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OPPONENT-PROCESS ADDITIVITY—I: RED/GREEN EQUILIBRIA[1]

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Abstract—A red/green equilibrium light is one which appears neither reddish nor greenish (i.e. either uniquely yellow, uniquely blue, or achromatic). A subset of spectral and nonspectral red/green equilibria was determined for several luminance levels, in order to test whether the set of all such equilibria is closed under linear color-mixture operations.

The spectral loci of equilibrium yellow and blue showed either no variation or visually insignificant variation over a range of 1–2 $\log_{10}$ unit. There were no trends that were repeatable across observers. We concluded that spectral red/green equilibria are closed under scalar multiplication; consequently they are invariant hues relative to the Bezold–Brücke shift.

The additive mixture of yellow and blue equilibrium wavelengths, in any luminance ratio, is also an equilibrium light. Small changes of the yellowish component of a mixture toward redness or greeness must be compensated by predictable changes of the bluish component of the mixture toward greenness or redness. We concluded that yellow and blue equilibria are complementary relative to an equilibrium white; that desaturation of a yellow or blue equilibrium light with such a white produces no Abney hue shift; and that the set of red/green equilibria is closed under general linear operations.

One consequence is that the red/green chromatic-response function, measured by the Jameson–Hurvich technique of cancellation to equilibrium, is a linear function of the individual's color-matching coordinates. A second consequence of linear closure of equilibria is a strong constraint on the class of combination rules by which receptor outputs are recoded into the red/green opponent process.

INTRODUCTION

In color-matching the two physical manipulations of light are additive mixture (wavelength by wavelength summation of the two spectral energy–density functions) and scalar multiplication (insertion or removal of neutral density filters, i.e. multiplication of the spectral energy–density function by a constant). Lights will be denoted by $a, b, c, \ldots$ and the two physical manipulations will be denoted by $\oplus$ and $*$. The additive mixture of a and b is denoted $a \oplus b$; the multiplication of a by a scale factor t (>0) is denoted $t * a$. The equivalence relation of metameric matching will be denoted by $a \sim b$.

Grassmann's laws (1853–4) include the invariance of metameric matching with respect to the operations $\oplus$ and $*$. More precisely:

(1) if $a \sim b$, then $t * a \sim t * b$;

(2) $a \sim b$ if and only if $a \oplus c \sim b \oplus c$.

A color theory must include not only the facts of metameric matching, but also those of color appearance. It is reasonable to ask whether there are equivalence relations different from $\sim$, based on color appearance, that also satisfy Grassmann's invariance laws.

Hering (1878) proposed that any hue can be described in terms of its redness or greenness and its yellowness or blueness. Moreover, red and green appear to be opposite poles of one aspect of hue, since one cannot experience both in a single color; and the same holds for yellow and blue. These two bipolar aspects are independent: red can be experienced simultaneously with either yellow or blue; and similarly for green.

The quantitative investigation of Hering's opponent-process theory began with an experiment by Jameson and Hurvich (1955). They measured the amount (in terms of intensity) of a standard light that had to be added to a spectral light to just cancel out the spectral light's redness, greenness, yellowness, or blueness. For example, to cancel the redness in a short-wavelength (violet) or a long-wavelength (orange) spectral light, a standard green was added to the spectral light. A nonreddish spectral light (i.e. one that is either greenish or uniquely yellow or blue) of course could not be cancelled by a green standard; if the light is greenish, then a red cancellation standard must be used. Jameson and Hurvich called the function which measures the intensity of the cancellation standard

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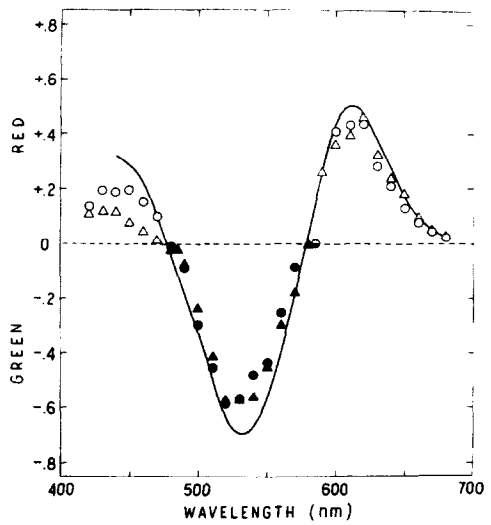

Fig. 1. Opponent-cancellation coefficients for redness (open symbols) and greenness (filled symbols) for observers H (circles) and J (triangles), for an equal-energy spectrum. Data replotted from Jameson and Hurvich (1955). The solid line is a linear functional for the CIE Standard Observer (Judd, 1951).

that has to be added to each spectral light to cancel its greenness or redness the chromatic-response function of the red/green opponent process. Their measurements for two observers are shown in Fig. 1. In a similar manner they measured the chromatic-response function of the yellow/blue opponent process, this time using standards that were yellow or blue (see Fig. 2). The solid curves in Figs. 1 and 2 are linear functions of the CIE tristimulus coordinates, proposed by Judd (1951) to describe the Hering theory.

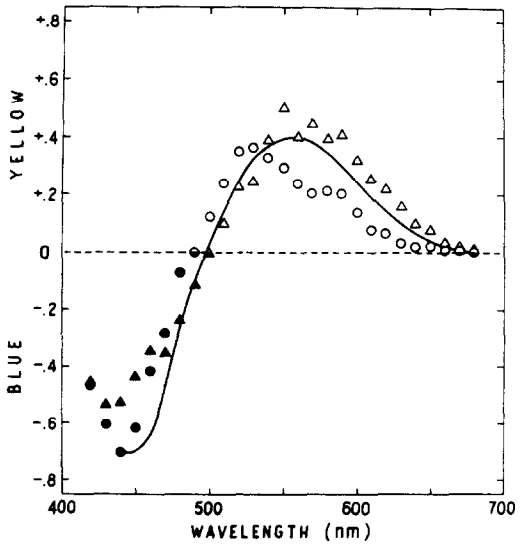

Fig. 2. Yellow/blue cancellation coefficient (details as in Fig. 1).

When the redness in a violet light has been cancelled by adding green, the endpoint of this procedure is a bluish white light that is neither reddish nor greenish; we call this light a red/green equilibrium color. Cancelling the redness of an orange light produces a yellowish white, which is also a red/green equilibrium color. Similarly, the cancellation endpoint obtained by cancelling the yellow in an orange light is a reddish color that is neither yellowish nor bluish. This is an example of a yellow/blue equilibrium color. A cancellation endpoint, or equilibrium color, is one that is either uniquely yellow, blue, or achromatic in the case of red/green cancellation; or uniquely red, green, or achromatic in the case of yellow/blue cancellation. We denote the set of all red/green equilibria by A_1 and the set of all yellow/blue equilibria by A_2.

Hurvich and Jameson implicitly assumed that each set of equilibria is closed under the linear operations of scalar multiplication and addition:

(i) if a is in A_i, then $t * a$ is in A_i $(i = 1, 2)$;
(ii) if a is in A_i, then b is in A_i if and only if $a \oplus b$ is in A_i $(i = 1, 2)$.

Krantz (1974) discussed these assumptions and showed that the two closure properties hold if and only if the corresponding chromatic-response function is a linear function of the tristimulus values based on metameric matching. More precisely, (i) and (ii) hold for A_i if and only if there exists a real-valued linear function ϕ_i such that

$$b \text{ is in } A_i \text{ if and only if } \phi_i(b) = 0.$$

The function ϕ_i is linear relative to color-mixture operations; that is, for any lights a, b and any s, $t \geq 0$,

$$\phi_i[(s * a) \oplus (t * b)] = s\phi_i(a) + t\phi_i(b). \qquad (1)$$

The function ϕ_i is consequently a linear function of any set of colorimetric primaries. It is measurable by the cancellation method; in fact, under these circumstances, the chromatic-response function (ϕ_i as a function of wavelength, for an equal-energy spectrum) is independent (except for a scale constant) of both the luminance level at which the measurement is made and the choice of the cancellation light.

Another way to regard (i) and (ii) is in terms of the classical Bezold–Brücke and Abney hue shifts. Property (i) asserts that equilibrium colors remain so (and thus show no Bezold–Brücke hue shift) with changing luminance; while property (ii) asserts that equilibrium colors remain so under desaturation with other equilibrium colors, in particular, under desaturation with a true equilibrium white. Thus, the equilibrium colors exhibit no Abney hue shift. In particular, the yellowish and bluish equilibrium wavelengths must remain in red/green equilibrium when they are used to mutually cancel yellow/blue; therefore, a suitable mixture of them is achromatic. In short, opposite-hued equilibria are complementary, relative to a properly chosen equilibrium white.

The cancellation experiment allows us to define new ways in which two lights can be equivalent. If two

lights are cancelled by the same light, then we call them cancellation-equivalent. Furthermore, if closure properties (i) and (ii) hold, then multiplying each light of a cancellation-equivalent pair by a scalar, or adding a third light to each will not destroy the cancellation-equivalence. In other words, the Grassmann laws (1) and (2) hold for cancellation-equivalence, if (i) and (ii) hold for cancellation endpoints. Cancellation-equivalence corresponds to ϕ_i-equality.

It is important to note that cancellation equivalence does not mean perceived equivalence. For example, if we desaturate a red light with an appropriate equilibrium white light, the desaturated light remains cancellation-equivalent to the original one, even though its apparent redness decreases.

Some evidence for closure property (i) is provided by the invariance of unique hues with respect to the Bezold–Brücke shift (Purdy, 1931). But there have been hardly any tests of (i) for nonspectral lights, nor has (ii) been tested directly. An indirect test for both (i) and (ii) is provided by the fit of Judd's linear functions to the Jameson and Hurvich cancellation data (Figs. 1 and 2). But this test is inconclusive. Among other things, the two observers differed from each other and from the ones used to establish the C.I.E. Standard Observer. Therefore, direct tests of (i) and (ii) are of interest. The tests are also interesting because of the implications of linearity for the physiology of opponent-color coding.

In this and the next paper (Larimer, Krantz and Cicerone, 1974), the closure properties (i) and (ii) are tested for both of the opponent processes. They are tested by determining the spectral equilibrium colors (i.e., equilibrium or "unique" blue, green and yellow wavelengths) and a non-spectral equilibrium red, at several luminance levels. Then the results of adding equilibrium yellow and blue, or equilibrium red and green, are examined. As Krantz (1974) has shown, these tests of the closure properties are exhaustive, since the spectral unique colors and any unique red act as primaries with respect to cancellation-equivalence. In other words, the closure properties (i) and (ii) hold if and only if the set of equilibrium lights lies on a straight line in the chromaticity diagram based on the individual's color-matching space. Therefore it suffices to test whether these four equilibrium points are invariant with luminance and whether all mixtures of them are in turn equilibrium points.

METHODS

Apparatus

The stimuli consisted of various intensity combinations of three monochromatic beams, superimposed optically by two beamsplitting cubes, as shown in the schematic diagram of Fig. 3. The field of view consisted of a circle, in Maxwellian view, subtending a visual angle of 2·6°; except for this circle, the field of view was dark.

The three monochromatic beams were produced by Bausch and Lomb grating monochromators with appropriate stray-light filters. Monochromators M1 and M2 (Fig. 3) were 500 mm instruments (cat. No. 33-86-45) illuminated by 18 A 100 W tungsten ribbon-filament lamps (underrun at

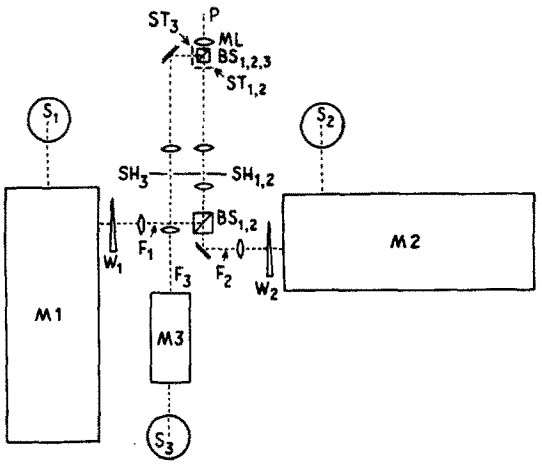

Fig. 3. A schematic diagram of the optical system. One channel combines beams from grating monochromators M1, M2 at beamsplitter $BS_{1,2}$. The second channel has a single monochromatic input from grating monochromator M3. S_1, S_2, S_3 are tungsten sources; W_1, W_2 neutral-density wedges; F_1, F_2, F_3 stray-light and neutral-density filters; $SH_{1,2}$ and SH_3 shutters; $ST_{1,2}$ and ST_3 stops. The channels are combined at beam-splitter $BS_{1,2,3}$ and the Maxwellian view is provided by lens ML. All 3 exit slits are imaged at P, the observer's pupil. (The horizontal distances are shown nearly to scale but the M3 channel is simplified by omission of a vertical segment of the light path and two auxiliary lenses.)

15 A). Light from M1 and M2 was mixed by beamsplitting cube $BS_{1,2}$, focused in the plane of shutter $SH_{1,2}$, and then projected onto a stop $ST_{1,2}$ with a circular aperture. Monochromator M3 was a Bausch and Lomb high-intensity (cat. No. 33-86-25-02) instrument, illuminated by a 6·6 A, 45 W tungsten iodine coil-filament lamp (underrun at 6 A). Light from M3 was focused at shutter SH_3 and then projected onto stop ST_3. The circular apertures of $ST_{1,2}$ and ST_3 were superimposed by beamsplitting cube $BS_{1,2,3}$, combining all three beams. The Maxwellian lens ML focused the exit slits of all three monochromators in register in the center of the observer's natural pupil. (The observer stabilized his head by biting on a dental impression.)

Wavelength calibrations were performed by centering the visible lines of a Hg vapor lamp on the exit slits of M1 and M2, and by passing light from M2 through M3. The half-band-widths were 6·6 nm for M1 and M2, 6·4 nm for M3.

Intensity was controlled by Kodak Wratten No. 96 circular wedges (W_1, W_2 for M1 and M2) and Wratten No. 96 filters placed at F_1, F_2, and F_3. Wedges and filters were calibrated every 25 nm *in situ*, with stray light filters in place, using a Photovolt photomultiplier tube.

By opening both shutters simultaneously, any desired combination of intensities, for any three wavelengths, could be presented as a stimulus.

Retinal illumination was measured at 580 nm by using the exit-slit image formed by lens ML to illuminate a test plate and measuring the illumination of the test plate by a (somewhat heterochromatic) match with a Macbeth illuminometer. Other wavelengths were matched in brightness relative to 580 nm using successive brightness judgments, mostly with 20 nm steps. As can be seen from this method, the luminance levels were only approximately equated across wavelengths; but this does not matter for the purposes

of this experiment. Relative luminance, for any fixed wavelength, was however very carefully controlled by careful filter calibrations.

Subjects

Two males and three females all with normal color vision served as observers in this experiment. Two of the female observers, PS and TC, were completely naive about the purpose of the experiment. Observers CC, DK, and JL were naive with respect to their own performance in the task. All observers used their right eyes.

Alignment

Subjects aligned themselves in the apparatus by adjusting the position of their bite bar while viewing a nearly white field of moderate intensity consisting of 650 nm light from M1 or M2 and 505 nm light from M2 or M1. The alignment proceeded first by finding a head position in the beam which maximized the apparent brightness of the circle and which also yielded a first approximation to good focus of the stop $ST_{1,2}$. Small adjustments were then made until no red or green fringes appeared at the edge of the circle. The M3 beam was not used in the experiments on red/green equilibria; its alignment in yellowness/blueness experiments is described in part II of this series.

Procedure

After alignment observers were dark adapted for 10 min after which the experimental session began. Stimuli were exposed for 1 sec with an intertrial interval of 20 sec of darkness. There was no fixation point, so at the beginning of the 1 sec exposure the observer had to make an eye movement to fixate the circle.

Staircase procedure

The sequence of stimulus presentations was determined by a staircase on the wavelength dimension. For example, to determine spectral equilibrium yellow, we might begin a sequence with a light of 565 nm. Generally, observers will call this wavelength greenish; a series of stimuli, say, 565, 570, 575, 580 nm, would generate a sequence of responses "greenish", "greenish", "greenish", "reddish", respectively. At each change of response (e.g. from "greenish" to "reddish") the direction of the staircase is reversed.

In one of the experiments to be described below a staircase was conducted using one monochromator while the other added a fixed-wavelength desaturant to the stimulus. In these experiments the desaturant is called the "addend" and the staircase or variable component to the mixture is called the "variable". Different staircases are defined by different luminance settings of the "variable" and "addend".[2]

In order to reduce the possibility that an observer might anticipate the next stimulus (Cornsweet, 1962; Nachmias and Steinman, 1965) several staircases were run simultaneously and the experimenter would switch among them irregularly, doing only one, two or three trials on a single staircase at one time. Thus the sequence which the observer saw might entail two trials on a low-luminance staircase to determine equilibrium yellow, followed by one trial on a high-luminance staircase to determine equilibrium blue, etc. All observers were uncertain what they would see next. Because of the moderate luminance and short stimulus exposures, the 20 sec intertrial interval was deemed sufficient to prevent adaptation effects across staircases, and it was verified that responses on a particular staircase at the very beginning of a session did not differ from those made after switching among staircases.

Response

The observer's task was to judge whether the stimulus he had seen on a trial was reddish or greenish. Often this judgement was difficult since the stimulus might appear to be neither greenish nor reddish. The observer was instructed to make the "best" guess possible, by responding to very minute hints of redness or greenness.

Practice plays a significant role in an observer's ability to respond to minute amounts of redness or greenness. At first, most observers perform reliably when the step size of the staircase is of the order of 5–10 nm. With practice all the observers in these experiments were able to perform consistently and reliably with a step size of 2–4 nm. Each observer had between 10 and 20 hr of practice under varying luminance conditions at this task before the data reported here were obtained. The practice sessions also provided useful initial estimates of the equilibrium loci.

EXPERIMENT 1: SCALAR MULTIPLICATION FOR SPECTRAL RED/GREEN EQUILIBRIA

Introduction

A direct test of (i), closure under scalar multiplication, was performed by determining the spectral loci (in the yellow and blue regions) of the red/green equilibria, at several luminances. If closure obtains, then these spectral loci should be independent of luminance.

These loci were determined using the staircase procedure described previously. On a single day a determination was made at all luminance levels. The experiment was repeated on each of 4 days. The exact spectral locus for a given luminance and hue on a single day was determined by linear interpolation to the wavelength that would have generated 50 per cent "too red" responses and 50 per cent "too green" responses. All staircases were obtained concurrently.

Results

The four daily determinations at each luminance–hue combination were averaged, and an estimate of the standard error was computed using the between-day variability. A wavelength-by-luminance plot of these loci for each observer is given in Fig. 4. Note that the wavelength scale is expanded in the blue and yellow spectral regions, with a break in between. The horizontal bars at each point are the 80 per cent confidence intervals for the mean based on between-day estimates of the standard error.

For linearity to obtain, the line connecting the various luminance levels at each unique hue should be vertical.

[2] Generally within a staircase brightness was maintained at a constant level with the exception that the luminance of the "variable" varied slightly with wavelength. This variability was due to imperfect compensation of small changes in the energy spectrum of the monochromators in any small region of the spectrum as well as differences in the luminosity functions among the observers. In all cases, these deviations were small.

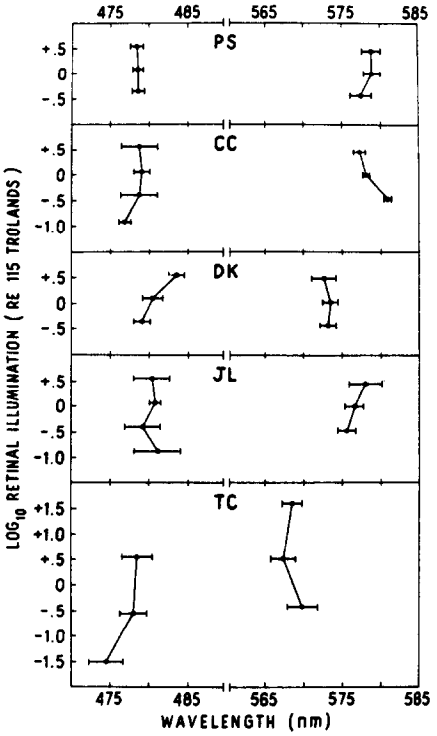

Fig. 4. Mean value of equilibrium blue (left) and equilibrium yellow (right) spectral loci, at various luminance levels, for five observers. Note the break in the wavelength abscissa between 485 and 565 nm. Error bars are 80 per cent confidence intervals based on between-session variability.

Only observers CC at yellow and DK at blue reliably[3] differ from the linearity prediction. For all other observers and for CC at blue and DK at yellow, the spectral loci of the equilibrium hues, yellow and blue, are independent of luminance.

Discussion

Except for CC's equilibrium yellow and DK's equilibrium blue, closure under scalar multiplication was strongly confirmed by this experiment. Had we chosen to plot the data on a 400–700 nm scale, then the deviations that did occur would appear trivial. And, in fact, they are visually trivial. The total shifts exhibited by CC and DK are both less than 5 nm, amounting to less than ± 1 step on the staircases. Within-session and between-session criterion shifts of one step are frequently seen. If one considers that a 0·5 log unit increment in luminance generates a marked increase in brightness, and also alters the saturation of blue or yellow, it is quite possible that the shifts of CC and DK are due to redness/greenness criterion shifts associated with these changes in brightness and saturation.

Furthermore, there is no indication of a trend that holds up across the five observers. (In part II of this series we show that a relatively small yellow/blue nonlinearity is detectable by our methods, since every observer shows the same trend.)

The luminance range spanned in this test covers a substantial part of the most interesting range for color vision. There would not be too much point, at this stage, in pursuing dimmer yellows, which would appear nearly white, or brighter blues (the latter would have been hard to obtain with our apparatus). Moreover, a factor of 10 to 100 is a very significant range for any system to operate linearly.

There is no question that the individual differences between observers, particularly in the yellow, are reliable. They must be caused either by differences in photopigment absorption functions or differences in the coefficients of the photopigments in the linear function ϕ_1.

EXPERIMENT 2: ADDITION OF RED/GREEN EQUILIBRIA

Introduction

A direct test of condition (ii) would be to take the equilibrium yellow and the equilibrium blue wavelengths and mix them in some luminance ratio. If the mixture were judged neither reddish nor greenish, then (ii) would be confirmed.

It is clear from Fig. 4, however, that the concept of "equilibrium wavelength" is a statistical one, subject to moderate day-to-day variability. It is not surprising to find a shift of, say, 3 nm between days; in which case the wavelength that is the 50 per cent point on one day may be judged "reddish" or "greenish," with perhaps 90 per cent consistency, on another day. These fluctuations are most likely due to criterion shifts; as discussed below, the stable criterion is a product of considerable practice. Therefore, it is far more reasonable to test (ii) by fixing one wavelength component (called the addend) and then using the staircase method on the other component (the variable). This method determines an equilibrium mixture with the same experimental design and the same statistical properties that characterized the determinations of single equilibrium wavelengths shown in Fig. 4. The test, then, would be to allow the addend to be (say) a blue equilibrium wavelength and to see whether the equilibrium yellow wavelength is the same with or without the blue addend and independent of the intensities of the blue and yellow components.

A further refinement of this idea was used, however, for three reasons. First, the above test requires prior determination of the equilibrium wavelength to use as addends. This requires temporal separation of the two experiments, with consequent possible criterion variation. Secondly, the above method would make the experiment particularly vulnerable to lack of statistical

[3] A significant variation from the prediction is operationally defined to be any plot where the 80 per cent confidence intervals do not overlap. From a purely statistical standpoint, the error term should be based on days × luminance interaction, but we felt that the between-days error was far more meaningful in the present context.

precision in the initial determination of the equilibrium addend. In Fig. 4, 80 per cent confidence intervals show error possibilities of $\pm 1{\cdot}3$ nm (median value of halfwidths). An error of only 2 nm would be extremely serious if the addend was bright and the variable was dim, since it would have to be compensated by a much larger deviation in the equilibrium staircase (see below). On the other hand, if the addend was dim and the variable was bright, then the 2 nm error in the addend would not matter; but by the same token, the dim addend would be expected to produce only small shifts, even if the additivity hypothesis were grossly wrong. These two problems suggest that the right method is to use not an equilibrium addend, but rather a series of addend wavelengths, on either side of the equilibrium point. One can then plot the 50 per cent point of the variable staircase, as a function of the addend wavelength, and one can test (ii) by observing whether this function goes through the expected point whose abscissa and ordinate are the two equilibrium wavelengths of Fig. 4.

The third reason for this last procedure can be seen if we analyze more precisely the expected results.

Let $a(\lambda)$ denote a light of wavelength λ and unit radiance. Let ϕ_1 be the linear function (equation 1) which is zero for red/green equilibria; in particular, for any mixture of two wavelengths, λ, μ,

$$b = [s * a(\lambda)] \oplus [t * a(\mu)]$$

we have

$$\phi_1(b) = s\phi_1(\lambda) + t\phi_1(\mu)$$

and

$$b \text{ is in } A_1 \text{ if and only if } \phi_1(b) = 0.$$

Here we abbreviate $\phi_1[a(\lambda)]$ by the simpler notation, $\phi_1(\lambda)$. Note however that ϕ_1, though linear in a, is not linear in λ. (The function $\phi_1(\lambda)$ corresponds to the curve in Fig. 1.) Thus, the equilibrium condition for two wavelengths λ, μ at radiances s, t is given by the equation

$$s\phi_1(\lambda) + t\phi_1(\mu) = 0. \tag{2}$$

Equation 2 describes a curve, for fixed s, t, of "complementary" wavelengths (λ, μ) relative to red/green equilibrium. In general, this curve will be nonlinear. If we restrict attention to wavelengths λ in a small region around equilibrium yellow, we expect that μ will vary in a region near equilibrium blue. If λ is below the equilibrium yellow point (greenish yellow) then μ will be below equilibrium blue (reddish blue). As λ becomes larger (orange) μ will also get larger (greenish blue). If $\phi_1(\lambda) = 0$ (equilibrium yellow) then also $\phi_1(\mu) = 0$ (equilibrium blue).

To see the steepness of the μ vs λ curve, we differentiate equation 2 with respect to λ:

$$s\phi_1'(\lambda) + t\phi_1'(\mu)(\mathrm{d}\mu/\mathrm{d}\lambda) = 0,$$

or

$$\frac{\mathrm{d}\mu}{\mathrm{d}\lambda} = -\frac{s\phi_1'(\lambda)}{t\phi_1'(\mu)}. \tag{3}$$

If s is large relative to t, then $\mathrm{d}\mu/\mathrm{d}\lambda$ is large: small shifts of a bright yellow toward greenish or orange must be compensated by large shifts of a dim blue toward violet or blue-green, respectively. Similarly, if the yellow is dim and the blue is bright, the curve of μ vs λ will be shallow.

At the point (λ^*, μ^*), where λ^* is monochromatic equilibrium yellow and μ^* is monochromatic equilibrium blue, the value of $\phi_1'(\lambda^*)\,\phi_1'(\mu^*)$ is a constant (negative, since ϕ_1 has opposite slopes at λ^* and μ^*, see Fig. 1) and therefore by equation 3, $\mathrm{d}\mu/\mathrm{d}\lambda$ at (λ^*, μ^*) is proportional to s/t, the radiance ratio of the yellow and blue components. All the curves for different luminance ratios must intersect at (λ^*, μ^*), with predictable relative slopes.

The third reason, then, for studying a series of addend wavelengths on either side of the equilibrium point is to provide a much more detailed handle on the additivity expressed by equations 1 and 2. In particular, we show that redness and greenness are not merely opponent or antagonistic attributes around equilibrium yellow; the greenness of a greenish yellow is also antagonistic to the redness of a violet-blue, etc., with linear rules of antagonism. Moreover, we get a useful metric to judge deviations from a perfect fit of linearity. A curve may go through (λ^*, μ), where $\mu \neq \mu^*$; but if that error, $\mu - \mu^*$, is small compared to the deviation that would be produced by a very small shift, $\lambda - \lambda^*$, then it can rightly be dismissed as unimportant.

The experimental determinations of μ vs λ curves for red/green equilibrium (Experiment 2) were carried out simultaneously with the determinations of λ^* and μ^* (equilibrium wavelengths) reported in Experiment 1. For each observer at least three and generally four luminance ratios were selected. Both yellowish and bluish addends were chosen at each ratio, and determinations were made at several luminance levels for each ratio. Each data point was determined once on two different days during the course of a 4-day experiment, and the determinations were averaged.

Results

The results of this experiment are presented graphically in Fig. 5 which plots the bluish wavelength, μ, against the yellowish wavelength, λ. The different symbols denote the various luminance ratios of the yellow to blue components of the mixture. The approximate ratios associated with each symbol in $\log_{10}$ units are: (a) X, 1·40; (b) ◇, 0·90 to 1·10; (c) □, 0·45 to 0·50; (d) ○, 0·00; (e) △, −0·45. The position of the bar on the symbols denotes the approximate retinal illumination from the yellow component of the mixture. Upper bar: high luminance; middle bar: medium luminance; lower bar: low luminance. The values of these retinal illuminances for the yellow component were respectively 324, 115 and 41 td. The solid six-pointed star is the locus of the spectral equilibrium hues, (λ^*, μ^*). The coordinates of this point were determined for each observer by averaging across luminance levels the

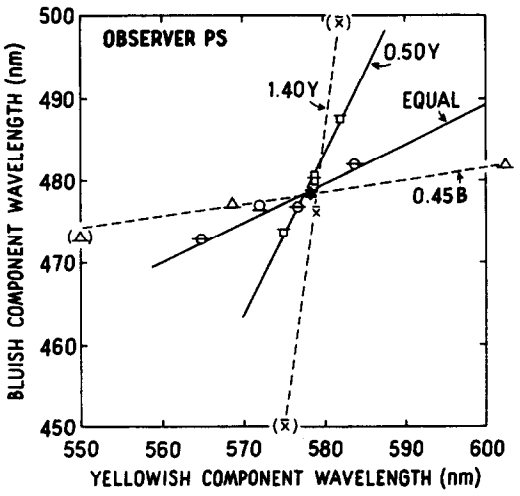

Fig. 5(a)

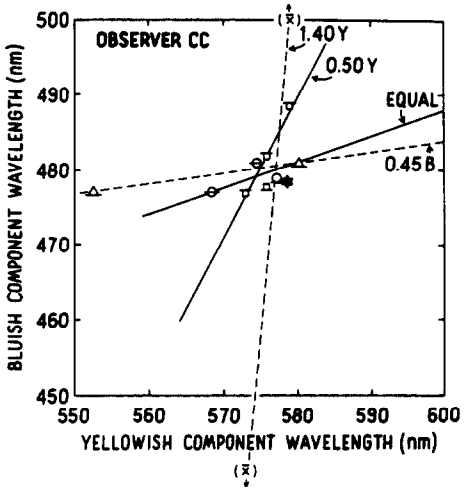

Fig. 5(b)

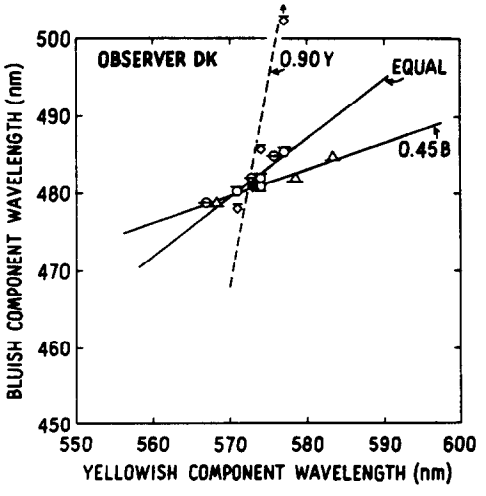

Fig. 5(c)

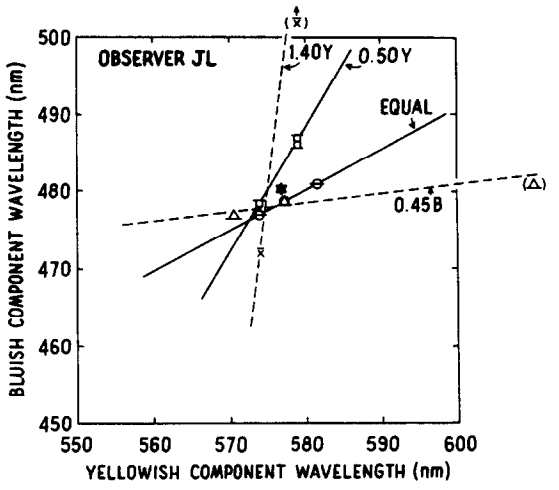

Fig. 5(d)

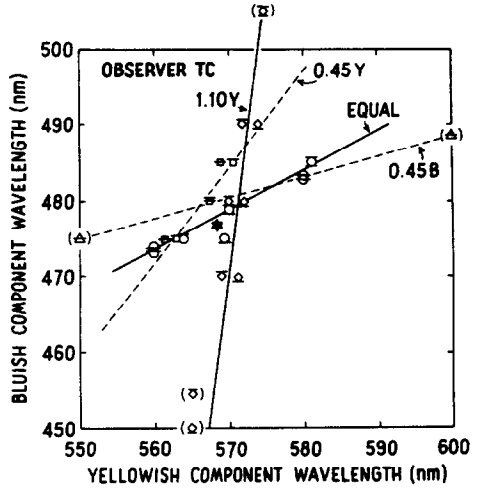

Fig. 5(e)

Fig. 5. Loci of yellow-plus-blue mixtures that are neither reddish nor greenish. Shape of symbol denotes approximate luminance ratio of yellow to blue [$\log_{10}$ (luminance Y/luminance B)]: X—1·40; ◇—0·90–1·10; □—0·45–0·50; ○—0·00; △——0·45. Position of bar on symbol denotes approximate retinal illumination from yellow component. Upper bar: high luminance; middle bar: medium luminance; lower bar: low luminance. (Respectively 324, 115 or 41 td.) Straight lines were fit by eye to each set of points having a fixed luminance ratio. Poorly determined points are indicated by parentheses. Heavy solid lines are more reliably determined than light dashed ones. The monochromatic equilibrium wavelengths (average from Fig. 4, for each observer) are the ordinate and abscissa of the point indicated by the large star. Under linearity, we predict that the locus for a given luminance ratio is independent of the luminance of the yellow component; that the different loci intersect at the star; and that the ratios of their slopes at the star are equal to the ratios of the corresponding luminance ratios.

determination of his spectral equilibria obtained in experiment 1.

Straight lines were fit by eye to each set of points having a fixed luminance ratio. The expected curve [equation (2)] seems to be approximated well enough by a straight line over the relatively small wavelength ranges involved. For two of the log ratios, 1·40 (X) and −0·45 (△), either the yellowish component (X) or the bluish component (△) was several times more intense than the other component. In these mixtures the less intense component slightly desaturated the mixture, but the appearance of the mixture would be described subjectively as either very yellow or very blue. At these ratios a slight shift away from the locus of the unique hue in the more intense component of the mixture caused a dramatic shift in the dim component. The extreme points on these curves were subject to considerable error. Some observers (PS, CC, JL and TC) were not able in every session to adjust the dimmer variable to cancel the redness or greenness in the more intense addend, and these mixtures are indicated by placing parentheses around the data point. We have drawn those lines as dashed to indicate that they are not as well determined as the solid lines which can be considered good linear approximations to the actual curves near the equilibrium hues.

The plots in Fig. 5 strongly confirm the expectation that mixtures in a fixed ratio lie on a single-valued curve and that these curves intersect at the locus of the equilibrium wavelengths, thus confirming property (ii). Property (i) implies that a change of overall luminance of the mixture should have no effect on the wavelength × wavelength locus of the data point. This prediction is also strongly confirmed, and for some observers (i.e. PS, DK, JL and TC) luminance differences of as much as 0·9 $\log_{10}$ units of otherwise identical mixtures yielded virtually congruent data points. Again, the three observers who deviate from the prediction do so in no systematic fashion and by amounts that are visually trivial.

Finally, in Fig. 6, we plot the logarithm of the slope, $d\mu/d\lambda$, against the logarithm of the luminance ratio, s/t, for all 19 lines (5 observers of Fig. 5). From equation 3, we expect that $\log(d\mu/d\lambda) = \log(s/t) - \log|\phi'_1(\lambda)/\phi'_1(\mu)|$. The predicted 45° line fits well, and the estimate value of $\log|\phi'_1(\lambda)/\phi'_1(\mu)|$ varies little across observers. The line drawn in Fig. 6 corresponds to $\phi'_1(\lambda)/\phi'_1(\mu) = -1{\cdot}9$, which is not far from the value that would be estimated from the ratio of the slopes of ϕ_1 around 580 nm and 475 nm in Fig. 1.

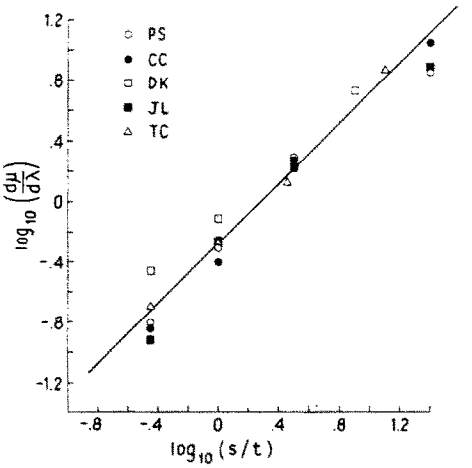

Fig. 6. The logarithm of the slope ($d\mu/d\lambda$) of each line in Fig. 5 is plotted against the logarithm of the luminance ratio (s/t = yellow/blue) that generated the line. From equation 3 the prediction is a 45° straight line with intercept $-\log|\phi'_1(\lambda)/\phi'_1(\mu)|$. The least-squares 45° line is shown.

Discussion

The results of experiment 2 strongly confirm the additivity property and reconfirm the results of experiment 1. When addends were chosen that were equilibria, the corresponding variable was always within 1 or 2 nm of the appropriate equilibrium wavelength determined in experiment 1, and this difference was always well within the between-day variability measured in experiment 1 (e.g. see observer JL's data points (578, 478) for "equal" and −0·45 luminance ratios).

GENERAL DISCUSSION: CONDITIONS FOR ADDITIVITY

Under the conditions of our experiments, additivity holds for red/green equilibria. Under what circumstances will it fail? A complete answer to this question constitutes a broad program of research. Nevertheless, we feel that it is necessary to mention some of the boundary conditions that may be important, at least in order to facilitate others' repeating our results.

Test-field parameters

We used a moderately large (2·6) centrally fixated field, with exposures of 1 sec. Some clue as to how field size, retinal locus, and exposure duration bear on the results may be obtained from previous studies. For example, Ingling, Scheibner and Boynton (1970) and Savoie (1973) failed to obtain results consistent with ours, in studies that involved a small (3′) or a brief (5 msec) test light, respectively. However, the logic of their methods was quite different also (see next section), and that may be as important as the test-field parameters.

Adaptation and surround

These have enormous and fairly well understood effects on color appearance, and hence on the spectral loci of the equilibria. A light which is achromatic or yellow in neutral conditions will appear greenish if the eye is red-adapted or if a reddish surround is present. Not only are the sets of equilibrium and achromatic lights different as a function of adaptation or surround, but possibly their additivity properties will change. For example, if the equilibrium yellow shifts to longer

wavelengths to compensate for an induced greenness, then it is possible that different shifts will be necessary at different luminance levels, thus violating property (i).

With a view to maximizing the chance of additivity being found, we employed a dark-adapted, dark surround condition. Equilibria after chromatic adaptation are studied in Part III of this series.

Fixation

Closely related to the above is the influence of fixation-point color (Jameson and Hurvich, 1967). There may be various ways of minimizing this influence; we chose to eliminate the fixation target. This had the disadvantage of forcing the subject to correct his fixation at the start of an exposure. Our choice of exposure duration and field size were made partly in order to counteract this latter disadvantage.

Perceptual learning

Deciding whether a bright blue light is reddish blue or greenish blue is not easy. For some observers, several sessions of practice are required before they learn what redness and greenness look like in very small amounts combined with blueness. Similar, but less pronounced effects exist for less saturated equilibrium colors, such as yellows or off-whites.

Unfortunately, the early stage of learning may not produce mere noise; it can produce apparent nonadditivity. The reason is this: near the equilibrium blue wavelength, blueness is dropping rapidly with increasing wavelength. Consequently, the greenness of wavelengths slightly longer than equilibrium blue is quite obvious; there is relatively little blueness left above 485 nm to veil the greenness. But on the short-wavelength side redness can be much harder to detect, in an intensely blue light. This is especially marked at higher luminances, where whiteness and blueness join in veiling redness. Consequently, the range of uncertainty is apt to be asymmetric about the "true" equilibrium blue: it may, for example, extend from 460 to 485 nm early in practice, for an observer whose eventual "true" equilibrium blue will be 478 nm. If the statistically determined equilibrium point falls near the midpoint of the uncertain range, say, at 472 nm, then nonadditivity results. For when the imperceptibly reddish 472 nm light is desaturated, with an equilibrium yellow addend, the redness of the mixture is easily detected, and the blue component of the equilibrium mixture promptly shifts to longer wavelengths.

It was quite important to wait for observers to become experienced in picking out small amounts of redness and greenness. We avoided any "reinforcement" of particular results; our instructions emphasized paying close attention to the stimuli and responding in terms of very slight residual redness or greenness, rather than guessing.

As a countervailing precaution, against allowing ourselves to decide that an observer was "asymptotic" with respect to learning when his data just happened to be pretty-looking, we adopted the practice of basing all conclusions of experiments 1 and 2 on a formal four-session experiment, designed and conducted for each observer after we had satisfied ourselves that the learning was complete.

Two other precautions are important. One must expose the observer to a moderately large range of wavelengths in any staircase, so that he sees some clearcut reds and greens. If he is unsure of every response, it becomes harder to pay attention and maintain a redness/greenness criterion. Secondly, one must give the observer an opportunity for additional warm-up sessions after a long layoff.

This perceptual learning phenomenon, and its fragility, indicate that our task has little relation to the conventional overlearned color names "red" and "green." "Reddish" and "greenish" are being used to denote sensations, not to name colors; they are being stretched to their very limit, applying to lights that are, at first glance, entirely yellow, blue, or white.

Statistical treatment of criterion

A final condition that must hold, if additivity is to be observed, is that criterion variability must be taken into account. Before attributing significance to any shift, one must assess its reliability; and even if it is reliable, one must calibrate it against criterion variability. If between-day shifts, at fixed luminance, are about as large as between-luminance shifts, then one must consider the possibility that the latter shifts have no more visual significance than the former, day-to-day fluctuations. An additional criterion is repeatability of shifts over several observers. If the same pattern of variation with luminance shows up for most observers, then it is less likely to be due to criterion shifts. Based on these principles we concluded that deviations from additivity, though sometimes statistically significant, probably have little visual significance.

In a way, precautions that minimize criterion variability are even disadvantageous, since they do not permit a full view of the size of visually irrelevant fluctuations. We felt it was important therefore to mix together staircases from widely varying conditions, within any one session, and to estimate variability on a between-days basis.

PREVIOUS TESTS OF ADDITIVITY

Yellow and blue equilibria

A number of workers have determined yellow and blue equilibrium wavelengths by methods similar to ours. Westphal (1909) used the method of limits to determine spectral loci of "Urblau" and "Urgelb" (psychologically fundamental blue and yellow) at 3 intensities. He also studied the effects of desaturation with "white" (natural daylight). His values for seven color-normal observers averaged about 478·4 and

574·5 nm and varied only slightly and non-systematically over a 1·3 log unit change in intensity. His white desaturant also had no effect. The range of values obtained in his sample was very similar to ours. (We do not know the exact parameters of his test light; his observers were daylight adapted.) Our experiment differs from his chiefly in the use of a more controlled psychophysical method and in its much greater variety of addition experiments (Experiment 2). That the results coincide is reassuring; the only discordant note is that he also measured "fundamental" green and red loci, and his results and conclusions for those determinations are quite different from ours (Larimer, Krantz and Cicerone, 1974).

Purdy (1931) repeated Westphal's experiment on his own eye, obtaining "fundamental" loci of 476 and 576 nm, with only slight variations at 10, 100 and 1000 td. He also made some observations with admixture of a "white" desaturant; he asserted that the hue of 580 nm is unchanged but that 480 and 470 nm become pinkish when white is added.

Boynton and Gordon (1965) used a color-naming method to determine equilibrium loci. Their equilibrium yellow and blue were defined by equality of "red" and "green" color-name scores. Since their color-name scores take salience of a hue component into account (1, 2 or 3 points are possible per trial), one would expect their "equilibrium blue" to occur at shorter wavelengths than those obtained by forced-choice methods and also to shift to even shorter wavelengths at high luminance. This prediction is based on the asymmetry in the salience of redness vs greenness in the 460–485 nm range, due to veiling of redness by blueness (see discussion of perceptual learning, supra). That is what happened, for all three of their observers: their 100-td blue equilibria were 465, 474 and 470 nm, while their 1000-td ones were 463, 462 and 463 nm, respectively. This result could conceivably be due to visual parameters (e.g. they use 0·3 sec flashes and a fixation point) but very likely it is due to the nature of the color-name score as just discussed. For equilibrium yellow, their observers fell within the usual range, and shifts were nonsystematic (575, 573 and 584 nm at 100 td; 573, 584 and 584 nm at 1000 td).

Our results can be seen as confirming the main conclusions of Westphal and Purdy, with a more elaborate multiple-staircase method and perhaps with more precisely specified conditions. We extend their results to a wider range of additive mixtures; and we find that, over such an extended range, the linearity hypothesis yields truly excellent predictions.

Invariant hues

To make clear the relation between our work and the work on hue shifts, we need to introduce an additional bit of formalism. We have already used A_1 to denote the set of all red/green equilibrium colors and A_2 for yellow/blue equilibria. Let C_r denote the set of lights that are constant in hue when radiance changes, and let C_w denote the set of lights that are constant in hue when a particular desaturating light w is added. In other words,

a is in C_r if and only if $a \sim_{HUE} t * a$, for all $t > 0$;

a is in C_w if and only if $a \sim_{HUE} a \oplus (t * w)$, for all $t > 0$.

The sets C_r and C_w are sets of invariant lights, respectively for the Bezold–Brücke effect and the Abney effect (relative to w as desaturant).

Two points should be noted about these newly defined sets. First, the definition depends on an empirical relation of hue matching, denoted here by $\sim_{HUE}$. This is a crucial element for any interpretations. Secondly, C_r or C_w may very well be empty. Nothing excludes this possibility. But A_1 and A_2 are surely not empty; we were bound, in our studies, to identify lights which were equilibrium lights.

To relate the sets A_i to C_r, we need to answer three interrelated questions:

(a) Is C_r nonempty? That is, are there invariant hues with respect to the Bezold-Brücke shift?

(b) Is A_i radiance-invariant, that is, does hypothesis (i) hold?

(c) Is A_i part of C_r?

The logical interrelations among questions (a)–(c) are as follows. Question (c) presupposes an affirmative answer to (a). If (a) and (c) are both answered positively, then obviously (b) is also affirmative. What is slightly less obvious is the converse: if (b) is answered affirmatively, then so are (a) and (c). The reason is found in the definition of hue match. We use the term "hue" in such a way that any two yellows that are neither reddish nor greenish match in hue. That is, any two yellow colors in A_1 are a hue match, and likewise for any two blues or any two whites. Therefore, if A_1 satisfies hypothesis (i), then any yellow light in A_1 that remains yellow (rather than going white or blue) with changes in radiance is in C_r, and likewise for any blue in A_1 that remains blue as radiance changes. A similar argument applies if A_2 satisfies (i). If both A_1 and A_2 satisfy (i), then every equilibrium color, including white, is hue-invariant, so A_1 and A_2 then are entirely contained in C_r.

Naturally, it is logically possible that there are invariant colors that are not equilibrium colors, whether or not hypothesis (i) holds. So it is possible that only (a) would be answered affirmatively.

Previous studies of the Bezold–Brücke effect yield answers to question (a) and only secondarily to question (c). Purdy (1931) found three monochromatic C_r lights (invariant wavelengths) at 474, 506 and 571 nm, as well as a mixture of long and short-wavelength light that was a C_r light (invariant bluish-red). He therefore claimed an affirmative answer to (a). He found that the three monochromatic C_r lights were nearly the same as "fundamental" blue, green and yellow, but that the red C_r light was bluish red, rather than "fundamental" red. For the A_1 equilibria, therefore, he concluded that (c) is affirmative.

Jameson and Hurvich (1951) made a series of invariant wavelength determinations in different adaptation states. For neutral adaptation, blue, green and yellow invariant points were located, confirming Purdy's result. For two observers, all three invariant points fall well within the distribution of our measurements of equilibrium wavelengths: 476, 499, 580 nm and 478, 494, 582 nm. The third observer yielded a more deviant result: 466, 491 and 587 nm. They did not test (c) directly, but the close correspondence to our equilibrium determinations for two observers is suggestive of an affirmative answer to (c).

Boynton and Gordon (1965) used three different criteria for hue matching: simultaneous matching for steady lights, simultaneous matching for 0·3 sec flashes, and a derived "match" based on color naming of 0·3 sec flashes. They found bluish, greenish and yellowish invariant wavelengths by all three methods, but the locations varied considerably with the method, and for no method did all three of their subjects produce invariant points near the equilibrium points. The closest correspondence was for the steady lights, in which Purdy's method was replicated and his results were generally well reproduced.

Savoie (1973) cast doubt on the existence of invariant hues, through his failure to find one in the yellow region of the spectrum, using a staircase hue-matching technique, with 5 msec flashes.

It is possible that this assortment of results corresponds to the variety of viewing conditions employed (bipartite vs homogeneous fields, fixation targets, flashes vs steady viewing, etc.). It is also possible that some of the "invariant" points are not truly invariant. Savoie found nonmonotonic variation of hue with radiance for a constant wavelength, and pointed out that given such nonmonotonicities, it is possible to find spurious "invariant" points if only two intensity levels are used (as was the case in the Hurvich and Jameson and Boynton and Gordon studies).

Our results, however, show that the yellow and the blue equilibrium wavelengths are, to an excellent approximation, radiance invariant. We set out to answer (b), rather than (a) as in the bulk of previous work; but our affirmative answer to (b) implies, as noted above, that (a) and (c) are also to be answered affirmatively, at least in so far as the yellowish and bluish red/green equilibria are concerned. Our results thus confirm Purdy's conclusions, though our initial goal, method, and viewing conditions were all quite different from his.

There is no question that both hue matching and color naming give very valuable information about the general features of the Bezold–Brücke shift. For all three of Boynton and Gordon's methods, and for all their subjects, the short-wavelength end of the spectrum became relatively bluer, compared to red, and the long-wavelength end became relatively yellower, compared to red, as luminance increased. The same results were obtained by Purdy; and indeed, this general qualitative finding can be quickly verified by anyone with access to a monochromator and a 1 $\log_{10}$ unit density filter.

Heteroluminous hue matching is an extremely difficult task, as Purdy noted, and it is possible that certain quantitative results obtained by this method have systematic errors. It is not clear, in general, how to identify "error". One principle, which we favor strongly, is the adoption of a definition of "hue match" such that any two equilibrium yellows are a hue match, and likewise for any two equilibrium greens and any two equilibrium blues. [Please note that adopting this principle does not force the result that (b) is answered affirmatively, since it would still be possible for two equilibria at different luminance levels to have different chromaticities.]

The Boynton and Gordon hue-matching procedures do not always satisfy this criterion. As we previously noted, one of their subjects had an equilibrium yellow of 584 nm at 1000 td and 573 nm at 100 td. But his hue-match (for the same 0·3-sec flashes) was between 584 nm at 1000 td and 587 at 100 td. The defined "hue match" (equal ratios of adjusted color-name scores) was in this case even more deviant: 584 nm, 1000 td, to 595 nm at 100 td. Their data show other instances which, while less severe, are nevertheless statistically and in some cases visually significant.

Savoie's hue matches cannot be subjected to this kind of internal analysis, since he did not determine a yellow equilibrium wavelength. The observer (principally Savoie himself) was required to judge only whether the comparison wavelength was "redder" or "greener" than the standard, with nothing at all said about "yellow". The consequences for hue matching of entirely ignoring yellowness as a perceptual quality are unpredictable.

We tried out Savoie's response mode in a successive matching technique, requiring the observer to judge whether the comparison stimulus was redder or greener than a standard equilibrium yellow. It was easy to verify that the 50 per cent "redder" point was invariant with luminance.

In sum, we observe that focusing on question (a), that of invariance of hue, requires hue matching, either directly, or indirectly by processing of other color responses. These methods are difficult, and in some instances they fail a test which we regard as a basic check on validity: like-colored equilibria should match in hue. The results of hue matching studies are equivocal with regard to the existence and the location of invariant hues. We, on the other hand, focus on question (b) and thus find answers to (a) and (c). On the other hand, our method has no way to locate nonequilibrium invariant hues, if there are any, nor does it assess the direction or the magnitude of hue shifts for noninvariant colors.

The relation between the equilibrium sets A_i and the set of C_w (no Abney effect) lights can be analyzed similarly into questions (a')–(c'):

(a') Is C_w nonempty: That is, are there invariant hues with respect to desaturation by w?

(b') Is A_i additive, that is, does hypothesis (ii) hold?
(c') Is A_i part of C_w?

We have answered (b') affirmatively for the set A_1 of red/green equilibria. If w is a white light in A_1, then we can define C_w using that white. For any a in A_1, $a \oplus (t * w)$ is also in A_1, and if a and $a \oplus (t * w)$ are both yellow, or both blue, then they match in hue by definition. Over a large range of values of t (intensities of desaturating light) this will hold satisfactorily, and so lights in A_1 can be considered part of C_w. Thus, (a') and (c') are answered affirmatively (for suitably selected white light). Conversely, if (a') and (c') are answered affirmatively, relative to C_w with w in A_i, then A_i satisfies at least partial additivity: (ii) holds when one component is white.

There seems to have been little precise work on the Abney hue shift. Clearly, the choice of white desaturant is crucial. On the basis of the present results, we can mix equilibrium yellow and blue to obtain a white, w, such that $t * w$ is in red/green equilibrium for a broad range of luminance levels; our yellow and blue equilibria are invariant hues relative to desaturation with such a white. Whether any other hues are invariant, and what the hue-shift properties of other "white" lights may be, cannot be inferred from the present results.

IMPLICATIONS OF ADDITIVITY

Colorimetric implications

As we noted briefly above, properties (i) and (ii) imply that there exists a function ϕ_1, which is a linear combination of the color-matching primaries (hence, of the pigment absorptions), such that $\phi_1(a) = 0$ for any A_1 equilibrium light. The function $\phi_1(a)$ is measured by the Jameson and Hurvich (1955) cancellation procedure, as shown by Krantz (1974). Suppose, e.g. that b_1 is a greenish light with $\phi_1(b_1) = -1$ by definition. Then for any reddish light a, we can choose t such that $a \oplus (t * b_1)$ is in A_1; hence

$$\phi_1[a \oplus (t * b_1)] = 0 = \phi_1(a) + t\phi_1(b_1).$$

Therefore, $\phi_1(a) = t$, the intensity of the cancellation standard.

The function ϕ_1 is independent of the choice of the cancellation light, except for changes of unit. For suppose that ϕ_1 and $\bar{\phi}_1$ were defined via cancellation with greenish lights b_1 and c_1 respectively. We would evaluate ϕ_1 and $\bar{\phi}_1$ for reddish lights a and a' by observing the following A_1 equilibria:

$$a \oplus (s * b_1),\ a \oplus (t * c_1),\ a' \oplus (u * b_1),\ a' \oplus (v * c_1).$$

By (i) and (ii), $(us^{-1} * a) \oplus (u * b_1) \oplus a' \oplus (v * c_1)$ is also in A_1; whence by (ii) and (i), $a \oplus (su^{-1}v * c_1)$ is in A_1. Therefore $su^{-1}v = t$ or $s/t = u/v$. It follows that ϕ_1 and $\bar{\phi}_1$ are proportional.

Krantz (1974) further showed that if A_1 and A_2 both satisfy (i) and (ii), then the cancellation functions ϕ_1 and ϕ_2 can be obtained directly from three-primary colorimetry, by using an A_1 light, an A_2 light, and an achromatic light as primaries.

If (i) and (ii) had been shown to be wrong, then the Jameson–Hurvich cancellation procedure would yield results that are not invariant with luminance or with choice of cancellation standard. In particular, the curve of Fig. 1, which is plotted for an equal-energy spectrum by use of a luminous-efficiency correction, would not be valid, since the measurements were made at equal luminance, not at equal energy.

In short, confirming (i) and (ii) simplifies several colorimetric problems.

Retinal mechanisms

Let us assume that the magnitude of a red/green opponent mechanism output, denoted f_1, is written as a function of three cone-photopigment quantum catches, α, β and γ:

$$f_1 = f_1(\alpha, \beta, \gamma).$$

The value of f_1 is (say) positive for reds, negative for greens, and zero for A_1 stimuli. The form of the function f_1 embodies whatever nonlinear input-output relations, inhibitory interactions, and summations are found in the red/green mechanism.[4]

The present results constrain f_1 sharply: it must be zero whenever a particular linear combination (ϕ_1) of the pigment catches α, β, γ is zero.

For example, suppose we were to assume that the cone outputs undergo a compressive power transformation before being combined linearly into an opponent system:

$$f_1 = k_1\alpha^{n_1} - k_2\beta^{n_2} + k_3\gamma^{n_3}, \tag{4}$$

where α, β, γ are short-wavelength, middle-wavelength, and long-wavelength cone responses, $k_i > 0$, and $1 > n_i > 0$, $i = 1,2,3$. The fact that (i) holds requires all the n_i to be equal to a common n. And the fact that (ii) also holds requires $n = 1$.

On the other hand, f_1 is by no means constrained to be a linear function of α, β, γ. (Only starting with the linear-power form of equation 4 were we able to deduce linearity.) Other forms that will do include:

$$f_1 = h(\phi_1), \tag{5}$$

where h is a nonlinear function and ϕ_1 is as above, linear in α, β, γ;

$$f_1 = h(k_1\alpha - k_2\beta) - h(k'_2\beta - k_3\gamma), \tag{6}$$

where h is nonlinear and

$$\phi_1 = k_1\alpha - (k_2 + k'_2)\beta + k_3\gamma. \tag{7}$$

[4] Savoie (1973) identified "hue" with the red/green mechanism, and so his reasoning about the red/green mechanism is based on hue matches rather than on equilibrium. We think this identification is wrong, since hue depends on both the red/green and yellow/blue mechanisms. Hue-matching data are relevant therefore to a combined theory of red green and yellow/blue mechanisms and their interactions (see Krantz, 1974); but not directly to the red/green mechanism alone.

There are many other possibilities. Nevertheless, a very powerful constraint has been obtained, in that many possibilities have been eliminated.

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Résumé—Une lumière d'équilibre rouge/vert n'apparait ni rouge ni verte (c'est à dire uniquement jaune, uniquement bleue ou achromatique). On détermine à divers niveaux de luminance un ensemble d'équilibres rouge/vert spectraux et non spectraux, afin de savoir si l'ensemble de tels équilibres est fermé dans des opérations linéaires de mélanges de couleurs.

Les lieux spectraux d'équilibre jaune et bleu ne montrent pas de variation, ou des variations visuellement insignifiantes, dans une marge de 1–2 $\log_{10}$ unités. Il n'y a pas de tendance répétable pour les observateurs. On déduit que les équilibres spectraux rouge/vert sont fermés vis-à-vis de la multiplication scalaire; ce sont donc des tonalités invariantes vis-à-vis de l'effet Bezold–Brücke.

Le mélange additif des longueurs d'onde d'équilibre jaune et bleu, à tout rapport de luminance, est aussi une lumière d'équilibre. De petits changements de la composante jaunâtre du mélange vers le rouge ou le vert doivent être compensés par des changements prévisibles de la composante bleuâtre du mélange vers le vert ou le rouge. On conclut que les équilibres jaune et bleu sont complémentaires vis-à-vis d'un blanc d'équilibre; que la désaturation d'un équilibre jaune ou bleu avec un tel blanc ne produit pas de décalage de tonalité d'Abney; et que l'ensemble des équilibres rouge/vert est fermé vis-à-vis des opérations linéaires générales.

En conséquence la fonction chromatique de réponse rouge/vert, mesurée par la technique de Jameson–Hurvich d'annulation à l'équilibre, est une fonction linéaire des coordonnées de mélanges de couleurs du sujet. Une seconde conséquence de la fermeture linéaire de l'équilibre est une forte contrainte sur la classe des règles de combinaison par lesquelles les réponses des récepteurs sont recodées dans le processus antagoniste rouge/vert.

Zusammenfassung—Ein gleichanteiliges rot-grünes Licht ist ein solches, das weder rötlich noch grünlich erscheint (d.h. entweder einheitlich gelb, einheitlich blau oder unbunt). Eine Reihe spaktraler und nichtspektraler Rot-Grün-Gleichgewichte wurde für mehrere Leuchtdichteniveaus bestimmt, um zu untersuchen, ob die Gesamtzahl aller dieser Gleichgewichte durch lineare Farbmischoperationen bestimmt ist. Die Farborte des gleichanteiligen Gelbs bzw. Blaus zeigten entweder überhaupt keine oder nur eine visuell nichtsignifikante Variation über einen Bereich von 1–2 logarithmischen Einheiten. Unter allen Versuchspersonen gab es keinerlei reproduzierbare Tendenzen. Wir schlossen, dass spektrale Rot-Grün-Gleichgewichte durch eine skalare Multiplikation bestimmt sind; folglich sind sie invariante Farbtöne bezüglich der Bezold-Brücke-Verschiebung.

Die additive Mischung gelber und blauer Gleichgewichtswellenlängen ergibt in jedem Leuchtdichteverhältnis ebenfalls ein gleichanteiliges Licht. Kleine Veränderungen der Gelb-Komponente einer Mischung in Richtung Rot oder Grün müssen durch vorherbestimmbare Änderungen der Blau-Komponente der Mischung in Richtung Grün oder Rot kompensiert werden. Wir schlossen, dass gelbe und blaue Gleichgewichte in Bezug auf ein gleichanteiliges Weiss komplementär sind; die Verringerung der Farbsättigung eines gelben oder blauen gleichanteiligen Lichtes durch Beimischung eines derartigen Weiss verursacht keine Abneysche Farbverschiebung; ausserdem folgerten wir, dass die Reihe der Rot-Grün-Gleichgewichte durch allgemeine lineare Operationen bestimmt wird.

Eine Konsequenz ist, dass die mit der Jameson–Hurvich-Technik des Gleichgewichtsabgleichs gemessene rot-grüne Farbübertragungsfunktion eine lineare Funktion der Farbvergleichskoordinaten der Versuchsperson ist. Eine zweite Konsequenz der linearen Gesetzmässigkeiten ist eine starke Einschränkung für die Klasse der Kombinationsregeln, nach denen die Ausgangssignale der Rezeptoren für diesen Mischprozess kodiert werden.

Резюме—Равновесный красно–зеленый свет не кажется ни красноватым, ни зеленоватым (Напр. или чистожелтым, или чистоголубым, или ахроматическим.)

Поднаборы спектральных и неспектральных равновесных красно-зеленых излучений были определены для нескольких значений яркости, чтобы проверить, являются ли наборы всех таких равновесий замкнутыми относительно линейных операций цветового смешения.

Линии спектрального равновесного желтого и голубого либо не изменились вообще, либо обнаружили зрительно незначительное отклонение в пределах одной–двух десятичных логарифмических единиц. Не было воспроизводимых различий между наблюдателями. Мы сделали вывод, что спектральные красно-зеленые равновесия замкнуты относительно скалярного произведения. Согласно этому они являются инвариантными оттенками относителжно сдвига Безольда–Брюкке.

Аддитивное смешение желтых и голубых равновесных излучений в любой яркосной пропорции, также является равновесным излучением. Небольшие изменения желтоватой компоненты смеси в сторону красноватого или зеленоватого должны быть компенсированы предсказуемыми изменениями голубоватой компоненты смеси в сторону зеленоватого или красноватого. Мы заключили, что желтое и голубое равновесие являются дополнительными относительно равновесного белого; что разбавление желтого или голубого равноовеснго света белым не приводит к сдвигу оттенка по Эбни; и что набор красно–зеленых равновесий замкнут относительно обычных линейных операций.

Первым выводом является то, что красно–зеленая функция цветовой чувствительности, измеренная методом вычитания до равенства Джемисон–Гурвича, является линейной функцией цветового сложения индивидуума. Вторым выводом из линейной замкнутости равновесий является сильное влияние на класс правил комбинирования, в соответствии с которыми ответы рецептора записываются методом красно–зеленых противоположных цветов.