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TECHNICAL NOTE

Observations on the statistical nature of terrestrial irradiation†

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The probability distribution of total, horizontal terrestrial solar radiation is non-Gaussian. Baker and Klink[1] have found the distribution for total *weekly* irradiation to be negatively skewed and occasionally bimodal; Bennett[2] reports the same characteristics for total *daily* irradiation. Recently, the present authors[3] have found that the probability distribution of *hourly* irradiation also possesses these characteristics. This study, which is preliminary, examined the rehabilitated SOLMET irradiance data[4] for Bismark, North Dakota; Columbia, Missouri; Madison, Wisconsin; and North Omaha, Nebraska. Because of the non-Gaussian nature of the distributions, the mean and variance values reported in statistical summaries, of which the work of Baker and Klink[1] and Getz and Nicholas[5] are examples, may not be the most appropriate nor useful statistics. In particular, they are of doubtful value in the design of solar conversion systems.

In a recent paper Mustacchi *et al.*[6] normalized hourly data (collected in Italy) by dividing measured levels by computed extraterrestrial radiation. The distribution of the resulting transformed variable was also found to be both bimodal and negatively skewed. In addition, when they performed an autoregressive analysis on hourly values of the transformed variable, the residual error distribution was very sharply peaked and clearly non-Gaussian although the resulting time series models were simple (1 or 2 autoresgressive parameters).

In this note we report the results of some recent similar work that has an improved potential for disclosing the significant statistical behavior of terrestrial irradiation. SOLMET hourly data were first divided by values obtained from a deterministic terrestrial clear sky model[7]. This normalization produces a variable that can be interpreted as an "attenuation" which accounts for the stochastic nature of irradiation received at the earth's surface due to unpredictable atmospheric effects. Knowledge of the probability distribution of this attenuation allows probability statements to be made about terrestrial irradiation, since the latter can be obtained by multiplying values obtained from a deterministic clear-sky model by the random attenuation. Thus, rather than introducing a bias, the normalization simply allows the separating out of random components and deterministic components of irradiation. Figure 1 is a histogram of this normalized variable constructed from the 12-noon data for each day in the month of March at Bismarck, North Dakota for the 23 (March) months in the period 1953–75. This shape is typical for results determined for other months[3]. The negative skew is clearly indicated. The normalized variable was analyzed using autoregressive techniques[8], which resulted in a simple time series model having only one autoregressive parameter. The model, however, also produced a non-Gaussian residual error distribution. For example, Fig. 2 illustrates the probability distribution of residual error resulting from regressing the data from two consecutive hours (12 noon and 1 p.m.) for the same dates used in Fig. 1. The shape of the error distribution is typical of results for all months and hours at the four locations indicated above.

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Because the results of Baker and Klink[1] and Bennet [2] for many more locations but larger time intervals (days and weeks) are similar to the present results using hourly data, it appears that the stochastic component of terrestrial irradiation possesses an inherent non-Gaussian probability distribution, independent of location and time. This conclusion has far reaching significance to the application and interpretation of solar irradiation data to the design of solar conversion systems. It is apparent that considerably more research is needed before the probabilistic nature of solar irradiance is more completely understood.

As a consequence of these results it is important to emphasize the following points regarding statistical analysis of irradiation data:

(a) The underlying non-Gaussian nature of the stochastic component of solar irradiation, as illustrated in both Figs. 1 and 2, must be recognized. In particular knowledge of only the mean and variance of these distributions may be of little utility in solar design.

(b) Existing time series analysis techniques which depend upon a Gaussian error structure are not appropriate for effective description of irradiance data.

We are currently conducting research on alternative methods to deal with time series which involve non-Gaussian error terms. In particular, we are evaluating a Markov process formulation combined with standard time series methods. In this structure, the states of the process represent gross atmospheric conditions as well as irradiation values. It also uses the clear sky nor-

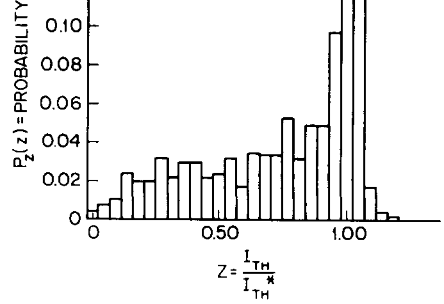

Fig. 1. Histogram for hourly normalized irradiance at Bismark, ND. Hour: 12 noon. Period: each March day, 1953–75. I_{TH} = Total hourly horizontal irradiance, Solmet[4], I^*_{TH} = Terrestrial, clear sky, total, hourly horizontal irradiance.

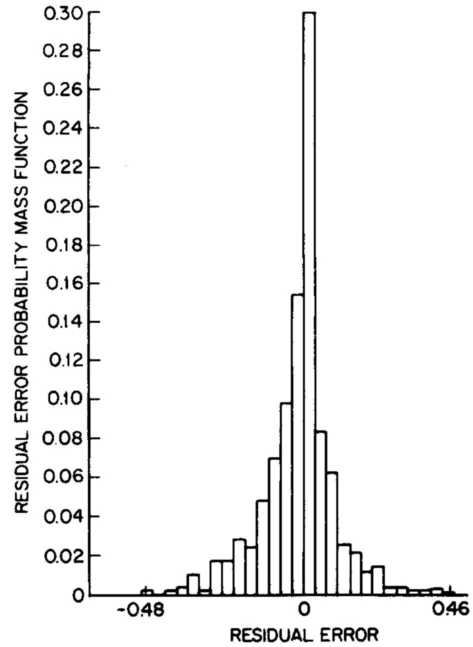

Fig. 2. Residual error histogram resulting from regressing the 1 p.m. vs the 12 noon hourly normalized irradiance at Bismark, ND. Period: each March Day 1953–75.

malization procedure, and shows great promise to provide a parsimonious yet useful stochastic description of terrestrial irradiance [3].

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