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TRANSIENT NUCLEATION AND BUBBLE GROWTH IN IMMISCIBLE LIQUID COMPOSITES INDUCED BY COUNTERDIFFUSION OF GASES

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ABSTRACT

Supersaturation, homogeneous nucleation, and subsequent bubble growth and motion in immiscible liquid layers induced by counterdiffusion of gases at different temperatures are studied analytically. The range of the critical embryo size to initiate transient nucleation is determined between spontaneous and infinitely-slow nucleation. In addition, the ranges of bubble departure size and terminal rising velocity are evaluated together with the degree of superheat required for heat transfer to be the controlling mechanism. Numerical results are obtained for two special cases: (i) a composite with negligible surface resistances to heat and mass transfer, and (ii) a composite with one side insulated. The mechanics of microexplosion of emulsions is explained.

Introduction

An immiscible liquid mixture exists in nature as well as in industry. For example, oil-water mixtures are observed in reserve underground or on the sea by accidental oil spills. In industry, water-in-oil (W/O) emulsions may be utilized in conventional liquid-fueled combustors for the purpose of reducing the various undesirable heterogeneous effects such as soot, smoke, and nitrogen oxides and enhancing beneficial effects such as saving fuel. Dryer [1] presented a historical review (up to 1976) on the applications of water addition to practical combustion systems and discusses in detail the fundamental aspects of combustion which are affected. Recent studies on the combustion of emulsified droplets are numerous, for example references 1-11. The effects of water addition on combustion can roughly be classified as (i) chemical kinetic (or dilution) effects that result in reductions in the formation of soot and NO_x formation and (ii) physical (or micro-explosion) effects that result in a secondary atomization process, shorter droplet lifetime and combustion time, a more uniform charge for combustion, and an upper bound on the droplet temperature. The phenomenon of micro-explosions of oil/water emulsion droplets suspended on fibers was first discovered by Ivanov and Nefedov [9] and recently observed by Dryer et al [10] and Jacques et al [3]. It constitutes a secondary atomization process caused by heterogeneous nucleation at the suspension fiber. The free secondary droplets produced through fiber-induced explosions undergo tertiary explosions [10] caused by homogeneous nucleation within the liquid phase. Thus, the existence of micro-explosions in both suspended and free droplets of emulsified liquid mixtures is confirmed. However, there is a marked difference in the micro-explosion phenomena between free and suspended emulsified droplets because the presence of the suspended filament changes the internal phase structure of the droplet. Free droplets are realistic in industrial applications although suspended droplets are often observed in experiments.

The characteristics of nuclear pool boiling from a horizontal wire to various emulsions were empirically determined by Mori et al [12]. Aside from shifting in the boiling curve in the presence of internal phase droplets, their important discoveries include (i) surface (heating) temperature overshooting at the inception and cessation of boiling and (ii) occurence of phase inversion (from O/W to W/O type) or oil separation during the course of raising surface heat flux as evidenced by a jump in the surface temperature. These indicate some basic differences in the processes of nucleation and bubble growth to exist in the suspended and free emulsions in heating.

Experimental study on steady-state isothermal counterdiffusion of two gases in immiscible liquid layers [13,14] detected the formation of bubbles at the liquid-liquid interface. A theoretical analysis yielded a crude qualitative criterion for bubble formation. However, repeated seeding at the liquid-liquid interface by adding artifical nuclei in the form of crushed glass particles produced continued bubble evolution in the composite layer. Law [5] proposed a model for the combustion of oil/water emulsion droplets. It is essentially identical to the model for the combustion of pure fuel droplets with more modifications or restrictions: (i) the entire system is assumed to be at steady state, (ii) the droplet temperature (at its maximum) and concentration are constant and uniform throughout the liquid phase with no distiction regarding the internal or dispersed phase, and (iii) both the fuel and water evaporate at the drop surface with the rates equal to their interior concentrations. The analysis yields the criterion for supersaturation under the assumption that nucleation occurs within the internal phase. It predicts micro-explosion at the droplet surface which is physically unrealistic and contradictory to the assumption. The model cannot predict when micro-explosion occurs or how bubbles are formed and grow. Avedesia [11] studied the problem of superheating and vaporization of internal phase droplets of a suspended emulsion. It has been found that, depending on the chemical structure of the emulsion components, the vaporization temperature of internal phase droplets of the emulsion may approach the homogeneous superheat limit of pure water. Thus, the superheat limit of the internal phase droplets within an emulsion may be lowered by the presence of the suspending filaments.

The subjects of boiling nucleation, boiling liquid supersaturation and bubble nucleation in liquids are reviewed in references 15, 16, and 17, respectively.

Basic research on micro-explosion of emulsions has only recently been initiated. Both references 5 and 11 emphasized "superheating" of internal phase droplets as its mechanism but devoted little effort on the direct cause i.e. nucleation and subsequent explosive bubble growth which eventually lead to the secondary and tertiary atomization processes of emulsions in combustion. The lack of such understanding has led to many inconsistencies reported in practical combustion applications of emulsified fuel.

The present work is to investigate the nature and characteristics of nucleation and bubble growth in immiscible liquid mixtures under transient heat and mass fluxes induced by counterdiffusion of gases at different temperatures. Simple system geometry such as binary-layer is employed. The layer model can be theoretically extended to the spherical composites with applications to emulsion droplets.

Analysis

Consider a two-layer composite of immiscible liquids A and B with the thickness L_1 and L_2 respectively. Initially, both layers are maintained at uniform temperature T_o and pressure P_o. Then, gases A and B at the temperature T_∞ and P_∞ respectively are suddenly introduced into the system. Gas A is in contact with layer 1 and gas B is in contact with layer 2, as shown in Fig. 1. Thus, gases A and B diffuse through the composite layer in opposite directions. It is assumed that these species obey Henry's law for the gas solubilities. Then, Fick's law for mass diffusion flux can be written as:

$$M_{ij} = -D_{ij}\ \partial P_{ij}/\partial x \tag{1}$$

Here, M_{ij} denotes the mass flux of i through j expresses as volume of i per unit area of j per unit time. D_{ij} represents the permeability factor, the product of mass diffusivity and solubility of the i in j in volumes of i per volume of j per unit pressure. P_{ij} is the partial pressure of i in j and x measures the distance from the liquid interface.

Both heat and mass balance equations read:

$$\frac{\partial T_j}{\partial t} = \alpha_j \frac{\partial^2 T_j}{\partial x^2} \tag{2}$$

provided the T_j and α_j are replaced by P_{ij} and K_{ij} respectively in the latter case. They are subject to the appropriate initial and boundary conditions:

$$T_1(x,o) = T_2(x,o) = T_o \tag{3}$$

$$T_1(o,t) = T_2(o,t); \quad k_1\frac{\partial T_1(o,t)}{\partial x} + k_2\frac{\partial T_2(o,t)}{\partial x} \tag{4}$$

$$-k_1\frac{\partial T_1(-L_1,t)}{\partial x} = h_1[T_\infty - T_1(-L_1,t)]; \quad -k_2\frac{\partial T_2(L_2,t)}{\partial x} = h_2[T_2(L_2,t)-T_o] \tag{5}$$

With T_j, k_j, h_j replaced by P_{ij}, K_{ij}, h_{Dj}, respectively in the mass transfer case

The Laplace transformation method is employed to obtain the exact solutions for T_j and P_{ij} in the Laplace domain. However, due to complexity in the expression resulting from the inverse Laplace transform and the fact that nucleation and bubble growth occur in the later part of the transient transfer phenomena, only the asymptotic solutions valid for large values of time are obtained for T_j and P_{ij}. This is accomplished by performing inverse Laplace transform on the expressions with hyperbolic functions being approximated by series expansion with the first two terms retained. One thus obtains:

$$\frac{T_j - T_o}{T_\infty - T_o} = \frac{a_j}{c_j} + (b_j - \frac{a_j}{c_j})e^{-c_j t} \tag{6}$$

$$\frac{P_{ij} - P_o}{P_\infty - P_o} = \frac{a_{ij}}{c_{ij}} + (b_{ij} - \frac{a_{ij}}{c_{ij}})e^{-c_{ij} t} \tag{7}$$

It must be noted that all coefficients are x-dependent. The instantaneous total pressure distribution P_j in the composite is the summation of the partial pressure profile of i.

The thermodynamic equation of Clausius-Clapeyron which represents the equilibrium pressure curve of more volatile liquid reads:

$$\frac{P_v - P_\ell}{T_v - T_s} \cong \frac{h_{fg}\rho_v}{T_s} \tag{8}$$

Here, P_v, T_v, and ρ_v signify the partial pressure, temperature and density of the vapor phase of more volatile liquid in an embryo, respectively, T_v is approximated as T_ℓ at the location of the embryo, while P_ℓ is the pressure of the liquid surrounding the embryo which is treated as P_o. T_s denotes the saturation temperature of more volatile liquid at P_ℓ and is thus equal to T_o. The Laplace-Kelvin equation relates the internal-to-external pressure difference and the equilibrium radius of an embryo r:

$$P_v + P_g + P_x - P_\ell = \frac{2\sigma}{r} \tag{9}$$

The embryo must reach a critical size r_e to initiate a spontaneous growth. It is assumed that the vapor and gases within the nucleus or bubble behave ideally and obey the thermodynamic equation of state for perfect gases.

The maximum total pressure in the composite occurs at the liquid interface, x=o. With the combination of Eqs. (8) and (9) and the instantaneous total pressure and temperature variations in the composite, one finds the location of nuceation, which corresponds to the location of the maximum total pressure in the composite, to be at the liquid interface x=o.

When the first cirtical embryo is formed, it will attempt to grow from this radius to its next stable radius [22]. The waiting time t_e required for an embryo to grow to the critical size r_e is found to be:

$$r_e^* = (J_1 - N_1\ e^{-t_e^*})\Delta T^* + (J_{A1} - N_{A1}\ e^{-E_1 t_e^*}) + (J_{B2} - N_{B2}\ e^{-E_2 t_e^*}) \tag{10}$$

The first term on RHS of Eq. (10) represents the contribution of heat transfer, while the other two terms are those by the diffusion of gases A and B, respectively. When r_e and t_e are replaced by r and t, respectively, Eq. (10) describes the time history of growth of an embryo to its critical size r_e. Two bounding cases of the nucleation corresponding to $t_e^* \to o$ and infinitely slow nucleation as $t_e^* \to \infty$. Equation (10) gives the condition of spontaneous nucleation as:

$$r_e^* = b_1\ \Delta T^* + b_{A1} + b_{B2} \tag{11}$$

The equation predicts the critical ΔT^* for a specified mass diffusion below which spontaneous nucleation of an embryo to become a nucleus of the radius r_e is impossible. In the other extreme case of $t_e^* \to \infty$, Eq. (10) gives:

$$r_e^* = J_1\ \Delta T^* + J_{A1} + J_{B2} \tag{12}$$

which represents the criterion for an infinitely slow nucleation process. Equations (11) and (12) can be combined to yield the range of nucleation size due to transient heat and mass diffusion as:

$$J_1\ \Delta T^* + J_{A1} + J_{B2} \geqq r_e^* \geqq b_1\ \Delta T^* + b_{A1} + b_{B2} \tag{13}$$

A comparison of the term due to heat transfer contribution to those contributed by mass diffusion in Eqs. (11) and (12) yields the expressions showing relative importance between heat and mass transfer:

$$\Delta T^* \geqq (b_{A1}+b_{B2})/b_1 \quad \text{and} \quad \Delta T^* \geqq (J_{A1}+J_{B2})/J_1 \tag{14}$$

for spontaneous nucleation and infinitely slow nucleation, respectively. The equal sign indicates some importance, while the unequal sign means heat transfer dominating. The same arguement can be applied to Eq. (10) to yield:

$$\Delta T^* = \frac{J_{A1} - N_{A1}\ e^{-E_1 t_e^*} + J_{B2} - N_{B2}\ e^{-E_2 t_e^*}}{J_1 - N_1\ e^{-t_e^*}} \tag{15}$$

valid for all nucleation speeds. In general, the larger is the magnitude of the denominator in Eqs. (14) and (15), the lower is the degree of superheat ΔT^* required for heat-transfer to be the dominating mechanism.

If heat and mass transfer continue, an embryo may grow beyond its critical size referred to as a bubble. The history of bubble growth is generally divided into three stages: initial, intermediate, and asymptotic stages. In the initial stage which corresponds to the small-time regime immediately following the bubble growth, the instantaneous bubble radius R is controlled by inertia, pressure difference across the bubble surface, viscosity, and surface tension, as described by the Rayleigh-Plesset bubble dynamic equation. After a brief duration of the intermediate stage, bubble growth is in the asymptotic stage which covers the major portion of the entire growth history. In the asymptotic stage, bubble growth is governed soley by the transport of heat and mass.

The fluxes of heat and mass transfer to a bubble at the liquid interface in the composite read:

$$\frac{1}{2}\left(k_2\frac{\partial T_2}{\partial x} - k_1\frac{\partial T_1}{\partial x}\right) = P_v\, h_{fg}\, \frac{dR}{dt} \tag{16}$$

and:

$$-D_{A1}\frac{\partial P_{A1}}{\partial x} = \rho_{A1}\frac{dR}{dt} \quad , \quad -D_{B2}\frac{\partial P_{B2}}{\partial x} = \rho_{B2}\frac{dR}{dt} \tag{17}$$

respectively. The application of mass conservation principle, thermodynamic equation of state and Claypeyeon equation (8) gives:

$$\rho_v + \rho_G = \frac{P_\infty\, T_s}{\bar{R}\, T_x\, T_s-(T_x-T_s)h_{fg}} + \frac{P_x}{\bar{R}\, T_x} \tag{18}$$

Equations (16), (17), and (18) are combined together with the temperature, pressure, and their gradients at the liquid interface from Eqs. (6) and (7). It yields the expression for dR/dt which is then integrated to produce the radius-time history of bubble growth. The resulting R(t) is very complicated and requires numerical integration. Since the bubble density ρ_b does not vary much with time, Eq. (18) may be replaced by:

$$\rho_v + \rho_G = \rho_b = \text{constant.}$$

One thenobtains:

$$R^*(t)-1 = (J_1^* \Delta T_1' + J_{A1}^* E_1 \Delta P_{A1}^* + J_{B2} E_2 \Delta P_{B2}^*)t^*$$
$$+ (b_1^*-J_1^*)\Delta T_1'(1-e^{-t^*}) + (b_{A1}^*-J_{A1}^*)\Delta P_{A1}^*(1-e^{-E_1 t^*}) + (b_{B2}^*-J_{B2}^*)\Delta P_{B2}^*(1-e^{-E_2 t^*}) \quad (19)$$

When a bubble grows to a critical size R_c at the liquid interface of a horizontal layer composite, it begins to rise through the upper liquid layer. R_c can be determined by a simple balance between the buoyancy and hydraulic force and the aid of Eq. (6) as:

$$t_c^* = \ell n(\frac{J_1-b_1}{J_1-3/4R_c^*}) \quad (20)$$

The restriction of the arguement of logarithms in Eq. (20) leads to

$$0.75/J_1 < R_c^* < 0.75/b_1 \quad (21)$$

The equations defines the range of departure size beyond which the bubble remains at the growing site. After a time of t_∞, the bubble ascends in the upper liquid layer at the terminal velocity of U_∞. The U_∞-t_∞ relation can be found by the balance of drag and buoyancy forces as:

$$t_\infty^* = \ell n(\frac{J_1-b_1}{J_1-1/8U_\infty^*}) \quad (22)$$

under the restriction:

$$0.125/b_1 > U_\infty^* > 0.125/J_1 \quad (23)$$

The equation specifies the range of terminal velocity beyondwhich the bubble ascension is still in an accelerating stage.

Results and Discussion

Theoretical expressions are obtained in the preceeding section for (i) temperature and pressure time history, (ii) location of the maximum total pressure, i.e. homogeneous nucleation site, (iii) growth history of an embryo and waiting time for nucleation (for an embryo to become a nucleus), (iv) growth history of a bubble and (v) departure time of a bubble and its rising terminal velocity. Two limiting cases are of special importance: one is the isothermal counterdiffusion of gases, corresponding to $\Delta T^*=o$. The phenomenon is observed in nature, some industrial processes and physiological processes such as "urticaria" and vertigo. The other corresponds to heat-transfer controlling case, i.e. for large values of ΔT^*. When ΔT^* is sufficiently high, mass transfer in the liquid composite may be neglected since the thermal diffusivity is about two-order of magnitude higher than mass diffusivity.

Numerical results are obtained for two special cases of important applications: (i) Both the heat and mass transfer coefficients at both gas-liquid interfaces are very large, i.e. Bi's = Sh's = ∞, indicating both the temperature and partial pressure at the gas-liquid interfaces are instantaneously brought up to those of the contacting gas. (ii) One gas-liquid interface is thermally as well as materially (to the gas diffusing from the opposite side) insulated but serves as the mass source for the gas diffusing toward the opposite side, for example $Bi_2 = Sh_{A2} = 0$ and $Bi_1 = Sh_{A1} = Sh_{B1} = Sh_{B2} = 2$. The latter case resembles the combustion of emulsified droplets except the system geometry (the insulated interface being considered as the droplet center). Both the Biot and Sherwood number of 2 correspond to natural convection case. Numerical results were obtained for an oil (1)/water (2) composite of L_1 = 2mm and L_2 = 1mm. Figure 2 illustrates the waiting time for nucleation t_e^* versus the critical radius r_e^* for both cases 1 and 2. For a given ΔP, and increase in ΔT^* signifies higher heat transfer through the system, while a high r_e^* indicates the formation of a small nucleus. It is observed in the figure that a system of higher ΔT^* requires less time to produce a given size of nucleus. In other words, a small nucleus may exist in a system with high ΔT^*. One also observes an increase in ΔT^* is characterized by a steeper $r_e^* - t_e^*$ curve and a broader range of r_e^* or r_e. A comparison of the results shows that r_e^* of case 2 is an order of magnitude higher that r_e^* of case 1, suggesting the possibility of a very rapid nucleation in case 2 or in a water/oil emulsified droplet. Since the critical nucleus size r_e in case 1 is an order of magnitude larger than that in case 2, it is easier for a nucleus to exist in case 2, even at a very small size. On the other hand, an embryo in case 1 needs higher ΔT^* to nucleate. $\Delta T^*=o$ corresponds to pure mass transfer process, while short horizontal lines at right indicate asymptotic values at infinite time. The two values of r_e^* at $t_e^*=o$ and $t_e^*=\infty$ define the range of r_e^* for a given ΔT^*, as specified by equation (13).

The effects of L_1/L_2 on the critical size r_c^* and the waiting time t_e^* are studied for T^*=30 and L_2=1 mm. In both cases, an increase in L_1/L_2 causes a substantial reduction in r_e^*, namely an enlargement of the critical size. Thickening of layer 1 with an increase in L_1/L_2 results in a longer time required for heat or mass penetration and thus a loner waiting time for nucleation of a given critical size. From the definition of b_i, one finds that an increase in α_1/α_2, D_{A1}/D_{A2} or D_{B2}/D_{B1} results in a shorter waiting time for nucleation due to an enhancement in heat or mass diffusion through liquid 1. The effects of these physical parameters on r_e^* and t_e^* are stronger in case 2 than case 1.

Of most importance is Fig. 3 illustrating the criterion for spontaneous nucleation which corresponds to zero waiting time or Eq. (11). It requires much higher ΔT^* in case 1 than case 2.

It is disclosed but not illustrated here due to page limitation that the growth curves for cases 1 and 2 are distinctly different: concaving upward in case 1 but concaving downward for case 2. In both cases, bubble growth rate is directly proportional to $\Delta T'$, ΔP_{A1}^* and ΔP_{B2}^* for both zero and infinite time, as can be realized from the time derivative of Eq. (19). This signifies that a bubble grows linearly with time during the initial stage and end of the bubble growth history(The bubble growth history can be classified into the initial, intermediate and asymptotic stages). The growth rate is high in the initial stage but very low in the asymptotic stage for case 1. It is reversed in case 2. The combination of very rapid nucleation process with high initial bubble growth rate is the mechanics of microexplosion phenomenon observed in emulsified drops.

In case a composite is placed in horizontal position, a bubble begins to rise after it has grown to a certain size R_c or R_c^* . The time required after the initiation of the transient heat and mass transfer processes t_c^* is related to the departure size R_c^* as shown in Fig. 4, a plot of Eq. (20). As expected, case 2 takes shorter waiting time than case 1 for the same departure size because of higher bubble growth rate.

Figure 5 is a graphical demonstration of Eq. (22) for the terminal velocity of a rising bubble U_∞^* versus the waiting time required in achieving the velocity t^*_∞. High heat and mass transfer rates to the rising bubble prompt a shorter waiting time for case 2 than case 1.

Conclusions

An analytical study is performed on supersaturation, nucleation, and bubble growth in two immissible liquid layers under the influence of transient heat and mass diffusion. Theoretical results are obtained in closed form for (i) the growth rates of an embryo and its subsequent form (called bubble) at the liquid interface where nucleation is most favorable, (ii) the waiting times for nucleation, bubble departure size and terminal rising velocity, and (iii) critical size for nucleation induced by transient heat and mass transfer, while that for spontaneous nucleation is described by equation (11). The degree of superheat required for heat transfer to be the controlling mechanism can be evaluated by Eqs. (14) and (15).

Also determined are the ranges of bubble departure size and terminal velocity of rising bubbles. It is disclosed that bubble growth in a composite approximately follows square-root law in the asymptotic stage. A composite with one-side insulated is characterized by high nucleation speed followed by rapid growth rate. Hence, this is the mechanism leading to microexplosion of emulsified drops. The analysis can be extended to diffusion of multiple gases in multiple liquid layer composites and spherical composites.

Nomenclature

a's $a_1=(2G_2-QX_1)/\phi$; $a_2=(2G_2-X_2)/\phi$; $a_{A1}=(2G_{A2}-Q_AX_1)/\phi_A$; $a_{A2}=(2G_{A2}-X_2)/\phi_A$
$a_{B1}=Q_B(2G_{B1}+X_1)/\phi_B$ $a_{B2}=(2Q_BG_{B1}+X_2)/\phi_B$

Bi_j Biot number on liquid j side, see Fig. 1

b's $b_1=[S_1G_2X_1^2+S_2H_2-QX_1(S_1X_1^2/6+S_2X_1/2+S_2/Bi_2]/\phi$; $b_2=S_2[0.5X_2^2/Bi_2+H_2-X_2(X_2^2/6+1/Bi_2)]/\phi$; $b_{A1}=[-Q_AX_1(S_{A1}X_1^2/6+S_{A2}X_1/2+S_{A2}/Sh_{A2})+S_{A1}X_1^2G_{A2}+S_{A2}H_{A2}]/\phi_A$; $b_{A2}=S_{A2}[-X_2(X_2^2/6+1/Sh_{A2})+0.5X_2^2/Sh_{A2}+H_{A2}]/\phi_A$; $b_{B1}=S_{B1}[Q_B(1/2Sh_{B1}+X_1^3/6)+G_{B1}X_1^2+X_1/Sh_{B1}+X_1/2+1/6]/\phi_B$; $b_{B2}=[S_{B1}(Q_B/2Sh_{B1}+X_2/Sh_{B1}+X_2/2+1/6)+S_{B2}X_2^2(G_{B1}+X_2/6)]/\phi_B$; $b_1^*=QS_2/(B_{i2}\phi)$; $b_{A1}^*=Q_AS_{A2}/(Sh_{A2}\phi_A)$; $b_{B2}^*=S_{B1}(1/Sh_{B1}+1/2)/\phi_B$

C's $C_1=C_2=2(G_2+QG_1)/\phi$; $C_{A1}=C_{A2}=2(G_{A2}+Q_AG_{A1})/\phi_A$; $C_{B1}=C_{B2}=2(Q_BG_{B1}+G_{B2})/\phi_B$

D_{ij} permeability of gas i through liquid j

E's $E_1=C_{A1}/C_1$; $E_2=C_{B2}/C_1$

G's $G_j=(1+1/Bi_j)/2$; $G_{ij}=(1+1/Sh_{ij})/2$

g gravitational acceleration

H's	$H_j=(1/3+1/Bi_j)/2$; $H_{ij}=(1/3+1/Sh_{ij})/2$
h_{fg}	latent heat of vaporization of more volatile liquid
h_j	heat transfer coefficient between liquid j and its surrounding gas
h_{Dj}	mass transfer coefficient between liquid j and its surrounding gas
J's	$J_j=a_j/C_j$; $J_{ij}=a_{ij}/C_{ij}$; $J_1^*=0.5Q/(G_2+QG_1)$; $J_{A1}^*=0.5Q_A/(G_{A2}+Q_AG_{A1})$; $J_{B2}^*=0.5/(G_{B2}+Q_BG_{B1})$
k_j	thermal conductivity of liquid j
M_{ij}	mass flux of gas i in liquid j
N's	$N_j=J_j-b_j$; $N_{ij}=J_{ij}-b_{ij}$
P_j	total pressure in liquid j; P_1, surrounding bubble; P_x, at liquid interface
P_o,P_∞	initial and final pressure of gas
P_g,P_v	pressure inside nucleus; P_g, of inert gas; P_v, of vapor phase of more volatile liquid
P_{ij}	partial pressure of gas i in liquid j, see Fig. 1
ΔP's	$\Delta P=P_\infty-P_o$; $\Delta P_A^* = D_{A1}\Delta P/(\rho_b C_{A1}L_1)$; $\Delta P_B^*= D_{B2}\Delta P/(\rho_b C_{B2}L_2)$
Q's	$Q= L_1K_2/(L_2K_1)$; $Q_A=L_1D_{A2}/(L_2D_{A1})$; $Q_B=L_1D_{B2}/(L_2D_{B1})$
$\bar{R}$	universal gas constant
R	instantenaous bubble radius; $R^*=R/r_e$; R_c, departure size; $R_c^*=B_1R_c\Delta T/L_1$
r	instantaneous embryo radius; r_e, nucleation radius; $r_r^*=(2\sigma/r_e-P_g-2P_o)/\Delta P$
S's	$S_j=L_j^2/\alpha_j$; $S_{ij}=L_j^2/D_{ij}$
Sh_{ij}	Sherwood number of gas i on liquid j side, see Fig. 1
T_j	temperature of liquid j; T_1, at nucleus; T_o, initial value; T_x, at liquid interface
T_v	vapor temperature of more volatile liquid inside embryo and bubble; T_s, saturated temperature at P_1
T_o,T_∞	initaial and final temperature of gases
ΔT	degree of superheat; $\Delta T^* = \rho_v h_{fg}\Delta T/(\Delta P T_s)$; $\Delta T' = K_1\Delta T/(\rho_b c_1 L_1 h_{fg})$
t	time; t_e, required for nucleation; $t_e^*=C_1t_e$; t_c, bubble departure time; $t_c^*=C_1t_c$; t_∞, ascending bubble reaching terminal velocity; $t_\infty^*=C_1t_\infty$
U_∞	terminal velocity; $U_\infty^*= \nu_1\beta_1U_\infty\Delta T/(gL_1^2)$
x	distance measured from liquid interface, positive in liquid 2; $X_j=x/L_j$
$\alpha_j\ \beta_j\ \nu_j$	thermal diffusivity, coefficient of thermal expansion and viscosity of liquid j
ρ	desity; $\rho_b = \rho_v+\rho_G$; $\rho_G = \rho_{A1}+\rho_{B2}$; ρ_{ij}, gas i in liquid j; ρ_v, vapor in bubble
σ	surface tension
ϕ's	$Q[S_1H_1+S_2G_1(1+2/Bi_2)]+S_1G_2(1+2/Bi_1)+S_2H_2$; ϕ_A, $Q_A[S_{A1}H_{A1}+S_{A2}G_{A1}(1+2/Sh_{A2})]+S_{A1}G_{A2}(1+2/Sh_{A1})+S_{A2}H_{A2}$; ϕ_B, $Q_B[S_{B1}H_{B1}+S_{B2}G_{B1}(1+2/Sh_{B2})]+S_{B1}G_{B2}(1+2/Sh_{B1})+S_{B2}H_{B2}$

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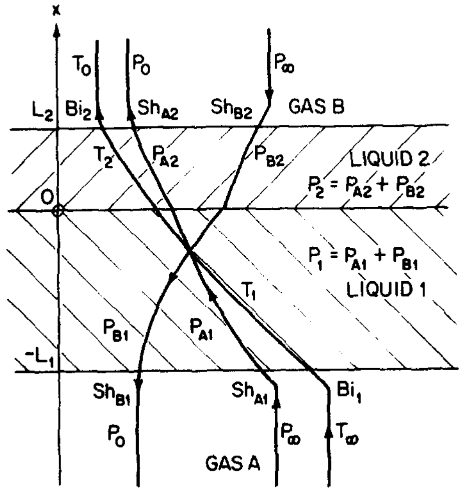

FIG.1 Schematic of Physical System

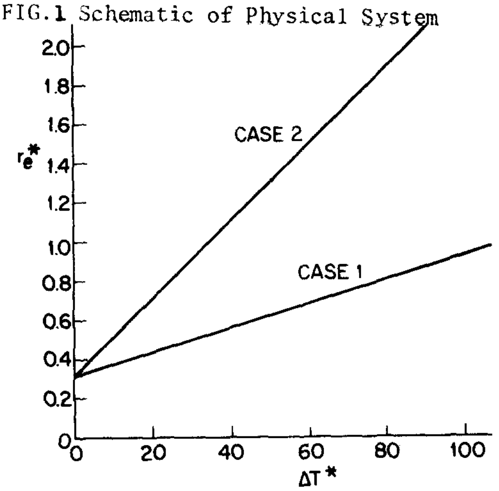

FIG.3 Criteria for Spontaneous Nucleation

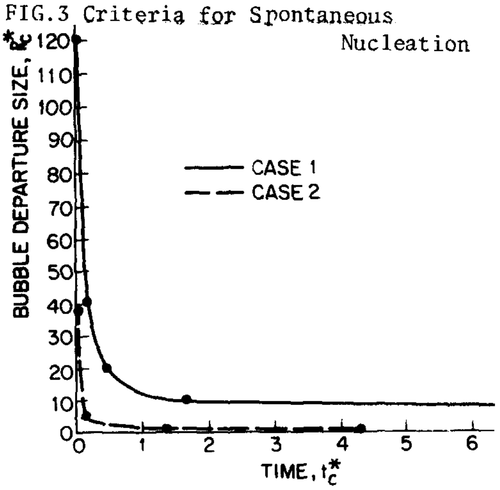

FIG.4 Bubble Departure Size

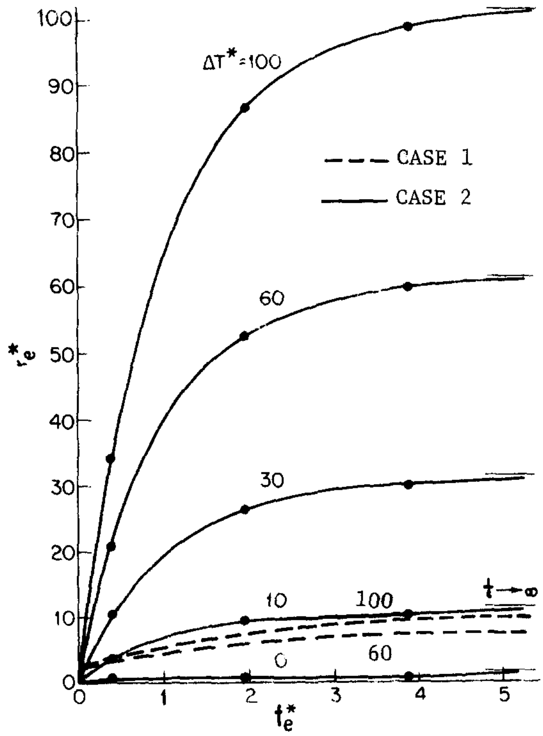

FIG.2
Effect of ΔT^* on Nucleation

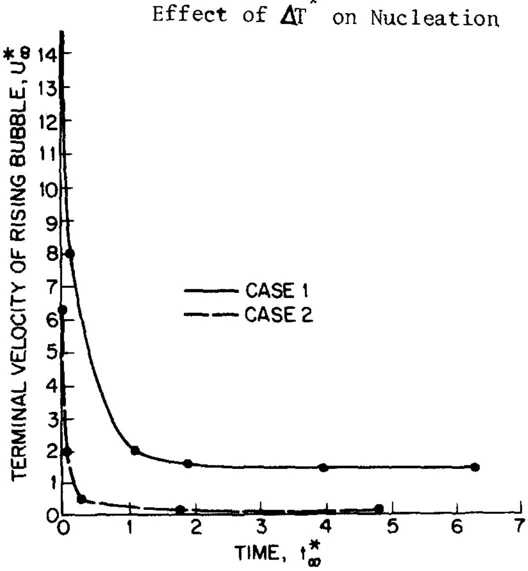

FIG.5
Terminal Velocity of Rising Bubbles