Buoyancy-Driven Turbulent Diffusion Flames

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A fundamental dimensionless number for pool fires,

$$\Pi_\beta = \left(\frac{\sigma_\beta}{1+\sigma_\beta}\right)\mathrm{Ra}_\beta,$$

is proposed. Here σ_β and Ra_β denote a flame Schmidt number and a flame Rayleigh number. The sublayer thickness of a turbulent pool fire, η_β, is shown in terms of Π_β to be

$$\frac{\eta_\beta}{l} \sim \Pi_\beta^{-1/3},$$

where l is an integral scale. The fuel consumption in a turbulent pool fire expressed in terms of η_β (Π_β) and correlated by the experimental data leads to

$$\frac{m'}{\rho D\,\mathrm{Ra}^{1/3}} = \frac{0.15B}{(1+0.05B)^{1/3}(1+B)^{1/3}},$$

where ρ is the density, D the mass diffusivity, Ra the usual Rayleigh number, and B the transfer number. The model agrees well with a previous model based on the stagnant film hypothesis.

NOMENCLATURE

a	function of B given by Eq. 40
b	first Schvab-Zeldovich property in laminar flow, or, fluctuating component in turbulent flow
$\tilde{b}$	first Schvab-Zeldovich property in turbulent flow
B	transfer number in laminar flow; mean transfer number in turbulent flow
c_p	specific heat of gas at constant pressure
c_l	specific heat of liquid
C_0, C_1	constants
D	mass diffusivity
D_β	modified mass diffusivity defined by Eq. 15
g	gravitational acceleration
g_i	gravitational acceleration vector
h	specific enthalpy
h_β	mass transfer coefficient
h_{fg}	heat of evaporation
k	thermal conductivity
l	integral scale
Le	Lewis number, α/D
m'	burning rate per unit length
m''	burning rate per unit area
M	molecular weight
Nu	Nusselt number, hl/k
Pr	Prandtl number, ν/α
q	heat flux
Q	heat release
Ra	Rayleigh number, $g(\Delta\rho/\rho)l^3/\nu\alpha$
Ra_β	flame Rayleigh number
s_{ij}	fluctuating rate of strain
S_{ij}	mean rate of strain
Sh_β	flame Sherwood number
T	temperature
u	longitudinal velocity; velocity fluctuation (rms)
u_i, u_j	velocity fluctuation
$\tilde{u}_i$	turbulent velocity
U	characteristic laminar velocity
$\overline{U}$	mean upward gas velocity
U_i	mean turbulent velocity
v_w	transverse velocity at wall
x, y	coordinate axes
Y	mass fraction

Greek Symbols

α	thermal diffusivity, $k/\rho c_p$
β	coefficient of thermal expansion

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γ parameter, Eq. A1
δ momentum boundary layer thickness
δ_β first Schvab-Zeldovich property boundary layer thickness
Δ difference
ϵ viscous dissipation
ϵ_β dissipation of the first Schvab-Zeldovich property
η_β flame Kolmogorov scale
θ temperature fluctuation
Θ_0 temperature of isobaric ambient
λ Taylor microscale for momentum
λ_β Taylor microscale for first Schvab-Zeldovich property
μ dynamic viscosity
ν kinematic viscosity
ν_O stoichiometric coefficient of oxidizer
ν_F stoichiometric coefficient of fuel
ν_β modified kinematic viscosity
Π_β dimensionless number for buoyancy-driven diffusion flames
ρ density
σ Schmidt number, ν/D
σ_β flame Schmidt number defined by Eq. 18
ψ parameter, Eq. A3

Script Symbols

$\mathscr{B}$ imposed buoyant production
$\mathscr{P}$ induced inertial production
$\mathscr{P}_\beta$ buoyant production of first Schvab-Zeldovich property

Superscripts

C convective
$C + R$ convective plus radiative
e effective
K conductive
R radiative
T total ($= C + R$)
$*$ modified

Subscripts

f flame
F fuel
g gas
l liquid
O oxidant
R reservoir
w wall
∞ ambient

INTRODUCTION

To date, the buoyancy-driven turbulent diffusion flame above a horizontal fuel in general and the turbulent pool fire in particular have remained analytically untractable. A hybrid (analytical plus dimensional) attempt first neglects the effect of buoyancy, considers a one-dimensional (heat plus species) formulation, and obtains the fuel consumption in terms of the stagnant film hypothesis. Next, it relates the fuel consumption to a power of the usual Grashof number (see, for example, Glassman [1]). A more realistic approach follows some dimensional arguments and suggest an empirical correlation that predicts the experiments on large-scale fires (see, for example Refs. 2–8 for experimental studies, Refs. 9–15 for various models, Refs. 16–22 for combined experimental and modeling efforts, and Refs. 23–31 for reviews on pool fires). The present study uses dimensional arguments to find appropriate scale lengths for turbulence, and, in terms of these scales, proposes a model for turbulent pool fires. This model throws further light on the experimental literature and is expected to be helpful in the presentation of future experimental results.

The study consists of four sections: following this introduction, we illustrate the theory of laminar pool burning in terms of the proposed dimensionless number; we then demonstrate the general nature of this number by developing a theory for turbulent pool fires in terms of the same number, which provides a comparison with experimental data; and we conclude with some final remarks.

LAMINAR DIFFUSION FLAME. A DIMENSIONLESS NUMBER

Although the objective of the study is to describe properties of buoyancy-driven turbulent diffusion flames and pool fires, here a brief dimensional review of laminar flames is useful as necessary background. Accordingly, we reconsider the pioneering work of Spalding [32, 33].

The balance of momentum integrated over the boundary layer thickness δ is

$$\frac{d}{dx}\int_0^{\delta}\rho u^2\,dy + \left(\mu\frac{\partial u}{\partial y}\right)_w = g\int_0^{\delta}(\rho_\infty - \rho)\,dy, \tag{1}$$

where ρ is the density, u the longitudinal velocity, μ the dynamic viscosity, and subscripts w and ∞ denote wall (fuel surface) and ambient conditions. Also, the balance of the first Schvab-Zeldovich (heat + oxidizer) property integrated over the boundary layer thickness δ_β is

$$\frac{d}{dx}\int_0^{\delta_\beta}\rho u(b_\infty - b)\,dy - B(\rho v)_w = \left(\rho D\frac{\partial b}{\partial y}\right)_w, \tag{2}$$

where v_w is the velocity normal to the fuel surface; $\mathrm{Le} = \alpha/D = 1$, α and D being thermal and mass diffusivities, respectively; b and the transfer number B [34-36] are defined as

$$b = (Y_O Q/\nu_O M_O + h)/h_{fg}^{\,e}, \tag{3}$$

and

$$B = b_\infty - b_w, \tag{4}$$

or, in terms of Eq. 3, explicitly,

$$B = (Y_{O\infty}Q/\nu_O M_O - h_w)/h_{fg}^{\,e}. \tag{5}$$

Here Y_O is the mass fraction of the oxidizer, $Y_{O\infty}$ is its ambient value, and Q is the heat released according to single global chemical reaction

$$\nu_F(\text{Fuel}) + \nu_O(\text{Oxidant}) \rightarrow \text{Products} + Q(\text{heat}), \tag{6}$$

where

$$\frac{Q}{\nu_O M_O} = \left(\frac{Q}{\nu_F M_F}\right)\left(\frac{\nu_F M_F}{\nu_O M_O}\right). \tag{7}$$

Here $Q/\nu_F M_F$ is the lower heating value (heat released per kilogram of fuel), $\nu_F M_F/\nu_O M_O$ is the stoichiometric fuel-to-oxidant ratio (kg fuel/kg oxidant); ν_F, ν_O, and M_F and M_O are the fuel and oxidant stoichiometric coefficients and molecular weights, respectively, h is the specific enthalpy relative to ambient temperature, $h_w = c_p(T_w - T_\infty)$, c_p is the specific heat, T_w and T_∞ are the fuel surface[1] and ambient temperatures, respectively, $h_{fg}^{\,e} = h_{fg} + c_l(T_w - T_R)$ is the effective heat of evaporation, including latent heat of evaporation h_{fg} and the sensible heat of liquid fuel, c_l is the specific heat of liquid fuel, and T_R is the temperature of the liquid fuel in the reservoir far from the fuel surface. When the sensible heat is ignored relative to latent heat, Eq. 5 is reduced to

$$B^* = (Y_{O\infty}Q/\nu_O M_O - h_w)/h_{fg}. \tag{8}$$

On dimensional grounds, Eq. 1 yields

$$U\frac{U}{l}\delta + \nu\frac{U}{\delta} \sim g\left(\frac{\Delta\rho}{\rho}\right)\delta, \tag{9}$$

U being a characteristic longitudinal velocity and l a length scale characterizing the direction of flow. Similarly, Eq. 2 yields

$$U\frac{B}{l}\delta_\beta - v_w B \sim D\frac{B}{\delta_\beta}. \tag{10}$$

In terms of the surface mass balance,

$$\rho v_w \sim \rho D\frac{B}{\delta_\beta}, \tag{11}$$

Eq. 10 may be rearranged as

$$U\frac{B}{l} \sim D(1 + B)\frac{B}{\delta_\beta^{\,2}}, \tag{12}$$

and, in terms of the Squire postulate[2] for buoyancy-driven flows,

$$\delta \sim \delta_\beta, \tag{13}$$

[1] Petty and coworkers [37, 38], conducting experiments witl different crude oils, observe throughout the burning proces that the fuel surface temperature remains unchanged. This fac refutes the interpretation of fuel burn as a distillation proces and is tacitly assumed in the present study.

[2] Often this hypothesis is misinterpreted. It postulates th secondary importance of the difference between δ and δ_β fc heat and mass transfer rather than suggesting their equality.

Eq. 9 becomes

$$U\frac{U}{l} + \nu\frac{U}{\delta_\beta^2} \sim g\left(\frac{\Delta\rho}{\rho}\right). \tag{14}$$

The Squire postulate has been well-tested in natural convection even for δ/δ_β differing considerably from unity. Its validity for the present problem is justified below (in the discussion of Fig. 3). Also, because of the same b-gradient involved with Eqs. 10 and 11, the factor $(1 + B)$ is independent of the dimensional arguments leading to Eq. 12. For notational convenience, let

$$D_\beta = D(1 + B). \tag{15}$$

Then, Eq. 12 is reduced to

$$U\frac{B}{l} \sim D_\beta\frac{B}{\delta_\beta^2}. \tag{16}$$

Clearly, Eqs. 14 and 16 can be directly obtained from the corresponding differential formulations, provided D_β is assumed for diffusivity in the latter.

A dimensionless number that describes buoyancy-driven diffusion flames may now be obtained by coupling Eqs. 14 and 16. Since velocity is a dependent variable for any buoyancy-driven flow, its elimination between these equations yields

$$\frac{l}{\delta_\beta^4}\left(1 + \frac{D_\beta}{\nu}\right) \sim \frac{g}{\nu D_\beta}\left(\frac{\Delta\rho}{\rho}\right), \tag{17}$$

which, in terms of a flame Schmidt number,

$$\sigma_\beta \sim \frac{\nu}{D_\beta} \tag{18}$$

and a flame Rayleigh number,

$$\mathrm{Ra}_\beta = \frac{g}{\nu D_\beta}\left(\frac{\Delta\rho}{\rho}\right)l^3, \tag{19}$$

may be rearranged as

$$\frac{l}{\delta_\beta} \sim \Pi_\beta^{1/4}, \tag{20}$$

where

$$\Pi_\beta \sim \left(\frac{\sigma_\beta}{1 + \sigma_\beta}\right)\mathrm{Ra}_\beta \tag{21}$$

is a fundamental dimensionless number for diffusion flames. Actually, the numeral one in Eq. 21 is an unknown constant because of the dimensional nature of the foregoing arguments. Equation 18 reflects this fact by its proportionality sign. Also, in view of

$$\frac{\Delta\rho}{\rho} \sim \frac{\rho_\infty - \rho_f}{\rho_f} = \frac{T_f - T_\infty}{T_\infty}, \tag{22}$$

the Rayleigh number may be more appropriately written as

$$\mathrm{Ra}_\beta = \frac{g(T_f - T_\infty)l^3}{\nu D_\beta T_\infty}. \tag{23}$$

Now, in terms of a (fuel) mass transfer coefficient h_β,

$$\mathrm{Sh}_\beta = \frac{h_\beta l}{D} \sim \frac{l}{\delta_\beta}, \tag{24}$$

Sh_β being a flame Sherwood number. Then, the fuel consumption in a laminar diffusion flame of size l,

$$\frac{m'}{\rho D} = \frac{m''_w l}{\rho D} = \mathrm{Sh}_\beta B \sim B\frac{l}{\delta_\beta} \tag{25}$$

(m''_w being the fuel consumption per unit area), may be written in terms of Eq. 20 as

$$\frac{m'}{\rho D} \sim B\Pi_\beta^{1/4}, \tag{26}$$

or, explicitly,

$$\frac{m'}{\rho D} \sim B\left(\frac{\sigma_\beta}{1 + \sigma_\beta}\right)^{1/4}\mathrm{Ra}_\beta^{1/4}, \tag{27}$$

or, in terms of the usual Rayleigh number for mass transfer,

$$\mathrm{Ra} = \frac{g}{\nu\alpha}\left(\frac{\Delta\rho}{\rho}\right)l^3, \tag{28}$$

as

$$\frac{m'}{\rho D\,\mathrm{Ra}^{1/4}} \sim B\left(\frac{\sigma_\beta}{1 + \sigma_\beta}\right)^{1/4}\left(\frac{D}{D_\beta}\right)^{1/4}. \tag{29}$$

Now, we introduce the definition of the usual Schmidt number,

$$\sigma = \frac{\nu}{D}, \tag{30}$$

and combine Eqs. 15 and 18 for

$$\frac{\sigma_\beta}{\sigma} \sim \frac{D}{D_\beta} = \frac{1}{1+B}. \tag{31}$$

Then, noting the proportionality and equality relations of Eq. 31, and equality replacing Eq. 29 may be written as

$$\frac{m'}{\rho D\,\mathrm{Ra}^{1/4}} = \frac{C_1 B}{(C_0 + B)^{1/4}(1+B)^{1/4}}, \tag{32}$$

where C_0 and C_1 remain to be determined from a computer/laboratory experiment, or, from an analytical solution. However, before proceeding to these coefficients, a number of important facts can be deduced from Eq. 32.

For small values of B,

$$\lim_{B\to 0}\left(\frac{m'}{\rho D\,\mathrm{Ra}^{1/4}}\right) \to B. \tag{33}$$

For $B > 1$, inertial effects are negligible. Either eliminating the inertial term of the momentum balance (Eq. 14), or, noting Eq. 31 and the definition of

$$\sigma_\beta = \left(\frac{\text{Viscous force}}{\text{Inertial force}}\right)\left(\frac{\text{Flow of } B}{\text{Diffusion of } B}\right), \tag{34}$$

and letting $\sigma_\beta \to \infty$ in

$$\frac{\sigma_\beta}{1+\sigma_\beta} = \frac{1}{1+\sigma_\beta^{-1}} \to 1, \tag{35}$$

Eq. 32 is reduced to

$$\frac{m'}{\rho D\,\mathrm{Ra}^{1/4}} \to B^{3/4}, \qquad B > 1, \tag{36}$$

a well-known but so far assumed to be an experimentally supported empirical result. For $B \gg 1$,

$$\lim_{B\to\infty}\left(\frac{m'}{\rho D\,\mathrm{Ra}^{1/4}}\right) \to B^{1/2}. \tag{37}$$

Hereafter, Eq. 32 is called LM (Laminar Model).

Now, for numerical values of C_0 and C_1, consider Spalding's classical study [33] based on an integral approach (with cubic profiles for both velocity and b-property), which, after a minor algebraic correction,[3] yields for the fuel consumption averaged over a length l

$$\frac{m'}{\rho D\,\mathrm{Ra}^{1/4}} = \frac{0.34115B}{\left[\frac{1}{a^2}\left(\frac{1+B}{5.5-a}\right)^2 + 0.6\sigma\frac{1}{a^3}\left(\frac{1+B}{5.5-a}\right)\right]^{1/4}}, \tag{38}$$

or, with some arrangement,

$$\frac{m'}{\rho D\,\mathrm{Ra}^{1/4}} = \frac{0.34115[a(5.5-a)]^{1/2}B}{\left[\left(1+\frac{0.6(5.5-a)\sigma}{a}\right)+B\right]^{1/4}(1+B)^{1/4}}, \tag{39}$$

where

$$a = \frac{2}{B}\left[\left(1+\frac{3}{2}B\right)^{1/2} - 1\right]. \tag{40}$$

A comparison of Eqs. 32 and 39 readily suggests that C_0 and C_1 are not actually constants but depend on B, as to be expected in view of the B-dependence of the b-profiles (Fig. 1). Thus,

$$\frac{m'}{\rho D\,\mathrm{Ra}^{1/4}} = \frac{C_1(B)B}{[C_0(B)+B]^{1/4}(1+B)^{1/4}}, \tag{41}$$

where

$$C_0(B) = 1 + \frac{0.6(5.5-a)\sigma}{a} \tag{42}$$

and

$$C_1(B) = 0.34115[a(5.5-a)]^{1/2}. \tag{43}$$

[3] Each factor $(3.25 + a)$ in Spalding needs to be replaced by $(5.5 - a)$.

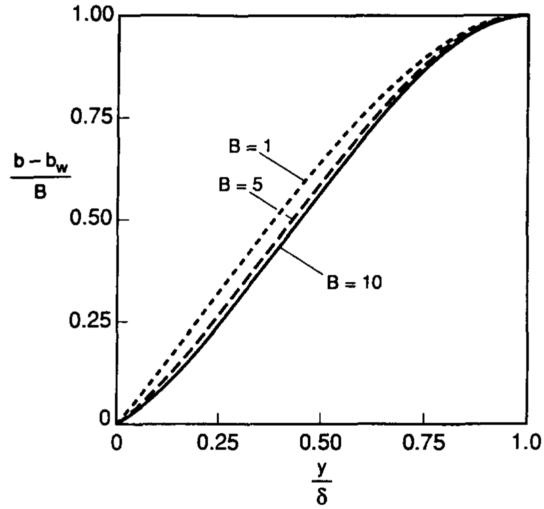


Fig. 1. b-profile versus dimensionless boundary layer thickness as a function of B.

Figure 2 shows the dependence of C_0 and C_1 on B. However, when Spalding's study is repeated with a linear b-profile, Eqs. 38 and 39 are reduced to

$$\frac{m'}{\rho D\,\mathrm{Ra}^{1/4}} = \frac{0.34115B}{\left[\left(\frac{1+B}{5.25}\right)^2 + 0.6\sigma\left(\frac{1+B}{5.25}\right)\right]^{1/4}}, \tag{44}$$

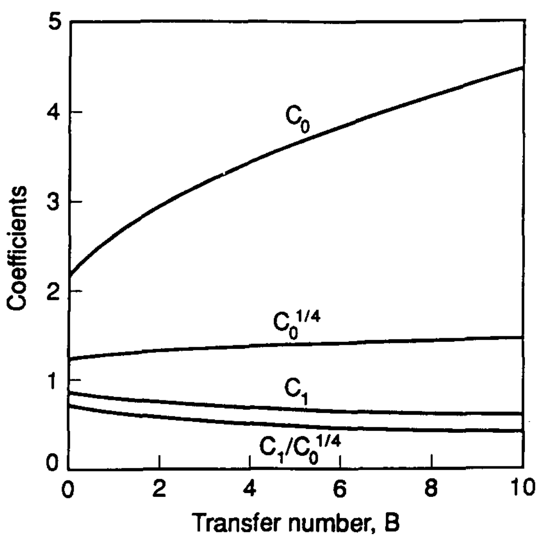


Fig. 2. C_0, $(C_0)^{1/4}$, C_1, and $C_1/(C_0)^{1/4}$ versus B.

and

$$\frac{m'}{\rho D\,\mathrm{Ra}^{1/4}} = \frac{0.34115(5.25)^{1/2}B}{\left[(1 + (0.6)(5.25)\sigma) + B\right]^{1/4}(1+B)^{1/4}}, \tag{45}$$

and the B-dependence[4] of Eq. 45 turns out to be identical to that of Eq. 32.

For an evaluation of the constants involved with LM, first consider the practical range of B. An inspection of the literature summarized in Table 1 taken from Kanury [39] reveals an approximate upper bound of 10 for both B and B^*. In this range, the fuel consumption versus B obtained from Spalding, as well as from the computational study of Kim, de Ris, and Kroesser [40] (KRK) are plotted in Fig. 3. Since Spalding employs the Squire postulate but KRK does not, the close agreement indicates the validity of this postulate also for buoyancy-driven diffusion flames. An overprediction of the burning rate by Spalding's approach should be expected in view of the fact that Spalding assumes a constant $\Delta\rho/\rho$ based on maximum buoyancy. Thus, it is reasonable to assume that KRK is numerically somewhat more accurate than Spalding because of its computational rather than approximate integral solution. Here, a least-square fitting of Eq. 32 to KRK over the range $0 \leq B \leq 10$ should be expected. Instead, a simpler curve-fitting procedure in parallel to that to be employed for turbulent flame is followed here.

In the limit of $B \to 0$, fuel consumption relations approach heat transfer correlations, that is

$$\lim_{B\to 0}\left(\frac{m'}{\rho DB}\right) \to \mathrm{Nu}. \tag{46}$$

The computational solution, available for the classical problem of natural convection next to a

[4]It is interesting to note that for small values of B, $a \to \frac{3}{2}$, and Eq. 39 is reduced to

$$\frac{m'}{\rho D\,\mathrm{Ra}^{1/4}} = \frac{0.34115(6)^{1/2}B}{\left[(1 + (0.6)(2.6667)\sigma) + B\right]^{1/4}(1+B)^{1/4}}$$

whose B-dependence is also identical to that of Eq. 32.

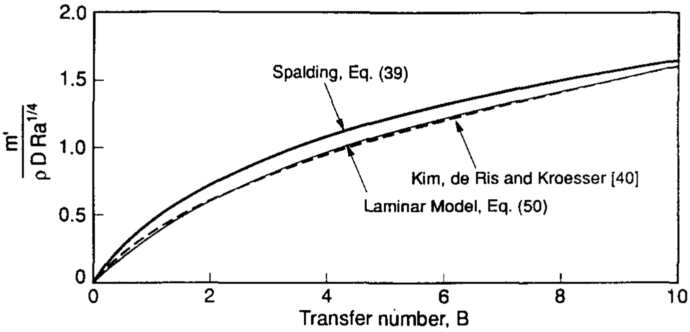


Fig. 3. Comparison of a number of models for laminar fuel consumption versus B.

vertical flat plate gives,[5] for Pr = 0.73,

$$\mathrm{Nu} = 0.52\,\mathrm{Ra}^{1/4} \tag{47}$$

(see, for example, Ede [42]). The coefficient 0.52 exactly matches that of Pohlhausen [43], which is based on the experimental results of Schmidt and Beckmann [44] (see, for example, McAdams [45, p. 171] and Saunders' approximate analysis [46]). Thus, in view of Eq. 32,

$$\lim_{B\to 0}\left(\frac{m'}{\rho\, DB}\right) \to \frac{C_1}{C_0^{\,1/4}}\,\mathrm{Ra}^{1/4}, \tag{48}$$

TABLE I

Mass Transfer Numbers for Combustion of Various Liquid Fuels in Air, $c_g \cong 0.31$ cal/g/°C, $T_\infty = 20$°C, $Y_{o\infty} = 0.232$, $T_R = 20$°C. From Kanury [39].

Fuel	B	B^*
n-Pentane	8.15	8.96
n-Hexane	6.70	8.67
n-Heptane	5.82	8.56
n-Octane	5.24	8.58
Iso-octane	5.56	9.41
N-Decane	4.34	8.40
n-Deodane	4.00	8.40
Octene	5.64	9.34
Benzene	6.05	7.47
Methanol	2.70	2.96
Ethanol	3.25	3.79
Gasoline	4.98	9.04
Kerosene	3.86	9.77
Light diesel	3.96	10.39
Medium diesel	3.94	11.14
Heavy diesel	3.91	11.61
Acetone	5.10	6.07
Toluene	6.06	8.58
Xylene	5.76	9.05

[5]A recent numerical study by Tsai and Liburdy [41] for natural convection from a horizontal disk leads to Nu = 0.903 $\mathrm{Ra}^{0.195}$ for Pr = 0.72.

Eqs. 46 and 47 yield

$$\frac{C_1}{C_0^{\,1/4}} = 0.52. \tag{49}$$

Then, $1/C_0 = 0.90$ is found by matching Eq. 32 to KRK at $B = 2$. With these values, Eq. 32 becomes

$$\frac{m'}{\rho D\,\mathrm{Ra}^{1/4}} = \frac{0.52B}{(1+0.90B)^{1/4}(1+B)^{1/4}}, \qquad B \le 10. \tag{50}$$

Figure 3 shows the close agreement between Eq. 50 and KRK despite one point-matching. Also interesting is the fact that the correlation is obtained with some mean values of C_0 and C_1 without taking their dependence on B into account. However, since the dependence of C_1 on B is weak and that of C_0 is weakened by its fractional power involved with Eq. 50, the result is not surprising. This is indeed an important fact because, under turbulent conditions, the dependence of these coefficients on B is difficult to estimate. Therefore, the model with some mean (constant) coefficients to be developed for turbulent flames in the next section can be safely utilized in the correlation of experimental data.

Next, the foregoing dimensional arguments on laminar diffusion flames are extended to buoyancy-driven turbulent diffusion flames and pool fires by following the recent developments on turbulent natural convection [47–50].

TURBULENT DIFFUSION FLAME

Following the usual practice, we decompose the instantaneous velocity and the first Schvab–Zeldovich (heat + oxidizer) property of a buoyancy-driven, turbulent diffusion flame into a temporal mean (denoted by capital letters) and fluctuations

$$\tilde{u}_i = U_i + u_i \quad \text{and} \quad \tilde{b} = B + b.$$

For a homogeneous pure shear flow (in which all averaged except U_i and B are independent of position and in which S_{ij} is a constant), the mean kinetic energy of velocity fluctuations and the root mean squre of the first Schvab–Zeldovich property yield

$$\mathscr{B} = \mathscr{P} + (-\epsilon) \tag{51}$$

and

$$\mathscr{P}_\beta = \epsilon_\beta, \tag{52}$$

where

$$\mathscr{B} = -g_i\overline{u_i\theta}/\Theta_0 \tag{53}$$

is the buoyant production (imposed),

$$\mathscr{P} = -\overline{u_iu_j}S_{ij} \tag{54}$$

is the inertial production (induced),

$$\epsilon = 2\nu\overline{s_{ij}s_{ij}} \tag{55}$$

is the dissipation of turbulent energy, and

$$\mathscr{P}_\beta = -\overline{u_ib}\frac{\partial B}{\partial x_i} \tag{56}$$

and

$$\epsilon_\beta = D_\beta\overline{\left(\frac{\partial b}{\partial x_i}\right)\left(\frac{\partial b}{\partial x_i}\right)} \tag{57}$$

are the production and dissipation of the first Schvab–Zeldovich property, respectively. Note that the incorporation of the boundary mass transfer into the b-balance is taken into account by considering a b-dissipation in terms of D_β. For buoyancy-driven flows, kinetic dissipation retains its usual form. For forced flows, as already shown by Spalding [33], this dissipation needs to be carried out in terms of $\nu_\beta = \nu(1 + B/\sigma)$.

On dimensional grounds, Eqs. 51 and 52 lead to

$$\mathscr{B} \sim \frac{u^3}{l} + \nu\frac{u^2}{\lambda^2}, \tag{58}$$

and

$$u\frac{b^2}{l} \sim D_\beta\frac{b^2}{\lambda_\beta^2}, \tag{59}$$

where λ and λ_β are the Taylor microscales associated with momentum and the first Schvab–Zeldovich property. Note that dimensional arguments lead to the same scales for homogeneous as well as nonhomogeneous flows.

Now, for a buoyancy-driven turbulent diffusion flame, following the Squire postulate, assume

$$\lambda \sim \lambda_\beta. \tag{60}$$

Then, eliminating the velocity between Eqs. 58 and 59, gives

$$\lambda_\beta \sim l^{1/3}(1 + \sigma_\beta)^{1/6}\left(\frac{D_\beta^3}{\mathscr{B}}\right)^{1/6}. \tag{61}$$

Under isotropy,

$$l \to \lambda_\beta \to \eta_\beta, \tag{62}$$

and Eq. 61 is reduced to a Kolmogorov microscale

$$\eta_\beta \sim (1 + \sigma_\beta)^{1/4}\left(\frac{D_\beta^3}{\mathscr{B}}\right)^{1/4}, \tag{63}$$

where, on dimensional grounds,

$$\mathscr{B} \sim gu\theta/\Theta_0, \tag{64}$$

Θ_0 being the temperature of isobaric ambient. The foregoing microscale is identical in form to that recently introduced by Arpaci [48, 50] for buoyancy-driven turbulent flows. Furthermore, for $\sigma_\beta \to 0$, Eq. 63 is reduced in form to the microscale discovered by Oboukhov [51] and Corrsin [52],

$$\eta_\beta \sim \left(\frac{D_\beta^3}{\mathscr{B}}\right)^{1/4}. \tag{65}$$

Also, for $\sigma_\beta \to \infty$, Eq. 63 is reduced in form to the scale discovered by Batchelor [53],

$$\eta_\beta \sim \left(\frac{\sigma_\beta D_\beta^{\,3}}{\mathscr{B}}\right)^{1/4} = \left(\frac{\nu D_\beta^{\,2}}{\mathscr{B}}\right)^{1/4}. \tag{66}$$

Now, following Taylor, assume

$$\theta \sim \Delta T, \tag{67}$$

ΔT being the imposed temperature difference, and note, for gaseous media,

$$\Theta_0^{-1} = \beta. \tag{68}$$

Then, Eq. 64 becomes

$$\mathscr{B} \sim g\beta u\,\Delta T, \tag{69}$$

or, in view of Eq. 59,

$$\mathscr{B} \sim g\beta D_\beta l\,\Delta T/\lambda_\beta^{\,2}. \tag{70}$$

Insertion of Eq. 70 into Eq. 61 leads to the Taylor microscale in terms of the buoyant force rather than buoyant energy,

$$\lambda_\beta \sim l^{1/4}(1+\sigma_\beta)^{1/4}\left(\frac{D_\beta^{\,2}}{g\beta\,\Delta T}\right)^{1/4}, \tag{71}$$

or, under the isotropy stated by Eq. 62, to the Kolmogorov microscale,

$$\eta_\beta \sim (1+\sigma_\beta)^{1/3}\left(\frac{D_\beta^{\,2}}{g\beta\,\Delta T}\right)^{1/3}. \tag{72}$$

Now, the Taylor and Kolmogorov scales for any σ_β may be rearranged in terms of Π_β as

$$\frac{\lambda_\beta}{l} \sim \Pi_\beta^{-1/4} \tag{73}$$

and

$$\frac{\eta_\beta}{l} \sim \Pi_\beta^{-1/3}. \tag{74}$$

Let the turbulent diffusion flame near a vertical fuel or the pool fire over a horizontal fuel be controlled by a turbulent sublayer. Assume the thickness of this layer be characterized by η_β. Then, following the development between Eqs. 25 and 32, the averaged fuel consumption is found to be

$$\frac{m'}{\rho D} = B\frac{l}{\eta_\beta} \sim B\Pi_\beta^{1/3}, \tag{75}$$

or, explicitly,

$$\frac{m'}{\rho D} \sim B\left(\frac{\sigma_\beta}{1+\sigma_\beta}\right)^{1/3} \mathrm{Ra}_\beta^{1/3}, \tag{76}$$

or, in terms of the usual Rayleigh number,

$$\frac{m'}{\rho D\,\mathrm{Ra}^{1/3}} \sim B\left(\frac{\sigma_\beta}{1+\sigma_\beta}\right)^{1/3}\left(\frac{D}{D_\beta}\right)^{1/3}. \tag{77}$$

Recalling Eq. 31, Eq. 77 may be rearranged as

$$\frac{m'}{\rho D\,\mathrm{Ra}^{1/3}} = \frac{C_1 B}{(C_0+B)^{1/3}(1+B)^{1/3}}, \tag{78}$$

where C_0 and C_1 are to be determined from experimental data (note that the constants of Eq. 78 are different than those of Eq. 32). The 1/3-power law of Rayleigh in pool fires is supported experimentally [19, 22, 54]. Hereafter, Eq. 78 is called TM (Turbulent Model).

Now, in a manner similar to those of laminar flames, the three distinct regimes of turbulent flames may be identified. For small values of B,

$$\lim_{B\to 0}\left(\frac{m'}{\rho D\,\mathrm{Ra}^{1/3}}\right) \to B. \tag{79}$$

For $B > 1$, inertial effects are negligible and Eq. 78 is reduced to

$$\frac{m'}{\rho D\,\mathrm{Ra}^{1/3}} \to B^{2/3}. \tag{80}$$

For $B \gg 1$,

$$\lim_{B\to\infty}\left(\frac{m'}{\rho D\,\mathrm{Ra}^{1/3}}\right) \to B^{1/3}. \tag{81}$$

The experimental data on small fires [4, 5, 10, 55] appears to correlate well with TM, as shown in Fig. 4. The original figure is taken from de Ris and Orloff [10] who rearranged Fig. 5 of Corlett [5] for ethane–nitrogen flames burning above a 10.16-cm-diameter burner and compared it with their model. The open symbols in the original figure for pure ethane are deleted here since they include a radiative heat transfer component towards the burner surface. Remaining data from Corlett represents the dominant convective component of the surface heat transfer. Half-filled

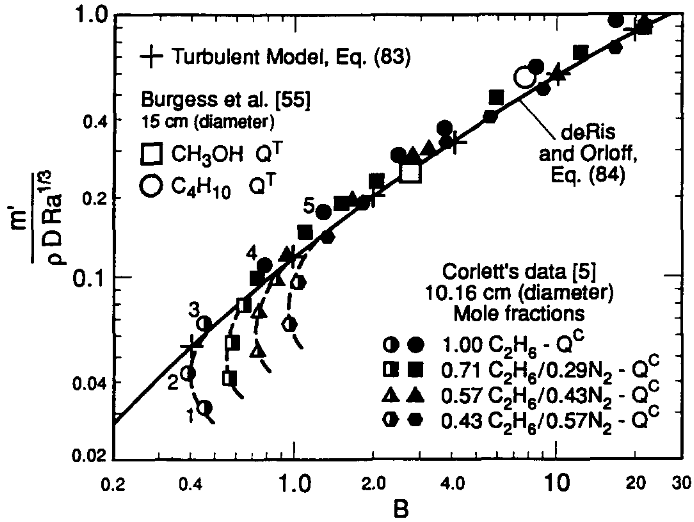


Fig. 4. Comparison of the model for turbulent fuel consumption with experimental data. (Original figure from de Ris and Orloff [10].)

symbols indicate increasing heat transfer with increasing velocity of gases leaving the burner surface. Also included in Fig. 4 are two data points from Burgess et al. [55] for liquid methanol and liquid butane, as shown by open symbols. The low B-range is in the vicinity of extinction of the flames.

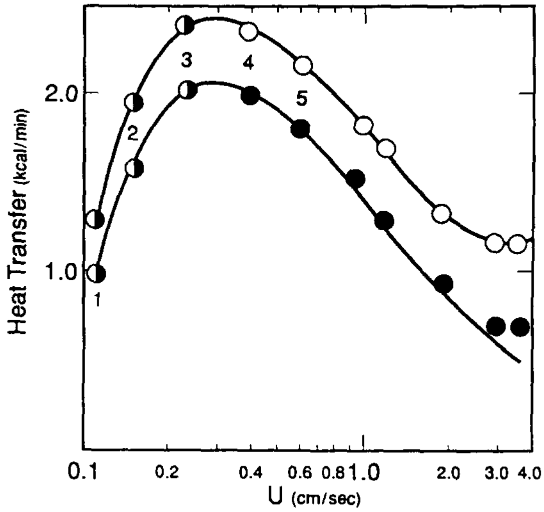
Now, consider convective and convective plus radiative heat transfer data versus upward gas velocity for pure ethane (from Fig. 1a of de Ris and Orloff [10]), as shown here in Fig. 5. Note that the foregoing convective part is the raw data used to construct information for pure ethane in Fig. 4. For convenience, let the pair of data points—one for convection and the other convection plus radiation—corresponding to about the same mass consumption rate be numbered as illustrated in Fig. 5. From definition of B (Eq. 5), for fixed fuel, burner geometry, and ambient conditions,

$$B \sim \frac{1}{Q^C/\bar{U} \quad \text{or} \quad Q^{C+R}/\bar{U}}, \tag{82}$$

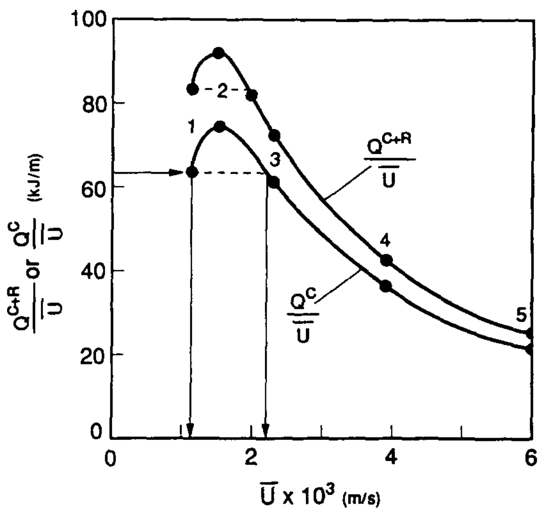
$\bar{U}$ being an average velocity for a single pair of data points. Following Botha and Spalding [56] on the interpretation of premixed propane-air flames on flat flame burners, first five pair of points are replotted in Fig. 6, $Q^C/\bar{U}$ or $Q^{C+R}/\bar{U}$ being associated with heat per unit mass of fuel and $\bar{U}$ characterizing the burning rate (fixed density and burner surface area factored out in both axes). Both $Q^C/\bar{U}$ vs. $\bar{U}$ and $Q^{C+R}/\bar{U}$ vs. $\bar{U}$ are rather similar, as expected from the raw data. Note that the points 1, 2, and 3 in Fig. 5 are transformed to a new curve (Fig. 6), where point 2 approximately corresponds to a maximum. Thus, between point 1 and a location slightly before point 3, the burning rate assumes double-values for each $Q^C/\bar{U}$ or $Q^{C+R}/\bar{U}$. Also, in terms of Eq. 82, Corlett's experiments give two burning rates for fixed B, in the range somewhat below point 3. The difference in the burning rates may be as high as 100%, as illustrated in Fig. 6, and diminish at the point of maximum. This fact leads to a strong scattering of data in the vicinity of extinction limit (Fig. 4). Consequently, the double-valued data in Fig. 4 is shown by dashed-lines and excluded from the proposed correlation.

Arpaci [50] has recently demonstrated, with a correlation on natural convection, the sensitivity of C_0 to experimental data. A preliminary attempt for the evaluation of C_0 and C_1 by a least-square fitting of Eq. 78 to Corlett's data demonstrates a similar sensitivity. Here, following the approach taken in the preceding section on laminar flames, the value of $C_1/C_0^{1/3} = 0.15$ is taken from the literature on natural convection.[6] Then, at $B = 5$, $1/C_0 = 0.05$ is evaluated by

[6]A relatively recent experimental study by Fujii and Imura [57] give $C_1 D_0^{1/3} = 0.16$ rather than 0.15 for a Rayleigh number range similar to that of Corlett. This coefficient appears to provide a slightly better fit for the experimental data.



(5)



(6)

Figs. 5 and 6. Double solution in the low-B range. (Figure 5 is from de Ris and Orloff [10].)

fitting Eq. 78 to Corlett's data. With these values, Eq. 78 becomes

$$\frac{m'}{\rho D\,\mathrm{Ra}^{1/3}} = \frac{0.15B}{(1 + 0.05B)^{1/3}(1 + B)^{1/3}}, \tag{83}$$

which agrees well with the correlation already given by de Ris and Orloff [10]

$$\frac{m'}{\rho D\,\mathrm{Ra}^{1/3}} = 0.15B\left[\frac{\ln(1 + B)}{B}\right]^{2/3}, \tag{84}$$

on the basis of the stagnant film theory coupled with the empirically assumed 2/3-power law. The maximum difference between two correlations remains less than 1.8% for the entire B-range. The agreement between two models, despite the fact that they are developed by following quite different arguments, is remarkable.

So far, the proposed models for laminar and turbulent flames and fires exclude any effect of radiation. Because of different intrinsic nature of radiation and conduction (or any diffusion), the Schvab–Zeldovich transformation used in the present study no longer applies to radiation-affected flames. On intuitive grounds, the emission effect of radiation (hotness of flame) has been already incorporated into the heat of combusion and the latent heat of evaporation by fractional lowering (say γ and ψ) of these properties (see Kanury [19]). Following the studies of Arpaci and coworkers [58–62] and Selamet and Arpaci [63], the optical thickness and scattering effects of radiation can be incorporated into γ and ψ. However, because of the lack of experimental data, no attempt is made here to demonstrate their influence on γ and ψ. The Appendix is a brief review of the emission effect alone and its influence on the fuel consumption.

CONCLUDING REMARKS

In this study a fundamental dimensionless number,

$$\Pi_\beta \sim \left(\frac{\sigma_\beta}{1 + \sigma_\beta}\right)\mathrm{Ra}_\beta,$$

is introduced for laminar and turbulent diffusio flames in general and pool fires in particular.

A boundary layer thickness δ_β and a sublaye thickness η_β for laminar and turbulent diffusio flames (and pool fires) are proposed in terms c Π_β. Two models built on these scales for the fue consumption in laminar and turbulent fires ar proposed. The analytical and computational stuc ies on laminar flames and the experimental dat on turbulent flames and pool fires are correlate with these models. The models lead, respectivel to the 1/4 and 1/3 power law for the Rayleig number, well-known for buoyancy-driven turbu lent flows. The previously proposed turbule model by de Ris and Orloff [10] based on tl stagnant film theory supports the same exponen For negligible inertial effects, the laminar fu consumption is shown to approach $B^{3/4}$, a well-known result, and the turbulent fuel consumption to that $B^{2/3}$, an overlooked result.

The intuitive approach involving implicit fractional reduction in the heat of combustion and the latent heat of evaporation for the radiation effect may be extended by interpreting these reductions in terms of the explicit effects of emission, absorption, and scattering of radiation. However, the experimental data on radiating pool fires lacks explicit information on these effects. Accordingly, the proposed correlation is for small fires with negligible radiation effect.

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APPENDIX: RADIATION EFFECT

As is well known, the primary effect of thermal radiation on high-temperature problems is on the heat flux. Studies by Arpaci and coworkers [58, 59, 61, 62, 64-66], and Selamet and Arpaci [63, 67] show the emission, absorption, and scattering (hotness, optical thickness, and refractive) effects on this flux. However, there is no experimental literature on pool fires separating these effects. Accordingly, the following development, in a manner similar to that of Kanury [19], assumes negligible absorption and scattering and illustrates the radiation effect on pool fires in terms of emission (flame temperature) alone.

We introduce

$$\gamma = \frac{Q^R}{Q}, \tag{A1}$$

where Q^R is the radiation emitted by the flame. Also, a thermal energy balance on the fuel surface

$$m'' h_{fg}{}^e = q_w{}^C + q_w{}^R \tag{A2}$$

rearranged in terms of[7]

$$\psi = \frac{q_w{}^R}{q_w{}^C} = \frac{q_w{}^R}{k(\partial T/\partial y)_w}, \tag{A3}$$

gives

$$m''\left(\frac{h_{fg}{}^e}{1+\psi}\right) = k\left(\frac{\partial T}{\partial y}\right)_w \tag{A4}$$

$q_w{}^R$ and $q_w{}^C$ being the radiative and convective fluxes incident on the fuel surface, respectively. Equation A4 may be interpreted as a reduced heat of evaporation. Then, in terms of Eqs. A1 and A4, the transfer number becomes

$$B = \left[\frac{Y_{Q\infty}Q(1-\gamma)}{\nu_O M_O} - c_p(T_b - T_\infty)\right]\frac{(1+\psi)}{h_{fg}{}^e} \tag{A5}$$

Experimental data on γ for some polymers is available in Kanury [19]. Also, data for a variety of other fuels is available, for example, in Corlet [4, 5], de Ris [30], and Burgess et al. [55]. One may note the different approaches followed by de Ris and Orloff [10] and Kanury [19] in the interpretation of radiation effects. However, bot eventually lead to the same result.

[7]Here $\psi \equiv \dfrac{\psi_{Kanury}}{1-\psi_{Kanury}}$, ψ_{Kanury} being given by Kanu [19].