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RESEARCH NOTE

SATELLITE MEASUREMENTS OF ATMOSPHERIC STRUCTURE BY REFRACTION

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The image of a star at zenith angle z, observed from a point within the atmosphere, is refracted through an angle R according to the textbook formula

$$R = \mu_0 r \sin z \int_1^{\mu_0} \frac{d\mu}{\mu[\mu^2(r_0 + h^2) - \mu_0^2 r^2 \sin^2 z]^{\frac{1}{2}}} \qquad r = r_0 + h_0 \tag{1}$$

where r is the radius to the center of the earth, μ is the index of refraction and h is the altitude. An arbitrary spherically stratified atmosphere is assumed. For rays which have passed completely through the atmosphere to a satellite observation point the curved portion of the ray is symmetrical with respect to the point at lowest altitude (indicated by subscript $_0$). The total angle is therefore double that given by equation (1) and since $z = 90°$, we have

$$R_s = 2\,\mu_o r \int_1^{\mu_0} \frac{d\mu}{\mu[(r_o + h)^2 - r^2 \mu_o^2]^{\frac{1}{2}}}\,. \tag{2}$$

The potential usefulness of equation (2) for the measurement of atmospheric density is revealed by considering an approximate form

$$R_s(h_o) = \sqrt{2r}\,k \int_{h_0}^{\infty} \frac{d\rho}{dh} \frac{dh}{\sqrt{h - h_o}} \tag{3}$$

where, in addition, μ has been replaced by the atmospheric density ρ according to Dale and Gladstone's Law

$$\mu - 1 = k\rho \tag{4}$$

Equation (3) is similar to Abel's Integral Equation and can be solved explicitly for the atmospheric density profile

$$\rho(h) = \frac{-1}{\pi\sqrt{2r}\,k} \int_h^{\infty} \frac{R_s(h_o)dh_o}{\sqrt{h_o - h}} \tag{5}$$

which can be converted to temperature and pressure profiles by the conventional manipulations of the equations of state and hydrostatic pressure.

The total angle of refraction measures the integral of all refraction effects on the portion of the ray within the atmosphere. The smoothing of lateral variations in the atmosphere along the ray can be studied by assuming a typical isothermal atmosphere for which the density formula is

$$\rho(h) = \rho_o e^{(h_0 - h)/H} \qquad H = \text{scale height} \tag{6}$$

The combined effect of the spherical atmosphere and the exponential density function tend to restrict refraction effects to the portion of the ray near the tangent point. It can be shown that in a typical case half the refraction occurs within a horizontal distance of approximately 350 km which may be taken as the effective length of a ray conceived as a 'probe'. Although the real atmosphere of the earth, for example, is neither spherically stratified nor isothermal exactly it has these properties in a gross sense so that the effective length of a refraction probe in the real atmosphere will not be appreciably different from 350 km. Examination of weather maps at any altitude shows that a probe of this size will resolve nearly all variations of meteorological significance. In a lateral direction perpendicular to the ray the resolution is extremely sharp. The magnitude of the refraction of rays passing through the earth's atmosphere at various tangent altitudes can be computed from the classical formula for an isothermal atmosphere.

$$R_s = 2R = 2\,k\,\rho_o \sqrt{\frac{\pi r}{2H}} \tag{7}$$

h_o, tangent altitude of ray, km	R_s, refraction angle, min
0	69·4
5	41·7
15	11·0
25	2·3

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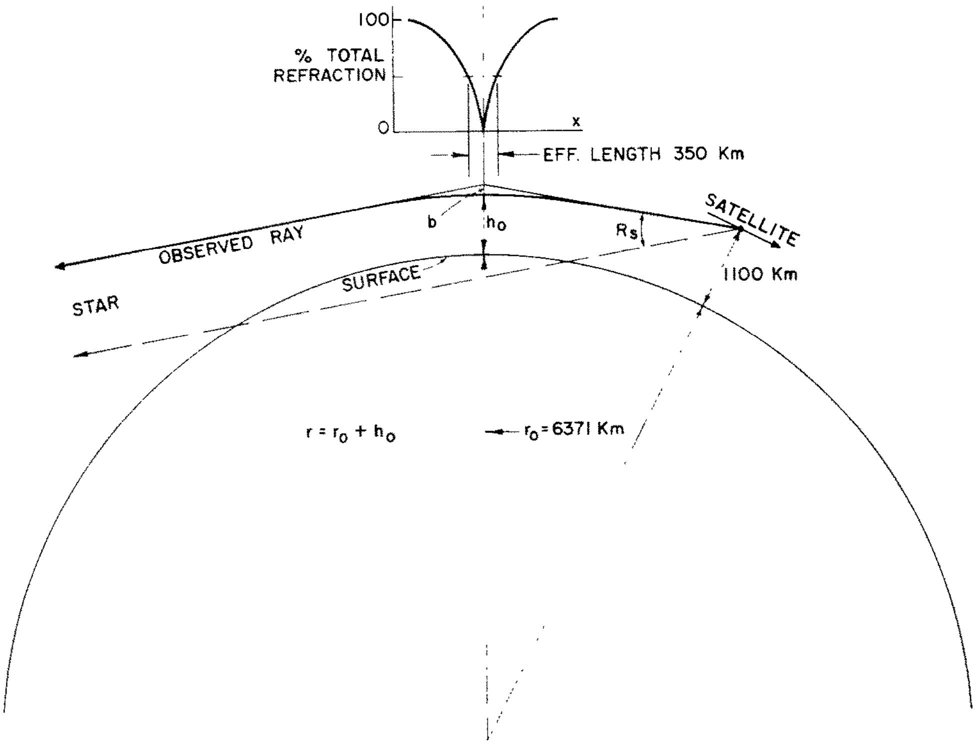
Since it is within the state of the art to measure angles to $\pm 0.1'$ of arc with automatic star trackers, it is proposed that a method of measuring upper air structure to altitudes of 25 km (25 mb) with satellite instrumentation is feasible. The errors in density at 25 km due to angle errors will be less than 5 per cent and will decrease with decreasing altitude. A lower limit, estimated to be about 5 km (500 mb) will usually be imposed by clouds and haze obscuring the star images.



The figure illustrates the experiment in the case of the earth with R_s, h_o and b exaggerated. The small upper figure shows per cent of total refraction as a function of horizontal distance x to illustrate the effective length of the probe. Dimension b, given by

$$b = (\mu_o - 1)r \tag{8}$$

is the distance between the tangent point of the refracted ray and the intersection of the two ends of the ray extended. Because the latter lines are known from the star position and the satellite observations, the altitude h_o of the density measurement can be calculated. Equation (8) is for a spherically stratified, but not necessarily isothermal atmosphere.

Equations (3), (5) and (8) are approximate formulas. Although the approximations are quite close it may prove desirable to use generalized functions in the final data processing.

The preliminary design of a system based on the foregoing ideas is being undertaken. Investigations of instrumentation, visibility and seeing, errors, geographical distribution of measurements and so on, are under way.

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