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MATRIX REPRESENTATION OF DECAY SCHEMES

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A matrix representation of the scheme of radioactive decay combines the compactness of a tabular form with the positional information of a graphical "decay scheme". The representation is particularly convenient for generation from computer files and printing with "personal computer" types of devices.

1. Introduction

As the quality and quantity of nuclear spectroscopic information have improved, methods of archiving and retrieving that information have evolved. When Strominger, Hollander and Seaborg [1] prepared the 5th edition of the "Table of Isotopes" (the last to appear in Reviews of Modern Physics), there was a considerable difference between knowledge about radiations (energies and intensities of various types of transitions) and about nuclear excited states (energies, spins, parities, half lives, etc.). Therefore it was useful to present both a list of radiations and a "disintegration scheme" showing levels and transitions among them. The difference increased in the 1960s when high resolution, low efficiency detectors (conversion electron spectrometers and Ge(Li) diodes) led to rapid expansion of the lists with relatively less information on correlations.

The gap between lists and decay schemes has closed to the point there is little need for the separate formats except as they fit the custom or other needs of the user. Almost all of the gamma rays listed in the 7th edition of the "Table of Isotopes" [2] are also included in level schemes.

The matrix formats described in the following sections combine the compactness of a list with the information on correlations contained in decay schemes. Furthermore, the matrices correspond to a simple computer format.

2. Matrix representation

The matrix form is used for both the computer data file and the printed output. The data file is prepared form a line file, each line of which describes a single transition, i.e. a format similar to that used at the National Nuclear Data Center [3]. The line file is convenient for editing but has little meaning outside its context. The decay scheme program assembles the gamma transitions among N levels into a 3-dimensional array ($N \times N \times 2$) in which the elements a_{ijk} are:

nominal energies of levels,	$i = j,\ k = 0,$
transition energies,	$i < j,\ k = 0,$
transition intensities,	$j < i,\ k = 0,$
uncertainties,	$k = 1.$

For unobserved transitions, intensities are listed as zero, and limits are given as negative values for the uncertainties. Transitions from other nuclides are stored in a 2-dimensional array ($N \times 2$) of intensities and uncertainties for each parent state. These working files are the basis for "what if" inquiries and the printed representation described below.

The arrangement of information in the printed form of a decay scheme is given in fig. 1. This arrangement, with feeding intensities on the left, is most appropriate for β^- decay in analogy with the convention of increasing Z from left to right. Other types of decay (feedings by positron emission, electron capture, and alpha decay) might be shown by feedings in a row at the top or a column at the right. In terms of regions, the matrix has the following form: diagonal – properties of states of a particular nuclide, lower left – measured properties of transitions between levels, upper right – inferred properties of transitions, extra rows or columns – transitions between nuclides.

As an example, part of the decay scheme [4–6] of ^{91}Sr is given in fig. 2. Lines have been drawn to show

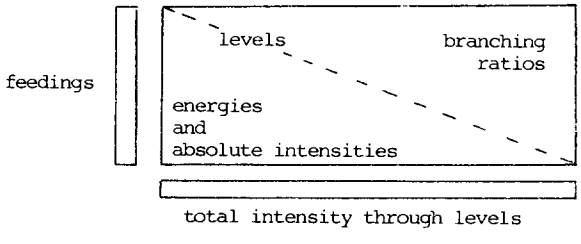

Fig. 1. Arrangement of information in matrix decay schemes.

2066.6 **3/2+**		34 M1 .43	<2 M2<40	<2 E1<.2	43 E1 4.6	303 M1=1	81 M2 155	70 E1 1	465 E1 3.7	<2 M3<10k	3 E1 .01
	1980.4 **(3/2-)**	<9 E1 <4	<26 E2 <32	<26 M1 <.6	85 M1 1.2	21 E1 1.9	153 E2 9.1	644 M1=1	97 M1 .08	<9 E3 <1k	<3 M1<.001
486.9 0.070	[illegible]	**1579.8** [illegible]			<.03	39	1	<.3	<1.2	961	<.09

2.0(1)	**2066.6** **5/2+**		34 M1 .43	<2 E1<.2	<2 E1<.2	43 E1 4.6	303 M1=1	81 E1 2.7	70 E1 1	465 E1 3.7	<2 E2<.003	3 M2 .09
0.34(2)		**1980.4** **(5/2-)**	<9 E1 <4	<26 M1 <.6	<26 M1 <.6	85 M1 1.2	21 E1 1.9	153 M1 .6	644 M1=1	97 M1 .08	<9 M2 <2	<3 E2<.002
32.6(16)	486.9 0.070	<0.003	**1579.8** **(5/2)+**			<.03 E1<.3	39 M1 1	1 E1 .14	<.3 E1<.02	<1.2 E1<.01	961 E2=5	<.09 M2<.004
0.09(2)	<.003	<.009		**1547.5** **(7/2)-**			<326 E1<200	250 M1 3	435 M1=1	<22 E2 <.2	315 E1 3	<33 M3<60k
1.9(1)	<.003	<.009			**1545.9** **(5/2-)**		<16 E1 <50	13 M1 .8	936 M1 11	21 M1 .08	<2 M2<5	30 E2=.1
0.14(2)	592.9 0.087	506.7 0.029	<.001			**1473.8** **3/2-**				402 M1 3.1	<19 E3	598 M1=.8
24.3(13)	761.4 0.62	675.1 0.007	274.4 1.25	<.03	<.03		**1305.3** **5/2+**	3 E1 3.6	8 E1 .29	107 E1 .78	879 E2=5	2 M2 .05
0.86(7)	879.7 0.165	793.7 0.052	392.9 0.033	360.3 0.023	358.8 0.025		118.36 0.083	**1186.9** **7/2-**	431 M1 .6	37 E2=.2	532 E1 .7	<3 M3<1400
1.1(4)	1141.1 0.144	1054.7 0.219	<0.01	621.5 0.04	620.2 1.78		379.6 0.218	261.21 0.534	**925.7** **5/2-**	62 M1 1	<2 M2<71	938 E2=5
4.7(7)	1413.6 0.95	1327.3 0.033	<0.04	<0.002	892.9 0.039	820.6 0.104	652.4 2.81	534 0.046	272.6 0.249	**653.1** **3/2-**	<0.5 E3	1000 E2=5
0.1(10)	<0.003	<0.003	1024.2 31.4	992.0 0.029	<0.004	<0.005	749.7 23.0	631.4 0.66	<0.007	<0.004	**555.9** **9/2+**	1000 M4
=30.8%	2066.7 0.007	<0.001	<0.003	<0.003	1545.9 0.057	1473.8 0.155	1305.3 0.05	<0.004	925.7 3.76	653.0 8.89	555.7 55.2	**000** **1/2-**
	2.0	0.34	32.7	0.09	1.9	0.26	26.2	1.24	4.01	8.89	55.2	100

Fig. 2. Partial decay scheme of $^{91}Sr(\beta^-)^{91}Y + ^{91m}Y$. Information is arranged in the body of the scheme as given in fig. 1. The inset at the top gives reduced transition rates (see text) for alternative spins for the two top levels.

the arrangement of the information in the computer output. Energy and J^π values of levels in ^{91}Y populated in the decay of ^{91}Sr are shown in the diagonal elements. Energies and intensities of gamma-ray transitions are given in the lower left elements. These energies are the observed values; for an unobserved transition only the upper limit for the intensity is given. Branching ratios (in parts per thousand) are given in the upper right. Empirical reduced transition rates (see definition in sect. 2.3) are given below the branching ratios. Absolute rates are rarely known, and these reduced transition rates are based on assigning a value to one of the transitions from each level. Each of the matrix entries is explained in more detail below.

2.1. Diagonal elements

Pertinent properties of the levels are given along the diagonal. The format is flexible in that the user can choose the number of lines per element according to particular needs. In the interest of compactness, the sizes should be approximately the same across the regions. If substantially more information is needed for a level than for transitions among levels, a separate row or column (in addition to feeding intensities) can be used. The reverse (more information on transitions) is not a problem since the off-diagonal elements constitute most of the matrix.

2.2. Energies and intensities

The off-diagonal entries are the substance of the decay scheme. The basic arrangement can accommodate entries as simple as the energies and intensities as given the lower left part of fig. 2 or as complicated as needed (uncertainties, angular correlation coefficients, etc.). The principle is that measured quantities appear in this part and inferred quantities be given in the upper right. There is little wasted space; any but the most complicated decay scheme can be given in a single figure with numerical entries of the minimum legible size.

2.3. Branching ratios

Although branching ratios can be obtained from intensities without making assumptions concerning models or nuclear structure, they cannot be interpreted without some assumptions. For example, the center of gravity of the 653- and 926-keV levels is 835 keV, which is very close to the energy (831.7 keV) of the first 2^+ state in ^{90}Sr [2]. One might interpret these levels as arising from a coupling of the $p_{1/2}$ proton to a quadrupole excitation of the core, which would imply enhanced E2 transitions to the ground state similar to the 5 single particle units [7] observed for the 2^+ to 0^+ transitions in neighboring even nuclides [2]. In contrast to this enhancement, many transitions are much slower than the Weisskopf estimate [7]. We have adopted empirical retardations of 10^5 for E1 transitions and 10^2 for M1. On this basis the 273-keV M1 transition form the 926-keV level is normal – unit rate on the empirical scale – if the 926-keV transition is a collective E2 of 5 single particle units (spu). Similarly, if the 1055-keV transition form the 1980-keV level to the 926-keV level is a "normal" M1 (0.01 spu), the weak 675-keV E1 is also normal (1.9×10^{-5} spu; 1.9 on the empirical scale).

The use of a printed matrix in a manner similar to a "spread sheet" program for "what if" questions is illustrated in the top 2 lines of fig. 2. The branching ratios remain fixed, but the reduced transition rates depend on assumptions as to be multipole character of the transitions. The choice of the normalizing transition is important but generally fairly clear. In any event, various choices can be explored.

3. Discussion

The printed matrix representation was created as an aid in experimental decay scheme studies. With a little experience it is easier to read than the traditional "lines and arrows" diagrams. It can be printed quickly, making updates convenient. Although the format for the data file is not a replacement for line files when frequent editing is required, the array format is compact and therefore attractive for on-line storage of mature decay schemes. One of the limits to the accuracy of gamma-ray spectroscopy in assaying complex mixtures of radionuclides is losses from coincidence summing. As the applications become more sophisticated it will be necessary to include coincidence relations in the data base which is used in the analysis. The array format includes that information with very little increase in the size of the data base. It does not include a list of gamma rays by energy, even for a single nuclide. However, it is intended for use with a computer, most of which have sorting programs available. The user could define the nuclides likely to be present in a particular mixture, obtain a list of energies and intensities (with nuclide labels), order the list, compare with experimental spectra, and refine the analysis by using gamma–gamma correlations to correct for coincidence effects.

References

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