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PERHAPS SCALAR NEUTRINOS ARE THE LIGHTEST SUPERSYMMETRIC PARTNERS

John S HAGELIN

Department of Physics, Maharishi International University, Fairfield IA 52556, USA

G L KANE* and S RABY†*

Randall Laboratory of Physics, University of Michigan, Ann Arbor, MI 48109, USA

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We consider the possibility that the scalar partners of the neutrinos ($\tilde{\nu}$) are the least massive supersymmetric partners, and show that this alternative is compatible with cosmological constraints, which put a significant lower bound on photino masses but not on $\tilde{\nu}$ masses Various consequences are examined the photon counting rate for $e^+e^- \to \gamma\tilde{\nu}\bar{\tilde{\nu}}$ may be large, the rate for $e^+e^- \to \tilde{W}^+\tilde{W}^-$ by $\tilde{\nu}$ exchange is enhanced, $Z^0 \to \tilde{\nu}\bar{\tilde{\nu}}$ increases $\Gamma(Z^0)$ by about 0 25 GeV, $W^\pm \to \tilde{\ell}^\pm\tilde{\nu}$ may be enhanced, the decay $\tau \to \tilde{\nu}_\tau \ell \bar{\tilde{\nu}}_\ell$ may be detectable, there can be additional contributions to the rare decay $K^+ \to \pi^+\tilde{\nu}\bar{\tilde{\nu}}$, restrictions on gluino masses, which depend on photinos interacting before they decay, have to be re-examined, scalar neutrinos have suitable characteristics as candidates for dark matter in the universe We discuss one currently fashionable class of models that can predict a light $\tilde{\nu}$

There is presently great interest in theories with spontaneously broken supersymmetry [1], because of the hope such theories offer [2] for alleviating the hierarchy problem associated with the weak interaction scale. In such theories, supersymmetric partners of all the known light particles should exist with masses less than about 1 TeV if the desired technical improvement in the hierarchy problem is indeed to be obtained Thus, one way to test supersymmetric models experimentally is to search for those supersymmetric partners [3]. A number of recent analyses have explored the cosmological [4,5] and terrestrial particle physics [6–9] implications of light gaugino-higgsino ("neutralino") states. In this paper we explore the possibility that the scalar neutrino ("sneutrino") is the lightest supersymmetric partner

One astrophysical requirement for any supersymmetric theory with an exact R symmetry is that the lightest supersymmetric partner (LSP), which is stable against

* Research supported in part by US DOE
† Present address T-8, MS B285, Los Alamos National Laboratory, Los Alamos, NM 87545, USA

decay, can annihilate readily enough so that its present cosmological mass density is reduced to acceptably low levels Goldberg [4] has pointed out that due to a P-wave suppression in the annihilation of Majorana gauginos, this requirement leads to significant lower bounds on the photino mass if the photino is the lightest supersymmetric partner A recent and detailed analysis by Ellis et al [5] places this lower bound at $\frac{1}{2}$ GeV if the LSP is predominantly a photino or at 5 GeV if the LSP is predominantly a higgsino. Sneutrinos, by contrast, can pair-annihilate via neutralino exchange (fig 1a) without P-wave or helicity suppression, with no lower bound on their mass if they are the LSP

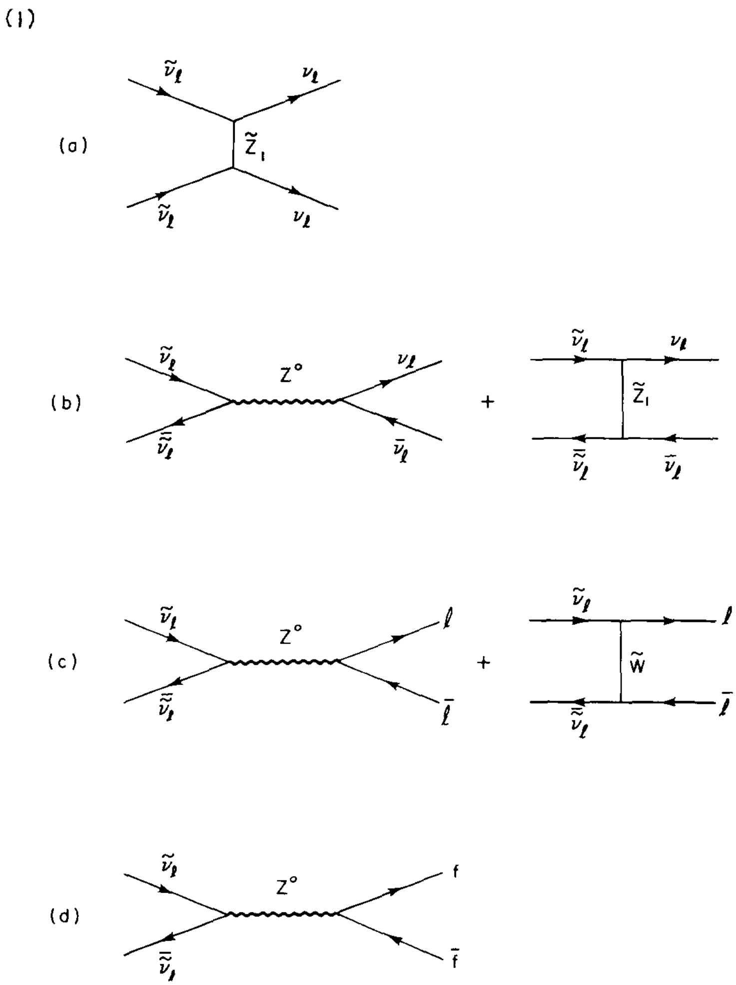

Fig 1 Diagrams contributing to sneutrino annihilation (a) $\tilde{\nu}_\ell + \tilde{\nu}_\ell \to \nu_\ell + \nu_\ell$, (b) $\tilde{\nu}_\ell + \bar{\tilde{\nu}}_\ell \to \nu_\ell + \bar{\nu}_\ell$, (c) $\tilde{\nu}_\ell + \bar{\tilde{\nu}}_\ell \to \ell + \bar{\ell}$, (d) $\tilde{\nu}_\ell + \bar{\tilde{\nu}}_\ell \to \mathrm{f} + \bar{\mathrm{f}}$, $\mathrm{f} \neq \nu_\ell, \ell$

In the minimal supersymmetric extensions of the standard model, the cross sections for $\tilde{\nu}$ annihilation corresponding to figs 1a–d are

$$(\sigma v_{\rm rel})_1 = \frac{1}{16\pi} \sum_{i=1}^{4} \left[\frac{(\alpha_i g_2 - \beta_i g_1)^2}{M_{\tilde{Z}_i}} \right]^2, \tag{1a}$$

$$(\sigma v_{\rm rel})_2 = \frac{s v_{\rm rel}^2}{192\pi} \left[\frac{4G_{\rm F}}{\sqrt{2}} + \sum_{i=1}^{4} \frac{(\alpha_i g_2 - \beta_i g_1)^2}{2M_{\tilde{Z}_i}^2} \right]^2, \tag{1b}$$

$$\begin{aligned}(\sigma v_{\rm rel})_3 = {} & \frac{G_{\rm F}^2 v_{\rm rel}^2}{4\pi} \sqrt{1 - \frac{4m^2}{s}} \left[4m^2 \sin^2\theta_{\rm w} \left(-\tfrac{1}{2} + \sin^2\theta_{\rm w} \right) \right. \\ & \left. + \tfrac{2}{3} \left(\tfrac{1}{4} - \sin^2\theta_{\rm w} + 2\sin^4\theta_{\rm w} \right)(s - m^2) \right] \\ & + \frac{G_{\rm F}^2 v_{\rm rel}^2}{2\pi} \sqrt{1 - \frac{4m^2}{s}} \left[\tfrac{1}{3} \left(-\tfrac{1}{2} + \sin^2\theta_{\rm w} \right)(m^2 - s) - \sin^2\theta_{\rm w} m^2 \right] \\ & \times \sum_{i=1}^{2} \frac{O_{\tilde{W}_i}^{+2} M_{\rm W}^2}{M_{\tilde{W}_i}^2} \\ & + \frac{G_{\rm F}^2 v_{\rm rel}^2}{24\pi} \sqrt{1 - \frac{4m^2}{s}} (s - m^2) \left[\sum_{i=1}^{2} \frac{O_{\tilde{W}_i}^{+2} M_{\rm W}^2}{M_{\tilde{W}_i}^2} \right]^2 \\ & + \frac{G_{\rm F}^2}{2\pi} m^2 \left(1 - \frac{4m^2}{s} \right)^{3/2} \left[\sum_{i=1}^{2} \frac{O_{\tilde{W}_i}^{+2} M_{\rm W}^2}{M_{\tilde{W}_i}^2} \right]^2, \end{aligned} \tag{1c}$$

$$\begin{aligned}(\sigma v_{\rm rel})_4 = {} & \frac{G_{\rm F}^2 v_{\rm rel}^2}{4\pi} \sqrt{1 - \frac{4m^2}{s}} \left[-4m^2 x_{\rm f} \left(T_3^{\rm f} - x_{\rm f} \right) \right. \\ & \left. + \tfrac{2}{3} \left(T_3^{{\rm f}\,2} - 2x_{\rm f} T_3^{\rm f} + 2x_{\rm f}^2 \right)(s - m^2) \right] \end{aligned} \tag{1d}$$

In eqs (1a–d) $v_{\rm rel}$ is the relative velocity of the annihilating particles, $x_{\rm f} \equiv Q_{\rm f} \sin^2\theta_{\rm w}$ and "m" always denotes the final state fermion mass Since the sneutrinos annihilate non-relativistically, higher powers in $v_{\rm rel}$ are not maintained. The quantities $M_{\tilde{Z}_i}$ are the eigenvalues corresponding to the four "neutralino" mass eigenstates

$$\tilde{Z}_i \equiv \alpha_i \tilde{W}^3 + \beta_i \tilde{B} + \gamma_i \tilde{H}_1^0 + \delta_i \tilde{H}_2^0 \qquad (i = 1 \quad 4), \tag{2}$$

of the Majorana mass matrix [5, 7, 8]

$$(\tilde{W}^3, \tilde{B}, \tilde{H}_1^0, \tilde{H}_2^0)\begin{pmatrix} M_2 & 0 & -\sqrt{\frac{1}{2}}\, g_2 v_1 & \sqrt{\frac{1}{2}}\, g_2 v_2 \\ 0 & M_1 & \sqrt{\frac{1}{2}}\, g_1 v_1 & -\sqrt{\frac{1}{2}}\, g_1 v_2 \\ -\sqrt{\frac{1}{2}}\, g_2 v_1 & \sqrt{\frac{1}{2}}\, g_1 v_1 & 0 & \varepsilon \\ \sqrt{\frac{1}{2}}\, g_2 v_2 & -\sqrt{\frac{1}{2}}\, g_1 v_2 & \varepsilon & 0 \end{pmatrix}\begin{pmatrix} \tilde{W}_3 \\ \tilde{B} \\ \tilde{H}_1^0 \\ \tilde{H}_2^0 \end{pmatrix} \tag{3}$$

arising from the Higgs vacuum expectation values $v_{1,2} \equiv \langle 0|H_{1,2}^0|0\rangle$ and the lagrangian terms

$$\mathcal{L} \supset \varepsilon\epsilon_{\alpha\beta}\tilde{H}_1^{\alpha}\tilde{H}_2^{\beta} - M_2\tilde{W}^a\tilde{W}^a - M_1\tilde{B}\tilde{B} \tag{4}$$

$\tilde{W}^a$ and $\tilde{B}$ are SU(2) and U(1) gauginos, and $\alpha, \beta(a)$ are doublet (triplet) SU(2) indices. For numerical purposes we shall assume $M_1 = (5\alpha_1/3\alpha_2)M_2$ where $\alpha_{1,2} \equiv g_{1,2}^2/4\pi$ are the SU(2) and U(1) couplings, which holds to leading order in the renormalization group equations if weak SU(2) × U(1) is eventually embedded in a unifying non-abelian group The quantities $M_{\tilde{w}_i}$ which appear in eq (1c) are the eigenvalues of the charged gaugino-higgsino mass matrix [5, 7, 8]

$$(\tilde{W}^+, \tilde{H}_1^+)_{\mathrm{L}}\begin{pmatrix} M_2 & g_2 v_2 \\ g_2 v_1 & -\varepsilon \end{pmatrix}\begin{pmatrix} \tilde{W}^- \\ \tilde{H}_2^- \end{pmatrix}_{\mathrm{L}} \tag{5}$$

and the quantities $O_{\tilde{w}_i}^+$ represent the coefficient of $\tilde{W}_{\mathrm{L}}^+$ in the ith charged eigenstate – the cosine or sine of the angle θ_+ which rotates the positively charged chiral fields in eq (5).

To compute the cosmological mass density of the sneutrino ($\rho_{\tilde{\nu}}$) we recall the rate equation for the number density of annihilating particles [10]

$$\frac{\mathrm{d}n}{\mathrm{d}t} = -3\frac{\dot{R}}{R}n - \langle\sigma v_{\mathrm{rel}}\rangle\left(n^2 - n_0^2\right), \tag{6}$$

where n is the actual number density at time t, n_0 is the number density of sneutrinos in thermal equilibrium, R is the cosmic scale factor, and angular brackets denote thermal average Following standard methods [10] one can rewrite (6) in the convenient form

$$\frac{\mathrm{d}f}{\mathrm{d}x} = \frac{M_{\tilde{\nu}}}{k^3}\left(\frac{8\pi^3 N_{\mathrm{F}} G}{45}\right)^{-1/2}\langle\sigma v_{\mathrm{rel}}\rangle\left(f^2 - f_0^2\right), \tag{7}$$

where $x \equiv kT/M_{\tilde{\nu}}$, $f(x) = n/T^3$, $f_0(x) = n_0/T^3$, k is the Boltzmann constant, $G = 1/M_{\mathrm{pl}}^2$ is Newton's constant and N_{F} counts the effective number of degrees of freedom at a given temperature

Replacing v_{rel} in eqs. (1a–d) by its thermal average ($v_{rel}^2 \simeq 6kT/M_{\tilde{\nu}} = 6x$), the cross section can be written

$$\langle \sigma v_{rel} \rangle = \tilde{a} + \tilde{b}x, \tag{8}$$

and the rate eq. (7) takes the general form

$$\frac{\mathrm{d}f}{\mathrm{d}x} = \frac{M_{\tilde{\nu}}}{k^3}\left(\frac{8\pi^3 N_F G}{45}\right)^{-1/2}(\tilde{a} + \tilde{b}x)(f^2 - f_0^2)$$

$$\equiv (a + bx)(f^2 - f_0^2) \tag{9}$$

Following the analytic approximation of Lee and Weinberg [10], we expect the scaled number density $f(x)$ in eq. (9) to remain approximately equal to its equilibrium value $f_0(x)$ until the temperature T drops to a freeze-out value of T_f where the annihilation rate is equal to the rate of change in f_0

$$\frac{\mathrm{d}f_0}{\mathrm{d}x} = (a + bx)f_0^2 \quad \text{at} \quad x = x_f, \tag{10}$$

and assume that thereafter it evolves approximately according to the equation

$$\frac{\mathrm{d}f}{\mathrm{d}x} = (a + bx)f^2, \tag{11}$$

subject to the initial conditions $f(x_f) = f_0(x_f)$ Since $x_f \ll 1$ for the sneutrino, we can use the non-relativistic approximation [10]

$$f_0(x) = 2k^3(2\pi x)^{-3/2}\mathrm{e}^{-1/x} \tag{12}$$

to solve for the freeze-out temperature Eqs (10) and (12) together give

$$x_f = \frac{1}{\ln\left(ax_f^{1/2} + bx_f^{3/2}\right) + \ln\left[2k^3/(2\pi)^{3/2}\right]} \tag{13a}$$

$$= \frac{1}{\ln\left(\tilde{a}x_f^{1/2} + \tilde{b}x_f^{3/2}\right) - \frac{1}{2}\ln\left(16\pi^6 N_F G/45M_{\tilde{\nu}}^2\right)} \tag{13b}$$

The present number density is obtained by integrating eq (12) from $x = x_f$ down to $x = 0$

$$f(0) = \frac{1}{ax_f + \frac{1}{2}bx_f^2} \tag{14}$$

The present mass density $\rho_{\tilde\nu}$ is then simply

$$\rho_{\tilde\nu} = 0.8\left(\frac{T_{\tilde\nu}}{T_\gamma}\right)^3 T_\gamma^3 M_{\tilde\nu}\frac{1}{ax_f + \frac{1}{2}bx_f^2}, \qquad (15)$$

where $(T_{\tilde\nu}/T_\gamma)^3$ accounts for the subsequent reheating of the photon temperature with respect to the temperature of $\tilde\nu$, due to the annihilation of particles with $M < x_f M_{\tilde\nu}$, and is tabulated [11] together with N_F in table 1. The "fudge factor" 0.8 is included to correct for the fact [10] that the analytic approximation (10), (11) to the full rate eq (7) gives a result which is approximately 25% too large. In terms of the coefficients $\tilde a$, $\tilde b$ appearing in the annihilation cross section, the mass density (15) reads

$$\rho_{\tilde\nu} = 5.0\times 10^{-40}\left(T_\gamma/2.8\,°\mathrm{K}\right)^3\left(\frac{43}{22N_F^{1/2}}\right)\frac{1}{\tilde a x_f + \frac{1}{2}\tilde b x_f^2}\quad \mathrm{g/cm^3}. \qquad (16)$$

It is now straightforward to incorporate into eq. (16) the results of a numerical analysis of the cross section (1a–d) (including the diagonalization of the mass matrices (3) and (5)) and to compare the present mass density $\rho_{\tilde\nu}$ with its cosmological upper bound We know from the rate of expansion of the universe that $\rho_{\tilde\nu} \leqslant 2\times 10^{-29}(\Omega h_0^2)$ g/cm³, where Ω is the density in units of the closure density

TABLE 1

$(T_{\tilde\nu}/T_\gamma)^3$ accounts for the subsequent reheating of the photon temperature with respect to the $\tilde\nu$ temperature due to the annihilation of particles with $M < x_f M_{\tilde\nu}$ N_F counts the effective number of degrees of freedom at a given temperature

T_f	N_f	$(T_{\tilde\nu}/T_\gamma)^3$
$m_e - m_\mu$	$\frac{43}{8}$	2.75
$m_\mu - m_\pi$	$\frac{57}{8}$	3.65
$m_\pi - T_H$	$\frac{69}{8}$	4.41
$T_H - m_s$	$\frac{205}{8}$	13.1
$m_s - m_c$	$\frac{247}{8}$	15.8
$m_c - m_\tau$	$\frac{289}{8}$	18.5
$m_\tau - m_b$	$\frac{303}{8}$	19.4
$m_b - m_t$	$\frac{345}{8}$	22.1
$m_t - M_W$	$\frac{387}{8}$	24.8
$> M_W$	$\frac{423}{8}$	27.1

This table is adapted from ref [11] Most of the notation is described in the text, with the exception that T_H is the temperature above which it is supposed that hadrons should be described in terms of quark and gluon degrees of freedom and h_0 is the Hubble parameter in units of 100 km s^{-1} Mpc^{-1} It is reasonable to believe that $\Omega h_0^2 \lesssim 1$ implying that $\rho_{\tilde{\nu}} \lesssim 2\times 10^{-29}$ g/cm^3

Fig 1a gives a large contribution to the annihilation cross section (1a) which is independent of s, v_{rel} and hence $M_{\tilde{\nu}}$. Physically this is because a Majorana neutralino can be exchanged, with an amplitude going as $1/M_{\tilde{Z}_i}$ (rather than $1/M_{\tilde{Z}_i}^2$), so $\sigma \sim 1/M_{\tilde{Z}_i}^2$ and no dependence on $M_{\tilde{\nu}}$ enters It is therefore apparent from eq (16) that $\rho_{\tilde{\nu}}$ is approximately independent of $M_{\tilde{\nu}}$. This contrasts with the familiar cases where the LSP is a Majorana neutralino [4,5] or even a heavy neutrino [10], where the annihilation cross section is proportional to M^2 (or to a final state mass^2). In such cases, the cosmological mass density grows at least as $1/M^2$ for small M, which leads to a lower bound on M In the present case, however, there is no lower bound on $M_{\tilde{\nu}}$ provided the contribution to the cross section from fig 1a is non-negligible*

In order for the contribution to $\langle \sigma v_{rel}\rangle$ from fig. 1a to be significant, it is necessary that the gaugino or higgsino mass terms M_2, ε in the lagrangian (4) be non-negligible In the limit of $M_2, \varepsilon \to 0$, there is a light photino and a light higgsino eigenstate

$$\tilde{\gamma} \equiv \frac{g_1 \tilde{W}^3 + g_2 \tilde{B}}{\sqrt{g_1^2+g_2^2}}, \qquad M_{\tilde{\gamma}} \approx \frac{8}{3}\frac{g_1^2}{g_1^2+g_2^2} M_2, \tag{17a}$$

$$\tilde{S}^0 \equiv \frac{v_2 \tilde{H}_1^0 + v_1 \tilde{H}_2^0}{v}, \qquad M_{\tilde{S}} \approx \frac{2 v_1 v_2}{v^2}\varepsilon, \tag{17b}$$

where we have introduced $v \equiv \sqrt{v_1^2+v_2^2}$ In this same limit the remaining two eigenstates are

$$\tilde{Z}_{\pm} \equiv \frac{g_1 \tilde{B} - g_2 \tilde{W}^3 \pm \sqrt{g_1^2+g_2^2}\,(v_1 \tilde{H}_1^0 - v_2 \tilde{H}_2^0)/v}{\sqrt{2(g_1^2+g_2^2)}}, \qquad M_{\tilde{Z}_\pm} \simeq M_Z = \sqrt{\tfrac{1}{2}(g_1^2+g_2^2)}\, v$$

Neither the photino ($\tilde{\gamma}$) nor the higgsino ($\tilde{S}^0$) contributes to sneutrino annihilation, while the two degenerate $\tilde{Z}_{\pm}$ neutralinos give contributions that are equal and opposite and hence cancel However if the higgsino mass parameter ε is $\gtrsim O(1$ GeV), this splits the degeneracy between the two $\tilde{Z}_{\pm}$ eigenstates enough so that the sneutrinos annihilate easily, in which case there is no lower bound on the $\tilde{\nu}$ mass from cosmology (see previous footnote) (In contrast to fig 1a which is "semi-weak," figs. 1b-d give weak contributions to the $\tilde{\nu}$ annihilation cross sections which also

* There is a separate constraint [11] that the sneutrino be non-relativistic during the time of helium synthesis Otherwise the expansion rate which depends on the total energy density and therefore on the number of relativistic particles present, would be too fast, causing the weak interactions to freeze out at a higher temperature resulting in more neutrons and a higher concentration of primordial ^{4}He This allows us to conclude that $M_{\tilde{\nu}} \gg$ few MeV

suffer from helicity and/or P-wave suppression. In the absence of fig. 1a, figs. 1b–d would lead to a lower bound on $M_{\tilde{\nu}}$ which is comparable to those derived for the photino as LSP) It therefore follows that by varying the parameters in the lagrangian (4), particularly ε, one can vary the strength of fig 1a and thereby adjust the present mass density of $\tilde{\nu}$ to any desired value up to or exceeding closure density This makes the sneutrino a potentially interesting dark matter candidate

There is considerable evidence that the dominant form of energy in the universe is neither luminous nor baryonic [12] This appears to be true on mass scales ranging from that of dwarf spheroidal galaxies ($\sim 10^7 M_\odot$) to rich clusters of galaxies ($\sim 10^{15} M_\odot$ where $M_\odot \sim 2 \times 10^{33}$ g) [12] Several elementary particles have been proposed as candidates for this dark matter. They typically lead to one of three distinct scenarios for galaxy formation. hot, warm or cold matter (see the review by Primack and Blumenthal [12]) Scalar neutrinos fall into the category of cold matter, thus joining the list of candidates which includes axions, photinos and massive gravitinos We briefly outline this scenario.

Let us define the temperature (T_{eq}) when the radiation energy density (ρ_r) *equals* the sneutrino energy density ($\rho_{\tilde{\nu}}$) For temperatures $T > T_{eq}$ the universe is radiation dominated We have

$$T_{eq} \cong 5 \Omega_{\tilde{\nu}} h_0^2 \quad \mathrm{eV}, \tag{18}$$

where h_0 (defined previously) satisfies $0\,5 \leqslant h_0 \leqslant 1$ and $\Omega_{\tilde{\nu}} \equiv \rho_{\tilde{\nu}}/\rho_c$ is the sneutrino energy density today in units of the closure density

$$\rho_c = \frac{3H_0^2}{8\pi G} = 1\,88 \times 10^{-29} h_0^2 \quad \mathrm{g/cm^3}$$

Initially at some temperature $T \gg T_{eq}$ sneutrinos are in thermal equilibrium with radiation However at the temperature $T_f \gg T_{eq}$, sneutrinos decouple, going out of both kinetic and chemical equilibrium. Typically $T_f \sim \frac{1}{20} M_{\tilde{\nu}}$ and sneutrinos are non-relativistic at this time.

Prior to decoupling, sneutrinos are prevented from clustering by radiation pressure. Density fluctuations which enter the horizon at this time oscillate acoustically However, for $T < T_f$ density fluctuations within the horizon can grow During the epoch $T_{eq} < T < T_f$ the clustering time scale ($\sim (G\rho_{\tilde{\nu}})^{-1/2}$) is much longer than the expansion time ($\sim (G\rho_r)^{-1/2}$) As a result fluctuations inside the horizon grow very slowly according to the relation [13]

$$\delta_{\tilde{\nu}} \equiv \frac{\delta\rho_{\tilde{\nu}}}{\rho_{\tilde{\nu}}} = \delta_{\tilde{\nu}}|_i \left(1 + \frac{3}{2}\frac{\rho_{\tilde{\nu}}(T)}{\rho_r(T)}\right), \tag{19}$$

which is valid for scales greater than the free streaming mass (M_{FS}) for sneutrinos

$(\delta_{\tilde{\nu}}|_i$ is the initial fluctuation spectrum at the time it enters the horizon Its origin is unclear. One possibility is that the largest contribution is imposed on sneutrinos by the dominant radiation background. If this is the case, then sneutrino fluctuations outside the horizon are given by [14]

$$\delta_{\tilde{\nu}}|_i = \tfrac{3}{4}\delta_r \tag{20}$$

In some recent scenarios based on an "inflationary" universe [15], it may naturally have the Zeldovich spectrum [16]

$$\delta_{\tilde{\nu}} \sim 10^{-4} \quad \text{(independent of scale)}$$

as it enters the horizon.)

The free streaming mass (M_{FS}) [17] is defined by the mass of sneutrinos in a volume determined by the distance (d_{FS}) sneutrinos can travel in a Hubble expansion time

We have

$$d_{\mathrm{FS}} \simeq \left(\frac{\langle v^2 \rangle}{G\rho_r(T)}\right)^{1/2}, \tag{21a}$$

$$M_{\mathrm{FS}}(T) = \tfrac{4}{3}\pi d_{\mathrm{FS}}^3(T)\rho_{\tilde{\nu}}(T). \tag{21b}$$

At $T = T_{\mathrm{eq}}$

$$M_{\mathrm{FS}}|_{\mathrm{eq}} \simeq 10^{-5}\mathrm{M}_{\odot}\left(\frac{M_{\tilde{\nu}}}{1\ \mathrm{GeV}}\right)^{-3}\Omega_{\tilde{\nu}}h_0^2, \tag{22}$$

where we have taken $T_f = \frac{1}{20}M_{\tilde{\nu}}$ This is to be compared to the mass of sneutrinos within the horizon at T_{eq}.

$$M_{\mathrm{H}}|_{\mathrm{eq}} \simeq 10^{16}\mathrm{M}_{\odot}h_0^{-4}\Omega_{\tilde{\nu}}^{-1/2} \tag{23}$$

It is these fluctuations on scales M for $M_{\mathrm{FS}} \leqslant M \leqslant M_{\mathrm{H}}$ which are the first to begin growing at $T < T_{\mathrm{eq}}$. They grow according to the relation [18]

$$\delta_{\tilde{\nu}}|_{T<T_{\mathrm{eq}}} = \delta_{\tilde{\nu}}|_{\mathrm{eq}}\left(\frac{T_{\mathrm{eq}}}{T}\right) \tag{24}$$

Given $T_{\mathrm{eq}}/T_0 \sim 2\,4 \times 10^4\Omega_{\tilde{\nu}}h_0^2$, we see that initial fluctuations $\delta_{\tilde{\nu}}|_i$ of order 10^{-4} can certainly become nonlinear by the present epoch (Note in hot or warm scenarios of dark matter, the fluctuations on scales of order $10^{15}\mathrm{M}_{\odot}$ or $10^{12}\mathrm{M}_{\odot}$ (resp) are the first to grow It is difficult, in these scenarios, to explain the presence of dark matter on small scales of order $10^7\mathrm{M}_{\odot}$.)

Baryons do not begin to cluster until after the temperature of hydrogen recombination ($T_{rec} \sim 0.3$ eV) They can then cluster about the dominant sneutrino background In the perturbative regime we have [19]

$$\delta_b + 2H\delta_b = 4\pi G\rho_{\tilde{\nu}}\delta_{\tilde{\nu}} \tag{25}$$

One finds that in a Hubble expansion time $\delta_b \simeq \delta_{\tilde{\nu}}$ This equation is valid for baryons within the horizon and on scales greater than the baryon free streaming mass ($M_{FSb} \simeq 10^5 M_{\odot}$ at recombination)

We thus have a picture of fluctuations on all scales from stars to clusters of galaxies becoming nonlinear at about the same epoch

It is not yet clear whether the large-scale structure of the universe can be explained in such a scenario. Recent computer simulations by Melott et al [20] are, however, encouraging.

Finally we note that if sneutrinos are relevant as dark matter candidates, we can place a rough upper limit on their mass This is because the sneutrino energy density today (or equivalently $\Omega_{\tilde{\nu}}$) is (as we discussed earlier) very sensitive to the higgsino mass parameter ε If we demand that sneutrinos are the LSP and that $\Omega_{\tilde{\nu}} \sim 0.1$ we find $M_{\tilde{\nu}} \leqslant 10$ GeV. In order to increase $\Omega_{\tilde{\nu}}$ we must decrease ε and hence $M_{\tilde{\nu}}$ For example $\Omega_{\tilde{\nu}} \sim 0.25$ implies $M_{\tilde{\nu}} \lesssim 5$ GeV and $\Omega_{\tilde{\nu}} \sim 1$ implies $M_{\tilde{\nu}} \lesssim 2$ GeV

Before we consider the various implications for particle physics of a light scalar neutrino, we will discuss the kinds of supersymmetric models which could predict a sneutrino for the LSP

In the minimal low energy supergravity model (MLES) [21] the sneutrino mass (for all three families) is given by the expression*

$$M_{\tilde{\nu}}^2 = m_g^2 - \tfrac{1}{2}M_Z^2\left(\frac{v_1^2 - v_2^2}{v^2}\right), \tag{26}$$

where m_g is the gravitino mass and $v_1 \gtrsim v_2$. Clearly sneutrinos are lighter than gravitinos. Gaugino and higgsino masses depend on the Majorana mass parameters M_3, M_2, M_1 for SU_3, SU_2, U_1 respectively and the Higgs mass parameter ε (eq (4)) It is always possible to choose these parameters such that sneutrinos are lighter than all gauginos and higgsinos.

Let us now consider squark and slepton masses For the latter we derive a simple constraint such that $M_{\tilde{\nu}} < M_{\tilde{\ell}}$. The slepton mass matrix is given by

$$M_{\tilde{\ell}}^2 = \begin{pmatrix} m_g^2 + m_\ell^2 + \frac{1}{4}(g_2^2 - g_1^2)(v_1^2 - v_2^2) & Am_g m_\ell \\ Am_g m_\ell & m_g^2 + m_\ell^2 + \frac{1}{2}g_1^2(v_1^2 - v_2^2) \end{pmatrix}, \tag{27}$$

* In the following analysis we have ignored corrections coming from gauginos These corrections for sleptons are proportional to α_1, α_2 and the gaugino mass terms M_1, M_2 For $M_1, M_2 \lesssim m_g$, these corrections are negligible

where m_ℓ is the corresponding lepton mass and A is an arbitrary parameter, typically of order 1 We find $M_{\tilde\nu} > M_{\tilde\ell}$ if

$$\frac{M_{\mathrm{w}}^2}{m_{\mathrm{g}} m_\ell}\left(\frac{1+3\tan^2\theta_{\mathrm{w}}}{2}\right)^{1/2}\left(\frac{v_1^2-v_2^2}{v^2}\right) > A \tag{28}$$

This constraint is easily satisfied For example even if $(v_1^2 - v_2^2)/v^2$ is as small as 0 14 and $m_\ell = m_\tau$ we find the constraint

$$\left(\frac{m_{\mathrm{g}}}{1\ \mathrm{GeV}}\right) A < 520 \tag{29}$$

The situation regarding squarks is more complicated If $M_3 \sim m_{\mathrm{g}}$ then squarks obtain significant renormalization group corrections to their tree-level masses These tend to increase the squark to slepton mass ratio In addition for third generation squarks, large Yukawa couplings also affect the running masses There is one range of parameters where we can simply analyze the squark spectrum, i e $A \ll 1$ and $M_3 \ll m_{\mathrm{g}}$ In this limit the sneutrino is lighter than all squarks [22]

We conclude that there is certainly a range of parameter space in the MLES model for which sneutrinos are the LSP

At what scale might we expect $M_{\tilde\nu}$? From eq (26) we see that $M_{\tilde\nu}$ can be as small as $\sim \frac{1}{10} m_{\mathrm{g}}$ without extreme fine tuning. We could thus reasonably expect $M_{\tilde\nu}$ to be of order 1–10 GeV.

We now would like to make some phenomenological remarks about the sneutrinos as LSP, beginning with the photon counting experiment $e^+e^- \to \gamma\tilde\nu\bar{\tilde\nu}$ The $\tilde\nu\bar{\tilde\nu}$ production cross section due to Z exchange is simply half of the conventional neutrino production cross section neglecting phase space [23]

$$\frac{\Gamma\left(e^+e^- \xrightarrow{Z} \tilde\nu\bar{\tilde\nu}\right)}{\Gamma\left(e^+e^- \xrightarrow{Z} \nu\bar\nu\right)} = \tfrac{1}{2}\left(1 - 4M_{\tilde\nu}^2/s\right)^{3/2} \tag{30}$$

However, the electron-sneutrino receives an extra contribution from $\tilde{\mathrm{W}}^\pm$ exchange which can be larger than the $\mathrm{W}^\pm$ contribution to the electron-neutrino cross section by the ratio $(M_{\mathrm{W}}/M_{\tilde{\mathrm{W}}})^4$ Since there are in fact two charged mass eigenstates (5) which contribute to sneutrino production through their $\tilde{\mathrm{W}}^\pm$ components, the exact cross section is quite model dependent.

$$\sigma(e^+e^- \to \tilde\nu\bar{\tilde\nu}) = \frac{G_{\mathrm{F}}^2 s\beta^{3/2}}{12\pi}\left\{\tfrac{1}{4} - \sin^2\theta_{\mathrm{w}} + 2\sin^4\theta_{\mathrm{w}} + \left(\tfrac{1}{2} - \sin^2\theta_{\mathrm{w}}\right)\sum_{i=1}^{2}\left(O_{\tilde{\mathrm{W}}_i}^{+2}\frac{M_{\mathrm{w}}^2}{M_{\tilde{\mathrm{W}}_i}^2}\right)\right.$$
$$\left. + \tfrac{1}{4}\sum_{i=1}^{2}\left(O_{\tilde{\mathrm{W}}_i}^{+2}\frac{M_{\mathrm{W}}^2}{M_{\tilde{\mathrm{W}}_i}^2}\right)^2\right\}, \tag{31}$$

where $\beta = 1 - 4M_{\tilde{\nu}}^2/s$ and the last two terms in brackets only contribute for $\tilde{\nu}_e$ production. In the limit of M_2, $\varepsilon \to 0$, the charged gaugino-higgsino mass eigenstates become the Dirac fermions $(\tilde{H}_2^-, \tilde{W}^+)$ and $(\tilde{W}^-, \tilde{H}_1^+)$ with masses $g_2 v_2$ and $g_2 v_1$, respectively It is the first of these, which is also presumably the lighter if $v_1 > v_2$ as expected, that contributes to $e^+e^- \to \tilde{\nu}\bar{\tilde{\nu}}$ so it is quite plausible that the cross section could be substantial Note. for ε small (i e the cosmologically interesting regime where $\Omega_{\tilde{\nu}} \simeq (\frac{1}{10}-1)$) and $v_1 \gtrsim 4v_2$, the cross section for $\tilde{\nu}_e$ production is roughly 100 times greater than for $\tilde{\nu}_\mu$ or $\tilde{\nu}_\tau$ production This is because the lightest charged wino eigenstate has mass $\lesssim 30$ GeV in this regime. The radiative production cross section is simply related to the bare process in the interesting "soft" photon limit

$$\frac{d^2\sigma(e^+e^- \to \gamma\tilde{\nu}\bar{\tilde{\nu}})}{dx_\gamma d(\cos\theta_\gamma)} \approx \frac{2\alpha}{\pi}\frac{\sigma(e^+e^- \to \tilde{\nu}\bar{\tilde{\nu}})}{x_\gamma \sin^2\theta_\gamma}, \tag{32}$$

where $x_\gamma \equiv 2E_\gamma/E_{\text{c m}}$. A signal for $e^+e^- \to \gamma +$ "unobserved" above the standard model prediction can be interpreted as additional neutrino species or as evidence for photinos or sneutrinos

It is also worth noting that if the $\tilde{\nu}$ is light, the cross section for $e^+e^- \to \tilde{W}^+\tilde{W}^-$ by $\tilde{\nu}$ exchange is enhanced, so that winos would be easier to observe up to the kinematic limit for producing them

If $M_{\tilde{\nu}} \lesssim O(10)$ GeV there is no significant phase space suppression for its contribution to Z decay Since a light $\tilde{\nu}$ contributes half as much as a conventional ν flavor, the $\tilde{\nu} + \bar{\tilde{\nu}}$ contribution is $\frac{1}{2}$ of the standard model ν prediction The latter is 6% per flavor, or 18% for three flavors, so the $\tilde{\nu}$ contribution is 9% for three flavors The expected Z width is then 1 09 times the standard model prediction, which is an increase of about 0 25 GeV, just due to $\tilde{\nu}$ contributions

Note that while photinos and sneutrinos both contribute to the photon counting experiment $e^+e^- \to \gamma +$ "unobserved" at low energies, the photino does not contribute to the Z width [8]. (Conversely, a light higgsino would contribute to the Z width but would not contribute significantly to $e^+e^- \to \gamma +$ "unobserved" at low energies [8]) Thus Z decay and photon counting at $\sqrt{s} \approx 30$ GeV provide complimentary information on supersymmetric particles

If $M_{\tilde{\nu}} \lesssim 1$ GeV, the decay $\tau \to \tilde{\nu}_\tau \ell \bar{\tilde{\nu}}_\ell$ ($\ell = \mu$ or e) becomes possible through $\tilde{W}^\pm$ exchange The implications of the non-observation of this decay have been studied in ref [24] If $M_{\tilde{W}}$ is large compared to M_W there are essentially no constraints, whereas if $M_{\tilde{W}} \lesssim M_W$ there are restrictions on $M_{\tilde{\nu}}$ which ensure that the decay becomes kinematically forbidden Specifically, if $M_{\tilde{W}} \lesssim M_W$ we demand that $M_{\tilde{\nu}_\tau} + M_{\tilde{\nu}_\ell} \gtrsim M_\tau$, but if $M_{\tilde{W}} > M_W$ lighter $M_{\tilde{\nu}}$ are allowed We feel that the decay $\tau \to \tilde{\nu}_\tau \ell \bar{\tilde{\nu}}_\ell$ should be looked for Its effect on the τ lifetime and the lepton spectrum can be found in ref. [24], note that this decay channel would increase the τ width and thus *decrease* the canonical lifetime.

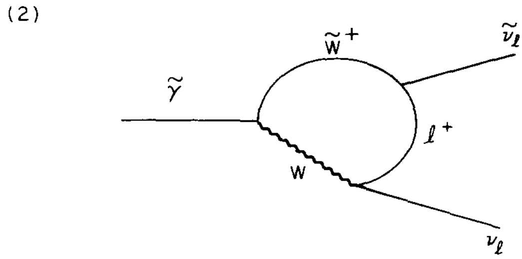

Fig 2 Diagram giving rise to an effective $\tilde{\gamma}\tilde{\nu}\nu$ vertex

If any sneutrino is lighter than $\frac{1}{2}(m_k - m_\pi)$ (which admittedly requires repugnant fine tuning in the context of most currently fashionable models) the decay $K^+ \to \pi^+ \tilde{\nu}\bar{\tilde{\nu}}$ becomes kinematically allowed. Neglecting phase space, each light sneutrino species contributes $\frac{1}{2}$ of a standard neutrino flavor through the Z-exchange channel, the $W^\pm$ "box" contribution is very model dependent.

Also, if $M_{\tilde{\nu}} < M_{\tilde{\gamma}}$ as assumed, the photino will decay through a loop diagram such as shown in fig 2 We can estimate the rate using the full one-loop analysis of Barnett et al [23], although they were calculating with the assumption that $\tilde{\nu} \to \tilde{\gamma}\nu$, the effective $\tilde{\gamma}\tilde{\nu}\nu$ vertex can be taken from their calculation If we write $\mathcal{M}_{\rm eff} = g_{\rm eff}\bar{u}P_R u$, then $g_{\rm eff} \simeq g_2^2 eF/16\pi\sqrt{2}$ where F is a function of the various masses, typically of order $\frac{1}{4}$ Then $g_{\rm eff} \simeq 5 \times 10^{-4}$ e, so $\Gamma(\tilde{\gamma} \to \tilde{\nu}\bar{\nu}) \simeq g_{\rm eff}^2 m_{\tilde{\gamma}}/32\pi \simeq 10^{-7}\frac{1}{4}\alpha m_{\tilde{\gamma}}$ This gives a lifetime $\tau_{\tilde{\gamma}} \lesssim 10^{-15}$ sec for $M_{\tilde{\gamma}} > 2$ GeV (ignoring corrections due to $M_{\tilde{\nu}} \neq 0$) Thus for most masses the photino decays very quickly – too rapidly to be observed

Finally, if the photino is unstable, various ways of searching for supersymmetric partners must be re-examined Here we will consider two such categories

(a) Often photinos are assumed to escape detectors, resulting in missing momentum Such analyses are *unchanged*, since the photino decays into final states $(\tilde{\nu}\bar{\nu})$ which are also invisible

(b) In beam dump experiments photinos are assumed to interact with an interaction cross section which is a few times the neutrino charged current cross section [25] If $\tilde{\gamma} \to \tilde{\nu}\bar{\nu}$ that situation is somewhat changed Both ν and $\tilde{\nu}$ interact, the $\tilde{\nu}$ as in fig 3 Since the Z-exchange contributions to the $\tilde{\nu}$ interaction requires no excitation of heavy squarks or sleptons with an associated kinematical suppression, it will dominate, if a signal is detected, it can be distinguished since it would have a y distribution characteristic of a scalar particle rather than that of a neutrino. Since both ν and $\tilde{\nu}$ from $\tilde{\gamma}$ decay will interact, there should be a signal Nevertheless previous analyses for gluino production might have to be reinterpreted. For one reason, the $\nu, \tilde{\nu}$ energies are somewhat degraded by the extra decay, and they may also emerge at large P_T so that fewer of them reach the detector, weakening the limit, however, since the FNAL group uses the presence of extra P_T as a possible

(3)

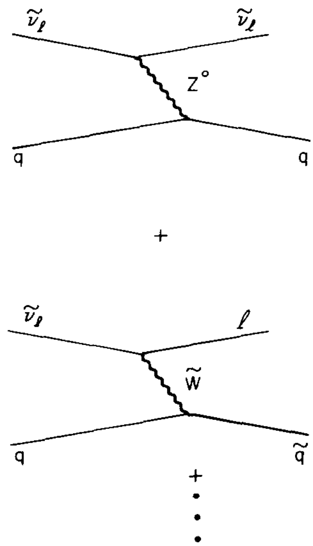

Fig 3 Neutral and charged current sneutrino interactions in a beam dump experiment

signal, this could work either way. Secondly, since the ν interaction will give both extra charged and extra neutral current events, and the $\tilde{\nu}$ will give some extra charged current events, the decision as to whether or not there are extra events will have to rely on absolute cross sections rather than the comparison of neutral versus charged current cross sections, at present this is very difficult experimentally We conclude that the existing limits on gluino masses may need some modification if sneutrinos are the LSP.

We have seen that it is easy in currently fashionable models for the sneutrino to be the lightest supersymmetric partner We have also shown that a light $\tilde{\nu}$ is very compatible with cosmological constraints, and provides a viable candidate for the "dark matter" in galaxies and galactic clusters. Finally, we have discussed certain interesting experimental implications of a light $\tilde{\nu}$ for particle physics

Note added

We have recently received a preprint by L Ibáñez [26] on the same subject. He comes to similar conclusions

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