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THE ANALYSIS OF GAMMA-GAMMA DIRECTIONAL CORRELATIONS IN A TRIPLE CASCADE*

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If a triple cascade exists among the nuclear levels of an isotope, the (1–3) directional correlation coefficients combined with those of the (1–2) and (2–3) cascades offer an effective way for determining mixing ratios or spin sequences. A formula is given to determine the mixing ratios for the intermediate transition directly, regardless of whether or not the other two transitions are mixed. Graphs are introduced for analyzing triple cascades.

1. Introduction

It has been pointed out[1]) that gamma–gamma directional correlations for a (1–3) cascade with the intermediate transition unobserved can be used to obtain multipolarities of transitions and spins of nuclear energy levels. Measurements on such a cascade demand good resolution of the detectors and the analysis involves spins of upper, lower and intermediate levels as well as the multipolarities of the first, third and the unobserved transitions. The advent of Ge(Li) detectors has provided adequate resolution for these measurements. Taylor et al.[2]) issued a set of tables for first–third correlations while Ramayya et al.[3]) generated graphs for several spins and multipolarities. The usefulness of these tables and graphs is limited by the inherent assumption of pure multipolarities in two of the three transitions. Even in the case of consecutive cascades, tables[2]) and graphs[4,5]) are constructed in such a way that the user must repeatedly choose a series of particular values for the other mixing ratio in order to determine the one in question or to select a proper spin sequence for the cascade. This is especially cumbersome when the A_{44} directional correlation coefficients have large uncertainties. Another common practice in determining mixing ratios of transitions is to begin by assuming a transition with higher multipolarity to be pure. Such an assumption is often unjustified. It has been pointed out[6]) that this assumption can be eliminated if one measures correlations in (1–3) cascades in addition to those in consecutive ones. In the present work, the reiteration method in ref. 6 is simplified by decoupling the mixing ratio of the unobserved transition from those of the first and third transitions. Graphs similar to those generated by Coleman[4]) and Arns and Wiedenbeck[5]) for consecutive cascades will be introduced for triple cascades. Since only one mixing ratio is involved, this representation can be used directly to analyze correlation data even when the multipolarities of the other transitions are mixed.

2. Decoupling of the intermediate transition

Our mixing ratios follow the sign convention of Dzhelepov et al.[7]) who suggested the sign of δ, given by the formulas of Biedenharn and Rose[8]), be reversed if the transition is the second in the cascade and otherwise left unchanged. Thus, the sign convention adopted here is opposite to that of Krane and Steffen[9]). Our notation follows that of ref. 10. The scheme is workable when a triple cascade as shown in fig. 1 exists among the levels of the isotope and the $\gamma_1-\gamma_2$, $\gamma_2-\gamma_3$ and $\gamma_1-\gamma_3$ directional correlations can all be determined. For $\gamma_1-\gamma_2$ cascade, one has

$$A_{kk}(\gamma_1\gamma_2) = A_k(\gamma_1)A_k(\gamma_2). \qquad (1)$$

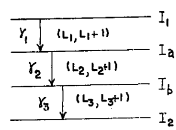

Fig. 1. Gamma transitions in a triple cascade.

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For the $\gamma_2-\gamma_3$ cascade, one has

$$A_{kk}(\gamma_2\gamma_3) = A'_k(\gamma_2)A_k(\gamma_3). \tag{2}$$

For the γ_1–γ_3 cascade, one has

$$A_{kk}(\gamma_1\gamma_3) = A_k(\gamma_1)U_{kk}(\gamma_2)A_k(\gamma_3). \tag{3}$$

The intermediate transition can be decoupled from the first and third transitions by multiplying eqs. (1) and (2) and then dividing by eq. (3),

$$R_k = \frac{A_k(\gamma_2)A'_k(\gamma_2)}{U_{kk}(\gamma_2)}, \tag{4}$$

where

$$R_k = \frac{A_{kk}(\gamma_1\gamma_2)A_{kk}(\gamma_2\gamma_3)}{A_{kk}(\gamma_1\gamma_3)}. \tag{5}$$

If the expressions for $A_k(\gamma_2)$, $A'_k(\gamma_2)$ and $U_{kk}(\gamma_2)$ are substituted into eq. (4), one obtains a fourth-order equation in the mixing ratios of the intermediate transition, δ_2,

$$\begin{aligned}(c_kf_k-g_kR_k)\delta_2^4 + 2(c_ke_k-b_kf_k)\delta_2^3 + \\ +(a_kf_k-4b_ke_k+c_kd_k+h_kR_k-g_kR_k)\delta_2^2 + \\ +2(a_ke_k-b_kd_k)\delta_2+(a_kd_k+h_kR_k) = 0,\end{aligned} \tag{6}$$

where

$$\begin{aligned}a_k &= F_k(L_2, L_2, I_b, I_a),\\ b_k &= F_k(L_2, L'_2, I_b, I_a),\\ c_k &= F_k(L'_2, L'_2, I_b, I_a),\\ d_k &= F_k(L_2, L_2, I_a, I_b),\\ e_k &= F_k(L_2, L'_2, I_a, I_b),\\ f_k &= F_k(L'_2, L'_2, I_a, I_b),\\ g_k &= (-1)^{I_a+I_b}[(2I_a+1)(2I_b+1)]^{\frac{1}{2}}(-1)^{L'_2}\begin{Bmatrix} I_b & I_b & k \\ I_a & I_a & L'_2\end{Bmatrix},\end{aligned}$$

and

$$h_k = (-1)^{I_a+I_b}[(2I_a+1)(2I_b+1)]^{\frac{1}{2}}(-1)^{L_2+1} \times \begin{Bmatrix} I_b & I_b & k \\ I_a & I_a & L_2\end{Bmatrix}. \tag{7}$$

It is evident from eqs. (6) and (7) that δ_2 is not functionally dependent on the quantities associated solely with the first and third transitions, i.e. L_1, L'_1, L_3, L'_3, I_1, I_2, δ_1 and δ_3, except via the ratios of experimentally deduced correlation coefficients, R_k.

A maximum of four solutions for δ_2 can result from each value of R_2. However, usually only one value of δ_2 is consistent with both the measured values of R_2 and R_4. Once δ_2 is determined, δ_1 and δ_3 or the upper and lower level spins can be deduced by conventional methods[8]) or from graphical representations[4,5]). Eq. (6) can be rearranged to read

$$\begin{aligned}R_k = [c_kf_k\delta_2^4+2(c_ke_k-b_kf_k)\delta_2^3 + \\ +(a_kf_k-4b_ke_k+c_kd_k)\delta_2^2 + \\ +2(a_ke_k-b_kd_k)\delta_2+a_kd_k] \times \\ \times(1+\delta_2^2)^{-1}(g_k\delta_2^2-h_k)^{-1}.\end{aligned} \tag{8}$$

Tables and graphs of R_k versus δ_2 can be constructed by using eq. (8). Also, R_2 versus R_4 parametric plots (with δ_2 as the parameter) can be generated. Thus R_k may turn out to be a very useful quantity in analyzing correlation data in a triple cascade.

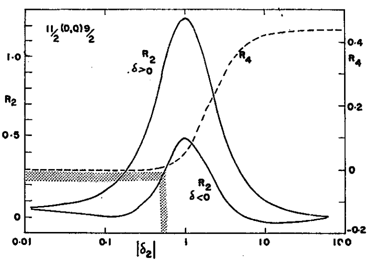

Fig. 2. R_2 and R_4 vs δ_2 for the 11/2(D,Q)9/2 intermediate transition.

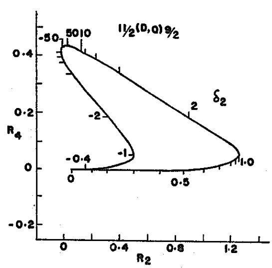

Fig. 3. R_4 vs R_2 for the 11/2(D,Q)9/2 intermediate transition. δ_2 is listed on the curve.

It is evident from the symmetry of the expression for R_k in eq. (8) that, if the sequence $I_a(L_2, L_2')I_b$ is reversed but the same R_k is maintained, δ_2 will only change its sign. Therefore, tables and graphs for $I_a(L_2, L_2')I_b$ can be used for the $I_b(L_2, L_2')I_a$ transition provided the sign of δ_2 obtained is reversed.

One can classify the graphs into three groups depending on whether the last factor $(g_k\delta_2^2-h_k)$ in eq. (8) can vanish for some values of δ_2. For group 1, g_k and h_k, which are essentially 6*j* symbols, have opposite signs. Thus, R_2 and R_4 should be finite for all δ_2. Graphs belonging to this group should have shapes similar to those in example 1. Graphs in group 2 allow either R_2 or R_4 to go to infinity for some δ_2. Graphs for group 3 can have R_2 and R_4 going to infinity for some values of δ_2.

3. Illustration, ^{175}Lu

As an example, the triple-cascade data[11]) for the (145–138–114) correlation in ^{175}Lu are listed in table 1. The mixing ratio for the 11/2(D, Q)9/2 transition can be calculated from eq. (6) or read from figs. 2 or 3. While the uncertainty in R_4 is large, the values of R_2 and R_4 as shown in figs. 2 and 3 favor $\delta(138)$ to be -0.540 ± 0.023 which agrees well with the value -0.55 ± 0.10 determined from nuclear orientation[12]). Figs. 4 and 5 show the corresponding curves for 2(D, Q)2 transitions.

All of the commonly encountered sequences up to 15/2 have been plotted and will be supplied upon request.

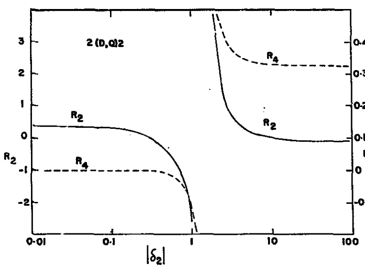

Fig. 4. R_2 and R_4 vs δ_2 for the 2(D,Q)2 intermediate transition.

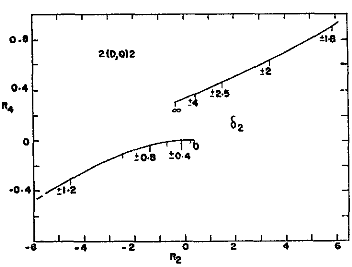

Fig. 5. R_4 vs R_2 for the 2(D,Q)2 intermediate transition. δ_2 is listed on the curves.

TABLE 1

Data used in the determination of mixing ratios of the intermediate transitions in triple cascades for ^{175}Lu.

Cascade (E in keV)	A_{22} / A_{44}	
(145–138)	-0.225 ± 0.004 -0.003 ± 0.006	
(138–114)	$+0.278\pm0.008$ $+0.080\pm0.011$	${}_nR_2=0.274\pm0.022$ ${}_nR_4=0.04\pm\infty$
(145–[138]–114)	-0.228 ± 0.007 -0.006 ± 0.009	Intermediate sequence 11/2(D,Q)9/2 $\delta(138)=-0.540\pm0.023$

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