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STEADY AXIALLY SYMMETRIC THREE-DIMENSIONAL THERMOELASTIC STRESSES IN FUEL RODS

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Steady axially symmetric three-dimensional thermoelastic stresses in solid rods having space dependent energy generation are given in terms of the Goodier and the Love–Galerkin or the Boussinesq–Papkovich potentials. Results find applications in nuclear technology.

1. Introduction

Although the subject of thermoelasticity has been well understood for more than a century, early considerations on three-dimensional problems are scarce and are usually of mathematical nature. The literature on these problems may be found in the texts by Parkus [1], Boley and Wiener [2], Nowacki [3] and Lur'e [4]. During the last two decades, increased attention has been given to three-dimensional problems, especially to those involving cylindrical geometry. These cylindrical problems find application in the rapidly developing field of nuclear technology. The literature, up to 1971, may be found in the comprehensive survey by Boley [5]. Also, for the latest developments on cylindrical geometry, one may refer, for example, to Chen [6] for the end effect, to Huang and Cozzarelli [7] for the cladding effect, to Sugano [8] for transient and anisotropic effects, to Spiga and Lorenzini [9] for the radial neutron distribution, to Amada and Yang [10] for the nonlinear thermal stresses resulting from phase changes in materials, to Thompson [11] for a general study on cylinders of arbitrary cross-section.

As is well known, the temperature distribution in three-dimensional problems associated with cylindrical geometry is available in terms of the Fourier–Bessel series (or integrals). The stresses resulting from this temperature distribution may be obtained analytically by the Goodier and Love–Galerkin displacement potentials. However, this approach involves rather lengthy algebraic manipulations and/or numerical computations (see, for example, Sundara Raja Tyengar and Chandrasekhara [12]). This fact led researchers to the development of a convenient but approximate mathematical procedure based on the use of Chebyshev polynomials (see, for example, Chen [13,14]). Another convenient approach, which has not received much attention, deals with the formulation rather than the solution of these problems. The first objective of the present study to simplify the formulation of a temperature problem with some nuclear application, and to show the relative simplicity of the stress solution to be obtained by the use of Goodier and Love–Galerkin potentials. The second, and equally important, objective of the study is to obtain the same stress solution in terms of the Goodier and Boussinesq–Papkovich potentials. Since the Boussinesq–Papkovich potential satisfies the harmonic equation, it is relatively easy to obtain a solution in terms of this potential.

2. Formulation of the problem

Let the cylindrical core of a heterogeneous nuclear reactor be made of a number of solid fuel rods surrounded by a coolant (fig. 1). The formulation of the problem is based on the assumptions that the fuel rods have (a) constant thermal conductivity, (b) constant coefficient of thermal expansion, and (c) negligible axial conduction, because

$$\frac{\text{Change in axial conduction}}{\text{Change in radial conduction}} \sim \frac{k(\theta/L^2)}{k(\theta/R^2)} = \left(\frac{R}{L}\right)^2 \ll 1,$$

θ, R and $2L$ being the temperature, radius and length of rods, respectively, (d) elastic stresses under all condi-

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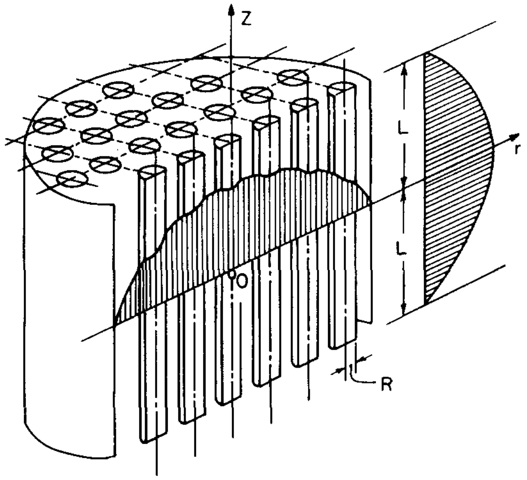

Fig. 1. Neutron flux distribution

tions, (e) energy generation proportional to neutron flux, (f) energy generation with negligible radial variation, because the cross-section of a fuel rod is negligibly small compared with the cross-section of the core, (g) energy generation varying sinusoidally in the axial direction, (h) negligible axial extrapolation of the energy generation, (i) negligible mechanical influence of cladding, (j) negligible effects from inertial and thermoelastic coupling terms.

Based on the foregoing assumptions, the axially symmetric three-dimensional temperature of the fuel rods satisfies, after neglecting the effect of axial conduction,

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \theta}{\partial r}\right)+\frac{U_0'''}{k}\cos\lambda^* z=0, \tag{1}$$

$$\theta(0,z)=\text{finite},\quad \theta(R,z)=T(R,z), \tag{2}$$

where r and z are the radial and axial variables, T is the coolant temperature, k the thermal conductivity of the fuel rods, U_0''' the maximum value of the energy generation, and $\lambda^*=\pi/2L$.

In the absence of any body force, elastic deformations associated with the foregoing temperature problem are governed by the displacement equations of equilibrium (see, for example, Timoshenko and Goodier [4],

$$\frac{\partial^2 V_i}{\partial x_j \partial x_j}+\frac{1}{1-2\nu}\frac{\partial}{\partial x_i}\left(\frac{\partial V_j}{\partial x_j}\right)-2\left(\frac{1+\nu}{1-\nu}\right)\beta\frac{\partial \theta}{\partial x_i}=0, \tag{3}$$

where V_i, β and ν denote the displacement vector, the coefficient of thermal expansion and the Poisson ratio of the fuel rods, respectively.

The solution of eq. (3), in view of its linearity, may be written as a superposition of two solutions,

$$V_i=V_i^{\mathrm{p}}+V_i^{\mathrm{c}}, \tag{4}$$

where V_i^{p} and V_i^{c} denote a particular solution of eq. (3) and the complementary solution of the homogeneous part of eq. (3), respectively. A particular solution of eq. (3) may be obtained in terms of the scalar displacement potential given by Goodier [5]

$$V_i^{\mathrm{p}}=\partial\phi/\partial x_i. \tag{5}$$

Inserting eq. (5) into eq. (3), the governing equation to be satisfied by this particular solution is found to be

$$\frac{\partial^2\phi}{\partial x_i \partial x_i}=\left(\frac{1+\nu}{1-\nu}\right)\beta\theta. \tag{6}$$

The complementary solution for the homogeneous part of eq. (3) may be found in terms of the displacement potential χ_i given by Love–Galerkin [6], [7],

$$V_i^{\mathrm{c}}=2(1-\nu)\frac{\partial^2\chi_i}{\partial x_j \partial x_j}-\frac{\partial}{\partial x_i}\left(\frac{\partial \chi_j}{\partial x_j}\right), \tag{7}$$

or that ψ_i given by Boussinesq–Papkovich [8,9],

$$V_i^{\mathrm{c}}=4(1-\nu)\frac{\partial^2\psi_i}{\partial x_j \partial x_j}-\frac{\partial}{\partial x_i}(x_j\psi_j), \tag{8}$$

where x_j is the position vector. The Love–Galerkin and Boussinesq–Papkovich potentials satisfy the biharmonic and harmonic equations

$$\frac{\partial^2}{\partial x_i \partial x_i}\left(\frac{\partial^2\chi_k}{\partial x_j \partial x_j}\right)=0, \tag{9}$$

$$\partial^2\psi_j/\partial x_i \partial x_i=0. \tag{10}$$

Axially symmetric problems require only the third component of these potentials, $\chi_i(0,0,\chi)$ and $\psi_i(0,0,\psi)$. Then, the stresses expressed in terms of Goodier and Love–Galerkin potentials, and Goodier and Boussinesq–Papkovich potentials become, respectively,

$$\sigma_r=2G\left[\left(\frac{\partial^2\phi}{\partial r^2}-\nabla^2\phi\right)+\frac{\partial}{\partial z}\left(\nu\nabla^2\chi-\frac{\partial^2\chi}{\partial r^2}\right)\right],$$
$$\sigma_\varphi=2G\left[\left(\frac{1}{r}\frac{\partial\phi}{\partial r}-\nabla^2\phi\right)+\frac{\partial}{\partial z}\left(\nu\nabla^2\chi-\frac{1}{r}\frac{\partial x}{\partial r}\right)\right],$$
$$\sigma_z=2G\left[\left(\frac{\partial^2\phi}{\partial z^2}-\nabla^2\phi\right)+\frac{\partial}{\partial z}\left((2-\nu)\nabla^2\chi-\frac{\partial^2\chi}{\partial z^2}\right)\right],$$
$$\tau=2G\left[\frac{\partial^2\phi}{\partial r\partial z}+\frac{\partial}{\partial r}\left((1-\nu)\nabla^2\chi-\frac{\partial^2\chi}{\partial z^2}\right)\right], \tag{11}$$

and

$$\sigma_r = 2G\left[\left(\frac{\partial^2\phi}{\partial r^2} - \nabla^2\phi\right) + 2\nu\frac{\partial\psi}{\partial z} - z\frac{\partial^2\psi}{\partial r^2}\right],$$

$$\sigma_\varphi = 2G\left[\left(\frac{1}{r}\frac{\partial\phi}{\partial r} - \nabla^2\phi\right) + 2\nu\frac{\partial\psi}{\partial z} - \frac{z}{r}\frac{\partial\psi}{\partial r}\right],$$

$$\sigma_z = 2G\left[\left(\frac{\partial^2\phi}{\partial z^2} - \nabla^2\phi\right) + 2(1-\nu)\frac{\partial\psi}{\partial z} - z\frac{\partial^2\psi}{\partial z^2}\right],$$

$$\tau = 2G\left[\frac{\partial^2\phi}{\partial r\partial z} + (1-2\nu)\frac{\partial\psi}{\partial r} - z\frac{\partial^2\psi}{\partial r\partial z}\right]. \quad (12)$$

3. Solution of the problem

The temperature problem is trivial. The solution of eq. (1) which satisfies eq. (2) is readily found to be

$$\theta(r,z) = \theta(R,z) + \left(U_0'''/4k\right)(R^2 - r^2)\cos\lambda^* z, \quad (13)$$

where the interface temperature $\theta(R,z)$ varies depending on (single or two phase, laminar or turbulent) flow conditions. However, the simple temperature distribution given by eq. (13) results in three-dimensional thermal stresses of considerable complexity. The prime objective of this study is to determine these stresses.

The linearity of the problem suggests separate consideration of stresses resulting from each term of eq. (13). For the fuel rods fixed at both ends, $\theta(R,z)$ yields a uniform compressive stress

$$\sigma_z = -(E\beta/L)\int_0^{2L}\theta(R,z)\,\mathrm{d}z. \quad (14)$$

Only the second term of eq. (13) gives axially symmetric three-dimensional stresses. The particular solution of this stress problem will now be determined in terms of the Goodier potential. Inserting the second term of eq. (13) into eq. (6) results in

$$\nabla^2\phi = m(R^2 - r^2)\cos\lambda^* z, \quad (15)$$

where $m = (U_0'''\beta/4k)(1+\nu)/(1-\nu)$. A particular solution of eq. (15) may be obtained by the method of the variation of parameters or, more conveniently, by the assumption that

$$\phi(r,z) = (Ar^2 + B)\cos\lambda^* z. \quad (16)$$

Introducing eq. (16) into eq. (15) gives

$$\phi(r,z) = \left(m/\lambda^{*4}\right)\left[4 + (\lambda^* r)^2 - (\lambda^* R)^2\right]\cos\lambda^* z. \quad (17)$$

Here the complementary solution will be obtained in terms of the Love–Galerkin potential (see section 4 for the alternative solution in terms of the Boussinesq–Papkovich potential). Let the third component of eq. (9),

$$\nabla^2\nabla^2\chi = 0, \quad (18)$$

be expressed in two steps as follows:

$$\nabla^2\chi = f, \quad (19)$$

$$\nabla^2 f = 0. \quad (20)$$

The appropriate solution of eq. (20) in polar coordinates may readily be obtained by the separation of variables. The result, noting that

$$\lim_{r\to 0} K_0(\lambda^* r) \to \infty$$

may be written as

$$f(r,z) = \begin{pmatrix}\sin\lambda^* z\\ \cos\lambda^* z\end{pmatrix} I_0(\lambda^* r). \quad (21)$$

Inserting eq. (21) into eq. (19) gives

$$\nabla^2\chi = \begin{pmatrix}\sin\lambda^* z\\ \cos\lambda^* z\end{pmatrix} I_0(\lambda^* r). \quad (22)$$

Since the right-hand side of eq. (22) is composed of product terms, an appropriate form of $\chi(r,z)$ is

$$\chi(r,z) = \begin{pmatrix}\sin\lambda^* z\\ \cos\lambda^* z\end{pmatrix} F(r). \quad (23)$$

Introducing eq. (23) into eq. (22) results in

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}F}{\mathrm{d}r}\right) - \lambda^{*2}F = I_0(\lambda^* r). \quad (24)$$

The solution of eq. (24) may be obtained by the variation of parameters (see, for example, Wylie and Barrett [10]). The result is

$$F(r) = \int^r \frac{I_0(\lambda^* r^*)K_0(\lambda^* r) - I_0(\lambda^* r)K_0(\lambda^* r^*)}{W|I_0(\lambda^* r^*), K_0(\lambda^* r^*)|} \times I_0(\lambda^* r^*)\,\mathrm{d}r^*, \quad (25)$$

where W is the usual Wronskian. Noting that

$$W|I_0(\lambda^* r^*), K_0(\lambda^* r^*)| = -1/\lambda^* r^*,$$

eq. (25) may be rearranged as

$$F(r) = -\lambda^* K_0(\lambda^* r)\int^r r^* I_0^2(\lambda^* r^*)\,\mathrm{d}r^* + \lambda^* I_0(\lambda^* r)\int^r r^* I_0(\lambda^* r^*)K_0(\lambda^* r^*)\,\mathrm{d}r^*. \quad (26)$$

The integrals involved in eq. (26) may be found in texts on Bessel functions (see, for example, Wylie and Barrett [10]). Using these integrals, eq. (26) is reduced to

$$F(r)=\tfrac{1}{2}(\lambda^* r)I_1(\lambda^* r). \tag{27}$$

Inserting eq. (27) into eq. (23) yields

$$\chi(r,z)=\frac{1}{2}\begin{pmatrix}\sin\lambda^* z\\ \cos\lambda^* z\end{pmatrix}(\lambda^* r)I_1(\lambda^* r). \tag{28}$$

Since eq. (20) is satisfied by eq. (21) and eq. (19) by eq. (28), and (eq. (18) is equivalent to eq. (19) plus eq. (20), then eq. (18) is satisfied by eq. (21) and eq. (28). Thus, the solution of eq. (18) may be written as

$$\chi(r,z)=\begin{pmatrix}\sin\lambda^* z\\ \cos\lambda^* z\end{pmatrix}\left[CI_0(\lambda^* r)+D(\lambda^* r)I_1(\lambda^* r)\right], \tag{29}$$

where coefficients C and D to be evaluated from the boundary conditions.

Leaving the boundary conditions in z to section 5(e), only the boundary conditions in r are considered here. Thus

$$\sigma_r(R,z)=\tau(R,z)=0. \tag{30}$$

Note from the first and last equations of eq. (11), and from eq. (17) and eq. (29) that the conditions given by eq. (30) are valid for all values of z provided eq. (29) has the form

$$\chi(r,z)=\left[CI_0(\lambda^* r)+D(\lambda^* r)I_1(\lambda^* r)\right]\sin\lambda^* z. \tag{31}$$

Now introducing eq. (17) and eq. (31) into the first and last equations of eq. (11), and using eq. (30) gives the following algebraic equations in terms of C and D:

$$[\lambda I_0(\lambda)-I_1(\lambda)]C+\lambda[(1-2\nu)I_0(\lambda)+\lambda I_1(\lambda)]D = 2mR^5/\lambda^4,$$

$$I_1(\lambda)C+[2(1-\nu)I_1(\lambda)+\lambda I_0(\lambda)]D=2mR^5/\lambda^4$$

where $\lambda=\lambda^* R$. Solving these equations for C and D yields

$$C=\left(\frac{2mR^5}{\lambda^4}\right)\left(\frac{2\nu\lambda I_0(\lambda)-\gamma I_1(\lambda)}{\lambda^2 I_0^2(\lambda)-\delta I_1^2(\lambda)}\right),$$

$$D=\left(\frac{2mR^5}{\lambda^4}\right)\left(\frac{\lambda I_0(\lambda)-2I_1(\lambda)}{\lambda^2 I_0^2(\lambda)-\delta I_1^2(\lambda)}\right),$$

where $\delta=\lambda^2+2(1-\nu)$ and $\gamma=\lambda^2-2(1-\nu)$.

Inserting the values of C and D into eq. (31) and the result together with eq. (17) into eq. (11) gives the stress distribution in the fuel rods as follows:

$$\frac{\sigma_r(r,z)}{\beta G(U_0'''R^2/k)} = \left(\frac{1+\nu}{1-\nu}\right)\Bigg\{\frac{2-\lambda^2(1-\rho^2)}{2\lambda^2} - \Big\{[2\nu\lambda I_0(\lambda)-\gamma I_1(\lambda)]\left[I_0(\lambda\rho)-\frac{1}{\lambda\rho}I_1(\lambda\rho)\right] + [\lambda I_0(\lambda)-2I_1(\lambda)][(1-2\nu)I_0(\lambda\rho)+\lambda\rho I_1(\lambda\rho)]\Big\} \times\left\{\lambda\left[\lambda^2 I_0^2(\lambda)-\delta I_1^2(\lambda)\right]\right\}^{-1}\Bigg\}\cos\pi\zeta, \tag{32}$$

$$\frac{\sigma_\varphi(r,z)}{\beta G(U_0'''R^2/k)} = \left(\frac{1+\nu}{1-\nu}\right)\Bigg\{\frac{2-\lambda^2(1-\rho^2)}{2\lambda^2} - \Big\{[2\nu\lambda I^0(\lambda)-\gamma I_1(\lambda)]\left[\frac{1}{\lambda\rho}I_1(\lambda\rho)\right] + [\lambda I_0(\lambda)-2I_1(\lambda)][(1-2\nu)I_0(\lambda\rho)]\Big\} \times\left\{\lambda\left[\lambda^2 I_0^2(\lambda)-\delta I_1^2(\lambda)\right]\right\}^{-1}\Bigg\}\cos\pi\zeta, \tag{33}$$

$$\frac{\sigma_z(r,z)}{\beta G(U_0'''R^2/k)} = \left(\frac{1+\nu}{1-\nu}\right) \times\Bigg\{-\frac{2}{\lambda^2}+\{[2\nu\lambda I_0(\lambda)-\gamma I_1(\lambda)]I_0(\lambda\rho) + [\lambda I_0(\lambda)-2I_1(\lambda)] \times[2(2-\nu)I_0(\lambda\rho)+\lambda\rho I_1(\lambda\rho)]\} \times\left\{\lambda\left[\lambda^2 I_0^2(\lambda)-\delta I_1^2(\lambda)\right]\right\}^{-1}\Bigg\}\cos\pi\zeta, \tag{34}$$

$$\frac{\tau(r,z)}{\beta G(U_0'''R^2/k)} = \left(\frac{1+\nu}{1-\nu}\right) \times\Bigg\{-\frac{\rho}{\lambda}+\{[2\nu\lambda I_0(\lambda)-\gamma I_1(\lambda)]I_1(\lambda\rho)$$

$$+[\lambda I_0(\lambda) - 2I_1(\lambda)]$$

$$\times [2(1-\nu) I_1(\lambda\rho) + \lambda\rho I_0(\lambda\rho)]\}$$

$$\times \left\{\lambda\left[\lambda^2 I_0^2(\lambda) - \delta I_1^2(\lambda)\right]\right\}^{-1}\Big\} \sin \pi\zeta \qquad (35)$$

where $\rho = r/R$ and $\zeta = z/2L$. Since the axial distribution of these stresses is trivial, only the radial distributions are plotted in figs, 2, 3, 4 and 5 for the values 0.05, 0.5, 0.8 and 1.0 of λ, and for the common value $\frac{1}{3}$ of ν corresponding to the elastic media.

A number of comments can be made now on the radial distribution of various stresses shown in these figures. Since the temperature of rods increases as $\rho \to 0$, the compressive normal stresses (σ_r, σ_φ and σ_z) found at the center of rods are to be expected. This result explains also the tension in σ_φ and σ_z as $\rho \to 1$. Note that a load free surface is also stress free. Accordingly, $\sigma_r \to 0$ and $\tau \to 0$ as $\rho \to 1$ which have been used as boundary conditions. Since the rods should be in equilibrium in the z-direction, σ_z satisfies

$$2\pi \int_0^R r\sigma_z \mathrm{d}r = 0,$$

as expected. Finally, $\tau \to 0$ as $\rho \to 0$ is a result of the assumed axial symmetry.

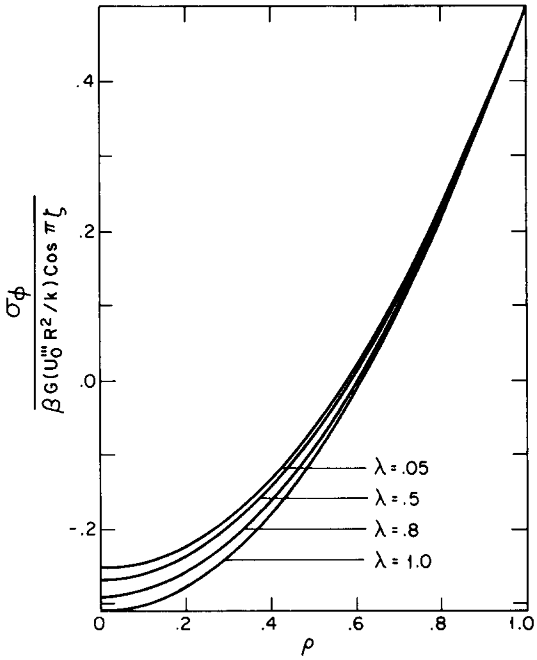

Fig. 3.

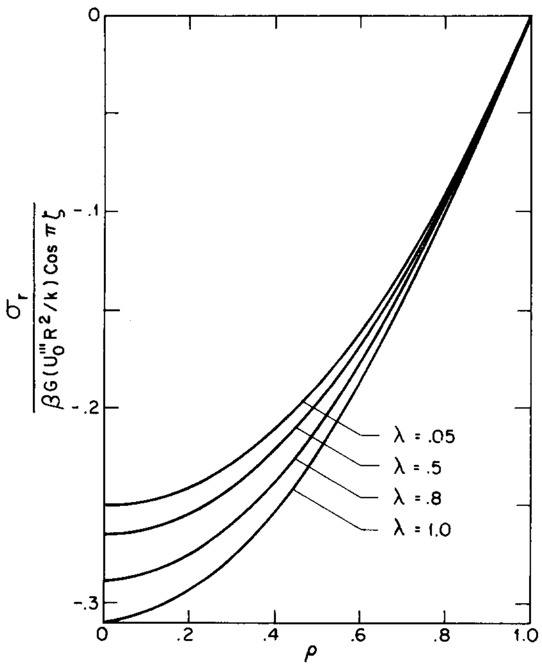

Fig. 2.

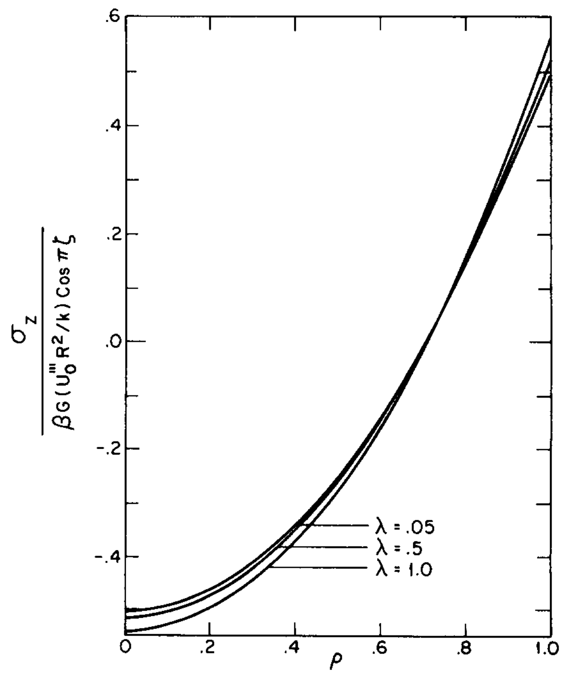

Fig. 4.

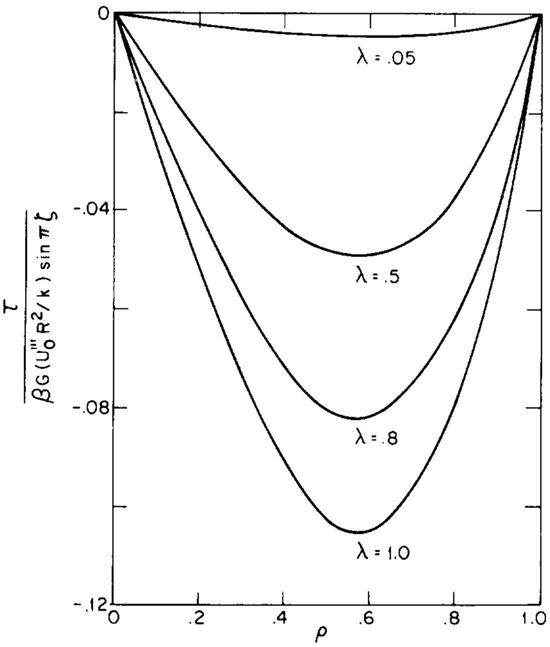

Fig. 5.

4. Alternative solution of the problem

As indicated in section 2, the complementary solution of the stress problem may also be obtained in terms of the Boussinesq–Papkovich potential which, for axially symmetric problems, satisfies

$$\nabla^2\psi = 0. \tag{36}$$

The appropriate harmonic function satisfying eq. (36) may easily be found, by the separation of variables and the consideration of eq. (12), as

$$\psi(r,z) = C_1 I_0(\lambda^* r)\sin\lambda^* z. \tag{37}$$

However ϕ or ψ should involve at least one more coefficient to be determined, together with C_1, from the use of eq. (30). Furthermore, since some terms of eq. (12) are multiplied by z, eq. (30) can only be satisfied by rearranging it in powers of z, and equating the coefficients of z to zero. This of course gives more than two relations, and suggests that the combination of ϕ and ψ should involve more than two coefficients. It follows that

$$\phi = \phi_0 + A_1\phi_1 + B_1\phi_2 + \dots,$$

$$\psi = C_1\psi_0 + \dots$$

where ϕ_0 and ψ_0 may be assumed equal to eq. (17) and eq. (37), respectively. Note that $\phi_1, \phi_2, \dots$ and $\psi_0, \dots$ are harmonic functions. There are a number of ways of constructing harmonic functions from a given harmonic function. For example, if ϕ_1 is harmonic,

$$\phi_2 = x_i\frac{\partial\phi_1}{\partial x_i} = r\frac{\partial\phi_1}{\partial r} + z\frac{\partial\phi_1}{\partial z} \tag{38}$$

is also harmonic. For the problem under consideration

$$\phi_1 = I_0(\lambda^* r)\cos\lambda^* z$$

is an appropriate harmonic function. Then eq. (38) readily reveals

$$\phi_2 = \lambda^* r I_1(\lambda^* r)\cos\lambda^* z - \lambda^* z I_0(\lambda^* r)\sin\lambda^* z.$$

After some algebra and by inspection of eq. (12), it is found that ϕ_0, ϕ_1, ϕ_2 and ψ_0 are sufficient in number for the present problem. Thus the new form of ϕ is

$$\begin{aligned}\phi(r,z) = {} & (m/\lambda^{*4})\left[4 + (\lambda^* r)^2 - (\lambda^* R)^2\right]\cos\lambda^* z \\ & + A_1 I_0(\lambda^* r)\cos\lambda^* z \\ & + B_1\left[\lambda^* r I_1(\lambda^* r)\cos\lambda^* z \right. \\ & \left. -\lambda^* z I_0(\lambda^* r)\sin\lambda^* z\right].\end{aligned} \tag{39}$$

Introducing eq. (37) and eq. (39) into the first and last equations of eq. (12), rearranging the result in powers of z, equating the coefficients of z to zero, and the consideration of eq. (30) give the values of A_1, B_1, and C_1. The result of this procedure indicates that $B_1 = C_1$. Thus obtained stresses are exactly identical to eqs. (32), (33), (34) and (35), and the details of the algebra are not given here.

5. Extension of the problem

The actual problem is somewhat more complicated than the model considered here. The addition of a number of effects to the present model, and their influence on the solution are briefly discussed below:

(a) The existence of fuel rods affects and depresses the neutron flux distribution at the locations of the fuel rods. This in turn gives rise to a radially distributed energy generation. This distribution is small relative to the axial distribution, and can be neglected. However, a radially symmetric distribution included into the energy equation presents neither conceptual nor algebraic difficulty. Only eqs. (1), (13), (15), (17), and (39), and the resulting stresses, should be modified accordingly. Radially asymmetric distributions are rather involved, and are beyond the scope of this study.

(b) The presence of a reflector leads to chopped radial and axial energy generations. This effect, however, can be included in the problem without any difficulty. The order of the algebra remains the same.

(c) The effect of cladding and that of the gap between rods and cladding present no conceptual but increased algebraic complexity. The temperature problem still remains simple. However, when included, the radial temperature distribution in cladding requires the consideration of the second complementary solutions for the Love–Galerkin and Boussinesq–Papkovich potentials in terms of the modified Bessel functions of the second kind.

(d) The effect of axial conduction considerably increases the algebraic complexity of both the temperature and stress problems. However, since this effect is small, a parametric study of its effect on temperature should be done first, rather than proceeding to the stress problem.

(e) The method of solution considered in this paper uses no axial boundary conditions. This method gives stresses whose mean, but not necessarily local, value is zero on the axial boundaries. However, these stresses diminish at very short (Saint-Venant) distances from the ends, and are negligible. At the expense of some algebra, these may be exactly eliminated by a suitable isothermal problem to be superimposed on the present problem.

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