THE SUPERSYMMETRIC QCD AND K_L-K_S MASS DIFFERENCE

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Assuming that globally supersymmetric QCD accounts for the discrepancy between the experimental value of the K_L-K_S mass difference and that obtained from the electroweak model, we extract contraints on the squark masses for different evaluations of the $K^0\bar{K}^0$ hadronic matrix element.

1. Introduction. In the past, the neutral kaon system has shown itself to be a sensitive seisometer [1] to the structure of the underlying quark dynamics as dictated by the standard electroweak model [2]. Perhaps the most stringent constraint which this system imposes on the model is that it should provide a natural explanation for the very small difference in mass between the long- and short-lived neutral kaons. As we all know, in the weak hamiltonian the suppression of the $\Delta S = 2$ operator responsible for the K_L-K_S mass difference is one of the consequences of the GIM mechanism [3]. If we only consider two generations of quarks and neglect strong radiative corrections then the evaluation of the familiar box graph of fig. 1 results in the effective weak hamiltonian [1]

$$H_{ew} = [(G_F/2\pi)\, m_c \cos\theta_c \sin\theta_c]^2(\bar{s}_L\gamma_\mu d_L)(\bar{s}_L\gamma^\mu d_L) + \text{h.c.}\,. \tag{1}$$

The K_L-K_S mass difference resulting from the electroweak box is then [‡1]

$$(m_L - m_S)_{ew} = 2\,\mathrm{Re}\langle\bar{K}^0|H_{ew}|K^0\rangle \tag{2}$$

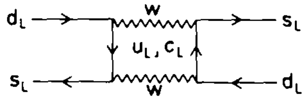

Fig. 1. Familiar electroweak box contribution to the $\Delta S = 2$ effective hamiltonian.

and whose value depends on the hadronic matrix element of the operator $(\bar{s}_L\gamma_\mu d_L)^2$. Recently, this has been calculated by one of us (J.T.) in several quark models [4] and can be related to the naive vacuum saturation result by

$$\langle\bar{K}^0|(\bar{s}_L\gamma_\mu d_L)^2|K^0\rangle = B\langle\bar{K}^0|(\bar{s}_L\gamma_\mu d_L)^2|K^0\rangle_{vs} = B\tfrac{1}{3}m_K f_K^2\,, \tag{3}$$

where $f_K \simeq 160$ MeV is the kaon decay constant. Recently [5], B has been extracted, due to the PCAC in the SU(3) limit, from the $\Delta I = 3/2$ amplitude of the $K^+ \to 2\pi$ decay. The resulting $B = 0.33$ is very close to the MIT-C value of ref. [4] and chiral perturbation analysis of ref. [6].

The upper bound on $B(=2 \pm 0.5)$ has also been found [7]. The values of the parameters B are given in refs. [4–7] and are quoted in table 1. Whence we obtain a value for the relative mass splitting from the electroweak model

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[‡1] We are using the phase convention $CPK^0 = -\bar{K}^0$.

Table 1
Coefficients B and C used in the text to convert from the vacuum saturation results. In the last case, the standard electroweak effect adequetely accounts for the discrepancy, so we can place no bounds on the supersymmetric contribution.

Type of calculation	B	C
vacuum saturation [1]	1	1
MIT bag model [4]	0.34	2.40
relativized harmonic oscillator quark model [4]	1.44	0.5
SU(3) and PCAC [5]	0.33	2.44
upper bound [7]	2	–

$$[(m_L - m_S)/m_K]_{ew} = B(0.42 \times 10^{-14}) , \qquad (4)$$

to be compared with the experimentally determined value [8]

$$[(m_L - m_S)/m_K]_{exp} = 0.71 \times 10^{-14} . \qquad (5)$$

In most quark models of interest this simple electroweak box contribution (4) alone is not sufficient to account for the experimental value (5). Previous investigations have removed the discrepancy by including the effects of the third [9] or a fourth [10] generation, or by invoking a left-right symmetric extension of the standard model [11]. In this letter we shall assume that the discrepancy is mostly due to the effects of the strong flavour-changing interactions present in supersymmetric QCD.

2. *The supersymmetric QCD contribution*. Since its inception ten years ago, supersymmetry [12] has found many applications in quantum field theory due to its remarkable renormalisation properties, and it is naturally of interest to ask if it is all relevant to physics at presently accessible energies. In the minimal globally supersymmetric extension of the standard model [13] one finds that there now exist flavour-changing strong interactions [14,15] given by the lagrangian

$$\mathcal{L}_s = i\sqrt{2} g_s \bar{\lambda}^\alpha [(d^0_{Li})^\dagger V(d)_{ij} T^\alpha d_{Lj} + (u^0_{Li})^\dagger V(u)_{ij} T^\alpha u_{Lj}] + \text{h.c.} , \qquad (6)$$

where the scalar supersymmetric partner of the left-handed quark u_{Li} (d_{Li}) in the ith generation is denoted by u^0_{Li} (d^0_{Li}) and the majorana gluino partners to the gluons are denoted by λ^α. The T^α are the generators of the fundamental representation of colour SU(3), g_s is the strong coupling constant and the super-Cabibbo matrices V(d) and V(u) parametrise the flavour-changing couplings [‡2].

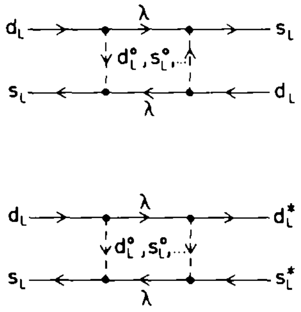

Fig. 2. Supersymmetric QCD box diagrams. The asterisks in the second box signify charge conjugation.

The two $\Delta S = 2$ box diagrams from supersymmetric QCD are shown in fig. 2, the second of which can be seen to be proportional to a majorana mass insertion for the gluino by a simple charge conjugation. Denoting the gluino mass by m and the ith scalar mass by m^0_i, the evaluation of these graphs results in the effective hamiltonian

$$H_s = \frac{\alpha_s^2}{6m^2} \sum_{i,j} (V^\dagger_{2i} V_{i1})(V^\dagger_{2j} V_{j1}) \times [G_1(x_i, x_j) O_1 - G_2(x_i, x_j) O_2] + \text{h.c.} , \qquad (7)$$

where α_s is the strong fine structure constant (as measured at the typical loop momentum scale), V stands for V(d) of (6), and the parametric integrals are

$$G_1(x_i, x_j) = 6 \int_0^1 d\alpha \int_0^{1-\alpha} d\beta\,(1 - \alpha - \beta) \times [1 - \alpha - \beta + \alpha x_i + \beta x_j]^{-1} ,$$

$$G_2(x_i, x_j) = 6 \int_0^1 d\alpha \int_0^{1-\alpha} \alpha\beta\,(1 - \alpha - \beta) \times [1 - \alpha - \beta + \alpha x_i + \beta x_j]^{-2} , \qquad (8)$$

‡2 In models with global supersymmetry such as we are considering, the flavour changing right-handed operators are suppressed relative to (6) [14]. However, in models based on local supersymmetry this need not be the case and hence there could be additional terms to consider [16].

where $x_i = (m_i^0/m)^2$ are dimensionless parameters. The four-quark operators have the form

$$O_1 = (\bar{s}_L\gamma_\mu T^\alpha T^\beta d_L)(\bar{s}_L\gamma^\mu T^\beta T^\alpha d_L)\,,$$

$$O_2 = (\bar{s}_L\gamma_\mu T^\alpha T^\beta d_L)(\bar{s}_L\gamma^\mu T^\alpha T^\beta d_L)\,, \quad (9)$$

which can be simplified via SU(3) identities and fierz rearrangement into

$$O_1 = \tfrac{11}{18}(\bar{s}_L\gamma_\mu d_L)^2\,, \quad O_2 = \tfrac{1}{9}(\bar{s}_L\gamma_\mu d_L)^2\,. \quad (10)$$

Whence from (2) and (3) we find that the relative mass splitting from supersymmetric QCD is

$$[(m_L - m_S)/m_K]_S = B(\alpha_s^2 f_K^2/162m^2)$$
$$\times \sum_{i,j} (V_{2i}^\dagger V_{i1})(V_{2j}^\dagger V_{j1})[11G_1(x_i,x_j) - 2G_2(x_i,x_j)]\,. \quad (11)$$

3. Limits on squark mass splitting. As we are assuming that the $\Delta S = 2$ processes in supersymmetric QCD are the most dominant corrections to the familiar electroweak box, then we can put an upper bound on their magnitude

$$[(m_L - m_S)/m_K]_S \leqslant [(m_L - m_S)/m_K]_{exp}$$
$$- [(m_L - m_S)/m_K]_{ew}\,. \quad (12)$$

In order to obtain a simplified form of (11) and to give us a guide as to the magnitude of (11) let us restrict ourselves to only two generations of squarks [in keeping with the two generations of quarks in (1)]. In this case V is a 2×2 rotation matrix parametrised by some angle θ. It can be shown [14] that in supersymmetric models with radiative scalar masses $\theta \simeq -\theta_C$ the usual Cabibbo angle. Further, if the squarks are almost degenerate, $m_2^0 \simeq m_1^0 \simeq m_0$, then to leading order we have

$$\sum_{i,j=1}^{2} (V_{2i}^\dagger V_{i1})(V_{2j}^\dagger V_{j1})[11G_1(x_i,x_j) - 2G_2(x_i,x_j)]$$
$$\simeq \cos^2\theta_c \sin^2\theta_c\,(x_2 - x_1)^2$$
$$\times [11F_1(x) - 2F_2(x)]\,, \quad (13)$$

where $x = m_0^2/m^2$ and we can explicitly calculate the functions

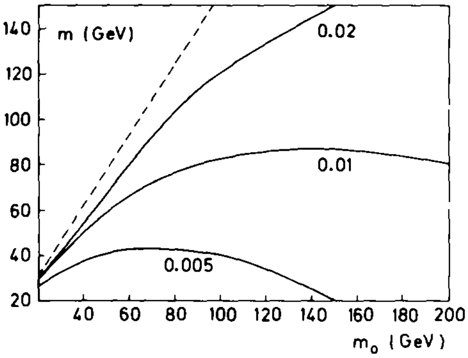

Fig. 3. Upperbounds of $(\Delta m_0/m_0)$ with vacuum saturation. For other quark models these numbers should be multiplied by C of table 1. The dashed line signifies where our analysis is no longer valid. The m is the gluino mass and m_0 is the average squark mass.

$$F_1(x) \equiv \partial^2 G_1/\partial x\,\partial y|_{y=x} = 1/x(x-1)^2 + 12/(x-1)^4$$
$$- 6(x+1)(\ln x)/(x-1)^5\,, \quad (14)$$

$$F_2(x) \equiv \partial^2 G_2/\partial x\,\partial y|_{y=x} = 1/x^2(x-1)^2$$
$$+ 6/x(x-1)^3 - 24/(x-1)^4$$
$$- 6(x+3)(\ln x)/(x-1)^5\,. \quad (15)$$

Hence we find that (12) provides us with a typical upper bound on the relative mass splitting of the squarks s_L^0 and d_L^0, viz.

$$|\Delta m_0/m_0| \leqslant m_0(\mathrm{GeV})(9/4B - \tfrac{4}{3})^{1/2} \times 10^{-4}$$
$$\times [x^3(11F_1(x) - 2F_2(x)]^{-1/2}\,, \quad (16)$$

where we have taken $\alpha_s \simeq 0.1$ at the loop scale. A contour plot of the upper bound in the vacuum saturation approximation ($B = 1$) for a range of squark and gluino masses is shown in fig. 3. It turns out that if $m > 1.56\ m_0$ (indicated by the dashed line) then the supersymmetric QCD contribution is negative and so our condition (12) is trivially satisfied. Thus bounds can only be extracted for masses to the right of the dashed line.

For other quark models we simply multiply these numbers for the upper bound by

$$C = [1 + \tfrac{49}{20}(1/B - 1)]^{1/2}\,, \quad (17)$$

which are also listed in table 1. For details of the calculations of the B we refer the reader to refs. [4–7].

4. Conclusions. We have examined the electroweak and supersymmetric QCD contribution to the $K_L - K_S$ mass difference in various quark models and found that upper bounds are placed on the relative mass splitting between the squarks. Typically, these bounds are of order 10^{-2} implying that if the scalars exist and are in the range examined then they are almost degenerate.

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