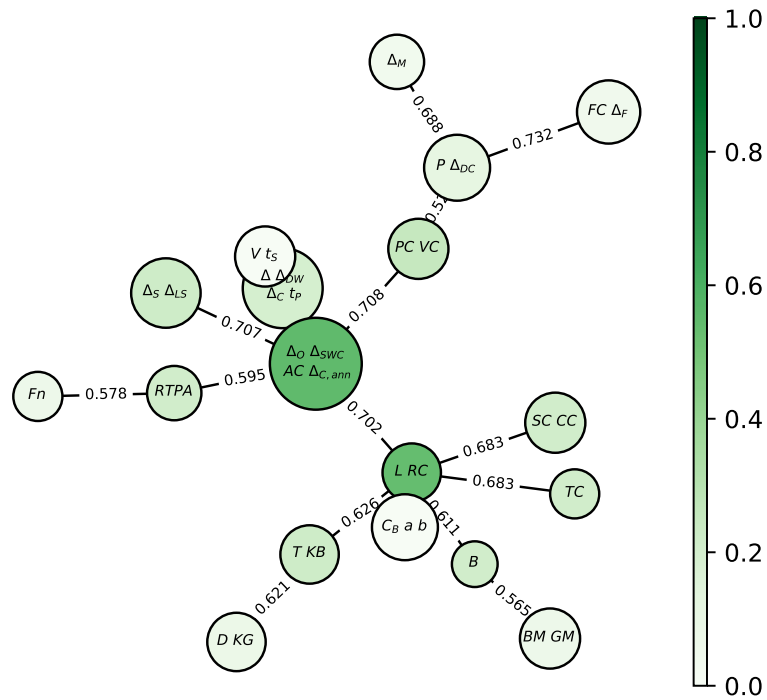


Figure 6.28: Eigenvector Centrality of Weighted Edge Network Modeling the Pareto Design Dataset of the Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.99, Trial 1

in variables with high closeness centrality.

Table 6.24: Average Ranked Eigenvector Centrality of K2 Scored Bulk Carrier Design Problem Chunked with a Correlation Threshold of 0.99

Full Dataset		Feasible Subset		Pareto Subset	
<i>AC</i>	0.488	<i>VC</i>	0.431	$\Delta_{C,ann}$	0.581
Δ_{SWC}	0.488	$\Delta_{C,ann}$	0.431	<i>RC</i>	0.571
<i>RC</i>	0.488	<i>t_S</i>	0.424	Δ_{SWC}	0.566
<i>PC</i>	0.488	<i>V</i>	0.424	Δ_O	0.419
Δ_O	0.488	<i>RTPA</i>	0.341	<i>AC</i>	0.419
$\Delta_{C,ann}$	0.488	Δ_{SWC}	0.329	<i>L</i>	0.36
<i>L</i>	0.323	<i>PC</i>	0.329	<i>PC</i>	0.318
<i>CC</i>	0.319	<i>RC</i>	0.329	<i>CC</i>	0.306
<i>SC</i>	0.319	Δ_F	0.245	<i>SC</i>	0.306
<i>KB</i>	0.313	<i>FC</i>	0.245	<i>VC</i>	0.264
<i>T</i>	0.313	<i>F_n</i>	0.237	<i>B</i>	0.227
<i>B</i>	0.27	<i>P</i>	0.21	<i>KB</i>	0.208
<i>t_P</i>	0.236	Δ_{DC}	0.21	<i>T</i>	0.208
Δ	0.236	Δ_M	0.187	<i>RTPA</i>	0.206
Δ_{DW}	0.236	Δ_{LS}	0.166	Δ_S	0.181
Δ_C	0.236	Δ_O	0.166	Δ_{LS}	0.181
Δ_S	0.233	<i>AC</i>	0.166	<i>TC</i>	0.177
Δ_{LS}	0.233	Δ_S	0.166	Δ	0.171
<i>RTPA</i>	0.231	<i>L</i>	0.153	Δ_{DW}	0.171
<i>V</i>	0.223	<i>KB</i>	0.15	Δ_C	0.171
<i>t_S</i>	0.223	<i>T</i>	0.15	<i>t_P</i>	0.171
<i>D</i>	0.173	Δ	0.119	<i>FC</i>	0.102
<i>KG</i>	0.173	Δ_C	0.119	Δ_F	0.102
<i>VC</i>	0.161	<i>t_P</i>	0.119	<i>D</i>	0.09
<i>GM</i>	0.151	Δ_{DW}	0.119	<i>KG</i>	0.09
<i>BM</i>	0.151	<i>TC</i>	0.118	Δ_M	0.085
<i>F_n</i>	0.145	<i>B</i>	0.094	<i>GM</i>	0.072
<i>a</i>	0.115	<i>SC</i>	0.056	<i>BM</i>	0.072
<i>b</i>	0.115	<i>CC</i>	0.056	<i>F_n</i>	0.065
<i>C_B</i>	0.115	<i>KG</i>	0.027	<i>P</i>	0.063
Δ_F	0.092	<i>D</i>	0.027	Δ_{DC}	0.063
Δ_M	0.092	<i>GM</i>	0.026	<i>a</i>	0.0
<i>FC</i>	0.092	<i>BM</i>	0.026	<i>t_S</i>	0.0
<i>TC</i>	0.086	<i>a</i>	0.0	<i>b</i>	0.0
<i>P</i>	0.067	<i>b</i>	0.0	<i>C_B</i>	0.0
Δ_{DC}	0.067	<i>C_B</i>	0.0	<i>V</i>	0.0

6.5 Lessons Learned

This analysis of Sen and Yang’s bulk carrier design K2 model robustly demonstrated the effectiveness of the framework proposed by this work. With the inclusion of the

chunking algorithm, Bayesian networks representing the design model were consistently and accurately developed. This case study did show that the consistency and accuracy of the chunking algorithm is dependent on the threshold of correlation used to identify the groups of chunked nodes. The best threshold for this work was a 0.99 correlation of K2 scores, but the differing consistencies between the full, feasible and Pareto subsets indicate that this threshold may not be the absolute best for use in design databases of differing size and resolution. Community detection using maximum modularity identified the disciplines of design within the model with access only to the structures learned from databases of design alternatives. Degree centrality of the full dataset networks identifies the direct relationships between variables as defined by the design model. In the networks modeling the later feasible and Pareto stages of design, degree centrality highlights the variables whose influence is so restricted by the constraints or optimality of objectives that they disconnect from the network all together. Betweenness Centrality also highlights the structure of the design problem, identifying nodes which connect their discipline communities to other communities in the network. Finally, closeness centrality and Eigenvector centrality work in tandem to identify the variables that drive variation among the feasible and Pareto-optimal design alternatives. Variables with the highest closeness centrality have distributions that are most effected by and have the most effect on the status of the rest of the model's variables. These are the variables that define the differences between design alternatives. Eigenvector centrality identifies variables which are closely connected to the variables with high closeness centrality. These variables drive the variation in the states of the most central nodes. These communities and centrality measures show the capacity for this methodology to build accurate and consistent networks from design data and provide insight into the model's design space.

CHAPTER VII

Conclusions and Future Work

7.1 Meeting the Research Objectives

Through the analysis of two case studies, this work shows the strength of using Bayesian networks and network science to understand complex design models. For both the six-variable, too-objective Osyczka and Kundu problem and the larger 33-variable, three-objective function bulk carrier design problem Bayesian networks were able to consistently learn representative network structures from databases of design alternatives. While the simple Osyczka and Kundu problem could be learned directly with no preprocessing the bulk carrier problem had redundant variables that needed to be consolidated before networks could be learned consistently. A key contribution of this work is the algorithm for chunking redundant nodes without the intervention of a design with expert knowledge of the problem. Using edge scoring and a measurement of network structure similarity, variables that would compete for all of the same edges in a greedy search-and-score algorithm are combined into a single node. The chunked nodes increase consistency and accuracy of network learning as well as highlight assumptions within the model. By calling attention to variables that have the same influences in the network, designers are made more aware of possible redundancies within the model that may or may not be congruent with the intended design problem. In our example, speed, V , and days at sea, t_s , are reasonable redundant

pairs, but chunks including both annual cargo, $\Delta_{C,ann}$, and outfit mass, Δ_O , should likely have more complexity and have indistinguishable influences in the model. The chunking algorithm allows Bayesian networks to learn network structures from the output from large complex design models without requiring intervention from a designer's perceived understanding of the design model.

Once learned from design datasets, the Bayesian networks were transformed into weighted edge networks preserving the information held in its conditional probability tables. Both a derivative of the log gamma K2 score and a modified match distance score were used to try to capture the strength of relationships held in each conditional probability table. While match distance produced weak representations of the tables, K2 scores presented robust characterizations of variable relationships. In the Osyczka and Kundu problem the edge weights scaled well with observations about the variable relationships and the design space made through manual analysis of the design problem, constraints and optimizations' effect on the design space. K2 scores of the bulk carrier networks reflected the relationships as detailed in the model for the full design data; for the feasible and Pareto networks, the K2 scores and network structure revealed the effects of the constraints and objectives that are more difficult to understand from the original presentation of the model.

Finally analysis of the weighted edge networks with traditional network theory tools showed the meaning of network metrics in terms of design and the design space. Degree centrality and the structure of the network reflected the complexities of influence within the models. The highly connected networks of the full design space are made mere spouse through the incorporation of constraints and optimization of the objective functions. The loss of edges at progressive stages of design minor the loss of influence those nodes have on the design space, whether heavily constrained or overpowered by mere controlling variables. Community detection using a greedy maximum modularity algorithm divides the network into communities reflecting the

two objective functions in the Osyczka and Kundu problem and the disciplines of ship design in the bulk carrier problem. Betweenness centrality measures the number of shortest paths a node is on in a network; in the design networks it highlights the nodes that connect to nodes outside their community, tying together the different disciplines within the design model. Closeness centrality, measuring strength of connection to all other nodes in a network, identified nodes which were most influenced or most influence able in the design model. These nodes contain the variables with variation between design alternatives in the design space. Especially in the Pareto-set these nodes are those that haven't been so constricted and change with the design along the Pareto front. Lastly, Eigenvector centrality accounts for the closeness of connections of a node's neighbors. This metric highlights the variables of the design problem which drive the changes to and variations of the variables with highest closeness centrality. Together the metrics and the structure of the networks reveal information about the design space and an understanding of the driving factors of a design at each design stage.

While this work tested the framework against two explicit and relatively simple design problems, the value of this work in application is for the analysis of complex, black box design models. The resulting networks structures can help a designer or stakeholder form a better mental model of an opaque, complex design model; centrality measurements highlight the variables which define the design spaces and the changing influence of variables as the design progresses through increasing constrained design stages. Better mental models support understanding of the model, from its complexities to its assumptions. This understanding aids informed decision making and the ability to build models that accurately reflect the problem to be solved.

7.2 Contributions

Each supporting the achievement of a research objective, three major novel contributions arise from this work:

1. The development of an algorithm to identify and combine redundant variables from large sets of data used to learn Bayesian networks
2. The development of use of scoring methods for use as edge weight scores and identifying a modified and normalized log gamma K2 score as an effective representation of the strength of relationships defined by conditional probability tables
3. Identifying the significance of network metrics as applied to weighted edge networks modeling the design space of complex design models

7.3 Future Work

This work presents several courses of future work, pertaining to both development of the framework and suitable future applications.

7.3.1 Observations as Constraints

One of the most interesting avenues for testing the power of this work would be to test the ability of the Bayesian networks to predict the networks or future design stages. One of the strongest features of Bayesian networks is their ability to update with observations; these observations remove the uncertainty from one or more nodes based on a known state of the node. An interesting question arises of whether a design constraint applied to a node's states in the same way and re-converted to a weighted edge network with the the observation in place, stopping some flow of information,

would produce networks similar to the feasible networks built from databases of the constraint applied before learning.

7.3.2 Integration of Chunking into Bayesian Network Learning

One area for increased efficiency in this process would be the integration of chunking into the Bayesian network learning algorithm. As the K2 edge weight score is a derivation of the K2 score used to score the goodness of networks, there could be a lot of efficiency gained by combining their calculation into a single step. In such a learning algorithm, the matrices for evaluating similarity could be populated as new edges are tested; then when two candidate edges provide very similar information, the nodes could be evaluated for chunking, rather than testing every pair of possible nodes, no matter how unrelated they are.

7.3.3 Exploration of Different Edge Weight Scoring

Finally, evaluating the use of additional edge weight scoring mechanisms could further increase the accuracy of the learned networks and network metrics. While K2 and match distance were explored in this work, many other established and novel scoring methods are available for learning Bayesian networks or measuring the difference between distributions. Scores developed from log-likelihood, like used in the BIC and AIC scoring functions, or Kullback-Leibler divergence could make interesting candidates for edge weights and may highlight other complexities of the conditional probability tables not captured by the K2 score.

APPENDIX

APPENDIX A

Bulk Carrier Design Model by Sen and Yang

Table A.1: Independent Design Variables

Variable	Units
length, L	m
draft, T	m
depth, D	m
block coefficient, C_B	-
beam, B	m
speed, V	knots

Table A.2: Model Parameters

Parameter	Value
η_1	4977.06
η_2	-8105.61
η_3	4456.51
ζ_1	-10847.2
ζ_2	12817.0
ζ_3	-6960.32

$$Fn = \frac{V}{\sqrt{gL}} \quad (\text{A.1})$$

$$\text{displacement, } \Delta = 1.025 \times L \times B \times T \times C_B \quad (\text{A.2})$$

Table A.3: Model Constants

Constant	Value	Units
round trip miles	5000	nm
fuel price	100	£/ton
cargo handling rate	8000	tons/day
g	9.8065	m/s ²

$$\text{steel mass, } \Delta_S = 0.034 \times L^{1.7} \times B^{0.7} \times D^{0.4} \times C_B^{0.5} \quad (\text{A.3})$$

$$\text{outfit mass, } \Delta_O = 1.0 \times L^{0.8} \times B^{0.6} \times D^{0.3} \times C_B^{0.1} \quad (\text{A.4})$$

$$a(C_B) = \eta_1 C_B^2 + \eta_2 C_B + \eta_3 \quad (\text{A.5})$$

$$b(C_B) = \zeta_1 C_B^2 + \zeta_2 C_B + \zeta_3 \quad (\text{A.6})$$

$$\text{sea days, } t_s = \frac{\text{round trip miles}}{24V} \quad (\text{A.7})$$

$$BM = \frac{(0.85C_B - 0.002)B^2}{T \times C_B} \quad (\text{A.8})$$

$$KG = 1.0 + 0.52D \quad (\text{A.9})$$

$$KB = 0.53T \quad (\text{A.10})$$

$$P = \frac{\Delta^{2/3} \times V^3}{b(C_B) \times Fn + a(C_B)} \quad (\text{A.11})$$

$$GM = KB + BM - KG \quad (\text{A.12})$$

$$\text{ship cost, } SC = 1.3(2000 \times \Delta_S^{0.85} + 3500 \times \Delta_O + 2400 \times P^{0.8}) \quad (\text{A.13})$$

$$\text{machinery mass, } \Delta_M = 0.17P^{0.9} \quad (\text{A.14})$$

$$\text{daily consumption } \Delta_{DC} = P \times 0.19 \times \frac{24}{1000} + 0.2 \quad (\text{A.15})$$

$$\text{fuel cost} = 1.05 \times \Delta_{DC} \times t_s \times \text{fuel price} \quad (\text{A.16})$$

$$\text{light ship mass, } \Delta_{LS} = \Delta_S + \Delta_M + \Delta_O \quad (\text{A.17})$$

$$\text{capital charges, } CC = 0.2 \times SC \quad (\text{A.18})$$

$$\text{fuel carried, } \Delta_F = \Delta_{DC} \times (t_s + 5) \quad (\text{A.19})$$

$$\text{deadweight, } \Delta_{DW} = \Delta - \Delta_{LS} \quad (\text{A.20})$$

$$\text{running costs, } RC = 40,000 \times \Delta_{DW}^{0.3} \quad (\text{A.21})$$

$$\text{port costs, } PC = 6.3 \times \Delta_{DW}^{0.8} \quad (\text{A.22})$$

$$\text{stores, water and crew, } \Delta_{SWC} = 2.0 \times \Delta_{DW}^{0.5} \quad (\text{A.23})$$

$$\text{voyage cost, } VC = FC + PC \quad (\text{A.24})$$

$$\text{cargo weight, } \Delta_C = \Delta_{DW} - \Delta_F - \Delta_{SWC} \quad (\text{A.25})$$

$$\text{port days, } t_p = 2 \left(\frac{\Delta_C}{\text{cargo handling rate}} + 0.5 \right) \quad (\text{A.26})$$

$$RTPA = \frac{350}{t_s + t_p} \quad (\text{A.27})$$

$$\text{annual cargo, } \Delta_{C,ann} = \Delta_C \times RTPA \quad (\text{A.28})$$

$$\text{annual cost, } AC = CC + RC + (\text{voyage costs} \times RTPA) \quad (\text{A.29})$$

$$\text{transportation cost, } TC = \frac{AC}{\Delta_{C,ann}} \quad (\text{A.30})$$

APPENDIX B

Oyszczka and Kundu Adjacency Matrices and Shortest Path Lengths

Table B.1: Average K2 Score Adjacency Matrix of Full Dataset Network of Oyszczka and Kundu Problem

	x_1	x_2	x_3	x_4	x_5	x_6	f_1	f_2
x_1	0.0	0.0	0.0	0.0	0.0	0.0	0.987	0.973
x_2	0.0	0.0	0.0	0.086	0.0	0.0	0.0	0.973
x_3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.005
x_4	0.0	0.086	0.0	0.0	0.0	0.08	0.678	0.86
x_5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.005
x_6	0.0	0.0	0.0	0.08	0.0	0.0	0.0	0.973
f_1	0.987	0.0	0.0	0.678	0.0	0.0	0.0	0.922
f_2	0.973	0.973	0.005	0.86	0.005	0.973	0.922	0.0

Table B.2: Average K2 Score Adjacency Matrix of Feasible Dataset Network of Osyczka and Kundu Problem

	x_1	x_2	x_3	x_4	x_5	x_6	f_1	f_2
x_1	0.0	0.156	0.0	0.0	0.0	0.0	0.615	0.0
x_2	0.156	0.0	0.0	0.0	0.0	0.0	0.0	0.0
x_3	0.0	0.0	0.0	0.102	0.0	0.0	0.0	0.039
x_4	0.0	0.0	0.102	0.0	0.0	0.0	0.0	0.0
x_5	0.0	0.0	0.0	0.0	0.0	0.689	0.0	0.85
x_6	0.0	0.0	0.0	0.0	0.689	0.0	0.0	0.657
f_1	0.615	0.0	0.0	0.0	0.0	0.0	0.0	0.0
f_2	0.0	0.0	0.039	0.0	0.85	0.657	0.0	0.0

Table B.3: Average K2 Score Adjacency Matrix of Pareto Dataset Network of Osyczka and Kundu Problem

	x_1	x_2	x_3	x_4	x_5	x_6	f_1	f_2
x_1	0.0	0.191	0.0	0.0	0.0	0.0	0.338	0.0
x_2	0.191	0.0	0.0	0.0	0.0	0.0	0.0	0.0
x_3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
x_4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
x_5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
x_6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.066
f_1	0.338	0.0	0.0	0.0	0.0	0.0	0.0	0.39
f_2	0.0	0.0	0.0	0.0	0.0	0.066	0.39	0.0

Table B.4: Average MD Score Adjacency Matrix of Full Dataset Network of Osyczka and Kundu Problem

	x_1	x_2	x_3	x_4	x_5	x_6	f_1	f_2
x_1	0.0	0.0	0.0	0.0	0.0	0.0	0.136	0.286
x_2	0.0	0.0	0.0	0.02	0.0	0.0	0.0	0.286
x_3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.194
x_4	0.0	0.02	0.0	0.0	0.0	0.018	0.13	0.151
x_5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.194
x_6	0.0	0.0	0.0	0.018	0.0	0.0	0.0	0.286
f_1	0.136	0.0	0.0	0.13	0.0	0.0	0.0	0.161
f_2	0.286	0.286	0.194	0.151	0.194	0.286	0.161	0.0

Table B.5: Average MD Score Adjacency Matrix of Feasible Dataset Network of Osyczka and Kundu Problem

	x_1	x_2	x_3	x_4	x_5	x_6	f_1	f_2
x_1	0.0	0.242	0.0	0.0	0.0	0.0	0.15	0.0
x_2	0.242	0.0	0.0	0.0	0.0	0.0	0.0	0.0
x_3	0.0	0.0	0.0	0.25	0.0	0.0	0.0	0.105
x_4	0.0	0.0	0.25	0.0	0.0	0.0	0.0	0.0
x_5	0.0	0.0	0.0	0.0	0.0	0.155	0.0	0.191
x_6	0.0	0.0	0.0	0.0	0.155	0.0	0.0	0.25
f_1	0.15	0.0	0.0	0.0	0.0	0.0	0.0	0.0
f_2	0.0	0.0	0.105	0.0	0.191	0.25	0.0	0.0

Table B.6: Average MD Score Adjacency Matrix of Pareto Dataset Network of Osyczka and Kundu Problem

	x_1	x_2	x_3	x_4	x_5	x_6	f_1	f_2
x_1	0.0	0.149	0.0	0.0	0.0	0.0	0.165	0.0
x_2	0.149	0.0	0.0	0.0	0.0	0.0	0.0	0.0
x_3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
x_4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
x_5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
x_6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.052
f_1	0.165	0.0	0.0	0.0	0.0	0.0	0.0	0.301
f_2	0.0	0.0	0.0	0.0	0.0	0.052	0.301	0.0

Table B.7: Average K2 Score Shortest Path Lengths of Full K2 Scores of Osyczka and Kundu Problem

	x_1	x_2	x_3	x_4	x_5	x_6	f_1	f_2
x_1	0.0	2.056	209.073	2.192	209.844	2.056	1.013	1.028
x_2	2.056	0.0	209.073	2.089	209.844	2.056	2.112	1.028
x_3	209.073	209.073	0.0	209.209	416.861	209.073	209.129	208.045
x_4	2.192	2.089	209.209	0.0	209.979	2.089	1.4	1.164
x_5	209.844	209.844	416.861	209.979	0.0	209.844	209.9	208.816
x_6	2.056	2.056	209.073	2.089	209.844	0.0	2.112	1.028
f_1	1.013	2.112	209.129	1.4	209.9	2.112	0.0	1.084
f_2	1.028	1.028	208.045	1.164	208.816	1.028	1.084	0.0

Table B.8: Average K2 Score Shortest Path Lengths of Feasible K2 Scores of Osyczka and Kundu Problem

	x_1	x_2	x_3	x_4	x_5	x_6	f_1	f_2
x_1	0.0	6.424	0.0	0.0	0.0	0.0	1.774	0.0
x_2	6.424	0.0	0.0	0.0	0.0	0.0	8.198	0.0
x_3	0.0	0.0	0.0	9.82	15.806	16.548	0.0	14.73
x_4	0.0	0.0	9.82	0.0	25.581	26.323	0.0	24.505
x_5	0.0	0.0	15.806	25.581	0.0	1.429	0.0	1.219
x_6	0.0	0.0	16.548	26.323	1.429	0.0	0.0	1.653
f_1	1.774	8.198	0.0	0.0	0.0	0.0	0.0	0.0
f_2	0.0	0.0	14.73	24.505	1.219	1.653	0.0	0.0

Table B.9: Average K2 Score Shortest Path Lengths of Pareto K2 Scores of Osyczka and Kundu Problem

	x_1	x_2	x_3	x_4	x_5	x_6	f_1	f_2
x_1	0.0	3.219	0.0	0.0	0.0	7.979	2.699	5.068
x_2	3.219	0.0	0.0	0.0	0.0	11.46	5.858	8.3
x_3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
x_4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
x_5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
x_6	7.979	11.46	0.0	0.0	0.0	0.0	5.348	3.151
f_1	2.699	5.858	0.0	0.0	0.0	5.348	0.0	2.369
f_2	5.068	8.3	0.0	0.0	0.0	3.151	2.369	0.0

Table B.10: Average MD Score Shortest Path Lengths of Full MD Scores of Osyczka and Kundu Problem

	x_1	x_2	x_3	x_4	x_5	x_6	f_1	f_2
x_1	0.0	6.994	8.659	10.224	8.646	6.996	7.573	3.497
x_2	6.994	0.0	8.658	9.846	8.645	6.995	9.854	3.496
x_3	8.659	8.658	0.0	11.889	10.31	8.66	11.519	5.162
x_4	10.224	9.846	11.889	0.0	11.876	9.878	7.525	6.727
x_5	8.646	8.645	10.31	11.876	0.0	8.647	11.506	5.149
x_6	6.996	6.995	8.66	9.878	8.647	0.0	9.856	3.498
f_1	7.573	9.854	11.519	7.525	11.506	9.856	0.0	6.357
f_2	3.497	3.496	5.162	6.727	5.149	3.498	6.357	0.0

Table B.11: Average MD Score Shortest Path Lengths of Feasible MD Scores of Osyczka and Kundu Problem

	x_1	x_2	x_3	x_4	x_5	x_6	f_1	f_2
x_1	0.0	4.136	0.0	0.0	0.0	0.0	7.194	0.0
x_2	4.136	0.0	0.0	0.0	0.0	0.0	11.331	0.0
x_3	0.0	0.0	0.0	4.065	10.357	8.558	0.0	4.762
x_4	0.0	0.0	4.065	0.0	14.197	12.398	0.0	8.602
x_5	0.0	0.0	10.357	14.197	0.0	6.534	0.0	5.349
x_6	0.0	0.0	8.558	12.398	6.534	0.0	0.0	4.193
f_1	7.194	11.331	0.0	0.0	0.0	0.0	0.0	0.0
f_2	0.0	0.0	4.762	8.602	5.349	4.193	0.0	0.0

Table B.12: Average MD Score Shortest Path Lengths of Pareto MD Scores of Osyczka and Kundu Problem

	x_1	x_2	x_3	x_4	x_5	x_6	f_1	f_2
x_1	0.0	4.086	0.0	0.0	0.0	12.538	5.494	8.498
x_2	4.086	0.0	0.0	0.0	0.0	16.242	9.621	12.635
x_3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
x_4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
x_5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
x_6	12.538	16.242	0.0	0.0	0.0	0.0	6.753	3.833
f_1	5.494	9.621	0.0	0.0	0.0	6.753	0.0	3.004
f_2	8.498	12.635	0.0	0.0	0.0	3.833	3.004	0.0

APPENDIX C

Unchunked Bulk Carrier Design Model Adjacency Matrices and Shortest Path Lengths

Table C.7: Average K2 Score Shortest Path Lengths of Full K2 Scores of Bulk Carrier Design Problem

<i>L</i>	0	1.94	2.93	4.16	1.19	2.32	1.06	2.99	1.91	1.98	3.94	4.06	3.1	2.22	2.83	2.14	4.7	2.24	2.17	3.27	4.79	3.81	1.7	2.36	4.0	4.01	2.98	3.56	2.83	4.17	4.85	4.53	3.21	2.72	2.02	4.01	
<i>T</i>	1.94	0	2.12	4.79	2.19	3.88	3.0	3.96	3.21	3.1	4.58	4.7	4.46	2.98	1.92	1.0	4.95	2.18	3.54	4.5	5.04	4.13	3.04	3.47	4.02	4.57	3.22	3.81	3.41	4.07	5.41	5.26	4.23	3.38	3.1	4.25	
<i>D</i>	2.93	2.12	0	5.68	2.99	4.74	3.99	4.94	4.41	3.99	5.47	5.59	5.46	3.94	1.0	2.92	5.89	3.71	4.54	5.27	5.98	5.02	4.23	4.42	4.93	5.39	4.06	4.72	4.23	4.89	6.23	6.08	5.08	4.33	4.09	5.09	
<i>CB</i>	4.16	4.79	5.68	0	3.54	4.75	5.08	4.31	4.88	2.37	1.12	1.15	5.79	5.68	5.13	4.82	4.36	3.66	3.81	4.12	2.37	4.31	4.25	3.23	5.26	4.57	4.49	4.15	3.66	5.86	5.86	4.98	3.93	3.24	5.6		
<i>B</i>	1.19	2.19	2.99	3.54	0	3.35	2.24	3.1	2.75	1.18	3.33	3.45	4.17	1.03	2.99	1.82	4.81	1.07	2.27	4.28	4.84	3.22	2.4	2.62	3.77	4.01	2.69	3.44	2.92	3.81	4.73	4.58	3.88	2.92	2.03	3.72	
<i>V</i>	2.32	3.88	4.74	4.75	3.35	0	1.47	3.66	3.74	3.47	4.54	4.66	1.34	4.37	4.74	4.13	2.77	4.24	3.53	1.22	2.69	2.76	3.6	3.2	2.67	3.74	2.57	2.86	2.23	3.24	3.9	3.54	1.09	2.13	2.64	3.6	
<i>F_n</i>	1.06	3.0	3.99	5.08	2.24	1.47	0	4.03	2.97	3.04	4.87	4.99	2.05	3.27	3.88	3.2	4.12	3.3	3.22	2.43	4.04	3.74	2.75	3.31	3.94	4.72	3.29	3.88	2.99	4.48	4.9	4.55	2.56	3.2	3.06	4.32	
<i>Δ_S</i>	2.99	3.96	4.94	4.31	3.1	3.66	4.03	0	2.99	2.15	4.1	4.22	4.58	4.12	4.94	4.15	4.97	4.14	2.77	4.7	5.07	4.14	2.95	2.94	4.0	1.11	3.05	3.06	2.21	3.8	2.73	2.53	3.25	2.2	1.18	4.08	
<i>Δ_Δ</i>	1.91	3.21	4.41	4.88	2.75	3.74	2.97	2.99	0	2.72	4.67	4.79	4.66	3.78	4.31	3.41	5.25	3.77	1.56	4.72	5.34	4.41	1.06	1.86	4.27	3.97	3.15	3.54	3.06	4.28	4.9	4.84	3.57	2.52	2.13	4.18	
<i>Δ_{Δ_O}</i>	1.98	3.1	3.99	2.37	1.18	3.47	3.04	2.15	2.72	0	2.15	2.27	4.34	2.2	3.99	2.85	4.24	2.24	1.49	3.83	4.26	2.14	2.08	3.19	3.09	2.72	2.88	2.19	3.22	3.91	3.75	3.02	1.97	1.07	3.75		
<i>a</i>	3.94	4.58	5.47	1.12	3.33	4.54	4.87	4.1	4.67	2.15	0	1.05	5.58	4.34	5.47	4.91	4.6	4.14	3.44	3.59	3.91	2.16	4.1	4.04	3.01	5.04	4.36	4.28	3.94	3.45	5.65	5.65	4.77	3.72	3.02	5.39	
<i>b</i>	4.06	4.7	5.59	1.15	3.45	4.66	4.99	4.22	4.79	2.27	1.05	0	5.7	4.46	5.59	5.03	4.72	4.26	3.56	3.71	4.03	2.28	4.22	4.16	3.13	5.16	4.48	4.4	4.06	3.57	5.77	5.77	4.89	3.84	3.14	5.51	
<i>t_S</i>	3.1	4.46	5.46	5.79	4.17	1.34	2.05	4.58	4.66	4.34	5.58	5.7	0	5.2	5.46	4.75	4.11	5.16	4.56	2.3	4.03	3.8	4.25	4.19	3.63	4.64	3.3	3.67	3.12	4.33	4.59	4.23	2.44	3.12	3.51	4.33	
<i>BM</i>	2.22	2.98	3.94	4.56	1.03	4.37	3.27	4.12	3.78	2.2	4.34	4.46	5.2	0	3.94	1.97	5.6	1.04	3.29	5.07	5.63	4.01	3.43	3.65	4.56	5.03	3.67	4.42	3.93	4.59	5.74	5.59	4.88	3.93	3.05	4.7	
<i>KG</i>	2.83	1.92	1.0	5.68	2.99	4.74	3.88	4.94	4.31	3.99	5.47	5.59	5.46	3.94	0	2.72	5.89	3.71	4.44	5.27	5.98	5.02	4.13	4.42	4.93	5.39	4.06	4.72	4.23	4.89	6.23	6.08	5.08	4.33	4.09	5.09	
<i>KB</i>	2.14	1.0	2.92	5.13	1.82	4.13	3.2	4.15	3.41	2.85	4.91	5.03	4.75	1.97	2.72	0	5.39	1.27	3.51	5.05	5.58	4.44	3.24	3.64	4.5	4.68	3.16	3.88	3.4	4.35	5.29	5.15	4.4	3.54	3.2	4.19	
<i>P</i>	4.7	4.95	5.89	4.82	4.81	2.77	4.12	4.97	5.25	4.24	4.6	4.72	4.11	5.6	5.89	5.39	0	5.29	4.72	2.79	1.0	3.17	4.86	4.03	2.12	5.26	3.58	3.44	3.62	3.23	5.19	5.13	3.51	3.22	3.9	4.6	
<i>GM</i>	2.24	2.18	3.71	4.36	1.07	4.24	3.3	4.14	3.77	2.24	4.14	4.26	5.16	1.04	3.71	1.27	5.29	0	3.3	4.86	5.39	3.81	3.46	3.65	4.32	4.9	3.48	4.11	3.72	4.28	5.52	5.38	4.71	3.76	3.08	4.5	
<i>SC</i>	2.17	3.54	4.54	3.66	2.27	3.53	3.22	2.77	1.56	1.49	3.44	3.56	4.56	3.29	4.44	3.51	4.72	3.3	0	4.33	4.82	3.61	2.5	1.0	3.75	3.61	2.76	2.98	2.62	3.73	4.53	4.48	3.02	1.97	1.91	3.79	
<i>Δ_F</i>	3.27	4.5	5.27	3.81	2.28	1.22	2.43	4.7	4.72	3.83	3.59	3.71	2.3	5.07	5.27	5.05	2.79	4.86	4.33	0	2.44	1.65	4.49	4.19	1.65	4.68	3.56	3.61	3.41	2.84	4.71	4.52	2.32	3.11	3.62	4.59	
<i>Δ_{Δ_M}</i>	4.79	5.04	5.98	4.12	4.84	2.69	4.04	5.07	5.34	4.26	3.91	4.03	4.03	5.63	5.98	5.58	1.0	5.39	4.82	2.44	0	2.17	4.96	4.13	1.12	5.37	3.77	3.64	3.81	3.05	5.38	5.32	3.61	3.31	3.99	4.8	
<i>Δ_{Δ_{DC}}</i>	3.81	4.13	5.02	3.37	3.22	2.76	3.74	4.14	4.41	2.46	2.16	2.28	3.8	4.01	5.02	4.44	3.17	3.81	3.61	1.65	2.17	0	4.03	3.27	1.07	4.12	2.98	2.47	3.29	1.41	4.31	4.31	3.5	2.46	3.06	4.01	
<i>FC</i>	1.7	3.04	4.23	4.31	2.4	3.6	2.75	2.25	1.06	2.14	4.1	4.22	4.25	3.43	4.13	3.24	4.86	3.46	2.5	4.49	4.96	4.03	0	2.08	3.89	3.35	2.94	3.31	2.41	3.89	4.26	4.1	3.25	2.2	1.08	3.97	
<i>CC</i>	2.36	3.47	4.42	4.25	2.62	3.2	3.31	2.94	1.56	2.08	4.04	4.16	4.19	3.65	4.42	3.64	4.03	3.65	1.0	4.19	4.13	3.27	2.08	0	3.06	3.49	2.16	2.39	2.52	3.13	4.13	3.98	2.52	1.47	2.08	3.19	
<i>Δ_F</i>	4.0	4.02	4.93	3.23	3.77	2.67	3.94	4.0	4.27	3.19	3.01	3.13	3.63	4.56	4.93	4.5	2.12	4.32	3.75	1.65	1.12	1.07	3.89	3.06	0	4.3	2.8	2.66	2.74	2.05	4.31	2.84	2.24	2.92	3.83		
<i>Δ_{D_W}</i>	4.01	4.57	5.39	5.26	4.01	3.74	4.72	1.11	3.97	3.09	5.04	5.16	4.64	5.03	5.39	4.68	5.26	4.9	3.61	4.68	5.37	4.12	3.35	3.49	4.3	0	2.93	2.23	1.86	3.19	1.62	1.42	3.77	2.74	2.27	3.96	
<i>RC</i>	2.98	3.22	4.06	4.57	2.69	2.57	3.29	3.05	3.15	2.72	4.36	4.48	3.3	3.67	4.06	3.16	3.58	3.48	2.76	3.56	3.77	2.98	2.94	2.16	2.8	2.93	0	1.23	1.11	2.08	3.11	2.95	2.04	1.09	1.97	1.03	
<i>PC</i>	3.56	3.81	4.72	4.49	3.44	2.86	3.88	3.06	3.54	2.88	4.28	4.4	3.67	4.42	4.72	3.88	3.44	4.11	2.98	3.61	3.64	2.47	3.31	2.39	2.66	2.23	1.23	0	2.05	1.07	2.39	2.33	1.96	1.12	2.24	2.26	
<i>Δ_{SWC}</i>	2.83	3.41	4.23	4.15	2.92	2.23	2.99	2.21	3.06	2.19	3.94	4.06	3.12	3.93	4.23	3.4	3.62	3.72	2.62	3.41	3.81	3.29	2.41	2.31	2.74	1.86	1.11	2.05	0	2.72	2.02	1.86	2.31	1.44	1.34	2.14	
<i>VC</i>	4.17	4.07	4.89	3.66	3.81	3.24	4.48	3.8	4.28	3.22	3.45	3.57	4.33	4.59	4.89	4.35	3.23	4.28	3.73	2.84	3.05	1.41	3.89	3.13	2.05	3.19	2.08	1.07	2.72	0	3.23	3.23	2.91	1.87	2.9	3.11	
<i>Δ_C</i>	4.85	5.41	6.23	5.86	4.73	3.9	4.9	2.73	4.9	3.91	5.65	5.77	4.59	5.74	6.23	5.19	5.52	4.48	4.52	5.32	4.31	4.1	3.98	4.31	1.42	2.95	2.33	1.86	3.23	1.0	4.04	3.18	3.18	4.14			
<i>t_P</i>	4.53	5.26	6.08	5.86	4.58	3.54	4.55	2.53	4.84	3.75	5.65	5.77	4.23	5.59	6.08	5.15	5.13	5.38	4.48	4.52	5.32	4.31	4.1	3.98	4.31	1.42	2.95	2.33	1.86	3.23	1.0	4.04	3.18	3.02	3.98		
<i>RTPA</i>	3.21	4.23	5.08	4.98	3.88	1.09	2.56	3.25	3.57	3.02	4.77	4.89	2.44	4.88	5.08	4.4	3.51	4.71	3.02	2.32	3.61	3.5	3.25	2.52	2.84	3.77	2.04	1.96	2.31	2.91	4.04	3.82	0	1.05	2.17	3.07	
<i>Δ_{C,ann}</i>	2.72	3.38	4.33	3.93	2.92	2.13	3.2	2.2	2.52	1.97	3.72	3.84	3.12	3.93	4.33	3.54	3.22	3.76	1.91	3.31	3.31	2.46	2.2	1.47	2.24	2.74	1.09	1.12	1.44	1.87	3.18	3.02	2.17	1.12	0	3.0	
<i>TC</i>	4.01	4.25	5.09	5.6	3.72	3.6	4.32	4.08	4.18	3.75	5.39	5.51	4.33	4.7	5.09	4.19	4.6	4.5	3.79	4.59	4.8	4.01	3.97	3.19	3.83	3.96	1.03	2.26	2.14	3.11	4.14	3.98	3.07	2.12	3.0	0	

Table C.8: Average K2 Score Shortest Path Lengths of Feasible K2 Scores of Bulk Carrier Design Problem

	<i>L</i>	<i>T</i>	<i>D</i>	<i>C_B</i>	<i>B</i>	<i>V</i>	<i>F_n</i>	Δ	Δ_S	Δ_O	<i>a</i>	<i>b</i>	<i>t_S</i>	<i>BM</i>	<i>KG</i>	<i>KB</i>	<i>P</i>	<i>GM</i>	<i>SC</i>	Δ_M	Δ_{DC}	<i>FC</i>	Δ_{LS}	<i>CC</i>	Δ_F	Δ_{DW}	<i>RC</i>	<i>PC</i>	Δ_{SWC}	<i>VC</i>	Δ_C	<i>t_P</i>	<i>RTPA</i>	$\Delta_{C,ann}$	<i>AC</i>	<i>TC</i>
<i>L</i>	0	3.31	6.94	0	5.43	2.85	1.07	6.55	7.62	5.78	0	0	2.85	6.76	7.55	3.51	6.23	7.89	7.23	6.33	7.24	4.86	6.94	6.21	5.3	6.3	3.02	5.01	4.0	4.85	6.41	7.43	3.95	4.26	4.88	4.18
<i>T</i>	3.31	0	4.53	0	4.5	6.03	4.37	5.18	6.95	5.44	0	0	6.14	5.83	5.14	1.02	8.69	7.03	6.74	8.8	9.71	7.32	6.27	5.72	7.76	4.94	1.41	3.91	2.64	5.81	5.05	6.07	5.43	4.52	4.39	2.57
<i>D</i>	6.94	4.53	0	0	8.25	9.15	7.98	8.9	9.79	7.88	0	0	9.53	9.16	1.02	3.92	11.94	8.88	9.92	12.05	12.96	10.57	9.11	8.9	11.01	8.66	5.37	7.63	6.36	9.33	8.77	9.79	8.81	8.19	7.57	6.53
<i>C_B</i>	0	0	0	0	0	0	0	0	0	1.23	1.32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
<i>B</i>	5.43	4.5	8.25	0	7.1	6.09	7.3	6.85	3.07	0	0	0	7.21	1.33	8.86	4.5	9.67	2.53	6.64	9.77	10.68	8.29	6.17	5.62	8.73	7.05	4.5	5.76	4.75	6.83	7.16	8.18	6.54	5.58	4.29	5.66
<i>V</i>	2.85	6.03	9.15	0	7.1	1.78	6.48	7.84	6.07	0	0	0	1.37	8.43	9.76	6.24	3.65	9.43	7.36	3.76	4.67	2.28	7.15	6.35	2.72	6.24	5.05	4.95	3.93	2.53	6.35	7.36	1.1	2.05	5.02	6.21
<i>F_n</i>	1.07	4.37	7.98	0	6.09	1.78	0	7.34	8.26	6.45	0	0	1.79	7.42	8.59	4.57	5.16	8.55	7.84	5.27	6.18	3.79	5.88	6.82	4.23	7.1	4.08	5.8	4.79	4.06	7.2	8.22	2.88	3.83	5.49	5.24
Δ	6.55	5.18	8.9	0	7.3	6.48	7.34	0	7.35	5.85	0	0	7.85	8.62	9.51	6.2	8.9	9.22	7.14	9.01	9.92	7.53	6.67	6.12	7.97	1.17	3.77	3.82	2.55	5.72	2.5	2.91	5.39	4.43	4.79	4.93
Δ_S	7.62	6.95	9.79	0	6.85	7.84	8.26	7.35	0	3.88	0	0	8.97	7.25	10.4	7.89	10.35	7.25	4.23	10.46	11.37	9.88	1.14	3.62	9.42	7.11	5.54	5.29	4.8	7.18	7.21	8.23	6.85	5.89	2.82	6.7
Δ_O	5.78	5.44	7.88	0	3.07	6.07	6.45	5.85	3.88	0	0	0	7.2	3.37	8.49	5.9	8.59	3.37	3.66	8.69	9.6	7.21	3.19	2.64	7.66	5.6	4.04	3.78	3.3	5.41	5.71	6.73	5.09	4.13	1.31	5.2
<i>a</i>	0	0	0	0	0	0	0	0	0	0	2.55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
<i>b</i>	0	0	0	0	0	0	0	0	0	0	2.55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
<i>t_S</i>	2.85	6.14	9.53	0	7.21	1.37	1.79	7.85	8.97	7.2	0	0	0	8.54	10.14	6.34	4.2	9.67	8.49	4.31	5.22	2.83	8.28	7.48	3.27	7.61	5.71	6.31	5.3	3.25	7.71	8.73	2.47	3.42	6.15	6.86
<i>BM</i>	6.76	5.83	9.16	0	1.33	8.43	7.42	8.62	7.25	3.37	0	0	8.54	0	9.78	5.83	10.99	1.2	7.04	11.1	12.01	9.62	6.57	6.02	10.6	8.38	5.83	6.91	6.07	8.16	8.49	9.5	7.86	6.9	4.69	6.99
<i>KG</i>	7.55	5.14	1.02	0	8.86	9.76	8.59	9.51	10.4	8.49	0	0	10.14	9.78	0	4.53	12.55	10.49	10.53	12.66	13.57	11.89	7.2	9.51	11.62	9.27	5.99	8.24	6.97	9.94	9.38	10.4	9.43	8.8	8.18	7.14
<i>KB</i>	3.51	1.02	3.92	0	4.5	6.24	4.57	6.2	7.89	5.9	0	0	6.34	5.83	4.53	0	8.98	7.03	7.76	9.09	10	7.61	7.21	6.74	8.05	5.96	2.43	4.93	3.65	6.6	6.07	7.08	6.11	5.49	5.41	3.59
<i>P</i>	6.23	8.69	11.94	0	9.67	3.65	5.16	8.9	10.35	8.59	0	0	4.2	10.99	12.55	8.98	0	11.9	9.88	2.7	1.02	2.5	9.67	8.86	1.4	8.66	7.57	7.37	6.36	3.19	8.77	9.79	4.72	4.47	7.53	8.73
<i>GM</i>	7.89	7.03	9.88	0	2.53	9.43	8.55	9.22	7.25	3.37	0	0	9.67	1.2	10.49	7.03	11.9	0	7.04	12.01	12.92	10.53	6.57	6.02	10.97	8.98	6.96	7.16	6.67	8.79	9.08	10.1	8.46	7.5	4.69	8.12
<i>SC</i>	7.23	6.74	9.92	0	6.64	7.36	7.84	7.14	4.23	3.66	0	0	8.49	7.04	10.53	7.76	9.88	7.04	0	9.99	10.9	8.51	3.55	1.02	8.95	6.9	5.33	5.08	4.59	6.71	7.0	8.02	6.38	5.42	2.35	6.49
Δ_M	6.33	8.8	12.05	0	9.77	3.76	5.27	9.01	10.46	8.69	0	0	4.31	11.1	12.66	9.09	2.7	12.01	9.99	0	3.51	2.4	9.78	8.97	1.3	8.77	7.68	7.47	6.46	3.29	8.87	9.89	4.82	4.58	7.64	8.83
Δ_{DC}	7.24	9.71	12.96	0	10.68	4.67	6.18	9.92	11.37	9.6	0	0	5.22	12.01	13.57	10.2	12.92	10.9	3.51	0	3.32	10.69	8.88	2.21	9.68	8.59	8.38	7.37	4.2	9.78	10.8	5.73	5.49	8.55	9.74	
<i>FC</i>	4.86	7.32	10.57	0	8.29	2.28	3.79	7.53	8.98	7.21	0	0	2.83	9.62	11.18	7.61	2.5	10.53	8.51	2.4	3.32	0	8.3	7.49	1.11	7.29	6.2	6.0	4.98	1.81	7.39	8.41	3.34	3.1	6.16	7.35
Δ_{LS}	6.94	6.27	9.11	0	6.17	7.15	7.58	6.67	1.14	3.19	0	0	8.28	6.57	9.72	7.21	9.67	6.57	3.55	9.78	10.69	8.3	0	2.93	8.74	6.42	4.86	4.6	4.12	6.5	6.53	7.55	6.17	5.21	2.14	6.01
<i>VC</i>	6.21	5.72	8.9	0	5.62	6.35	6.82	6.12	3.62	2.64	0	0	7.48	6.02	9.51	6.74	8.86	6.02	1.02	8.97	9.88	7.49	2.93	0	7.93	5.88	4.31	4.06	3.57	5.69	5.99	7.0	5.36	4.4	1.33	5.47
Δ_F	5.3	7.76	11.01	0	8.73	2.72	4.23	7.97	9.42	7.66	0	0	3.27	10.6	11.62	8.05	1.4	10.97	8.95	1.3	2.21	1.11	8.74	7.93	0	7.73	6.64	6.44	5.42	2.26	7.84	8.85	3.78	3.54	6.6	7.8
Δ_{DW}	6.3	4.94	8.66	0	7.05	6.24	7.1	1.17	7.11	5.6	0	0	7.61	8.38	9.27	5.96	8.66	8.98	6.9	8.77	9.68	7.29	6.42	5.88	7.73	0	3.53	3.58	2.31	5.47	1.33	1.74	5.15	4.19	4.55	4.69
<i>RC</i>	3.02	1.41	5.37	0	4.5	5.05	4.08	3.77	5.54	4.04	0	0	5.71	5.83	5.99	2.43	7.57	6.96	5.33	7.68	8.59	6.2	4.86	4.31	6.64	3.53	0	2.5	1.23	4.4	3.64	4.66	4.07	3.11	2.98	1.16
<i>PC</i>	5.01	3.91	7.63	0	5.76	4.95	5.8	3.82	5.29	3.78	0	0	6.31	6.91	8.24	7.93	7.16	5.08	7.46	7.37	8.38	6.0	4.6	4.06	6.44	3.58	2.5	0	1.28	4.18	3.69	4.71	3.86	2.9	2.73	3.66
Δ_{SWC}	4.0	2.64	6.36	0	4.75	3.93	4.79	2.55	4.8	3.3	0	0	5.3	6.07	6.97	3.65	6.36	6.67	4.59	6.46	7.37	4.98	4.12	3.57	5.42	2.31	1.23	0	3.17	2.41	3.43	2.85	1.88	2.24	2.38	
<i>VC</i>	4.85	5.81	9.33	0	6.83	2.53	4.06	5.72	7.18	5.41	0	0	3.25	8.16	9.94	6.6	3.19	8.79	6.71	3.29	4.2	1.81	6.5	5.69	2.26	5.47	4.4	4.18	3.17	0	5.58	6.6	2.44	1.28	4.36	5.55
Δ_C	6.41	5.05	8.77	0	7.16	6.35	7.2	2.5	7.21	5.71	0	0	7.71	8.49	9.38	6.07	8.08	7.0	8.87	9.78	3.87	6.53	5.99	7.84	1.33	3.64	3.69	2.41	5.58	0	1.02	5.26	4.3	4.66	4.8	
<i>t_P</i>	7.43	6.07	9.79	0	8.18	7.36	8.22	2.91	8.23	6.73	0	0	8.73	9.5	10.4	7.08	9.79	10.1	8.02	9.89	10.8	8.41	7.55	7.0	8.85	1.74	4.66	4.71	3.43	6.6	1.02	6.27	5.31	5.67	5.81	
<i>RTPA</i>	3.95	5.43	8.81	0	6.54	1.1	2.88	5.39	6.85	5.09	0	0	2.47	7.86	9.43	6.11	4.72	8.46	6.38	4.82	5.73	3.34	6.17	5.36	3.78	5.15	4.07	3.86	2.85	2.44	5.26	6.27	0	1.16	4.03	5.23
$\Delta_{C,ann}$	4.26	4.52	8.17	0	5.58	2.02	3.83	4.43	5.89	4.13																										

Table C.9: Average K2 Score Shortest Path Lengths of Pareto K2 Scores of Bulk Carrier Design Problem

	<i>L</i>	<i>T</i>	<i>D</i>	<i>C_B</i>	<i>B</i>	<i>V</i>	<i>F_n</i>	Δ	Δ_S	Δ_O	<i>a</i>	<i>b</i>	<i>t_S</i>	<i>BM</i>	<i>KG</i>	<i>KB</i>	<i>P</i>	<i>GM</i>	<i>SC</i>	Δ_M	Δ_{DC}	<i>FC</i>	Δ_{LS}	<i>CC</i>	Δ_F	Δ_{DW}	<i>RC</i>	<i>PC</i>	Δ_{SWC}	<i>VC</i>	Δ_C	<i>t_P</i>	<i>RTPA</i>	$\Delta_{C,ann}$	<i>AC</i>	<i>TC</i>		
<i>L</i>	0	4.38	6.35	0	4.42	0	6.67	6.86	8.41	5.66	0	0	0	6.43	7.27	5.07	104	7.18	7.07	11.1711	11.1777	7.8	8.92	6.16	8.72	6.85	1.85	4.7	2.95	5.86	7.42	8.32	4.9	4.63	4.6	3.26		
<i>T</i>	4.38	0	2.75	0	5.11	0	7.37	7.55	8.84	6.1	0	0	0	7.12	3.44	1.15	10.747	8.7	7.77	11.8711	11.8784	9.35	6.86	9.42	7.55	2.54	5.4	3.64	6.56	8.12	9.02	5.59	5.32	5.3	3.95			
<i>D</i>	6.35	2.75	0	0	7.08	0	9.34	9.52	10.818	0.7	0	0	0	9.09	1.15	2.06	12.719	8.4	9.74	13.8413	13.8410	45.11	32.83	13.399	5.2	4.51	7.37	5.61	8.53	10.9	10.997	5.6	7.29	7.27	5.92			
<i>C_B</i>	0	0	0	0	0	0	0	0	0	1.33	1.48	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
<i>B</i>	4.42	5.11	7.08	0	0	0	6.85	7.31	8.06	5.03	0	0	1.44	2.01	8.0	5.8	10.492	7.6	7.25	11.6211	11.638	23	8.57	6.34	9.18	7.31	2.57	5.44	3.4	6.31	7.88	8.78	5.07	4.8	4.51	3.99		
<i>V</i>	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
<i>F_n</i>	6.67	7.37	9.34	0	6.85	0	7.35	8.63	6.15	0	0	0	0	8.86	10.268	0.6	10.539	6.1	7.29	11.6611	11.668	27	9.14	6.38	9.21	7.35	5.11	5.76	3.72	6.35	7.91	8.82	1.77	4.84	5.09	6.53		
Δ	6.86	7.55	9.52	0	7.31	0	7.35	10.458	2.5	9.63	10.7	0	0	9.33	10.458	9.5	10.761	0.77	3.7	8.24	4.85	8.32	1.21	5.3	5.94	3.91	5.45	2.46	3.36	5.58	4.46	5.28	6.71	6.71	6.71	6.71		
Δ_S	8.41	8.84	10.810	8.06	0	0	8.63	7.73	0	5.71	0	0	0	10.7	11.739	3.3	10.331	0.82	3.93	11.4611	11.478	0.7	1.25	3.33	9.02	7.72	6.85	7.22	5.47	6.15	8.29	9.19	6.86	4.34	4.92	8.27		
Δ_O	5.66	6.1	8.07	0	5.03	0	6.15	6.33	5.71	0	0	2.82	0	7.05	8.99	6.79	8.69	7.79	9.82	9.82	6.43	6.22	5.08	7.37	6.32	4.37	3.95	2.99	4.51	6.89	7.79	4.37	3.55	1.32	5.79			
<i>a</i>	0	0	0	1.33	0	0	0	0	0	0	0	2.82	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
<i>b</i>	0	0	0	1.48	0	0	0	0	0	0	2.82	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
<i>t_S</i>	0	0	0	0	0	1.44	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
<i>BM</i>	6.43	7.12	9.09	0	2.01	0	8.86	9.33	10.7	7.05	0	0	0	0	101	7.81	12.5	1.24	9.26	13.6313	13.6410	2410	588	35	11.199	32	4.58	7.45	5.41	8.32	9.89	10.707	0.8	6.82	6.52	6.0	6.84	
<i>KG</i>	7.27	3.44	1.15	0	8.0	0	10.2610	4.511	7.38	9.9	0	0	0	101	0	2.75	13.6310	7.610	6.6614	7.6113	7.712	24.9	75	12.3110	44.5	4.3	8.29	6.53	9.45	11.011	9.18	4.8	8.22	8.19	6.84	6.84		
<i>KB</i>	5.07	1.15	2.06	0	5.8	0	8.06	8.25	9.33	6.79	0	0	0	7.81	2.75	0	11.438	5.6	8.46	12.5612	12.569	17	104	7.55	10.118	24	3.23	6.09	4.33	7.25	8.81	9.71	6.28	6.02	5.99	4.64	4.64	
<i>P</i>	104	10.7412	710	10.490	0	0	10.539	6.3	10.338	6.9	0	0	0	12.5	13.6311	4.30	13.25	8.46	2.7	1.14	2.5	10.847	7.8	1.31	9.62	8.75	9.4	7.37	4.18	10.1911	0.98	7.5	7.37	10.17	10.17	10.17	10.17	
<i>GM</i>	7.18	7.87	9.84	0	2.76	0	9.61	10.7	10.827	7.9	0	0	0	1.24	10.768	5.6	13.250	101	14.3814	13.910	9.911	33.91	11.94	10.7	5.33	8.2	6.16	9.07	10.6411	5.47	8.3	7.57	7.27	6.75	6.75	6.75	6.75	
<i>SC</i>	7.07	7.77	9.74	0	7.25	0	7.29	5.08	3.93	5.99	0	0	0	9.26	10.668	4.6	8.46	101	0	9.59	9.6	6.21	4.44	1.14	7.15	5.07	5.51	6.16	4.12	4.28	5.64	6.54	5.51	2.44	4.67	6.93	6.93	
Δ_M	11.1711	8.713	8.40	11.620	0	0	11.6610	7.611	4.69	8.2	0	0	0	13.6314	7.612	5.62	14.389	5.9	0	2.93	3.63	11.978	9.1	2.45	10.759	8.8	10.538	5.31	11.3212	2.29	8.9	7.66	8.5	11.3	11.3	11.3		
Δ_{DC}	11.1711	8.713	8.40	11.630	0	0	11.6610	7.711	4.79	8.2	0	0	0	13.6414	7.612	5.61	14.389	6	2.93	0	3.63	11.988	9.2	2.45	10.769	8.9	10.538	5.32	11.3312	2.29	8.9	7.67	8.5	11.3	11.3	11.3		
<i>FC</i>	7.78	8.48	10.450	8.23	0	0	8.27	7.37	8.07	6.43	0	0	0	10.2411	3.79	1.7	2.5	10.996	2.1	3.63	3.63	0	8.58	5.52	1.18	7.37	6.49	7.14	5.11	1.92	7.93	8.84	6.5	4.27	5.11	7.91	7.91	
Δ_{LS}	8.92	9.35	11.320	8.57	0	0	9.14	8.24	1.25	6.22	0	0	0	10.5812	2.4104	10.8411	3.34	4.4	11.9711	9.88	5.8	0	4.44	5.3	8.23	7.36	7.73	5.98	6.66	8.8	9.7	7.37	4.85	5.43	8.78	8.78		
<i>CC</i>	6.16	6.86	8.83	0	6.34	0	6.38	4.85	3.93	5.08	0	0	0	8.35	9.75	7.55	7.78	9.1	1.14	8.91	8.92	5.52	4.44	0	6.47	4.84	4.6	5.25	3.21	3.6	5.41	6.31	4.6	1.53	3.76	6.02	6.02	
Δ_F	8.72	9.42	11.390	9.18	0	0	9.21	8.32	9.02	7.37	0	0	0	11.1912	3.110	11.131	11.947	1.5	2.45	1.18	9.53	6.47	0	8.31	7.44	8.08	6.05	2.87	8.88	9.78	7.44	5.22	6.05	8.85	8.85	8.85		
Δ_{DW}	6.85	7.55	9.52	0	7.31	0	7.35	1.21	7.72	6.32	0	0	0	9.32	10.448	2.4	9.62	10.7	5.07	10.7510	7.67	3.7	8.23	4.84	8.31	0	5.29	5.94	3.9	5.44	1.25	2.15	5.57	4.45	5.27	6.7	6.7	
<i>RC</i>	1.85	2.54	4.51	0	2.57	0	5.11	5.3	6.85	4.37	0	0	0	4.58	5.43	3.23	8.75	5.33	5.51	9.88	9.89	6.49	7.36	4.6	7.44	5.29	0	3.14	1.39	4.57	5.86	6.76	3.33	3.07	3.32	1.42	1.42	
<i>PC</i>	4.7	5.4	7.37	0	5.44	0	5.76	5.94	7.22	3.95	0	0	0	7.45	8.29	6.09	9.4	8.2	6.16	10.5310	5.37	1.4	7.73	5.25	6.05	5.94	3.14	0	2.03	5.22	6.51	7.41	3.98	3.71	3.42	4.56	4.56	
Δ_{SWC}	2.95	3.64	5.61	0	3.4	0	3.72	3.91	5.47	2.99	0	0	0	5.41	6.23	4.33	7.37	6.17	4.12	8.5	8.5	5.11	5.98	3.21	6.05	3.9	1.39	2.03	0	3.19	4.47	5.37	1.95	1.68	1.93	2.8	2.8	
<i>VC</i>	5.86	6.56	8.53	0	6.31	0	6.35	5.45	6.15	4.51	0	0	0	8.32	9.45	7.25	4.18	9.07	4.28	5.31	5.32	1.92	6.66	3.6	2.87	5.44	4.57	5.22	3.19	0	6.01	6.91	4.57	2.35	3.19	5.99	5.99	
Δ_C	7.42	8.12	10.9	0	7.88	0	7.91	2.46	8.29	6.89	0	0	0	9.89	11.018	8.1	10.1910	6.45	6.64	11.3211	3.33	9.3	8.7	6.31	9.78	2.15	6.76	7.41	5.37	6.91	1.13	6.14	5.02	5.84	7.27	7.27	7.27	
<i>t_P</i>	8.32	9.02	10.990	8.78	0	0	8.82	3.36	9.19	7.79	0	0	0	7.08	8.48	6.28	8.75	7.83	5.51	9.89	9.89	6.5	7.37	4.6	7.44	5.57	3.33	3.98	1.95	4.57	6.14	7.04	0	3.07	3.32	4.75	4.75	
<i>RTPA</i>	4.9	5.59	7.56	0	5.07	0	1.77	5.58	6.86	4.37	0	0	0	6.82	8.22	6.09	7.37	7.27	4.67	7.67	4.27	4.85	1.53	5.22	4.54	3.07	3.71	1.68	2.35	5.02	5.92	3.07	0	2.23	4.48	4.48		
$\Delta_{C,ann}$	4.63	5.3	7.29	0	4.51	0	5.09	5.28	4.92	1.32	0	0	0	6.52	8.12	5.99	7.37	7.27	4.67	8.5	8.5	5.11	5.43	3.76	6.05	5.27	3.32	3.42	1.93	3.19	5.84	6.74	3.32	2.23	0	4.73	4.73	
<i>AC</i>	3.26	3.95	5.92	0	3.99	0	6.53	6.71	8.27	5.79	0	0	0	6.0	6.84	4.64	10.176	7.5	6.93	11.3	11.3	7.91	8.78	6.02	8.85	6.7	1.42	4.56	2.8	5.99	7.27	8.17	4.75	4.48	4.73	4.73		

Table C.10: Average MD Score Shortest Path Lengths of Full MD Scores of Bulk Carrier Design Problem

<i>L</i>	0	3.85	5.83	7.38	2.15	3.49	1.59	4.71	3.21	3.57	6.98	7.83	4.18	3.63	5.48	4.03	7.44	3.81	4.12	5.68	7.62	6.12	2.86	4.2	6.69	6.26	5.7	6.01	4.9	6.82	7.54	7.05	5.91	4.77	3.45	6.87	
<i>T</i>	3.85	0	3.55	8.34	3.5	6.58	5.44	6.5	5.54	5.22	7.94	8.79	6.91	4.67	2.77	1.91	8.03	3.36	6.32	7.91	8.36	7.09	5.28	5.8	6.9	7.46	5.68	6.48	5.86	6.92	8.6	8.5	7.58	5.95	5.43	6.86	
<i>D</i>	5.83	3.55	0	9.92	5.37	8.43	7.42	8.16	7.89	7.06	9.52	10.379	2.4	6.65	2.0	5.45	9.72	6.23	8.19	9.25	10.5	8.45	7.32	7.66	8.4	8.91	7.28	7.86	7.31	8.21	10.18	10.6	9.15	7.66	7.14	8.45	
<i>CB</i>	7.38	8.34	9.92	0	5.89	7.97	8.47	6.9	8.14	3.95	1.66	2.27	9.23	7.26	10.267	9.4	7.53	6.89	6.19	5.91	6.47	3.7	6.97	7.15	5.04	8.25	7.4	6.92	6.97	5.67	9.02	8.98	8.32	6.52	5.44	8.58	
<i>B</i>	2.15	3.5	5.37	5.89	0	5.43	3.74	4.67	4.37	1.93	5.49	6.34	6.02	1.48	5.29	2.4	7.54	1.66	4.02	7.01	7.81	5.5	3.7	4.13	6.14	6.17	4.8	5.42	4.58	6.13	7.18	7.06	6.49	4.71	3.23	5.98	
<i>V</i>	3.49	6.58	8.43	7.97	5.43	0	2.18	5.67	6.07	5.6	7.57	8.42	1.76	6.91	8.4	6.56	4.68	6.63	6.02	7.55	4.73	4.99	5.3	5.02	4.62	5.76	4.24	4.56	3.66	5.48	5.98	5.5	2.71	3.73	4.4	5.42	
<i>F_n</i>	1.59	5.44	7.42	8.47	3.74	2.18	0	6.08	4.8	5.16	8.07	8.92	2.88	5.23	7.07	5.62	6.63	5.4	5.65	4.37	6.68	6.15	4.38	5.34	6.46	7.15	5.38	5.96	4.76	6.91	7.49	6.97	4.77	5.2	4.82	6.56	
<i>Δ_S</i>	4.71	6.5	8.16	6.9	4.67	5.67	6.08	0	4.36	3.23	6.5	7.35	6.75	6.15	8.41	6.1	7.71	6.06	4.29	7.41	7.9	6.56	3.16	4.47	6.23	1.62	4.71	4.58	3.29	5.74	4.23	3.93	5.24	3.44	1.63	5.89	
<i>Δ_Δ</i>	3.21	5.54	7.89	8.14	4.37	6.07	4.8	4.36	0	4.5	7.74	8.59	6.59	5.86	7.69	5.69	8.22	5.72	2.56	7.86	8.41	7.07	1.69	2.49	6.74	5.89	5.23	5.51	4.87	6.63	7.42	7.29	5.86	4.06	3.26	6.41	
<i>Δ_O</i>	3.57	5.22	7.06	3.95	1.93	5.6	5.16	3.23	4.5	0	3.55	4.4	6.4	3.42	7.01	4.15	6.58	3.49	2.52	6.46	6.77	4.24	3.3	3.51	5.1	4.73	4.62	4.63	3.61	5.21	6.08	5.94	5.14	3.33	1.77	5.8	
<i>a</i>	6.98	7.94	9.52	1.66	5.49	7.57	8.07	6.5	7.74	3.55	0	2.22	8.83	6.86	9.86	7.54	7.13	6.49	5.79	5.51	6.07	3.3	6.57	6.75	4.64	7.85	7.0	6.52	6.57	5.27	8.62	8.58	7.92	6.12	5.04	8.18	
<i>b</i>	7.83	8.79	10.372	2.27	6.34	8.42	8.92	7.35	8.59	4.4	2.22	0	9.68	7.71	10.718	39	7.98	7.34	6.64	6.36	6.92	4.15	7.42	7.6	5.49	8.7	7.85	7.37	7.42	6.12	9.47	9.43	8.77	6.97	5.89	9.03	
<i>t_S</i>	4.18	6.91	9.24	9.23	6.02	1.76	2.88	6.75	6.59	6.4	8.83	9.68	0	7.5	9.04	6.98	6.44	7.32	7.03	3.96	6.49	6.25	5.8	6.14	5.77	6.84	5.11	5.41	4.6	6.51	6.79	6.31	4.31	4.85	5.35	6.29	
<i>BM</i>	3.63	4.67	6.65	7.26	1.48	6.91	5.23	6.15	5.86	3.42	6.86	7.71	7.5	0	6.6	2.78	8.53	1.67	5.5	8.13	8.86	6.62	5.18	5.62	7.22	7.65	6.01	6.63	5.93	7.12	8.52	8.41	7.93	6.15	4.71	7.18	
<i>KG</i>	5.48	2.77	2.0	10.265	2.9	8.4	7.07	8.41	7.69	7.01	9.86	10.719	0	6.6	0	4.67	10.215	8.7	8.01	9.5	10.558	9.8	7.52	7.74	8.74	10.310	28.9	36	7.87	7.36	8.53	10.289	36	7.87	7.36	8.53	
<i>KB</i>	4.03	1.91	5.45	7.94	2.4	6.56	5.62	6.1	5.69	4.15	7.54	8.39	6.98	2.78	4.67	0	7.92	1.48	5.89	8.04	8.25	6.73	5.36	5.31	6.6	7.02	5.12	5.79	5.33	6.49	7.86	7.84	7.13	5.36	4.83	6.29	
<i>P</i>	7.44	8.03	9.72	7.53	7.54	4.68	6.63	7.71	8.22	6.58	7.13	7.98	6.44	8.53	10.217	9.2	0	7.84	7.28	3.87	1.66	4.74	7.57	6.13	3.08	7.86	5.51	5.1	5.9	4.77	7.85	7.61	6.31	4.91	6.25	6.68	
<i>GM</i>	3.81	3.36	6.23	6.89	1.66	6.63	5.4	6.06	5.72	3.49	6.49	7.34	7.32	1.67	5.87	1.48	7.84	0	5.37	7.62	8.18	6.25	5.25	5.36	6.53	7.29	5.56	6.07	5.53	6.47	8.06	8.02	7.37	5.59	4.72	6.74	
<i>SC</i>	4.12	6.32	8.18	6.19	4.02	6.07	5.65	4.29	2.56	2.52	5.79	6.64	7.03	5.5	8.01	5.89	7.28	5.37	0	7.37	7.46	5.82	4.14	1.71	5.79	5.53	4.51	4.68	4.34	5.8	6.91	6.81	5.04	3.24	3.21	5.69	
<i>Δ_M</i>	5.68	7.91	9.25	5.91	7.01	2.55	4.37	7.41	7.86	6.46	5.51	6.36	3.96	8.13	9.5	8.04	3.87	7.62	7.37	0	3.51	2.55	7.01	6.63	2.55	7.81	5.98	5.64	5.87	4.36	7.65	7.32	5.09	5.31	5.95	7.15	
<i>Δ_{ΔC}</i>	7.62	8.36	10.5	6.47	7.81	4.73	6.68	7.9	8.41	6.77	6.07	6.92	6.49	8.86	10.558	2.5	1.66	8.18	7.46	3.51	0	3.38	7.76	6.32	1.7	8.05	5.84	5.44	6.23	4.71	8.18	7.94	6.5	5.09	6.44	7.02	
<i>FC</i>	6.12	7.09	8.45	3.7	5.5	4.99	6.15	6.56	7.07	4.24	3.3	4.15	6.25	6.62	9.98	6.73	4.74	6.25	5.82	2.55	3.38	0	6.4	4.99	1.68	6.25	4.63	3.77	5.11	2.14	6.49	6.43	5.56	3.76	5.11	5.81	
<i>Δ_{LS}</i>	2.86	5.38	7.52	6.97	3.7	5.3	4.38	3.16	1.69	3.3	6.57	7.42	5.8	5.18	7.52	5.36	7.57	5.25	4.14	7.01	7.76	6.4	0	3.4	6.09	4.79	4.77	5.11	3.84	6.17	6.42	6.2	5.34	3.54	1.59	5.94	
<i>CC</i>	4.2	5.8	7.66	7.15	4.13	5.02	5.34	4.47	2.49	3.51	6.75	7.6	6.14	5.62	7.74	5.31	6.13	5.36	1.71	6.03	6.32	4.99	3.4	0	4.65	5.23	3.39	3.59	3.82	4.71	6.23	6.04	4.13	2.33	3.39	4.57	
<i>Δ_F</i>	6.69	6.9	8.4	5.04	6.14	4.62	6.46	6.23	6.74	5.1	4.64	5.49	5.77	7.22	8.92	6.6	3.08	6.53	5.79	2.55	1.7	1.68	6.09	4.65	0	6.38	4.34	3.9	4.59	6.32	5.08	3.42	4.77	5.52			
<i>Δ_{DW}</i>	6.26	7.46	8.91	8.25	6.17	5.76	7.15	1.62	5.89	4.73	7.85	8.7	6.84	7.65	9.12	7.02	7.86	7.29	5.53	7.81	8.05	6.25	4.79	5.23	6.38	0	4.41	3.4	2.82	4.88	2.64	2.31	5.87	4.1	3.25	5.59	
<i>RC</i>	5.7	5.68	7.28	7.4	4.8	4.24	5.38	4.71	5.23	4.62	7.0	7.85	5.11	6.01	7.35	5.12	5.51	5.56	4.51	5.98	5.84	4.63	4.77	3.39	4.34	4.41	0	1.8	1.79	3.09	4.73	4.51	3.31	1.7	3.39	1.18	
<i>PC</i>	6.01	6.48	7.86	6.92	5.42	4.56	5.96	4.58	5.51	4.63	6.52	7.37	5.41	6.63	8.22	5.79	5.1	6.07	4.68	5.64	5.44	3.77	5.11	3.59	3.9	3.4	1.8	0	3.02	1.63	3.62	3.61	3.04	1.6	3.56	2.98	
<i>Δ_{SWC}</i>	4.9	5.86	7.51	6.97	4.58	3.66	4.76	3.29	4.87	3.61	6.57	7.42	4.6	5.93	7.58	5.33	5.9	5.53	4.34	5.87	6.23	5.11	3.84	3.82	4.59	2.82	1.79	3.02	0	4.08	3.04	2.88	4.14	2.41	2.28	2.97	
<i>VC</i>	6.82	6.92	8.21	5.67	6.13	5.48	6.91	5.74	6.63	5.21	5.27	6.12	6.51	7.12	8.74	6.49	4.77	6.47	5.8	4.36	4.71	1.14	6.17	4.71	3.15	4.88	3.09	1.63	4.08	0	4.87	4.87	4.5	2.71	4.59	4.26	
<i>Δ_C</i>	7.54	8.6	10.189	0.2	7.18	5.98	7.49	4.93	7.29	6.08	8.62	9.47	6.79	8.52	10.317	8.6	7.85	8.06	6.91	7.65	8.18	6.49	6.42	6.04	6.2	6.34	4.51	3.61	2.88	4.87	0	1.65	6.35	4.46	4.84	5.9	
<i>t_P</i>	7.05	8.5	10.6	8.98	7.06	5.5	6.97	3.93	7.29	5.94	8.58	9.43	6.31	8.41	10.287	8.4	7.31	8.02	6.81	7.32	7.94	6.42	6.04	6.2	6.04	6.32	2.31	4.51	3.61	2.88	4.87	0	1.65	6.35	4.46	4.61	5.69
<i>RTPA</i>	5.91	7.58	9.15	8.32	6.49	2.71	4.77	5.24	5.86	5.14	7.92	8.77	4.31	7.93	9.36	7.13	6.31	7.37	5.04	5.09	6.5	5.56	5.34	4.13	5.08	5.87	3.31	3.04	4.14	4.5	6.35	6.0	6.0	4.48	4.61	5.69	
<i>Δ_{C,ann}</i>	3.47	5.95	7.66	6.52	4.71	3.73	5.2	3.44	4.06	3.33	6.12	6.97	4.85	6.15	7.86	4.83	6.25	4.72	3.21	5.95	6.44	5.01	1.59	3.39	3.42	4.1	1.7	1.6	2.41	4.76	4.48	1.8	0	1.98	2.88		
<i>TC</i>	6.87	6.86	8.45	8.58	5.98	5.42	6.56	5.89	6.41	5.8	8.18	9.03	6.29	7.18	8.53	6.29	6.68	6.74	5.69	7.15	7.02	5.81	5.94	4.57	5.52	5.59	1.18	2.98	2.97	4.26	5.9	5.69	4.48	2.88	4.57	0	

Table C.11: Average MD Score Shortest Path Lengths of Feasible MD Scores of Bulk Carrier Design Problem

	<i>L</i>	<i>T</i>	<i>D</i>	<i>C_B</i>	<i>B</i>	<i>V</i>	<i>F_n</i>	Δ	Δ_S	Δ_O	<i>a</i>	<i>b</i>	<i>t_S</i>	<i>BM</i>	<i>KG</i>	<i>KB</i>	<i>P</i>	<i>GM</i>	<i>SC</i>	Δ_M	Δ_{DC}	<i>FC</i>	Δ_{LS}	<i>CC</i>	Δ_F	Δ_{DW}	<i>RC</i>	<i>PC</i>	Δ_{SWC}	<i>VC</i>	Δ_C	<i>t_P</i>	<i>RTPA</i>	$\Delta_{C,ann}$	<i>AC</i>	<i>TC</i>					
<i>L</i>	0	4.81	8.85	0	8.0	3.81	1.47	9.6	11.338	37	0	0	3.81	9.91	102	5.18	8.45	11.3510	8.48	6.8	10.176	5.6	10.269	0.4	7.26	9.23	4.42	7.39	6.02	6.73	9.41	11.145	7.5	6.24	7.26	5.57					
<i>T</i>	4.81	0	5.54	0	6.8	8.61	6.28	7.5	10.317	95	0	0	8.57	8.72	6.71	1.87	12.3410	5.6104	12.5714	0.610	4.59	2.4	8.24	11.157	1.4	1.91	5.72	3.93	8.21	7.32	9.04	8.13	6.73	6.46	3.06						
<i>D</i>	8.85	5.54	0	0	10.6311	9.610	1.811	8.613	2.710	210	0	0	12.1911	7.219	4.42	15.7712	8.213	5.917	4.813	8.812	2	11.7914	5.711	5.5	6.68	107	8.28	12.1411	6.713	4	11.7010	9.98101	7.83								
<i>C_B</i>	0	0	0	0	0	0	0	0	0	1.65	2.24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0			
<i>B</i>	8.0	6.8	10.630	0	10.378	9.1	10.729	6.6	3.9	0	0	0	10.331	1.92	11.8	6.8	13.7	9.34	13.9315	4.211	8.18	5.79	7.54	12.5110	3.56	8.1	8.39	7.14	9.74	10.5312	2.69	7	8.4	5.76	7.96						
<i>V</i>	3.81	8.61	11.960	0	10.370	2.34	9.78	11.849	0.7	0	0	0	1.77	12.2813	1.28	9.9	5.0	12.8	11.165	2.2	6.71	3.11	10.779	3.6	3.8	9.42	7.56	6.2	3.58	9.59	11.321	9.4	3.46	7.58	8.71						
<i>F_n</i>	1.47	6.28	10.180	0	8.91	2.34	0	10.5512	1	9.29	0	0	2.34	10.8211	3.46	6.5	6.98	5.79	10.195	8.9	5.35	6.98	5.57	10.3712	0.94	2.8	5.71	7.97	7.04												
Δ	9.6	7.5	11.860	0	10.729	7.8	10.550	10.638	2.7	0	0	0	11.3	12.0	13.029	3.7	12.4312	10.3612	6.514	15.0	5.49	5.6	8.56	11.241	7.5	5.59	5.37	3.58	7.86	4.0	4.69	7.86	6.38	6.78	6.74						
Δ_S	11.3310	3.13	2.70	0	9.66	11.8412	1	10.630	5.76	0	0	0	13.079	4.8	14.4411	9.914	7.49	4.8	6.4	14.9716	4.612	8.51	7.8	5.33	13.5510	2.08	4	7.58	7.05	10.2	10.4412	17.10	2	8.72	4.26	9.55					
Δ_O	8.37	7.95	10.210	0	3.9	9.07	9.29	8.27	5.76	0	0	0	10.313	1.73	11.378	6.8	11.983	7.2	5.44	12.2	13.7	10.9	4.69	3.64	10.797	9.1	6.05	5.23	4.7	7.44	8.09	9.81	7.44	5.96	1.86	7.2					
<i>a</i>	0	0	0	1.65	0	0	0	0	0	0	0	3.89	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
<i>b</i>	0	0	0	2.24	0	0	0	0	0	0	3.89	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
<i>t_S</i>	3.81	8.57	12.190	0	10.331	1.77	2.34	11.3	13.0710	310	0	0	12.2413	3.58	9.5	5.71	13.669	12.395	9.3	7.43	3.82	12.0	10.594	5.2	10.938	0.7	9.09	7.72	4.41	11.112	8.43	7.1	5.07	8.81	9.22						
<i>BM</i>	9.91	8.72	11.720	0	1.92	12.2810	8.212	0	9.48	3.73	0	0	12.240	12.898	7.2	15.481	8.4	9.17	15.7	17.2	13.598	4.1	7.37	14.2911	6.48	7.1	8.96	8.43	11.1611	8.213	5.411	11.79	6.9	5.59	9.86						
<i>KG</i>	10.2	6.71	1.94	0	11.8	13.1211	3.413	0.214	4.411	3.70	0	0	13.3512	2.890	5.58	16.9313	9.914	7.617	16.18	6.515	0.413	3.712	9.615	7.412	6.67	8.5	11.249	4.5	13.3	12.8414	5.612	9.9312	1.411	1.89	0						
<i>KB</i>	5.18	1.87	4.42	0	6.8	8.99	6.65	9.37	11.998	6.8	0	0	8.95	8.72	5.38	0	12.8210	5.611	1.9	13.0514	5.410	9.310	9.210	11.639	10.1	11.639	10.1	3.77	7.58	5.79	9.51	9.18	10.919	18	8.49	8.32	4.92				
<i>P</i>	8.45	12.3415	7.70	0	13.7	5.0	6.98	12.4314	7.411	9.80	0	0	5.71	15.4816	9.312	8.20	15.7	14.063	6.6	1.72	3.63	13.6712	2.618	9.12	0.710	8.410	2.38	8.5	4.57	12.2413	9.76	6.05	10.4811	1.69	6.8	5.59	10.92				
<i>GM</i>	11.3510	5.612	8.20	0	3.76	12.8	12.2712	0	9.48	3.72	0	0	13.691	1.84	13.9910	5.615	7	9.17	15.9317	4.213	8.18	4.1	7.37	14.5111	6.39	7.7	8.95	8.42	11.1611	8.113	5.411	11.69	6.8	5.59	10.92						
<i>SC</i>	10.8410	4	13.590	0	9.34	11.1611	5.510	3.66	4	5.44	0	0	12.399	1.7	14.7611	1.9	14.069	1.7	0	14.2915	7.812	1.75	5.33	1.8	12.879	9.9	8.13	7.31	6.78	9.52	10.1711	9	9.52	8.04	3.58	9.28					
Δ_M	8.68	12.5715	9.90	0	13.935	2.2	7.21	12.6514	9.712	2	0	0	5.93	15.7	17.1613	0.53	6.6	15.9314	2.90	5.03	3.51	13.9	12.491	7.7	12.2911	0.710	4.59	0.8	4.79	12.4714	19.7	0.9	6.27	10.7112	2.2						
Δ_{DC}	10.1714	0.617	4.80	0	15.426	7.1	8.7	14.1516	4.613	7	0	0	7.43	17.2	18.6514	5.41	7.2	17.4215	7.85	0.3	5.0	15.3913	9.83	2.6	13.7912	5.611	9.410	5.76	2.9	13.9615	6.98	5.8	7.76	12.2	13.71						
<i>FC</i>	6.56	10.4513	8.80	0	11.813	1.1	5.09	10.5412	8.5109	0	0	0	3.82	13.5915	0.410	9.33	6.3	13.8112	1.73	5.1	5.0	11.7810	3.71	7.4	10.188	9.6	8.34	6.96	2.68	10.3612	0.8	4.97	4.16	8.59	10.1						
Δ_{LS}	10.269	2.4	12.2	0	8.59	10.7711	0.39	5.6	1.78	4.69	0	0	12.0	8.41	13.3710	9.213	6.78	4.1	5.33	13.9	15.3911	7.80	4.26	12.48	9.19	7.33	6.51	5.98	9.13	9.37	11.1	9.13	7.65	3.19	8.48						
<i>CC</i>	9.04	8.24	11.790	0	7.54	9.36	9.75	8.56	5.33	3.64	0	0	10.597	3.7	12.9610	1	12.267	3.7	1.8	12.4913	9.810	3.74	2.6	0	11.078	2	6.33	5.52	4.98	7.72	8.37	10.1	7.73	6.25	1.78	7.48					
Δ_F	7.26	11.1514	5.70	0	12.513	8	5.79	11.2413	5.510	7.90	0	0	4.52	14.2915	7.411	6.318	9	14.5112	8.71	7.77	3.26	1.74	12.4811	0.70	10.889	6.5	9.03	7.66	3.38	11.0512	7.85	6.7	4.85	9.29	10.8						
Δ_{DW}	9.23	7.14	11.5	0	10.359	4.2	10.191	7.5	10.267	91	0	0	10.9311	6.412	6.69	0.1	12.0711	6.39	9.9	12.2913	7.910	1.89	19	8.2	10.880	5.23	5.01	3.21	7.5	2.25	2.94	7.5	6.02	6.42	6.38						
<i>RC</i>	4.42	1.91	6.68	0	6.81	7.56	5.89	5.59	8.4	6.05	0	0	8.07	8.71	7.85	3.77	10.849	7.7	8.13	11.0712	5.68	9.6	7.33	6.33	9.65	5.23	3.81	2.02	6.3	5.41	7.13	6.31	4.83	4.55	1.15						
<i>PC</i>	7.39	5.72	10.7	0	8.39	7.58	8.35	5.37	7.58	5.23	0	0	9.09	8.96	11.247	5.8	10.238	9.5	7.31	10.4511	1.94	8.34	6.51	5.52	9.03	5.01	3.81	1.79	5.66	5.18	6.91	5.66	4.18	3.74	4.96						
Δ_{SWC}	6.02	3.93	8.28	0	7.14	6.2	6.98	3.58	7.05	4.7	0	0	7.72	8.43	9.45	5.79	8.85	8.42	6.78	9.08	10.576	9.6	4.98	7.66	3.21	2.02	1.79	4.28	3.39	5.12	4.29	2.81	3.2	3.17							
<i>VC</i>	6.73	8.21	12.140	0	9.74	3.58	5.57	7.86	10.2	7.44	0	0	4.41	11.1613	3	9.51	4.57	11.69	5.2	4.79	6.29	2.68	9.13	7.72	3.38	7.5	6.3	5.66	4.28	7.07	9.4	3.36	1.47	5.94	7.45						
Δ_C	9.41	7.32	11.670	0	10.539	5.9	10.374	0	10.448	0.9	0	0	11.1111	8.212	8.49	18	12.2411	10.1712	4.713	9.610	3.69	3.7	8.37	11.052	2.25	5.4	5.18	3.39	7.67	0	1.73	7.68	6.2	6.59	6.56						
<i>t_P</i>	11.149	0.4	13.4	0																																					

Table C.12: Average MD Score Shortest Path Lengths of Pareto MD Scores of Bulk Carrier Design Problem

	<i>L</i>	<i>T</i>	<i>D</i>	<i>C_B</i>	<i>B</i>	<i>V</i>	<i>F_n</i>	Δ	Δ_S	Δ_O	<i>a</i>	<i>b</i>	<i>t_S</i>	<i>BM</i>	<i>KG</i>	<i>KB</i>	<i>P</i>	<i>GM</i>	<i>SC</i>	Δ_M	Δ_{DC}	<i>FC</i>	Δ_{LS}	<i>CC</i>	Δ_F	Δ_{DW}	<i>RC</i>	<i>PC</i>	Δ_{SWC}	<i>VC</i>	Δ_C	<i>t_P</i>	<i>RTPA</i>	$\Delta_{C,ann}$	<i>AC</i>	<i>TC</i>			
<i>L</i>	0	5.42	8.04	0	5.44	0	7.35	8.91	10.897	31	0	0	0	7.53	9.5	6.51	12.568	62	9.26	14.021	4.319	38	11.587	86	10.798	91	2.29	5.82	3.76	7.35	9.76	11.125	85	6.09	5.87	3.47			
<i>T</i>	5.42	0	3.72	0	6.3	0	8.2	9.76	11.387	79	0	0	0	8.39	4.81	1.82	13.4	9.47	10.111	4.871	5.161	231	212	0.78	71	11.649	76	3.14	6.67	4.61	8.19	10.611	97	6.7	6.94	6.72	4.32		
<i>D</i>	8.04	3.72	0	0	8.91	0	10.82	12.38	13.99	10.410	0	0	0	11.011	8.3	2.63	16.02	12.089	12.731	7.712	8.414	69	11.331	251	12.385	76	9.29	7.22	10.813	23	14.599	32	9.56	9.34	6.94	0	0	0	
<i>C_B</i>	0	0	0	0	0	0	0	0	0	1.62	2.24	0	0	2.09	10.387	38	12.983	18	9.32	14.451	4.739	8	10.977	93	11.219	37	3.16	6.69	4.22	7.77	10.231	158	5.91	6.16	5.61	4.34	0	0	0
<i>B</i>	5.44	6.3	8.91	0	0	0	7.42	9.37	10.276	33	0	0	1.74	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
<i>V</i>	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
<i>F_n</i>	7.35	8.2	10.820	7.42	0	0	8.42	10	6.78	0	0	0	0	9.51	12.289	29	12.031	59	8.37	13.5	13.788	85	10.7	6.98	10.268	42	5.47	6.07	3.6	6.82	9.27	10.631	5	5.2	5.34	6.66	0	0	0
Δ	8.91	9.76	12.380	9.37	0	0	8.42	10	105	8.33	0	0	0	11.471	8.410	85	12.12	12.556	6.71	13.591	3.878	94	10.756	6.37	10.351	71	7.03	7.62	5.16	6.91	3.58	4.94	6.92	5.95	6.9	8.21	0	0	0
Δ_S	10.891	13.813	13.990	10.270	0	0	10	105	0	7.48	0	0	0	12.371	5.461	46	12.881	3.45	5.05	1.73	5.05	1.111	1.105	9.01	9.25	7.13	7.67	10.912	2.68	5.6	6.4	10.19	0	0	0	0	0	0	
Δ_O	7.31	7.79	10.410	6.33	0	0	6.78	8.33	7.48	0	0	0	0	8.42	11.878	88	10.869	5.1	7.92	12.331	2.617	6.8	8.18	6.53	9.09	8.33	5.78	4.96	3.91	5.65	9.18	10.545	27	4.76	1.79	6.97	0	0	0
<i>a</i>	0	0	0	1.62	0	0	0	0	0	0	3.86	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
<i>b</i>	0	0	0	2.24	0	0	0	0	0	0	0	3.86	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
<i>t_S</i>	0	0	0	0	0	1.74	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
<i>BM</i>	7.53	8.39	11.010	2.09	0	0	9.51	11.471	2.378	42	0	0	0	12.470	3.73	17.481	3.551	4.191	8.951	9.241	3.116	13.061	102	13.311	11.475	25	8.78	6.31	9.86	12.321	3.688	0.1	8.25	7.7	6.43	0	0	0	0
<i>KB</i>	9.5	4.81	1.83	0	10.380	0	12.281	3.841	5.461	1.870	0	0	0	9.48	3.73	0	14.491	0.561	11.2	15.961	6.241	3.113	1.69	8	12.721	0.894	23	7.76	5.69	9.28	11.7	13.067	7.9	8.03	7.81	5.41	0	0	0
<i>P</i>	12.561	13.4	16.020	12.980	0	0	12.031	12.12	8.810	8.60	0	0	0	15.071	4.814	490	16.161	0.553	5.8	1.75	3.53	13.589	5.1	1.77	12.121	0.041	1.639	16	5.21	12.97	14.331	0.538	0.4	9.07	12.22	0	0	0	0
<i>GM</i>	8.62	9.47	12.090	3.18	0	0	10.591	2.551	3.459	5.1	0	0	0	1.8	13.551	0.561	1.60	12.5	17.621	9.112	9.814	14.111	14.391	2.556	33	9.86	7.39	10.941	3.4	14.769	0.9	9.33	8.79	7.51	0	0	0	0	0
<i>SC</i>	9.26	10.112	7.30	9.32	0	0	8.37	6.71	5.05	7.92	0	0	0	11.421	14.191	1.2	10.551	12.5	0	12.021	2.317	3.8	5.74	1.75	8.79	6.71	7.38	7.97	5.5	5.34	7.57	8.93	6.87	3.17	6.13	8.56	0	0	0
Δ_M	14.021	4.871	4.90	14.450	0	0	13.5	13.591	4.351	2.330	0	0	0	16.541	8.951	9.63	5.8	17.621	2.020	3.92	4.99	15.041	0.983	23	13.591	12.5	13.1	10.636	6.8	14.441	5.8	11.999	5.1	10.531	3.69	0	0	0	0
Δ_{DC}	14.315	1.617	7.70	14.750	0	0	13.781	3.871	4.631	2.610	0	0	0	16.831	2.416	241.75	17.911	2.313	9.2	5.28	15.331	2.63	5.2	13.871	2.791	3.810	9.169	14.731	16.081	2.289	8.8	10.821	3.97	0	0	0	0	0	
<i>FC</i>	9.38	10.231	2.840	9.8	0	0	8.85	8.94	9.7	7.68	0	0	0	11.9	14.311	3.13	3.53	12.987	3.8	4.99	5.28	0	10.4	6.33	1.76	8.94	7.86	8.45	5.98	2.04	9.8	11.157	3.5	4.87	5.89	9.04	0	0	0
Δ_{LS}	11.581	2.071	4.690	10.970	0	0	10.7	10.751	7.3	8.18	0	0	0	13.061	15.131	16.13	5.81	14.574	1.45	7.4	15.041	5.331	10.4	5.74	11.811	0.759	7	9.95	7.83	8.36	11.6	12.969	19	6.3	7.1	10.89	0	0	0
<i>CC</i>	7.86	8.71	11.330	7.93	0	0	6.98	6.37	5.05	6.53	0	0	0	10.2	12.799	9.51	11.1	1.75	10.981	2.66	3.3	5.74	0	7.74	6.37	5.98	6.58	4.11	4.3	7.22	8.58	5.47	1.77	4.74	7.17	0	0	0	
Δ_F	10.791	1.641	4.250	11.210	0	0	10.261	3.511	1.19	0.9	0	0	0	13.311	5.721	7.21	7.7	14.398	7.9	3.23	3.52	1.76	11.81	7.4	0	10.359	27	9.86	7.39	3.45	11.212	5.68	7.6	6.28	7.3	10.45	0	0	0
Δ_{DW}	8.91	9.76	12.380	9.37	0	0	8.42	1.71	105	8.33	0	0	0	11.471	3.841	0.851	2.121	5.56	7.1	13.591	3.878	94	10.756	6.37	10.350	7.03	7.63	5.16	6.91	1.87	3.23	6.92	5.95	6.9	8.22	0	0	0	0
<i>RC</i>	2.29	3.14	5.76	0	3.16	0	5.47	7.03	9.01	5.78	0	0	0	5.25	7.22	4.23	11.046	3.3	7.38	12.5	12.797	8.6	9.7	5.98	9.27	7.03	0	3.94	1.88	5.82	7.88	9.24	3.97	4.21	4.35	1.18	0	0	0
<i>PC</i>	5.82	6.67	9.29	0	6.69	0	6.07	7.62	9.25	4.96	0	0	0	8.78	10.757	7.6	11.639	8.6	7.97	13.1	13.388	4.5	9.95	6.58	9.86	7.63	3.94	0	2.47	6.42	8.48	9.84	4.56	4.81	4.24	5.12	0	0	0
Δ_{SWC}	3.76	4.61	7.22	0	4.22	0	3.6	5.16	7.13	3.91	0	0	0	6.31	8.69	5.69	9.16	7.39	5.5	10.631	0.915	9.8	7.83	4.11	7.39	5.16	1.88	2.47	0	3.95	6.01	7.37	2.09	2.34	2.47	3.06	0	0	0
<i>VC</i>	7.35	8.19	10.810	7.77	0	0	6.82	6.91	7.67	5.65	0	0	0	9.86	12.279	28	5.21	10.945	3.4	6.68	6.96	2.04	8.36	4.3	3.45	6.91	5.82	6.42	3.95	0	7.76	9.12	5.31	2.83	3.86	7.01	0	0	0
Δ_C	9.76	10.611	3.230	10.230	0	0	9.27	3.58	10.919	18	0	0	0	13.681	0.513	0.614	3.314	7.68	9.3	15.8	16.081	1.512	9.68	5.8	12.563	23	9.24	9.84	7.37	9.12	1.7	7.77	6.8	7.75	9.07	0	0	0	
<i>t_P</i>	11.121	1.971	4.590	11.580	0	0	10.634	4.94	12.261	0.540	0	0	0	8.01	10.787	7.9	10.539	0.67	11.991	2.287	3.5	9.19	5.47	8.76	6.92	3.97	4.56	2.09	5.31	7.77	9.13	0	3.7	3.84	5.15	0	0	0	
<i>RTPA</i>	5.85	6.7	9.32	0	5.91	0	1.5	6.92	8.5	5.27	0	0	0	8.25	11.028	0.3	9.04	9.33	3.17	9.51	9.9	4.87	6.3	1.77	6.28	5.95	4.21	4.81	2.34	2.83	6.8	8.							

APPENDIX D

Bulk Carrier Design Model Chunked with Threshold of 0.99 Adjacency Matrices and Shortest Path Lengths

Table D.7: Average K2 Score Shortest Path Lengths of Full K2 Scores of Bulk Carrier Design Problem

<i>L</i>	0	1.7	1.0	3.1	2.1	2.2	1.0	1.7	3.5	2.2	1.3	2.0	2.0	2.6	2.0	2.0	2.0	3.1	2.6	2.4	2.4	3.1	2.6	2.4	2.4	2.0	2.7
<i>T</i>	1.7	1.0	2.8	2.8	2.7	1.2	1.0	0	4.0	1.2	2.0	3.8	2.4	3.8	2.4	1.3	1.3	1.3	3.1	2.6	2.4	2.4	2.4	2.4	2.4	2.0	2.7
<i>D</i>	1.0	1.0	3.7	3.7	3.2	2.2	0	1.0	4.5	2.2	2.3	4.8	2.4	4.8	3.4	2.4	2.4	2.4	3.6	2.4	3.4	3.4	3.4	3.4	2.4	2.1	
<i>C_B</i>	3.1	2.8	3.7	0	2.9	3.0	3.7	2.7	2.8	1.7	0	3.0	3.4	3.7	2.8	2.5	1.3	2.5	1.3	2.8	2.4	1.3	2.7	1.7	1.7	3.8	
<i>B</i>	1.3	1.5	2.2	2.9	0	3.1	2.3	2.9	2.2	1.9	2.9	3.1	1.0	2.2	1.5	4.3	1.0	2.0	4.0	4.3	4.0	2.2	2.0	2.9	1.9	2.6	
<i>V</i>	2.1	2.7	3.2	3.0	3.1	0	1.1	2.1	2.3	1.4	3.0	3.0	0	3.5	3.2	2.7	1.4	2.4	2.3	2.5	2.4	2.1	1.4	1.4	1.4	3.7	
<i>F_n</i>	1.1	2.7	2.1	3.7	2.3	1.1	0	3.2	2.1	2.5	3.3	2.1	3.3	2.1	2.7	3.5	3.5	2.1	2.3	3.5	3.2	2.5	2.5	2.1	1.1	3.7	
<i>Δ</i>	2.6	2.4	3.4	2.7	2.9	2.1	3.2	0	2.0	1.1	2.7	2.7	3.2	3.4	3.4	3.2	1.4	3.5	3.4	3.5	2.0	1.4	3.5	0	1.5	3.5	
<i>Δ_S</i>	1.0	2.4	2.1	2.8	2.3	3.2	2.1	2.4	3.6	3.2	2.0	3.6	3.6	0	2.0	3.6	2.0	1.1	1.1	2.3	2.0	2.0	2.2	1.1	1.1	3.4	
<i>Δ_O</i>	2.0	1.3	2.4	1.7	1.9	1.4	2.5	1.1	1.1	0	1.7	1.7	1.4	2.2	2.4	1.3	2.6	2.4	1.1	1.1	2.4	1.1	1.1	1.0	0	2.4	
<i>Δ_S</i>	3.1	2.8	3.7	0	2.9	3.0	3.7	2.7	2.8	1.7	0	3.0	3.4	3.7	2.8	2.5	3.4	2.4	1.3	2.5	1.3	2.8	2.4	1.3	2.7	3.8	
<i>a</i>	3.1	2.8	3.7	0	2.9	3.0	3.7	2.7	2.8	1.7	0	3.0	3.4	3.7	2.8	2.5	3.4	2.4	1.3	2.5	1.3	2.8	2.4	1.3	2.7	3.8	
<i>b</i>	3.1	2.8	3.7	0	2.9	3.0	3.7	2.7	2.8	1.7	0	3.0	3.4	3.7	2.8	2.5	3.4	2.4	1.3	2.5	1.3	2.8	2.4	1.3	2.7	3.8	
<i>t_S</i>	2.1	2.7	3.2	3.0	3.1	0	1.1	2.1	2.3	1.4	3.0	3.0	0	3.5	3.2	2.7	1.4	2.4	2.3	2.5	2.4	2.1	1.4	1.4	1.4	3.7	
<i>BM</i>	2.2	1.2	2.2	3.4	1.0	3.5	3.3	3.2	3.2	2.2	3.4	3.4	3.5	0	2.2	1.2	4.8	0	2.9	4.4	3.2	2.9	4.4	3.2	2.2	2.2	
<i>KB</i>	1.7	0	1.0	2.8	1.5	2.7	2.7	2.4	3.7	3.7	3.2	3.4	3.4	3.5	0	1.0	4.5	2.2	2.3	4.8	4.5	4.8	2.1	2.3	4.8	2.1	
<i>KG</i>	1.0	1.0	0	3.7	2.2	3.2	2.1	3.4	2.1	2.4	3.7	3.7	3.7	3.2	2.2	0	1.0	4.5	2.2	2.3	4.8	4.5	4.8	2.1	2.3	4.8	
<i>P</i>	3.5	4.0	4.5	2.5	4.3	1.4	2.5	3.4	3.6	2.6	2.5	2.5	1.4	4.8	4.5	4.0	0	4.0	1.2	2.0	3.8	4.0	3.8	2.4	2.0	3.8	
<i>GM</i>	2.2	1.2	2.2	3.4	1.0	3.5	3.3	3.2	3.2	2.2	3.4	3.4	3.5	0	2.2	1.2	4.8	0	2.9	4.4	4.8	4.4	3.2	2.9	4.4	3.2	
<i>SC</i>	1.3	2.0	2.3	2.4	2.0	2.5	2.3	1.4	2.0	1.1	2.4	2.4	2.5	2.9	2.3	2.0	3.7	2.9	0	3.2	3.7	3.2	2.0	0	3.2	1.1	
<i>ΔM</i>	4.2	3.8	4.8	1.3	4.0	2.4	3.5	3.5	3.6	2.4	1.3	1.3	2.4	4.4	4.8	3.8	1.2	4.4	3.2	0	3.5	2.4	2.4	2.4	2.4	4.8	
<i>ΔDC</i>	3.5	4.0	4.5	2.5	4.3	1.4	2.5	3.4	3.6	2.6	2.5	2.5	1.4	4.8	4.5	4.0	0	4.8	3.7	1.2	0	1.2	3.6	3.7	1.2	3.4	
<i>FC</i>	4.2	3.8	4.8	1.3	4.0	2.4	3.5	3.5	3.6	2.4	1.3	1.3	2.4	4.4	4.8	3.8	1.2	4.4	3.2	0	1.2	3.6	3.7	1.2	3.4	2.4	
<i>ΔLS</i>	1.0	2.4	2.1	2.8	2.2	2.3	2.1	2.0	0	1.1	2.8	2.8	2.3	3.2	3.2	2.1	2.4	3.6	3.2	2.0	3.6	3.6	0	2.0	3.6	2.0	
<i>CC</i>	1.3	2.0	2.3	2.4	2.0	2.5	2.3	1.4	2.0	1.1	2.4	2.4	2.5	2.9	2.3	2.0	3.7	2.9	0	3.2	3.7	3.2	2.0	0	3.2	1.1	
<i>ΔF</i>	4.2	3.8	4.8	1.3	4.0	2.4	3.5	3.5	3.6	2.4	1.3	1.3	2.4	4.4	4.8	3.8	1.2	4.4	3.2	0	1.2	0	3.6	3.2	0	3.5	
<i>ΔDW</i>	2.6	2.4	3.4	2.7	2.9	2.1	3.2	0	2.0	1.1	2.7	2.7	2.1	3.2	3.4	2.4	3.4	3.2	1.4	3.5	3.4	3.5	2.0	1.4	3.5	0	
<i>RC</i>	2.0	1.3	2.4	1.7	1.9	1.4	2.5	1.1	1.1	0	1.7	1.7	1.4	2.2	2.4	1.3	2.6	2.2	1.1	2.4	2.6	2.4	1.1	1.1	1.1	1.1	
<i>PC</i>	2.0	1.3	2.4	1.7	1.9	1.4	2.5	1.1	1.1	0	1.7	1.7	1.4	2.2	2.4	1.3	2.6	2.2	1.1	2.4	2.6	2.4	1.1	1.1	1.1	1.1	
<i>ΔSWC</i>	2.0	1.3	2.4	1.7	1.9	1.4	2.5	1.1	1.1	0	1.7	1.7	1.4	2.2	2.4	1.3	2.6	2.2	1.1	2.4	2.6	2.4	1.1	1.1	1.1	1.1	
<i>VC</i>	3.1	2.6	3.6	2.6	3.1	2.6	3.7	2.3	2.3	1.2	2.6	2.6	2.6	3.4	3.6	2.6	3.4	2.0	1.5	2.3	1.2	1.2	1.2	1.2	1.2	1.2	
<i>ΔC</i>	2.6	2.4	3.4	2.7	2.9	2.1	3.2	0	2.0	1.1	2.7	2.7	2.1	3.2	3.4	2.4	3.4	3.2	1.4	3.5	3.4	3.5	2.0	1.4	3.5	0	
<i>t_P</i>	2.6	2.4	3.4	2.7	2.9	2.1	3.2	0	2.0	1.1	2.7	2.7	2.1	3.2	3.4	2.4	3.4	3.2	1.4	3.5	3.4	3.5	2.0	1.4	3.5	0	
<i>RTPA</i>	3.1	2.4	3.4	2.8	2.9	1.1	2.2	1.5	2.2	1.0	2.8	2.8	1.1	3.2	3.4	2.4	2.5	3.2	2.2	3.4	2.5	3.4	2.2	2.2	3.4	2.4	
<i>ΔC_{ann}</i>	2.0	1.3	2.4	1.7	1.9	1.4	2.5	1.1	1.1	0	1.7	1.7	1.4	2.2	2.4	1.3	2.6	2.2	1.1	2.4	2.6	2.4	1.1	1.1	1.1	1.1	
<i>AC</i>	2.0	1.3	2.4	1.7	1.9	1.4	2.5	1.1	1.1	0	1.7	1.7	1.4	2.2	2.4	1.3	2.6	2.2	1.1	2.4	2.6	2.4	1.1	1.1	1.1	1.1	
<i>TC</i>	2.7	1.0	2.1	3.8	2.6	3.7	3.7	3.5	3.4	2.4	3.8	3.8	3.7	2.2	2.1	1.0	5.0	2.2	3.1	4.8	5.0	4.8	3.4	3.1	4.8	3.5	

Table D.8: Average K2 Score Shortest Path Lengths of Feasible K2 Scores of Bulk Carrier Design Problem

	<i>L</i>	<i>T</i>	<i>D</i>	<i>C_B</i>	<i>B</i>	<i>V</i>	<i>F_n</i>	Δ	Δ_S	Δ_O	<i>a</i>	<i>b</i>	<i>t_S</i>	<i>BM</i>	<i>KG</i>	<i>KB</i>	<i>P</i>	<i>GM</i>	<i>SC</i>	Δ_M	Δ_{DC}	<i>FC</i>	Δ_{LS}	<i>CC</i>	Δ_F	Δ_{DW}	<i>RC</i>	<i>PC</i>	Δ_{SWC}	<i>VC</i>	Δ_C	<i>t_P</i>	<i>RTPA</i>	$\Delta_{C,ann}$	<i>AC</i>	<i>TC</i>		
<i>L</i>	0	2.4	4.9	0	4.6	2.1	1.1	4.2	3.5	3.5	0	0	2.1	6.2	4.9	2.4	4.7	6.2	4.8	4.4	4.7	4.4	3.5	4.8	4.4	4.2	2.9	2.9	4.0	4.2	4.2	3.2	4.0	3.5	3.8			
<i>T</i>	2.4	0	2.5	0	3.1	3.8	3.3	2.7	2.8	2.8	0	0	3.8	4.7	2.5	0	5.1	4.7	4.1	4.9	5.1	4.9	2.8	4.1	4.9	2.7	1.4	1.4	3.1	2.7	2.7	3.9	3.1	2.8	2.7			
<i>D</i>	4.9	2.5	0	0	5.6	6.3	5.8	5.2	5.3	5.3	0	0	6.3	7.3	0	2.5	7.6	7.3	6.6	7.4	7.6	7.4	5.3	6.6	7.4	5.2	4.0	4.0	4.0	5.6	5.2	5.2	6.4	5.6	5.3	5.2		
<i>C_B</i>	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
<i>B</i>	4.6	3.1	5.6	0	0	4.9	5.5	2.9	3.0	3.0	0	0	4.9	1.6	5.6	3.1	5.4	1.6	4.3	5.2	5.4	5.2	3.0	4.3	5.2	2.9	1.7	1.7	1.7	3.3	2.9	2.9	4.2	3.3	3.0	4.4		
<i>V</i>	2.1	3.8	6.3	0	4.9	0	1.1	4.5	4.2	4.2	0	0	0	6.6	6.3	3.8	2.5	6.6	5.5	2.3	2.5	2.3	4.2	5.5	2.3	4.5	3.3	3.3	3.3	2.0	4.5	4.5	1.1	2.0	4.2	4.0		
<i>F_n</i>	1.1	3.3	5.8	0	5.5	1.1	0	5.0	4.4	4.4	0	0	1.1	7.1	5.8	3.3	3.6	7.1	5.7	3.4	3.6	3.4	4.4	5.7	3.4	5.0	3.8	3.8	3.1	5.0	5.0	2.2	3.1	4.4	4.0			
Δ	4.2	3.7	5.2	0	2.9	4.5	5.0	0	2.5	2.5	0	0	4.5	4.5	5.2	2.7	5.0	4.7	3.9	4.7	5.0	4.7	2.5	3.9	4.7	0	1.2	1.2	1.2	2.8	0	0	3.8	2.8	2.5	3.9		
Δ_S	3.5	2.8	5.3	0	3.0	4.2	4.4	2.5	0	0	0	0	4.2	4.6	5.3	2.8	4.8	4.6	1.3	4.5	4.8	4.6	0	1.3	4.6	2.5	1.3	1.3	2.4	2.5	2.5	3.4	2.4	0	3.8	0	3.8	
Δ_O	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
<i>a</i>	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
<i>b</i>	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
<i>t_S</i>	2.1	3.8	6.3	0	4.9	0	1.1	4.5	4.2	4.2	0	0	0	6.6	6.3	3.8	2.5	6.6	5.5	2.3	2.5	2.3	4.2	5.5	2.3	4.5	3.3	3.3	2.0	4.5	4.5	1.1	2.0	4.2	4.0	4.0	4.0	
<i>BM</i>	6.2	4.7	7.3	0	1.6	6.6	7.1	4.5	4.6	4.6	0	0	6.6	0	7.3	4.7	7.0	6.0	6.8	7.0	6.8	7.0	6.8	4.6	6.0	6.8	4.5	3.3	3.3	3.3	4.9	4.5	5.8	4.9	4.6	6.0	6.0	6.0
<i>KG</i>	4.9	2.5	0	0	5.6	6.3	5.8	5.2	5.3	5.3	0	0	6.3	7.3	0	2.5	7.6	7.3	6.6	7.4	7.6	7.4	5.3	6.6	7.4	5.2	4.0	4.0	4.0	5.6	5.2	6.4	5.6	5.3	5.2	5.2	5.2	
<i>KB</i>	2.4	0	2.5	0	3.1	3.8	3.3	2.7	2.8	2.8	0	0	3.8	4.7	2.5	0	5.1	4.7	4.1	4.9	5.1	4.9	2.8	4.1	4.9	2.7	1.4	1.4	3.1	2.7	2.7	3.9	3.1	2.8	2.7	2.7	2.7	
<i>GM</i>	6.2	4.7	7.3	0	1.6	6.6	7.1	4.5	4.6	4.6	0	0	6.6	0	7.3	4.7	7.0	6.0	6.8	7.0	6.8	7.0	6.8	4.6	6.0	6.8	4.5	3.3	3.3	3.3	4.9	4.5	5.8	4.9	4.6	6.0	6.0	6.0
<i>P</i>	4.7	5.1	7.6	0	5.4	2.5	3.6	5.0	4.8	4.8	0	0	2.5	7.0	7.6	5.1	0	7.0	6.1	1.3	0	2.7	4.8	6.1	2.7	5.0	3.7	3.7	3.7	2.9	5.0	3.6	2.9	4.8	5.3	5.3	5.3	
<i>SC</i>	4.8	4.1	6.6	0	4.3	5.5	5.7	3.9	1.3	1.3	0	0	5.5	6.0	6.6	4.1	6.1	6.0	6.0	5.9	6.1	5.9	1.3	0	5.9	3.9	2.7	2.7	2.7	3.7	3.9	3.9	4.7	3.7	1.3	5.1	5.1	5.1
Δ_M	4.4	4.9	7.4	0	5.2	2.3	3.4	4.7	4.5	4.5	0	0	2.3	6.8	7.4	4.9	1.3	6.8	5.9	0	1.3	1.3	4.5	5.9	1.3	4.7	3.5	3.5	3.5	2.6	4.7	3.3	2.6	4.5	5.0	5.0	5.0	
Δ_{DC}	4.7	5.1	7.6	0	5.4	2.5	3.6	5.0	4.8	4.8	0	0	2.5	7.0	7.6	5.1	0	7.0	6.1	1.3	0	2.7	4.8	6.1	2.7	5.0	3.7	3.7	3.7	2.9	5.0	3.6	2.9	4.8	5.3	5.3	5.3	
<i>FC</i>	4.4	4.9	7.4	0	5.2	2.3	3.4	4.4	2.5	0	0	0	2.3	6.8	7.4	4.9	1.3	6.8	5.9	1.3	2.7	0	4.6	5.9	1.3	4.7	3.5	3.5	3.5	2.6	4.7	3.3	2.6	4.6	5.0	5.0	5.0	
Δ_{LS}	3.5	2.8	5.3	0	3.0	4.2	4.4	2.5	0	0	0	0	4.2	4.6	5.3	2.8	4.8	4.6	1.3	4.5	4.8	4.6	0	1.3	4.6	2.5	1.3	1.3	2.4	2.5	2.5	3.4	2.4	0	3.8	0	3.8	
<i>CC</i>	4.8	4.1	6.6	0	4.3	5.5	5.7	3.9	1.3	1.3	0	0	5.5	6.0	6.6	4.1	6.1	6.0	6.0	5.9	6.1	5.9	1.3	0	5.9	3.9	2.7	2.7	3.7	3.9	3.9	4.7	3.7	1.3	5.1	5.1	5.1	
Δ_F	4.4	4.9	7.4	0	5.2	2.3	3.4	4.7	4.5	4.5	0	0	2.3	6.8	7.4	4.9	1.3	6.8	5.9	1.3	2.7	0	4.6	5.9	1.3	4.7	3.5	3.5	3.5	2.6	4.7	3.3	2.6	4.6	5.0	5.0	5.0	
Δ_{DW}	4.2	2.7	5.2	0	2.9	4.5	5.0	0	2.5	2.5	0	0	4.5	4.5	5.2	2.7	5.0	4.5	3.9	4.7	5.0	4.7	2.5	3.9	4.7	0	1.2	1.2	1.2	2.8	0	0	3.8	2.8	2.5	3.9		
<i>RC</i>	2.9	1.4	4.0	0	1.7	3.3	3.8	1.2	1.3	1.3	0	0	3.3	3.3	4.0	1.4	3.7	3.3	2.7	3.5	3.7	3.5	1.3	2.7	3.5	1.2	0	0	1.6	1.2	1.2	2.5	1.6	1.3	2.7	2.7	2.7	
<i>PC</i>	2.9	1.4	4.0	0	1.7	3.3	3.8	1.2	1.3	1.3	0	0	3.3	3.3	4.0	1.4	3.7	3.3	2.7	3.5	3.7	3.5	1.3	2.7	3.5	1.2	0	0	1.6	1.2	1.2	2.5	1.6	1.3	2.7	2.7	2.7	
Δ_{SWC}	2.9	1.4	4.0	0	1.7	3.3	3.8	1.2	1.3	1.3	0	0	3.3	3.3	4.0	1.4	3.7	3.3	2.7	3.5	3.7	3.5	1.3	2.7	3.5	1.2	0	0	1.6	1.2	1.2	2.5	1.6	1.3	2.7	2.7	2.7	
<i>VC</i>	4.0	3.1	5.6	0	3.3	2.0	3.1	2.8	2.4	2.4	0	0	2.0	4.9	5.6	3.1	2.9	4.9	3.7	2.6	2.9	2.6	2.4	3.7	2.6	2.8	1.6	1.6	1.6	0	0	2.8	2.8	1.2	0	2.4	2.7	
Δ_C	4.2	2.7	5.2	0	2.9	4.5	5.0	0	2.5	2.5	0	0	4.5	4.5	5.2	2.7	5.0	4.5	3.9	4.7	5.0	4.7	2.5	3.9	4.7	0	1.2	1.2	1.2	2.8	0	0	3.8	2.8	2.5	3.9		
<i>t_P</i>	4.2	2.7	5.2	0	2.9	4.5	5.0	0	2.5	2.5	0	0	4.5	4.5	5.2	2.7	5.0	4.5	3.9	4.7	5.0	4.7	2.5	3.9	4.7	0	1.2	1.2	1.2	2.8	0	0	3.8	2.8	2.5	3.9		
<i>RTPA</i>	3.2	3.9	6.4	0	4.2	1.1	2.2	3.8	3.4	3.4	0	0	1.1	5.8	6.4	3.9	3.6	5.8	4.7	3.3	3.6	3.3	3.4	4.7	3.3	3.8	2.5	2.5	2.5	1.2	3.8	3.8	0	1.2	3.4	3.8		
$\Delta_{C,ann}$	3.5	2.8	5.3	0	3.0	4.2	4.4	2.5	0	0	0	0	4.2	4.6	5.3	2.8	4.8	4.6	1.3	4.5	4.8	4.6	0	1.3	4.6	2.5	1.3	1.3	2.4	2.5	2.5	3.4	2.4	0	3.8	0	3.8	
<i>AC</i>	3.0	3.1	5.6	0	3.3	2.0	3.1	2.8	2.4	2.4	0	0	2.0	4.9	5.6	3.1	2.9	4.9	3.7	2.6	2.9	2.6	2.4	3.7	2.6	2.8	1.6	1.6	1.6	0	2.8	2.8	1.2	0	2.4	2.7	2.7	
<i>TC</i>	3.8	2.7	5.2	0	4.4	4.0	4.0	3.9	3.8	3.8	0	0	4.0	6.0	5.2	2.7	5.3	6.0	5.1	5.0	5.3	5.0	3.8	5.1	5.0	3.9	2.7	2.7	2.7	2.7	2.7	3.9	3.9	3.8	2.7	3.8	0	

Table D.10: Average MD Score Shortest Path Lengths of Full MD Scores of Bulk Carrier Design Problem

<i>L</i>	0	3.5	3.2	6.3	2.4	4.9	2.1	3.4	6.3	6.3	4.6	3.6	3.1	4.6	5.4	2.3	3.4	5.2	4.8	3.5	5.0	3.2	5.0	4.8	4.8	5.5	3.9	3.4	4.7									
<i>T</i>	3.5	0	2.6	6.1	2.5	5.3	4.4	4.9	6.1	6.1	5.0	1.3	2.6	0	5.8	5.8	5.1	4.4	5.0	5.0	5.2	2.2	4.9	3.7	4.9	5.2	5.2	3.7	4.9	3.4								
<i>D</i>	3.2	2.6	0	8.7	4.8	7.7	5.0	6.5	8.7	8.7	6.4	3.9	0	2.6	7.8	7.5	5.2	6.6	7.4	7.6	4.9	7.3	6.0	7.4	7.6	7.6	8.2	6.4	6.5	6.1								
<i>CB</i>	6.3	6.1	8.7	0	5.9	4.8	6.5	4.5	3.9	0	4.8	7.2	8.7	6.1	4.4	7.2	3.9	4.4	4.4	2.8	4.5	3.9	4.1	4.3	3.6	4.5	4.5	4.1	2.3	3.9	5.1							
<i>B</i>	2.4	2.5	4.8	5.9	0	4.3	4.0	4.6	3.7	3.4	5.9	5.9	4.3	1.5	4.8	2.5	5.5	3.7	5.5	3.6	3.6	4.4	3.0	4.5	3.2	5.0	4.4	5.4	3.6	3.4	4.2							
<i>V</i>	3.2	5.0	6.4	4.8	4.3	0	1.6	4.4	4.7	4.5	4.8	4.8	0	5.5	6.4	5.0	1.4	5.5	4.7	1.4	2.2	4.5	4.2	2.8	4.6	2.7	2.9	4.2	2.7	3.5	4.5	4.0						
<i>F_n</i>	1.6	5.1	4.8	6.5	4.0	1.6	0	5.7	3.6	4.7	6.5	6.5	1.6	5.1	4.8	5.1	3.0	5.1	4.6	3.0	3.0	3.0	3.0	4.6	5.0	4.0	4.6	5.5	4.3	5.1	4.7	5.5						
<i>Δ</i>	4.9	5.3	7.7	4.5	4.6	4.4	5.7	0	3.3	3.1	4.5	4.5	4.4	5.8	7.7	5.3	5.7	5.8	3.4	3.3	4.5	0.7	1.7	3.5	0.2	0.2	4.0	2.2	3.1	4.6	4.6	4.6						
<i>Δ_S</i>	2.1	4.4	5.0	5.5	3.7	4.7	3.6	3.3	0	2.3	5.5	5.5	4.7	4.9	5.0	4.4	5.6	4.9	2.2	5.6	5.6	4.7	0.2	2.2	3.5	3.4	3.4	4.9	3.1	2.3	4.5	4.5						
<i>Δ_O</i>	3.4	4.9	6.5	3.9	3.4	4.5	4.7	3.1	2.3	0	3.9	3.9	4.5	4.7	6.5	4.9	5.5	4.4	2.1	0.2	4.2	3.3	3.0	2.9	2.8	3.1	3.3	3.3	3.4	1.6	0	4.2						
<i>Δ_S</i>	6.3	6.1	8.7	0	5.9	4.8	6.5	4.5	3.9	0	4.8	7.2	8.7	6.1	4.4	7.2	3.9	4.4	4.4	2.7	5.4	4.1	2.8	4.5	4.1	3.6	4.5	4.5	4.1	2.3	3.9	5.1						
<i>a</i>	3.2	5.0	6.4	4.8	4.3	0	1.6	4.4	4.7	4.5	4.8	4.8	0	5.5	6.4	5.0	1.4	5.5	4.7	1.4	2.2	4.5	4.6	2.1	4.2	2.8	4.2	2.7	3.5	4.5	4.0	4.0						
<i>b</i>	3.6	1.3	3.9	7.2	1.5	5.5	5.1	5.8	4.9	4.7	7.2	7.2	5.5	0	3.9	1.3	6.6	0	4.9	6.6	6.6	6.4	4.7	4.9	6.2	5.6	3.4	5.6	6.7	4.9	4.7	4.6						
<i>BM</i>	3.2	2.6	0	8.7	4.8	6.4	4.8	7.7	5.0	6.5	8.7	8.7	6.4	3.9	0	2.6	7.8	7.5	5.2	6.6	7.4	7.6	4.9	7.3	6.0	7.4	7.6	8.2	6.4	6.5	6.1	6.1						
<i>KB</i>	3.5	0	2.6	6.1	2.5	5.0	5.1	5.3	4.4	4.9	6.1	6.1	5.0	1.3	2.6	0	5.8	1.3	4.9	5.8	5.8	5.8	5.8	5.8	5.8	5.8	5.2	5.2	5.5	3.7	4.9	3.4	3.4					
<i>P</i>	4.6	5.8	7.8	4.4	5.5	1.4	3.0	5.7	5.6	5.5	4.4	4.4	1.4	6.6	7.8	5.8	0	6.6	5.5	0	1.8	5.6	5.7	1.7	5.6	3.6	5.5	4.1	3.3	5.6	5.6	4.1	4.2	5.5	4.7			
<i>GM</i>	3.6	1.3	3.9	7.2	1.5	5.5	5.1	5.8	4.9	4.7	7.2	7.2	5.5	0	3.9	1.3	6.6	0	4.9	6.6	6.6	6.4	4.7	4.9	6.2	5.6	3.4	5.6	4.2	5.9	5.6	6.7	4.9	4.7	4.6			
<i>SC</i>	3.1	4.9	6.3	3.9	3.7	4.7	4.6	3.4	2.2	0.3	3.9	3.9	4.7	4.9	6.3	4.9	5.5	4.9	0	5.5	5.5	4.4	2.3	0.4	4.2	3.0	3.2	3.1	3.1	3.4	3.4	3.4	1.6	0.3	4.2			
<i>Δ_M</i>	4.6	5.8	7.8	4.4	5.5	1.4	3.0	5.7	5.6	5.5	4.4	4.4	1.4	6.6	7.8	5.8	0	6.6	5.5	0	1.8	5.6	5.7	1.7	5.6	3.6	5.5	4.1	3.3	5.6	5.6	4.1	4.2	5.5	4.7			
<i>Δ_{DC}</i>	4.6	5.8	7.8	4.4	5.5	1.4	3.0	5.7	5.6	5.5	4.4	4.4	1.4	6.6	7.8	5.8	0	6.6	5.5	0	1.8	5.6	5.7	1.7	5.6	3.6	5.5	4.1	3.3	5.6	5.6	4.1	4.2	5.5	4.7			
<i>FC</i>	5.4	5.1	7.5	2.7	5.5	2.2	3.9	4.7	4.7	4.4	2.7	2.7	2.2	6.4	7.5	5.1	1.8	6.4	4.4	1.8	1.8	0	4.7	4.6	0.2	4.6	3.1	4.3	3.4	1.6	4.6	4.6	4.1	2.8	4.4	4.3		
<i>Δ_{LS}</i>	2.3	4.4	5.2	5.4	3.6	4.5	3.6	3.1	0.2	2.1	5.4	5.4	4.5	4.7	5.2	4.4	5.6	5.6	4.7	2.3	5.6	5.6	4.7	0	2.0	4.5	3.3	3.3	3.1	2.0	3.5	3.3	3.3	4.9	3.1	2.1	4.5	
<i>Δ_F</i>	3.4	5.0	6.6	4.1	3.6	4.6	4.9	3.3	2.2	0.2	4.1	4.1	4.6	4.9	6.6	5.0	5.7	4.9	0.4	5.7	5.7	4.6	2.0	0	4.4	3.5	3.2	3.0	3.0	3.3	3.5	3.5	3.5	1.7	0.2	4.3		
<i>Δ_W</i>	5.2	5.0	7.4	2.8	5.4	2.1	3.7	4.5	4.6	4.2	2.8	2.8	2.1	6.2	7.4	5.0	1.7	6.2	4.2	1.7	1.7	0.2	4.5	4.4	0	4.5	2.9	4.1	3.2	1.7	4.5	4.5	4.0	2.6	4.2	4.1		
<i>RC</i>	4.8	5.2	7.6	4.5	4.4	4.2	5.5	0.2	3.4	3.3	4.5	4.5	4.5	4.2	5.6	7.6	5.2	5.6	3.4	5.6	5.6	4.6	3.3	3.5	4.5	0	3.3	0.9	1.6	3.4	0	0	3.9	2.1	3.3	4.4		
<i>PC</i>	5.0	4.9	7.3	4.1	4.5	4.6	5.9	0.7	3.2	2.9	4.1	4.1	4.6	5.6	7.3	4.9	5.5	5.6	3.2	5.5	5.5	4.3	3.1	3.0	4.1	0.9	2.9	0	2.0	2.8	0.9	0.9	3.6	1.8	2.9	4.1		
<i>Δ_{SWC}</i>	3.2	3.7	6.0	4.3	3.2	2.7	4.0	1.7	2.2	2.8	4.3	4.3	2.7	4.2	6.0	3.7	4.1	4.1	3.4	2.0	3.0	3.2	1.6	1.8	2.0	0	2.3	1.6	1.6	3.7	1.9	2.8	3.0	3.0	3.0	3.0		
<i>VC</i>	5.0	4.9	7.4	3.6	5.0	2.9	4.6	3.5	3.5	3.1	3.6	3.6	2.9	5.9	7.4	4.9	3.3	5.9	3.1	3.3	3.3	1.6	3.5	3.3	1.7	3.4	0	3.4	3.4	3.4	1.6	3.1	4.2	4.2	4.2	4.2		
<i>Δ_C</i>	4.8	5.2	7.6	4.5	4.4	4.2	5.5	0.2	3.4	3.3	4.5	4.5	4.5	4.2	5.6	7.6	5.2	5.6	3.4	5.6	5.6	4.6	3.3	3.5	4.5	0	3.3	0.9	1.6	3.4	0	0	3.9	2.1	3.3	4.4		
<i>Δ_C</i>	4.8	5.2	7.6	4.5	4.4	4.2	5.5	0.2	3.4	3.3	4.5	4.5	4.5	4.2	5.6	7.6	5.2	5.6	3.4	5.6	5.6	4.6	3.3	3.5	4.5	0	3.3	0.9	1.6	3.4	0	0	3.9	2.1	3.3	4.4		
<i>RTPA</i>	5.5	5.5	8.2	4.1	5.4	2.7	4.3	4.0	4.9	3.4	4.1	4.1	2.7	6.7	8.2	5.5	4.1	4.1	4.1	4.1	4.1	4.1	4.1	4.1	4.1	4.1	4.1	4.1	4.1	4.1	4.1	4.1	4.1	4.1	4.1	4.1	4.1	4.1
<i>Δ_{C,ann}</i>	3.9	3.7	6.4	3.6	3.5	5.1	2.2	3.1	1.6	2.3	3.1	3.3	3.5	4.9	6.4	4.2	4.2	4.2	4.2	4.2	4.2	4.2	4.2	4.2	4.2	4.2	4.2	4.2	4.2	4.2	4.2	4.2	4.2	4.2	4.2	4.2	4.2	4.2
<i>AC</i>	3.4	4.9	6.5	3.9	3.4	4.5	4.7	3.1	2.3	0	3.9	3.9	4.5	4.7	6.5	4.9	5.5	4.4	2.1	0.2	4.2	3.3	3.0	2.9	2.8	3.1	3.3	3.3	3.4	1.6	0	4.2	4.2	4.2	4.2	4.2	4.2	4.2
<i>TC</i>	4.7	3.4	6.1	5.1	4.2	4.0	5.5	4.6	4.5	4.2	5.1	5.1	4.0	4.6	6.1	3.4	4.7	4.6	4.2	4.7	4.7	4.7	4.7	4.7	4.7	4.7	4.7	4.7	4.7	4.7	4.7	4.7	4.7	4.7	4.7	4.7	4.7	4.7

BIBLIOGRAPHY

BIBLIOGRAPHY

- Akaike, H. (1974), A new look at the statistical model identification, *Institute of Electrical and Electronics Engineers. Transactions on Automatic Control, AC-19*, 716–723, doi:10.1109/tac.1974.1100705.
- A/S, H. E. (2018), Hugin Researcher.
- Cooper, G. F., and E. Herskovits (1992), A Bayesian method for the induction of probabilistic networks from data, *Machine Learning, 9*(4), 309–347, doi:10.1007/BF00994110.
- Devine, T. E. (2016), Using and Interpreting the Bayesian Optimization Algorithm to Improve Early Stage Design of Marine Structures., Ph.D. thesis, University of Michigan, Ann Arbor.
- Dijkstra, E. W. (1959), A note on two problems in connexion with graphs, *Numerische Mathematik, 1*(1), 269–271, doi:10.1007/BF01386390.
- Friebe, M., D. Skahen, and S. Aksu (2019), A framework to improve the naval survivability design process based on the vulnerability of a platform’s systems, *Ocean Engineering, 173*, 677–686, doi:10.1016/j.oceaneng.2018.12.074.
- Gillespie, J. W. (2012), A Network Science Approach to Understanding and Generating Ship Arrangements in Early-Stage Design, Thesis, University of Michigan.
- Goodrum (2020), Conceptually Robust Knowledge Generation in Early Stage Complex Design, Thesis.
- Heckerman, D. (1995), Learning Bayesian networks: The combination of knowledge and statistical data, p. 47.
- Jason E. Bartolomei (2007), Qualitative Knowledge Construction for Engineering Systems: Extending the Design Structure Matrix Methodology in Scope and Procedure, Ph.D. thesis, Massachusetts Institute of Technology.
- Kendall, M. G. (1948), *Rank correlation methods.*, C. Griffin, London.
- KJ MacCallum (1982), Understanding relationships in marine systems design.
- Kjærulff, U. B., and A. L. Madsen (2008), *Bayesian Networks and Influence Diagrams: A Guide to Construction and Analysis*, Information Science and Statistics, Springer-Verlag, New York.

- Maurer, M. S. (2007), Structural awareness in complex product design, Ph.D. thesis, Technischen Universität München, München, oCLC: 213389577.
- Nannapaneni, S., C. Liang, and S. Mahadevan (2017), Bayesian network approach to multidisciplinary, multi-objective design optimization under uncertainty.
- Newman, M. (2018), *Networks*, 2nd edition ed., Oxford University Press, Oxford.
- Newman, M. E. J. (2001), Scientific collaboration networks. II. Shortest paths, weighted networks, and centrality, *Physical Review E*, 64(1), 016,132, doi: 10.1103/PhysRevE.64.016132, publisher: American Physical Society.
- Nowacki, H. (2010), Five decades of Computer-Aided Ship Design, *Computer-Aided Design*, 42(11), 956–969, doi:10.1016/j.cad.2009.07.006.
- Osyczka, A., and S. Kundu (1995), A new method to solve generalized multicriteria optimization problems using the simple genetic algorithm, *Structural Optimization*, 10(2), 94–99, doi:10.1007/BF01743536.
- Parker, M. C. (2014), A Contextual Multipartite Network Approach to Comprehending the Structure of Naval Design, Thesis, University of Michigan.
- Parker, M. C., and D. J. Singer (2015), Analyzing the dynamic behavior of marine design tools using network theory, *Ocean Engineering*, 106, 227–237, doi: 10.1016/j.oceaneng.2015.06.035.
- Parraguez, P., S. D. Eppinger, and A. M. Maier (2015), Information Flow Through Stages of Complex Engineering Design Projects: A Dynamic Network Analysis Approach, *IEEE Transactions on Engineering Management*, 62(4), 604–617, doi: 10.1109/TEM.2015.2469680.
- Pasqual, M. C., and O. L. de Weck (2012), Multilayer network model for analysis and management of change propagation, *Research in Engineering Design*, 23(4), 305–328, doi:10.1007/s00163-011-0125-6.
- Pelikan, M. (2002), Bayesian Optimization Algorithm: From Single Level to Hierarchy, Ph.D. thesis, University of Illinois at Urbana-Champaign, Urbana, Illinois.
- Rigterink, D. T. (2014), Methods for Analyzing Early Stage Naval Distributed Systems Designs, Employing Simplex, Multislice, and Multiplex Networks, Thesis, University of Michigan.
- Rubner, Y., C. Tomasi, and L. J. Guibas (), The Earth Mover’s Distance as a Metric for Image Retrieval, p. 20.
- Saridakis, K. M., and A. J. Dentsoras (2008), Soft computing in engineering design – A review, *Advanced Engineering Informatics*, 22(2), 202–221, doi: 10.1016/j.aei.2007.10.001.

- Schwarz, G. (1978), Estimating the dimension of a model, *The Annals of Statistics*, 6(2), 461–464.
- Sen, P., and J.-B. Yang (1998), Multiple Objective Decision Making, in *Multiple Criteria Decision Support in Engineering Design*, pp. 113–175, Springer London, London, doi:10.1007/978-1-4471-3020-8₄.
- Shahan, D. W., and C. C. Seepersad (2012), Bayesian Network Classifiers for Set-Based Collaborative Design, *Journal of Mechanical Design*, 134(7), 071,001–071,001–14, doi:10.1115/1.4006323.
- Shields, C. P. F., and D. J. Singer (2017), Naval Design, Knowledge-Based Complexity, and Emergent Design Failures.
- Steven D. Eppinger, and Tyson R. Browning (2012), *Design Structure Matrix Methods and Applications*, The MIT Press, Cambridge, Massachusetts.
- Steward, D. V. (1981), The design structure system: A method for managing the design of complex systems, *IEEE Transactions on Engineering Management*, EM-28(3), 71–74, doi:10.1109/TEM.1981.6448589.