American Journal of ORTHODONTICS

Volume 75, Number 6 **June, 1979**

ORIGINAL ARTICLES

The inappropriateness of conventional cephalometrics

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The cephalometric method today delivers a formal reduction of craniofacial morphology in point and line, a conventional representation of some considerable clinical use. The properties of form which it abstracts have been used to study craniofacial growth, to diagnose deviations of an individual from population norms, and to plan orthodontic treatment and assess its progress and results. The extensive literature on cephalometric and craniometric methods includes many papers on sources of error in the method, such as enlargement difficulties and errors in positioning of the subject, and on problems in interpretation. For instance, think of the difficulty of quantifying three dimensions from images, or pairs of images in two. These are technical problems which can generally be controlled through calibration and care. But, however useful it may be in the study of craniofacial growth and in clinical orthodontics, current cephalometrics has conceptual as well as technical handicaps.

These more fundamental difficulties may be traced to discrepancies between the technology of today and the research style of an earlier era.* During the development of modern biostatistics, shortly after the turn of the century, human craniums were used extensively as a source of raw data. The human skull, after all, has several features which appeal to the biostatistician developing tools for his trade: It is very complex; it varies widely in shape during ontogeny, through phylogeny, and between species; it is an important and widely studied region for clinical investigation; it has long been a prime indicator of racial identity and biological relationships in prehistory; and, for many of us, it has always had a certain morbid fascination. Although all these factors were known to

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This article comprised the Cordwainers Lecture delivered at the Institute of Dental Surgery, University of London, May 17, 1978.

*For the ideas expressed in this paragraph, we are indebted to Dr. David Carlson, Assistant Research Scientist in the Center for Human Growth and Development, and Assistant Professor of Anatomy, The University of Michigan. Dr. Carlson has elaborated on similar ideas in his forthcoming essay, "Skeletal Biology and Human Evolution."

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early biostatisticians, there was little consideration of ontogenetic biologic processes, since no living skull could be properly measured on the inside. After their invention, cephalograms provided a standardized representation of the living skull, equally complex, equally variable, and more clinically relevant. Nevertheless, roentgenographic cephalometrics is subject to the same limitations as craniometry of dry skulls when used for research purposes. The techniques of both cephalometrics and craniometry have been based not on the biology of craniofacial growth but only on the properties of dried skulls.

We propose in this article to explore these more fundamental difficulties.

There is no theory of cephalometrics

What kind of information lies in the radiographic cephalogram and its tracings? Just what are we looking at in the cephalogram, and what do we expect from it? The raw data within the cephalogram are points and curves representing anatomic shadows structured in a complicated fashion, both logically and biologically. How best do we study these points and curves? Some cephalometric points lie on one curve (gonion), some on two curves (menton) or none (ear-rod "porion"). Some "points" are not points on the skull at all. Articulare, for example, the intersection of two shadows, is in fact a line looked at down its length. It is not a landmark or an intersection, for the curves used to define it never cross in space. In the literature of geometry and statistics there is no body of knowledge providing a formal framework for discourse about points on a curve or varying numbers of curves on a point. A theory, if it existed, should give definitive instructions on how to proceed with analysis in these different situations. Current cephalometrics provides no such theory, only conventions. How can we think clearly about these matters of cephalometric points and curves, or use them well in our work, when there is no underlying theory for points and curves?

Landmarks and curves. In conventional cephalometrics one measures the face and cranium only after locating on the projected images a large number of "landmarks," special points operationally defined. Form is characterized in terms of distance between pairs of landmarks, distances between landmarks and lines through landmarks, and angles between pairs of lines through landmarks. "Size" is then a matter of some predetermined distances, and "shape" is a matter of distance ratios and angles. Analysis proceeds in terms of sample means and covariances of these quantities. There results a structure of statistical inference for the ordination of subjects and comparison with "norms."

Any landmark is either "anatomic" or "extremal." Anatomic landmarks are true biologic loci identified by some feature of the local morphology. Examples include tips of cusps, nasion, and sella turcica. Other landmarks, however, not differentiated by local properties, are defined implicitly by the maximum or minimum of some geometric property and so we call them extremal. Menton is the *lowest* point of the mandibular symphysis; pogonion is the *most anterior* point on the chin; condylion the *most superior posterior* point on the condyle; etc. As the mandible rotates, the positions of all extremal points on it are altered. Then such points cannot be located until an orientation, let us say the horizontal, is fixed. Now all the standard orientations are themselves operationally defined in terms of landmarks too. The Frankfort plane, for instance, passes through the top of porion and the bottom of orbitale, but "top" and "bottom" are themselves defined in terms of the orientation of that one subject to earth. Malplacement of the horizontal orientation may be due to a mistake in positioning the subject, to inappropriateness of the

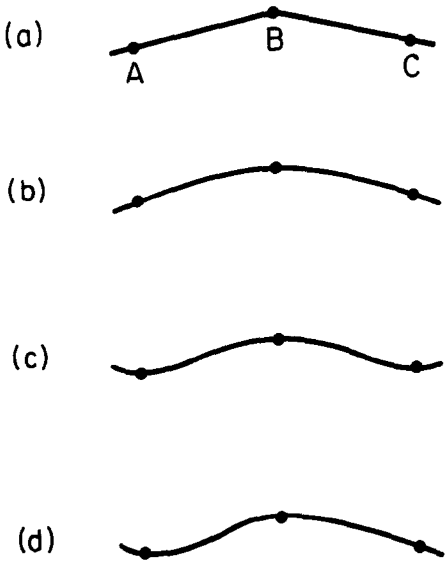

Fig. 1. Geometry of a configuration of landmarks. **a,** Polygonal geometry for a configuration of three neighboring landmarks. **b, c,** and **d,** Three possible outline forms: "arc," "bulge," and "wave."

landmarks for that subject, or perhaps because of disproportionate growth or asymmetry. This makes the whole scheme susceptible to a pernicious form of mismeasurement in which error in the orientation is propagated to affect the positions of all the orientation-dependent landmarks instead. This problem is of considerable importance in cross-sectional analysis.

The raw data, then, are points and curves, but the "derived data" of the tracing are entirely points. Such points do not define the curves but, rather, happen to lie upon them. Cephalometric points all have names; ordinary geometric points do not. Landmark points tell you where you are on a curve, while ordinary geometric points tell you only *that* you are on a curve. Both have two coordinates, but the coordinates are twice as informative for the one as for the other. Landmarks are used to convey information about particular loci, not about curving anatomy, for, as a single point, a landmark cannot represent the shape or relative position of an entire curved feature or its own changes which occur with remodeling.

Straight lines, or what do we do about bulges? Coordinates, of course, are not themselves points; they are instructions, relations between our calibrating instruments and the craniofacial object before us. For curves through a landmark, the coordinate pair lacks a necessary further instruction: where to go on the curve either side of the landmark. Irrespective of all the familiar cephalometric errors of orientation and identification, for analysis of morphology and the change of morphology, any scheme involving only landmarks and lacking information about curving is inadequate in principle.

Consider, for instance, the three landmarks shown in Fig. 1, *a* which manifest an angle of 160 degrees ($\angle$ABC). The true outline through these three points can vary with infinite subtlety around the landmarks, and the meaning of that angle changes with it. Fig. 1, *b, c,* and *d* shows three of the possibilities. These are very different forms—arc, bulge,

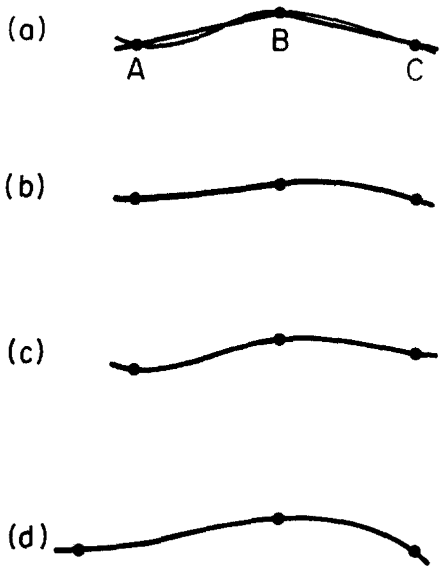

Fig. 2. Three ways in which the landmarks of Fig. 1, *d* might have moved in the course of increasing angle ∠ABC. **a,** Starting position. **b, c,** and **d,** Point B drops; point C rises; point A moves aside.

wave, we might call them—but they all have the same angle, the same landmark description. In the study of growth or remodeling, the problem is equally difficult. If we are told that the angle ∠ABC in Fig. 2, *a* increases with growth, we can decide among the possibilities shown in Fig. 2, *b, c,* and *d* (and others) by auxiliary information on distances. The curves of these figures correspond to quite different states after the *next* increment of time, so it is crucial that we discriminate them. Fig. 1 can be read, too, as a history of remodeling in which the landmarks have not moved at all. Clearly, a description in terms of landmarks captures nothing whatever of the changes in this case. In cephalometric practice we cannot separate the admixture of *these* two possibilities—landmark movement and landmark redefinition so the true understanding of change is lost to us.

Any collection of individual landmarks obscures the continuous variation of form in between. The form is not delineated somehow as in Fig. 3, *a,* by all possible segments between parts of landmarks and the associated collection of distance-ratios and angles; it is delineated rather, as in Fig. 3, *b,* by the curving of the outline around the whole. Those are the data; the head contains no polygons. Landmarks, progress markers on the circuit of the form, do not define it but merely lie upon it.

"Size" representation. The difficulties generated by loss of shape information extend to the very notion of size itself. In the absence of a representation of the curving form, size is measured by straight-line distances between points. Now that straight line along which the distance is measured has no biologic reality, for it describes neither direction of growth, lines of stress, directions of muscle pull, nor the lineaments of form in between. In fact, it may pass outside the form entirely (as frequently does the segment condylion-gnathion used for "length of mandible"). We believe that a sound measure of size must take into account the curving of the form being measured by going along arcs or up the

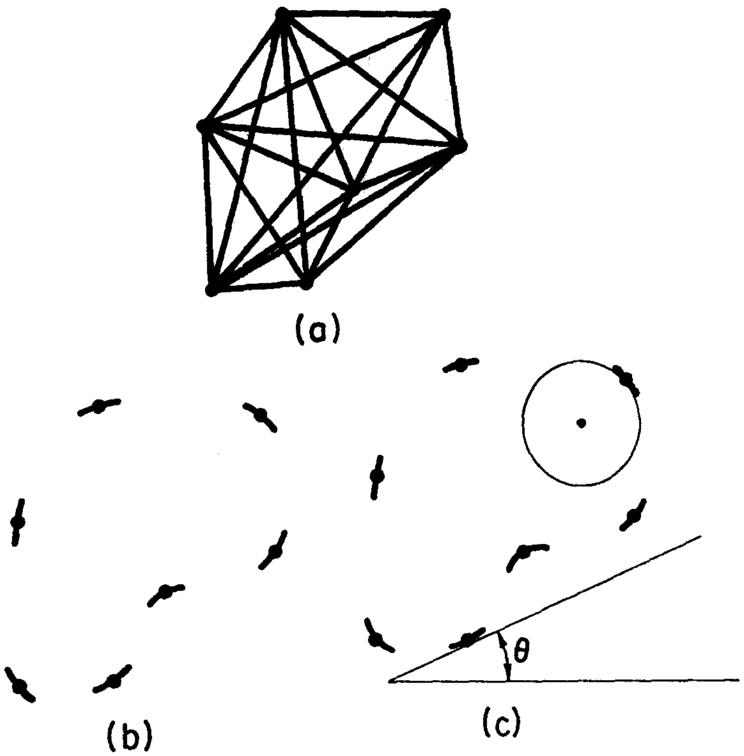

Fig. 3. Tangent angle and curvature as information. **a,** The totality of lines and angles available from constructions on landmarks only. **b,** Additional information at each landmark, omitted from the representation in **a. c,** Geometric meaning of tangent angle and curvature at landmarks.

middle of extended structures. Straight-line distance works only for structures which are clearly extended in one consistent direction, like the limbs.

A better measure of length for bent forms was explained in the anthropologic literature a few years ago,[8] following a basic construction invented surprisingly recently by the visual psychologist Harry Blum.[1, 2, 5] The medial axis, or "skeleton," of a curving boundary is defined as the locus of centers of circles which touch (for example, are tangent to) the boundary at two distinct points. This notion has all the intuitive features one would expect of a "line down the middle." In particular, it captures both the general curving of form, in the bending of that medial axis, and the "width" of the form from side to side, in the radii of those circles centered on the axis. Fig. 4 shows the skeleton of a mandibular trace on a cephalogram, as determined by hand using dividers. The two large dots are branch points indicating the choices which have to be made if length is to be measured intelligently. Such an analysis is quite impossible in terms of landmarks only, since information about curving is required, information that is lost when one relies on landmarks alone. In fact, landmarks do not enter into this analysis at all; in computing the skeleton, no points of the outline are treated differently from any others.

Studying growth changes. Growth itself is not compatible with the current cephalometric scheme, for as landmarks are carried along on a field of continuous spatial deformation, the change between them is not at all summarized by any set of distances and

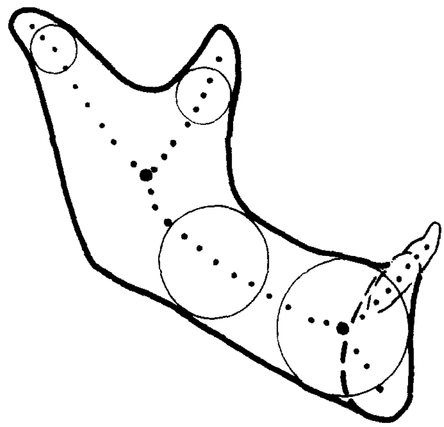

Fig. 4. Skeleton, or medial axis, of a mandible in sagittal projection. The dots indicate the locus of points with no nearest neighbor upon the outline. Centered at each dot there is a circle which is tangent to the outline at two different places (for the larger dots, three places). The curving of the skeleton and the variation of the radii along it are separate measures of the curving of the mandibular form.

directions, however hallowed. One can only speculate on how encumbered our concepts of craniofacial growth have become with the inevitable sella-nasion orientation. In the current state of analysis, based on polygons, we have difficulty searching for structural relationships invariant under the normal range of differential growth rates, for we have not measured these rates, even indirectly. Heads and faces actually change by growth and remodeling in many places, not just by displacement at the corners of polygons. As D'Arcy Thompson[11] pointed out in 1917, lines that are straight today become curved tomorrow, and so the polygons we measure today do not grow into the polygons we measure later. Then to compare polygons is not to measure growth accurately.

Growth is somewhere between the landmarks, not localized "at" them in any sense. To understand the changes of growth and remodeling, we need to know how each landmark is being moved away from the others. It would suffice even to know how each is carried away from those on either side, changing local distances and directions simultaneously. Such an analysis is necessarily formulated in terms of the curved form changes between landmarks, where growth is altering the relationship of any pair of landmarks in a geometrically intricate fashion.

When data do not capture the curving of form, the problem of classifying facial types over a succession of ages is especially difficult. The forms of a face and all of its parts change because of differential growth in many places and in several directions. The facial types we assign and use should properly be constructed as invariant with respect to ordinary growth, for facial types depict the results of invariant patterns of growth. Thus, the failure to record curvature information denies a fair representation of facial types as well as losing localized remodeling details.

Conventional cephalometric procedures often misinform

The lack of a theory of points and curves together forces cephalometrics to rely on points only, of the two different sorts, for its data base. In the preceding discussion we

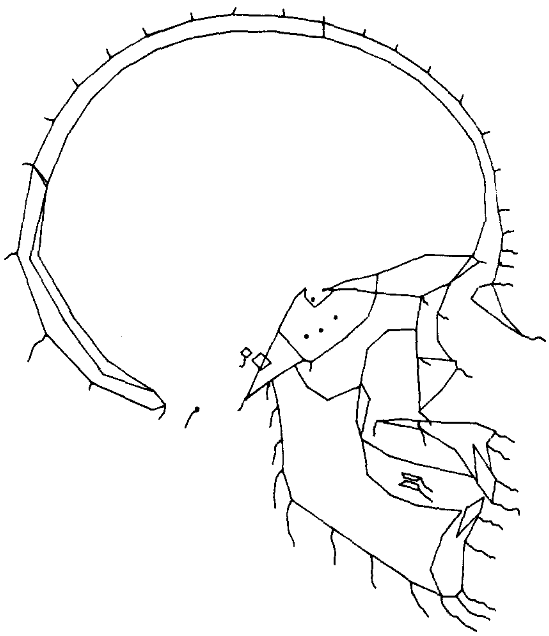

Fig. 5. A complete cephalometric history of the means for a population of 60 girls to age 14. Each "whisker" tracks one landmark through successive cephalograms (only the first of which is drawn in full). Registration is upon sella, with orientation upon a line from base of occipital bone to center of palate. (From Walker and Kowalski: AM. J. OOTHOD. **61**:221-230, 1972.)

saw how the basic notions of size and shape were distorted by the restriction to point data and driven away from their intuitive foundation in the curving of form. That loss of data was forced by the reduction to points, and the errors it introduces into the measurement of growth are irrecoverable thereafter. There are other inappropriate customs in cephalometric practice which are not forced by the reduction to landmarks but, rather, compound our difficulties by mismanagement of the reduced data with which we are left. We sketch these under four general headings: fabrication, camouflage, confusion, and subtraction.

Fabrication. Let us consider the superimposition of tracings. Successive tracings of cephalograms are placed upon each other according to some rule and the motion of corresponding landmarks is plotted, or points of a single cephalogram are each moved a bit according to some population average motion and the resulting future state is drawn out in line. Such a line following the progress or a single landmark will here be called a *track*. Fig. 5 presents an example of a population mean cephalometric history displayed by this method. In principle, such a diagram seems to evade the objections that we presented for the previous techniques. The "motions" of landmarks are now just a sample from a continuum all the way around an outline. The little tracks show how growth of all landmarks proceeds away from some central point of the skull (usually the sella). From this, can we not somehow infer the divergences between neighboring landmarks which, according to theory, are the real mechanism of change? No, we cannot. For any of these techniques, statistical treatment is quite problematical. All location measures are beset by the measurement error of the standard location and orientation in addition to their own. There is then a functional correlation among any pair of displacement measurement errors

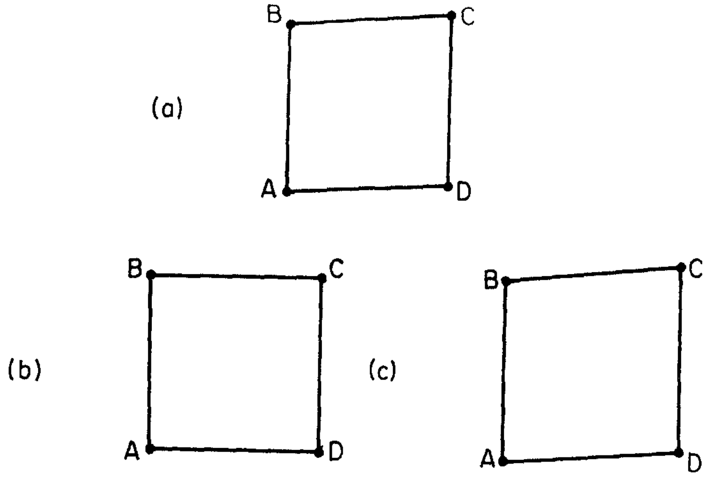

Fig. 6. A hypothetical ontogenetic sequence, (**a** to **c**), of cephalometric polygons.

which is of constantly changing magnitude and direction. Points near the site of registration move less than points farther away, and all points generally move away from the "fixed point." Both these trends are anatomically quite meaningless. The divergence between neighboring landmarks is much less than their common translation, for when a bone is translated by growth in one region (for example, at the mandibular condyle), all landmarks on its far side are passively moved in addition to growing on their own. Then the structure of variation of the outermost tracks is a statistically intractable composite of translations and remodeling there and elsewhere; to untangle it, one must start over by other analytical procedures.

Behind these statistical difficulties is an even more basic flaw. The conclusions we draw about growth necessarily vary, quantitatively and qualitatively, with the registration rule. The superimposition of two images generally proceeds by specifying one registration that is presumed to be unmoving and one direction that is presumed not to be rotating. What is drawn out is a record of two quantities assembled in polar coordinates: the distance from the point of registration to the landmark of interest (which distance is a summation of rates and directions of all the growth in between) and the angle among the landmark and the two points making up the lines of orientation. Any revision of the orientation, then, and also any measurement error in the coordinate system will change this track more or less drastically as the landmark is a greater or lesser distance from the fixed point. The track is, in short, "fabricated." It is a function of the changing coordinate system we choose.

Camouflage. Once the tracks have been fabricated, the superpositioning can be wielded to give an analysis of shape change (that is, growth) in which biologic reality is thoroughly camouflaged. Depending on the orientation and registration we choose to hold fixed (Frankfort horizontal, sella-nasion, or *whatever*), the progress of particular landmarks can appear to be curving up or curving down, speeding up or slowing down on its track. Now all the registrations contain *exactly the same geometric information*—the motion of all points relative to a moving coordinate system—and we can pass from one to another by adjusting trajectories to take into account the simple formulas for change of

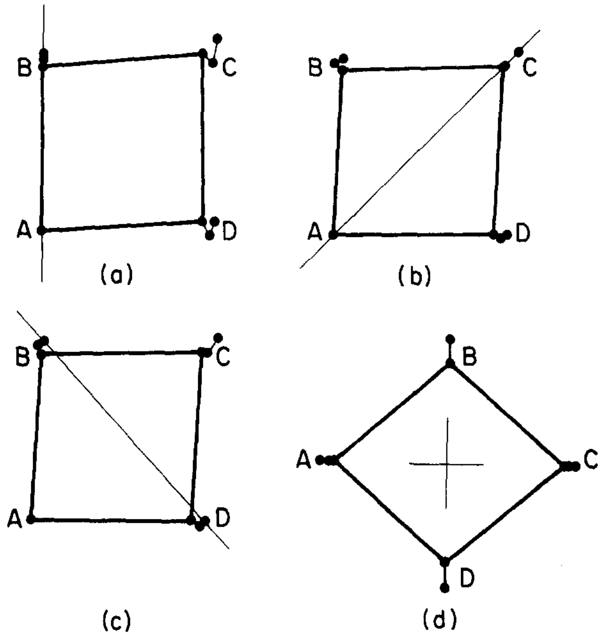

Fig. 7. Summaries of the data of Fig. 6 by four different registration rules. Upon a drawing of the earliest form, the trajectories of the corners, variously oriented and registered, are traced in broken lines. **a,** Registered upon *A,* oriented upon *AB.* **b,** Registered upon *A,* oriented upon *AC.* **c,** Registered upon *A,* oriented upon *BC.* **d,** Oriented on *AC,* registered at the intersection of *AC* and *BD.* Only for the last analysis, which is not registered on a landmark, is the visual summary biologically suggestive.

polar coordinates. The shape of the little curved tracks, then, has no biologic meaning whatsoever, for we can make any track go to a fixed point simply by registering directly upon it, without any loss of information. There is, then, *no best registration*. That the information content of alternative registrations is equivalent does not save us from statistical fallacies, inasmuch as all our statistical models are fundamentally linear. A track which is straight or in any way well behaved in one analysis will be curved and difficult to analyze in another.

This can be shown best by means of an artificial example. Fig. 6, *a, b,* and *c* shows three successive states of a quadrilateral of landmarks in a hypothetic developmental sequence. Selecting point A for registration and the line AB for orientation, we obtain the tracks shown in Fig. 7, *a;* selecting point A and the line AC, we obtain 7, *b;* registering on A and orienting on line BD, we obtain 7, *c*. The raw data are unchanged, but the registrations are related to each other in nonlinear ways. The simplest way of viewing these data is shown in Fig. 7, *d,* oriented on lines AC and BD with their intersection for register. The shape change was, in fact, produced by an extension along line BD, followed by an equal extension along line AC. This is not so unreasonable a model for certain changes in the face—but how unpromising *a, b,* and *c* looked for the same data!

It is no accident that the registration for Fig. 7, *d* is on a constructed point rather than a landmark. Any pattern in which the points used for registration or orientation are involved will be wholly misconstrued by registration or orientation upon them. Their spatial variation has been arbitrarily restricted to a line or to a point, while all other points are free to

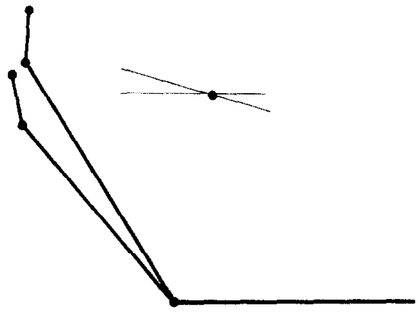

Fig. 8. Movement of condylar axis in a coordinate system registered on gonion and oriented on mandibular plane. There is an apparent center of "rotation" of the condyle, located at the intersection of the perpendicular bisectors of segments connecting homologous points "before" and "after." This point has no biologic meaning, being wholly an artifact of the registration procedure.

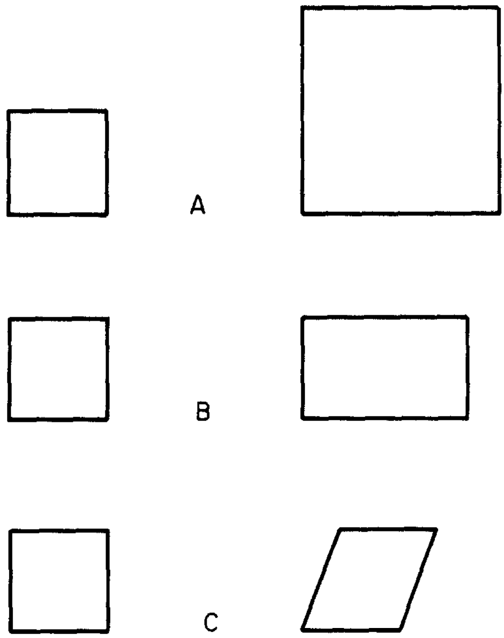

Fig. 9. The levels of geometric complexity of growth. **a**, Change in size only, shape preserved, description by scalars. **b**, Change of size-ratios only, axes preserved, description by vectors. **c**, Change of sizes in all directions, orientations not preserved, description by tensors.

vary over a region of non-zero area. In particular, registration at sella with orientation on sella-nasion is predicated upon the postulate that growth there is functionally unrelated to change in the parts of the face that are of orthodontic interest, that the geometric straight line from sella to nasion does not geometrically bend over the course of growth. Likewise, use of the Frankfort horizontal requires an assumption of linearity of growth all along the line from porion to orbitale. In fact, the crucial points of either of these registrations do not move smoothly with respect to the other registration, and certainly both systems are

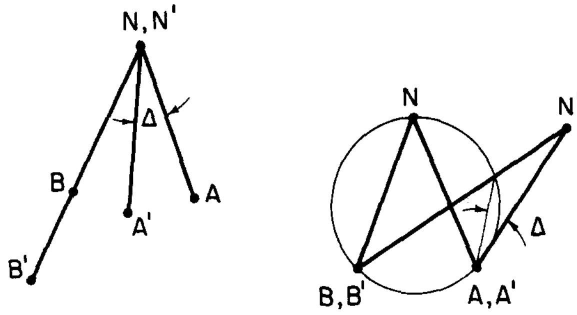

Fig. 10. The impossibility of attributing change in angle to particular sites or directions of growth. Consider the triangle *ANB* and its image *A′N′B′* at some later date. **a,** Registering on *N*, orienting on *NB*: $\Delta = \angle A'N'B' - \angle ANB$ is interpreted in terms of the relative movement of point *A* away from line *NB*. **b,** Registering on *A* and *B* (by change of scale if necessary), Δ is interpreted using the amount of relative motion of *N* radially away from the circle through *N, A,* and *B*.

correlated with developments elsewhere in the craniofacial complex, which thereby become systematically mismeasured.

How can any such assumption of "featureless" registration, registration not implicated in the conclusions we draw, be defended? No known registration yields smooth linear tracks for the landmarks distant from the fixed point, because the growth rates vary irregularly in space and time over the developing face and cranium. In the absence of any metric theory describing how the landmarks are borne apart by the growth in between them, the appearance of consistency among tracks is a matter of chance. As all registrations express the same geometric content, the preferred choice can be only an accident of near linearities over an extended region—averages of nearly balanced nonlinearities acting everywhere throughout. This is a most evanescent statistical advantage. In particular, a registration which works for normal cephalograms can be expected to fail systematically for cases with any notable imbalance. That is, we will be at the greatest disadvantage in those subjects who are most important to us—our patients with severe malocclusions. However we proceed, nonlinearities always enter the data analysis. Because we do not yet have a good grasp of the functional constraints relating sizes and rates of growth throughout the face, it is very difficult to decide whether any registration is concealing or camouflaging crucial covariation. There is no essential difference among them; all share this fundamental statistical flaw.

Confusion. Distinct from the difficulties just explored are certain problems in the handling of data which derive from fundamental confusions about the behavior of geometric objects in the plane of the cephalogram. We have found three major problems of this sort, in conceptions of rotations, "controlling for size," and measuring changes in angle.

Attention has been directed recently to studies of "rotations" of parts of the head and face during growth. At first glance these "rotations" seem very significant and simple enough to understand. However, the motion of a rotating object observed in a rotating coordinate system is just plain complicated in any third coordinate system (Fig. 8). Even in a conjunction of two rotations at constant speed, the apparent net center of rotation moves and the apparent net angular velocity varies and may even change its sign (from

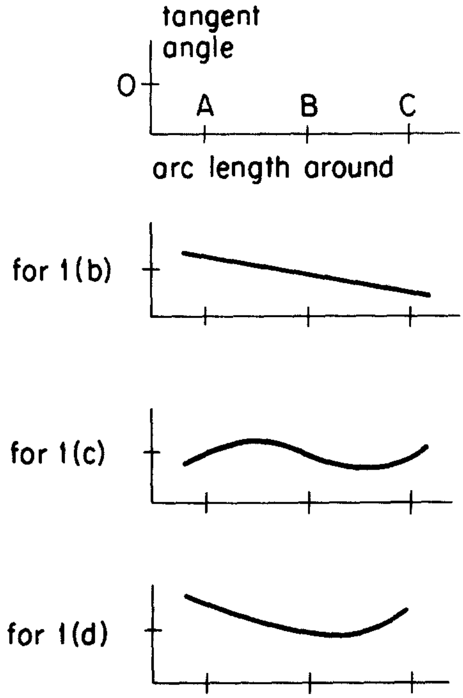

Fig. 11. Sketches of tangent angle as a function of arc length for the three curves **(b, d, d)** of Fig. 1. They can be discriminated very easily.

clockwise to anticlockwise, or vice versa). This elementary prohibition is directly relevant to studies of parts which rotate as they grow (for example, the maxilla and the mandible with respect to the cranial base). Whatever results are obtained are set within a rotating system themselves and cannot be expected to be consistent from analysis to analysis. For measuring the growth rotation of the mandible, for instance, the quantification obtained by superposition on the Frankfort plane or the palatal plane has no geometric relationship to the same rotation superimposed on the cranial base. Therefore, when we speak simply about growth rotations of the mandible, we have not stated the crucial parameter, the system against which it is presumed to rotate. Perhaps we pursued this faulty logic in prior studies of growth rotation because of the many useful kinesiologic studies of mandibular movements. However, centers of rotation determined during mandibular opening and closing can properly use a single angular coordinate. So-called growth rotations are essentially much more complex, for all parts, and not just one, are changing through time.

Two other notable confusions deserve mention here. Some speak of "size and shape" as two separate measurable concepts and often refer to the measurement of shape "controlling for size." The orthodontist, in separating size and shape, is obviously oversimplifying. By *size change,* he means proportional increase in all dimensions (Fig. 9); by *shape change,* he means everything else. But size and shape are functionally correlated. "Size" is one predetermined distance, but "shape" is not "everything else," for "everything else" involves the same points that were used to measure size. We cannot "control" for one and examine the other, for the other has been controlled as well. The large

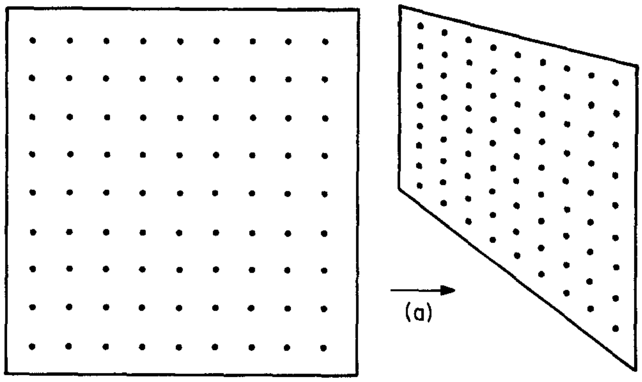

Fig. 12. Depiction of a pair of shapes as a distortion, after the manner of D'Arcy Thompson. From information about the homologies of points and curves, we compute a smooth function (here sampled at points of a square grid) which describes the distortion as a mapping everywhere inside.

biometric literature on allometry convinces us that shape, however measured, generally has a significant regression on size, however measured, for a myriad of biologic analyses of interest. There can be many shape variables in an analysis—differences, angles, ratios all over the craniofacies. Even if one of these is perfectly uncorrelated with size, all the rest are more or less correlated and confounded. Analysis of size and shape as separate quantities is necessarily confused.

A final confusion deals with the notion of changes in an angle. Any measured angle is a function of three points of two coordinates each, totaling 6 degrees of freedom. We draw change in angle, though, with only one coordinate, a curved arrow around the vertex. In summarizing, we are in effect registering on the vertex and orienting on one of the other rays of the angle as we so frequently do with ANB registering at nasion. Let us, instead, register on the other two landmarks involved. We can do this by altering the scale of one of the figures; as angle is invariant under change of scale. Then (Fig. 10), whenever the vertex moves along the unique circle through the three points of the initial configuration, there is no change in angle at all (by the inscribed angle theorem of Euclidean geometry). The change in angle is measuring movement of the vertex perpendicular to that circle, but that direction is a function of all three points; the measured change in angle is a very complex function of the relative motions of the three landmarks involved, not just one or two as we so often assume. Growth movements of nasion, therefore, have an important effect on changes in the angle ANB. In registering on a moving point, we lose touch with geometry. Later in this article we shall present a method of analyzing shape change which circumvents this fallacy by measuring no angles at all, by reducing all change in shape to differential changes in size.

Subtraction. Growth may be defined as localized differential translation, and it can be measured by superposing locally. Because growth sites themselves may be localized (as, for example, at synchondroses or sutures), some registrations and orientations are better for understanding growth in one particular place. However, not all growth occurs at the landmarks we may have chosen; it occurs generally in many places, usually between landmarks. For instance, gnathion does not grow away from sella; rather, it is translated by all the growth change between gnathion and sella. In general, apparent movement of

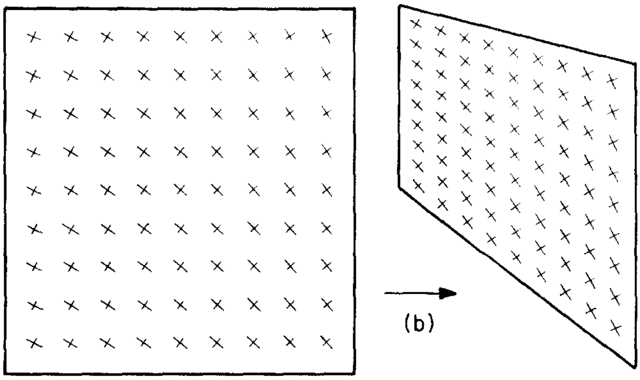

Fig. 13. A sample of homologous directions at 90 degrees in both images of Fig. 12. At every point of the image pair there exists just one such cross, an algebraic function of the form of the distortion locally.

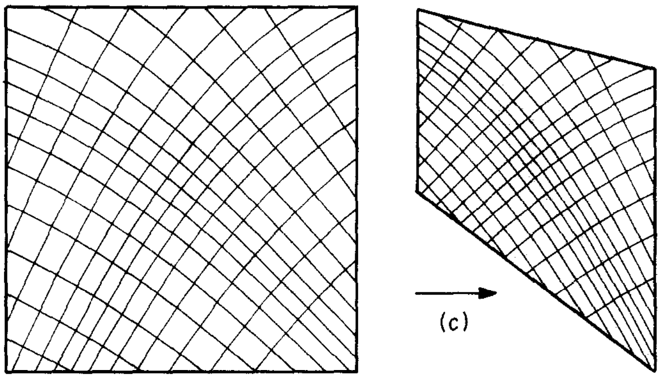

Fig. 14. A "biorthogonal" coordinate system, whose curves are aligned everywhere with the tensor field of crosses sampled in Fig. 13. This coordinate system explicitly depicts the shape change.

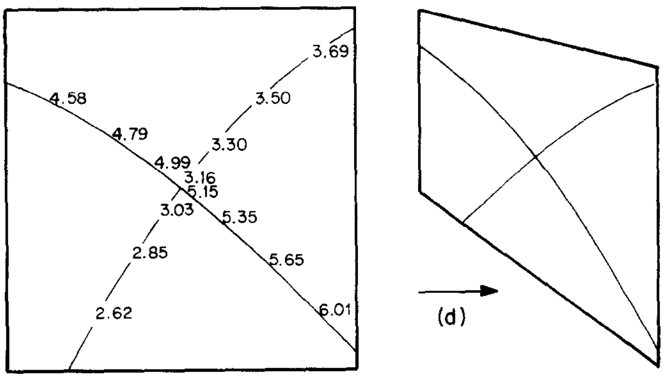

Fig. 15. Summary of the biorthogonal analysis of Fig. 14. Two coordinate curves and ratios of corresponding lengths sampled along them. (The scale of the right-hand form has been reduced sixfold for drawing). The curves and the gradients of size change along them summarize the shape change in a manner quite independent of predetermined measurements of the shapes separately.

implants is translation caused by growth in many places, in loci not restricted to the straight line between a cephalometrician's favorite registration points. Remodeling accounts for further growth changes and is ignored by subtracting two distances with one common point of superpositioning. Because of the work of Enlow[7] and others, it is now accepted that form moves through bones, remodeling as it passes. We need methods which treat of the changes of form directly rather than depending on fixed points alone, whether landmarks or implants.

Let us summarize this manifold critique. Conventional cephalometrics is inadequate for the description of individual cephalograms because of its inability to apprehend curved form; it is limited to landmark-based indices. The problem of recognizing stable patterns over time is especially difficult, as there exist local directional growth gradients all over the face which the technique cannot take into account. For description of growth or shape change, the usual techniques are all dependent upon registration and orientation, which distort irrevocably the very record of change we would like to examine and which have a systematic error interacting with particular abnormalities of form.

Are there better customs for arriving at numbers whose statistics and comparisons are meaningful?

Plato is reputed to have said: "It is an easy thing to fault another's oration; it is a much more difficult task to produce a better one in its place." Our task is to devise a scheme which will minimize the misinformation retrieved from the cephalogram and maximize the biologically relevant data made available. An additional task is to understand the irreducible errors of measurement which remain. One of us (F.L.B.) is a biometrician who has been studying the geometry of shape change for some time. The other (R.E.M.), an orthodontist, has felt that the well of conventional cephalometrics ran dry some time ago. Our collaborations these past 2 years are an attempt to apply Bookstein's notions to the problems of Moyers' field. Let us now return to some earlier figures and the problems to which they were directed.

Measuring shapes separately. The indices associated with Fig. 3, *a*, the actual quantitative data currently extracted from cephalograms, are not very useful for analyzing change, for they do not even distinguish the situations of Fig. 1. We could take a giant step toward adequacy of measurement by noting, as suggested in Fig. 3, *b*, the tangent angle and curvature in a little neighborhood around each landmark, in addition to the landmark position itself. The tangent angle is the azimuth of a straight line lying along the outline at the landmark, as in Fig. 3, *c*; the curvature is the inverse of the radius of the circle closest fitting to the outline there and indicates how rapidly the tangent angle is varying with distance along the outline. It is precisely these two measures that are embodied in the little hachures which make Fig. 3, *b* so much more informative than Fig. 3, *a*.

No extant cephalometric scheme includes these two simple measures, for they arise mathematically as limits from the construction of segments through pairs and triples of points arbitrarily close and so override the basic notion that landmarks should be individual loci at some finite spacing. Nevertheless, the data required for computation of such figures as 3, *b* already exist. Tangent angle and curvature can be estimated quite easily from digitized cephalogram tracings of the sort described by Riolo and associates[9] and Walker and Kowalski.[12] Hitherto these data banks have never been processed in this way but only for extraction of the old indices.

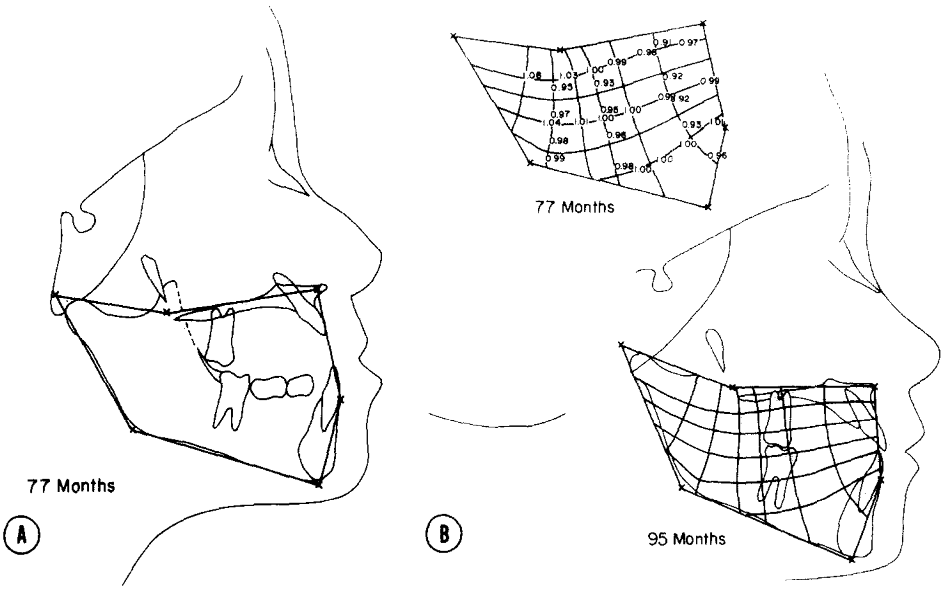

Fig. 16. Biorthogonal analysis of craniofacial growth and remodeling in an Angle Class III patient from the age of 77 months to the age of 95 months. The analysis is of a six-sided polygon through condylion, posterior nasal spine, A point, infradentale, menton, and gonion. **A,** The polygon at 77 months, with the tracing of the 77-month cephalogram. Scales of polygon and tracing are not quite the same; here the polygon is drawn by the computer program to a scale slightly larger than that of the tracing. **B,** Biorthogonal grid pair for 18 months of treatment by Fränkel FR-III. Upper and lower grids are drawn in exact relative scale, so that each dilatation entered upon the upper grid is the actual ratio by which a segment of the 77-month grid is to be multiplied to yield the corresponding segment in the 95-month grid. The 95-month polygon has been approximately superimposed over the 95-month tracing, which is to slightly different scale. The grid indicates that the maxilla is translated forward without itself growing in length and that there is no size increase elsewhere in the polygon.

Landmarks do not define the form; they only serve as pointers to hold our conceptual place upon it. We need not be satisfied to measure tangent angle and curvature only at landmarks; ideally, we would want the history of these quantities all the way round. At this juncture mathematics comes to our rescue with a handy fact: the two parameters we introduced are redundant, for curvature is merely the derivative of tangent angle with respect to arc length along the curve. The information we want is contained in a single function, tangent angle, defined all around the outline of the cephalogram. The arcs of Fig. 1 are then represented as in Fig. 11, where they can be told apart quite trivially. Landmark data can sample this tangent-angle function at points located reliably. They provide useful information, not about their own isolated locations but about the shape of the outline in their vicinity. In fact, the outline could as well be measured in the absence of any landmarks at all, as by the skeleton of Fig. 4, for which not a single anatomic locus need be specified in advance. The skeleton is self-measuring; it bears its own distinct features. As the tangent-angle function goes around the outside of the form, so the skeleton passes up the middle, capturing in its curvatures the subtleties of form which quite elude statistics based on landmark position.

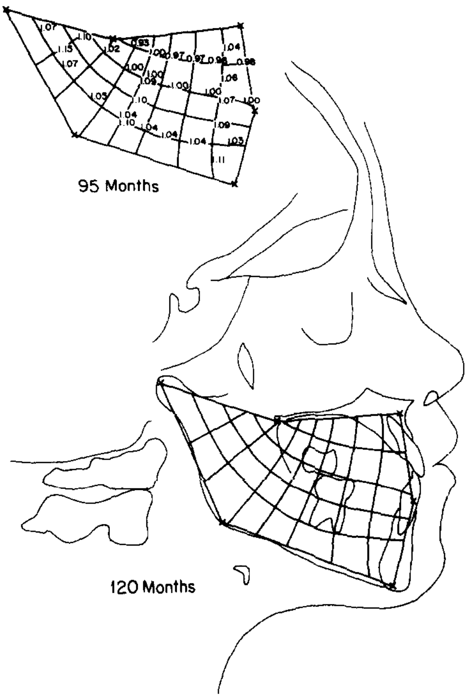

Fig. 17. Biorthogonal grid pair for the same patient between the ages of 95 months and 120 months, under treatment with a chin-cup. The two polygons are drawn to exact relative scale, with the 120-month polygon approximately superimposed over the 120-month tracing. Again the maxilla is translated forward without growing in length. At every point in the mandible, growth rate lengthwise (in the direction of the arc from menton to condylion) is always the minimum of all the local directional growth rates; the mandible is growing "downward and backward."

Measuring shape change. We suggest that subtraction methods for shape change, with all their flaws and fallacies be superseded by the method of biorthogonal grids, or change coordinates, which we have developed[3-6] after a profound insight of D'Arcy Thompson.[11] Below we explain the basic themes of the method in terms of two principles and five steps of implementation.

PRINCIPLE 1. Shape change is not measured by subtracting measurement A from measurement B but by measuring the operation of shape change itself, that is, distortion. This is the main idea of Thompson's famous method of 1917, the Cartesian grid.

PRINCIPLE 2. Our method reduces change in shape to simple changes in size along certain specifically computed directions. The report of shape change is in terms of these growth gradients and the locus of the coordinate curves on which they lie.

These principles will become more clear as we analyze an artificial example, the shape change which transforms a square into a polygon.

Step 1: Obtain landmark points and the curves between them which are biologically homologous for a growth series (Fig. 12).

Step 2: Compute the distortion mapping inside the outline data recorded. Special computer programs are needed for this step.[5, 6, 10] In effect, we are mapping the image of all growth in the cephalogram by interpolating the "sample" values of the mapping supplied by the homology between the two images.

Step 3: At every point inside the first outline, compute the unique pair of directions which are perpendicular to each other and which grow into directions which are still perpendicular (Fig. 13). This is done by simple algebraic manipulation of the mapping function from Step 2. The figure shows only a sample of these perpendicular directions.

Step 4: Compute homologous pairs of extended curves passing all the way across the image which are parallel everywhere to the directions from Step 3 (Fig. 14). These curves, by construction, intersect at right angles in the later image as they did in the earlier. We call this figure the *biorthogonal grids* for the problem. The intersections of the curves, themselves homologous between the images, provide a new picture of the distortion analogous to that of Step 2; this time, however, the starting grid is not square, aligned with a single form and its separate coordinate system, but expresses the natural coordinates of the distortion itself.

Step 5: With all the angles fixed at 90 degrees, growth is embodied in the stretching or shrinking of lengths in the latter figure relative to their homologues. These dilatations are graded along the curves of the grid. Reports of a selected few of these gradients, as decimal ratios written upon the local arcs to which they pertain, summarize the shape change and make it possible to proceed with statistical analysis, just as we have been doing with the scalars (Fig. 15).

A clinical example. The biorthogonal, or change-coordinate, technique has applications throughout craniometrics whenever forms are to be compared, whether in studies of normal growth, abnormal growth, population variation, or treatment effects. We close this brief discussion with an example displaying the contrast between two treatment programs for the same Class III patient (Figs. 16 and 17). For the purpose of demonstrating the change-coordinate technique, and because the effects we will diagram are so considerable, we represent the maxillary-mandibular complex by a crude six-sided polygon. For subtler changes, of course, the curving representations we have put forth above are preferred.

The first grid pair displays the effect of treatment by a Fränkel appliance (FR-III) from the age of 77 months to the age of 95 months. The grids indicate a consistent regimen of growth gradients throughout the region, with translation of the maxilla, stasis of mandibular length, and apparent general decrease in lower face height. Fig. 17, presenting the second grid pair for this patient, shows the effect of chin-cup traction for 25 months after removal of the FR-III appliance. At first glance, the coordinate grid here has a very similar aspect, but the principal gradients of growth are quite reversed. Now the vertical growth gradients uniformly *exceed* the horizontal; the forward growth of the mandible has been less than the vertical size increase, that is, the mandible has been rotated downward and backward.

In general, as here, the biorthogonal method directly displays those curves along which growth is proceeding most quickly and most slowly. It is a direct display of change only, not of any aspects of the separate forms being compared. That this patient's malocclusion is called Class III at both times has no role in the computations and is less important than the change observed. The clear contrasts which the method displays are, of course, a measure of therapeutic progress; their direct display is the greatest advantage of the technique.

Conclusion

In this article we have attempted to explain the stasis afflicting cephalometrics nowadays and to propose several new techniques which speak to its current incapacities. The reformation of cephalometrics is clearly overdue. Modern computational geometry can handle the curving constructs we have suggested—tangents, "skeletons," biorthogonal grids—for any computer-based representation of a cephalogram. Our ambition is the rectification of imbalance between computerized biomedical shape data and the archaic tools of morphometrics—ruler and protractor—bequeathed us by the Greeks. We hope that the new cephalometrics will spur craniofacial science to a much-needed sophistication of measurement for the most stubborn outstanding problems.

Summary

1. Cephalometric conventions today may have little basis in either biology or biometrics.

2. There is no theory of cephalometrics, only conventions which involve landmarks and straight lines only. These fail to capture the curving of form and its changes, exclude proper measures of size for bent structures, and misrepresent growth, portraying it as vector displacement rather than a generalized distortion.

3. Conventional cephalometric procedures misinform by *fabrication* of misleading geometric quantities, by *camouflage,* particularly of remodeling, by *confusion* about what is happening (analysis of rotations, treating shape separately from size, and registering angles on landmarks as vertices), and by *subtraction* as a representation of growth.

4. We suggest that the present systems offer little real hope of improvement sufficient to meet our needs in craniofacial growth research. We call attention to three possible techniques to be included in future cephalometric conventions: (1) tangents and curvatures, (2) Blum's medial axis ("skeleton"), and (3) biorthogonal grids.

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