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Brief Paper

Graphical Stability Criteria for Nonlinear Multiloop Systems*

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Key Word Index—Feedback; nonlinear systems; stability criteria; Popov criterion; multivariable systems.

Summary—In this paper a description of a certain class of multiloop systems, called the Standard Multiloop Form, is introduced. This description is expressed explicitly in terms of certain scalar subsystems and can be shown to include many of the common descriptions of multiloop systems. The stability criteria presented in this paper involve the individual Nyquist plots of the linear scalar subsystems and a certain positivity condition on the nonlinear subsystems. The method allows for relatively convenient computations. The derivation depends on the hyperstability concept introduced by V. M. Popov.

1. *Introduction*

NONLINEAR stability theory has received the attention of researchers for many years. In 1961 V. M. Popov introduced a method of stability analysis based on the use of the frequency domain[1], which greatly simplified the analysis for systems having a particular structure, namely, systems having a linear time-invariant 'plant', one nonlinear element, and a single feedback loop. Since the introduction of Popov's method and the associated Circle Condition, many researchers have generalized the Popov method to include systems having multiple nonlinearities and multiple feedback loops, e.g. [1–6]. It is the purpose of this paper to introduce a method of analysis which leads to a convenient graphical interpretation in the frequency domain. There have been a few recent results[7–10] where an attempt has been made to preserve a graphical interpretation; in these works various system representations were considered but the difficulty of handling large-scale multiloop systems remains.

2. *System description*

Stability criteria will be derived for systems having a particular structure called the Standard Multiloop Form.

2.1 *Definition.* A system of equations of the form

$$\dot{x}_i(t) = A_i x_i(t) + b_i f_i(t)$$
$$y_i(t) = c_i' x_i(t)$$
$$f_i(t) = \sum_{j=1}^{n} \psi_{ij}\left[u_j(t) - \sum_{k=1}^{n} \phi_{jk}(y_k, t), t\right]$$
$$\text{for } i = 1, \ldots, n \qquad (2.1)$$

is said to be in the *Standard Multiloop Form.* The function $x_i(t)$ is an n_i-vector, and $f_i(t)$ and $y_i(t)$ are scalar functions.

The function $u_i(t)$ is a scalar input function to the ith subsystem. The time varying and nonlinear functions ψ_{ij} and ϕ_{ij} are assumed to be continuous functions of their arguments with $\phi_{ij}(0) = 0$ and $\psi_{ij}(0) = 0$. It shall be assumed throughout that $n > 1$.

The transfer function for the ith linear subsystem is given by by

$$G_i(s) = \frac{Y_i(s)}{F_i(s)} = c_i'(sI - A_i)^{-1} b_i, \qquad (2.2)$$

and a block diagram of the system is shown in Fig. 1. It may be shown that a great many multiloop feedback systems may be placed in this form through suitable selection of the nonlinearities ψ_{ij} and ϕ_{ij} [11, 12].

In this work the stability of the multiloop system is considered only for zero inputs.

2.2 *Definition.* The system (2.1) is *globally stable with degree* γ if, for $u_i(t) \equiv 0$, $i = 1, \ldots, n$, and for any $x_i(0)$, $i = 1, \ldots, n$, there exist numbers $K_i > 0$ such that

$$\|x_i(t)\| \le K_i \, e^{-\gamma t}, \; t \ge 0,$$
$$i = 1, \ldots, n.$$

The definition implies global asymptotic stability in the sense of Liapunov if $\gamma > 0$.

3. *Stability criteria*

The stability criteria are based upon a variation of a lemma developed by V. M. Popov for single-loop systems[1, 11].

3.1 *Basic lemma.* If the linear systems

$$\dot{x}_i(t) = A_i x_i(t) + b_i f_i(t)$$
$$y_i(t) = c_i' x_i(t) + d_i f_i(t), \; i = 1, \ldots, n$$

with transfer functions $G_i(s) = c_i'(sI - A_i)^{-1} b_i + d_i$ are each irreducible, i.e. controllable and observable, and the functions $G_i(s)$ have all poles in $\mathrm{Re}\, s < 0$, and the Nyquist plots of $G_i(s)$ lie in $\mathrm{Re}\, s \ge 0$, then there exist $\mu_i > 0$ and $\nu_i > 0$, $i = 1, \ldots, n$ such that

$$\sum_{i=1}^{n} \mu_i \|x_i(t)\|^2 \le \sum_{i=1}^{n} \nu_i \|x_i(0)\|^2 + \int_0^t \sum_{i=1}^{n} f_i(\tau) y_i(\tau)\, d\tau \qquad (3.1)$$

holds for all $t \ge 0$.

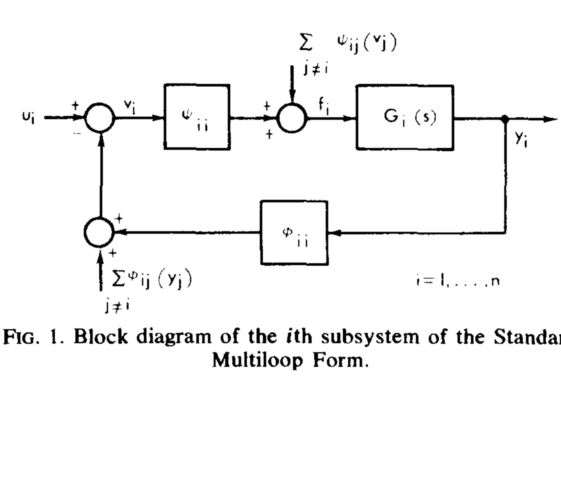

FIG. 1. Block diagram of the ith subsystem of the Standard Multiloop Form.

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To illustrate the graphical stability criteria, consider the following special case of (2.1)

$$\begin{aligned} \dot{x}_i(t) &= A_i x_i(t) + b_i f_i(t) \\ y_i(t) &= c_i' x_i(t) \\ f_i(t) &= -\sum_{j=1}^{n} \phi_{ij}(y_j(t), t) \\ i &= 1, \dots, n. \end{aligned} \tag{3.2}$$

3.2 *Theorem*. The system (3.2) with irreducible transfer functions (2.2) is globally stable with degree γ if the pair $[A_i + \gamma I_i, b_i]$ is controllable and the pair $[A_i + \gamma I_i, c_i]$ is observable for $i = 1, \dots, n$, where I_i is the $n_i \times n_i$ identity matrix, and:

If $G_i(s)$ *has all poles in* $\mathrm{Re}\, s < -\gamma$: One of the following holds:

(a) The Nyquist locus of $G_i(s-\gamma)$ does not encircle nor enter the closed disk D_i, where $0 < p_i q_i$; $p_i < q_i$;

(b) The Nyquist locus of $G_i(s-\gamma)$ is inside the closed disk D_i, where $p_i < 0 < q_i$;

(c) The Nyquist locus of $G_i(s-\gamma)$ lies in the closed half plane where $\mathrm{Re}\, s \geq -(1/q_i)$, where $0 = p_i < q_i$;

(d) The Nyquist locus of $G_i(s-\gamma)$ lies in the closed half plane where $\mathrm{Re}\, s \leq -(1/p_i)$, where $p_i < q_i = 0$

If $G_i(s)$ *has* $N_i \geq 0$ *poles in* $\mathrm{Re}\, s > -\gamma$: The Nyquist locus of $G_i(s-\gamma)$ encircles the disk D_i exactly N_i times in the counterclockwise direction and does not enter the disk; where $0 < p_i < q_i$.

For the nonlinear functions

$$\alpha_{ii} + p_i \leq \frac{\phi_{ii}(y_i)}{y_i} \leq \beta_{ii} + p_i \leq q_i, \quad i = 1, \dots, n$$

$$-\beta_{ij} \leq \frac{\phi_{ij}(y_j)}{y_j} \leq \beta_{ij}, \; i \neq j \tag{3.3}$$

and also for $i \neq j$

$$\frac{1}{n-1}\alpha_{ii}\left(1 - \frac{1}{q_i - p_i}\beta_{ii}\right) - \frac{\beta_{ji}^2}{q_j - p_j} \geq 0$$

$$\left[\frac{1}{n-1}\alpha_{ii}\left(1 - \frac{1}{q_i - p_i}\beta_{ii}\right) - \frac{\beta_{ji}^2}{q_j - p_j}\right]$$
$$\times \left[\frac{1}{n-1}\alpha_{jj}\left(1 - \frac{1}{q_j - p_j}\beta_{jj}\right) - \frac{\beta_{ij}^2}{q_i - p_i}\right]$$
$$-\frac{1}{4}\left[\beta_{ij} + \beta_{ji} + 2\sum_{k=1}^{n}\frac{\beta_{ki}\beta_{kj}}{q_k - p_k}\right]^2 \geq 0 \tag{3.4}$$

where $\alpha_{ii} \geq 0$ for $i = 1, \dots, n$.

Proof. Let $Y_i(s)$ and $\tilde{Y}_i(s)$ denote the Laplace transform of $y_i(t)$ and $\tilde{y}_i(t)$, etc., and note that $Y_i(s-\gamma)$ is the Laplace transform of $e^{\gamma t} y_i(t)$. Define the following variables for $i = 1, \dots, n$

$$\tilde{F}_i(s) \triangleq F_i(s-\gamma) + p_i Y_i(s-\gamma)$$

$$\tilde{Y}_i(s) \triangleq Y_i(s-\gamma) + \frac{1}{q_i - p_i}\tilde{F}_i(s).$$

Then

$$\frac{\tilde{Y}_i(s)}{\tilde{F}_i(s)} = \frac{G_i(s-\gamma)}{1 + p_i G_i(s-\gamma)} + \frac{1}{q_i - p_i} \triangleq \tilde{G}_i(s).$$

Now define $\tilde{X}_i(s) \triangleq X_i(s-\gamma)$, where $X_i(s)$ is the Laplace transform of $x_i(t)$. Now

$$X_i(s) = (sI - A_i)^{-1} b_i F_i(s)$$

and

$$\tilde{X}_i(s) = (sI - A_i - \gamma I)^{-1} b_i F_i(s-\gamma).$$

Thus

$$(sI - A_i - \gamma I)\tilde{X}_i(s) = b_i \tilde{F}_i(s) - p_i b_i c_i' \tilde{X}_i(s)$$

and

$$\tilde{X}_i(s) = (sI - A_i + p_i b_i c_i' - \gamma I)^{-1} b_i \tilde{F}_i(s)$$

and $\tilde{Y}_i(s) = c_i' \tilde{X}_i(s) + [(1/q_i - p_i)]\tilde{F}_i(s)$, which has a realization

$$\begin{aligned} \dot{\tilde{x}}_i &= (A_i - p_i b_i c_i' + \gamma I)\tilde{x}_i + b_i \tilde{f}_i \\ \tilde{y}_i &= c_i' \tilde{x}_i + \frac{1}{q_i - p_i}\tilde{f}_i \\ i &= 1, \dots, n \end{aligned} \tag{3.5}$$

which is minimal by hypothesis. The conditions imposed on $G_i(s)$ by the theorem ensure, by the Nyquist condition, that the poles of $\tilde{G}_i(s)$ are in $\mathrm{Re}\, s < 0$ for $i = 1, \dots, n$. After some simplification

$$\mathrm{Re}\, \tilde{G}_i(s) = \mathrm{Re}\left\{\frac{(1 + q_i G_i(s-\gamma))(1 + p_i \overline{G_i(s-\gamma)})}{(q_i - p_i)|1 + p_i G_i(s-\gamma)|^2}\right\}$$

where the bar denotes complex conjugate. In cases (a) and (b):

$$\mathrm{Re}\, \tilde{G}_i(s) = \frac{p_i q_i}{(q_i - p_i)|1 + p_i G_i(s-\gamma)|^2} \times \left\{\left|G_i(s-\gamma) + \frac{q_i + p_i}{2p_i q_i}\right|^2 - \left[\frac{q_i - p_i}{2p_i q_i}\right]^2\right\}.$$

The requirements on the Nyquist locus of $G_i(s)$ guarantee that $\mathrm{Re}\, \tilde{G}_i(j\omega) > 0$ for all ω, making the standard allowances for poles s_0 such that $\mathrm{Re}\, s_0 = \gamma$. For case (c):

$$\mathrm{Re}\, \tilde{G}_i(s) = \mathrm{Re}\, G_i(s-\gamma) + \frac{1}{q_i},$$

so that $\mathrm{Re}\, \tilde{G}_i(j\omega) \geq 0$ for all ω; and for case (d):

$$\mathrm{Re}\, \tilde{G}_i(s) = \frac{-1}{|1 + p_i G_i(s-\gamma)|^2}\mathrm{Re}\left\{G_i(s-\gamma) + \frac{1}{p_i}\right\}$$

so that again $\mathrm{Re}\, G_i(j\omega) \geq 0$ for all ω by hypothesis. Therefore, the functions $\tilde{G}_i(s)$, $i = 1, \dots, n$, satisfy the conditions of Lemma 3.1. It follows that there exist numbers $\mu_i > 0$ and $\nu_i > 0$ such that

$$\sum_{i=1}^{n} \mu_i \|\tilde{x}_i(t)\|^2 \leq \sum_{i=1}^{n} \nu_i \|\tilde{x}_i(0)\|^2 + \int_0^t \sum_{i=1}^{n} \tilde{f}_i(\tau)\tilde{y}_i(\tau)\,d\tau$$

$$e^{2\gamma t}\sum_{i=1}^{n} \mu_i \|x_i(t)\|^2 \leq \sum_{i=1}^{n} \nu_i \|x_i(0)\|^2 + \int_0^t \sum_{i=1}^{n} \tilde{f}_i(\tau)\tilde{y}_i(\tau)\,d\tau$$

$$\sum_{i=1}^{n} \mu_i \|x_i(t)\|^2 \leq e^{-2\gamma t}\sum_{i=1}^{n} \nu_i \|x_i(0)\|^2 + e^{-2\gamma t}\int_0^t \sum_{i=1}^{n} \tilde{f}_i(\tau)\tilde{y}_i(\tau)\,d\tau.$$

for all $t \geq 0$.

Now

$$\begin{aligned} \sum_{i=1}^{n} e^{-2\gamma t}\tilde{f}_i \tilde{y}_i = &-\sum_{i=1}^{n} p_i\left(1 + \frac{p_i}{q_i - p_i}\right)y_i^2 \\ &+ \sum_{i=1}^{n}\left(1 + \frac{2p_i}{q_i - p_i}\right)y_i \sum_{j=1}^{n}\phi_{ij}(y_j) \\ &- \sum_{i=1}^{n}\frac{1}{q_i - p_i}\left\{\sum_{j=1}^{n}\phi_{ij}(y_j)\right\}^2 \\ &\triangleq Q(y). \end{aligned}$$

Define

$$\tilde{\phi}_{ii}(y_i) = \phi_{ii}(y_i) - p_i y_i$$
$$\tilde{\phi}_{ij}(y_j) = \phi_{ij}(y_j),\ i \neq j$$

then

$$Q(y) = \sum_{i=1}^{n}\sum_{j=1}^{n} y_i\tilde{\phi}_{ij}(y_j) - \sum_{i=1}^{n}\frac{1}{q_i - p_i}\left\{\sum_{j=1}^{n}\tilde{\phi}_{ij}(y_j)\right\}^2.$$

Now

$$Q(y) = \sum_{i=1}^{n}\sum_{j=1}^{n}\left\{\frac{1}{n-1}y_i\tilde{\phi}_{ii} + \frac{1}{n-1}y_j\tilde{\phi}_{jj} + y_i\tilde{\phi}_{ij} + y_j\tilde{\phi}_{ji}\right\}$$
$$-\sum_{i=1}^{n}\frac{1}{q_i - p_i}\{\tilde{\phi}_{i1}^2 + \cdots + \tilde{\phi}_{in}^2$$
$$+ 2\tilde{\phi}_{i1}\tilde{\phi}_{i2} + \cdots + 2\tilde{\phi}_{i1}\tilde{\phi}_{in} + \cdots + 2\tilde{\phi}_{i,n-1}\tilde{\phi}_{in}\}.$$
$$= \sum_{i=1}^{n}\sum_{j=i+1}^{n}\left\{\frac{1}{n-1}\tilde{\phi}_{ii}\left[y_i - \frac{1}{q_i - p_i}\tilde{\phi}_{ii}\right]\right.$$
$$+ \frac{1}{n-1}\tilde{\phi}_{jj}\left[y_j - \frac{1}{q_j - p_j}\tilde{\phi}_{jj}\right] + y_i\tilde{\phi}_{ij} + y_j\tilde{\phi}_{ji}$$
$$\left. - \frac{1}{q_i - p_i}\tilde{\phi}_{ij}^2 - \frac{1}{q_j - p_j}\tilde{\phi}_{ji}^2 - 2\sum_{k=1}^{n}\frac{1}{q_k - p_k}\tilde{\phi}_{ki}\tilde{\phi}_{kj}\right\}.$$

Consider the term

$$\tilde{\phi}_{ii}\left[y_i - \frac{1}{q_i - p_i}\tilde{\phi}_{ii}\right].$$

If $y_i = 0$, then any terms involving y_i vanish and cannot lessen the value of the expression; therefore, let $y_i \neq 0$ for all i. By hypothesis,

$$\frac{\phi_{ii}}{y_i} \leq \beta_{ii} + p_i \leq q_i;$$

thus

$$\frac{\tilde{\phi}_{ii}}{y_i} \leq \beta_{ii} \leq q_i - p_i.$$

It follows that

$$0 \leq \alpha_{ii} \leq \frac{\tilde{\phi}_{ii}}{y_i} \leq \beta_{ii} \leq q_i - p_i$$

and

$$\tilde{\phi}_{ii}\left[y_i - \frac{1}{q_i - p_i}\tilde{\phi}_{ii}\right] = \frac{\tilde{\phi}_{ii}}{y_i}\left[y_i^2 - \frac{1}{q_i - p_i}y_i\tilde{\phi}_{ii}\right]$$
$$\geq \alpha_{ii}\left[y_i^2 - \frac{1}{q_i - p_i}\beta_{ii}y_i^2\right]$$
$$= \alpha_{ii}\left[1 - \frac{\beta_{ii}}{q_i - p_i}\right]y_i^2 \geq 0$$

since $\beta_{ii} \leq q_i - p_i$.

Now consider the term $\tilde{\phi}_{ki}(y_i)\tilde{\phi}_{kj}(y_j)$. If $y_iy_j > 0$, then

$$\tilde{\phi}_{ki}\tilde{\phi}_{kj} = y_iy_j\frac{\tilde{\phi}_{ki}\tilde{\phi}_{kj}}{y_iy_j} \leq y_iy_j\beta_{ki}\beta_{kj}.$$

If $y_iy_j < 0$, then

$$\tilde{\phi}_{ki}\tilde{\phi}_{kj} \leq -y_iy_j\beta_{ki}\beta_{kj}$$

so in all cases

$$\tilde{\phi}_{ki}\tilde{\phi}_{kj} \leq \beta_{ki}\beta_{kj}|y_iy_j|.$$

Note finally

$$y_i\tilde{\phi}_{ij} \geq -\beta_{ij}|y_iy_j|$$

and

$$\tilde{\phi}_{ij}^2 \leq \beta_{ij}^2y_j^2.$$

It follows that

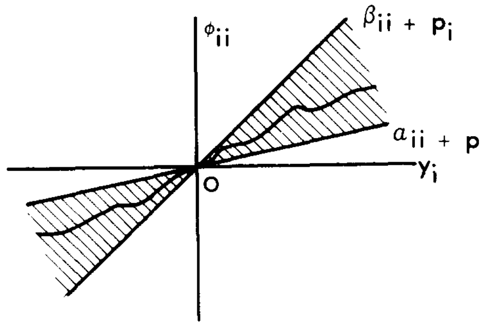
$$Q(y) \geq \sum_{i=1}^{n}\sum_{j=i+1}^{n}\left\{\frac{1}{n-1}\alpha_{ii}\left[1 - \frac{\beta_{ii}}{q_i - p_i}\right]y_i^2\right.$$
$$+ \frac{1}{n-1}\alpha_{jj}\left[1 - \frac{\beta_{jj}}{q_j - p_j}\right]y_j^2 - \beta_{ij}|y_iy_j| - \beta_{ji}|y_jy_i|$$
$$\left. - \frac{\beta_{ij}^2}{q_j - p_j}y_j^2 - \frac{\beta_{ji}^2}{q_j - p_j}y_i^2 - 2\sum_{k=1}^{n}\frac{\beta_{ki}\beta_{kj}}{q_k - p_k}|y_iy_j|\right\}$$
$$= \sum_{i=1}^{n}\sum_{j=i+1}^{n}\begin{bmatrix}y_i\\ y_j\end{bmatrix}\begin{bmatrix}\frac{1}{n-1}\alpha_{ii}\left[1 - \frac{\beta_{ii}}{q_i - p_i}\right] - \frac{\beta_{ji}^2}{q_j - p_j} & \pm\frac{1}{2}\left[\beta_{ij} + \beta_{ji} + 2\sum_{k=1}^{n}\frac{\beta_{ki}\beta_{kj}}{q_k - p_k}\right]\\ \pm\frac{1}{2}\left[\beta_{ij} + \beta_{ji} + 2\sum_{k=1}^{n}\frac{\beta_{ki}\beta_{kj}}{q_k - p_k}\right] & \frac{1}{n-1}\alpha_{jj}\left[1 - \frac{\beta_{jj}}{q_j - p_j}\right] - \frac{\beta_{ij}^2}{q_i - p_i}\end{bmatrix}\begin{bmatrix}y_i\\ y_j\end{bmatrix}$$

The $\pm$ signs are taken due to the $|y_iy_j|$ terms. The inequalities (3.4) guarantee that each matrix is non-negative definite. Q.E.D.

The conditions of Theorem 3.2 require that the Nyquist locus for each $G_i(s - \gamma)$ remain outside of a "forbidden" region in the complex plane. This forbidden region is either the inside or outside of a critical disk. In the special situation where $p_i = 0$ or $q_i = 0$, the disk degenerates into a half plane. This condition is in the familiar form of the Circle Criterion used in the analysis of single-loop nonlinear systems.

In theorem 3.2 the inequalities (3.3) require that the graph of each of the nonlinearities ϕ_{ij} lie in certain sectors which are illustrated in Fig. 2. The inequalities require that $0 \leq \alpha_{ii} < \beta_{ii}$ for $i = 1, \ldots, n$ but the number p_i, taken from the Nyquist plot of $G_i(s - \gamma)$, may be positive or negative and has the effect of "rotating" the sector for ϕ_{ii}. Consequently $\phi_{ii}(y_i)$ may lie in any of the four quadrants.

A close examination of Theorem 3.2 in this case reveals that the uncoupled subsystems (i.e., if $\phi_{ij}(v_j) = 0$, $i \neq j$) are



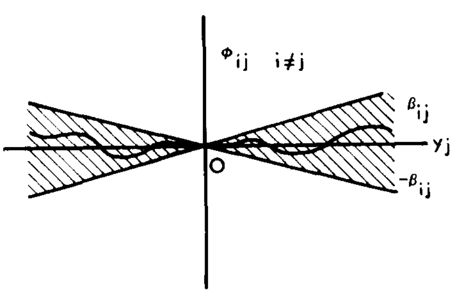


FIG. 2. Sector conditions for nonlinearities.

necessarily globally stable. It can be shown[11] that there always exist nontrivial gain sectors for the interconnection nonlinearities $\phi_{ij}(y_j)$, $i \neq j$, such that the interconnected system is globally stable. In particular, under the stated assumptions, if n globally stable subsystems are interconnected the multiloop system will be globally stable for sufficiently small interconnections. As one might expect, the stable sectors for the interconnections necessarily become smaller as the number of loops increases.

4. *Conclusion*

This paper has focused on a multiloop system as an interconnection of scalar subsystems. This viewpoint has made it possible to obtain conditions for global stability which involve the Nyquist plot for each of the scalar linear subsystems separately; a certain inequality involving the nonlinearities must also be satisfied. As a special case, conditions where the constraints on the nonlinearities can be expressed in terms of sector conditions were also considered. The main advantage of the method is that the results are explicitly expressed in terms of the properties of the scalar subsystems which define the particular interconnection; most methods, e.g. in[1–6], do not focus on the scalar subsystems explicitly.

In addition, the following points might be noted:

(1) The frequency response criteria may be interpreted graphically.

(2) The frequency conditions can easily be satisfied first by proper choice of the parameters p_i and q_i, $i = 1, \ldots, n$; the positivity inequality (3.4) then constitutes the required condition for global stability.

(3) Conditions involving controllability and observability apply individually to each linear subsystem, not to the interconnection.

Extensions of the work described here should be mentioned. Stability conditions for more general forms of (2.1) can be developed[11, 12], and bounded-input bounded-state stability can be inferred from the same graphical criteria[11].

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