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GREEN'S FUNCTIONS FOR PLANAR THERMOELASTIC CONTACT PROBLEMS - INTERIOR CONTACT

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Introduction

We considered the Green's function for exterior thermoelastic contacts in a previous article [1]. Here we give the Green's function for the interior contact problem. For interior contacts, such as in problems of the Hertz type, it is convenient to write the governing integral equations on the contact zones and to match derivatives of normal displacements rather than the displacements themselves. The Green's function again consists of two parts: A thermal field corresponding to a combination of a heat source and a sink, positioned at the surfaces of the bodies, and a mechanical field induced by a pair of concentrated forces. The latter is readily obtained from the Flamant solution [2] but, for the sake of completeness, we write the fields in detail.

Heat Source and Sink

Consider the temperature distributions

$$T_1 = -\frac{\lambda}{\pi k_1}\log r, \quad T_2 = \frac{\lambda}{\pi k_2}\log r \qquad (1,2)$$

where λ is a constant. If $\lambda > 0$, these temperatures represent a heat source of strength λ acting at the surface of the upper solid, and a heat sink of the same strength at the surface of the lower solid. The components of heat flux derived from (1) and

(2) are

$$q_x^{(1)} = -q_x^{(2)} = \frac{\lambda}{\pi}\frac{x}{r^2}, \quad q_y^{(1)} = -q_y^{(2)} = \frac{\lambda}{\pi}\frac{y}{r^2} \tag{3,4}$$

The free expansion displacements corresponding to (1) and (2) are

$$u_x^{(1)} = -\frac{\lambda\delta_1}{\pi}\{x(\log r-1)+y(\pi-\theta)\} \tag{5}$$

$$u_x^{(2)} = \frac{\lambda\delta_2}{\pi}\{x(\log r-1)-y(\theta-\pi)\} \tag{6}$$

$$u_y^{(1)} = -\frac{\lambda\delta_1}{\pi}\{y(\log r-1)-x(\pi-\theta)\} \tag{7}$$

$$u_y^{(2)} = \frac{\lambda\delta_2}{\pi}\{y(\log r-1)+x(\theta-\pi)\} \tag{8}$$

Of particular interest toward formulating the interior contact problem are the temperature discontinuity across the interface, the heat flux transmitted by the interface and the gap developing between the solids due to the free expansion displacements. Shifting the source-sink configuration to the point $(\xi,0)$ on the interface, the results are

$$\frac{d\tau(x)}{dx} = \frac{d}{dx}[T_2(x,0)-T_1(x,0)] = \frac{\lambda}{\pi}\frac{k_1+k_2}{k_1k_2}\frac{1}{x-\xi} \tag{9}$$

$$q_y^{(1)}(x,0) = q_y^{(2)}(x,0) = \lambda\delta(x-\xi) \tag{10}$$

$$\frac{dg(x)}{dx} = [u_y^{(1)}(x,0)-u_y^{(2)}(x,0)] = \lambda(\delta_1-\delta_2)H(x-\xi) \tag{11}$$

If a source-sink combination with the density $\Lambda(x)$ is distributed over the interval (a,b) on the interface,

$$\frac{d\tau(x)}{dx} = \frac{1}{\pi}\frac{k_1+k_2}{k_1k_2}\int_a^b \frac{\Lambda(\xi)d\xi}{x-\xi} \tag{12}$$

$$q_y(x,0) = \Lambda(x) \tag{13}$$

$$\frac{dg(x)}{dx} = 0, \qquad x<a$$

$$= (\delta_1-\delta_2) \int_a^x \Lambda(\xi)d\xi, \quad a<x<b$$

$$= (\delta_1-\delta_2) \int_a^b \Lambda(\xi)d\xi, \quad b<x \tag{14}$$

Concentrated Forces

Suppose that concentrated normal forces are applied to each of the solids. Both forces are of magnitude f_y and act in a tensile direction. Since their directions are opposite, they satisfy Newton's third law. If the forces are applied at the origin, the induced displacements and stresses in the coordinate system shown in Fig. 1 of the previous article [1] are given by the following expressions:

$$u_x^{(1)} = -\frac{f_y}{2\pi\mu_1}\{\frac{1}{2}(\kappa_1-1)\theta + \frac{xy}{r^2}\} \tag{15}$$

$$u_x^{(2)} = \frac{f_y}{2\pi\mu_2}\{\frac{1}{2}(\kappa_2-1)\theta + \frac{xy}{r^2}\} \tag{16}$$

$$u_y^{(1)} = \frac{f_y}{2\pi\mu_1}\{\frac{1}{2}(\kappa_1+1)\log r + \frac{x^2}{r^2}\} \tag{17}$$

$$u_y^{(2)} = -\frac{f_y}{2\pi\mu_2}\{\frac{1}{2}(\kappa_2+1)\log r + \frac{x^2}{r^2}\} \tag{18}$$

$$\sigma_{xx}^{(1)} = -\sigma_{xx}^{(2)} = \frac{2f_y}{\pi}\frac{x^2y}{r^4} \tag{19}$$

$$\sigma_{xy}^{(1)} = -\sigma_{xy}^{(2)} = \frac{2f_y}{\pi}\frac{x}{r^2}(1 - \frac{x^2}{r^2}) \tag{20}$$

$$\sigma_{yy}^{(1)} = -\sigma_{yy}^{(2)} = \frac{2f_y}{\pi}\frac{y}{r^2}(1 - \frac{x^2}{r^2}) \tag{21}$$

TABLE I

Temperature discontinuity, heat flux, gap and normal tractions induced by various distributed actions. Note that the column headings are in the middle of the table.

Vortex $\Omega(x)$	$\int_{-\infty}^{x} \Omega(\xi)d\xi$	$-\frac{1}{\pi}\frac{k_1k_2}{k_1+k_2}\int_{-\infty}^{\infty}\frac{\Omega(\xi)d\xi}{x-\xi}$	0	$2M(\delta_1-\delta_2)\frac{k_1k_2}{k_1+k_2}\int_{-\infty}^{x}\Omega(\xi)d\xi$
Dislocation $B_y(x)$	0	0	$-\int_{-\infty}^{x} B_y(\xi)d\xi$	$\frac{2M}{\pi}\int_{-\infty}^{\infty}\frac{B_y(\xi)d\xi}{x-\xi}$
Type & density	$\tau(x)$ $\frac{d\tau(x)}{dx}$	$q_y(x,0)$	$g(x)$ $\frac{dg(x)}{dx}$	$\sigma_{yy}(x,0)$
Source-sink $\Lambda(x)$	$\frac{1}{\pi}\frac{k_1+k_2}{k_1k_2}\int_{-\infty}^{\infty}\frac{\Lambda(\xi)d\xi}{x-\xi}$	$\Lambda(x)$	$(\delta_1-\delta_2)\int_{-\infty}^{x}\Lambda(\xi)d\xi$	0
Forces $F_y(x)$	0	0	$\frac{1}{2\pi M}\int_{-\infty}^{\infty}\frac{F_y(\xi)d\xi}{x-\xi}$	$F_y(x)$

For the pair of concentrated forces shifted to the point $(\xi,0)$, the quantities of interest are the mismatch in the derivatives of normal displacements

$$\frac{dg(x)}{dx} = \frac{d}{dx}[u_y^{(1)}(x,0)-u_y^{(2)}(x,0)] = \frac{f_y}{2\pi M}\frac{1}{x-\xi} \tag{22}$$

and the normal tractions

$$\sigma_{yy}(x,0) = f_y\delta(x-\xi)$$

If the pair of forces is distributed with the density $F_y(x)$ on (a,b),

$$\frac{dg(x)}{dx} = \frac{1}{2\pi M}\int_a^b \frac{F_y(\xi)d\xi}{x-\xi} \tag{23}$$

$$\sigma_{yy}(x,0) = F_y(x) \tag{24}$$

Conclusion

A summary of the results is given in Table I, where the limits of the integrals are written so that several zones of the types encountered in contact problems are automatically accommodated.

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