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TRACKING AND STABILITY OF MULTI-UNIT
TRUCK COMBINATIONS

FINAL REPORT
MVMA PROJECT NUMBER 9165

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16. Abstract This report presents pertinent information for developing truck combinations that possess improved tracking and greater dynamic stability. It examines how geometric parameters and tire characteristics influence conflicting performance requirements for tracking and articulation stability. It also explores the benefits of kinematically steered wheels on full trailers. Trailer wheels that steer in response to draw bar angles are shown to provide good tracking in selected designs, but such a steering arrangement tends to reduce articulation stability. The analysis indicates that, at highway speeds, rapid steering efforts (such as required in a sudden maneuver to avoid an unexpected obstacle) can lead to amplified motions of full trailers equipped with steering arrangements designed to provide excellent low-speed tracking.			
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1.0 INTRODUCTION

This report, prepared by the Engineering Research Division of The University of Michigan's Transportation Research Institute (UMTRI) for the Motor Vehicle Manufacturers Association (MVMA), is intended to guide the development of truck combinations that will provide both improved tracking and increased stability margins in dynamic maneuvers.

Energy and economic considerations provide strong motivations for improving the efficiency of the long-haul trucking enterprise in the U.S. With the largest fraction of long-haul freight being carried in cube-full loads, there is reason for gaining a broader acceptance of doubles and triples combinations in this service. Research has clearly demonstrated, however, that doubles and triples, as conventionally configured, can exhibit a tendency to prematurely roll over the last trailer of the combination in a dynamic steering maneuver. Further, assuming conventional coupling devices, both experiment and analysis indicate that better tracking performance means poorer stability qualities and vice versa. Clearly, the long-term health and potential growth of the American trucking enterprise depends, in part, on the ability of the trucking community to achieve high levels of productivity without adopting configurations that unduly compromise their safety quality and their compatibility with the geometry of the highway network. The provisions of the recently-enacted Surface Transportation Assistance Act of 1982 virtually guarantee that a broad expansion in the use of multi-unit truck combinations will begin soon. Accordingly, it is increasingly important that research be conducted to guide the development of vehicle systems which provide more nearly optimal solutions to both tracking and stability requirements.

It is relevant to observe that the truck size and weight study sponsored by the Federal Highway Administration [1] has characterized the basic behavioral properties of all of the conventional vehicle combinations used in the U.S. such that the groundwork has been laid for the study of schemes for improving upon conventional performance. In this regard, research has been recently completed at UMTRI which provides a clearer understanding of the factors governing the rearward amplification of lateral acceleration in

highway trains [2,3]. Thus, a substantial base of research results and analysis methods has been produced recently, enabling a serious study of the tradeoff between tracking and stability.

The results presented herein represent an initial step towards an ultimate goal of developing new concepts for optimizing the tradeoff between offtracking and the articulation stability of multi-unit truck combinations. With regard to this ultimate goal, this study has investigated the use of kinematically-steered wheels on the dolly and trailer axles of full trailers.

The reported investigation consists of two parts: (1) an examination of the role played by geometric parameters of conventional configurations in determining the conflicting performance attributes of tracking and articulation stability (see Section 2) and (2) an exploration of benefits deriving from kinematically-steered wheels on full trailers (see Section 3). The report concludes that, although kinematic steering of trailer wheels can provide "perfect" low-speed tracking, these steering arrangements are probably unsuitable for operation at highway speeds. The concept of steering trailer wheels in response to drawbar angle seems to be viable only if the steering gain (level) is adjusted according to vehicle speed, possibly utilizing some form of control mechanism.

2.0 THE CONFLICT BETWEEN LOW-SPEED TRACKING AND DYNAMIC PERFORMANCE

This section treats the articulated vehicle which employs conventional trailer coupling mechanisms. Accordingly, the vehicle under consideration is a tractor-semitrailer unit coupled with a traditional fifth wheel and towing one or more full trailers. A full trailer is made up of a semitrailer and a converter dolly--the dolly using a single, wagon-tongue connection to its leading element and employing a conventional axle.

2.1 Low-Speed Offtracking

When an articulated vehicle tracks a steady-state circular trajectory at low speed, each axle of the train subtends a circular path whose radius is smaller than that of the preceding axle. Figure 1 illustrates this phenomenon for a three-axle tractor-semitrailer. The "offtracking" (OT) is defined as the difference in the turn radius of the first and last axle. An expression for OT, according to the notation employed in Figure 1, can be derived to yield

$$OT = R_1 - R_3 = R_1 - \sqrt{R_1^2 + KO^2 - L_1^2 - L_2^2} \quad (1)$$

Figure 2 illustrates a generalized scheme for labeling the significant length parameters (ignoring the rather insignificant kingpin offset dimensions, for example, KO). Using this notation, a generalized expression for the offtracking of the rearmost axle of a multiply articulated vehicle is given by the following equation:

$$OT = R_1 - \sqrt{R_1^2 - \sum_{i=1}^{n-1} (L_{i1}^2 + L_{i2}^2 - L_{i3}^2) + L_{n1}^2 + L_{n2}^2} \quad (2)$$

where n is the number of units in the train, with i=1 denoting dimensions which apply to the tractor-semitrailer and i>1 denoting dimensions applying to full trailers.

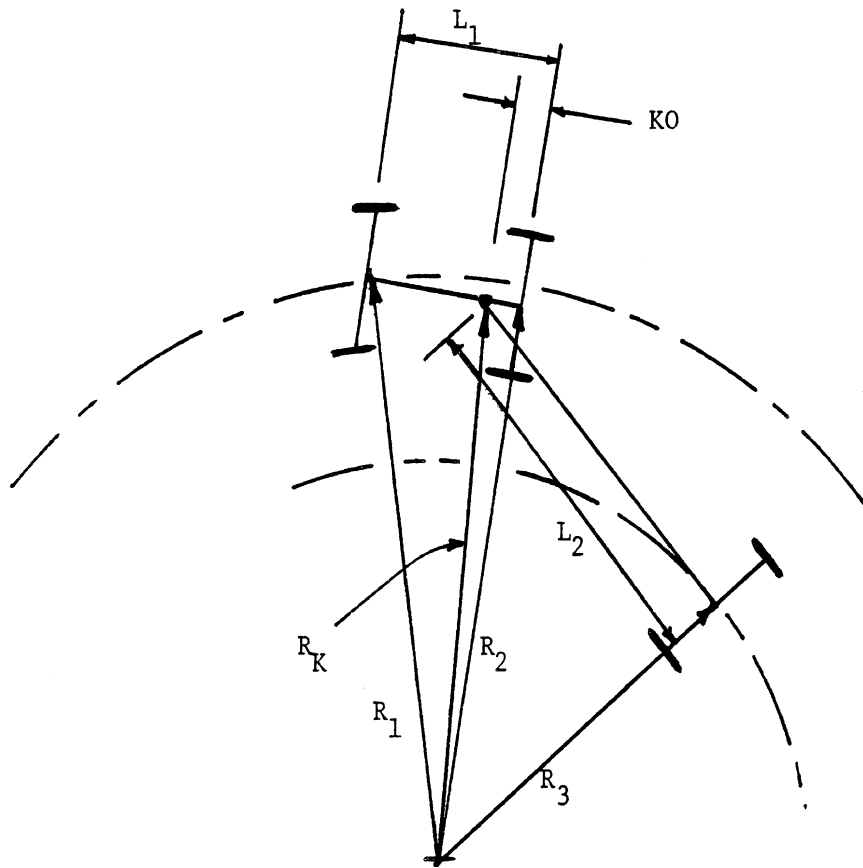


Figure 1. Maximum low-speed offtracking of a tractor-semitrailer

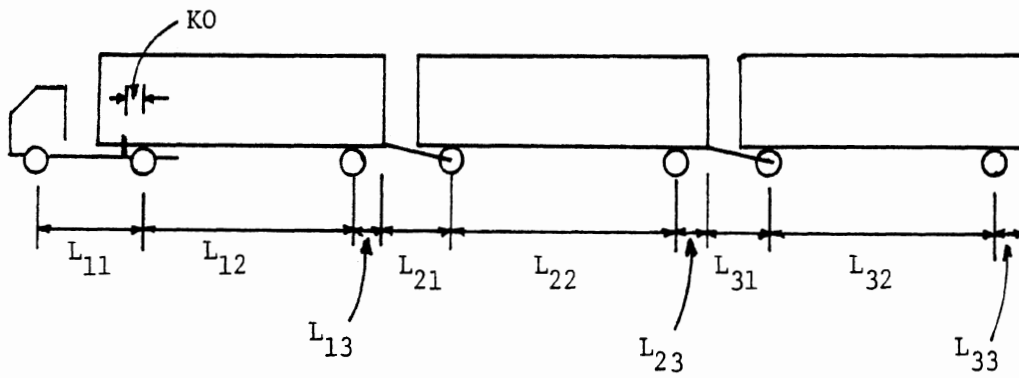


Figure 2. Definition of length dimensions applicable to low-speed offtracking calculation.

Equation (2) illustrates the advantage of adding articulation joints to reduce offtracking. For example, consider two vehicles, each of the same overall wheelbase (first axle to last axle). One vehicle is a single-unit truck and the other is assumed to be composed of a very large number of units, each with individual wheelbases approaching zero. The single-unit truck will exhibit the maximum offtracking for a vehicle of this length:

$$OT(1) = R_1 - \sqrt{R_1^2 - L_{11}^2} \quad (3)$$

The second vehicle, however, will have no offtracking since each length dimension approaches zero. That is:

$$OT(2) = R_1 - \sqrt{R_1^2} = 0 \quad (4)$$

Thus it can be seen that, for a vehicle of a given length, steady-state offtracking is reduced by each additional articulation joint. When economic incentives promote the use of long vehicles to increase freight capacity, practical issues of maneuverability in confined spaces, as in terminal yards and urban environments, promote the use of multiple articulation joints.

In practice, the offtracking exhibited by long vehicles on real roads is not simply a function of this steady-state offtracking performance, but is also determined by the arc length of the curved path being followed by the lead axle. In effect, there is a transient offtracking phenomenon whose analysis is considerably more complicated than the prediction of the offtracking which occurs in a "zero-speed" steady turn. Although the transient phenomenon is amenable to calculation by computer, given any prescribed path for a leading axle, Jindra [4] has developed a generalized solution for two specific paths of a lead axle, namely, a 90-degree turn and a 180-degree turn. These solutions, as computed for the rear axle of a single-unit vehicle, are shown in Figure 3. Note that the results are given in terms of a nondimensional offtracking, r/R (where r is the turn radius of the rear axle and R is the radius of the prescribed turn being followed by the lead or steering axle), plotted as (1) a function of the angle, θ , traversed by the trailing axle and (2) a function of the nondimensional ratio, $\lambda = \ell/R$, where ℓ is the wheelbase of the vehicle.

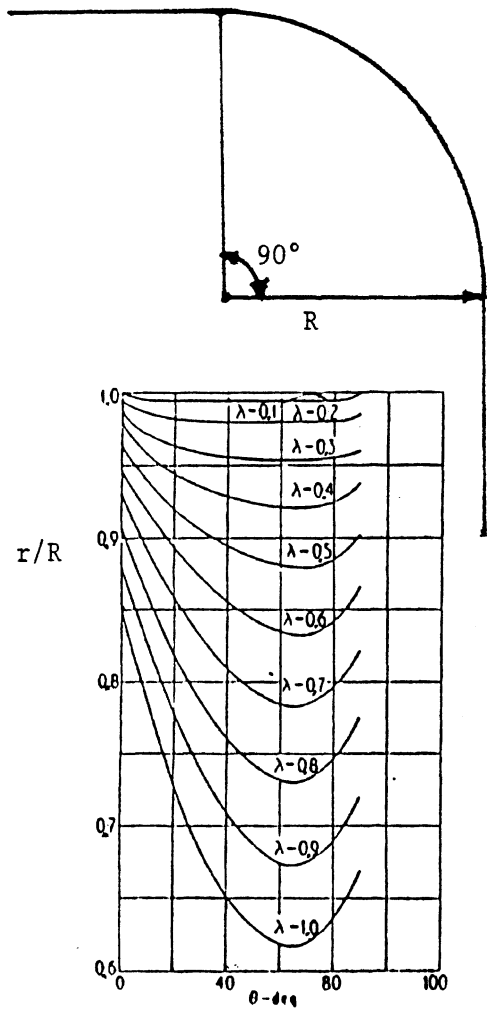


Figure 3a. Offtracking of a single-unit vehicle in a 90° turn [4].

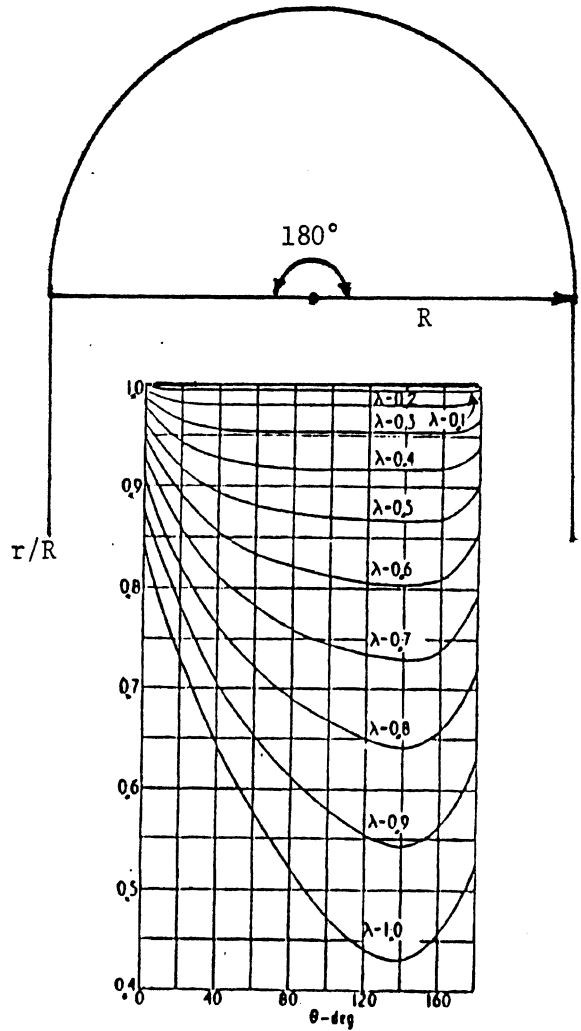


Figure 3b. Offtracking of a single-unit vehicle executing a 180° turn [4].

Jindra shows that the results plotted in Figure 3 are also applicable to a tractor-semitrailer, in that these curves give the offtracking of the axle on a semitrailer, relative to the path prescribed for a kingpin or fifth wheel comparable to the path prescribed for the lead axle of the towing vehicle. Further, by using the concept of an "equivalent wheelbase," Jindra argues that the curves presented in Figure 3 yield the approximate transient offtracking between the leading and last axle of a multiply articulated highway train. To apply Figure 3 to this general case, it is necessary to treat the ratio, λ , as

$$\lambda = L_{eq}/R$$

where L_{eq} is the equivalent wheelbase as defined by

$$L_{eq} = \left[\sum_{i=1}^{n-1} (L_{i1}^2 + L_{i2}^2 - L_{i3}^2) + L_{n1}^2 + L_{n2}^2 \right]^{1/2}$$

Jindra's solution indicates that:

1. The influence of articulation joints is mathematically identical in the low-speed transient response as it is in steady-state response.
2. Smaller intended turn radii (relative to the effective wheelbase) results in longer transients in terms of degrees of turn.
3. For a fixed turn radius, shorter effective wheelbases (smaller λ) result in less offtracking at any point (degrees) in the turn.

The above discussion only applies to vehicles which have single axles, front and rear. For the sake of completeness (although the point is not crucial to the multiple-articulation issue), we should note that tandem axles, as well as dual-wheel assemblies, also affect offtracking, particularly on low-friction roadways, since both generate a turn-resistant yaw moment. Morrison [5] has shown that, in small radii turns on low-friction surfaces,

widely spaced tandem axles can significantly increase trailer offtracking.

2.2 High-Speed Offtracking

While low-speed offtracking is characterized by each axle of the vehicle tracking a smaller radius than the axle preceding it, high-speed offtracking has the opposite quality. Generally, it can be expected that articulated commercial vehicles will exhibit an outboard, rather than inboard, offtracking at highway speeds. For multiply articulated vehicles, this offtracking may become sufficiently large that the increase in the width of the vehicle's swept path is significant to safety quality.

Figure 4 shows the general condition of a semitrailer in a high-speed, steady turn [6]. The reference radius (R) is measured to the kingpin of the semitrailer. (Since we have previously designated inboard offtracking as positive, the outboard offtracking shown in the figure is shown as -OT.) From the geometry of the figure, it can be shown that, for small angles:

$$OT = L^2/(2R) - L \cdot \alpha$$

where L is the wheelbase of the trailer and α is the slip angle at the rear axle of the trailer. Given the required static moment balance in yaw and in pitch, it can be shown that:

$$\alpha = a_y \cdot F_z/C_\alpha = (F_z/C_\alpha)(V^2/Rg)$$

where a_y is lateral acceleration in g's, V is forward velocity, F_z is the load on the trailer tires, and C_α is the total cornering stiffness of the tires mounted on the axle of the trailer. Combining these equations yields the following expression for offtracking at speed:

$$OT = L^2/2R - V^2L/Rg \cdot F_z/C_\alpha \quad (5)$$

where positive values of OT indicate inboard offtracking and negative values indicate outboard offtracking.

Equation 5 shows that offtracking at speed consists of an inboard,

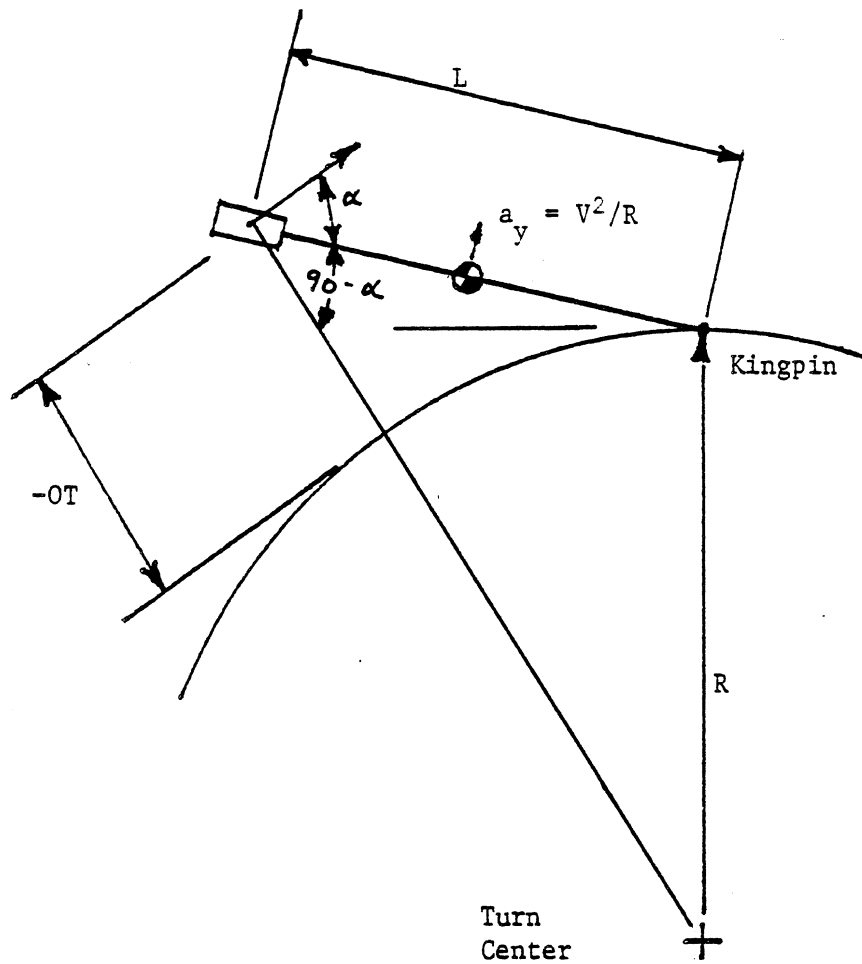


Figure 4. Geometry of the high-speed offtracking of a semitrailer.

zero-speed offtracking component (the first term) and a velocity-related component (second term) with an outboard polarity. For turns at fixed radii, the outboard component increases strongly with speed and is more pronounced for trailers which use lower stiffness tires.

Equation 5 also demonstrates that for a particular speed, load, and cornering stiffness, there is a critical trailer length (L_{cr}) which results in maximum outboard offtracking (minimum OT given the sign convention used herein). Differentiating OT with respect to L and setting the result to zero, we find that:

$$L_{cr} = (F_z/C_\alpha)(V^2/g) \quad (6)$$

Equation (6) shows that there is a trailer wheelbase dimension, L_{cr} , at which high-speed offtracking maximizes for given values of speed and F_z/C_α . (Note that L_{cr} is not a function of either path radius or lateral acceleration, per se.)

All of the preceding discussion has been concerned with the outboard offtracking of a single trailer relative to its lead point (kingpin for a semitrailer). On noting that a dolly can be treated in the same manner (considering its lead point as the pintle hitch), it can be shown that the overall offtracking of a multiply articulated vehicle at speed is approximately as follows:

$$OT = OT_{\text{zero speed}} - V^2/Rg \cdot (L_1 F_{z1}/C_{\alpha 1} + L_2 F_{z2}/C_{\alpha 2} + L_3 F_{z3}/C_{\alpha 3} + \dots) \quad (7)$$

On assuming a vehicle which uses the same tires on all axles, each carrying the same load, we find that

$$OT = OT_{\text{zero speed}} - (V^2/Rg) (F_z/C_\alpha) (L_1+L_2+L_3+\dots)$$

where the L's are the "wheelbases" (hitch to axle lengths) of each of the dollies and semitrailers making up the train. It is particularly of interest to note that the sum of all these lengths is approximately equal to the

overall length of the trailer train. Thus, we see that the outboard component of offtracking at speed is not a function of the number of articulation joints, but only of overall length. Note, however, that the articulated vehicle will still track further outboard at any given lateral acceleration because of its smaller, inboard component at zero-speed. Figure 5 illustrates this offtracking phenomenon and demonstrates why multiple articulation joints lead to larger outboard offtracking at increased speed.

2.3 Dynamic Yaw Response: Yaw Damping and Rearward Amplification

The dynamic yaw response of multi-articulated vehicles has been studied by a number of investigators [1-3, 7-16]. In recent years, the investigation [9] of the safety quality of the double-bottom fuel-hauling fleet in the State of Michigan has led to a broader, national interest in the performance of multi-articulated vehicles. Additional UMTRI studies following this original work include References [10-13].

The dominant performance property distinguishing the dynamic yaw response of multi-articulated vehicles from that of other commercial vehicles is "rearward amplification." It has been found that, in transient turning maneuvers, the rear unit of a multi-articulated vehicle may well experience a maximum lateral acceleration level which substantially exceeds the maximum lateral acceleration of the lead unit of the vehicle. In general, the lateral acceleration level of each unit may be expected to increase over that of the preceding unit, in typical dynamic maneuvers. It appears that it is this rearward amplification quality which leads to a tendency toward rear trailer rollover, a tendency well documented by the accident record.

Rearward amplification is a frequency-sensitive phenomenon, tending to be more pronounced in maneuvers where the steer input has a relatively high frequency content. Multiply articulated vehicles are multi-degree-of-freedom dynamic systems, characterized by several oscillatory dynamic modes of motions, some of which may be very lightly damped. When such a system is excited by inputs whose frequency content is near the natural frequency of these lightly damped modes, a very strong, resonant-like response occurs. The natural frequencies of the lightly damped modes of multi-articulated vehicles

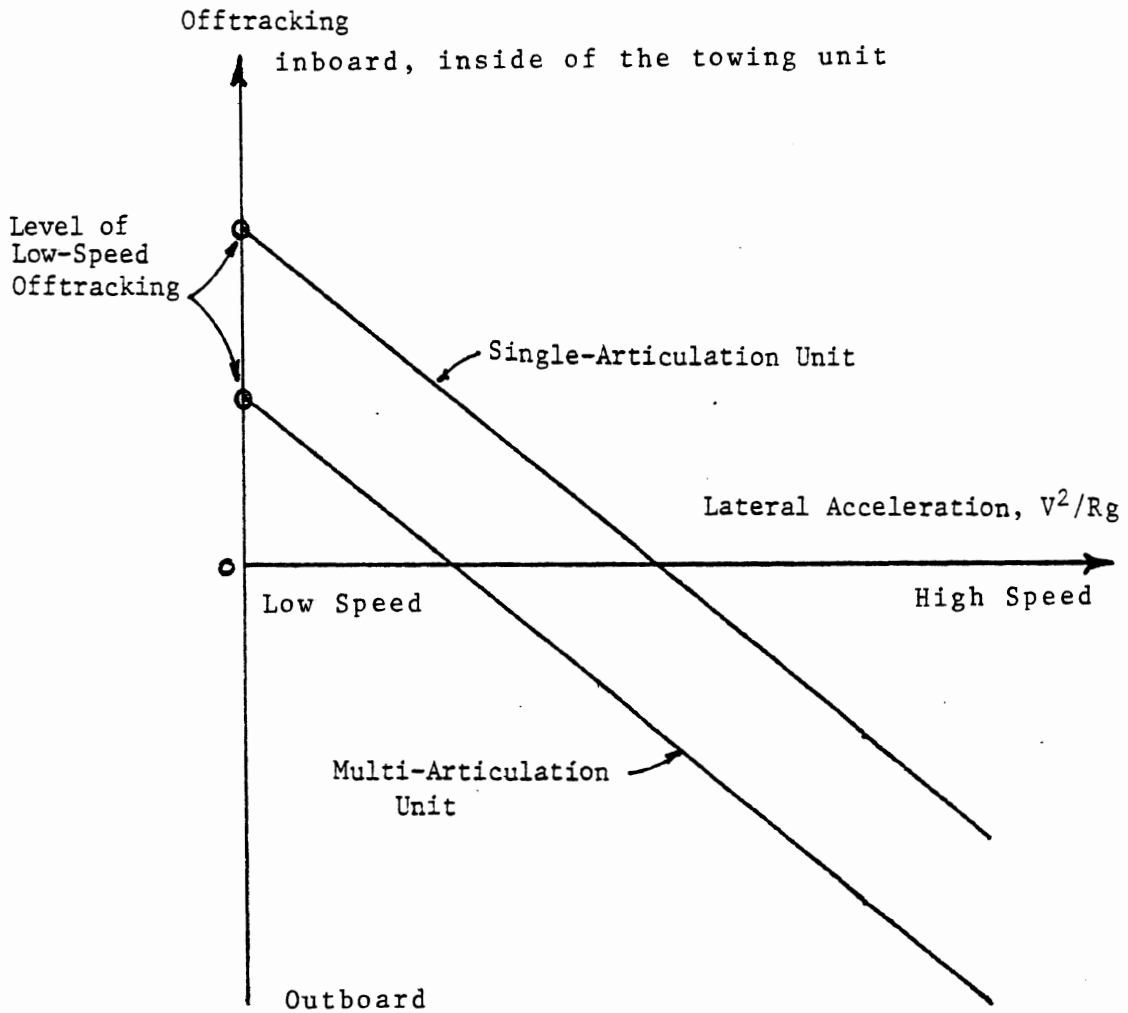


Figure 5. Offtracking of single- and multi-articulated vehicles of similar length.

tend to be higher than the steering frequencies used in normal driving, but low enough to be excited in emergency or evasive maneuvers.

It is useful to begin a consideration of the directional dynamics of articulated vehicles by examining the characteristic roots of the dynamic system. Figure 6 displays the root locus plot of a "Michigan double" as a function of forward speed. We note that this system has four (the number of articulation joints plus one) natural modes of oscillation, as indicated by the four root loci. One mode is seen to be heavily damped, two modes show an intermediate level of damping, with a fourth mode being rather lightly damped. The lightly damped mode has a natural frequency in the vicinity of 4.5 to 5 rad/sec (.7 to .8 Hz). All modes tend to become less damped as speed increases (a finding consistent with the properties of all pneumatic-tired vehicles), but only the heavily damped mode changes its frequency significantly with speed.

When the results of Figure 6 are compared to results calculated for the tractor-semitrailer only, we find that the two loci shown as dashed lines remain unchanged, and the two loci shown as solid lines disappear. This implies that it is the full trailer which is lightly damped and the tractor-semitrailer which is well damped, and that the multiply articulated vehicle is dynamically "decoupled" at the pintle hitch connection. Results for truck-full trailer combinations and for trains with more than one full trailer are consistent in that (1) the two modes associated with a given full trailer tend to be lightly damped, (2) the addition of more full trailers does not affect the dynamic behavior of units ahead of the added trailers, and (3) the modes of motion associated with each full trailer become less and less damped moving rearward in the train.

The "decoupling" phenomenon is explained intuitively by the fact that the lateral force at the pintle hitch, required to steer the "wagon-tongue" type of dolly is rather small--so small, in fact, that the yaw performance of the full trailer cannot "feed forward" and affect the yaw performance of leading units. The decoupled nature of the system enhances our ability to understand rearward amplification, since it means that each full trailer unit may be analyzed individually.

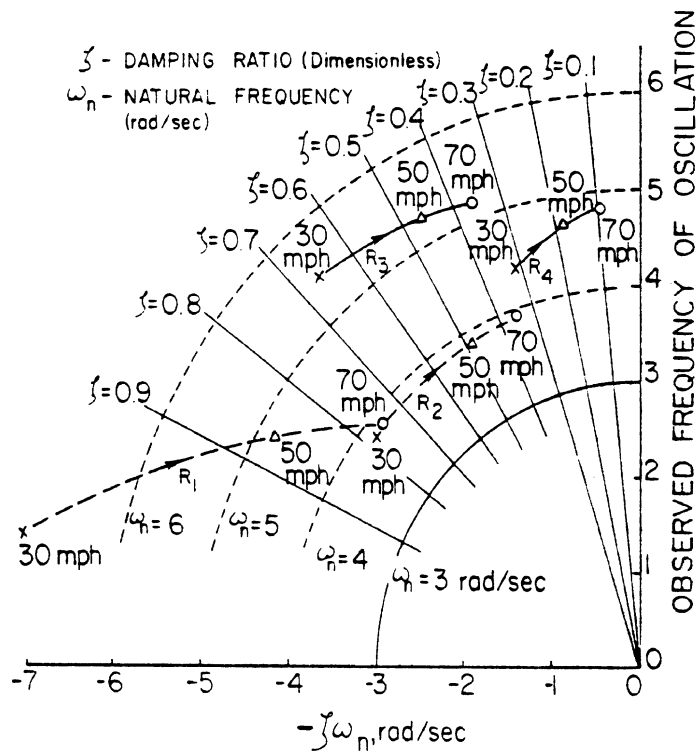


Figure 6. Loci of the roots, $R_1, R_2, R_3,$ and $R_4,$ of the characteristic equation indicating the influence of forward velocity on the natural frequencies $\omega_n,$ and damping ratios, $\zeta,$ of the basic modes of vehicle motion.

References [2,3] point out that, in the frequency domain, rearward amplification is equivalent to the magnitude of the transfer function between the lateral accelerations of the centers of gravity of the first and last units. Further, due to decoupling, this overall transfer function can be expressed as the product of localized transfer functions between centers of gravity and pintle hooks along the length of the train. This concept is illustrated in Figure 7, where the mass centers of the tractor, the first full trailer, and the second full trailer, are identified as points 1, 3, and 5, respectively, and the pintle hitches at the rear of the semitrailer and at the rear of the first full trailer are identified as points 2 and 4, respectively.

The analysis of rearward amplification can, in general, be reduced to an analysis which derives the local transfer functions. For this purpose, we point out a dichotomy in transfer function types, viz.:

1. Towing unit transfer functions are the transfer functions between the c.g. of a unit and the pintle hitch point on the rear of that unit.
2. Towed unit transfer functions are the transfer functions between a pintle (or tongue) hitch point at the front of a unit and the c.g. of that unit.

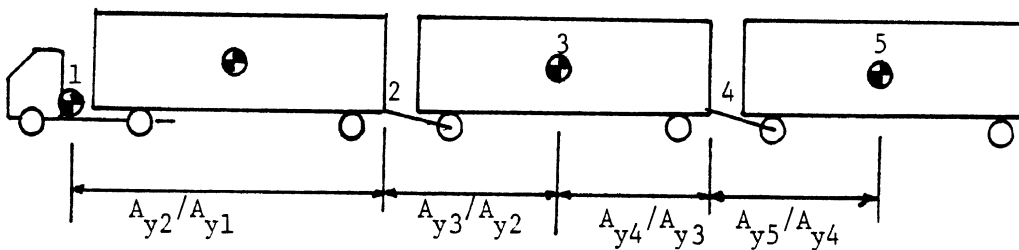
Analysis [2,3] has shown that the four local transfer functions of interest are of the forms given in Table 1. (Table 2 contains the definitions of symbols used.) The transfer functions presented in Table 1 are all expressed in terms of (a) vehicle properties (masses, lengths, tire parameters, etc.), (b) forward velocity, u , and (c) the frequency of excitation (i.e., steering input). All of these quantities have a significant influence on the property that we define as rearward amplification.

Examination of Table 1 shows that the only type of towed unit is, by definition, a full trailer. Tractor-semitrailer combinations*, straight trucks, and full trailers can all be towing units.

*Tractor-semitrailers are considered as a unit since they do not "decouple" at their hitch point.

Towing Unit Transfer Functions: A_{y2}/A_{y1} and A_{y4}/A_{y3}

Towed Unit Transfer Functions: A_{y3}/A_{y2} and A_{y5}/A_{y4}



$$\text{Overall Transfer Function} = A_{y5}/A_{y1} = (A_{y2}/A_{y1}) \times (A_{y3}/A_{y2}) \times (A_{y4}/A_{y3}) \times (A_{y5}/A_{y4})$$

Figure 7. A definition of localized transfer functions for a triple-trailer combination.

Table 1. Amplification Factors

Note: For each towing unit, the symbol x_{pc} represents the distance from the c.g. to the pintle hitch.

1. Towing Unit: Straight Truck

Rearward amplification between the c.g. of a straight truck and its pintle hitch

$$\frac{A_{yp}}{A_{yc}} = (1 + \Delta A)$$

where

$$\Delta A = \left[\frac{\frac{-x_{pc}}{u} j\omega \left(\frac{m_1 u}{\Sigma C_\alpha} j\omega + 1 \right)}{1 - \frac{I_1}{x_{11} \Sigma C_\alpha} \omega^2 + \frac{j\omega \cdot \Sigma x^2 C_\alpha}{x_{11} u \Sigma C_\alpha}} \right]$$

2. Towing Unit: Tractor-Semitrailer

a. Rearward amplification between the c.g. of a semitrailer and its pintle hitch connection to the unit being towed

$$\frac{A_{yp}}{A_{yc}} = (1 + \Delta A)$$

where

$$\Delta A = \left[\frac{\frac{-x_{pc} j\omega}{u} \left(\frac{u m_2 j\omega x_{2A}}{\Sigma(x_{2i} + x_{2A}) C_{\alpha 2i}} + 1 \right)}{1 - \left(\frac{I_2 \omega^2}{\Sigma(x_{2i} + x_{2A}) C_{\alpha 2i}} \right) + j\omega \left(\frac{\Sigma x_{2i} (x_{2i} + x_{2A}) C_{\alpha 2i}}{u \Sigma(x_{2i} + x_{2A}) C_{\alpha 2i}} \right)} \right]$$

b. Note that for typical tractor-semitrailers (3), the rearward amplification between the c.g. of the tractor and the c.g. of the semitrailer may range from a maximum of approximately 1.2 to a minimum of approximately 0.8 in the frequency range from 0 to 3.5 rad/sec. Vehicles with short semitrailers tend to have maximum amplification factors greater than 1.0 at frequencies in the range from 1 to 4 rad/sec. Vehicles with longer semitrailers tend to have amplification factors of 1.0 at low frequencies with their amplification factors falling off to approximately 0.8 in the neighborhood of 3 rad/sec. For first-order estimates of overall rearward amplification, a reasonable compromise is to assign an amplification factor of 1.0 between the c.g. of the tractor and the c.g. of the semitrailer if this amplification factor is not known from prior work.

3. Towing Unit: Full Trailer

Rearward amplification between the c.g. of a full trailer and the pintle hitch connection to the unit it is towing

$$\frac{A_{yp}}{A_{yc}} = (1 + \Delta A)$$

where

$$\Delta A = \left[\frac{\frac{-x_{pc}}{u} j\omega \left(\frac{m_T u}{\Sigma C_\alpha} j\omega + 1 \right)}{\left(1 - \frac{I_T}{x_{BT} \Sigma C_\alpha} \omega^2 \right) + \frac{j\omega \Sigma x^2 C_\alpha}{x_{BT} u \Sigma C_\alpha}} \right]$$

(The amplification factor for a towed full trailer is given next in Item 4.)

4. Towed Unit: Full Trailer

Rearward amplification between the pintle hitch connection to the towing unit and the c.g. of the full trailer

$$\frac{A_{yc}}{A_{yp}} (j\omega) = \frac{1}{1 - \left(\frac{\omega}{\omega_{nc}} \right)^2 + j 2\zeta_c \frac{\omega}{\omega_{nc}}}$$

where

$$\omega_{nc} = \sqrt{\frac{\Sigma C_\alpha}{m_T} \frac{1}{l}}$$

$$\zeta_c = \frac{1}{2k_A} \sqrt{\frac{\Sigma C_\alpha}{m_T}} (l)$$

See Equations (8) and (9) for determining the maximum value of A_{yc}/A_{yp} .

Table 2. Symbols, Subscripts, and Definitions

<u>Motion Variables - Definitions</u>		<u>Force and Moment Coefficients Used in Linear Equations of Motion</u>	
v	lateral velocity	F_v	the rate of change of lateral force with respect to v
r	yaw rate	F_r	the rate of change of lateral force with respect to r
ψ	heading angle	F_ψ	the rate of change of lateral force with respect to ψ
y	lateral displacement	F_y	the rate of change of lateral force with respect to $(y_A - y_T)$
A_y	lateral acceleration	F_δ	the rate of change of lateral force with respect to δ
Γ	articulation angle	T_v	the rate of change of yaw moment with respect to v
α	tire slip angle	T_r	the rate of change of yaw moment with respect to r
δ	steering angle of front wheels	T_ψ	the rate of change of yaw moment with respect to ψ
<u>Parameters - Definitions</u>		T_y	the rate of change of yaw moment with respect to $(y_A - y_T)$
u or \dot{x}_A	forward velocity	T_δ	the rate of change of yaw moment with respect to δ
C_α	the sum of the cornering stiffnesses of all the tires mounted on a designated axle	<u>Operators and Frequency Response Quantities</u>	
x	longitudinal distance between points indicated by subscripts, e.g., x_{BA} is the distance from point B to point A	p or ($\dot{}$)	indicates differentiation with respect to time
m	mass	ω	frequency, rad/sec
I	yaw moment of inertia	j	complex number equal to $\sqrt{-1}$
m_1, m_2, m_T	masses of straight trucks, semitrailers, and full trailers, respectively	ϕ	phase
I_1, I_2, I_T	moments of inertia of straight trucks, semitrailers and full trailers, respectively	K	amplitude
A	pinch hitch of a full trailer, also fifth wheel of a tractor-semitrailer, generally the articulation joint closest to the front of the vehicle	A_{y2}/A_{y1}	lateral acceleration transfer function between points 1 and 2
P	pinch hitch of any towing unit	A	amplification factor for a towing unit
C	c.g. of any towing unit	Y_o	open-loop transfer function
1,2,3 etc.	rear axles of a full trailer starting from the rear axle closest to the front of the trailer; also, these numbers are used in a double subscript notation (4) to denote the j^{th} axle on the i^{th} unit of a train. For example, x_{13} is the distance from the center of gravity of a 3-axle tractor to its rearmost axle	Y_c	closed-loop transfer function
ℓ	wheelbase	N	numerator
<u>Special Summations</u>		D	denominator
ΣC_α	the sum of the cornering stiffnesses of all the tires on a full trailer or straight truck	Y_z	quantity pertaining to complex conjugate zeros
$\Sigma_f C_\alpha$	the sum of the cornering stiffnesses of the tires on the front axle(s) of a full trailer or straight truck	Y_p, Y_{po}	quantity pertaining to complex conjugate poles
$\Sigma_r x_{T\alpha} C_{\alpha T}$	the sum of the products of the distance from the c.g. to each rear axle with its cornering stiffness for full trailers	ω_{nc}	natural frequency
$\Sigma x^2 C_\alpha$	the sum of the products of the square of the distance from the c.g. to each axle times the cornering stiffness for that axle (see T_i in Fig. 2 for full trailer situations)	ζ_c	damping ratio
		ω_{max}	frequency at maximum gain
		G_{max}	maximum gain for a full trailer
		<u>Special Points Used in Subscripts</u>	
		<u>Points</u>	<u>Location</u>
		B	turntable of a full trailer
		T	c.g. of a full trailer

The towed unit transfer function has a maximum value, corresponding to a maximum rearward amplification of c.g. motion relative to the pintle hitch motion, at frequency ω_{\max} , given by:

$$\omega_{\max} = \omega_{nc} \sqrt{1 - 2\zeta_c^2} \quad \text{for } \zeta_c < .707 \quad (8)$$

The maximum rearward amplification gain occurring at this frequency can be shown to be:

$$G_{\max} = \left(2\zeta_c \sqrt{1 - \zeta_c^2} \right)^{-1} \quad (9)$$

Thus, from the equation for ζ_c (see item 4 in Table 1) we observe that towed unit amplification is large (because damping is small) when (1) velocity is high, (2) tire stiffness relative to vehicle mass is low, and (3) the vehicle wheelbase is short. More complex formulations than those of Table 1 show that towed unit amplification is also a rather mild function of towbar length. The nature of this function is such that there is a worst-case towbar length at which local amplification maximizes [3]. Shorter or longer towbars will reduce the local amplification, slightly. However, towbar length is a minor consideration in estimating rearward amplification.

Calculations for three types of full trailers (standard 27-foot van with dolly, the Michigan double pup trailer, and the California long-tongue pup trailer) yield maximum amplifications ranging from 1.2 to 1.6 at frequencies ranging from 2.7 to 4 rad/sec.

With respect to the rearward amplification associated with towing units, examination of Table 1 shows that the general form of the transfer function from the c.g. to the pintle hitch is:

$$A = 1 + \frac{(-x_{pc}/u)(j\omega)(j\omega\tau + 1)}{1 - (\omega/\omega_n)^2 + j2\zeta\omega/\omega_n} \quad (10)$$

where the specific definitions of τ , ω_n , and ζ depend upon whether the towing unit is a straight truck, a tractor-semitrailer, or a full trailer. The quantity τ represents the forward velocity divided by the generalized

cornering coefficient. From the numerator of Equation (10), we see that (1) stiff tires relative to the mass of the vehicle are again advantageous for reducing rearward amplification and (2) a short distance from the c.g. back to the pintle hitch reduces amplification. The denominator of the equation corresponds to the classical second-order system which can resonate near ω_n if ζ is small. Typically, for commercial vehicles, ω_n tends to be above 6 to 7 rad/sec, well above the frequencies which the driver can effectively generate. However, if ζ is small, its influence can be important in the frequency range associated with emergency maneuvering.

As indicated in Figure 7, the overall rearward amplification can be determined from the product of the individual transfer functions, namely, at any given frequency the gain of the local transfer functions may be determined and multiplied to determine the overall rearward amplification for that frequency. This finding implies that the issue of "tuning" among units of a vehicle may be critical. If a multi-articulated vehicle is made up of several identical trailers, its overall amplification may be large, even though local amplifications are small, since they will all peak near the same frequency. For example, a conventional triple consisting of a tractor and three 28-foot trailers has been found to exhibit the following amplification factors at 3.15 rad/sec, the frequency at which maximum gain is exhibited by each trailer:

Tractor c.g. to semi c.g.	1.13
Semi c.g. to pintle	1.36
1st trailer pintle to c.g.	1.08
1st trailer c.g. to pintle	1.38
2nd trailer pintle to c.g.	1.08

The product of these factors totals 2.48, a large amplification at the relatively low frequency of 3.15 rad/sec.

A graphical presentation of the manner in which the two local amplifications of a truck-full trailer combine to produce an amplification for the complete vehicle system is given in Figure 8. Here we see that the truck's amplification is very high at high frequencies, but that the trailer's amplification peaks at less than 3 rad/sec. The combined effect is total

Rearward Amplification
(Gain)

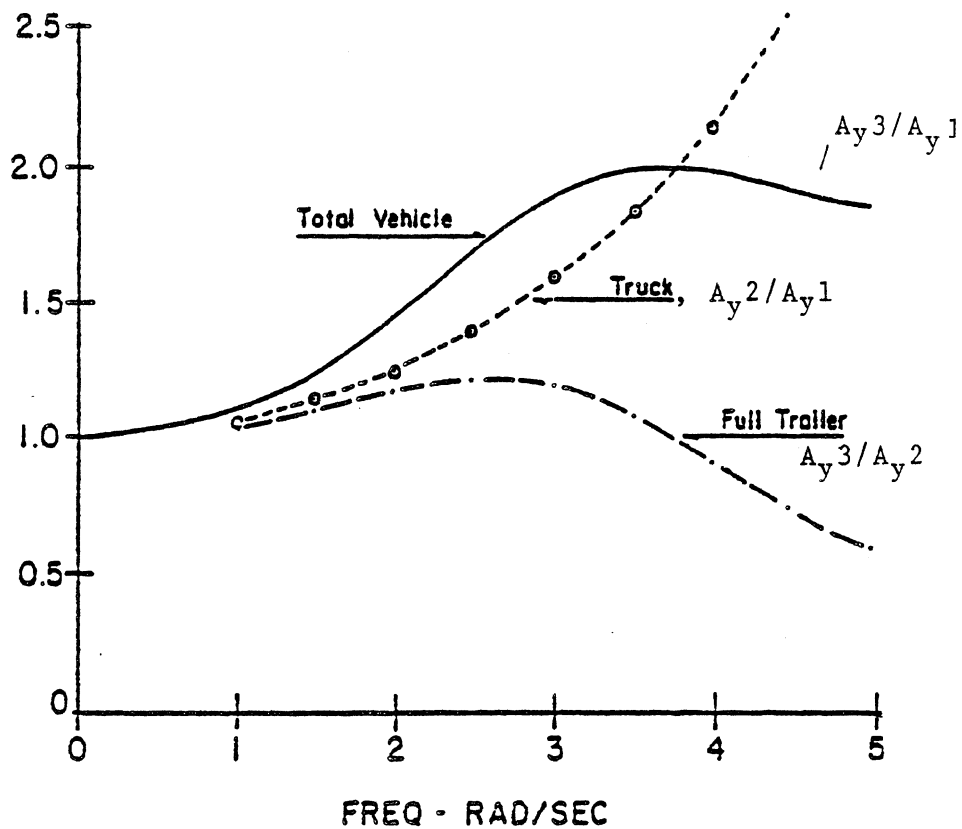


Figure 8. Rearward amplification response (A_{y3}/A_{y1}) as determined by the product of truck and trailer characteristics, A_{y2}/A_{y1} and A_{y3}/A_{y2} respectively.

amplification which peaks between 3 and 4 rad/sec at a magnitude of about 2.

The findings discussed above show that the use of full trailers in commercial vehicle trains inherently leads to reductions in yaw stability and corollary increases in rearward amplification, but that various vehicle properties (e.g., tire stiffness, mass distribution, geometry, and "symmetry") have a strong influence on the specific level of amplification that results.

2.4 Conclusion

The multiply articulated commercial vehicle train, particularly that which uses conventional full trailers, exhibits a fundamental conflict between desirable and undesirable performance properties. The conflicting performance properties are as follows:

Desirable Performance

1. Improved low-speed offtracking and increased maneuverability in confined areas

Undesirable Performance Properties

1. Degraded high-speed offtracking (resulting directly from improved low-speed offtracking and the use of trains having increased overall length)
2. Reduced dynamic stability (due to the introduction of lightly damped dynamic modes) resulting in increased rearward amplification leading to increased rollover potential for the trailing unit

The amount of low speed offtracking of a long trailer can be greatly reduced by adding articulation joints. For example, in a tight turn with a radius of 50 feet a semi-trailer with 40 feet between the 5th wheel and the rear axle will offtrack the 5th wheel by 20 feet; while if this hypothetical semi-trailer were to be divided into 20 foot semis, the second of the two semis would off-track by less than 9 feet. These results are well known to truck drivers and are the basis for desiring to use articulated vehicles.

High speed offtracking is not large on long radius curves designed for highway speeds. However, trailer wheels may track outside of the tractor wheels, especially on exit ramps if they are traversed at high speed. Drivers, who may not be aware of this phenomenon, may cause their vehicles to strike curbs or other roadside objects if they are not warned of this danger. The addition of an articulation joint contribution is no more than 1/2 foot of outboard offtracking.

Rearward amplification is highly dependent upon the lengths of the units involved in a combination vehicle. For example, a turnpike double with trailer wheel bases of 432 inches will have a maximum rearward amplification of approximately 1.1. while a conventional 65' double with trailer wheel bases of 252 inches will have a maximum rearward amplification of about 2.1 [12]. The turnpike double will perform almost ideally in obstacle avoidance maneuvers, but its offtracking at low speed will be much worse than that of a conventional 65' double, which, due to rearward amplification, runs a greater risk of rolling over the rear trailer in the event that an obstacle avoidance maneuver is required. The differences between the performance properties of conventional 65' doubles and longer turnpike doubles clearly illustrate the conflict between low speed offtracking and high speed maneuverability.

3.0 THE ADVANTAGES AND DISADVANTAGES OF FULL TRAILERS IN WHICH REAR WHEELS ARE STEERED AS WELL AS FRONT WHEELS

The combination vehicles discussed in Section 2.0 employ full trailers that are "steered" in a wagon-tongue fashion by the drawbar attached to their dollies. In this instance, the front axle (i.e., the dolly axle) has a steer angle, δ_f , that is equal to the articulation angle, Γ , between the drawbar and the longitudinal axis of the semitrailer portion of the full trailer (see Figure 9). In addition, the rear wheels of these trailers are nonsteering and remain parallel to the longitudinal plane of the semitrailer. In contrast to this arrangement, full trailers can be configured with wheels supported on inboard kingpins such that the front and rear wheels of the trailer can be steered in proportion to the articulation angle between the drawbar and the longitudinal axis of the trailer (see, for example, Figure 10).

The tracking and stability of full trailers utilizing so-called Ackerman steering systems may be analyzed individually in the same manner as the conventional full trailer with wagon-tongue steering. The lateral forces at the pintle hitches are small because the steering system layout is such that the moments about the steering pivot are small, similar to the moment which exists about the fifth wheel mounted on the conventional "wagon-tongue" dolly. In order to examine the advantages and disadvantages of four-wheel steering, we assume that the steer angles of the front and rear wheels (δ_f and δ_r , respectively) are proportional to the articulation angle, Γ , viz.,

$$\delta_f = G_f \Gamma \quad (11)$$

$$\delta_r = G_r \Gamma \quad (12)$$

where

G_f is the "gain" of the steering mechanism which steers the front wheels

G_r is the gain of the steering mechanism which steers the rear wheels

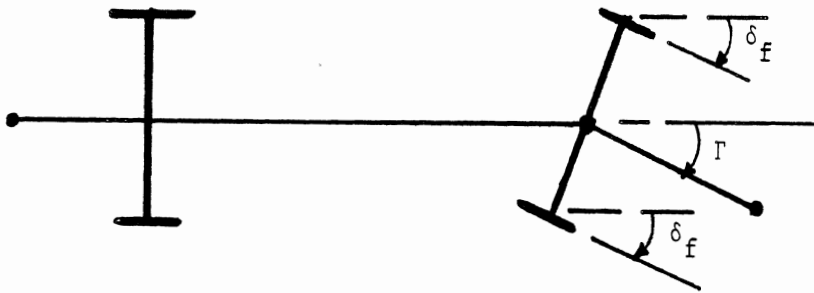
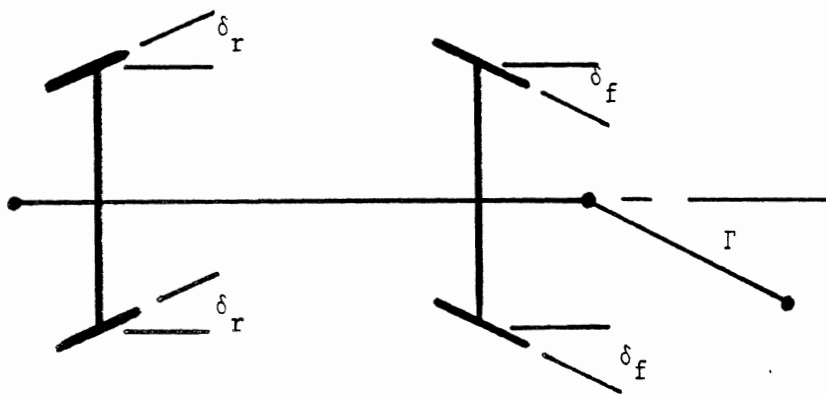


Figure 9. Wagon-tongue steering.



Note: As shown, δ_r has a positive value.

Figure 10. Front and rear steer angles produced by a steering linkage attached to the drawbar.

Note that the details of the layout of the steering system are not needed to study the advantages and disadvantages of steering the rear wheels along with the front wheels. Thus, Equations (11) and (12) suffice, together with linearized equations of motion, to investigate rearward amplification as influenced by rear-wheel steering.

Appendices A and B outline the theory developed to examine the offtracking and rearward amplification attributable to steering the front and rear wheels at specified gain levels. These results are applied below to evaluate the tracking advantages and the rearward amplification disadvantages that derive from such an arrangement.

3.1 Advantages in Tracking

A kinematic analysis shows that a four-wheel steering system can provide "perfect" low-speed tracking such that the rear pintle on the trailer follows the path of the front pintle when turning on a fixed radius at very low speed [3].

The full trailer shown in Figure 11 represents a trailer on which the wheels are steered about in-board kingpins by means of a steering linkage activated by a drawbar that pivots about a central kingpin. As illustrated in Figure 11, X_{KP} is the distance from the central kingpin to the front axle and X_{PH} is the distance from the rear axle to the rear pintle hitch. The symbol " X_{DB} " represents the drawbar length and ℓ represents the wheelbase.

The development in Appendix A shows that perfect low-speed tracking can be achieved by the proper adjustment of the dimensions, X_{DB} , X_{KP} , and X_{PH} , relative to the wheelbase, ℓ , and of the steering gains, G_f and G_r . In general, the use of symmetric steering gains, front and rear, requires that the drawbar be approximately as long as the wheelbase when $G_f = G_r = 0.5$. On the other hand, shorter drawbars suffice when $G_f = G_r = 1.0$. The primary advantage of using smaller values of steering gain stems from the ability to cause intermediate points on the trailer to track on a radius comparable to

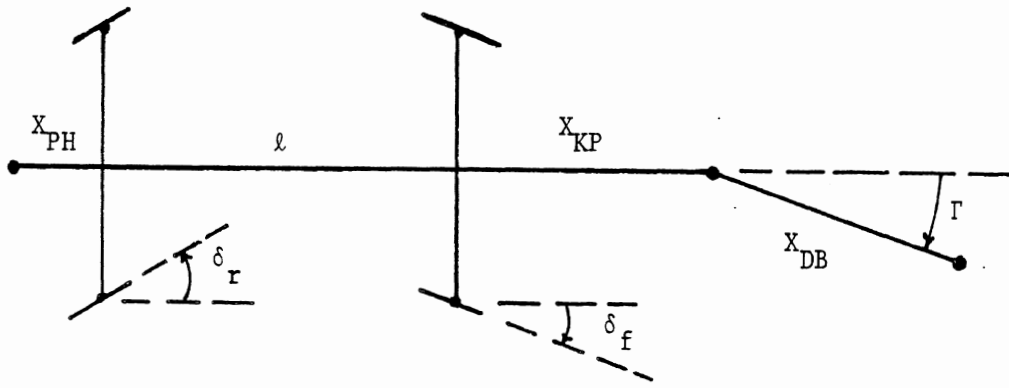


Figure 11. Full trailer with drawbar steering.

those being tracked by the fore and aft hitch points.

The developed equations show that shorter drawbars can be used by shifting the turn center forward of a point opposite the mid-wheelbase of the trailer. This objective can be accomplished by employing a rear steering gain which is larger than that used at the front. The equations also show that the rear pintle of a trailer with nonsteering rear wheels will tend to track the path traced by the front pintle provided the front steering gain is in the neighborhood of 0.5 and the drawbar is approximately the length of the wheelbase dimension. This achievement is, however, substantially compromised by the tracking of the front axle of the trailer.

3.2 Disadvantages Due to Rearward Amplification

The amount of rearward amplification can be very large for trailers with wheels that are kinematically steered in response to drawbar motion. For example, consider a typical 28-foot full trailer as might be incorporated in a doubles or triples combination. Let us compare three cases: (1) let the drawbar length, X_{DB} , be equal to the wheelbase, ℓ , and let $G_f = G_r = 0.5$; (2) let $X_{DB} = \ell$, $G_f = 0.5$, and $G_r = 0$; (3) let $x_{DB} = 6.1'$, $G_f = 1.0$, and $G_r = 0$. The first two cases provide perfect tracking and the final case represents current practice in trailer layout.

Per the analysis developed in Appendix B and using the parameters given in Table 3, we find that, in the second case, the response of the rear hitch point to motion of the front pintle hitch has a maximum steady-state gain of approximately 1.7 at 2.1 rad/sec at 55 mph (see Figure 12). The gain shown in Figure 12 for the first case is nearly as large as that of the second case, and the maximum gains for both of these steering arrangements are much greater at frequencies comparable to those used in emergency maneuvers (approximately 2 to 3.15 rad/sec) than the gains attained in the third case which corresponds to a typical layout of a conventional full trailer. In summary, the rearward amplification (gains) of the trailers with perfect tracking are sufficiently large to make these units marginally suitable for use at highway speeds and probably unsuitable for use in doubles and triples combinations.

Table 3. Parametric Values for Two Modified and One Conventional Full Trailer

Basic parameters applicable to all three trailers:

$$m = 35000/g = 1090 \text{ lb-sec}^2/\text{ft}, X_{if} = \ell/2 = 11.4 \text{ ft}$$

$$I = 90,000 \text{ ft-lb-sec}^2, X_{ir} = \ell/2 = 11.4 \text{ ft}$$

$$C_{\alpha} = 162,000 \text{ lb/rad (both axles)}$$

$$\ell = 22.8 \text{ ft} \quad u = 80.667 \text{ ft/sec (55 mph)}$$

$$X_{KP} = 0$$

Parameters distinguishing the three trailers:

Case No.	X_{DB} (ft)	G_f	G_r	X_{ic} (ft)
1	22.8	0.5	+0.5	11.4
2	22.8	0.5	0	11.4
3	6.1	1.0	0	13.6 ($X_{PH} = 2.2'$)

The above basic and distinguishing parameters lead to the following values of stability derivatives

$$F_v = 4016.5 \text{ lbs/(ft/sec)}$$

$$F_r = T_v = 0 \text{ lbs/(rad/sec) or ft-lbs/(ft/sec)}$$

$$T_r = 521,986 \text{ ft-lbs/(rad/sec)}$$

	Case 1	Case 2	Case 3
lbs/rad F_{ψ}	0	121,513	464,754
lbs/ft F_y	0	3,553	26,557
lbs/rad F_r	0	81,000	162,000
ft-lbs/rad T_{ψ}	2,770,200	1,385,100	5,298,197
ft-lbs/ft T_y	81,000	40,500	302,754
ft-lbs/rad T_r	1,846,800	932,400	1,846,800

x_{db} = drawbar length, ft.
 l = wheelbase, ft.

- Case 1: $x_{db}=l=22.8$, $G_f=Gr=0.5$
 (perfect tracking, front and rear steering)
- Case 2: $x_{db}=l=22.8$, $G_f=0.5$, $Gr=0$
 (perfect tracking, front steering)
- Case 3: $x_{db}=6.1$, $l=22.8$, $G_f=1.0$, $Gr=0$
 (conventional trailer)

rearward
 amplification
 $|Y_{c/A}|$

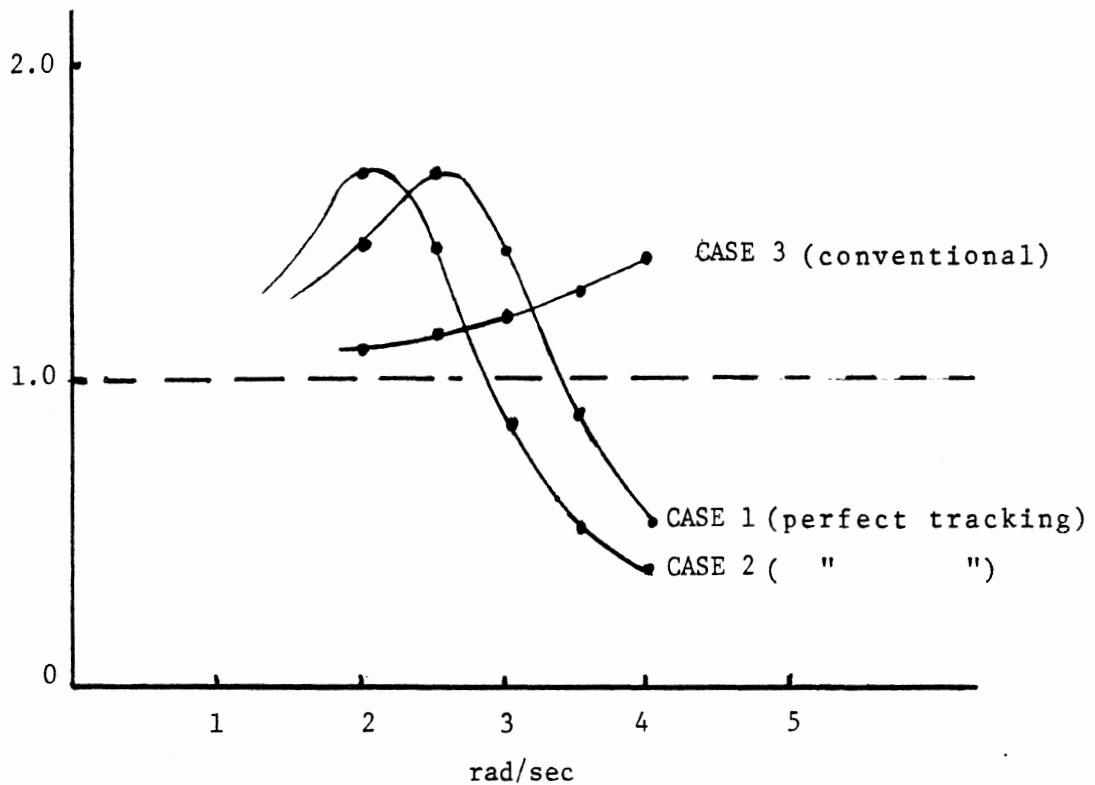


Figure 12. Comparisons of rearward amplifications, showing high amplification for trailers that track perfectly at low speed.

4.0 CONCLUDING REMARKS

With respect to conventional full trailers, the work performed in this study confirms the following conflicts between offtracking and dynamic maneuvering capability: (1) longer trailers have less rearward amplification, but more offtracking than short trailers; (2) if the hitch location on a towing unit is moved closer to the center of gravity of the unit, rearward amplification will be decreased, but offtracking will be increased. A surprising finding, that has been demonstrated using the theory presented here, is that there is a "worst-case" drawbar length which maximizes rearward amplification [3]. This finding is frequency dependent such that the worst-case length is a function of the steering quickness. Nevertheless, these results confirm those found by Hazemoto [16] and they indicate that shortening the drawbar may decrease rearward amplification slightly, although the influence of moderate changes in drawbar length will not have an important influence on rearward amplification.

The analysis methods developed in this study provide the means for evaluating the low-speed tracking characteristics and the high-speed maneuvering capability of trailers with kinematic (drawbar) steering systems (see Appendices A and B). The analyses performed herein indicate that trailers with drawbar steering systems, which provide perfect low-speed tracking, tend to have undesirably large amounts of rearward amplification in high-speed maneuvering such as that required in emergency obstacle-avoidance situations (see Figure 12). These findings provide evidence refuting the position that kinematic steering of trailer wheels will provide a good compromise between low-speed offtracking and rearward amplification.

Nevertheless, there may be situations in which developers truck combinations will wish to judge for themselves the trade offs between rearward amplification and offtracking for a particular trailer arrangement.

In that case, equations 4, 5, and 6 of appendix A can be used to compute offtracking for various drawbar angles; and equations B 11 to B 17 and equations B 24 through B 30 of appendix B can be used to analyze rearward amplification for various velocities.

If operation speeds are kept to modest levels, one might be interested in investigating very long trailers with a small amount of rear wheel steering added to improve low speed offtracking. However, trailers, which have (a) conventional wagon-tongue steering and (b) unacceptable levels of rearward amplification at their maximum operation speed, do not appear to be good candidates for the application of kinematic arrangements for steering the rear wheels.

Given that gains in both low speed offtracking and obstacle avoidance performance are not attainable using drawbar steering systems, we are inclined to recommend that developers also consider other means for improving the directional performance of full trailers. In that regard, we believe that innovative dolly concepts and hitching arrangements are reasonable areas for future research aimed at reducing rearward amplification without sacrificing the ability to maneuver successfully at low speeds in places with restrictive geometry.

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APPENDIX A

THE OFFTRACKING CHARACTERISTICS OF FOUR-WHEELED STEERED TRAILERS

This appendix derives the equations which describe the low-speed offtracking behavior of the most general configuration of a four-wheeled trailer, namely, a trailer whose front and rear wheels can be steered by a kinematic linkage attached to a drawbar which pivots about a central kingpin. In theory, such a trailer can be designed so as to steer the front and rear wheels to angles which are proportional to the articulation angle of the drawbar by differing amounts, front and rear. In all cases, however, the steering links must be arranged such that the rear wheels steer to the left when the front wheels steer to the right and vice versa.

If we assume that the turning radius of the trailer is large relative to the length of the wheelbase and drawbar, it becomes possible to neglect the finite track width of the trailer and thereby collapse a four-wheeled trailer into the two-wheeled trailer shown in Figure A-1. (Note that in drawing this figure, we must show turn radii whose lengths are not much greater than the length dimensions of the trailer.) In Figure A-1, we show the front wheels being steered to an angle which is less than the articulation angle, Γ , of the drawbar with the rear wheels being steered to an angle greater than the articulation angle. To describe this differential steering, we define a front and rear steering gain, respectively, such that

$$\delta_f = G_f \Gamma$$

and

$$\delta_r = G_r \Gamma$$

where

$$G_f = \partial \delta_f / \partial \Gamma$$

and

$$G_r = \partial \delta_r / \partial \Gamma$$

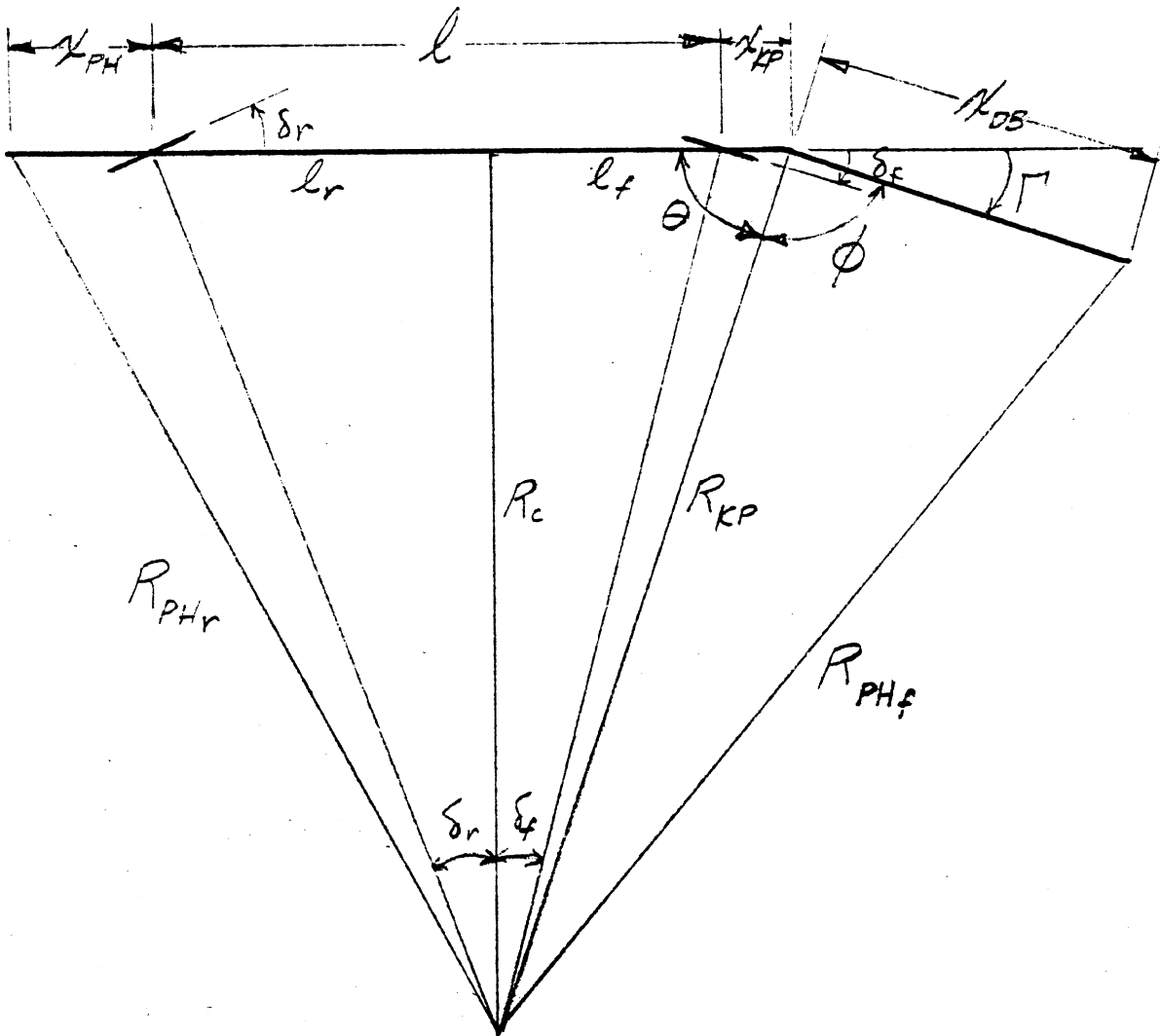


Figure A-1. Definition of the geometry of a four-wheel steered trailer, when $\delta_r > \delta_f$.

(Note that a positive articulation angle causes a positive front-wheel displacement and a positive rear-wheel displacement, with the latter positive displacement involving wheels steered to the left.)

Since "zero-speed" offtracking implies that the front and rear tires must operate at a zero slip angle, an inspection of Figure A-1 shows the following:

- 1) When $G_f = G_r$, i.e., $\delta_r = \delta_f$, the turn center must lie on a point opposite the midpoint of the wheelbase
- 2) When $G_f > G_r$, the turn center will lie aft of mid-wheelbase and when $G_f < G_r$, the turn center will lie forward of mid-wheelbase
- 3) In general, for designs which incorporate kingpin and pintle locations as shown, the objective of having the rear pintle hitch follow in the track of the forward pintle hitch means that the rear wheels must be steered in excess of the front wheels.

The offtracking of the trailer can be defined as

$$OT = R_{PH_f} - R_{PH_r}$$

Note that when $OT = 0$, we are assured that every unit in a train of trailers will follow in the path established by the lead pintle connection. On the other hand, we are not necessarily assured that the tires will track in the circular path being traced by the pintles.

To evaluate the offtracking, as defined, Figure A-1 can be examined to obtain the following geometric relationships:

$$R_{PH_f}^2 = R_{KP}^2 + X_{DB}^2 - 2R_{KP}X_{DB} \cos \phi \quad (1)$$

$$R_{KP}^2 = R_c^2 + (\ell_f + X_{KP})^2 \quad (2)$$

$$R_{PH_r}^2 = R_c^2 + (\ell_r + X_{PH})^2 \quad (3)$$

Note that

$$\theta + \phi + \Gamma = 180^\circ$$

Therefore,

$$\phi = 180^\circ - (\theta + \Gamma)$$

and

$$\cos \phi = -\cos(\theta + \Gamma)$$

Since

$$\theta = \tan^{-1} \frac{R_c}{\ell_f + X_{KP}}$$

$$\ell_f = R_c \tan \delta_f = R_c \tan G_f \Gamma$$

$$\ell_r = R_c \tan \delta_r = R_c \tan G_r \Gamma$$

$$\ell = \ell_f + \ell_r = R_c (\tan G_f \Gamma + \tan G_r \Gamma)$$

we find that

$$R_c = \ell / (\tan G_r \Gamma + \tan G_f \Gamma)$$

$$\ell_r = \ell [\tan G_f \Gamma / (\tan G_r \Gamma + \tan G_f \Gamma)]$$

$$\ell_f = \ell [\tan G_r \Gamma / (\tan G_r \Gamma + \tan G_f \Gamma)]$$

Equations (1), (2), and (3) can now be expressed as

$$R_{PH_f}^2 = R_{KP}^2 + X_{DB}^2 + 2 \cdot R_{KP} X_{DB} \cdot$$

$$\cos\{\tan^{-1}[\ell / (\ell \tan G_f \Gamma + X_{KP} [\tan G_r \Gamma + \tan G_f \Gamma])] + \Gamma\} \quad (4)$$

$$R_{KP}^2 = [\ell / (\tan G_r \Gamma + \tan G_f \Gamma)]^2 + [\ell (\tan G_f \Gamma / (\tan G_r \Gamma + \tan G_f \Gamma)) + X_{KP}]^2 \quad (5)$$

$$R_{PH_r}^2 = [\ell / (\tan G_r \Gamma + \tan G_f \Gamma)]^2 + [\ell (\tan G_r \Gamma / (\tan G_r \Gamma + \tan G_f \Gamma)) + X_{PH}]^2 \quad (6)$$

In their above form, Equations (4), (5), and (6) do not provide much guidance as to how to select values of the design variables (viz., ℓ , X_{DB} , X_{KP} , X_{PH} , G_f , and G_r) in order to achieve good or perfect offtracking. By numerical means, it is possible to solve for OT as a function of the articulation angle, Γ , and to demonstrate that good or perfect tracking can be achieved for a proper combination of design variables. To first order, one finds that doubling the articulation angle, Γ , reduces the turn radius by half. If an offtracking error exists at a finite articulation angle, then doubling the articulation angle approximately doubles the offtracking error. For example, if one assumes a trailer with the following design variables, namely, $\ell = 62$ in, $X_{DB} = 50$ in, $X_{KP} = 2.5$ in, $X_{PH} = 24.4$ in, $G_f = 1$, and $G_r = 1$, the results given in Table A-1 are obtained.

Table A-1

$\Gamma \sim \text{deg}$	$R_{PH_f} \sim \text{ft}$	$R_{PH_r} \sim \text{ft}$	OT $\sim \text{ft}$
5	29.97	29.88	.09
10	15.54	15.36	.18

As indicated, Equations (4), (5), and (6) can be used to calculate and check the offtracking of a given design, particularly when different steering gains are used front and rear. However, it is useful to consider some special cases, in order to derive certain insights and some rules of thumb regarding the requirements for good or perfect tracking.

Consider the case where $G_f = G_r = G$, such that the turn center is located opposite the mid-wheelbase and the rear wheels track the front wheels in a steady, zero-speed turn. Let us assume, first, a symmetric trailer, such that $X_{PH} = X_{KP} = X$. (This arrangement produces a trailer that can be towed from either end.) Equations (4), (5), and (6) reduce to the following expressions, viz.:

$$R_{PH_f}^2 = R_{KP}^2 + X_{DB}^2 + 2 \cdot R_{KP} \cdot X_{DB} \cdot \cos\{\Gamma + \tan^{-1}[\ell/2 \tan G\Gamma(X + \ell/2)]\} \quad (7)$$

$$R_{KP}^2 = [\ell/2 \tan G\Gamma]^2 + [\ell/2 + X]^2 \quad (8)$$

$$R_{PH_r}^2 = [\ell/2 \tan G\Gamma]^2 + [\ell/2 + X]^2 = R_{KP}^2 \quad (9)$$

For zero offtracking, Equation (7) must be equal to Equation (9), and, on substituting Equation (8) into Equation (7), one finds that

$$0 = X_{DB}^2 + 2X_{DB} \{[\ell/2 \tan G\Gamma]^2 + [\ell/2 + X]^2\}^{1/2} \cos \{\Gamma + \tan^{-1}[\ell/2 \tan G\Gamma(X + \ell/2)]\} \quad (10)$$

Equation (10) must be satisfied for all values of articulation angle, Γ , if perfect tracking is to be established (i.e., $R_{PH_r} = R_{PH_f}$) by a symmetrical trailer, viz., $G_f = G_r$ and $X_{KP} = X_{PH}$. Nevertheless, for a given wheelbase, ℓ , three design variables, X_{DB} , X , and G , remain to be selected such that the geometrical requirement specified by Equation (10) is met. Accordingly, it is useful to make further simplifications to obtain more physical insight than is offered by Equation (10).

Assume now that $X_{PH} = X_{KP} = X = 0$, and that $G = 0.5$. Equation (10) reduces to the following expression:

$$0 = X_{DB} + \ell \{[1 + (1/\tan.5\Gamma)^2]^{1/2} \cos [\Gamma + \tan^{-1}(1/\tan.5\Gamma)]\} \quad (11)$$

Examination of the quantity within the braces shows that, irrespective of the value of Γ , this quantity always has the value of minus one. Accordingly, Equation (11) reduces to the simple result that

$$X_{DB} = \ell$$

when

$$X_{KP} = X_{PH} = X = 0$$

and

$$G_f = G_r = G = 0.5$$

This finding corresponds to the geometry diagrammed in Figure A-2. We observe that

$$\phi = 180^\circ - \Gamma - (90^\circ - .5\Gamma)$$

or

$$\phi = 90^\circ - (1-.5)\Gamma = 90^\circ - .5\Gamma$$

Further, we note that $R_{PH_r} = R_{KP} = R_{PH_f}$ for perfect tracking. On applying the law of cosines, we see that

$$R_{PH_f}^2 = R_{PH_r}^2 + X_{DB}^2 - 2R_{PH_r} X_{DB} \cos \phi \quad (12)$$

Since

$$\cos(90^\circ - .5\Gamma) = \sin .5\Gamma$$

and

$$\sin .5\Gamma = \ell/2 R_{PH_r}$$

Equation (12) reduces to

$$X_{DB} = \ell$$

Consider now the case where $X_{KP} = 0$ and $G_f = G_r = 1.0$, but X_{PH} is finite and positive as shown in Figure A-3. From the indicated geometry, we see that

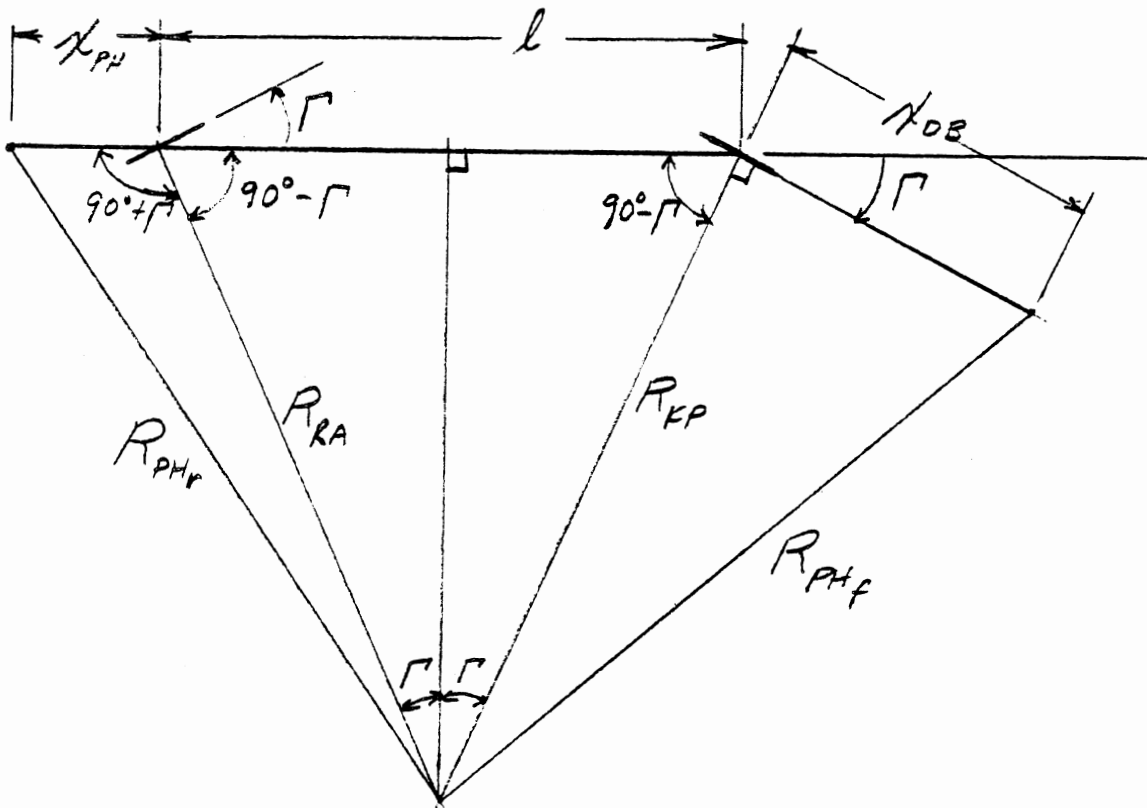


Figure A-3. Trailer geometry providing perfect tracking when $X_{KP} = 0$, $G_f = G_r = 1.0$, and $X_{PH} > 0$.

$$R_{KP} = R_{RA}$$

where

R_{RA} = length of radius from the turn center to the rear axle

We also note that

$$R_{PH_f}^2 = R_{KP}^2 + X_{DB}^2 \quad (13)$$

and

$$R_{PH_r}^2 = X_{PH}^2 + R_{RA}^2 - 2X_{PH} R_{RA} \cos(90^\circ + \Gamma) \quad (14)$$

Since

$$\cos(90^\circ + \Gamma) = -\sin \Gamma$$

and

$$\sin \Gamma = \ell/2 R_{KP} = \ell/2 R_{RA},$$

Equation (14) reduces to

$$R_{PH_r}^2 = X_{PH}^2 + R_{RA}^2 + X_{PH} \ell \quad (15)$$

On equating Equations (13) and (15), we obtain the relationship

$$X_{DB}^2 = X_{PH}^2 + X_{PH} \ell$$

to be satisfied for perfect tracking. If, for example, we let

$$X_{PH} = \ell/4$$

then we find that a drawbar length of

$$X_{DB} = \sqrt{5}/4 \ell = .559\ell$$

will cause the rear pintle to track the front pintle when $X_{KP} = 0$ and $G_f = G_r = 1.0$. However, in contrast to the geometry diagrammed in Figure A-2, where the axle centers track the path of the pintles, Figure A-3 shows that the axle centers will track inside the path of the pintles.

Consider a third case, in which $X_{KP} = 0$, $G_f = G_r = 0.5$, and $X_{PH} > 0$. The applicable geometry is shown in Figure A-4. By means of the law of cosines, we see that

$$R_{PH_f}^2 = R_{KP}^2 + X_{DB}^2 - 2R_{KP}X_{DB} \cos \phi$$

and

$$R_{PH_r}^2 = R_{RA}^2 + X_{PH}^2 - 2X_{PH}R_{RA} \cos(90^\circ + .5\Gamma)$$

Since

$$R_{KP} = R_{RA}$$

$$\phi = 90^\circ - .5\Gamma$$

$$\cos \phi = \cos(90^\circ - .5\Gamma) = \sin(.5\Gamma)$$

and

$$\cos(90^\circ + .5\Gamma) = -\sin(.5\Gamma),$$

we find that zero offtracking requires that

$$X_{DB}^2 - X_{DB}l = X_{PH}^2 + X_{PH}l$$

Solving for X_{DB} yields:

$$X_{DB} = l/2 + \sqrt{l^2/4 + X_{PH}^2 + X_{PH}l}$$

If we, again, assume $X_{PH} = l/4$, we find that

$$X_{DB} = \ell/2 + \sqrt{\ell^2/4 + \ell^2/16 + \ell^2/4} = 5\ell/4,$$

demonstrating that reductions in steering gain require longer drawbars to produce zero offtracking.

Consideration of other special cases does not appear to offer insight over and above that provided by the special cases just considered. Examination of Figures A-1 through A-4 shows that the designer can reduce the length of the required drawbar by increasing the steering gain at the expense of increased offtracking of the axle centers. On the other hand, the designer can also attempt to equalize the radial distances to the fore and aft pintles by reducing the steering gain at the front wheels and increasing the gain at the rear wheels. If he/she chooses to vary X_{KP} , X_{PH} , X_{DB} , G_f , and G_r to find a geometrical layout conducive to good tracking, it is necessary to resort to Equations (4), (5), and (6) to determine the magnitudes of the design variables which produce either zero or a finite offtracking error.

It should be noted that this analysis of the offtracking of a four-wheel steering trailer reduces to an analysis applicable to a front-wheel-steering trailer by letting $G_r = 0$. To simplify matters, consider a trailer whose rear pintle and central kingpin are located such that $X_{PH} = X_{KP} = 0$. To obtain perfect tracking, i.e., with respect to the rear pintle tracing the path of the forward pintle, Figure A-5 shows that it is necessary for X_{DB} to be equal to ℓ and for G_f to be 0.5. Clearly, in this instance, the front axle center will track on a larger radius than is tracked by the fore and aft pintles.

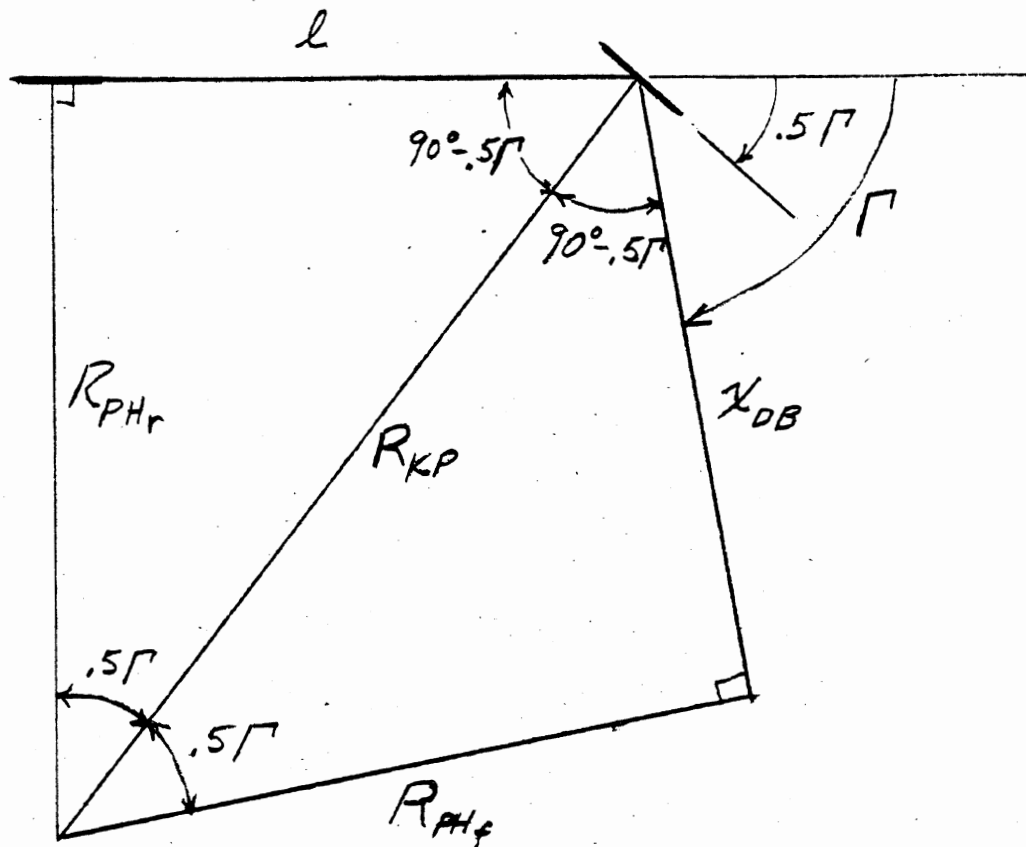


Figure A-5. Geometry of a front-wheel steering trailer which results in $R_{PHr} = R_{PHf}$ when $X_{KP} = X_{PH} = 0$.

APPENDIX B

TRANSFER FUNCTIONS DESCRIBING TRAILERS WITH DRAWBAR STEERING SYSTEMS

The equations developed here pertain to full trailers whose wheels are steered in proportion to the articulation angle between the drawbar and the longitudinal axis of the trailer. This work is an extension of earlier results [2] and it has increased our understanding of the importance of drawbar length [3]. As in the earlier work, the primary assumption is that the forces at the pintle hitches are small, that is, the trailer is steered by the motion of its forward pintle hitch and a full trailer applies a negligible force to the unit ahead of it in a vehicle train.

The basic quantities used in this analysis are illustrated in Figure B1. The quantities shown in Figure B1 have been used to develop the following equations describing a "drawbar steering system."

Equations Describing Drawbar Steering

$$\Gamma = \frac{y_A - y_K}{X_{DB}} - \psi \quad (B1)$$

$$y_K = y_i + (X_{if} + X_{KP})\psi \quad (B2)$$

$$\delta_f = G_f \Gamma \quad (B3)$$

$$\delta_r = G_r \Gamma \quad (B4)$$

With this type of steering, the slip angles of the front and rear wheels are as follows:

$$\alpha_f = v/r + X_{if}r/u - \delta_f \quad (B5)$$

$$\alpha_r = v/u - X_{ir}r/u + \delta_r \quad (B6)$$

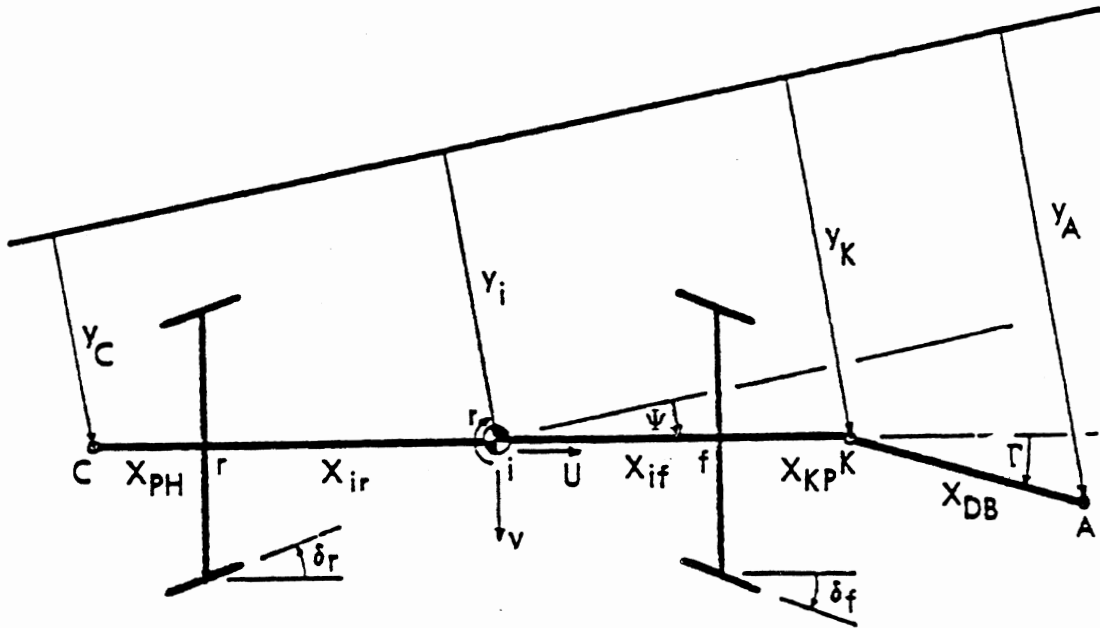


Figure B1. Illustration defining basic variables and geometric quantities.
 (Note: δ_r is shown with a positive value corresponding to $G_r > 0$.)^r

In writing the equations of motion, describing the response of the full trailer to the motion of point A (that is, y_A), it is convenient to use so-called "stability derivatives." In this context, the treatment of multiple axles at the front or the rear of the trailer is readily incorporated in the stability notation. The equations of motion (see Table B1) may be treated in two parts:

- 1) the motion of y_i with respect to that of y_A , and
- 2) the motion of y_c with respect to that of y_i .

Table B1. Linearized Equations of Motion for a Full Trailer

(1) y_i with respect to y_A

$$m(\dot{v}+ur) = -F_v v - F_r r - F_\psi \psi + F_y (y_A - y_i) \quad (B7)$$

$$I\dot{r} = -T_v v - T_r r - T_\psi \psi + T_y (y_A - y_i) \quad (B8)$$

$$\dot{\psi} = r \quad (B9)$$

$$\dot{y} = v + u\psi \quad (B10)$$

where

m is the trailer mass

I is the trailer yaw moment of inertia

u is the forward velocity

v is the sideslip (lateral velocity)

r is the yaw rate

ψ is the heading angle

y_i is the lateral displacement of the c.g.

y_A is the lateral displacement of the forward pintle hitch

The stability derivatives are as follows:

(Note that " Σ " means the summation over all wheels, " Σ_f " means summation over all front axles, and " Σ_r " means summation over all rear axles. Also, the " C_α 's" are tire cornering stiffnesses.)

$$F_v = \Sigma C_\alpha / u \quad (B11)$$

$$F_r = T_v = (\Sigma_f X_{if} C_{\alpha f} - \Sigma_r X_{ir} C_{\alpha r}) / u \quad (B12)$$

$$F_y = (\Sigma_f G_f C_{\alpha f} + \Sigma_r G_r C_{\alpha r}) / X_{DB} = F_\Gamma / X_{DB} \quad (B13)$$

$$F_\psi = X_{iA} F_y \quad (B14)$$

$$T_r = (\Sigma_f X_{if}^2 C_{\alpha f} + \Sigma_r X_{ir}^2 C_{\alpha r}) / u \quad (B15)$$

$$T_y = (\Sigma_f G_f X_{if} C_{\alpha f} + \Sigma_r G_r X_{ir} C_{\alpha r}) / X_{DB} = T_\Gamma / X_{DB} \quad (B16)$$

$$T_\psi = X_{iA} T_y \quad (B17)$$

Table B1 (Cont.)

where X_{iA} is the distance from point i to point A
 and X_{ir} and X_{if} define the distances from the c.g. to the
 rear axles and to the front axles, respectively.

(2) y_c with respect to y_i

Rewriting (B7) and (B8),

$$m(\dot{v}+ur) = -F_v v - F_r r + F_\Gamma \Gamma \quad (B18)$$

$$I\dot{r} = -T_v v - T_r r + T_\Gamma \Gamma \quad (B19)$$

where

$$F_\Gamma = \sum_f G_f C_{\alpha f} - \sum_r G_r C_{\alpha r} = F_y X_{DB} \quad (B20)$$

$$T_\Gamma = \sum_f X_{if} G_f C_{\alpha f} + \sum_r X_{ir} G_r C_{\alpha r} = T_y X_{DB} \quad (B21)$$

$$\ddot{y}_i = \dot{v} + ur \quad (B22)$$

$$\ddot{y}_c = \dot{v} + ur - X_{ic} \dot{r} \quad (B23)$$

The equations given in Table B1 can be transformed to the Laplace or frequency domain and expressed as transfer functions. The results are as follows:

$$(1) \quad Y_{i/A} = N/(N+D) \quad (B24)$$

where

$$N = F_y I s^2 + (F_y T_r - T_y F_r) s + (T_y F_v - T_v F_y) u \quad (B25)$$

$$D = s[(m s + F_v)(I s^2 + T_r s + T_\psi) - (T_v)((m u + F_r) s + F_\psi)] \quad (B26)$$

$$(2) \quad Y_{c/i} = 1 - A \quad (B27)$$

where

$$A = N_A/D_A = X_{ic} s r(s)/(s v(s) + u r(s)) \quad (B28)$$

which reduces to

$$N_A = - X_{ic} s(m T_r s + F_v T_r - T_v F_r) \quad (B29)$$

$$D_A = F_r I s^2 + (T_r F_r - T_r F_r) s + (F_v T_r - T_v F_r) u \quad (B30)$$

Special cases of particular interest are (1) the "neutral steer" trailer for which $F_r = T_v = 0$ (this applies to trailers with equal loads and tires at all wheel positions) and (2) trailers with traditional dollies for which $G_f = 1$ and $G_r = 0$.