Foundations for auto shredders*

F. E. RICHART, Jr

W. J. Emmons Professor of Civil Engineering, University of Michigan, Ann Arbor, MI, USA

R. D. WOODS

Professor of Civil Engineering, University of Michigan, Ann Arbor, MI, USA

Unbalanced forces developed by hammer wear and impact must be resisted by auto shredder foundations. Methods for estimating the impact forces are described. Because of different soil conditions, a concrete mat, a concrete block, and a pile-supported foundation system were adopted at three different construction sites. The design procedures involved in determining the dynamic response for each type of foundation are illustrated by examples. Vibration measurements made on the pile-supported foundation after construction permitted comparisons of prototype motions with design predictions.

INTRODUCTION

Automobile shredders consist of rows of rotating hammers which pass between a slotted anvil as shown in Fig. 1. The automobile body which is fed into this system is reduced to scrap metal by the impact and shearing forces developed when the body interferes with the hammer motion. Thus, during the process large impact forces are developed by the machine. Also, because of uneven hammer wear, large steady-state unbalanced rotating forces are produced. These dynamic forces are transmitted through the foundation system into the underlying soil.

This paper describes briefly the design of three types of auto shredder foundations, (a) a rigid concrete mat, (b) a deeply embedded rigid concrete block, and (c) a pile-supported concrete mat. Each foundation system reduces the machine vibration to tolerable levels.

DYNAMIC FORCES

Uneven wear of the hammers produces an unbalanced force vector rotating about the shaft. The limits for this type of unbalanced force are established by the machine manufacturer and the control of the magnitude of this force depends upon the owner's maintenance procedure. Hammers must be replaced periodically because of wear, and the machine can be nearly balanced after each hammer replacement but will become more unbalanced with time of operation.

Each type of shredder has a different allowable unbalance, depending on the number and size of the hammers and the operating speed. For a machine carrying 34 hammers, two rows of 9 each weighing 240 lb (1067 N) and two rows of 8 at 144 lb (640 N) each, and operating at 700 rpm, the vertical and horizontal (i.e. centrifugal) force amounts to 56400 lb (2.51×10^5 N). Another machine with 34 hammers each weighing 450 lb (2002 N) and operating at 600 rpm has a limiting force of 112500 lb (5.0×10^5 N).

Impact forces are developed when each row of hammers hits the auto body, thus the frequency of impacts is *four times* the operating frequency. The limiting value of this impact force depends upon the impulse required to stop the hammer at the point of impact. This occurs occasionally when hard chunks of metal cannot be shredded by one impact. Figure 2(a) shows a single hammer and the dynamic forces which act on it. The point of impact is taken as 12 in. (0.30 m) from the centerline of the 4 in. (0.10 m) diameter hammer bolt. The moment of the impact force ($M = Q_I r_q$) tends to rotate the hammer about the hammer bolt. This rotation is resisted by the frictional moment fFr_b at the hammerbolt plus the inertial resistance of the hammer to being rotated about the hammerbolt. At a rotating speed of 600 rpm the centrifugal force developed by each hammer is:

$$F = \frac{W}{g} r_1 \omega^2 = \frac{450}{386} (28.5 + 8.5)(2\pi \times 10)^2$$
$$= 1.70 \times 10^5 \text{ lb } (7.56 \times 10^5 \text{ N})$$

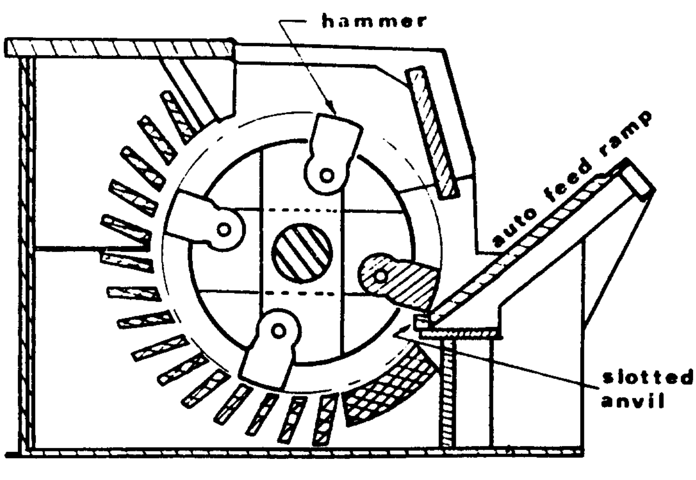

Figure 1. Cross-section of auto shredder

* Paper taken from the proceeings of the first International Conference on Soil Dynamics and Earthquake Engineering, Southampton, UK, 13-15 July 1982.

0261-7277/83/010054-06 $2.00

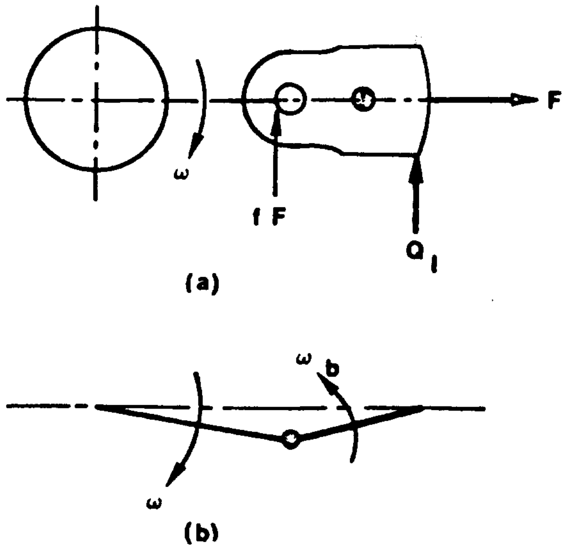

Figure 2. Hammer action, (a) forces on hammer, (b) motion of hammer and arm

Then the impact force which can be developed just to overcome *frictional resistance* is:

$$Q_f = fF[r_b/r_q] = 0.15 \times 1.70 \times 10^5 \times 1.5/12$$
$$= 3200\,\text{lb}\ (1.42 \times 10^4\,\text{N})$$

for each hammer.

The dynamic force needed to rotate the hammer about the hammerbolt depends on the time interval involved. At the contact point the linear velocity is:

$$v_c = r_c\omega = (28.5 + 12) \times 2\pi \times 10 = 2545\,\text{in./s}\ (64.6\,\text{m/s})$$

Then if the contact point is brought to rest in a distance of 20 in. (0.51 m) which is assumed as the compressed height of a mashed auto body, then the time interval needed for this contact point to be stopped is 0.008 s. During this 0.008 s the hammerbolt has travelled through an angle of $\theta_b = 0.008 \times 2\pi \times 10 = 0.50$ rad. Then if the hammer must swing through an angle of 60° before it can pass by the hard material (see Fig. 2b), the average angular rotation of the hammer about the hammerbolt is:

$$\omega_b = \frac{\theta n}{\Delta t} = \frac{\pi/3}{0.008} = 131\,\text{rad/s}$$

From considerations of impulse and momentum, the average impact force can be estimated from:

$$I_b \omega_b = Q_I r_q \Delta t$$

or

$$Q_i = \frac{90 \times 131}{12 \times 0.008} = 1.23 \times 10^5\,\text{lb}\ (5.47 \times 10^5\,\text{N})$$

This discussion of the impact forces illustrates that the magnitude of the force transmitted to the anvil can be significant and must be considered.

Additional dynamic forces are transmitted to the foundation by vibrating conveyors. Typical values are 13750 lb (6.12×10^4 N) for horizontal dynamic force and 11000 lb (4.89×10^4 N) for vertical dynamic force, both at 720 rpm. However, differeent conveyors will have different force and frequency outputs.

MAT FOUNDATION FOR SHREDDER

The soil at the site was loose fine sand and silt and the water table was near the surface. Thus clean fill was required and both the natural soil and the fill were compacted with surface vibratory compaction equipment.

For this installation a rigid concrete foundation mat 3.5 ft (1.067 m) thick was chosen. The mat provided the large surface contact area which was the important criterion for resisting the overturning moments. The mass of the mat was of secondary importance. General plan dimensions of the mat are shown in Fig. 3 and significant data are listed below:

Weight of mat and machinery	$= 1.70 \times 10^6$ lb (7.56×10^6 N)
Plan area	$= 1842$ ft² (171 m²)
Radius of circle with same area	$r_0 = 24.2$ ft (7.38 m)
Mass moment of inertia for rotation of foundation about line on base parallel to axis of shredder	$I_0 = 1.050 \times 10^7$ ft lb s² 1.424×10^7 mN s²)
Radius of circle having same moment of inertia as plan	$r_0 = 27.0$ ft (8.23 m) for rocking

The soil properties were influenced by the confining pressures developed by the weight of the installation plus the fill. The soil properties were:

Shear modulus	$G = 13500$ lb/in² (9.3×10^7 N/m²)
Poisson's ratio	$\nu = 1/3$
Saturated unit weight	$\gamma_{sat} = 125$ lb/ft³ (1.96×10^4 N/m³)

The shredder for this installation had an allowable unbalanced force of 56400 lb (2.51×10^5 N) rotating at 720 rpm from wear of 34 hammers. Two rows of 9 hammers each weighing 240 lb (1067 N) and two rows of 8 each weighing 144 lb (640 N) each constituted the hammer system. The average impulse forces which could act over a time interval of 0.085 s were 7.8×10^4 lb (3.46×10^5 N) for the 240 lb (1067 N) and 4.7×10^4 lb (4.70×10^4 N) for the 144 lb (640 N) hammers.

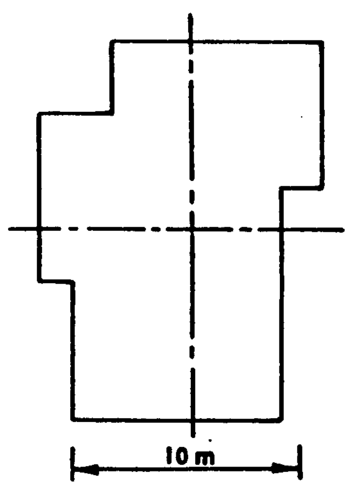

Figure 3. Plan of mat foundation

Four vibratory conveyors each operating at 360 rpm were aligned perpendicular to the shredder axis. Thus these vibratory forces could be superposed on the shredder forces, with the final resultant force depending on the phase relationship of each vibratory motion. The vertical and horizontal components of dynamic conveyor forces were:

Conveyor No.	Dead weight (lb*)	Horizontal force (lb*)	Vertical force (lb*)
1	13000	±18000	±10500
2	3300	±4850	±2800
3	3200	±4850	±2800
4	4800	±5500	±3200

* Note: No. of Newtons = No. lb × 4.448.

Vertical motion

The procedures for evaluation of dynamic motions of the foundation are described in detail in Ref. 1, thus only the basic elements are treated here. The mat foundation was treated as an effective rigid circular foundation resting at the surface of an elastic half-space.

For respone to ***vertical vibrations***, the mass ratio, B_Z, and damping ratio, D_Z, are:

$$B_Z = \frac{(1-\nu)}{4}\,\frac{W}{\gamma_{sat}(r_0)^3} = \frac{0.67 \times 1.7 \times 10^6}{4 \times 125(24.2)^3} = 0.16$$

$$D_Z = \frac{0.425}{\sqrt{B_Z}} = 1.06$$

The static vertical displacement caused by the 56400 lb (2.51×10^5 N) vertical unbalanced force was:

$$z_s = \frac{(1-\nu)\,Q_z}{4GR_0} = \frac{0.67 \times 56400}{4 \times 13500(24.2 \times 12)}$$

$$= 0.0024 \text{ in. } (6.1 \times 10^{-6} \text{ m})$$

and the dynamic magnifications factor, M_Z, amounted to 1.0. The low value of dynamic magnification factor follows from the high value of damping ratio in vertical motion and indicates that the dynamic motion is essentially the same as the static displacement. For the unbalanced force at 720 rpm, the dynamic motion amounted to $A_Z = z_s M_Z =$ 0.0024 in. (6.1×10^{-5} m). If ***all*** the vertical components of the conveyor forces were ***in phase*** they would produce a sinusoidal motion of 0.0009 in. (2.3×10^{-6} m). This sinusoidal motion would be superposed on that developed by the shredder to give the total motion. However, it is possible to arrange the phases of the conveyor motions to minimize the resultant force output.

Rigid-body rocking of the foundation

Because of the high water table at the site, it was necessary to mount the shredder on a pedestal to provide space for conveyors beneath. The centerline of the shredder was located 11.33 ft (3.45 m) above the top of the concrete mat. Thus large overturning moments were introduced by the unbalanced forces and the mat dimensions were selected to provide resistance to these forces.

The mass-ratio B_ψ for rigid body rocking, and associated damping ratio, D_ψ, are:

$$B_\psi = \frac{3(1-\nu)\,I_g g}{8\gamma_{sat}(r_0)^5} = \frac{3(0.67)\,1.05 \times 10^7 \times 32.2}{8 \times 125(27.0)^5} = 0.047$$

$$D_\psi = \frac{0.15}{(1B_\psi)\sqrt{B_\psi}} = 0.66$$

$$M \approx 1.0;\ \psi_s = \frac{3(1-\nu)\,M_\psi}{8Gr_0^3} = \frac{3 \times 0.67 \times 56400 \times 14.83}{8 \times 13500 \times 144(27)^3}$$

$$= 5.5 \times 10^{-6} \text{ rad}$$

Then:

$$A_\psi = M_\psi \psi_s = 5.5 \times 10^{-6} \text{ rad}$$

or

$$A_x = 14.83(12)\,A_\psi = 0.00098 \text{ in. } (2.5 \times 10^{-6} \text{ m})$$

Thus, the amplitude of horizontal motion at the shredder centerline amounts to about 0.001 in. (2.5×10^{-6} m) at the operating speed of 720 rpm.

These two calculations show that the mat develops high values of geometrical damping in vertical and rocking motions. The impulsive loads produced smaller motions than those calculated above, and the solution by the phase-plane method (see Ref. 1) will not be treated here.

BLOCK FOUNDATION FOR SHREDDER

The site for this shredder installation had competent stiff clay (G = 21000 lb/in^2) (1.45×10^8 N/m^2), $\nu = 0.4$, γ_{sat} = 125 lb/ft^3 (1.96×10^4 N/m^3) at a depth of 21.5 ft (6.55 m) below grade. Between the thin surface crust and the stiff clay was a soft alluvium with an effective value of G of 1/20 of that for the stiff clay.

One economically feasible solution for this foundation was to use a mass concrete block foundation as shown in Fig. 4. The response of this embedded block to the imposed dynamic loads was studied using Novak's analysis for embedded foundation.[2]

This shredder had two rows of 8 and two rows of 9 hammers, each weighing 450 lb (2002 N). The unbalanced radial force had an amplitude of 112500 lb (5.0×10^5 N) at 600 rpm. Impact forces developed as each hammer was temporarily stopped were as noted in the previous section on dynamic forces.

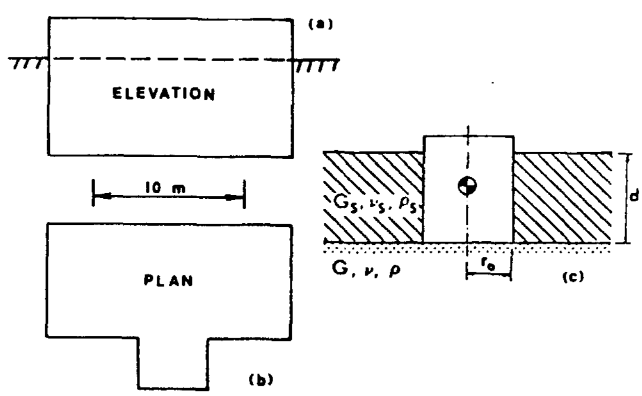

Figure 4. Block foundation, (a) elevation, (b) plan, (c) equivalent cylindrical foundation

Figure 4(c) shows the equivalent cylindrical foundation of radius, r_0, which is embedded a depth, d, into soil having a shear modulus, G_s, and Poisson's ratio, ν_s. Beneath the base of the foundation the soil had the properties G and ν. For this equivalent foundation, the values of r_0, d and h_0 were:

$$d = 21.5\,\text{ft}\ (6.55\,\text{m});\ r_0 = 22\,\text{ft}\ (6.71\,\text{m});$$

$$h_0 = 37\,\text{ft}\ (11.28\,\text{m})$$

and the dead weight, W, and the mass moment of inertia, I_g, about the C.G. were:

$$W = 6.40 \times 10^6\,\text{lb}\ (2.85 \times 10^7\,\text{N})$$

and

$$I_g = 6.26 \times 10^8\,\text{in lb s}^2\ (7.07 \times 10^7\,\text{mN s}^2)$$

Vertical vibrations

Novak's expressions for the spring and damping factors in vertical vibration include the influence of the soil acting along the sides of the foundation as well as that below the base. Thus:

$$k_{zz} = 4Gr_0/(1-\nu) + 2.7G_s d$$

$$c_{zz} = 3.4r_0^2\sqrt{\rho G/(1-\nu)}$$

Following through the calculations for damping ratio:

$$D = c_{zz}/2\sqrt{K_{zz}m};\ M = 1/(2D\sqrt{1-D^2}),\ z_s = Q_z/k_{zz}$$

leads to:

$$A_z = Mz_s = 0.0031\,\text{in.}\ (7.9 \times 10^{-6}\,\text{m})$$

Rocking and horizontal vibrations

The horizontal force Q_x was applied above the center of gravity of the foundation and above the center of soil resistance. Consequently, coupled rocking and horizontal motions were developed. The motion depends upon the resonant frequencies and damping of the soil-foundation system and the frequency of the exciting force and moment. The calculations follow the procedure clearly described in Ref. 2 and are not included here. The resulting vibrations can be described by a horizontal translation of the center of gravity of amplitude, u_g, and a rotation about the center of gravity, ψ_g. At the frequency of 600 rpm:

$$u_g = 0.003\,\text{in.}\ (7.6 \times 10^{-5}\,\text{m})$$

and

$$\psi_g = 5.6 \times 10^{-6}\,\text{rad}$$

Thus this particular block foundation was considered to perform satisfactorily.

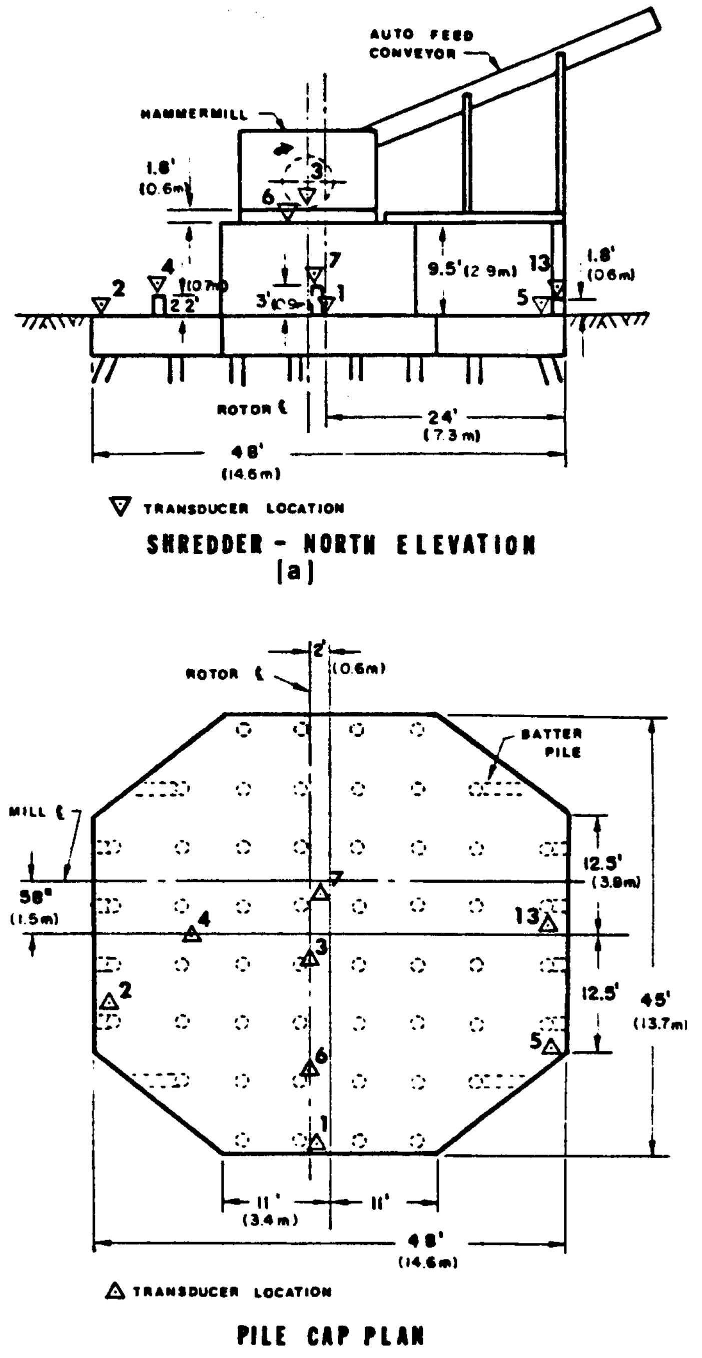

Figure 5. Pile-supported shredder, (a) north elevation, (b) pile cap plan

PILE-SUPPORTED FOUNDATION FOR SHREDDER

For this installation the soil at the site consisted of an upper crust of competent material then a zone of soft cohesive soil overlying a bed of firm sand. Thus piles were required to bypass the soft zone and to transmit the static and dynamic loads to the sand.

The shredder used at this site had the same dynamic force outputs as those described for the mat foundation. This shredder was mounted on a pile cap and pedestal system as shown in Fig. 5, with the centerline of the shredder at a distance of 13.81 ft (4.21 m) above the top of the pile cap. Thus, operation of the shredder developed steady state forces at 720 rpm which caused vertical, horizontal, and rocking motions of the foundation, and developed impact forces at a frequency of four times the operating speed.

Figure 5(a) shows the final pile pattern selected following several cycles of analysis and modification of the geometry. The key resisting members of this system were the 52 concrete-filled 12 in. (0.30 m) OD pipe piles, and secondary restraint was provided by clean, cohesionless soil compacted to a dense condition against the vertical faces of the pile cap. Pile loading tests were run in the field on a pile which had been driven to capacity then redriven to minimize the effects of subsequent dynamic loads. Repeated cyclic loading about the static load provided values for the vertical stiffness of the individual piles. From these cyclic loading tests, the vertical static stiffness of a single pile 50 ft (15.24 m) long was found to be $k_p = 2.5\times10^6$ lb/in. (4.38×10^8 N/m). This value was used for each pile when calculating the combined effects of all piles for the vertical and rocking modes of vibration.

Vertical vibrations

The dead weight of the components of the foundation system was 1.0×10^6 lb (4.5×10^6 N) for the pile cap, 3.5×10^6 lb (15.57×10^6 N) for the pedestals, and 2.25×10^5 lb (10.0×10^5 N) for the shredder and motor. The total mass to participate in translational vibrations was $m = 4080$ lb s²/in (7.14×10^5 N s²/m). Then, using the vertical stiffness of the 52 piles, and the steady-state unbalanced force, the static vertical deflection was:

$$z_s = Q_z/k_z = 56400/52\times2.5\times10^6\times0.65$$
$$= 0.00067\text{ in. } (1.7\times10^{-5}\text{ m})$$

with an undamped natural frequency of $f_n = 28.4$ Hz. Thus the magnification factor, M, was about 1.2, and the dynamic motion amounted to 0.0008 in. (2.0×10^{-5} m). In the equation above for Z_s, the number 0.65 represents the pile group effect.

For average values of *impact* loads of $Q_I = 1.0\times10^{-5}$ lb (4.45×10^5 N) acting over a time interval of 0.008 s, the phase-plane procedure[1] gave an estimated vertical motion of 0.001 in. (2.54×10^{-5} m) at the frequency of 2800 rpm. This force-time pattern corresponded to the action of one hammer. For the phase-plane solution, a damping factor of $c = 6000$ lb-s/in. (1.05×10^6 N s/m) per pile was established from the PILAY program.[3]

The soil adjacent to the vertical face of the foundation was not considered to add to the stiffness or damping of the system because of the underlying soft soil.

Rocking vibration

Because of the horizontal unbalanced force applied at a distance of 17.3 ft (5.27 m) above the base of the pile cap, the overturning moment was $T_\psi = 9.76\times10^5$ ft lb (1.32×10^6 mN).

The mass moment of inertia of the foundation system in rocking about the centerline of the base was $I_\psi = 8.5\times10^6$ ft lb s² (11.52 mN s²) and the resisting spring constant provided by vertical deformation of the 52 piles as the foundation rotated was $k_{\psi p} = 2.88\times10^{12}$ in lb/rad (3.25×10^{11} mN/rad). Thus, the static rotation was $\psi_s = T_\psi/k_{\psi p} = 4.0\times10^{-6}$ rad. This rotation contributes a horizontal motion of $x_s = 0.001$ in. (2.54×10^{-5} m) at the centerline of the shredder. The natural frequency in rocking about this base centerline was $f_n = 26.7$ Hz. Then the dynamic magnification factor was 1.25, even for the undamped case, and the horizontal dynamic motion at the shredder centerline would be $A_x = 0.00125$ in. (3.2×10^{-5} m).

MEASUREMENTS ON PILE-SUPPORTED SHREDDER

Opportunities for comparing performance of constructed facilities with predicted performance are rare. However, the real test of any analytical technique lies in how well measurements match prediction. The pile-supported shredder described above was the subject of such a comparison. Vibration measurements were made while the shredder was idling and while it shredded cars.

Instrumentation

Velocity transducers and a strip chart recorder were used to make the vibration measurements. The velocity transducers were Electro-Tech, 4.5 Hz units, two of which detected vertical motions and one detected horizontal motions. The strip chart recorder was a Hewlett-Packard Model 320, dual channel, hot-pen writing, amplifier recorder.

Measurements

The locations at which measurements were made are shown in Figs. 5(a) and (b). By recording two transducers simultaneously, it was possible to compare phases and determine the mode of motion as well as amplitude. Figure 6(a) shows the vertical motion at opposite ends of the pile cap while no cars were being shredded (idling condition) and shows that these two points were 180° out-of-phase. The lower trace in Fig. 6(a) at location 2 shows a different signature because that location was near a support for a vibrating conveyor which was operating at all times.

Figure 6(b) shows vertical (upper) and horizontal (lower) motion at location 3 near the axis of the shredder when cars were being shredded. This record shows that the two directions of motion occurred at different frequencies. The horizontal motion was at the rotational speed of the machine (12 Hz) while the vertical motion was at four times that speed.

Discussion

From these vibration measurements, it was concluded that when the shredder was running at idle, the predominant motion was rocking at a frequency of about 12 Hz. The maximum vertical displacement was about 0.00165 in.

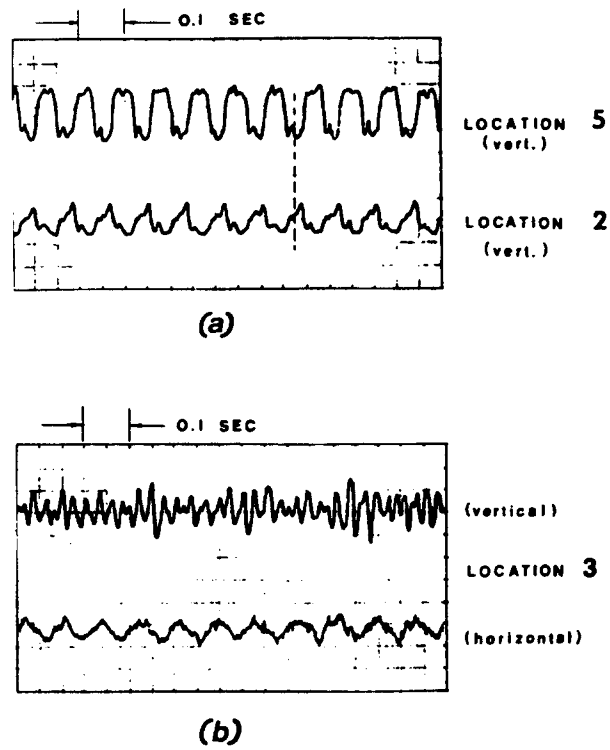

Figure 6. Velocity-time traces, (a) vertical motions from locations 2 and 5, (b) vertical and horizontal motions from location 3

(4.2×10^{-5} m) peak and the maximum horizontal rocking displacement was 0.0019 in. (4.8×10^{-5} m) peak. The axis for rocking was estimated to be about 15 ft (4.57 m) below the pile cap.

When cars were being shredded, the mode of motion at maximum amplitude was vertical translation. This vibration occurred at about 48 Hz or four times the primary frequency of the shredder. This frequency represents the rate at which hammers shear through metal at the anvil. The maximum vertical displacement in this mode was 0.0026 in. (6.6×10^{-5} m) peak.

CONCLUSION

Analytical results for a mat, block, and pile-supported foundation system have been described by examples. The most important factor is identifying the maximum loads and associated frequencies which act as excitation. Then translational, and coupled rocking and horizontal modes of vibration must be studied. The analytical procedure to be used depends on the geometry of the system, and elastic solutions are acceptable because of the small strains developed in the soil. A critical parameter in the analysis is the shear modulus of the soil, which should be established by *in situ* measurements if possible.

Measurements were made on the pile-supported foundation during idling and during shredding. During idling the unbalanced force at about 12 Hz produced vertical, and rocking and horizontal motions. At location 2 at the edge of the pile cap, the maximum vertical displacement was 0.0017 in. (4.3×10^{-5} m) during idling, and for the same condition, the maximum horizontal displacement at location 3 was 0.0019 in. (4.83×10^{-5} m). The predicted vertical motion was 0.0022 in. (5.58×10^{-5} m) from the combined effects of vertical and rocking motions developed by the steady-state unbalanced force (idling).

The most significant finding from the field measurements was the vertical motion of 0.0026 in. (6.6×10^{-5} m) at a frequency of about 48 Hz, or four times the operating speed. Based on data supplied by the shredder manufacturer, the expected mode should have been rocking at about 12 Hz. Because the vertical motion caused by the impact loads was about 2.6 times greater than that estimated in the example, methods of measuring or estimating the impact forces need to be improved.

REFERENCES

1 Richart, F. E., Jr, Hall, J. R. Jr and Woods, R. D. *Vibrations of Soils and Foundations*, Prentice-Hall, Inc., Englewood Cliffs, NJ, 1970

2 Novak, M. 'Effect of soil on structural response to wind and earthquake', *Int. J. Earthquake Engg. and Struc. Dyn.*, 1974, 3 (1), 79

3 PILAY, Computer Code for Pile Vibrations in Layered Soils. By M. Novak and F. Aboul-Ella, SACDA, University of Western Ontario, London, Ontario, N6A 5B9, Canada

4 Kuhlemeyer, R. L. 'Static and dynamic laterally loaded piles', Res. Rep. CE76-9, University of Calgary, Canada, 65 pp., March, 1976