SUPERSYMMETRY AND PARASTATISTICS

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Received 18 October 1982
Revised manuscript received 23 March 1983

An analysis is made of the supersymmetry of parafields in Wess–Zumino-type models with two cases in which parabosons and parafermions form a supermultiplet. In case one the symmetry is realized by either the normal superalgebra or an infinite Lie algebra as in ordinary supersymmetry. In case two the infinite Lie algebra is intrinsic to the supersymmetry. With appropriate symmetry breaking, formulations of these types may be used to explain the generation problem by postulating unobserved parabosons as supersymmetric partners. The relevance to these models of an infinite Lie algebra constructed from supersymmetry is mentioned.

1. Introduction. The basic idea of supersymmetry is the mixture of particles of different statistics; normally taken to be bosons and fermions. It is natural to ask if this concept can be extended to parafields which generalize normal quantum statistics [1,2]. While there are no observed paraparticles in nature, the possibility exists for unobserved particles which obey this statistics.

Parastatistical fields obey double commutation relations [1,2]. In particular, parafermi relations contain only commutators as opposed to anticommutators. Analogous double commutators among fermionic quantities were introduced in a treatment of the supersymmetry algebra by defining commutators of fermionic generators as members of the algebra [3]. The resulting Lie algebra is infinite dimensional and similar to a Kac–Moody algebra [4,5].

Due to the similarity of parastatistical commutation relations and the infinite Lie algebra derived from supersymmetry, it is anticipated that there are representations of the infinite algebra (for fixed momentum) realized with parafields [6].

In this article we present two modifications of the Wess–Zumino model [7] containing parafields. In the first case the parameters are either anticommuting c-numbers (ordinary supersymmetry) or commuting c-numbers supplemented by a Klein transformation on the fermionic generators (infinite Lie algebra [3]). The second case utilizes c-number parameters obeying commutation relations of a generalized parastatistical type. A discussion of these cases follows in section 3.

2. Wess–Zumino lagrangians with parafields.

Case 1. Following Green, parafields are decomposed into components:

$$\phi = \phi_1 + \phi_2 + \ldots + \phi_p \,,$$

$$\psi = \psi_1 + \psi_2 + \ldots + \psi_p \,, \tag{1}$$

such that

$$\{\phi_1, \phi_2\} = [\psi_1, \psi_2] = 0 \,, \tag{2a}$$

$$\{\phi_1, \psi_2\} = \{\phi_2, \psi_1\} = 0 \,, \quad \text{etc} \,, \tag{2b}$$

at a space like separation, and each component satisfies normal boson and Majorana fermion equal time commutation relations ($i = 1,2, \ldots, p$)

$$[\phi_i(x,t), \dot{\phi}_i(x',t)] = \mathrm{i}\delta(x - x') \,,$$

$$\{\psi_i^\alpha(x,t), \psi_i^\beta(x',t)\} = -(\gamma_4 C)^{\alpha\beta}\delta(x - x') \,, \tag{3}$$

where $C(=\gamma_2\gamma_4)$ is the charge conjugation matrix [‡1].

[‡1] $C = (\gamma_2\gamma_4)$ all γ-matrices in Pauli–Dirac representation.

Note that eqs. (1)–(3) are equivalent [2] to the equal time double commutation relations of Green [1];

$$[\{\phi(x,t),\dot{\phi}(x',t)\},\phi(x'',t)]$$
$$= -\mathrm{i}2\delta(x'-x'')\phi(x,t)\,,\quad \text{etc}\,, \tag{4}$$

and

$$[[\psi^\alpha(x,t),\psi^\beta(x',t)],\psi^\delta(x'',t)]$$
$$= 2(\gamma_4 C)^{\alpha\delta}\delta(x-x'')\psi^\beta(x',t)$$
$$-2(\gamma_4 C)^{\beta\delta}\delta(x'-x'')\psi^\alpha(x,t)\,,\quad \text{etc}\,,$$

which define parafields. For simplicity we restrict to the $p = 2$ Green ansatz with the generalization to arbitrary p obvious.

Taking A, B, ψ to be $p = 2$ parafields in the Wess–Zumino model (A scalar; B pseudoscalar; ψ Majorana) we have the lagrangian

$$\mathcal{L} = -\tfrac{1}{4}\{\partial_\mu A,\partial_\mu A\} - \tfrac{1}{4}\{\partial_\mu B,\partial_\mu B\} - \tfrac{1}{4}[\psi,\not\partial\psi]$$
$$-\tfrac{1}{4}m^2\{A,A\} - \tfrac{1}{4}m^2\{B,B\} - \tfrac{1}{4}m[\overline{\psi},\psi]\,, \tag{5}$$

where we neglect interactions for simplicity. The interaction terms must be appropriately symmetrized in order to satisfy locality [2]. The fields in eq. (5) are $p = 2$ parafields as given in eqs. (1) and (2).

By a method similar to that in ref. [7] we take Majorana fermionic charges given by

$$S^\alpha = -\mathrm{i}\int S_4^\alpha \mathrm{d}^3x$$
$$= -\frac{\mathrm{i}}{2}\int \{\gamma_\lambda\partial^\lambda(A - \mathrm{i}\gamma_5 B) + m(A + \mathrm{i}\gamma_5 B), \gamma_4\psi\}^\alpha \mathrm{d}^3x. \tag{6}$$

Note the symmetrization in eq. (5) and eq. (6) eliminate fields of different Green index in bilinear terms. From S^α and the commutation relations of parafields [eqs. (3) or (4)] we have the transformations;

$$[S^\alpha, A_j] = \psi_j^\alpha\,,\quad [S^\alpha, B_j] = (\mathrm{i}\gamma_5\psi_j)^\alpha\,,$$
$$\{S^\alpha,\psi_j^\beta\} = \mathrm{i}[\partial_\mu(A - \mathrm{i}\gamma_5 B)\gamma^\mu + m(A + B\mathrm{i}\gamma_5)]_j^{\alpha\xi}C^{\xi\beta}\,, \tag{7}$$

where $j = 1,2$. The anticommutator,

$$\{S^\alpha, S^\beta\} = \mathrm{i}(\gamma_\mu C)P^\mu\,, \tag{8}$$

is the same as normal supersymmetry and gives the standard supersymmetry algebra for anticommuting parameters ϵ. The use of commuting ϵ parameters, supplemented by a Klein transformation [8] on the fermionic generators, leads to an infinite Lie algebra as shown in ref. [3]. In eq. (8), P^μ is constructed using symmetrized bilinears as in eq. (5). It should be noted that S^α in eq. (6) is a normal Majorana fermionic generator as contrasted with a parafermionic generator. The latter possibility is considered in case 2.

Case 2. We define parafermionic parameters, ϵ_{ij}^α, by commutation relations (no sum on repeated Green indices heretofore)

$$\{\epsilon_{ij},\epsilon_{ij}\} = 0\,, \tag{9a}$$

and

$$[\epsilon_{ij},\epsilon_{ik}] = 0\quad (j \neq k)\,,$$
$$\{\epsilon_{ij},\epsilon_{kj}\} = 0\quad (i \neq k)\,. \tag{9b}$$

Consider the lagrangian in eq. (5) assuming

$$[A_i,\psi_j] = [B_i,\psi_j] = 0\,, \tag{10}$$

with para commutation relations, eq. (2a), among bosonic and fermionic Green components. Define conserved generators;

$$S_{11} = -\mathrm{i}\int [\langle (A_1 + \mathrm{i}B_1)/\sqrt{2}, \psi_1^+\rangle$$
$$+ \langle (A_1 - \mathrm{i}B_1)/\sqrt{2}, \psi_1^-\rangle]\,\mathrm{d}^3x\,, \tag{11a}$$

$$S_{22} = -\mathrm{i}\int [\langle (A_2 + \mathrm{i}B_2)/\sqrt{2}, \psi_2^+\rangle$$
$$+ \langle (A_2 - \mathrm{i}B_2)/\sqrt{2}, \psi_2^-\rangle]\,\mathrm{d}^3x\,, \tag{11b}$$

$$S_{12} = -\mathrm{i}\int [\langle (A_1 - \mathrm{i}B_1)/\sqrt{2}, \psi_2^+\rangle$$
$$+ \langle (A_1 + \mathrm{i}B_1)/\sqrt{2}, \psi_2^-\rangle]\,\mathrm{d}^3x\,, \tag{11c}$$

$$S_{21} = -\mathrm{i}\int [\langle (A_2 - \mathrm{i}B_2)/\sqrt{2}, \psi_1^+\rangle$$
$$+ \langle (A_2 + \mathrm{i}B_2)/\sqrt{2}, \psi_1^-\rangle]\,\mathrm{d}^3x\,, \tag{11d}$$

where

$$\langle\phi,\psi\rangle \equiv \partial_\lambda\phi(\gamma_\lambda\gamma_4\psi)\,, \tag{12}$$

and

$$\psi_i^\pm = \tfrac{1}{2}(1 \pm \gamma_5)\psi_i \, . \tag{13}$$

Note $\psi_i^\pm$ are chiral Majorana fields with Green index i. In eqs. (11) we take the massless case for simplicity. It is trivial to include mass terms in which all fields have the same mass. The following relative para commutation relations are assumed between the fields in eqs. (11) and between fields and parameters:

$$[A_1, B_1] = [A_2, B_2] = \{A_1, B_2\} = \{A_2, B_1\} = 0 \, , \tag{14a}$$

$$\{\epsilon_{ij}, \psi_j\} = 0 \, , \quad [\epsilon_{ij}, \psi_k] = 0 \quad (j \neq k) \, , \tag{14b}$$

$$[\epsilon_{ij}, A_i] = [\epsilon_{ij}, B_i] = 0 \, ,$$

$$\{\epsilon_{ij}, A_k\} = \{\epsilon_{ij}, B_k\} = 0 \quad (i \neq k) \, . \tag{14c}$$

With these relations we have the transformations

$$[\bar{\epsilon}_{jj} S_{jj}, (A_k \pm \mathrm{i}B_k)/\sqrt{2}] = \delta_{jk}\bar{\epsilon}_{jj}\psi_k^\mp \, , \tag{15a}$$

$$[\bar{\epsilon}_{jl} S_{jl}, (A_k \pm \mathrm{i}B_k)/\sqrt{2}] = \delta_{jk}\bar{\epsilon}_{jl}\psi_l^\pm \quad (j \neq l) \, , \tag{15b}$$

$$[\bar{\epsilon}_{jj} S_{jj}, \psi_k^\pm] = \mathrm{i}\partial_\mu [(A_k \mp \mathrm{i}B_k)/\sqrt{2}]\gamma^\mu \epsilon_{jj}^\pm \, , \tag{15c}$$

$$[\bar{\epsilon}_{jl} S_{jl}, \psi_k^\pm] = \mathrm{i}\partial_\mu [(A_l \pm \mathrm{i}B_l)/\sqrt{2}]\gamma^\mu \epsilon_{jl}^\pm \quad (j \neq l) \, , \tag{15d}$$

where

$$\epsilon_{ij}^\pm = \tfrac{1}{2}(1 \pm \gamma_5)\epsilon_{ij} \quad (i, j = 1,2) \, . \tag{16}$$

Define charges

$$S_1 = (S_{11} + S_{12}) \, , \quad S_2 = (S_{21} + S_{22}) \, , \tag{17a,b}$$

and observe easily the commutation relations ($i = 1,2$)

$$\{S_j, S_j\} = \mathrm{i}(\gamma_\mu C)^{\alpha\beta} P^\mu \, , \tag{18a}$$

and

$$[S_1^\alpha, S_2^\beta] = 0 \, . \tag{18b}$$

Note that eq. (18a) is the normal supersymmetric commutation relation for each Green component whereas eq. (18b) is a result of the parastatistics of the fields. The proof of eq. (18a) follows as in the usual manner; we show the proof of eq. (18b) in the appendix.

Eqs. (18) are parafermi commutation relations for fixed momentum and imply the double commutation relation for

$$S = S_1 + S_2 \, ; \tag{19}$$

$$[[S^\alpha, S^\beta], S^\delta] = 2\mathrm{i}S^\alpha(\gamma_\mu C)_{\beta\delta}P^\mu - 2\mathrm{i}(\gamma_\mu C)_{\alpha\beta}P^\mu S^\beta \, . \tag{20}$$

As is described in ref. [3], the repeated commutator of operators on the RHS of eq. (20) leads to an infinite Lie algebra of the Kac–Moody type. Because neither the commutator, $[S^\alpha, S^\beta]$, nor the anticommutator, $\{S^\alpha, S^\beta\}$, can be written linearly in terms of the generators of the transformations, the infinite Lie algebra [3] is intrinsic to this symmetry.

Note that by singling out S^α as a generator of the infinite Lie algebra, we are considering a subalgebra of the total supersymmetry algebra. The algebra generated by S_1, S_2 and the Poincare group is closed allowing commutators and anti-commutators. The algebra generated by S^α and the Poincare group is closed allowing double commutators of the parafermi type as in eq. (20). As a result, the corresponding parafermionic parameters, ϵ^α,

$$\epsilon^\alpha = \epsilon_{11}^\alpha + \epsilon_{12}^\alpha + \epsilon_{21}^\alpha + \epsilon_{22}^\alpha \, , \tag{21}$$

should satisfy the double commutator

$$[[\epsilon^\alpha, \epsilon^\beta], \epsilon^\delta] = 0 \, . \tag{22}$$

3. Discussion. The commutation relations of independent fields as s been discussed extensively in the past [8]. It has been proven that fermionic fields can be changed from relative anticommutators to relative commutators by appropriate Klein transformations without changing physical content. However, this is not the case for bosons because there is no conserved boson number operator (with the exception of conserved charges associated with the bosons such as the strangeness quantum number). Therefore the existance of a paraboson in supersymmetric relation with parafermions (case 2) may imply significant experimental consequences. For example, an arbitrary Yukawa interaction between parafermions and parabosons is not allowed due to the violation of locality [2]. However, it may be possible to construct local interactions between supersymmetric partners with gauge bosons which are supersinglets. Certainly this possibility exists within superspace formulation [9] of supersymmetry.

The superspace concept may be applied to parasupersymmetry by defining a parafermionic, rather than fermionic, extension of Minkowski space. The c-number parameters, θ^α, would satisfy the condition

$$[[\theta^\alpha, \theta^\beta], \theta^\delta] = 0 \, . \tag{23}$$

Also, θ should have appropriate relative pararelations with parafields and parafermionic generators.

The formulation discussed in case 2 is different from standard parafield theories in that the Green index is not confined in a form shown in eqs. (2) and (4). The possibility of Green components as independent fields implies double commutation relations among components which are more general than Green's ansatz. This possibility will be discussed further in a forthcoming publication.

In conclusion, we mention possible phenomenological implications of these ideas. First, due to the existance of a wider class of representations, the infinite Lie algebra (allowing the possibility of paraparticles) may explain multiplicities of various particles. For example, the generation problem may be understood by a model in which parabosons are related to parafermions prior to symmetry breaking. In such a model the generation symmetry would be an internal symmetry between independent Green fields. In another direction, the parastatistical nature of certain particles connected in supersymmetry and the liberation of the Green index (as in case 2) suggest the possibility that the breaking of supersymmetry may split Green component masses. This could mean that the experimental non-existance of paraparticles is simply a low energy effect, and only at sufficiently high energies all Green components are observed.

The authors would like to acknowledge P. Tataru-Mihai for sending his manuscript before publication. We are also indebted to Jacques Leveille and Hans Wospakrik for useful information and discussions. This work is supported in part by the US Department of Energy.

Appendix. Proof of eq. (18b), $[S_1^\alpha, S_2^\beta] = 0$. From eqs. (11) and (17) we have

$$[S_1^\alpha, S_2^\beta] = \sum_{\epsilon=\pm 1} (I_{1\alpha\beta}^{(\epsilon)} + I_{2\alpha\beta}^{(\epsilon)} + I_{3\alpha\beta}^{(\epsilon)} + I_{4\alpha\beta}^{(\epsilon)}) , \quad \text{(A1)}$$

where

$$I_{1\alpha\beta}^{(\epsilon)} = \int \mathrm{d}^3x\, \mathrm{d}^3x'$$
$$\times \{[\partial_\lambda[(A_1 + \mathrm{i}\epsilon B_1)/\sqrt{2}], \partial_\nu[(A_1' - \mathrm{i}\epsilon B_1')/\sqrt{2}]]$$
$$\times (\gamma_\lambda\gamma_4\psi_1^{(\epsilon)})_\alpha(\gamma_\nu\gamma_4\psi_2'^{(\epsilon)})_\beta\} , \quad \text{(A2)}$$

$$I_{2\alpha\beta}^{(\epsilon)} = \int \mathrm{d}^3x\, \mathrm{d}^3x'$$
$$\times \{[\partial_\lambda[(A_2 - \mathrm{i}\epsilon B_2)/\sqrt{2}], \partial_\nu[(A_2' + \mathrm{i}\epsilon B_2')/\sqrt{2})]]$$
$$\times (\gamma_\lambda\gamma_4\psi_2^{(\epsilon)})_\alpha(\gamma_\nu\gamma_4\psi_1'^{(\epsilon)})_\beta\} ,$$

$$I_{3\alpha\beta}^{(\epsilon)} = \int \mathrm{d}^3x\, \mathrm{d}^3x'$$
$$\times [\partial_\lambda[(A_1 + \mathrm{i}\epsilon B_1)/\sqrt{2}]\partial_\nu[(A_2' - \mathrm{i}\epsilon B_2')/\sqrt{2}]$$
$$\times \{(\gamma_\lambda\gamma_4\psi_1^{(\epsilon)})_\alpha, (\gamma_\nu\gamma_4\psi_1'^{(-\epsilon)})_\beta\}] ,$$

$$I_{4\alpha\beta}^{(\epsilon)} = \int \mathrm{d}^3x\, \mathrm{d}^3x'$$
$$\times [\partial_\lambda[(A_2 - \mathrm{i}\epsilon B_2)/\sqrt{2}]\partial_\nu[(A_1' + \mathrm{i}\epsilon B_1')/\sqrt{2}]$$
$$\times \{(\gamma_\lambda\gamma_4\psi_2^{(\epsilon)})_\alpha, (\psi_\nu\gamma_4\psi_2'^{(-\epsilon)})_\beta\}] . \quad \text{(A2 con'd)}$$

By using canonical commutation relations for fields with the same Green index and parastatistical relations, eq. (2), between fields of different index, we obtain

$$I_{1\alpha\beta}^{(+)} = -\int \mathrm{d}^3x \sum_l \Big(\tfrac{1}{4}\sum_A (\gamma_A C)_{\alpha\beta}$$
$$\times [(C^{-1}\tfrac{1}{2}(1+\gamma_5)\gamma_4\gamma_l\gamma_A\tfrac{1}{2}(1+\gamma_5))_{\beta'\alpha'}$$
$$+ (C^{-1}\tfrac{1}{2}(1+\gamma_5)\gamma_A\gamma_l\gamma_4\tfrac{1}{2}(1+\gamma_5))_{\beta'\alpha'}]$$
$$\times \psi_1^{\alpha'}\partial_l\psi_2^{\beta'}\Big), \quad \text{(A3)}$$

where the Fierz transformation,

$$X_{\alpha\eta}Y_{\beta\xi} = \frac{1}{4}\sum_A (\gamma_A C)_{\alpha\beta}(Y^\mathrm{T}C^{-1}\gamma_A X)_{\xi\eta} , \quad \text{(A4)}$$

has been used with C = the charge conjugation matrix. In eq. (A4),

$$A = 1, \gamma_5, \gamma_\mu, \mathrm{i}\gamma_\mu\gamma_5, \sigma_{\mu\nu} . \quad \text{(A5)}$$

From the projection operator $\frac{1}{2}(1+\gamma_5)$ in eq. (A3) one has immediately that the sum is restricted to $A = 1$, γ_5, $\sigma_{\mu\nu}$. Because of the symmetry of $C_{\gamma A}$ (symmetric for $A = \gamma_\mu, \sigma_{\mu\nu}$; antisymmetric for $A = 1, \gamma_5, \gamma_\mu\gamma_5$), the summation is further restricted to $A = \sigma_{\mu\nu}$ with

$$(\mu, \nu) = (4, k \neq l) ,$$
$$= (k, l) , \quad \text{(A6)}$$

for the coefficient of $\psi_1^{\alpha'}\partial_l\psi_2^{\beta'}$, where $k, l \neq 4$. Similarly,

$$I_{2\alpha\beta}^{(+)} = -\int \mathrm{d}^3x \sum_l \Big(\frac{1}{4}\sum_A (\gamma_A C)_{\alpha\beta}$$

$$\times [(C^{-1}\tfrac{1}{2}(1+\gamma_5)\gamma_4\gamma_l\gamma_A)_{\beta'\alpha'}$$

$$+ (C^{-1}\tfrac{1}{2}(1+\gamma_5)\gamma_A\gamma_l\gamma_4)_{\beta'\alpha'}]\,\psi_2^{\alpha'}\partial_l\psi_1^{\beta'}\Big)\,, \qquad \text{(A7)}$$

where, by arguments similar to above, A is restricted to $\sigma_{\mu\nu}$ with (μ, ν) in eq. (A6). One easily concludes

$$I_{1\alpha\beta}^{(\epsilon)} + I_{2\alpha\beta}^{(\epsilon)} = 0 \quad (\epsilon = \pm 1)\,, \qquad \text{(A8)}$$

from partial integration and commutivity of ψ_1 and ψ_2 for $\epsilon = +1$. A similar argument applies for $\epsilon = -1$. Continuing, we have

$$I_3^{(\epsilon)} + I_4^{(\epsilon)} = -\int \mathrm{d}^3x$$

$$\times\, \partial_\lambda[(A_1 + \mathrm{i}\epsilon B_1)/\sqrt{2}]\,\partial_\nu[(A_2 - \mathrm{i}\epsilon B_2)/\sqrt{2}]$$

$$\times\, [\tfrac{1}{2}(1 \pm \gamma_5)(\gamma_\lambda\gamma_4\gamma_\nu - \gamma_\nu\gamma_4\gamma_\lambda)C]_{\alpha\beta}\,, \qquad \text{(A9)}$$

which vanishes directly for $\lambda = \nu$ or $\lambda = 4$, $\nu = l$ or $\lambda = l$, $\nu = 4$ ($l = 1, 2, 3$). The remaining case, $(\lambda, \nu) = (k, l)$ ($k \neq l$; $k, l = 1, 2, 3$), vanishes by partial integration.

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