

COMPARISON BETWEEN KELLER'S AND UFIMTSEV'S  
THEORIES FOR THE STRIP

T. B. A. Senior and P. L. E. Uslenghi  
Radiation Laboratory, The University of Michigan  
Ann Arbor, Mich. 48108

Abstract

The high-frequency backscattered far field produced by a plane electromagnetic wave obliquely incident on a perfectly conducting infinite strip is considered. A comparison between the results obtained by applying Keller's and Ufimtsev's asymptotic theories is performed. It is shown that Ufimtsev's expansion is incorrect beyond the leading term, but that the discrepancy is numerically small. The implications in caustic matchings for more complicated shapes, such as finite cones and disks, are briefly discussed.

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The geometrical theory of diffraction of Keller is a systematic improvement of geometrical optics that permits the direct determination of the high-frequency field expansion at any distance from the scattering body. In contrast, the asymptotic theory of Ufimtsev attempts a systematic improvement of the physical optics surface currents, by introducing "non-uniform" contributions and the far field is subsequently obtained by integration over the surface of the scatterer. A discussion of the two theories may be found in Beckmann [1]. Since both theories are heuristic in nature, a preferential choice between them can only be made by comparing their predictions with the asymptotic expansion of the exact solution to the scattering problem. This comparison is possible for a very limited number of geometrically-simple bodies for which both the exact solution and its high-frequency approximation are known: one such body is the strip, and is considered in the following.

We examine the two-dimensional backscattering of a plane electromagnetic wave obliquely incident on a perfectly conducting strip occupying the region  $(-a \leq y \leq a ; -\infty < z < \infty)$  of the  $x = 0$  plane. The primary fields are given by

$$\left. \begin{array}{l} \underline{E}_{\text{inc.}} \\ \underline{H}_{\text{inc.}} \end{array} \right\} = \hat{z} \exp \left\{ ik(x \cos \Phi + y \sin \Phi) \right\}, \text{ for } \left\{ \begin{array}{l} E \\ H \end{array} \right\} \text{ polarization,} \quad (1)$$

where the time-dependence factor  $e^{-i\omega t}$  is omitted. The backscattered far fields can be written as

$$\left. \begin{array}{l} \underline{E}_{\text{b.s.}} \\ \underline{H}_{\text{b.s.}} \end{array} \right\} = \hat{z} \begin{array}{l} P_E \\ P_H \end{array} \sqrt{\frac{2}{\pi k \rho}} e^{ik\rho - i\frac{\pi}{4}}, \quad (2)$$

for E and H polarizations respectively, where  $\rho$  is the distance of the observation point from the center of the strip.

At high frequencies ( $ka \gg 1$ ), the far-field coefficients  $P_E$  and  $P_H$  can be asymptotically expanded in inverse fractional powers of  $ka$  to yield:

$$P_E = -\frac{\sin(2ka \sin \Phi)}{2 \sin \Phi} - \frac{i}{2} \cos(2ka \sin \Phi) - \frac{\exp(i2ka + i\frac{\pi}{4})}{8ka \sqrt{\pi ka} \cos \Phi} F_E + O\left\{(ka)^{-\frac{5}{2}}\right\}, \quad (3)$$

$$P_H = \frac{\sin(2ka \sin \Phi)}{2 \sin \Phi} - \frac{i}{2} \cos(2ka \sin \Phi) - \frac{\exp(i2ka - i\frac{\pi}{4})}{\sqrt{\pi ka} \cos \Phi} F_H + O\left(\frac{1}{ka}\right). \quad (4)$$

Aside from an incorrect order term in (3), expressions (3) and (4) were derived using the geometrical theory of diffraction by Karp and Keller [2], who found that

$$F_E = F_H = 1. \quad (5)$$

The results (3 - 5) are undoubtedly correct. For E polarization, they have been independently derived by Lüneburg and Westpfahl [3] and by Millar [4], whereas for H polarization they follow as particular cases of more general formulas obtained by Karp and Russek [5] and by Khaskind and Vainshteyn [6]: for a detailed discussion, see [7].

Alternative expansions for  $P_E$  and  $P_H$  have been provided by Ufimtsev [8], whose formulas contain Fresnel-type integrals. We have expanded these integrals for  $ka \rightarrow \infty$ , and have obtained the expressions shown in (3) and (4) with

$$(F_E)_{Uf.} = \frac{1}{\sqrt{2} \cos \frac{\Phi}{2}}, \quad (F_H)_{Uf.} = \frac{1 + \frac{1}{2} \cos \Phi}{\sqrt{2} \cos \frac{\Phi}{2}}. \quad (6)$$

Results (6) differ from (5) and are therefore incorrect. However,  $(F_E)_{Uf.}$  increases monotonically from a minimum value of 0.707 at  $\Phi = 0$  to a maximum value of unity at  $\Phi = \pi/2$ , whereas  $(F_H)_{Uf.}$  decreases monotonically from a maximum value of 1.06 at  $\Phi = 0$  to a minimum value of unity at  $\Phi = \pi/2$ . Thus, even though Keller's and Ufimtsev's expansions for the strip do not agree beyond the leading term, the discrepancy between the second-order terms is numerically small.

For other scattering shapes, the differences between the two theories can be more pronounced. For example, Ufimtsev's second-order term in the oblique back-scattering from a circular disk of radius  $a$  exhibits a dependence on both  $ka$  and  $\Phi$  (see [9]) which differs from that provided by Keller's theory. In all cases in which an exact solution to a scattering problem is known, its asymptotic expansion has always been found to coincide with the predictions of the geometrical theory supplemented, when necessary, by boundary-layer techniques. We therefore conclude that Ufimtsev's theory does not, in general, lead to the correct asymptotic expansion, even though it may yield good numerical estimates.

An erroneous angular dependence in the second-order terms of Ufimtsev's expansions is especially inconvenient when a uniform transition is desired between the on-axis (caustic) expansion and the off-axis expansion of the field backscattered from a body of revolution such as, for example, a finite right circular cone or a circular disk [9 - 11]. In such cases, the functional dependence of the second-order terms on the angle  $\Phi$  between the direction of observation and the axis of symmetry of the scatterer is crucial in determining the choice of special functions (Bessel functions, Fresnel integrals, etc.) which appear in the uniform far-field expansion.

Finally, we point out that in a recent work [12] Ufimtsev recognizes that his previous results [7] are incorrect beyond the leading term. He succeeds in finding the correct higher-order terms only by employing function-theoretic results.

### References

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