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AN APPROACH FOR SIMULATING REGIONAL MIGRATION PATTERNS

H. ELAYAT
Department of Industrial and Systems Engineering, The University of Michigan-Dearborn, Dearborn, MI 48128, U.S.A.

E. A. ELSAYED
Department of Mechanical, Industrial and Aerospace Engineering, Rutgers University, Piscataway, NJ 08854, U.S.A.

and

M. RAGAB
School of Business Administration, University of Windsor, Canada

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Abstract—In this paper, GERTS III is presented as a possible approach for simulating regional migration patterns. Fourteen years of aggregate Canadian census data were compiled and a mathematical model was constructed to describe the patterns of inter-regional migration viewed as a discrete time Markov process. A GERTS III network was developed to analyze and describe this process. Measures of population mobility, degree of mobility and retention were derived. The implications for policy makers are highlighted.

INTRODUCTION

The socio-economic consequences of regional disparity have become an important concern for governments at various levels in Canada. The crux of the problem lies in the lack of alignment between population mobility patterns on one hand and the distribution and characteristics of resources on the other. Other things being equal, migration is the central force underlying population mobility.

Modelling migration would result in further measurement and facilitate the prediction of population movement patterns, and as such would be a crucial input to the economic planning process. For the purposes of this discussion, migration is operationalized in terms of distance and duration. Thus, a migration is defined as a move from a given region to any of a number of regions and residing there for a period of time.

Because of the uncertainty inherent in migration, it can only be adequately described in probabilistic terms. Accordingly, the dynamics of migration can be best modeled in stochastic terms. Consequently, great use has been made of Markov chains in formulating and analyzing migration models. Rogers[1, 2] and Willis[3]—in addition to their own—reported numerous studies that structured migration as a Markov process.

While it is instructive to view migration as a Markov process, the difficulty lies in the development of analytical solutions to derive the desired statistics. Such difficulty increases exponentially as the number of states increases, which is generally the case in migration. One possible alternative is simulation. This paper develops a simulation approach to the migration phenomenon structured as a Markov process. The simulation approach proposed here is an adaptation of GERTS III methodology[4, 5, 6, 8, 11]. The paper then proceeds to derive a number of statistics that can answer significant questions raised by policy makers.

To make the analysis concrete, the approach will be applied to the Canadian migration scene. This paper attempts to model migration among the provinces of Canada. Canada consists of ten provinces. Because some provinces have generally the same socio-economic characteristics, they can be treated as one region. The country is therefore divided into five regions. Census data regarding the residence in each region were collected from Statistics Canada for 14 years.

This paper is divided into two parts. The first is a presentation on GERT and GERTS III methodology in the context of Markov processes. The second part contains a case study, namely Canadian Migration patterns and a set of relevant statistics for the policy maker.

MODELING MARKOV PROCESSES WITH GERT AND GERTS METHODOLOGY

For simplicity, consider a two-state Markov process with transition probability matrix

$$P = \begin{vmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{vmatrix}$$

where P_{ij} = the probability transition from state i to state j.

This Markov process can be represented by a GERT network (Fig. 1). In this network, node 1 represents state 1 and node 2 represents state 2.

The previous GERT network can be easily generalized to represent an n-state Markov process (Fig. 2). This network consists of n nodes each representing a state.

GERTS III simulation of Markov processes

Howard[9] noted that recurrent states in a Markov process have statistics that are important in many applications. The following are of particular importance:

(a) The mean first passage time, $\bar{\theta}_{ij}$, which is the expected number of transitions it will take the system to reach state j for the first time if the system is in state i at time zero.

(b) The n step as well as the steady state probabilities.

(c) $\phi_{ij}(k/n)$, the probability that a system started in state i will after n transitions, be in state j and have made k special transitions.

(d) The mean state occupancy $\bar{v}_{ij}(n)$, which is the expected number of times state j is entered through time n given that the system is started in state i at time zero.

The above information is difficult, if not impossible, to obtain using the standard GERT techniques. This difficulty led to the use of GERTS III as a simulation approach for analyzing Markov processes. The following is an example of how a two-state Markov process is modeled using GERTS III, and of how to obtain all the relevant information discussed earlier. In Fig. 3,

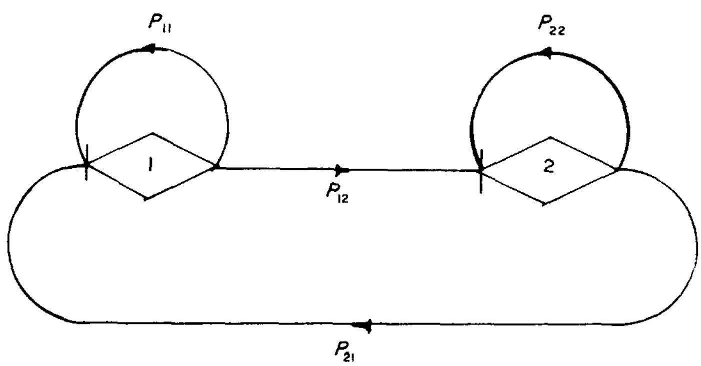

Fig. 1. A GERT network for a two-state Markov process.

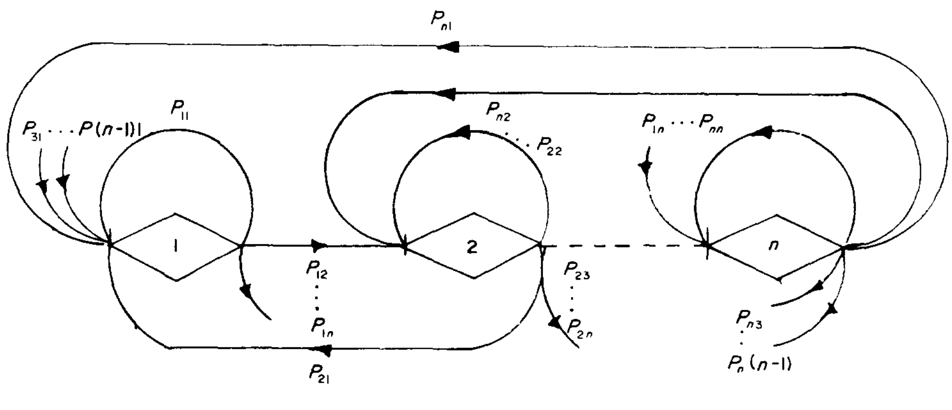

Fig. 2. A GERT network for an n state Markov process without trapping states.

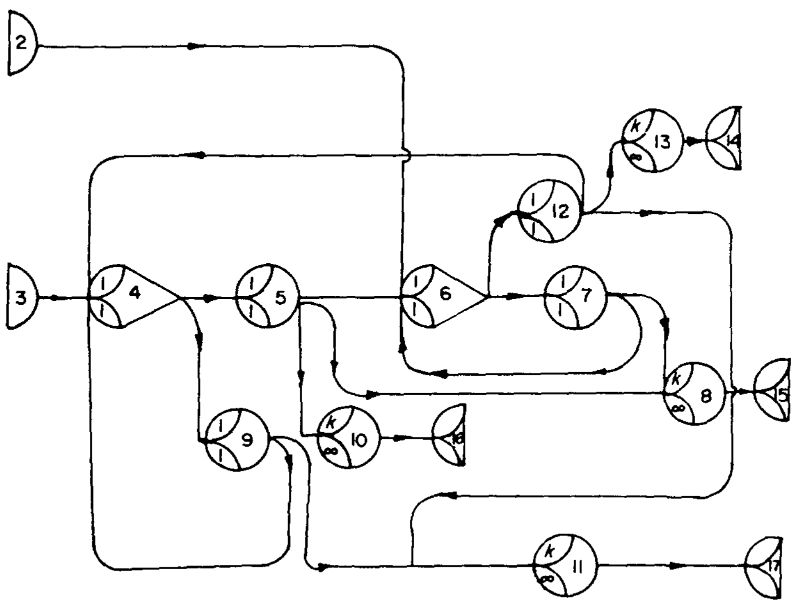

Fig. 3. A GERTS III network for a two state Markov process.

nodes 2 and 3 are source nodes. Nodes 14–17 are sink nodes. Nodes 4 and 6 represent states 1 and 2, respectively. Nodes 5, 9, 7 and 12 are statistics nodes that count the number of transitions from nodes 4 to 5, 4 to 4, 6 to 6, and 6 to 5 respectively. Nodes 8, 10, 11 and 13 are statistics nodes used for evaluating $\phi_{ij}(k/n)$ for $i = 1, 2$ and $j = 1, 2$. More specifically, if one is interested in evaluating $\phi_{11}(k/n)$ for the special class of transitions 1 to 1 and 2 to 1, then one would collect statistics at node 17 while letting k assume different values ($k = 1, 2, 3, \ldots$) at node 11 and infinity at nodes 8, 10 and 13. Also, if one is interested in evaluating $\phi_{11}(k/n)$ for the special case of transitions 2 to 1 only, one would then collect statistics at node 14 while allowing k to assume different values ($k = 1, 2, \ldots$, etc.) at node 13 and infinity at nodes 8, 10 and 11. In both these cases the time it takes a transaction to proceed from nodes 3 to 4 is set as zero while the time from node 2 to 6 is set as infinity. In a similar manner $\phi_{12}(k/n)$, $\phi_{21}(k/n)$ and $\phi_{22}(k/n)$ can be evaluated using the appropriate statistics nodes.

In order to evaluate $\bar{v}_{11}(n)$, one would collect statistics at node 4 while letting k assume different values ($k = 1, 2, \ldots$) at node 11 and infinity at nodes 8, 10 and 13. Similarly if one needs to evaluate $\bar{v}_{12}(n)$ one would collect statistics at node 6 while changing the value of k at node 8. In a similar manner one would be able to evaluate $\bar{v}_{21}(n)$ and $\bar{v}_{22}(n)$.

To evaluate $\bar{\theta}_{11}$ and $\bar{\theta}_{21}$ statistics are maintained at node 17 for transactions starting at nodes 3 and 2 respectively. Similarly, to evaluate $\bar{\theta}_{12}$ and $\bar{\theta}_{22}$ statistics are maintained at node 15 for transactions starting at nodes 3 and 2 respectively.

SIMULATING CANADIAN MIGRATION PATTERNS

The migration patterns of different cohorts in Canada are viewed as a discrete time Markov process. The Canadian provinces will be treated as different Markov states. These provinces are grouped as follows: (1) Atlantic Provinces (include Nova Scotia, New Brunswick, P.E.I. and Newfoundland), (2) Quebec, (3) Ontario, (4) The Prairies (include Manitoba, Saskatchewan and Alberta), and (5) British Columbia.

Aggregate data for: (1) the overall population; (2) the male and female population; and (3) age groups were collected from 14 consecutive years (1960–73). For each year, the number of persons residing in each area was recorded. Using the restricted generalized least square estimator[10], the transition probability matrices for these regions were evaluated.

GERTS III networks of interprovincial migration

The GERTS III network is presented in Fig. 4. The probabilistic nodes 11–15 represent the Atlantic provinces, Quebec, Ontario, the Prairies and British Columbia, respectively. Nodes

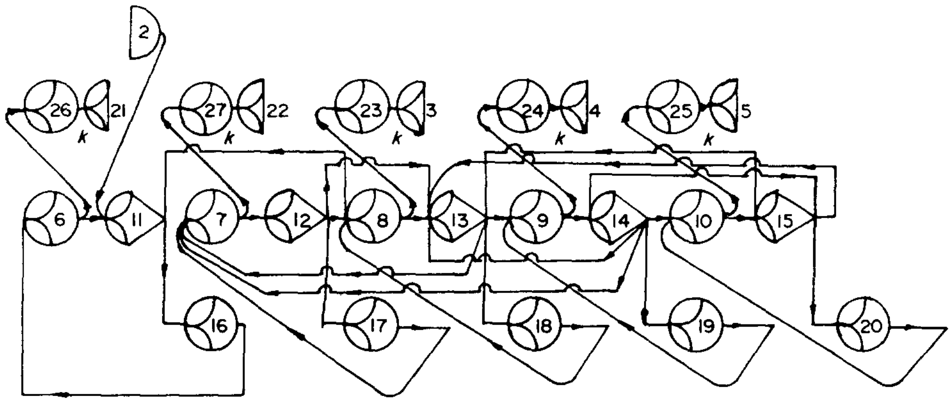

Fig. 4. GERTS III network for interprovincial migration.

21–25 are sink nodes. Node 2 is a source node connected to either nodes 11, 12, 13, 14 or 15, depending on the statistics one is interested in. Deterministic nodes 3–5, 26 and 27 control the realization count k of the sink nodes 23–25, 21 and 22 respectively. Suppose one is interested in collecting statistics for state 1 (node 11), then node 2 is connected to node 11 and in this case node 6 is deterministic, while the count k on nodes 3–5 and 27 is set to infinity so that sink nodes 22–25 are not realized, i.e. only node 21 will be realized. After a simulation of 1000 transitions, the following statistics for inter-regional migration were obtained:

(a) The expected number of years it takes a resident of a province to migrate to another province for the first time (Table 2).

(b) The equilibrium distribution (the steady state probabilities). This provides information regarding the long-term implication of current mobility trends (Table 1).

(c) The probability that a present resident of a particular region will continue to be a resident of this region after n years and in the interim he has resided k years only (not necessarily consecutive) in this region (Table 3).

(d) The expected number of years a resident lives in a given province (not necessarily consecutive) during n years (Table 4).

RESULTS

The transition probability matrices (Table 1) provide a great deal of information about the mobility of migrant classes. Their diagonal elements provide an immediate dimension along which the degree of overall mobility of different groups can be contrasted. Thus, with respect to degree of mobility, the five regions can be ranked from highest to lowest in this order: Quebec, Ontario, British Columbia, The Prairies, the Atlantic Provinces. Looking at migrant groups and comparing males and females as an illustration, the data suggests that Quebec females are more mobile than males, the reverse is true for Ontario. For age groups, younger groups seem to be more mobile than the older group and *vice versa* across the board.

The equilibrium distribution (Table 1) provides information about the long term implications of current mobility trends. Based on these trends, it appears that males favor British Columbia as a destination while females favor Quebec as a destination. As for age groups, the 15–19 group favors Ontario as a destination; the 50–54 group favors British Columbia.

Mean first passage time (Table 2) provides a measure of a particular kind of contiguity in the sense that it is based on interchange probabilities rather than distance. Rogers[7] labels this spatial measure of proximity as "migrant distance". In this regard, it can be seen that the "migrant distance" from Quebec to the Prairies for both males and females is considerably "shorter" (17.55 and 11.5 respectively) than from the Prairies to Quebec (47.64 and 98.8 respectively). Also, females appear to have a shorter "migration distance" from Quebec to the Prairies than males.

The probabilities in Table 3 provide additional measures of mobility. The lower the value of $\phi_{ii}(k/n)$ for $k = n$, the higher the mobility of the residents in a particular region.

Table 1. Transition probability matrices and equilibrium distributions for Canada

A. For the Total Population

	A	Q	O	P	B
A	.997		.003		
Q		.796	.142	.062	
O		.124	.856		.040
P		.093	.006	.901	
B			.141	.020	.839

A = Atlantic Provinces
Q = Quebec
O = Ontario
P = Prairies
B = British Columbia

Equilibrium Vector:

	A	Q	O	P	B
a =	0.0	.226	.489	.162	.123

B. 1. For the Male Population

	A	Q	O	P	B
A	.995		.005		
Q		.804	.165	.031	
O		.124	.831		.045
P		.087	.001	.912	
B			.145	.018	.837
a =	0	.327	.419	.018	.837

2. For the Female Population

	A	Q	O	P	B
A	.999		.001		
Q		.788	.129	.083	
O		.123	.842		.035
P		.098	.011	.891	
B			.137	.022	.841
a =	0	.339	.319	.272	.070

C. For Different Age Groups

1. Age Group 15–19 years

	A	Q	O	P	B
A	.887		.113		
Q		.891	.041	.068	
O		.047	.899		.054
P		.054	.059	.887	
B			.059	.042	.899
a =	0	.236	.358	.215	.191

2. Age Group 35–39 years

	A	Q	O	P	B
A	.945		.055		
Q		.911	.077	.012	
O		.034	.924		.042
P		.130	.025	.962	
B			.051	.022	.927
a =	0	.177	.402	.189	.232

3. Age Group 50–54 years

	A	Q	O	P	B
A	.984		.016		
Q		.995	.004	.001	
O		.007	.933		.060
P		.001	.007	.992	
B			.029	.005	.966
a =	0	.289	.175	.228	.308

Table 2. The expected number of years it takes a resident of a province to migrate to another province for the first time

A. For the Overall Population

	A	Q	O	P	B
A	5.16	-	-	-	-
Q	-	3.29	10.07	12.92	-
O	-	14.575	2.58	36.93	-
P	-	46.95	49.42	4.69	-
B	-	-	72.23	85.26	11.62

"-" is considered infinity

B. 1. For the Male Population

	A	Q	O	P	B
A	3.94	-	-	-	-
Q	-	3.69	12.21	17.55	-
O	-	9.38	2.98	-	5.95
P	-	98.8	104.12	8.09	-
B	-	-	53.29	67.55	11.29

2. For the Female Population

	A	Q	O	P	B
A	1.00	-	-	-	-
Q	-	4.01	12.08	11.85	-
O	-	10.34	3.42	-	6.24
P	-	47.64	62.73	6.75	-
B	-	-	120.96	157.63	25.04

C. For Different Age Groups

1. Age Group 15-19 years

	A	Q	O	P	B
A	3.9	-	-	-	-
Q	-	4.082	33.9	21.8	-
O	-	20.55	2.82	24.3	11.99
P	-	26.73	33.75	4.43	-
B	-	-	39.46	46.58	5.69

2. Age Group 35-39 years

	A	Q	O	P	B
A	4.2	-	-	-	-
Q	-	6.47	48.3	87.9	-
O	-	10.82	2.14	-	21.76
P	-	115.2	110.7	6.15	-
B	-	-	52.4	62.7	4.6

3. Age Group 50-54 years

	A	Q	O	P	B
A	26.38	-	-	-	-
Q	-	1.04	20.59	-	-
O	-	217.25	6.35	-	34.04
P	-	-	143.1	3.49	-
B	-	-	32.77	35.3	1.94

Table 3. $\phi_{ii}(k/n)$. The probability that a resident of a particular region will continue to be a resident of this region after *n* years and in the interim he has resided *k* years only (not necessarily consecutive) in this region

A. Atlantic Provinces $\phi_{55}(k/n)$

k \ n	0	1	2	3	4	5
0	1	0	0	0	0	0
1	0	.997	0	0	0	0
2	0	0	.994	0	0	0
3	0	0	0	.991	0	0
4	0	0	0	0	.993	0
5	0	0	0	0	0	.992

B. Quebec $\phi_{22}(k/n)$

k \ n	0	1	2	3	4	5
0	1	0	0	0	0	0
1	0	.804	.031	.022	.008	.007
2	0	0	.665	.020	.034	.025
3	0	0	0	.503	.057	.044
4	0	0	0	0	.390	.058
5	0	0	0	0	0	.342

C. Ontario $\phi_{33}(k/n)$

k \ n	0	1	2	3	4	5
0	1	0	0	0	0	0
1	0	.862	.013	.019	.015	.013
2	0	0	.711	.038	.034	.024
3	0	0	0	.616	.040	.044
4	0	0	0	0	.504	.041
5	0	0	0	0	0	.425

D. Prairies $\phi_{44}(k/n)$

k \ n	0	1	2	3	4	5
0	1	0	0	0	0	0
1	0	.909	.006	.005	.002	.003
2	0	0	.812	.011	.005	.007
3	0	0	0	.733	.011	.014
4	0	0	0	0	.685	.013
5	0	0	0	0	0	.605

E. British Columbia $\phi_{55}(k/n)$

k \ n	0	1	2	3	4	5
0	1	0	0	0	0	0
1	0	.849	.003	.003	.007	.007
2	0	0	.734	.008	.003	.006
3	0	0	0	.597	.011	.004
4	0	0	0	0	.521	.011
5	0	0	0	0	0	.424

Table 4. $\bar{v}_{ii}(n)$. The expected number of years a resident lives in a given Province (not necessarily consecutive) during n years

A. For the Overall Population

at the end of the nth year

	1	2	3	4	5	6
A	1	1.99	2.99	3.98	4.97	5.96
Q	1	1.80	2.5	3.05	3.5	4.02
O	1	1.86	2.58	3.25	3.84	4.39
P	1	1.91	2.73	3.48	4.18	4.82
B	1	1.85	2.59	3.19	3.74	4.19

B. 1. The Male Population

	1	2	3	4	5	6
A	1	1.99	2.99	3.98	4.96	5.94
Q	1	1.82	2.51	3.09	3.59	4.05
O	1	1.85	2.55	3.18	3.80	4.34
P	1	1.92	2.77	3.54	4.25	4.88
B	1	1.83	2.56	3.16	3.68	4.11

2. The Female Population

	1	2	3	4	5	6
A	1	2	2.99	3.99	4.99	5.99
Q	1	1.79	2.46	3.03	3.50	3.94
O	1	1.86	2.60	3.25	3.85	4.39
P	1	1.90	2.72	3.42	4.06	4.65
B	1	1.84	2.81	3.43	3.90	4.39

C. For Different Age Groups

1. Age Group 15-19 years

	1	2	3	4	5	6
A	1	1.88	2.87	3.85	4.82	5.81
Q	1	1.89	2.69	3.44	4.09	4.66
O	1	1.91	2.76	3.49	4.2	4.817
P	1	1.89	2.69	3.42	4.06	4.636
B	1	1.90	2.73	3.46	4.12	4.72

2. Age Group 35-39 years

	1	2	3	4	5	6
A	1	1.94	2.85	3.87	4.90	5.92
Q	1	1.96	2.75	3.51	4.22	4.88
O	1	1.96	2.83	3.65	4.42	5.1
P	1	1.97	2.91	3.81	4.67	5.5
B	1	1.93	2.79	3.59	4.35	5.05

3. Age Group 50-54 years

	1	2	3	4	5	6
A	1	1.99	2.96	3.91	4.85	5.75
Q	1	1.99	2.98	3.97	4.96	5.94
O	1	1.93	2.78	3.59	4.4	5.12
P	1	1.99	2.98	3.96	4.93	5.88
B	1	1.97	2.92	3.84	4.72	5.58

The value of $\bar{v}_{ii}(n)$ can be viewed as a measure of the region's "retention" of its residents. For the overall population, it appears that Quebec has a measure of retention of 4.02 years (out of 6 years) compared to 5.96 years for the Atlantic region. The same line of reasoning can be used for making observations about age and sex groupings.

CONCLUSION

This paper demonstrated GERTS methodology as a simulation approach for modeling migration patterns in stochastic terms, namely as a Markov process. A number of useful statistics were derived for helping policy makers ascertain migration trends and their implications. It is worth noting that the same approach can be used for simulating more micro migration patterns such as inter-city population movement in a given province or inter-zone mobility within a certain city.

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