

The Center and Range of the Probability Interval as Factors Affecting Ambiguity Preferences

SHAWN P. CURLEY AND J. FRANK YATES

University of Michigan

Ambiguous decision situations are characterized as having probabilities that are uncertain. The uncertainty is due to the common, real-world deficiency of information about the process by which the outcomes are determined. Thirty lotteries having uncertain probabilities were constructed by varying the centers and the ranges of the intervals within which the imprecise probabilities of winning could lie. Pairs of the lotteries were presented as choice alternatives to subjects, with each pair having lotteries with the same interval center but differing interval ranges. Ambiguity avoidance, the selection of the less ambiguous option, was found to increase with the interval center C , with ambiguity indifference occurring for values of $C \leq 0.40$. No evidence of ambiguity seeking as the prevalent behavior was obtained. Ambiguity avoidance did not significantly increase with the interval range R , but an interaction effect between C and the ranges R_1 and R_2 of the choice pair was obtained. This effect of the ranges could not be described simply by knowledge of the difference $R_1 - R_2$; knowledge of both individual values was necessary. The theoretical implications of these results are discussed. © 1985 Academic Press, Inc.

Decisions in the natural environment often involve less than perfect information about the processes by which outcomes may be determined. For example, suppose a physician is choosing between two potential treatments for a diagnosed disease. One treatment has been used extensively, so that the physician has good information about its success rate, which is well known to be 30%. Alternatively, a new treatment is available about which the physician has little information. Should the physician express indifference between the treatments if the best guess as to the success rate of the latter, more uncertain treatment is also 30%? How should the physician behave if the latter treatment has a best-guess success rate of 35%?

Many situations, as in this medical example, involve options which differ in the decision maker's uncertainty about the available information. These situations share a common structure which can be abstracted and formalized as in Fig. 1. Each option is simulated by a gamble G having a probability P of a favorable outcome, shown as the positive amount

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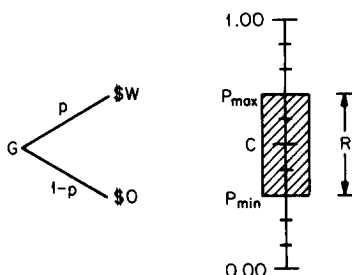


FIG. 1. Common lottery structure G and the notation used to describe the features of the lottery.

$\$W$, and the complementary probability of a less favorable outcome, shown as no payoff. As illustrated, it is only known that P lies within the interval $[P_{\min}, P_{\max}]$, which is centered at $C = (P_{\min} + P_{\max})/2$ and has range $R = (P_{\max} - P_{\min})$. All of the gambles used in the present study are operationalized using this form.

Inherent in this formulation are two distinct types of uncertainty: uncertainty as to which outcome will occur, and uncertainty as to the likelihood of the outcomes. Although distinguishable types of uncertainty have been the subject of recent speculation (Howell & Burnett, 1978; Kahneman & Tversky, 1982), most decision models and empirical studies in the literature continue to assume that probability theory provides an adequate characterization of all the decision maker's uncertainty. Knight (1964) and Ellsberg (1961) questioned this assumption in certain situations. Whereas one's uncertainty about decision outcomes is captured by probabilities, the uncertainty of "ambiguity," using Ellsberg's terminology, is not. Ambiguity is defined as uncertainty about the processes by which outcomes are determined, and has been characterized as uncertainty about the outcome probabilities themselves.

For clarification, Fig. 2 illustrates probability intervals for several lotteries, differing in center C and range R . The first two intervals were among those used by Ellsberg (1961), and are both centered at $C = 0.50$ (Fig. 2a, 2b). Since ambiguity presumably increases as the range R increases (Becker & Brownson, 1964), these lotteries differ in ambiguity. The lottery represented in Fig. 2a might, for instance, correspond to tossing a fair coin. This is a well-known process which has no ambiguity. In contrast, the lottery represented in Fig. 2b most closely corresponds to a state of "ignorance," which is probably rare in actual natural situations (Coombs, Dawes, & Tversky, 1970). However, this lottery, having the maximum possible range of $R = 1.00$, implies the maximal ambiguity possible for gambles of this type. As another example, the lottery in Fig. 2c is centered at $C = 0.30$, and might represent the second, newer treatment in the clinical decision example which opened this paper. With an

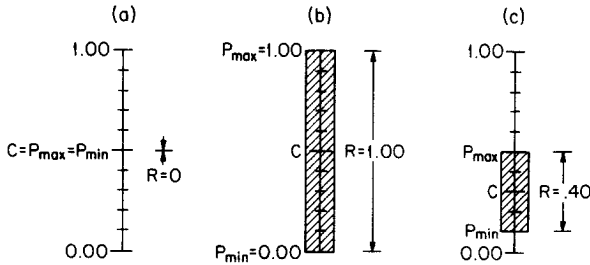


FIG. 2. Examples of lotteries with differing centers (C) and ranges (R).

intermediate range of $R = 0.40$, there is the supposition of an intermediate amount of ambiguity for this gamble.

Ellsberg (1961) suggested that many people, given a choice, would tend to select the option having lower ambiguity, exhibiting *ambiguity avoidance*. This conjecture has been supported empirically in a variety of contexts: choices among lotteries (Becker & Brownson, 1964; Yates & Zuckowski, 1976), foreign investment choices (MacCrimmon, 1968), clinical treatment choices (Curley, Eraker, & Yates, in press; Hamm & Bursztajn, 1979), almanac-type questions (Goldsmith & Sahlin, 1981), and judgments of likelihood based on inference (Einhorn & Hogarth, in press). The strength of the phenomenon has been evidenced by subjects' willingness to pay to avoid ambiguity (Becker & Brownson, 1964), and by the reluctance of subjects to change their behavior after the presentation of prescriptive counterarguments (MacCrimmon, 1968; MacCrimmon & Larsson, 1979; Slovic & Tversky, 1974).

Consider two gambles G_1 and G_2 , constructed as in Fig. 1 such that they share a common center C , but have different ranges R_1 and R_2 . As a convention, G_1 is designated to have the larger interval range, so that $R_1 - R_2 > 0$. Having the greater range, G_1 presumably is the more ambiguous lottery of the pair. The present study focused on the effects of the three parameters which characterize these gambles— C , R_1 , and R_2 —upon individuals' behavior when given a choice between G_1 and G_2 . In particular, three hypotheses were tested: (a) whether ambiguity avoidance increases as C increases; (b) whether, as C decreases, there is some value of C beyond which ambiguity avoidance will cease to predominate and, also, whether ambiguity seeking might obtain for even smaller values of C ; and (c) whether the levels of R_1 and R_2 affect ambiguity preferences and, if so, whether knowledge of the difference $R_1 - R_2$ is sufficient to describe any effects. We first turn to the claims of existing models regarding these proposals.

Several models would predict none of the stated effects. Among these are expected utility theory (von Neumann & Morgenstern, 1953; Luce & Raiffa, 1957) and subjective expected utility theory (Savage, 1972). Since

Ellsberg (1961) proved that persons' nonindifference to ambiguity is not captured within models which use probabilities as the sole measure of uncertainty, these models would predict ambiguity indifference in all situations. Also not predictive of reactions to ambiguity, without some further assumptions, are models in the spirit of Karmarkar's (1978), in which decision weights are a function of the probability P . In contrast, prospect theory (Kahneman & Tversky, 1979), which also uses weights which are a function of P , does make a further assumption which allows for ambiguity avoidance. Specifically, it is claimed that the presence of ambiguity depresses the weight function, presumably with the amount of depression being proportional to the extent of ambiguity. Although prospect theory is not specific in its predictions, at the least, no effect of C upon ambiguity preferences is indicated.

More relevant to the issues regarding ambiguity preferences and their relation to the gamble parameters C , R_1 , and R_2 are various theories which have been proposed specifically to account for subjects' behavior in ambiguous situations. The first is that of Ellsberg (1961), which proposes a two-stage gamble evaluation process. Applied to gambles of the form sketched in Fig. 1, the first stage consists of reducing the range R of possible probability values to a subset of P values which seem "reasonable," and are not ruled out. Among these is $P_{\min'}$, which is the lowest value in this subset, and is such that $P_{\min} \leq P_{\min'} \leq C$. This value depends both on the amount of ambiguity in the situation as well as on the individual's attitude toward ambiguity. In Ellsberg's second stage, the subject behaves as if maximizing a convex linear combination of the center C and the minimum reasonable probability $P_{\min'}$. Specifically, let

$$S = \rho C + (1 - \rho)P_{\min'}, \quad (1)$$

where ρ is a measure of the individual's "degree of confidence" in the center C as reflective of the perceived likelihood of the favorable outcome. The model proposes that an individual chooses G_2 over G_1 if $S_2 > S_1$. In the present task, where both gambles share a common center C and $R_1 > R_2$, Equation (1) implies that an individual chooses G_2 over G_1 when:

$$\rho_2 C + (1 - \rho_2)P_{\min', 2} > \rho_1 C + (1 - \rho_1)P_{\min', 1}. \quad (2)$$

Note that the range R , which is the only parameter which distinguishes G_1 and G_2 , affects the choice both through ρ and $P_{\min'}$. In particular, since G_1 has the greater range, $\rho_1 \leq \rho_2$ and $P_{\min', 1} \leq P_{\min', 2} \leq C$ by assumption. These inequalities clearly preclude ambiguity seeking under any conditions. In fact, it is straightforward to show that the model predicts ambiguity indifference only when $\rho_1 = \rho_2 = 1.0$, or when $P_{\min', 1} = C$ while $\rho_2 = 1.0$.

Regarding the other hypotheses of interest, there is no reason to expect, from this model, any C effects. The model does predict range effects in that the extent of ambiguity is influenced by the parameter R ; but not predicted is that $R_1 - R_2$ is sufficient in explaining these effects. It should also be noted that, although the model allows neither the possibility of ambiguity seeking at small values of C , nor effects of varying the parameter C , Ellsberg in a later note (Becker & Brownson, 1964, footnote 4) did indicate a belief that both of these claims would obtain.

Related to Ellsberg's model are two models which allow for ambiguity seeking through the inclusion of an additional parameter which differentially weights values of P in the interval $[P_{\min}, P_{\max}]$ that are less than C as compared to values of P in this interval that are greater than C (Einhorn & Hogarth, in press; Toda & Shuford, 1965). Although Einhorn and Hogarth's model concerns probability judgments, rather than choices, and Toda and Shuford's model considers only the case of $C = 0.50$, the models appear to predict that range effects would obtain in a manner similar to the Ellsberg (1961) model. Less clearly indicated by such a differential weighting process are effects of the level of C , including the possibility of ambiguity seeking at low values of C . Einhorn and Hogarth do claim that the differential weighting parameter varies as C varies, thus allowing for these hypothesized effects.

Another model which resembles Ellsberg's (1961) model is that of Gardenfors and Sahlin (1982, 1983). Their model, in the present situation, is essentially a special case of Ellsberg's model, in which $\rho = 0$ in Equation (1). Thus, the choice model in Equation (2) reduces to choosing G_2 over G_1 if $P_{\min', 2} > P_{\min', 1}$. Being a special case, the predictions of this model for the three hypotheses of this study are similar to those of the Ellsberg model. One difference is their specific conceptualization of those probabilities included in the "reasonable" subset of $[P_{\min}, P_{\max}]$ leading to the specification of $P_{\min'}$. They propose a "wave effect" such that $P_{\min'}$ tends toward C as C decreases. As such, their model predicts that ambiguity avoidance would decrease as C decreases. However, since the probability interval for G_2 is a subset of that for G_1 , it is always the case that $P_{\min', 1} \leq P_{\min', 2}$. Thus, like Ellsberg's model, the Gardenfors and Sahlin model would not predict ambiguity seeking under any conditions in the present type of situation.

The final model to be considered is that of Becker and Brownson (1964). They proposed that the extent of ambiguity, and ambiguity avoidance, is a linear function of the difference $R_1 - R_2$. Becker and Brownson found support for this, our third hypothesis, holding C constant at 0.50. No effect of varying C is claimed in this model. The present study sought to replicate their finding at $C = 0.50$, and to test its generalizability to other values of C .

In this regard, the positive correlation between $R_1 - R_2$ and ambiguity preferences has seemingly been contradicted by data reported by Larson (1980) and Yates and Zukowski (1976). However, these latter two studies adopted an operational definition of an ambiguous probability as one described by a well-specified second-order probability distribution. Such a distribution need not, and in these studies did not, involve uncertainty about the process of outcome determination. Whether second-order distributions could profitably be used in decision theory is the subject of considerable debate (Borch, 1975; De Finetti, 1977; Marschak *et al.*, 1975), but this debate is distinct from the present line of research. Being well specified, these distributions do not involve uncertainty about the process by which outcomes are determined, and, as such, lack ambiguity.

METHOD

Subjects

Eighty undergraduates at the University of Michigan participated in fulfillment of a requirement of several introductory psychology courses. Subjects worked individually through a response booklet after the procedure was described to them.

Materials

Instructions. A pamphlet described the study for the subjects. It was explained that the experiment involved pairs of lotteries, each having two possible outcomes—to win \$5 or nothing; and that preference was to be indicated for one of the lotteries in each pair. Also described was the procedure by which the subject might actually play one of the selected lotteries. This was an incentive for the subject to think carefully about the choices, more so than if they were completely hypothetical.

The common procedure by which all the lotteries would be played involved six steps, which were explained in the instruction booklet. Lottery *L* on the left side of Figure 3 was used to exemplify these steps:

1. The subject receives a bag of unknown contents, except as specified under "Initial Bag."
2. The subject selects red or white as the winning chip color.
3. The subject adds the number of chips designated in the "You Add" section of the display, bringing the total number of chips in the bag to 100, as displayed in the "Final Bag" section, which summarizes the lottery.
4. The subject draws a chip without looking.
5. If a winning chip is drawn, the subject receives \$5; if otherwise, nothing is received.
6. The subject may check that the bag contains what it is said to contain.

SAMPLE

<u>Lottery L</u>	<u>Lottery R</u>
Initial Bag: $\frac{0-20 \text{ WHITE chips}}{0-20 \text{ RED chips}}$ $\frac{\quad}{20}$ Total chips	Initial Bag: Empty
You Add: $\frac{45 \text{ WINNING chips}}{35 \text{ LOSING chips}}$	You Add: $\frac{25 \text{ WINNING chips}}{75 \text{ LOSING chips}}$
Final Bag: $\frac{45-65 \text{ WINNING chips}}{35-55 \text{ LOSING chips}}$ $\frac{\quad}{100}$ Total chips	Final Bag: $\frac{25 \text{ WINNING chips}}{75 \text{ LOSING chips}}$ $\frac{\quad}{100}$ Total chips
<div style="border: 1px solid black; padding: 5px; width: 60px; margin: 0 auto;">WIN</div> <div style="border: 1px solid black; padding: 5px; width: 60px; margin: 0 auto;">?</div> <div style="border: 1px solid black; padding: 5px; width: 60px; margin: 0 auto;">LOSE</div>	<div style="border: 1px solid black; padding: 5px; width: 60px; margin: 0 auto;">WIN</div> <div style="border: 1px solid black; padding: 5px; width: 60px; margin: 0 auto;">LOSE</div>
CHOICE: Lottery L _____	Lottery R _____
RATING: Lottery L - - - - - - - - Lottery R	

FIG. 3. Sample response sheet.

Finally, the instructions explained the responses that would be required.

Response booklet. Each page in the response booklet displayed a pair of lotteries and a response area. A sample sheet is shown in Fig. 3. Two responses were required for each pair: a forced choice of one lottery or the other, and a "strength of preference" rating, which was translated into a 21-point scale, with -10 corresponding to a strong preference for the higher range lottery and +10 corresponding to the complementary preference for the lower range lottery of the pair.

Lottery pairs. There were 30 lotteries used in the study, all of the form illustrated in Fig. 1. These lotteries shared common outcomes, with $\$W = \5 , but they differed in their values for the center C and the range R of the probability interval. The interval centers were varied from $C = 0.10$ to $C = 0.90$; the ranges of these intervals were also varied to span the domain of possible values, from $R = 0.00$ to $R = 1.00$. The specific lotteries employed are displayed in Fig. 4, which describes the number of winning chips, out of 100 total chips, which comprised each of the lotteries. Each lottery in this matrix format is characterized by its center C and its range R . The 30 lottery pairs presented to the subjects are denoted by connecting arcs in the matrix. Five of these pairs, those connected by double arcs, were presented twice to each subject.

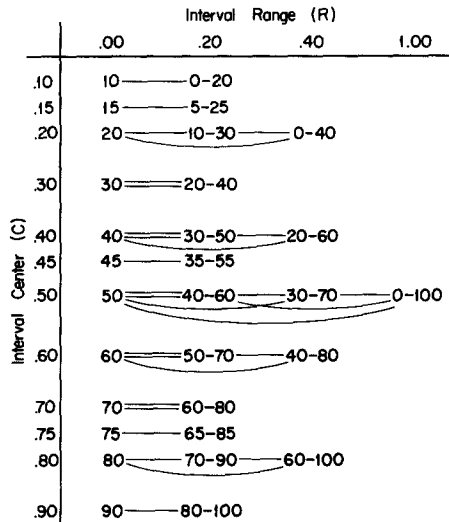


FIG. 4. The number of winning chips, out of 100 total chips, for each of the thirty lotteries used in the study. Each lottery is characterized by its interval range R , the columns of the display, and its interval center C , the rows of the display. Arcs connect the 30 lottery pairs included in the study.

Procedure

Each subject responded to all 30 lottery pairs in Fig. 4. The order of the pairs in the response booklets was varied randomly under the constraint that pairs presented twice were separated from each other by at least three other pairs. The left-right placement of the lotteries within each pair was also varied to make two forms of the response booklet. For the replicated pairs, the two lotteries were left-right reversed within each response booklet; for the nonreplicated pairs, the two lotteries were left-right reversed between the two forms, so that the higher range lottery of the pair was on the left side of the response sheet for exactly 15 of the 30 pairs.

Each subject was presented with a copy of the instructions, which the subject read and kept for reference throughout the experiment. In the instruction booklet, and verbally, it was emphasized (a) that the fairness of each gamble was guaranteed by the procedure, and (b) that, at the end of the experiment, the subject would have a chance to play one of the lotteries and possibly win \$5. As a final check, the subject completed the sample response sheet displayed in Fig. 3. Note that this is not one of the lottery pairs included in the response booklet. The subject then completed the response booklet with the instruction to proceed through it from beginning to end without going back to check previous responses.

Upon completion of the booklet, feedback was provided and a coin

toss was called by the subject. If the call was successful, one of the response pages was randomly selected, and the lottery chosen by the subject on that page was played according to the lottery procedure, with the subject either winning \$5 or nothing.

RESULTS

No systematic differences were found between males and females or between the two forms of response booklet used. Three subjects gave at least one strength of preference rating that was inconsistent with their choice: one subject had 28 out of 30 inconsistencies, one had 11 inconsistencies, and one had 4 inconsistencies. These subjects were excluded from the analyses, leaving the sample size at $N = 77$. The consistency of the subjects was believed acceptable; based on the five pairs with replications, the questionnaire had a Spearman-Brown reliability coefficient of .92.

Effect of the Interval Center

As illustrated in Fig. 4, the interval center C was varied for three fixed levels of lottery pair ranges R_1 and R_2 : $R_1 = 0.20$ versus $R_2 = 0.00$; $R_1 = 0.40$ versus $R_2 = 0.00$; and $R_1 = 0.40$ versus $R_2 = 0.20$. Figure 5 shows the mean strength of preference ratings as a function of C for each of the three combinations of R_1 and R_2 . Figure 6 contains analogous graphs, with mean rating being replaced by the percentage of subjects choosing the less ambiguous lottery of the pair, that having range R_2 . As shown, similar results obtained with both aggregate measures of ambiguity preference.

A one-way analysis of variance (ANOVA) and trend analyses were applied to each of the three sets of strength of preference ratings in Fig. 5. For $R_1 = 0.20$ versus $R_2 = 0.00$, there was a significant effect for C on mean rating [$F(11, 825) = 9.378, p < .001$]; trend analyses indicated significant first-order linear [$F(1, 825) = 90.499, p < .001$] and fifth-order [$F(1, 825) = 4.332, p < .05$] effects. For $R_1 = 0.40$ versus $R_2 = 0.00$, there was also a significant effect for C [$F(4, 300) = 8.223, p < .001$], but only a significant linear trend [$F(1, 300) = 29.517, p < .001$]. For $R_1 = 0.40$ versus $R_2 = 0.20$, the effect for C was not significant [$F(4, 304) = 1.808, p > .10$].

These results support the first hypothesis, that ambiguity avoidance increases with C , for range comparisons of $R_1 = 0.20$ versus $R_2 = 0.00$ and $R_1 = 0.40$ versus $R_2 = 0.00$. This relationship did not obtain for $R_1 = 0.40$ versus $R_2 = 0.20$, a result to which we return in the next section.

The second hypothesis concerned the existence of ambiguity indifference and ambiguity seeking behaviors for low values of C . A region of ambiguity indifference was identified, in that the null hypothesis of no

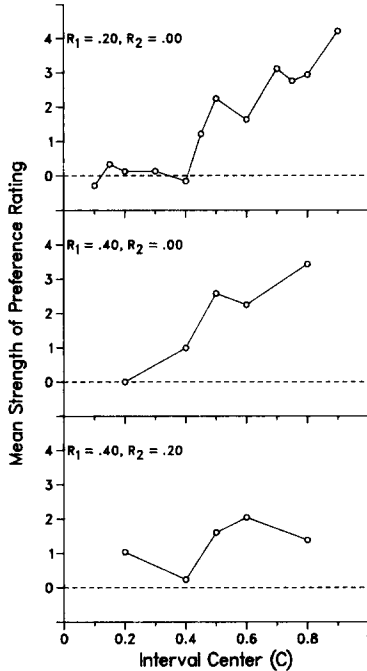


FIG. 5. Mean strength of preference ratings as a function of the interval center C for three combinations of range comparison: $R_1 = 0.20$ versus $R_2 = 0.00$; $R_1 = 0.40$ versus $R_2 = 0.00$; and $R_1 = 0.40$ versus $R_2 = 0.20$. The dashed line indicates the point of ambiguity indifference.

reaction to ambiguity was not rejected. For both the pairs with $R_1 = 0.20$ versus $R_2 = 0.00$ and with $R_1 = 0.40$ versus $R_2 = 0.00$, ambiguity indifference was accepted for values of $C \leq 0.40$ [two-tailed t , $p > .05$]. For values of $C \geq 0.45$, ambiguity avoidance predominated [$p < .01$]. This same pattern obtained for pairs with $R_1 = 0.40$ versus $R_2 = 0.20$, but recall that the C effect in this case was not significant. For no values of C , even at the smallest value of $C = 0.10$, was ambiguity seeking the significantly prevalent behavior.

Effect of the Interval Range

The range of the probability interval was varied from $R = 0.00$ to $R = 1.00$ at a fixed value of $C = 0.50$. The mean strength of preference ratings and percentages of subjects choosing the less ambiguous lottery, that having range R_2 , are indicated in Table 1 for all the ranges R_1 and R_2 used in the study at $C = 0.50$. A one-way ANOVA applied to the ratings did not indicate a significant difference in these mean values [$F(5, 380) = 1.199$, $p > .25$]. Still, the extent of ambiguity avoidance was sizeable.

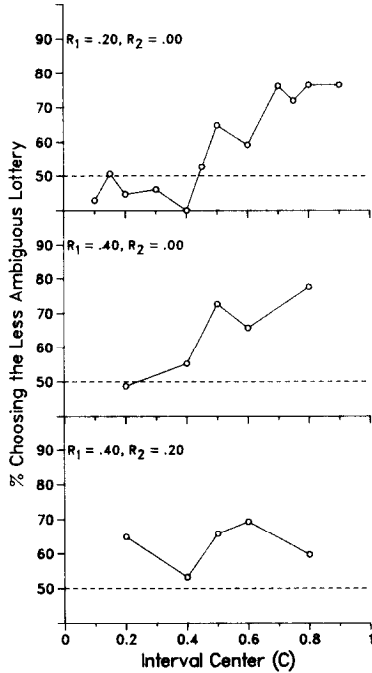


FIG. 6. Percentages of subjects choosing the less ambiguous lottery (R_2) in each pair as a function of the interval center C for three combinations of range comparison: $R_1 = 0.20$ versus $R_2 = 0.00$; $R_1 = 0.40$ versus $R_2 = 0.00$; and $R_1 = 0.40$ versus $R_2 = 0.20$. The dashed line indicates the point of ambiguity indifference.

Even the lowest mean rating, that for $R_1 = 0.40$ versus $R_2 = 0.20$, significantly differed from 0 [two-tailed t , $p < .01$]. Similar conclusions were indicated by analyses of the choice frequencies.

Our reanalysis of the data reported by Becker and Brownson (1964) did reveal a significant effect of varying the ranges R_1 and R_2 [$F(9, 126) = 11.164$, $p < .001$]. This range interval effect in their data was not replicated in the present study. The pattern of their data, however, is consistent with that obtained in the present study, shown in Table 1. Namely, the extent of ambiguity avoidance monotonically increases both as R_1 increases, holding R_2 constant, and as R_2 decreases, holding R_1 constant.

Of additional interest is the difference in subjects' responses between situations with $R_1 = 0.20$ versus $R_2 = 0.00$ and those with $R_1 = 0.40$ versus $R_2 = 0.20$ (see Figs. 5 and 6). Becker and Brownson (1964) proposed that the extent of ambiguity avoidance, under the present conditions, is an approximately linear function of the difference $R_1 - R_2$. The present data, in which both situations share the value $R_1 - R_2 = 0.20$,

TABLE 1
 MEAN STRENGTH OF PREFERENCE RATINGS AND CHOICE PERCENTAGES FOR LOTTERIES
 CENTERED AT $C = 0.50$ WITH RANGES R_1 AND R_2

R_2	R_1		
	0.20	0.40	1.00
	Mean strength of preference ratings ^a		
0.00	2.21	2.57	2.87
0.20	—	1.60	2.81
0.40		—	2.04
1.00			—
	% Choosing the less ambiguous lottery		
0.00	64.9	72.7	76.3
0.20	—	65.8	73.7
0.40		—	70.1
1.00			—

^a High ratings favor the less ambiguous lottery, that with range R_2 .

cast doubt upon this hypothesis. In particular, an interaction between the center C and the ranges R_1 and R_2 is indicated. This was verified with a two-way ANOVA applied to the ratings data for the 3×5 factorial design embedded in the complete set of lottery pairs shown in Fig. 4. The pairs in the design are those having centers of $C = 0.20, 0.40, 0.50, 0.60, 0.80$ and having paired ranges of $R_1 = 0.20$ versus $R_2 = 0.00$; $R_1 = 0.40$ versus $R_2 = 0.00$; and $R_1 = 0.40$ versus $R_2 = 0.20$. The mean responses to these pairs form a subset of the points shown in Fig. 5. The analysis indicated a significant main effect for the interval center C [$F(4, 304) = 11.48, p < .001$] and a significant interaction of the interval center C and the range pair R_1 and R_2 [$F(8, 608) = 2.04, p < .05$]. The main effect for the ranges was not significant [$F(2, 152) = 1.94, p > .10$].

DISCUSSION

The major results of the study are summarized relative to the predictions expressed in the models previously presented. First, the extent of ambiguity avoidance was found to increase with the probability interval center C for range pairs $R_1 = 0.20$ versus $R_2 = 0.00$ and $R_1 = 0.40$ versus $R_2 = 0.00$, but not for pairs $R_1 = 0.40$ versus $R_2 = 0.20$. Although not part of Ellsberg's original formulation (1961), this finding is consistent with his later comments (Becker & Brownson, 1964). It also agrees with Einhorn and Hogarth's (in press) and Gardenfors and Sahlin's (1982, 1983) explicit prediction of a relationship between ambiguity avoidance and C . However, the failure to find such a relationship for pairs in which $R_1 = 0.40$ and $R_2 = 0.20$ is troublesome to these models.

Second, for values of $C \leq 0.40$, ambiguity avoidance was not signifi-

cantly the most prevalent behavior; however, for no values of C used in the study ($C \geq .10$) was ambiguity seeking predominant. This finding fails to substantiate the additional conjecture of Ellsberg (Becker & Brownson, 1964) that ambiguity seeking might predominate for small values of C . Perhaps this hypothesis remains viable for values of C smaller than those studied here. However, note that, in lotteries of the present form, lower values of C are constrained since the interval centers and ranges cannot be manipulated completely independently of each other. At the extreme, lottery pairs having $R_1 - R_2 = 1.00$ can only have interval centers at $C = 0.50$. Similarly, lottery pairs centered at $C < 0.10$ have a maximum value of $R_1 - R_2 = 2C < 0.20$. This constraint limits further pursuit of this issue by means of the present procedure.

Third, no main effect of interval ranges on ambiguity preferences was found; but a significant $C \times (R_1, R_2)$ interaction did obtain. Furthermore, knowledge of the range difference $R_1 - R_2$ was not sufficient to account for the observed effect. This clearly contradicts both of the stated claims of the Becker and Brownson (1964) study: (a) that ambiguity, in situations like these, is a simple function of the range, and (b) that ambiguity avoidance is a function of the difference $R_1 - R_2$. Of these two, the claim of the sufficiency of $R_1 - R_2$, although stated, was not explicitly tested by those authors and it is not a necessary conclusion of their data or analysis. Our finding of an absence of a range effect, however, is clearly inconsistent with their data, as well as the predictions of Ellsberg's model (1961) and of the models related to it.

It is worth reiteration, in discussing possible range effects, that the interval centers and ranges are not independent in the present procedure. In general, highly discrepant values of R_1 and R_2 are achievable only for intermediate values of C . Thus, any effects on ambiguity preferences of the interval ranges have an applicable domain which is limited to alternatives with nonextreme interval centers. So, it is likely that for decisions in a natural environment, any main effect of R_1 and R_2 may be negligible under certain conditions. Still, to be noted is that the difference between R_1 and R_2 consistently elicits reaction; ambiguity avoidance, not ambiguity indifference, predominated for all levels of R_1 and R_2 at $C = 0.50$.

Of further interest is the observed interaction effect. One possible interpretation of this result is that subjects' reactions to ambiguity vary with C only when one of the alternatives has no ambiguity, that is, when $R_2 = 0$, independent of the magnitude of R_1 . Although the data so suggest, this strong interpretation is premature. The reanalysis of the Becker and Brownson (1964) data does not support this conclusion; and, combined with the consistency of the pattern of results in Table 1 with their data, the above hypothesis remains to be proven.

Regarding the methodology of this experiment, in contrast to studies such as those of Becker and Brownson (1964) and Yates and Zukowski (1976), rating scales rather than a pricing paradigm were used to obtain measures of strength of preference. The rating scale measure was suggested by a preliminary version of the present experiment that we performed using pricing techniques derived from Becker, DeGroot, and Marschak (1964). That earlier experiment demonstrated the asymmetry of allowable prices that obtains for lotteries not having interval centers of $C = 0.50$. To illustrate, for $C = 0.30$, a person setting the value of a lottery at less than its "expectation" of $(0.30)\$W$ (refer to Fig. 1) has a smaller interval within which to express a strength of preference than an individual setting the value of the same lottery at more than $(0.30)\$W$. This fact is inconsequential if one can assume an expectation theory of decision such as SEU; however, as Ellsberg (1961) and others have demonstrated, such a theory cannot describe people's behavior in ambiguous situations. Thus, a pricing response method would run the clear risk of systematic bias; this eventuality does not pertain to the rating response method which was employed.

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