Time-domain analysis of wave exciting forces on floating bodies at zero forward speed

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1. INTRODUCTION

The problem of determining the exciting forces acting on a rigid floating body due to uni-directional waves has been extensively studied. The problem is usually formulated in the frequency domain by assuming that the incident waves are sinusoidal of fixed frequency. Results for more general wave systems may then be found using superposition and Fourier analysis. In this paper, the problem will be formulated directly in the time domain. Numerical techniques are developed for bodies of arbitrary shape. The solutions in the time domain and frequency domain are related through the use of Fourier transforms.

Time-domain analysis has been used by several authors to solve the radiation problem. For axisymmetric bodies, Newman[1] determined the impulse response functions for right circular cylinders of various radius to draft ratios. Beck and Liapis[2] and Liapis and Beck[3] use the technique to solve the radiation problem for arbitrary three-dimensional bodies at zero speed and with forward speed respectively.

The use of time-domain analysis to solve the exciting force problem has not been widely investigated. Wehausen[4] developed the analogue to the Haskind relations for zero forward speed in the time domain. Using these relations, the exciting force is determined as an integral over the body surface of combinations of the incident wave potential and the radiated wave potentials. The solution of the diffraction problem remains of interest because the Haskind integral relations cannot be used to calculate local phenomena such as wave elevations or hydrodynamic pressures.

Direct solution of the diffracted wave problem in the time domain is interesting for several reasons. Possibly, the most important reason is that the method used here may be extended to the cases of steady forward speed and/or arbitrary maneuvers of the body without any serious modifications to the approach. To the contrary, frequency domain methods involve very difficult Green function evaluations at steady forward speed and are not meaningful for general transient maneuvers. The time domain approach is also very useful in verifying and clarifying the relationship between frequency domain results and time domain results. Frequency domain solutions are often used to develop time domain simulations for body motions without great care regarding the restrictions and assumptions of such a step. The time-domain approach allows the direct calculation of the transient solutions.

In this paper the methods developed in Beck and Liapis[2] and Liapis and Beck[3] will be extended to solve the exciting force problem. The analytic basis for the problem is presented in Section 2 followed by a discussion of the boundary conditions for the diffraction problem in Section 3. Numerical methods and results are then presented in Sections 4 and 5, respectively.

2. MATHEMATICAL FORMULATION

Consider an arbitrary three-dimensional body floating on the free surface of an incompressible ideal fluid. Let $0xyz$ be a right-handed coordinate system with the x-y plane coincident with the calm water level. For ship shapes the x-axis points toward the bow, the z-axis is positive upwards, and the origin is placed at midship. Plane, sinusoidal waves are incident to the vessel. The waves are travelling in a direction which makes an angle β to the x-axis.

The incident wave amplitude at the origin as a function of time is given by $\zeta_0(t)$. The amplitude is assumed small so that a linear theory may be developed. The flow is considered irrotational so there exists a velocity potential such that in the fluid domain

$$\nabla^2\Phi = 0$$

and

$$\Phi = \Phi_0(P, t) + \Phi_7(P, t) \tag{1}$$

where Φ_0 = incident wave potential
Φ_7 = diffracted wave potential
$P = (x, y, z)$

For convenience the subscripts 0 and 7 are used to denote the incident and diffracted waves respectively. The subscripts $1, 2, \ldots, 6$ are reserved for the radiation potentials in the 6 degrees of freedom. This leads to an efficient notation for numerical techniques when both the radiation and exciting force problems are solved simultaneously.

The incident wave amplitude at the origin, $\zeta_0(t)$, is related to the incident wave potential by

$$\zeta_0(t) = -\frac{1}{g}\frac{\partial}{\partial t}\Phi_0(0, t) \tag{2}$$

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where g = acceleration of gravity. On the free surface, both Φ_0 and Φ_7 must satisfy the free surface boundary condition

$$\frac{\partial^2\Phi}{\partial t^2} + g\frac{\partial\Phi}{\partial z} = 0 \text{ on } z = 0 \tag{3}$$

The boundary condition on the body surface, S_0, is

$$\frac{\partial\Phi}{\partial n} = 0$$

so that

$$\frac{\partial\Phi_0}{\partial n} = -\frac{\partial\Phi_7}{\partial n} \text{ on } S_0 \tag{4}$$

where n = unit normal on the body surface out of the fluid domain. The incident wave potential is assumed to be zero at $t = -\infty$. This implies that the initial conditions on Φ_7 are

$$\begin{aligned} &\Phi_7 \to 0 \qquad t \to -\infty \\ &\frac{\partial}{\partial t}\Phi_7 \to 0 \qquad t \to -\infty \end{aligned} \tag{5}$$

In addition, the radiation condition requires

$$\nabla\Phi_7 \to 0 \text{ as } r \to \infty$$

An integral equation which must be solved to determine the diffraction potential can be derived using the Green theorem and a Green function for an impulsive source below the free surface. Beck and Liapis[2] give the Green function, which was first derived by Finkelstein,[5] as

$$\begin{aligned} G(P,Q,t-\tau) &= \left(\frac{1}{r} - \frac{1}{r'}\right)\delta(t-\tau) \\ &\quad + H(t-\tau)\tilde{G}(P,Q,t-\tau) \end{aligned}$$

where

$$\begin{aligned} \tilde{G}(P,Q,t-\tau) &= 2\int_0^\infty dk\sqrt{gk}\,\sin(\sqrt{gk}\,(t-\tau)) \\ &\quad \times e^{k(z+\zeta)} J_0(kR) \\ P &= (x,y,z) \\ Q &= (\xi,\eta,\zeta) \\ r^2 &= (x-\xi)^2 + (y-\eta)^2 + (z-\zeta)^2 \\ r'^2 &= (x-\xi)^2 + (y-\eta)^2 + (z+\zeta)^2 \\ R^2 &= (x-\xi)^2 + (y-\eta)^2 \\ \delta(t) &= \text{delta function} \\ H(t) &= \text{unit step function} \end{aligned} \tag{6}$$

The Green function gives the potential at a point P and time t due to an impulsive source at Q created at time τ. The $(1/r - 1/r')\,\delta(t-\tau)$ term represents the impulsive source plus its negative image. Together they satisfy a $\Phi = 0$ boundary condition on the free surface. The impulsive source generates a Cauchy-Poisson type wave system which is represented by $\tilde{G}(P,Q,t-\tau)$. The Green function satisfies the following problem:

$$\begin{aligned} &\nabla^2 G = -4\pi\,\delta(P-Q)\,\delta(t-\tau) \\ &\frac{\partial^2}{\partial t^2}G + g\frac{\partial}{\partial z}G = 0 \text{ on } z = 0 \\ &G \text{ and } \frac{\partial G}{\partial t} = 0 \text{ for } t - \tau < 0 \end{aligned} \tag{7}$$

An integral equation which must be solved to determine Φ_7 on the body surface is obtained by applying the Green theorem to the volume of fluid bounded by the body surface, the free surface, and the surrounding surface at infinity and then integrating both sides with respect to τ from $-\infty$ to t^+. The integrals over the free surface and the surfaces at infinity vanish because of the boundary conditions. Using the boundary condition on the body (4) gives the final result:

$$\begin{aligned} &\Phi_7(P,t) + \frac{1}{2\pi}\int_{-\infty}^{t} d\tau \iint_{S_0} dS_Q \Phi_7(Q,\tau) \\ &\quad \times \frac{\partial}{\partial n_Q} G(P,Q,t-\tau) \\ &= \frac{1}{2\pi}\int_{-\infty}^{t} d\tau \iint_{S_0} dS_Q G(P,Q,t-\tau) \\ &\quad \times \frac{\partial}{\partial n_Q}\Phi_0(Q,\tau) \qquad P \varepsilon S_0 \end{aligned} \tag{8}$$

The hydrodynamic forces acting on the body are found by using the linearized Bernoulli's equation, and integrating the pressure over the body surface as follows

$$\begin{aligned} F_j(t) &= F_{j0}(t) + F_{j7}(t) \\ &= -\rho\iint_{S_0} ds n_j \frac{\partial\Phi_0}{\partial t} - \rho\iint_{S_0} ds n_j \frac{\partial\Phi_7}{\partial t} \end{aligned} \tag{9}$$

where

F_{j0} = Froude-Krylov exciting force in the jth direction
F_{j7} = Diffraction exciting force in the jth direction
$j = (1, 2, 3)$ forces along the x, y, z axis respectively
$j = (4, 5, 6)$ moments about the x, y, z axis respectively
n_j = generalized unit normal
$(n_1, n_2, n_3) = \underline{n}$
$(n_4, n_5, n_6) = \underline{r} \times \underline{n}$
$\underline{r} = (x, y, z)$

3. LINEAR SYSTEM THEORY AND THE DIFFRACTION BODY BOUNDARY CONDITION

Because the hydrodynamic problem is linear, linear system theory may be used to put the expression for the Froude-Krylov force and the diffraction exciting force in the form of a convolution of the arbitrary incident wave amplitude with a kernel function which is independent of the incident wave amplitude.

For the Froude-Krylov exciting force we seek a function, $K_{j0}(P,t)$ such that the force on the body as a function of time can be written in the form:

$$\begin{aligned} F_{j0}(t) &= \int_{-\infty}^{\infty} d\tau\zeta_0(t-\tau)\iint_{S_0} ds_Q\hat{p}(Q,\tau)n_j \\ &= \int_{-\infty}^{\infty} d\tau\zeta_0(t-\tau)K_{j0}(\tau) \end{aligned} \tag{10}$$

The function $\hat{p}(P,t)$ is effectively the time history of the pressure at a point P caused by an incident wave whose elevation is an impulse at the origin at time $t = 0$. The wave is similar to the wave system produced in the Cauchy Poisson problem of Lamb[6] except that the waves are travelling as opposed to the waves resulting from an initial disturbance. The elevation at the origin has the unique property of being zero for all time except $t = 0$ where it is infinite.

The function $\hat{p}$ is real and satisfies the condition that $\hat{p}(P, t) \to 0$ as $t \to -\infty$. It does not satisfy the usual causality condition of being equal to zero for t less than 0. This does not imply that the system is anticipatory in nature. The condition results from the fact that when a wave arrives at the origin it has already contacted the body (and exerted a pressure) at some earlier time. The time delay increases as the wavelength decreases and hence there is no absolute limit except $t = -\infty$.

To determine the impulse response function for the Froude-Krylov exciting force, $\zeta_0(t)$ is assumed to be a unit amplitude sinusoidal wave of the form

$$\zeta_0(t) = e^{i\omega t} \tag{11}$$

For sinusoidal waves with an amplitude given by (11), the velocity potential is

$$\Phi_0 = \frac{ig}{\omega} e^{kz} e^{-ik\alpha} e^{i\omega t} \tag{12}$$

where ω = wave frequency
$k = \omega^2/g$
$\alpha = x \cos\beta + y \sin\beta$
β = wave heading angle

An explicit expression for $\hat{p}(P, t)$ is found by equating the forms of the Froude-Krylov exciting force given by (9) and (10) and substituting equations (11) and (12). An inverse Fourier transform is then taken to obtain $\hat{p}(P, t)$ as a function of time. The inverse Fourier transform is defined by the transform pair:

$$F(\omega) = \int_{-\infty}^{\infty} dt f(t) e^{-i\omega t}$$
$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega F(\omega) e^{i\omega t} \tag{13}$$

In order to take the inverse Fourier transform, negative values of the frequency range must be considered. Since $\hat{p}(P, t)$ is real, the continuation of (12) must be complex conjugate symmetric with respect to ω. The final result is:

$$\hat{p}(P, t) = \frac{\rho g}{\pi} \mathrm{Re}\left\{ \int_0^{\infty} d\omega \, e^{kz} e^{i(\omega t - k\alpha)} \right\} \tag{14}$$

To solve for the diffraction potential, linear system theory is again used. First the normal velocity induced on the body surface by the incident wave is put in the form

$$\frac{\partial}{\partial n} \Phi_0(P, t) = \underline{n} \cdot \int_{-\infty}^{\infty} d\tau \underline{K}(P, t - \tau) \zeta_0(\tau) \tag{15}$$

where

$\underline{n}$ = unit normal to body surface $= (n_1, n_2, n_3)$
$\zeta_0(\tau)$ = arbitrary incident wave amplitude at the origin

The vector function, $\underline{K}(P, t)$, is effectively the impulse response function for the incident wave velocity. When it is convolved with the arbitrary incident wave amplitude, it gives the induced velocity on the body surface. $\underline{K}$ is a real function which goes to zero as $t \to -\infty$. Unlike a normal impulse response function it is not zero for $t < 0$. As with the incident wave kernel, this occurs because there is an induced velocity on the body surface before the impulsive wave forms at the origin.

The diffracted wave potential may also be defined in terms of an impulse response function

$$\Phi_7(P, t) = \int_{-\infty}^{\infty} d\tau \hat{\phi}_7(P, t - \tau) \zeta_0(\tau) \tag{16}$$

where $\hat{\phi}_7(P, t)$ is the impulse response function for the diffraction potential. $\hat{\phi}_7(P, t)$ is found by substituting (15) and (16) into (8). After interchanging the orders of integration with respect to time, and several changes of variables the final result is

$$\begin{aligned} \hat{\phi}_7(P, t) &+ \frac{1}{2\pi} \int_{-\infty}^{\infty} d\tau \iint_{S_0} ds_Q \hat{\phi}_7(Q, \tau) \times \frac{\partial}{\partial n_Q} G(P, Q, t - \tau) \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} d\tau \iint_{S_0} dS_Q G(P, Q, t - \tau) \times (\underline{n} \cdot \underline{K}(Q, \tau)) \end{aligned} \tag{17}$$

Equation (17) could be derived in an alternate manner by assuming $\zeta_0(t)$ is an impulsive wave such that $\zeta_0(t) = \delta(t)$. Equation (15) is then reduced to

$$\frac{\partial \Phi_0}{\partial n} = \underline{n} \cdot \underline{K}(P, t) \tag{18}$$

Substituting (18) into (8) then results in (17). In this form, it is clear that $\hat{\phi}_7(P, t)$ is the diffracted potential due to an impulsive wave.

To determine $\underline{K}(P, t)$ the known result for a sinusoidal wave is used.
Assuming $\zeta_0(t) = e^{i\omega t}$ $\quad -\infty < t < \infty$, then

$$\frac{\partial \Phi_0}{\partial n} = \underline{n} \cdot \int_{-\infty}^{\infty} d\tau \underline{K}(P, t - \tau) e^{i\omega\tau} \tag{19}$$

$\partial\Phi_0/\partial n$ may also be found by taking the directional derivative of the incident wave potential:

$$\frac{\partial \Phi_0}{\partial n} = \underline{n} \cdot [(\underline{i} \cos\beta + \underline{j} \sin\beta + \underline{k} i)\omega \, e^{kz} e^{-ik\alpha} e^{i\omega t} \tag{20}$$

Equating (19) and (20) and taking the inverse Fourier transform defined by (13) results in

$$\underline{K}(P, t) = \mathrm{Re}\left\{ \begin{bmatrix} \underline{i} \cos\beta \\ \underline{j} \sin\beta \\ \underline{k}\, i \end{bmatrix} \frac{1}{\pi} \int_0^{\infty} d\omega\omega \, e^{kz} e^{-ik\alpha} e^{i\omega t} \right\} \tag{21}$$

As with the Froude-Krylov exciting force, in taking the inverse Fourier transform the function of frequency is made complex conjugate symmetric in order to insure that $\underline{K}(P, t)$ is real.

For small values of z, equation (21) may be difficult to evaluate numerically because of the oscillatory nature of the integrand. The problem may be avoided by using a non-impulsive input wave.

Assume the input wave has the form

$$\zeta_0(t) = \sqrt{\frac{aL}{\pi g}} e^{-at^2} \qquad -\infty < t < \infty \tag{22}$$

where L is some characteristic length. This form of the input wave is very convenient because all derivatives of

$\zeta_0(t)$ are finite and continuous. Increasing the size of the constant a will adjust the input to as close to an impulsive wave as desired.

In addition, the Fourier transform of (22) has the simple form

$$F(\zeta_0(t)) = \sqrt{[L/g]}\,e^{-\omega^2/4a} \tag{23}$$

where $F(\ldots)$ denotes the Fourier transform defined by (13).

The velocity potential for this input is found by solving (8) directly. The input normal velocity, $\partial\Phi_0/\partial n$, is determined from (15) as follows:

$$\begin{aligned}\frac{\partial \Phi_0}{\partial n} &= \underline{n}\cdot\int_{-\infty}^{\infty} d\tau \underline{K}(P, t-\tau)\sqrt{\frac{aL}{\pi g}}\,e^{-a\tau^2}\\ &= \underline{n}\cdot\nabla\Phi_0\end{aligned} \tag{24}$$

where $\underline{K}(P, t-\tau)$ is given in (21). Equation (24) may be evaluated using (21), (20) and the fact that the Fourier transform of a convolution is the product of the individual Fourier transforms. The real part of the inverse Fourier transform of the product is then taken to give:

$$\nabla\Phi_0(P,t) = \frac{1}{\pi}\sqrt{\frac{L}{g}}\,\mathrm{Re}\left\{\begin{bmatrix}\underline{i}\cos\beta\\ \underline{j}\sin\beta\\ \underline{k}\,i\end{bmatrix}\int_0^{\infty} d\omega\omega\, e^{kz}\,e^{-\omega^2/4a}\,e^{-ik\alpha}\,e^{i\omega t}\right\} \tag{25}$$

Note that as opposed to (21) the integrand in (25) contains the factor $e^{-\omega^2/4a}$. This exponential decay makes the integral easy to evaluate even when z is small.

Physically, this choice of $\zeta_0(t)$ corresponds to a wave system with exponentially small short wave content. Using (25), the solution to integral equation (8) is the diffraction potential for an incident wave amplitude of the form

$$\zeta_0(t) = \sqrt{\frac{aL}{\pi g}}\,e^{-at^2}.$$

To find $\hat{\phi}_7(P,t)$, the impulse response function for the diffracted wave potential, recall that for a linear system the Fourier transform of the output is equal to the Fourier transform of the input times the Fourier transform of the impulse response function. Thus, it can be determined that

$$\begin{aligned}\hat{\phi}_7(P,t) &= F^{-1}\left(\frac{F(\phi(P,t))}{F(\zeta_0(t))}\right)\\ &= F^{-1}\left(\frac{F(\phi(P,t))}{\sqrt{[L/g]}\,e^{-\omega^2/4a}}\right)\end{aligned} \tag{26}$$

where $F^{-1}(\ldots)$ = Inverse Fourier transform
$\phi(P,t)$ = solution of (8) using (25) to determine the induced normal velocities.

The exciting force due to the diffracted waves is found by substituting (16) into (9) which results in

$$\begin{aligned}F_{j7}(t) &= -\rho\iint_{S_0} ds n_j\frac{\partial}{\partial t}\int_{-\infty}^{\infty} d\tau\hat{\phi}_7(P, t-\tau)\zeta_0(\tau)\\ &= \int_{-\infty}^{\infty} d\tau K_{j7}(\tau)\zeta_0(t-\tau)\end{aligned} \tag{27a}$$

$$\text{where } K_{j7}(t) = -\rho\iint_{S_0} ds n_j\frac{\partial}{\partial t}\hat{\phi}_7(P,t) \tag{27b}$$

The value of $K_{j7}(t)$ may be easily found by interchanging the order of integration over the body surface with the time differentiation and recalling that differentiation with respect to time is equivalent to multiplying the Fourier transform by $i\omega$. If the diffracted wave exciting force due to a wave amplitude given by

$$\sqrt{\frac{aL}{\pi g}}\,e^{-at^2}$$

is defined as the time derivative of the function

$$g_j(t) = -\rho\iint_{S_0} ds n_j\phi(P,t) \tag{28}$$

it can then be shown using (27b) and (26) that:

$$K_{j7}(t) = F^{-1}\left(\frac{i\omega F(g_j(t))}{\sqrt{[L/g]}\,e^{-\omega^2/4a}}\right) \tag{29}$$

In numerically evaluating (29) care must be taken because at large ω the quotient approaches 0 over 0. To determine the proper limit, it is noted that the numerator simply represents the Fourier transform of the exciting force due to the incident wave given by (22). Thus, (29) may be rewritten as

$$K_{j7}(t) = F^{-1}\left(\frac{X_j(\omega)\sqrt{[L/g]}\,e^{-\omega^2/4a}}{\sqrt{[L/g]}\,e^{-\omega^2/4a}}\right) \tag{30}$$

where $X_j(\omega)$ represents the traditional complex frequency-domain diffraction force response at frequency ω. While not proven here, it is reasonable to expect $X_j(\omega)$ to be absolutely integrable and the inverse Fourier transform to be well defined. Numerically, for large values of ω, the term $i\omega F(g_j(t))$ in (29) cannot be expected to have exactly the same exponential decay as the denominator. To obtain reasonable numerical results, a cut-off frequency is used and the value of the arbitrary constant a must be chosen sufficiently large that the numerator $(i\omega F(g_j(t)))$ becomes small before the exponential in the denominator. It has been found that for reasonable values the parameter a has no effect on the results. Specifically, a value for a was chosen such that the expression $e^{-\omega^2/4a}$ was no smaller than 0.1 for values of ω within the range of interest. Typical values of aL/g were around 1.0.

To summarize, the total exciting force in the jth direction acting on the vessel at zero forward speed for arbitrary long crested seas is given by

$$\begin{aligned}F_j(t) &= \int_{-\infty}^{\infty} d\tau\zeta_0(t-\tau)K_{j0}(\tau)\\ &\quad+\int_{-\infty}^{\infty} d\tau\zeta_0(t-\tau)K_{j7}(\tau)\end{aligned} \tag{31}$$

$K_{j0}(\tau)$ is found by direct integration of the function $\hat{p}(P,t)$ (eqn. (14)) over the body surface as given in (10). $K_{j7}(\tau)$ is determined by evaluating the Fourier transform given in (29). The time derivative of the function $g_j(t)$ is the non-impulsive exciting force in the jth direction due to an incident wave amplitude of the form

$$\sqrt{\frac{aL}{\pi g}}\,e^{-at^2}$$

from direction β. This force is found by evaluating (28) where $\phi(P,t)$ is the solution to the integral equation (8) with the induced velocity on the body surface given by (25).

4. NUMERICAL METHODS

It has been shown that the diffracted wave potential and resulting forces due to an arbitrary incident wave on an arbitrary body may be found using relations (16), (26) and (27). These relations require that the potential $\phi(P,t)$ due to an incident wave of the form

$$\zeta_0(t)=\sqrt{\frac{aL}{\pi g}}\,\mathrm{e}^{-at^2}$$

be found. This potential may be computed using a numerical discretization of equation (8). The surface integrals are performed by dividing the body surface into M plane panels which have constant potential strengths. The convolution integrals in time are performed by a trapezoidal rule from t_0 to t_N. The discretized integral equation takes the form of a system of algebraic equations given as

$$\begin{aligned}\phi(P_m,t_N)&+\frac{1}{2\pi}\sum_{i=1}^{M}\phi(Q_i,t_N)\iint_{S_i}\mathrm{d}S_Q\,\frac{\partial}{\partial n_Q}\left(\frac{1}{r}-\frac{1}{r'}\right)\\&=\frac{1}{2\pi}\sum_{i=1}^{M}\left\{\iint_{S_i}\mathrm{d}S_Q\left(\frac{1}{r}-\frac{1}{r'}\right)\underline{n}\cdot\nabla\Phi_0(Q,t_N)\right.\\&\quad+\Delta t\sum_{n=1}^{N-1}\left[\iint_{S_i}\mathrm{d}S_Q\tilde{G}(P_m,Q,t_{N-n})\underline{n}\right.\\&\quad\cdot\nabla\Phi_0(Q,t_n)-\phi(Q_i,t_n)\iint_{S_i}\mathrm{d}S_Q\\&\quad\left.\left.\times\frac{\partial}{\partial n_Q}\tilde{G}(P_m,Q,t_{N-n})\right]\right\}\qquad m=1,2,\ldots,M\end{aligned}\tag{32}$$

where $\underline{n}\cdot\nabla\Phi_0$ is given by (24). The end points of the trapezoidal rule summation do not appear because of the choice of initial time at which $\Phi(Q,t_0)=0$ and the condition that $G(P,Q,t)=0$ for $t<0$. The Rankine source terms are calculated using analytic expressions following Hess and Smith. The integrals involving the wave terms of the Green function are performed using 2×2 Gaussian quadrature. The numerical methods used to evaluate $\tilde{G}(P,Q,t)$ are discussed in detail in Beck and Liapis[2] and King[7].

5. RESULTS

The technique was tested on a floating hemisphere since accurate results are available from other methods for comparison. Taking advantage of the body symmetry, the integral equation (32) was solved with panels on 1/4 of the submerged hemisphere's surface.

Figures 1 and 2 show the Froude-Krylov and diffraction force impulse response functions for a sphere of radius R, respectively. The two modes of motion of heave and sway are shown. In Figure 1, the Froude-Krylov exciting force is even about $t=0$ for heave and it is odd for sway. This is the result of the impulsive wave pressure which is an even function in time about $t=0$. Since n_3 is always positive, the heave Froude-Krylov force exhibits a very large hump and is even about $t=0$. The function n_2 is odd in space so that the sway impulse response function is also odd and displays many humps. The area under the K_{j0} curve is proportional to the zero frequency Froude-Krylov exciting force. For this reason, the integral of $K_{20}(t)$ with respect to time is zero and the heave curve integrates to 1 because of the nondimensionalization.

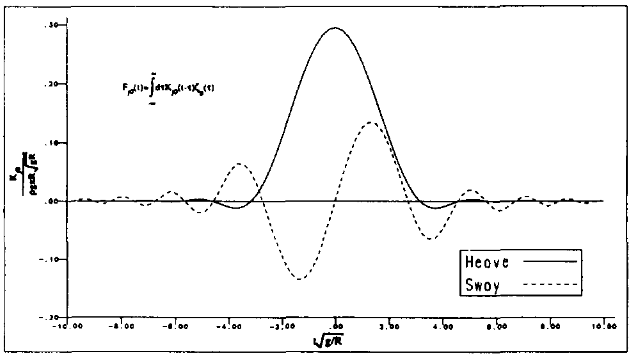

Figure 1. The Froude-Krylov impulse response function for a sphere in heave and sway

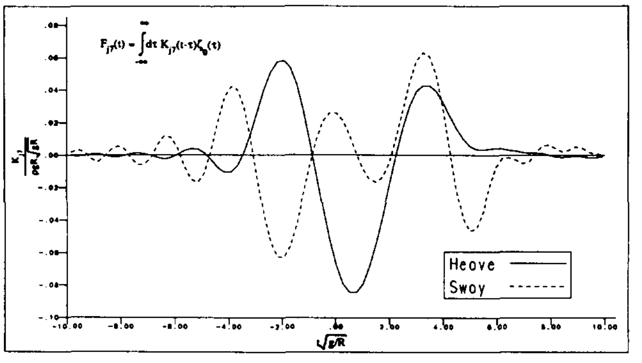

Figure 2. The diffraction impulse response function for a sphere in heave and sway

The diffraction exciting force impulse response functions shown in Figure 2 do not display the same nice symmetry properties as the Froude-Krylov forces. Because of the memory effects of the diffracted wave system, the results are neither odd or even. In addition, the curves are more oscillatory due to interference effects.

Figures 3 and 4 present the amplitude and phase of the total exciting force versus kR for the sphere in sway and heave respectively. Three sets of results are shown in the figures. The solid lines are frequency-domain results obtained from the time-domain results by taking the Fourier transform of equation (30). The plus signs are the results of Cohen[8] using a multipole expansion. The circles were computed by Breit[9] using a higher order panel method. As can be seen the results agree very closely.

The results of Figures 3 and 4 are somewhat misleading because all three methods predict the Froude-Krylov part of the exciting force with equivalent accuracy. Since this component of the force dominates for about half the kR range shown, the results compare extremely well. Figures 5 and 6 show only the diffraction exciting force component. In these figures Breit's results are not

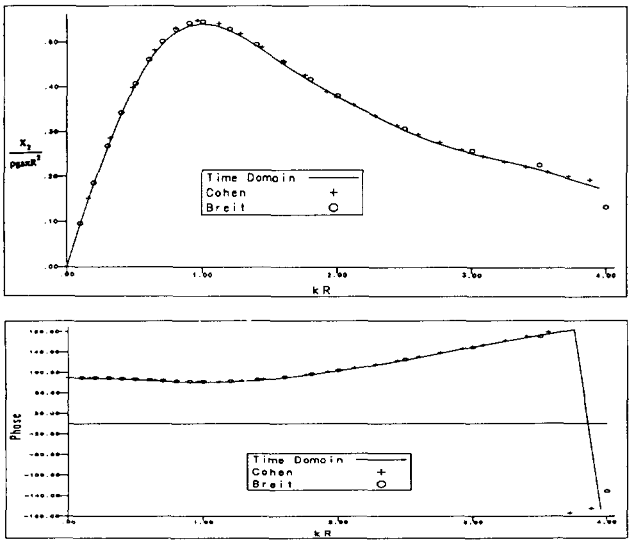

Figure 3. The amplitude and phase of the total sway exciting force acting on a sphere versus nondimensional wave frequency (kR). The solid line was obtained by Fourier transform of the impulse response function. Cohen's results were calculated using multipole expansions. Breit's results were computed using a higher-order panel method in the frequency domain

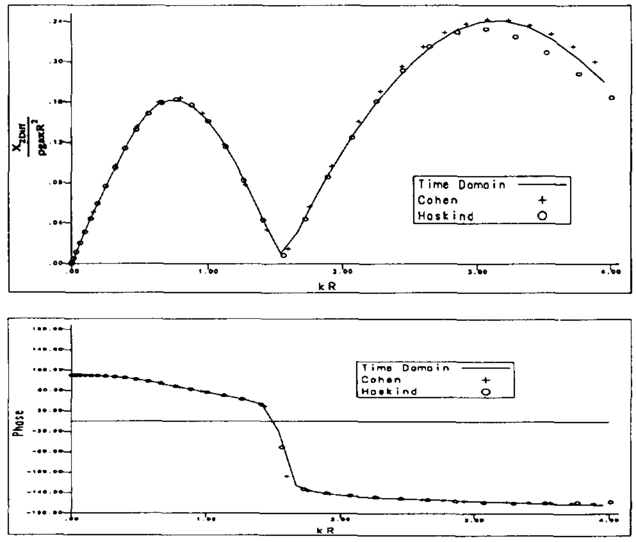

Figure 5. The amplitude and phase of the diffraction sway exciting force acting on a sphere versus nondimensional wave frequency (kR). The solid line was obtained by Fourier transform of the impulse response function shown in Figure 2. Cohen's results were calculated using multipole expansions. The circles were computed using the Haskind relations in the time domain

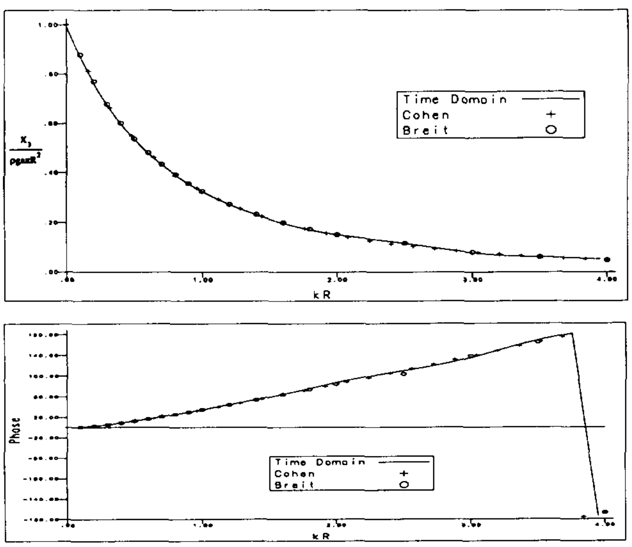

Figure 4. The amplitude and phase of the total heave exciting force acting on a sphere versus nondimensional wave frequency (kR). The solid line was obtained by Fourier transform of the impulse response function. Cohen's results were calculated using multipole expansions. Breit's results were computed using a higher-order panel method in the frequency domain

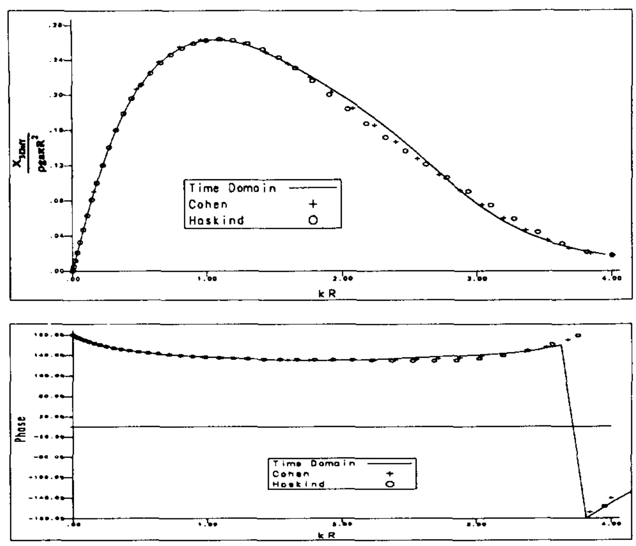

Figure 6. The amplitude and phase of the diffraction heave exciting force acting on a sphere versus nondimensional wave frequency (kR). The solid line was obtained by Fourier transform of the impulse response function shown in Figure 2. Cohen's results were calculated using multipole expansions. The circles were computed using the Haskind relations in the time domain

shown since he only computed the total exciting force. They are replaced by results using the Haskind relations developed by Wehausen.[4] The Haskind relations determine the diffraction exciting force in terms of the incident wave potential and the solution to the forced oscillation problem. The results presented in Figures 5 and 6 were computed by Liapis using the solution to forced heave or sway given in Beck and Liapis[2] (1987). The agreement is again good. At high frequencies differences begin to appear in the sway amplitude. Furthermore, there is a slight hump in the heave time-domain curve around $kR = 2.2$. The reason for this hump is not known.

To investigate the accuracy of the convolution integral for the exciting forces (equation (31)), the exciting force in a random sea was computed using equation (31) and a frequency-domain addition. The pseudo-random sea shown in Figure 7 was produced by summing six unit sine curves of the wavelengths shown on the figure.

The incident elevation is given as

$$\zeta_0(t) = \sum_{i=1}^{6} \sin(\sqrt{[k_i g]}\, t),$$

where k_j is the wave number of the jth component.

The heave exciting force as a function of time acting on the sphere due to the pseudo-random excitation of Figure 7 is shown in Figure 8. The data labeled as frequency domain were computed by summing (with proper amplitude and phase) the exciting forces due to each of the 6 individual sine wave components. The time domain line was obtained using (31) and the impulse response functions for heave presented in Figures 1 and 2. The two sets of results agree except near $t = 0$, where there is a starting transient in the time domain. It should be noted that by the assumptions used to compute the frequency domain results, the excitation continues indefinitely from $t = -\infty$ to $t = \infty$. The time domain approach correctly predicts the starting transient. It may be noted that for the examples shown, the sphere had 65 panels on 1/4 of the submerged surface. With regard to convergence of the numerical scheme it was found that the results followed closely the findings of Beck and Liapis[2] for the problem of wave radiation by the same body.

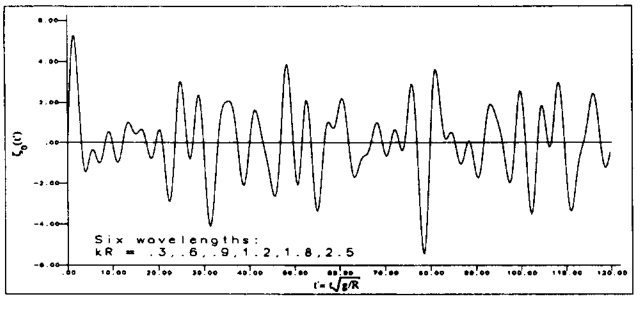

Figure 7. Irregular wave amplitude versus nondimensional time. The wave amplitude is formed by summing six sine waves of the frequencies shown. The starting phases were all set equal to zero

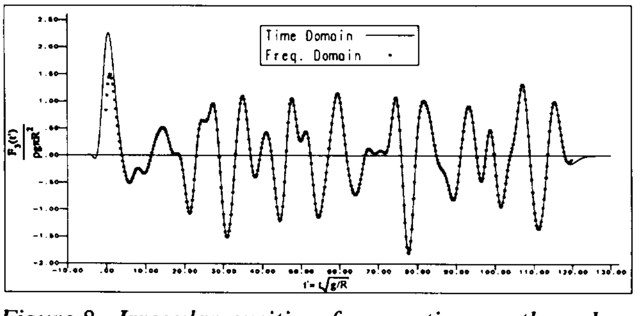

Figure 8. Irregular exciting force acting on the sphere due to the wave amplitude shown in Figure 7. The solid line is obtained by convolution integral in the time domain. The dots are obtained by summing the exciting forces due to the six sine waves in the frequency domain

CONCLUSIONS

Solving the exciting force problem directly in the time domain is a viable alternative to the more conventional frequency-domain approach. At zero forward speed the frequency-domain approach is probably computationally faster because of the convolution integrals and time stepping which is required in the time domain. The next step is to extend the theory to include forward speed.

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REFERENCES

1 Newman, J. N. Transient axisymmetric motion of a floating cylinder. *Journal of Fluid Mechanics*, 1985, **9**, 190–199

2 Beck, R., and Liapis, S. Transient motions of floating bodies at zero forward speed. *Journal of Ship Research*, 1987, **31**, 3, 164–176

3 Liapis, S., and Beck, R. Seakeeping computations using time-domain analysis. In *Proceedings* of the fourth international conference on numerical ship hydrodynamics, 34–54. Washington, D.C.: National Academy of Sciences, 1985.

4 Wehausen, J. V. Initial-value problem for the motion in an undulating sea of a body with fixed equilibrium position. *Journal of Engineering Mathematics*, 1967, **1**, 1–19

5 Finkelstein, A. B. The initial value problem for transient water waves. *Communications on Pure and Applied Mathematics*, 1957, **10**, 511–522

6 Lamb, H. *Hydrodynamics*, Cambridge University Press, 1932.

7 King, B. K. Time-domain analysis of wave exciting forces on ships and bodies. Ph.D. Diss., The University of Michigan, 1987.

8 Cohen, S. B. A time domain approach to three-dimensional free surface hydrodynamic interactions in narrow basins. Ph.D. Diss., The University of Michigan, 1986.

9 Breit, S. R. A higher-order panel method for surface wave radiation and diffraction by a spheroid. In *Proceedings* of the fourth international conference on numerical ship hydrodynamics, 200–215. Washington, D.C.: National Academy of Sciences, 1985.