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Short Communication

Shape of a worn slider

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A slider can be expected to wear in due time to a shape that gives a uniform contact pressure and consequently uniform friction stress. Computation of the worn shape is discussed using the theory of elasticity. The results show that the trailing edge may be worn down more than the leading edge. The worn shape at a sharp corner may be either convex or concave depending on the elastic constants, the friction coefficient and the corner angle.

1. Introduction

Unless a slider is machined exactly to the required shape, the pressure distribution between the slider and the base will not be uniform during the early stages of motion. However, if it is assumed that the rate of wear is higher at larger pressures, the slider will eventually wear to a shape that gives a uniform pressure under a constant load. Then by Coulomb's law the friction stress will also be uniform.

The purpose of this paper is to illustrate how the shape of a worn slider can be determined using theory of elasticity. We assume that there are no plastic deformations and that the temperatures do not reach a level such that the thermal stresses become important. For simplicity we start first with a rectangular slider moving on a half-space and take the problem as two dimensional. We assume that the elastic moduli of the slider are much larger than the moduli of the half-space and we take the slider as rigid although it can wear. Next we study the worn corner of an elastic slider on a half-space by means of asymptotic analysis. In both problems the boundary conditions are written in the operating state, *i.e.* when the slider transmits forces. However, in the case of the elastic corner the worn shape is that of the stress-free state or when the slider is unloaded. In the case of a rigid slider the shapes for the operating and stress-free states are the same. Moreover, the shape of the rigid slider is the same as the shape of an elastic slider but in the operating state.

2. The worn shape of an elastically rigid slider

The two-dimensional situation considered is shown in Fig. 1. A slider that is elastically rigid in comparison to the base against which it is pressed is vertically guided and moves to the right. (The assumption that the slider is rigid is discussed in Section 4.) The slider reaches a steady state profile when the distribution of contact pressure becomes uniform. It is then a simple matter to compute the shape of the worn slider. This is done by taking advantage of the known elasticity solutions for the two-dimensional half-space subjected to a uniform pressure p and uniform shearing tractions q over the interval $(-a, a)$.

The Airy stress function for the pressure loading is (ref. 1, p.106)

$$\phi = \frac{p}{2\pi}(r_2^2\theta_2 - r_1^2\theta_1) \tag{1}$$

in which the counterclockwise angles θ_1 and θ_2 are counted as positive. The displacements in polar coordinates for the biharmonic function $\phi = r^2\theta$ are

$$u_r = \frac{\kappa - 1}{2\mu} r\theta \tag{2}$$

$$u_\theta = -\frac{\kappa + 1}{2\mu} r \log r \tag{3}$$

where μ denotes the shear modulus, $\kappa = 3 - 4\nu$ for plane strain and ν is Poisson's ratio. Using eqns. (2) and (3), the vertical displacement at the surface $x = 0$ can be obtained by superposing the contributions of the two terms in eqn. (1). Thus

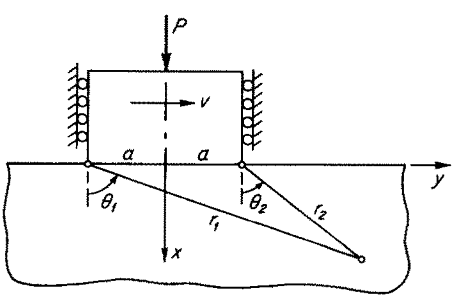

Fig. 1. Geometry of the elastically rigid slider.

$$u_x(0, y) = -\frac{p(\kappa+1)}{4\pi\mu}(r_2 \log r_2 + r_1 \log r_1) \tag{4}$$

The required results for a shear load can be extracted from an exercise problem given by Timoshenko and Goodier (ref. 1, p. 147). Thus the Airy stress function is

$$\phi = \frac{q}{2\mu}\{r_2^2(\log r_2 \cos 2\theta_2 - \theta_2 \sin 2\theta_2) + r_2^2 \log r_2 -$$

$$- r_1^2(\log r_1 \cos 2\theta_1 - \theta_1 \sin 2\theta_1) - r_1^2 \log r_1\} \tag{5}$$

where q is positive if the tangential tractions applied to the half-space act in the direction of increasing y. The Airy stress function

$$\phi = r^2(\log r \cos 2\theta - \theta \sin 2\theta)$$

corresponds to the displacements

$$u_r = -\frac{1}{2\mu}r\{(2 \log r + 1) \cos 2\theta - 2\theta \sin 2\theta\} \tag{6}$$

$$u_\theta = \frac{1}{2\mu}r\{(2 \log r + 1) \sin 2\theta + 2\theta \cos 2\theta\} \tag{7}$$

and $\phi = r^2 \log r$ yields

$$u_r = \frac{\kappa - 1}{2\mu}r(\log r - 1) \tag{8}$$

$$u_\theta = \frac{\kappa + 1}{2\mu}r\theta \tag{9}$$

Superposing these results to obtain the vertical displacement in the interval $|x| < a$ ($\theta_1 = \pi/2$ and $\theta_2 = -\pi/2$) for the Airy stress function (5) we have

$$u_x(0, y) = -\frac{q(\kappa - 1)}{8\mu}(r_2 - r_1) \qquad |y| < a \tag{10}$$

The vertical displacement of the half-space is computed from eqns. (4) and (10) by noting that when $x = 0$

$$r_1 = a + y \tag{11}$$

$$r_2 = a - y \tag{12}$$

and that

$$p = P/2a \tag{13}$$

$$q = fp \tag{14}$$

where P is the total force pressing the slider against the base and f denotes the coefficient of kinetic friction. Thus

$$u_x(0, y) = \frac{P}{8\mu}\left\{\frac{\kappa + 1}{\pi}\left(\log \frac{a^2}{a^2 - y^2} + \frac{y}{a}\log \frac{a - y}{a + y}\right) + f(\kappa - 1)\frac{y}{a}\right\} \qquad |y| < a \tag{15}$$

if a constant term corresponding to a rigid body displacement is added to render the arguments of the logarithmic terms dimensionless.

Equation (15) is also the expression for the profile of the worn slider. The shapes for $\kappa = 2$ ($\nu = \frac{1}{4}$) and a few values of the friction coefficient are shown in Fig. 2. The rather unexpected result is that the slider is worn down more at its trailing than at its leading edge. The slopes at the edges are infinite.

3. The corner of an elastic slider

The problem becomes much more difficult if the slider is elastic because the deformations of the slider must also be found. However, it is possible to find the worn shape near a sharp corner from simple asymptotic analysis.

The situation considered is shown in Fig. 3. The mechanical boundary conditions are

$$\sigma_{r\theta}^{(1)}(r,0) = \sigma_{r\theta}^{(2)}(r,0) = fp \tag{16}$$

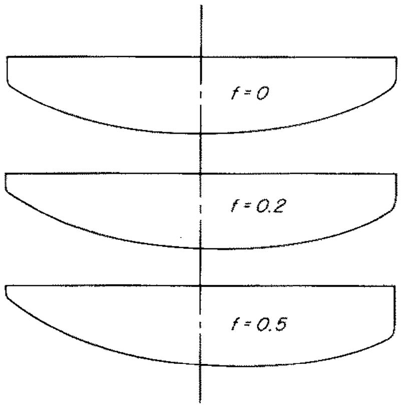

Fig. 2. Profile of the worn slider for various coefficients of friction.

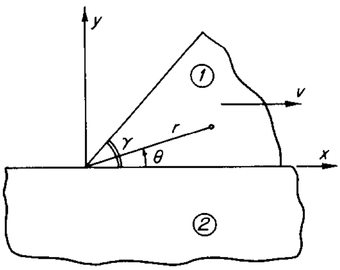

Fig. 3. Geometry of the corner of an elastic slider.

$$\sigma_{\theta\theta}^{(1)}(r,0) = \sigma_{\theta\theta}^{(2)}(r,0) = -p \tag{17}$$

$$\sigma_{r\theta}^{(1)}(r,\gamma) = \sigma_{\theta\theta}^{(1)}(r,\gamma) = 0 \tag{18}$$

$$\sigma_{r\theta}^{(2)}(r,-\pi) = \sigma_{\theta\theta}^{(2)}(r,-\pi) = 0 \tag{19}$$

Equation (16) allows us to take into account both directions of sliding by permitting f to assume both positive and negative values. Thus $f > 0$ corresponds to motion of the slider to the right and $f < 0$ to motion to the left. There will be a gap g_0 between the worn slider and the half-space if the slider is not pressed against the base. This gap disappears when the slider is loaded by forces corresponding to the operating condition. Thus

$$g_0(r) = u_\theta^{(2)}(r,0) - u_\theta^{(1)}(r,0) \tag{20}$$

The quantity $g_0(r)$ is precisely what defines the worn shape near the sharp corner.

The mathematical problem is readily set up using the catalogues of elastic fields that are suitable for the asymptotic analysis. The sets of elastic fields denoted by $\{A\}, ..., \{D\}$ are given in ref. 2 and the sets $\{E\}, ..., \{H\}$ in ref. 3. The first four sets correspond to stresses that are proportional to $r^{-\lambda}$ The stresses in the last four sets contain the terms $r^{-\lambda}$ and $r^{-\lambda} \log r$. It is clear from eqns. (16) and (17) that we must choose

$$\lambda = 0 \tag{21}$$

The simultaneous equations for the unknown coefficients A_1, B_1, .., H_1 and $A_2, B_2, ..., H_2$ associated with the elastic fields in the two bodies can be written by inspection if use is made of the catalogues. The eight boundary conditions yield 16 equations because the terms containing $\log r$ must satisfy the boundary conditions by themselves. Neither the system of the equations nor the values of most of the coefficients are of immediate interest. When only the dominant terms are retained the gap in the stress-free state is

$$g_0(r) = \left[\frac{1}{2\mu_2}\{F_2(\kappa_2 + 1) + H_2\} - \frac{1}{2\mu_1}\{F_1(\kappa_1 + 1) + H_1\}\right] r \log r \tag{22}$$

and the simultaneous equations yield for the required coefficients

$$F_1 = \frac{p\{\sin 2\gamma + f(1 - \cos 2\gamma)\}}{2(1 - \cos 2\gamma - \gamma \sin 2\gamma)} \tag{23}$$

$$F_2 = p/2\pi \tag{24}$$

$$H_1 = H_2 = 0 \tag{25}$$

Equation (22) shows that there are two possible shapes. If the term in square brackets in eqn. (22) is positive the slider wears to a convex shape near the corner. However, if the term is negative the shape is concave.

It is also possible to reason from the asymptotics that thermal effects do not influence the worn shape in the vicinity of a sharp corner. The boundary condition on the heat fluxes in the contact region is

$$q_\theta^{(1)}(r,0) - q_\theta^{(2)}(r,0) = v|f|p \tag{26}$$

where the right-hand side is the rate per unit area at which heat is generated by friction. For the right-hand side to be constant requires that $\lambda = -1$ in the thermal problem, but the displacements then contain the terms with r^2 and $r^2 \log r$ [3] and they are of higher order than $r \log r$ in eqn. (22) for small r.

4. Conclusion

We can distinguish between the worn shape in the operating state when the slider transmits forces and the shape in the stress-free state when the slider is not loaded. It becomes clear from the derivation for the rigid slider that eqn. (15) also gives the shape of an elastic slider in the operating state. Consequently the operating shape depends neither on the overall geometry of the slider nor on its elastic constants. Similarly, nothing needs to be said about the details of loading or how the slider is guided. If the slider is lifted off the base it will undergo deformations as the pressure and shearing tractions on its worn face are removed, and these deformations which lead to a change in shape depend on all the factors mentioned. Of course the stress-free shape is nearly the same as the operating shape if the slider is much more rigid than the base.

An interesting conclusion is that finding the shape of an elastic slider in the operating state requires the solution of an elasticity problem for the sliding base only.

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