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SCATTERING OF ELECTROMAGNETIC WAVES BY ELECTRON ACOUSTIC WAVES

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Abstract—It is shown that a large amplitude electromagnetic wave can parametrically excite low-frequency electrostatic modified electron acoustic waves which are unique to three-component plasmas containing ions, hot electrons and a group of cold electrons. The growth rates and thresholds of the decay instabilities are obtained. Application of our results in the auroral region of the ionosphere is illustrated.

1. INTRODUCTION

The stimulated scattering of electromagnetic waves in a two-component unmagnetized plasma was first studied in detail by Drake *et al.* (1974). They considered the stimulated Raman, Brillouin and Compton scattering processes in which coherent electromagnetic waves are scattered off Langmuir, ion acoustic and quasi-modes, respectively. Yu *et al.* (1974) and Stenflo (1981, 1985) extended the problem by including the effects of an external magnetic field. In the latter case, the electromagnetic waves can be scattered off a large variety of low-frequency modes which can exist in a two-component magnetized plasma.

Space and laboratory plasmas often contain more than two species of charged particles. Electron distributions which can be interpreted as being composed of more than one temperature are common. For example, a Maxwellian distribution with an enhanced tail of high energy electrons can be considered as consisting of two groups of electrons, one hotter than the other (Bezzerides *et al.*, 1978; Mozer *et al.*, 1979; Temerin *et al.*, 1982; Fontheim *et al.*, 1982; Nishiguchi *et al.*, 1985). Physically, the phenomenon can appear when the electron thermalization time is much longer (say, due to some inhibition process) than the time scale of interest. The temperature differential of the electron fluids can be of several causes. They can be from the mixing of originally spatially separated plasma, such as during injection of electron streams from the geomagnetic tail into the polar ionsophere. They can also be from kinetic instabilities in which electrons in certain velocity ranges are preferentially heated, and from the velocity-dependence of the Coulomb collision mechanism. Electrons trapped self-consistently in the electrostatic field of a nonlinear structure can also appear as having two temperatures (Levine and Crawford, 1980).

It has been shown that a plasma containing multi-temperature electrons can give rise to linear and nonlinear phenomena which are unique to such plasmas (Yu and Shukla, 1983). The purpose of this paper is to investigate the scattering instabilities in a plasma containing distinct hot and cold electrons. In particular, we concentrate on the scattering of a large amplitude electromagnetic wave by the electrostatic electron acoustic waves (Yu and Shukla, 1983; Ashour-Abdalla and Okuda, 1986) which occurs only in such plasmas.

2. INSTABILITY ANALYSIS

Consider the parametric interaction of a coherent electromagnetic pump wave

$$E_0 \exp(i\mathbf{k}_0 \cdot \mathbf{x} - i\omega_0 t) + \text{c.c.},$$

in the presence of an external magnetic field $B_0\,\hat{z}$. The pump wave $(\omega_0, \mathbf{k}_0)$ interacts with a low-frequency mode $(\omega, \mathbf{k})$ to generate sidebands $(\omega_\pm = \omega \pm \omega_0, \mathbf{k}_\pm = \mathbf{k} \pm \mathbf{k}_0)$. The latter in turn beat with the pump wave, yielding a low-frequency ponderomotive force which reinforces the low-frequency waves. Generalizing the parametric instability formulation of Stenflo (1981, 1985) for a magnetized plasma consisting of ions and a mixture of cold and warm

* Deceased.

electrons, we find the following dispersion relation for $\omega_0 \gg \omega_{pe}, \Omega_e$

$$(1+X_i)^{-1} + \left(\sum_{j=h,c} X_j\right)^{-1} = k^2 \sum_{+,-} |\mathbf{k}_\pm \times \mathbf{v}_0|^2 / k_\pm^2 D_\pm, \quad (1)$$

where X_i is the ion susceptibility, X_h and X_c are the susceptibilities associated with the hot (h) and cold (c) electron components, $\mathbf{v}_0 = q_e \mathbf{E}_0 / m_e \omega_0$, m_e is the electron mass,

$$D_\pm = k_\pm^2 c^2 - \omega_\pm^2 + \omega_{pe}^2 + i\nu_e \omega_{pe}^2/\omega_0,$$

c is the velocity of light, ω_{pe}, Ω_e and ν_e are the electron plasma, gyro and collision frequencies, respectively.

For the three-wave decay interaction, we assume D_+ to be off-resonant. Thus, equation (1) becomes (Drake *et al.*, 1974)

$$1 + X_c + X_h + X_i = \frac{(1+X_i)(X_c+X_h)k^2 v_0^2 \sin^2\phi}{2\omega_0(\omega - \Delta + i\Gamma_-)}, \quad (2)$$

where $\Delta = c^2(2\mathbf{k}\cdot\mathbf{k}_0 - k^2)/2\omega_0$, $\Gamma_- = \nu_e \omega_{pe}^2/2\omega_0^2$ and ϕ is the angle between $\mathbf{k}_-$ and E_0. In deriving (2), the pump wave dispersion relation, $\omega_0^2 = k_0^2 c^2 + \omega_{pe}^2$, has been used.

We are interested in low-frequency electrostatic waves which are unique to three-component plasmas. Four physically distinct waves shall be considered. First, let us consider the case where the external magnetic field is negligible. Assuming that the hot particles are in equilibrium, we have for $\omega \ll k v_{tj}$

$$X_i = (k\lambda_{Di})^{-2}, \quad (3)$$

$$X_h = (k\lambda_{De})^{-2}, \quad (4)$$

where $v_{tj} = (T_j/m_j)^{1/2}$ is the thermal velocity and $\lambda_{Dj} = (T_j/4\pi n_{0j} e^2)^{1/2}$ is the Debye length.

For the cold electrons, we neglect any thermal effects and obtain

$$X_c = -\omega_{pc}^2/\omega^2, \quad (5)$$

where $\omega_{pc} = (4\pi n_{0c} e^2/m_e)^{1/2}$ is the plasma frequency of the cold electron component.

The linear low-frequency dispersion realtion is then (Yu and Shukla, 1983)

$$\omega^2 = \omega_{a1}^2 \equiv k^2 c_s^2/(1+k^2\lambda_s^2), \quad (6)$$

where $c_s = [\delta T_i/(1+\delta+\alpha)m_e]^{1/2}$, with $\delta = n_{0c}/n_{0h}$ and $\alpha = T_i/T_h$, is the modified electron acoustic speed, and $\lambda_s = [T_i/4\pi n_{0h} e^2(1+\delta+\alpha)]^{1/2}$ is the modified Debye length.

Substituting (3), (4) and (5) into (2), we obtain the nonlinear dispersion relation

$$(\omega + i\Gamma_- - \Delta)(\omega + i\Gamma - \omega_{a1}) = -\frac{k_0^2 v_0^2 \omega_{a1} b^2}{\omega_0 a}, \quad (7)$$

where Γ is the damping rate of the low-frequency waves, introduced in a phenomenological manner. We have defined $a = b + (k\lambda_h)^{-2}$, $b = 1 + (k\lambda_i)^{-2}$ and have taken the limit corresponding to maximum coupling: $\phi = \pi/2$ and $\mathbf{k} = 2\mathbf{k}_0$. Thus, setting $\omega \sim \omega_{a1} + i\gamma$, and $\Delta \sim \omega_{a1}$, we obtain the growth rate

$$\gamma = -\frac{1}{2}(\Gamma_- + \Gamma) + \frac{1}{2}\left[(\Gamma_- - \Gamma)^2 + \frac{4k_0^2 v_0^2 \omega_{a1} b^2}{\omega_0 a}\right]^{1/2}, \quad (8)$$

and the instability threshold condition

$$v_0^2 > \frac{\Gamma\Gamma_- a\omega_0}{k_0^2 b^2 \omega_{a1}}. \quad (9)$$

In the same manner, we can treat the case in which the ions are cold, so that $X_i = -\omega_{pi}^2/\omega^2$. We note that since n_{0c} can be much smaller than n_{0i}, ω_{pi} can be comparable to ω_{pc}. The linear dispersion relation is

$$\omega^2 = \omega_{a2}^2 \equiv \frac{(n_{0c} + n_{0i} m_e/m_i) k^2 v_{te}^2}{n_{0h}(1+k^2\lambda_h^2)}. \quad (10)$$

Clearly, the condition $\delta_0 = (n_{0c} + n_{0i} m_e/m_i)/n_{0h} \ll 1$ is required for the present theory to be valid. The nonlinear dispersion relation (2) yields an instability growth rate (for $\omega_r \sim \omega_{a2}$, where $\omega = \omega_r + i\gamma$)

$$\gamma = -\frac{1}{2}(\Gamma_- + \Gamma) + \frac{1}{2}\left[(\Gamma_- - \Gamma)^2 + \frac{4k_0^2 v_0^2(\omega_r^2 - \delta k^2 v_{te}^2)(\omega_{pi}^2 - \omega_r^2)}{\omega_0 \omega_r^3 (1+k^2\lambda_h^2)}\right]^{1/2}. \quad (11)$$

The threshold is

$$v_0^2 > \frac{\Gamma\Gamma_- \omega_0 \omega_r^3 (1+k^2\lambda_h^2)}{k_0^2(\omega_r^2 - \delta k^2 v_{te}^2)(\omega_{pi}^2 - \omega_r^2)}. \quad (12)$$

The growth rate here is in general smaller than that given by equation (8).

We now consider decay channels in the presence of an external magnetic field. For $\Omega_{i,e} \neq \omega \ll k_z v_{te}$ and $k\rho_i \gg 1$, where ρ_j is the Larmor radius, hot ion and electron components can again be considered to be in equilibrium. Thus, the appropriate dielectric susceptibilities are given by (3) and (4). On the other hand, for $\omega \ll \Omega_c$, the susceptibility for the cold electron component is

$$X_c = (\omega_{pc}^2/\Omega_e^2) k_\perp^2/k^2 - (\omega_{pc}^2/\omega^2) k_z^2/k^2. \quad (13)$$

In the absence of nonlinear interaction, we obtain for $\delta k_\perp^2 \rho_e^2 \ll 1$ the linear dispersion relation of the electron-acoustic waves in a three component magnetized plasma (Ashour-Abdalla and Okuda, 1986)

$$\omega = \omega_{a1} k_z / k \equiv \omega_{a3}. \quad (14)$$

Note that these waves exist for $\delta \ll 1$ and $k_z \rho_i > (1+\alpha)/\alpha\delta$. Despite the resemblence between (6) and (14), the conditions under which the latter is derived are quite different.

To investigate the scattering instability, we insert (3), (4) and (13) into (2), and find

$$(\omega - \Delta + i\Gamma_-)(\omega - \omega_{a3} + i\Gamma) = \frac{k^2 v_0^2 b}{4a\omega_{a3}\omega_0}\left(a\omega_{a3}^2 - \frac{\omega^2}{k^2\lambda_h^2}\right). \quad (15)$$

Letting $\omega = \omega_{a1} k_z / k + i\gamma \approx \Delta$, one can find from (15) the growth rate and threshold conditions, which are similar to (8) and (9). Here, we note that the growth rate much above the threshold is

$$\gamma \approx (\omega_{a3}/a\omega_0)^{1/2}(bkv_0/2). \quad (16)$$

Finally, we consider the case of a magnetized plasma in which the ions are cold. For $\omega \ll \Omega_i$ and $k_\perp^2 \rho_i^2 \ll 1$, the dielectric susceptibility of the cold ions is

$$X_i = (\omega_{pi}^2/\Omega_i^2) k_\perp^2 / k^2 - (\omega_{pi}^2/\omega^2) k_z^2 / k^2. \quad (17)$$

For this case, the linear dispersion relation found from (4), (13) and (17) is

$$\omega^2 = k_z^2 v_{te}^2 \delta_0 / (1 + k^2\lambda_h^2 + k_\perp^2 \rho_s^2) \equiv \omega_{a4}^2, \quad (18)$$

where

$$\rho_s^2 = (1 + m_e n_{0c}/m_i n_{0i})\omega_{pi}^2 \lambda_h^2 / \Omega_i^2.$$

These electron-acoustic waves exist if $\delta_0 \ll 1$ and

$$k_\perp^2 \rho_s^2 \delta_0 \ll (m_e/m_i)^2 (1 + k^2\lambda_h^2 + k_\perp^2 \rho_s^2).$$

Substituting (4), (13) and (17) into (2), one can investigate the scattering of the pump wave by the long-wavelength mode given by (18). The growth rate of the three-wave decay interaction much above threshold is found to be

$$\gamma^2 = \frac{k_z^2 v_0^2}{4\omega_0\omega_{a4}} \times \frac{[(1+k^2\lambda_h^2)\omega_{a4}^2 - \delta k_z^2 v_{te}^2](\omega_{pi}^2 - Q\omega_{a4}^2)}{(1+k^2\lambda_h^2 + k_\perp^2\rho_s^2)}, \quad (19)$$

where $Q = (1 + k_\perp^2 \omega_{pi}^2 / k^2 \Omega_i^2) k^2 / k_z^2$.

3. APPLICATION

As an illustration, we apply our results to the auroral region of the ionosphere at an altitude of 500 km. We take the typical plasma parameters: $N_{oh} \sim 10^5\,\mathrm{cm}^{-3}$, $B_0 = 0.3\,\mathrm{G}$, $T_h \sim 0.1\,\mathrm{eV}$ and $T_i \sim T_h/2$. Thus, one finds $\omega_{pe} \sim 1.7\times10^7\,\mathrm{s}^{-1} \sim 3\Omega_e$, $v_{ei} \sim 45\,\mathrm{s}^{-1}$, $v_{te} \sim 1.6\times10^7\,\mathrm{cm\,s}^{-1}$ and $\lambda_{De} \sim 1\,\mathrm{cm}$. For $\delta = 0.1$ and $k_z \sim k = 2\times10^{-3}\,\mathrm{cm}^{-1}$, we have $\omega_{a3} \sim 2\times10^3\,\mathrm{s}^{-1}$. For a pump frequency $\omega_0 \sim 6\times10^7\,\mathrm{s}^{-1}$ and the quiver velocity $V_0(=eE_0/m_e\omega_0) \sim 4\times10^6\,\mathrm{cm\,s}^{-1}$ (this corresponds to an electric field $E_0 \sim 0.14\,\mathrm{V\ cm}^{-1}$), the e-folding time calculated from (16) is roughly 40 ms.

Thus, it seems that the electron-acoustic modes could be easily generated in the ionosphere by a ground-based transmitter used for active stimulation of auroral plasmas (Wong *et al.*, 1981).

4. SUMMARY

In this paper, we have investigated the stimulated scattering of a coherent electromagnetic wave by electrostatic modes that can exist only in a three-species plasma. The parametrically excited electron-acoustic waves can account for the low-frequency electrostatic fluctuations frequently observed in space plasmas (Wong *et al.*, 1981; Ashour-Abdalla and Okuda, 1986). Such an enhanced level of low-frequency electrostatic fluctuations can cause anomalous resistivity leading to rapid plasma heating. These effects are, therefore, relevant to plasma heating experiments (Wong *et al.*, 1981).

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